Parton Distributions on a Quantum Computer

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Parton Distributions---
a Great challenge both in
Exp. and Th.

LaMET is the most popular
theoretical approach so far.

Large Momentum Effective Theory (LaMET)

X. Ji, PRL 2013

- Quasi-PDF: Equal time correlator for a proton with a large momentum

Wilson line

$$\tilde{q}(x,\mu^2,P^z) \quad = \quad \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle \qquad \lambda^\mu = (0,0,0,1)$$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle \qquad (\lambda^2 = 0)$$

UV, PQCD

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

Analogous to HQET: need power corrections & matching

This year's highlights

- Precision frontier: no Wilson line, Rapid Interpolators, DA…
- Higher dimensional frontier: TMD (CS kernel), GPD…
- Flavor frontier: gluon…

Lattice QCD

- Path Integral, Monte Carlo, sign problem and real time dependence challenging

An Alternative: Quantum Computation

- See how a state evolves under unitary transformation, including real time evolution
- Matrix elements can be computed

Toy: PDF in QED_2

- Linear, confining potential. Positronium mimics meson in 3+1D QCD
- Working in Minkowski space, accessing lightcone correlators directly

$$\mathcal{O}(z_1, z_2) = \bar{\psi}(z_1 n) n \cdot \gamma W_n(z_1 n, z_2 n) \psi(z_2 n) \qquad n^\mu = (1, -1)$$

$$D(z_1 - z_2) = \frac{1}{2n \cdot P} \langle P | \mathcal{O}(z_1, z_2) | P \rangle$$

$$f(x) = \int_{-\infty}^{\infty} \frac{dz n \cdot P}{2\pi} e^{-izn \cdot Px} D(z)$$

Super renormalizable

$$f(x, a) = f(x) + \mathcal{O}(a)$$

Kogut Susskind Fermions in 1+1 d

$$\psi(x) \;=\; (\phi(n), \phi(n+1))^T$$

$$\{\phi(n), \phi^\dagger(k)\} \;=\; \delta_{n,k} \quad , \quad \{\phi(n), \phi(k)\} \;=\; 0 \quad .$$

(similar to 3+1d equal time $\{\psi_a(\mathbf{x}), \psi_b^\dagger(\mathbf{y})\} \;=\; \delta^{(3)}(\mathbf{x}-\mathbf{y})\;\delta_{ab}$.)

$$\hat{H} \;=\; \frac{i}{2a_n}\sum_n \left[\, \phi^\dagger(n)\phi(n+1) \;-\; \phi^\dagger(n+1)\phi(n) \,\right]$$

$$i\frac{d}{dt}\phi(n) \;=\; \left[\, \phi(n), \hat{H} \,\right] \;=\; \frac{i}{2a_n}\left(\, \phi(n+1) - \phi(n-1) \,\right)$$

$$\frac{d}{dt}\psi(x) \;=\; \alpha_x \frac{d}{dx}\psi(x) \quad , \quad \alpha_x \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma_0 \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \gamma_x \;=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad \{\gamma_\mu, \gamma_\nu\} \;=\; 2g_{\mu\nu}$$

Jordon Wigner Mapping

Occupied (unoccupied) fermion state at site n as spin up (down)

$$\phi(n) = \left(-i\hat{Z}_1\right)\left(-i\hat{Z}_2\right)\ldots\left(-i\hat{Z}_{n-1}\right)\sigma_n^-$$
$$\phi^\dagger(n) = \left(+i\hat{Z}_1\right)\left(+i\hat{Z}_2\right)\ldots\left(+i\hat{Z}_{n-1}\right)\sigma_n^+$$

$$\phi(0) = \sigma^- \otimes I \otimes I \otimes I \quad , \quad \phi^\dagger(0) = \sigma^+ \otimes I \otimes I \otimes I$$
$$\phi(1) = -i\hat{Z} \otimes \sigma^- \otimes I \otimes I \quad , \quad \phi^\dagger(1) = +i\hat{Z} \otimes \sigma^+ \otimes I \otimes I$$
$$\phi(2) = -\hat{Z} \otimes \hat{Z} \otimes \sigma^- \otimes I \quad , \quad \phi^\dagger(2) = -\hat{Z} \otimes \hat{Z} \otimes \sigma^+ \otimes I$$
$$\phi(3) = +i\hat{Z} \otimes \hat{Z} \otimes \hat{Z} \otimes \sigma^- \quad , \quad \phi^\dagger(3) = -i\hat{Z} \otimes \hat{Z} \otimes \hat{Z} \otimes \sigma^+$$

$$\{\phi(1), \phi^\dagger(1)\} = I \quad \{\phi(1), \phi(1)\} = 0 \quad \{\phi(0), \phi^\dagger(1)\} = 0$$

$$\hat{H} = -\frac{1}{2a}\sum_n \left[\, \sigma_n^+ \sigma_{n+1}^- + \sigma_{n+1}^+ \sigma_n^- \,\right] + \frac{m}{2}\sum_n (-)^n \hat{Z}_n$$

Hamiltonian of QED_2

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi, \qquad D_\mu = \partial_\mu + ieA_\mu$$

$$\Pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)} = F^{\mu 0}.$$

$\Pi^0 = 0$ implies A^0 is not dynamical.

$$\frac{\partial\mathcal{L}}{\partial A_0} = J^0 - \partial_\mu \Pi^\mu = 0$$

(Gauss's law, a quantum constraint)

$A^1=0$ (the axial gauge) (open boundary condition in x)

$$\begin{aligned}\mathcal{H} &= \pi\partial_0\psi - \mathcal{L} \\ &= \frac{1}{2}E^2 - i\bar{\psi}\gamma^1\partial_1\psi + m\bar{\psi}\psi,\end{aligned}$$

Wilson Line

$$W_n(z_1 n, z_2 n) = e^{-ie\int_{z_2}^{z_1} n\cdot A(z)dz}$$

Gauge invariant

$$O'(x,y) = \overline{\psi}(x) n\cdot\gamma Q_n(x)\overline{Q}_n(y)\psi(y)$$

$$\mathcal{L}' = \mathcal{L} + \overline{Q}_n(x) i n\cdot D Q_n(x)$$

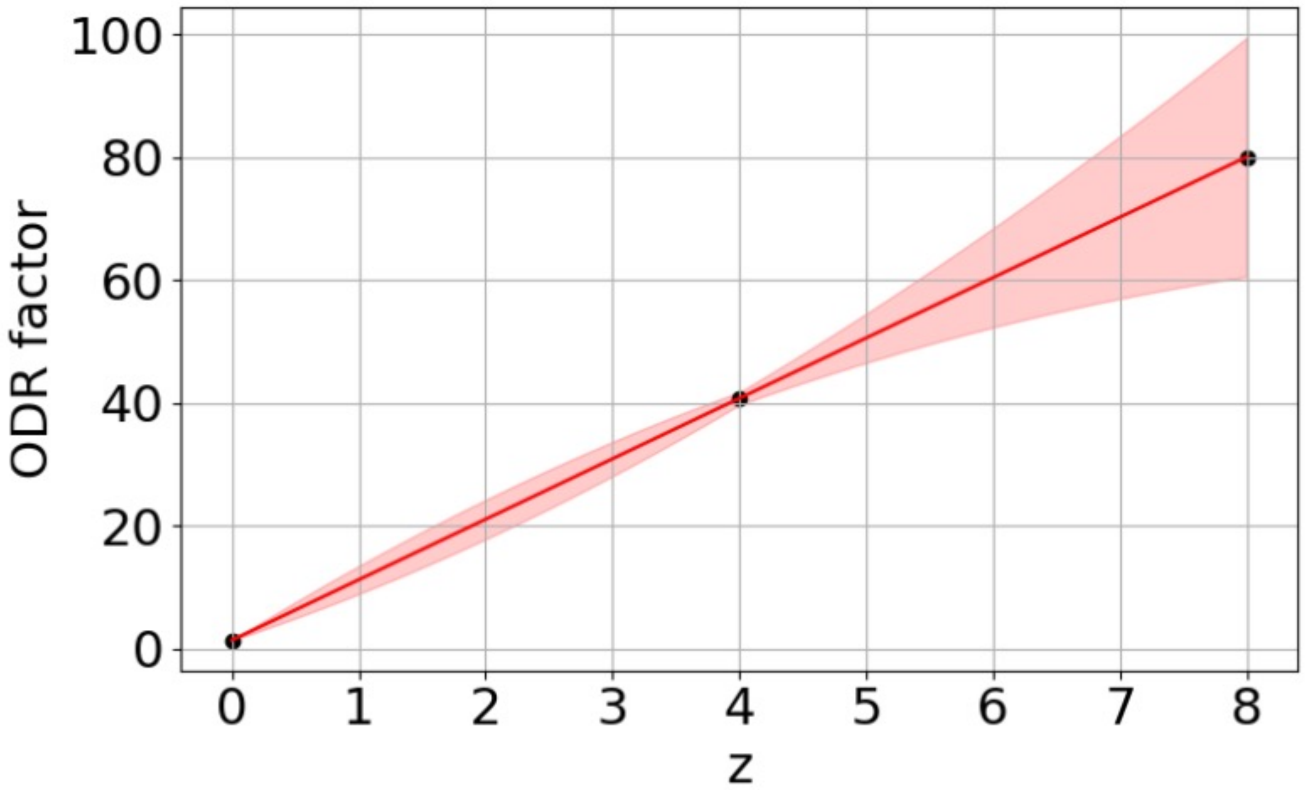
$$\delta J^\mu = -e\overline{Q}_n(x) n^\mu Q_n(x)$$

Wilson line carries the same charge as the fermion

Quantum Computation

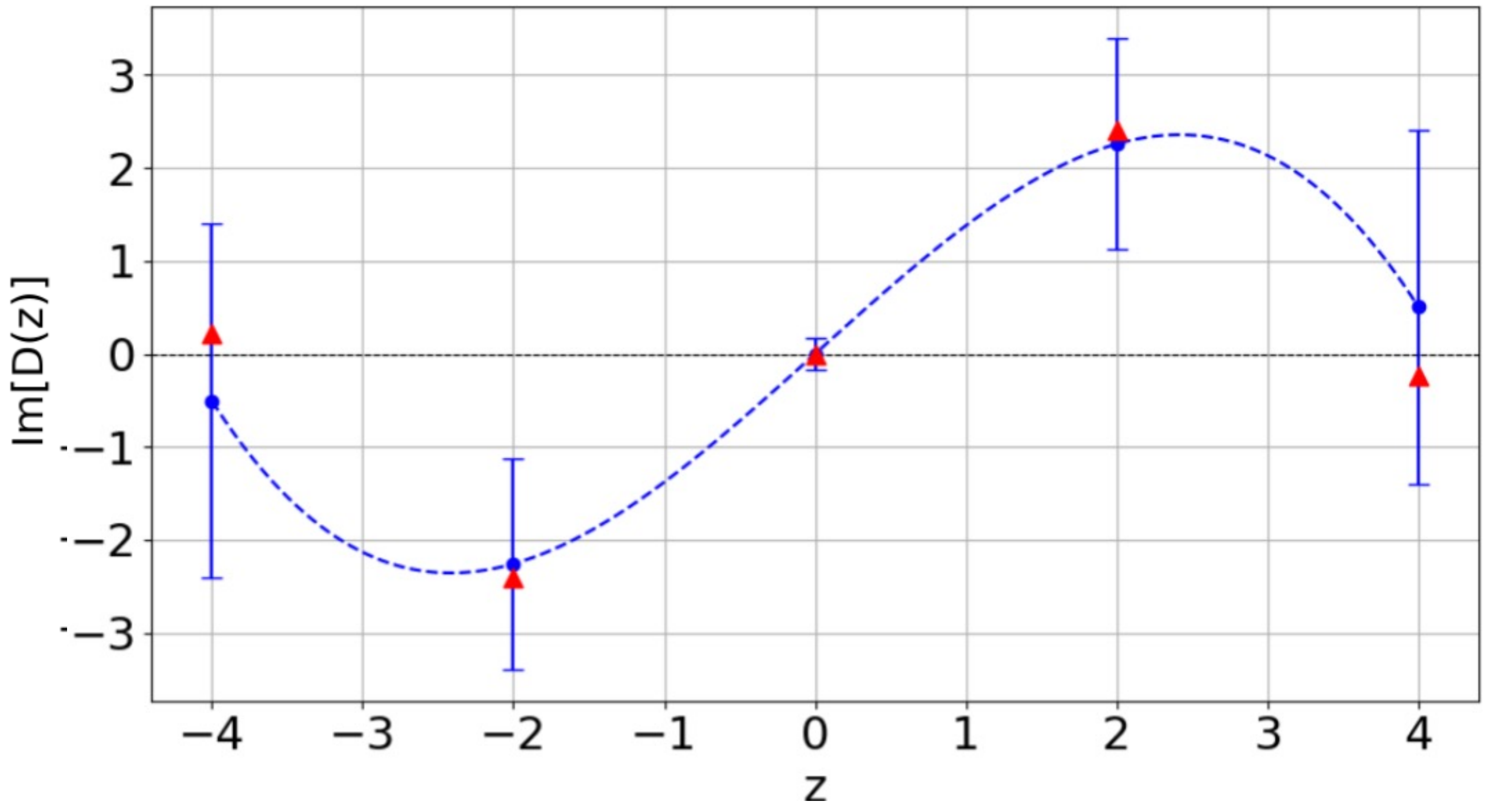
- IBM cloud quantum computers
- 11 qubits: 5 physical spatial sites, 10 staggered fermion sites, 1 ancillary qubit; Zigzag Wilson lines
- 2-qubit gate depth ~5K reduced to ~500 (very critical!). Mainly from changing state preparation, operator choice, and doubling the step size for time evolution and Wilson line. Each reduces the gate depth by half.

Error Mitigations



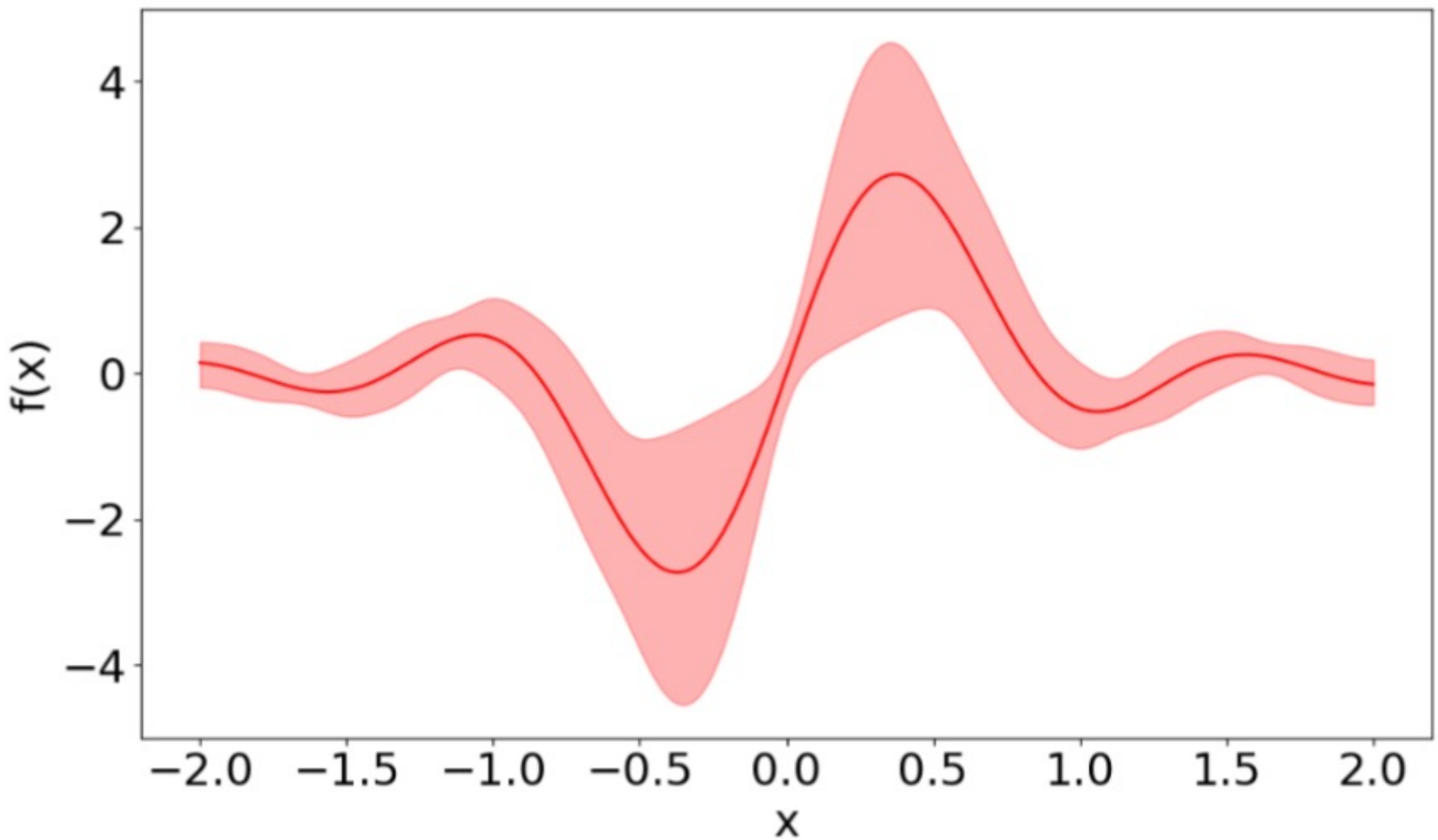
Dynamical decoupling (DD), Pauli twirling (PT) with randomization = 100, and Operator Decoherence Renormalization (ODR).

Result: Lightcone Correlator



8,000 shot result from IBM quantum computers.
Triangles from a classical simulator.

First "Quantum" PDF Result



$$\int_{-1}^{1} dx\, x f(x) = 1.04^{+0.66}_{-0.76}, \qquad \int_{0}^{1} dx\, f(x) = 1.37^{+0.89}_{-1.02}$$

So What?

Renormalon Ambiguity in LaMET

$$\tilde{q}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

- OPE: separation of long and short distance physics not strict in MS-bar scheme
- Matching kernel: IR Borel resummation leads to $\Lambda_{\rm QCD}^2/P_z^2$ ambiguity to be canceled by power corrections
- Large momentum expansion fails when the parton momentum is small

Quantum PDF is IR Renormalon Ambiguity Free

$$q(x,\frac{1}{a}) = \int_{-1}^{1} \frac{dy}{|y|} \mathcal{C}\Big(\frac{x}{y}, \mu a\Big) q(y,\mu) + \mathcal{O}\Big(a\Lambda_{\rm QCD}\Big)$$

$$q(x) + \delta q(x, 1/a) = q(x, \frac{1}{a})$$

$$q(x,\epsilon) + \delta q(x, 1/a, \epsilon) = q(x, \frac{1}{a})$$

$$q(x,\mu) + \delta q(x, 1/a, \mu) = q(x, \frac{1}{a})$$

No soft mode in $\delta q(x, 1/a, \mu)$ or $\mathcal{C}\Big(\frac{x}{y}, \mu a\Big)$.

No IR renormalon ambiguity. Can access x near 0 and 1.

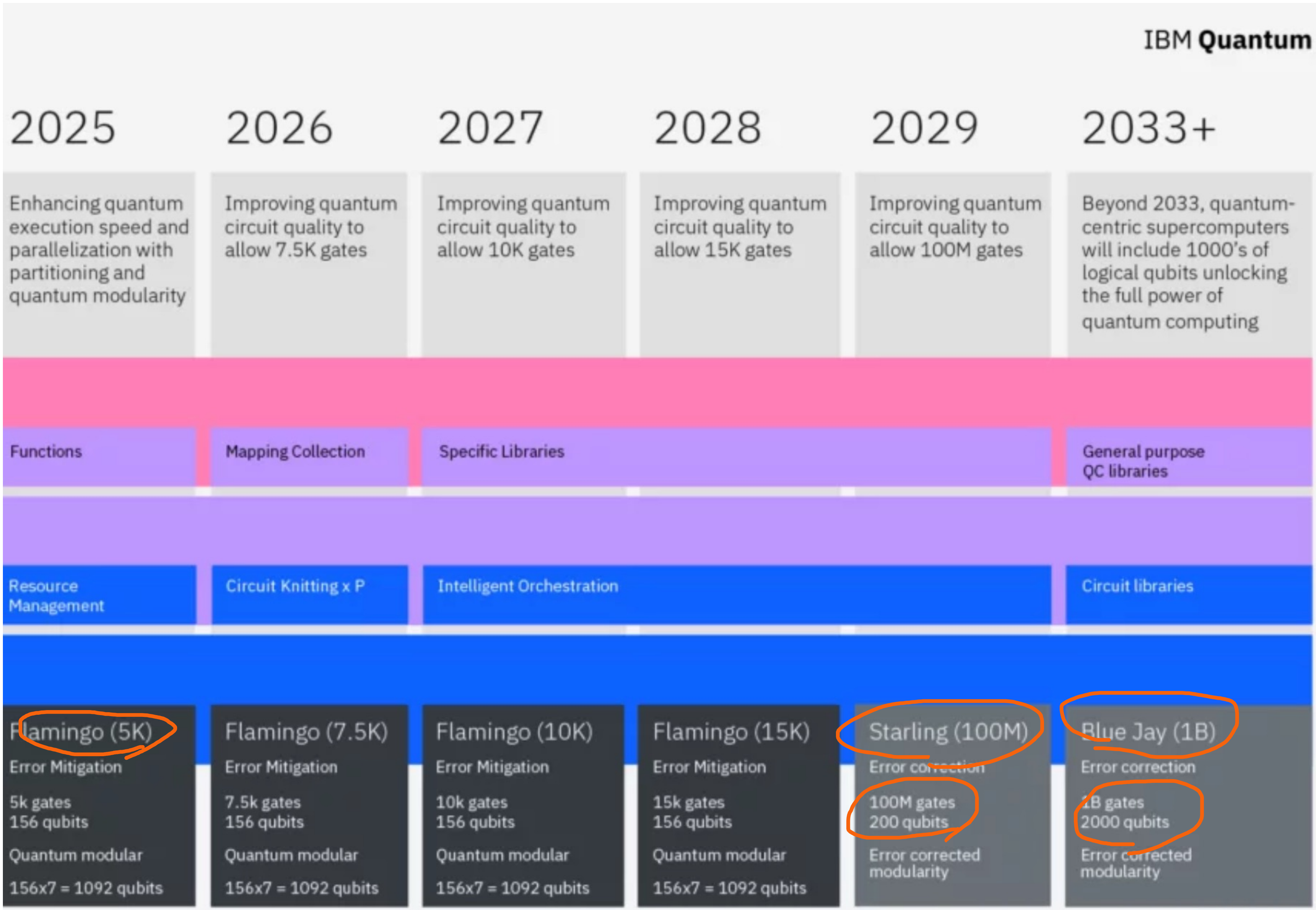
Other Possible Advantages of Quantum PDFs

- Accessing nonvalance partons with same expanses as valence ones (no disconnected diagram complications as in lattice QCD)
- No IR renormalon in GPD and TMD?

Outlook

- The future is a great unknown…

IBM Quantum Roadmap



Just Hype?

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"for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit"
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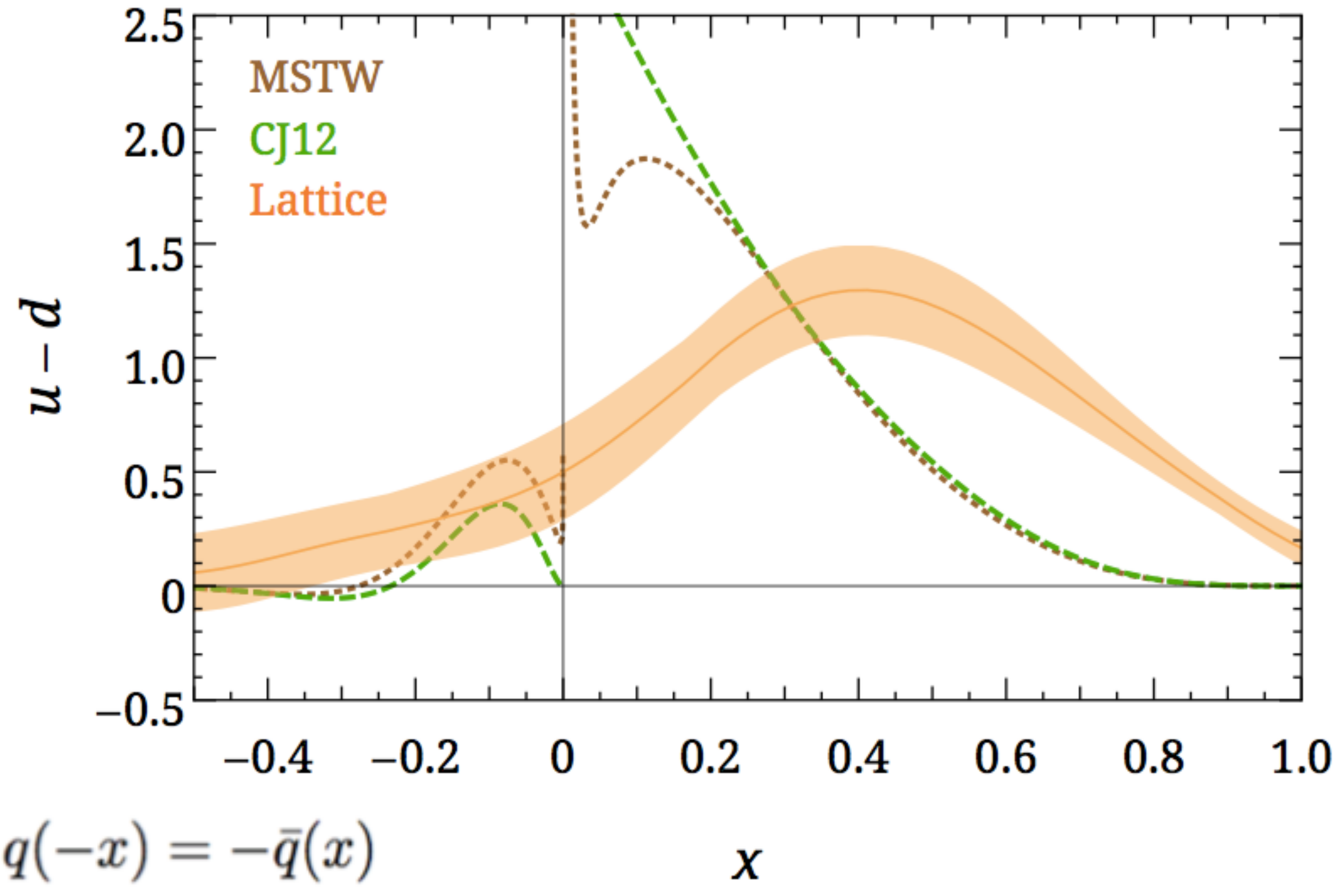
1^{st} LaMET Calculation

Lin, JWC, Cohen, Ji (1402.1462)



$$q(-x) = -\bar{q}(x)$$

Just Enjoy the Ride!

Backup

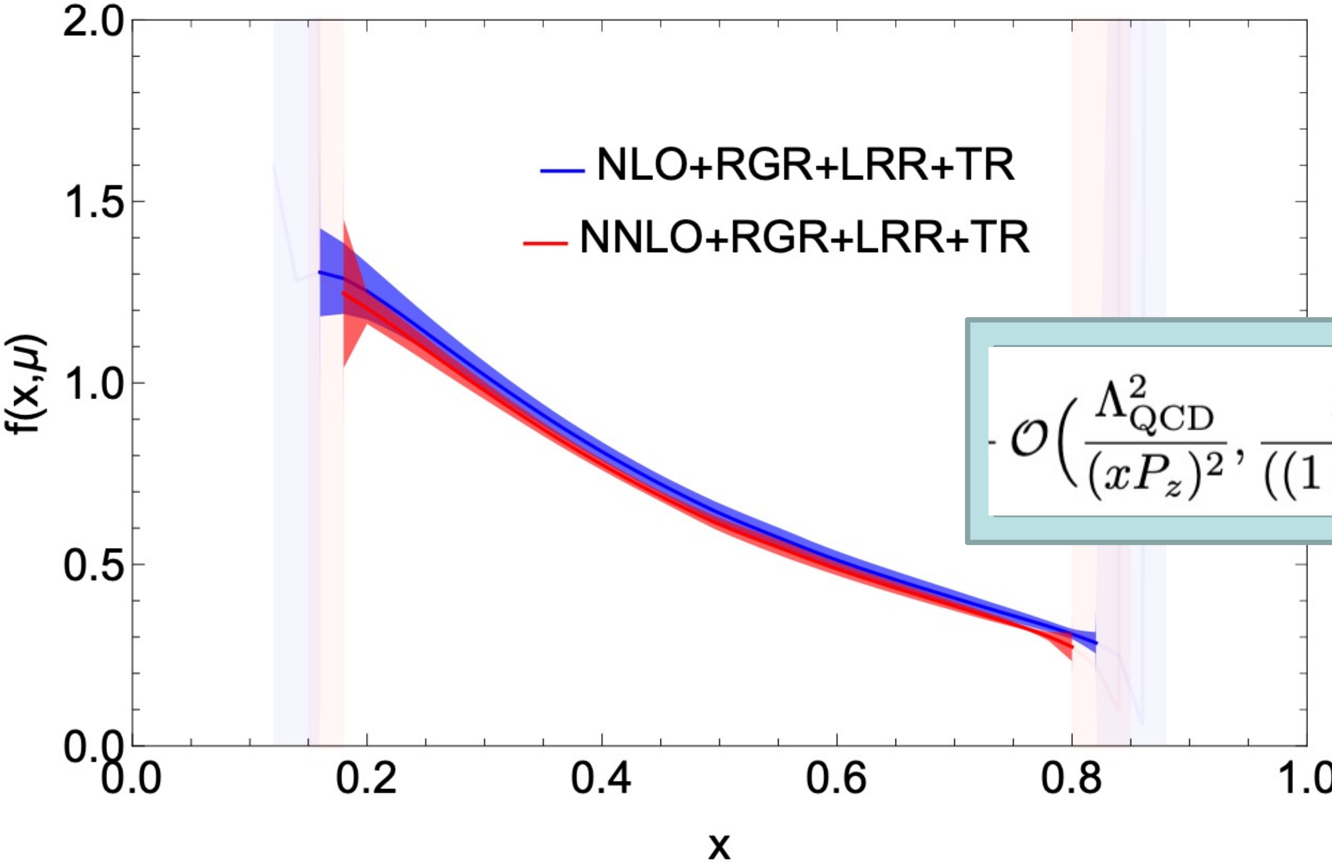
Outlook

- Clear demonstration of quantum advantage on a problem with great scientific impact: PDF with 3+1D QCD near $x = 0$ or $x = 1$

Remaining Errors

- Statistical errors (standard errors): inversely proportional to the number of shots
- Remove finite volume effect and lattice spacing, need ~3 and ~2 times more sites
- 2-qubit gate depth scales with 3 powers of the spatial volume
- Machines available in 2029!

Gymnastics of Resummations



$$\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

Xiangdong Ji, Yizhuang Liu, Yushan Su, Rui Zhang (2410.12910)

NNLO kernel: ZY Li, YQ Ma, JW.Qiu; LB Chen, W Wang, R Zhu

Resummations: RGR: ln(x) powers; TR: ln(1-x) powers; LRR: leading renormalon