

Moments of the Pion Generalized Parton Distributions from Nonlocal Correlators

Fei Yao

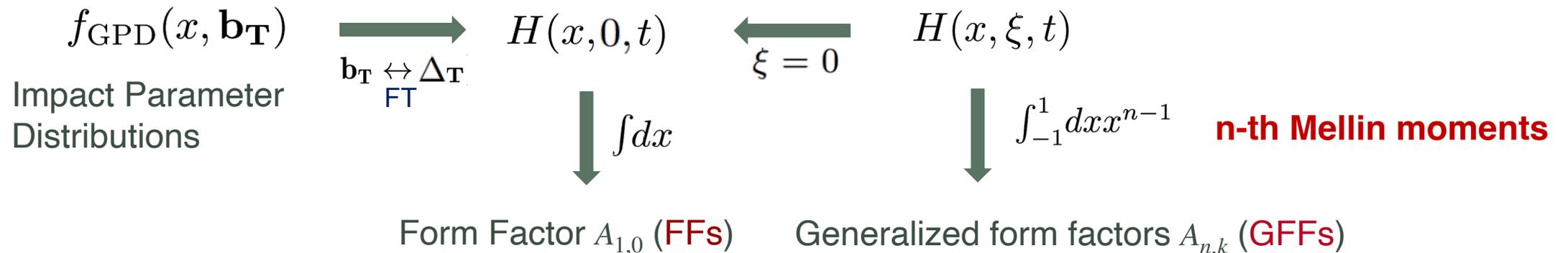
In collaboration with Xiang Gao, Swagato Mukherjee, Qi Shi and Yong Zhao

Introduction

- Pion Generalized Parton Distributions (**GPD**)

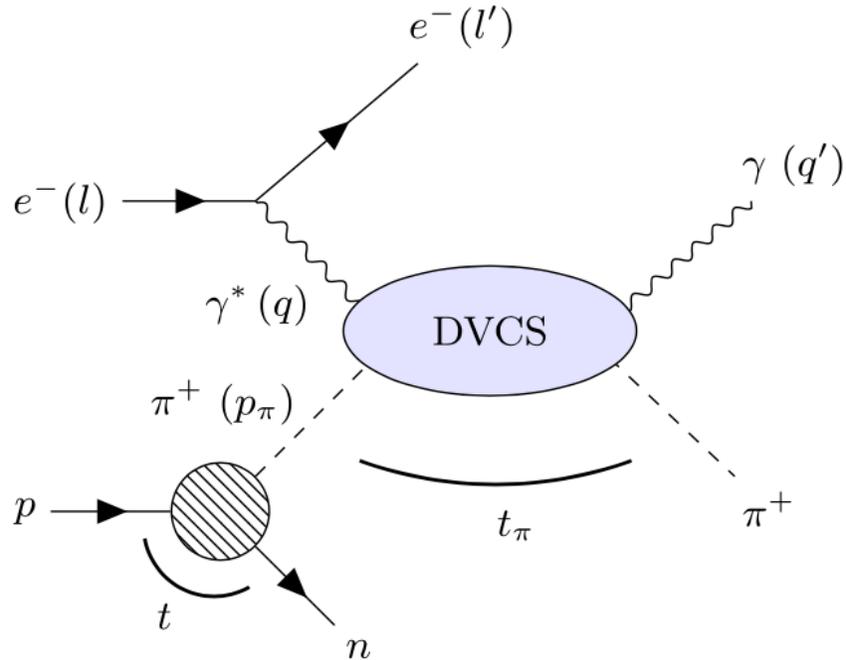
$$H^\pi(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle \pi(p_f) \left| \bar{\psi} \left(-\frac{z^-}{2} \right) \gamma^+ \mathcal{L} \left(-\frac{z^-}{2}, \frac{z^-}{2} \right) \psi \left(\frac{z^-}{2} \right) \right| \pi(p_i) \right\rangle$$

x	$\Delta^\mu = (p_f - p_i)^\mu$	$-t = \Delta^2$	$\xi = \frac{(p_i^+ - p_f^+)}{(p_i^+ + p_f^+)}$	$P^\mu = \frac{(p_i^\mu + p_f^\mu)}{2}$
momentum fraction	momentum transfer	momentum transfer squared	skewness	average hadron momentum



Experimental access to Pion GPDs

- No fixed pion target \rightarrow no direct $e\pi$ scattering
- Pion GPDs accessible only **indirectly** (via pion pole deep virtual Compton scattering)



Chávez et al., PRL 128 (2022) 202501

- No experimental data available yet.
- Future prospects: JLab 12 GeV, EIC, AMBER, EicC.

From first principles: Lattice QCD

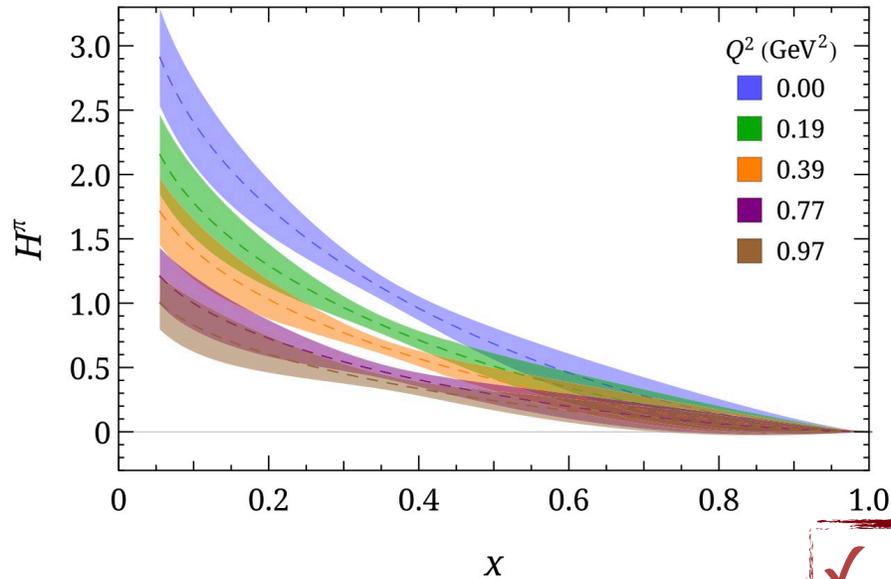
Pion GPDs from lattice QCD

- Mellin moments of pion GPD from **local operators**

[1] QCDSF/UKQCD Collab., *PoS LAT2005* (2006) 360;
 [2] QCDSF/UKQCD Collab., *Eur.Phys.J.C* 51 (2007).

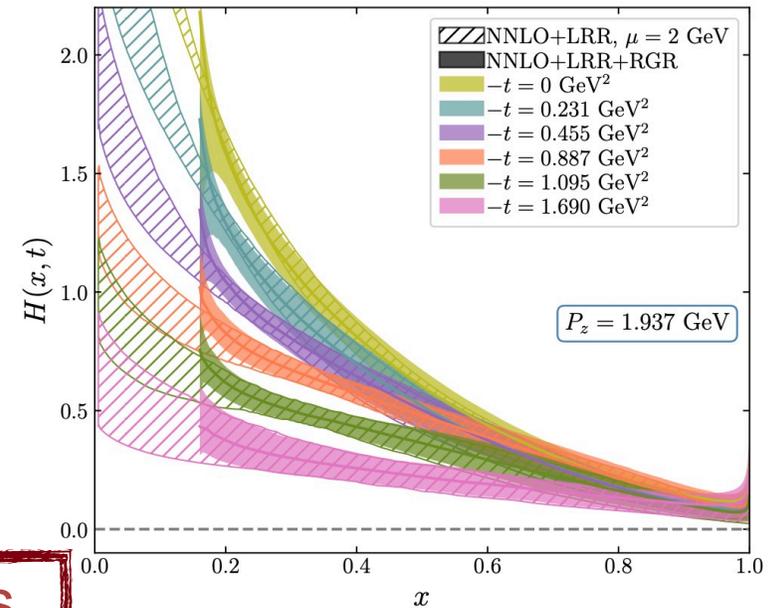
$$\begin{aligned}
 \langle \pi^+(p') | \mathcal{O}^{\mu\mu_1\mu_2\cdots\mu_n} | \pi^+(p) \rangle &= \langle \pi^+(p') | \bar{u}(0) \gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \cdots i\overleftrightarrow{D}^{\mu_n} u(0) | \pi^+(p) \rangle \\
 &= 2 P^{\{\mu} P^{\mu_1} \cdots P^{\mu_n\}} A_{n+1,0}(\Delta^2) + 2 \sum_{i=1, \text{odd}}^n \Delta^{\{\mu} \Delta^{\mu_1} \cdots \Delta^{\mu_i} P^{\mu_{i+1}} \cdots P^{\mu_n\}} A_{n+1,i+1}(\Delta^2)
 \end{aligned}$$

- LaMET approach to x-dependent GPDs



H.-W. Lin, *Phys. Lett. B* 846, 138181 (2023)

✓ Zero skewness



H.T. Ding et al., *JHEP* 02, 056 (2025)

Moments from nonlocal correlators

T. Izubuchi et al., Phys. Rev. D 98, 056004 (2018); Y.-S. Liu et al., Phys. Rev. D 100, 034006 (2019).

Operator product expansion (OPE) of nonlocal operators:

- ✔ **Pion and nucleon PDFs** X. Gao et al., Phys.Rev.D 109 (2024); X. Gao et al., Phys.Rev.D 107 (2023); X. Gao et al., Phys.Rev.D 104 (2021); J. Karpie et al., JHEP 11, 178 (2018); B. Joo et al., Phys. Rev. D 100, 114512 (2019) ...

$$\tilde{Q}(\lambda, \mu^2 z^2) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} a_{n+1}; \quad a_{n+1} = \int_{-1}^1 dx x^n q(x, \mu)$$

- ✔ **Pion and nucleon GPDs (zero skewness)**

S. Bhattachary et al., JHEP 01, 146 (2025);
 X. Gao et al., Phys. Rev. D 109, 094506 (2024);
 H.-T. Ding et al., PoS SPIN2023, 024 (2024);
 X. Gao et al., Phys.Rev.D 104 (2021) 11, 114515

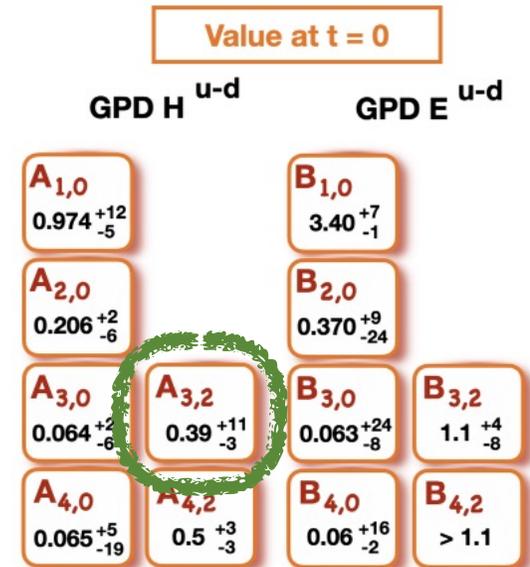
$$\tilde{H}(z, P_z, t) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} A_{n,0}(t); \quad A_{n,0}(t) = \int_{-1}^1 dx x^{n-1} H(x, t, \mu)$$

- ✔ **Only nucleon GPDs (nonzero skewness)**

$$\tilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2 \mu^2); \quad H_n(\xi, t, \mu) = \int_{-1}^1 dx x^{n-1} H(x, \xi, t, \mu)$$

- $H_3 = A_{3,0} + \xi^2 A_{3,2}$ becomes larger as ξ increases.
- Only LL-resummed considered.

Opposite trend for pion case!!



HadStruc Collab., JHEP 02, 056 (2025)

Theoretical framework

- OPE of nonlocal operators:

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 z^2\right)$$



✓ Renormalized matrix elements



✓ n-th moment of pion GPDs.



✓ Perturbative matching matrix (non-diagonal)

Valence-quark pion matrix element and its LI decomposition

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 z^2\right)$$



✓ Renormalized matrix elements

Ratio scheme:

$$\widetilde{H}^R(z, P_z, \xi, t) = \frac{\widetilde{H}(z, P_z, \xi, t)}{\widetilde{H}(z, 0, 0, 0)}$$

A. Radyushkin, Phys. Rev. D 98, 014019 (2018)

- Valence-quark matrix element:

$$M^\mu(z, P, \Delta) = \left\langle \pi(p_f) \left| \bar{\psi}(-z/2) \gamma^\mu L(-z/2, z/2) \psi(z/2) \right| \pi(p_i) \right\rangle^{u-d}$$

- Lorentz-invariant (LI) amplitudes $A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$:

$$M^\mu(z, P, \Delta) = P^\mu A_1 + z^\mu m^2 A_2 + \Delta^\mu A_3$$

- **Traditional quasi-GPD** defined by γ^0 is **frame dependent**: S. Bhattacharya et al., PRD 106 (2022) 114512

$$\widetilde{H}(z, P_z, \xi, t) = \frac{1}{P^0} M^0(z, P, \Delta) = A_1 + \frac{\Delta^0}{P^0} A_3$$

- **LI quasi-GPD**: combining $M^0(\gamma^0)$ and $M^x(\gamma^x)$

$$\widetilde{H}_{\text{LI}}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_1 + \frac{z \cdot \Delta}{z \cdot P} A_3$$

Consistency with the γ^0 definition will be shown later!

Polynomiality of GPD moments

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 z^2\right)$$



✓ n-th moment of pion GPDs.

Polynomiality of GPD moments (ξ -dependent):

$$H_n(\xi, t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} \boxed{A_{n,k}(t)} (2\xi)^k \pm \text{mod}(n+1, 2) (2\xi)^n C_n(t)$$

D-term contribution (for even n)

GFFs

GFFs

H. B. O'Connell et al., PLB 354, 14 (1995)

1. monopole-like parameterization

$$A_{n,k}(t) = \frac{A_{n,k}(0)}{1 + t/M_{n,k}^2}$$

G. Lee et al., PRD 92, 013013 (2015),

2. z -expansion (model-independent)

$$A_{n,k}(t) = \sum_{l=0}^2 a_{n,k,l} \mathbf{z}^l, \quad \mathbf{z}(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}$$

$\mathbf{z}(t)$ is introduced to improve the convergence of the series expansion.

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2\mu^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)$$

RGE: $\mu^2 \frac{d}{d\mu^2} C(z^2\mu^2) = \gamma(\alpha_s(\mu)) C(z^2\mu^2),$

with $\gamma(\alpha_s) = \gamma_0 \alpha_s + \gamma_1 \alpha_s^2$, $\beta(\alpha_s) \equiv \mu \frac{d\alpha_s}{d\mu} = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{2(2\pi)^2} \alpha_s^3$. **up to NNLO**

Note that both $C(z^2\mu^2)$ and anomalous dimension $\gamma(\alpha_s)$ are matrices.

✓ Perturbative matching matrix (non-diagonal)

$$c_{k,k-n} = \begin{pmatrix} c_{1,0} & 0 & 0 & 0 & 0 \\ 0 & c_{2,0} & 0 & 0 & 0 \\ c_{3,2} & 0 & c_{3,0} & 0 & 0 \\ 0 & c_{4,2} & 0 & c_{4,0} & 0 \\ c_{5,4} & 0 & c_{5,2} & 0 & c_{5,0} \end{pmatrix} \quad \gamma_1^{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.246 & 0 & 0 & 0 \\ -0.068 & 0 & 0.352 & 0 & 0 \\ 0 & -0.059 & 0 & 0.422 & 0 \\ -0.027 & 0 & -0.054 & 0 & 0.476 \end{pmatrix}$$

✓ Derive from Ref. [Y.Ji, F.Yao and J.H. Zhang, Arxiv:2504.09367]

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^k H_n(\xi, t, \mu) \xi^{k-n} c_{k,k-n}(z^2\mu^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)$$

RGE: $\mu^2 \frac{d}{d\mu^2} C(z^2\mu^2) = \gamma(\alpha_s(\mu)) C(z^2\mu^2),$

with $\gamma(\alpha_s) = \gamma_0 \alpha_s + \gamma_1 \alpha_s^2, \quad \beta(\alpha_s) \equiv \mu \frac{d\alpha_s}{d\mu} = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{2(2\pi)^2} \alpha_s^3.$

Note that both $C(z^2\mu^2)$ and anomalous dimension $\gamma(\alpha_s)$ are matrices.

✓ Perturbative matching matrix (non-diagonal)

	LL-RGR	NLL-RGR
$\xi = 0$	✓	✓
$\xi \neq 0$	✓	✗

✓ Analytical solution

✗ No analytical solution but numerical one exists.

Lattice calculation

- **Asymmetric (non-Breit) approach:** computationally economical

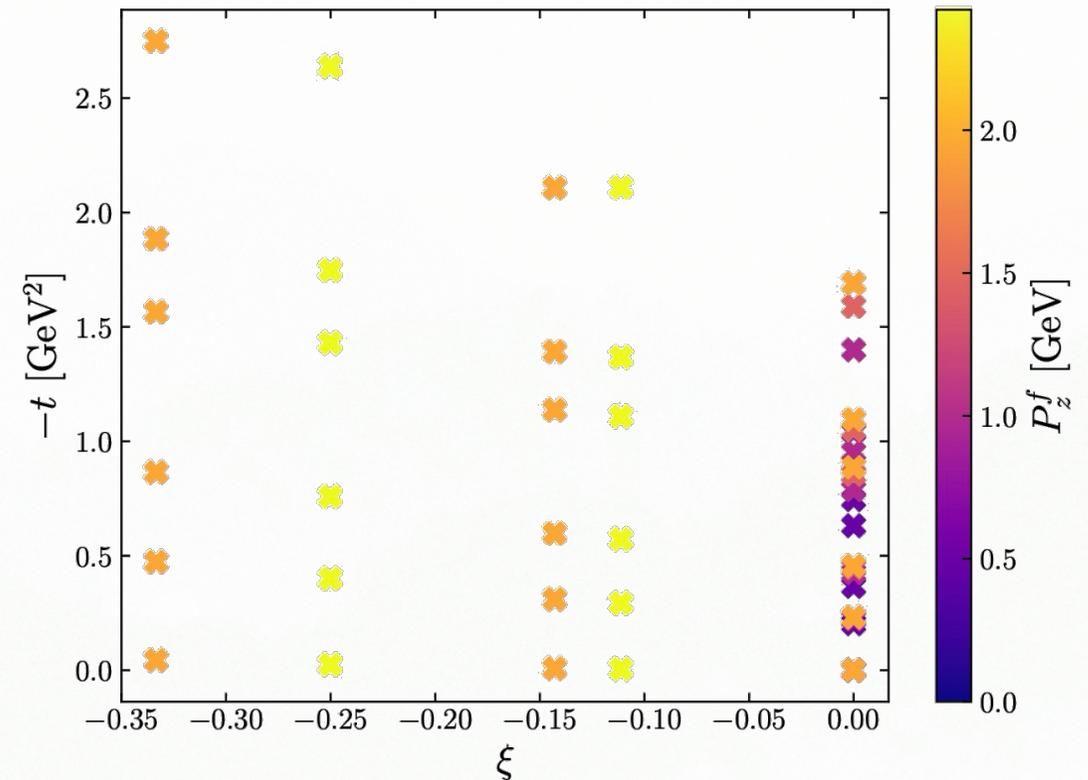
S. Bhattacharya et al., PRD 106 (2022) 114512

$$\vec{p}_f = (0, 0, P_z), \quad \vec{p}_i = \vec{p}_f - \vec{\Delta} = (-\Delta_x, -\Delta_y, P_z - \Delta_z)$$

- **Lattice setup:** HISQ gauge ensembles

HotQCD collab., PRD 75, 054502 (2007)

- $a = 0.04$ fm
- Pion mass: 300 MeV
- $P_z = \{0.968, 1.453, 1.937, 2.428\}$ GeV
- $\xi = \{-0.33, -0.25, -0.14, -0.11, 0\}$
- $-t$ up to 2.748 GeV^2

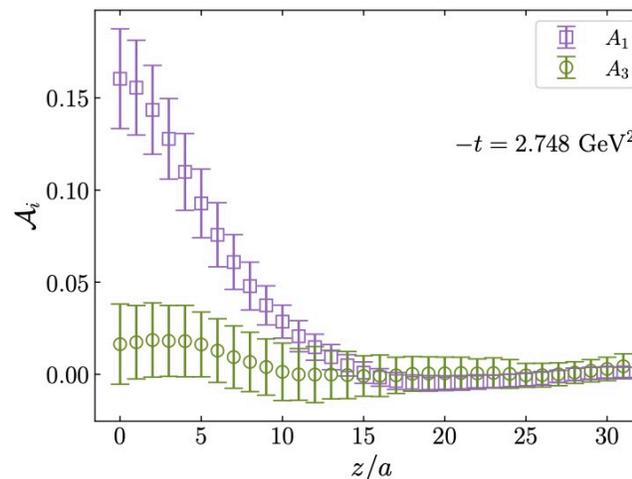
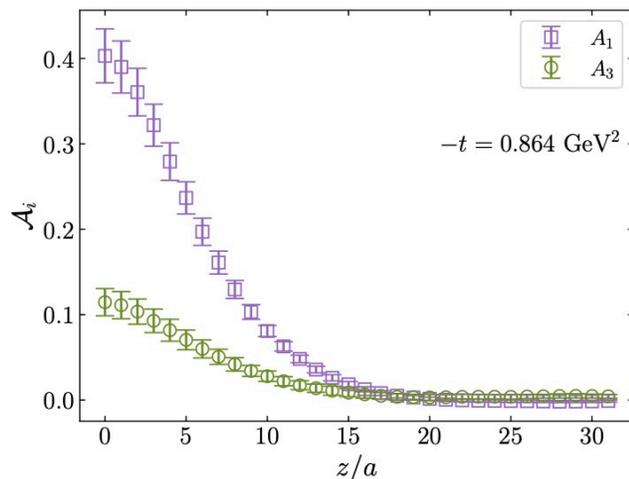


The comparison of \widetilde{H}_{LI} and \widetilde{H}

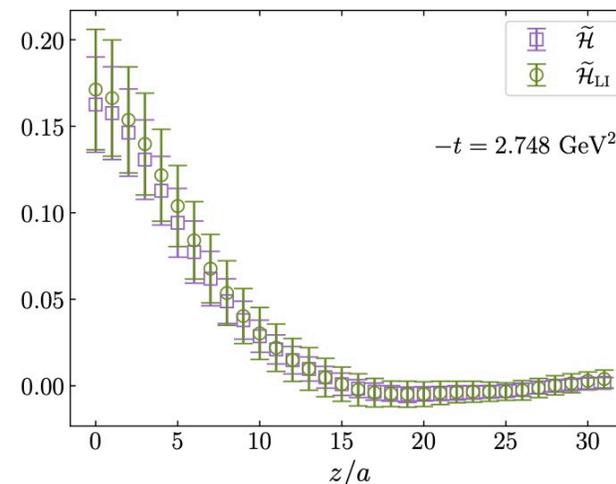
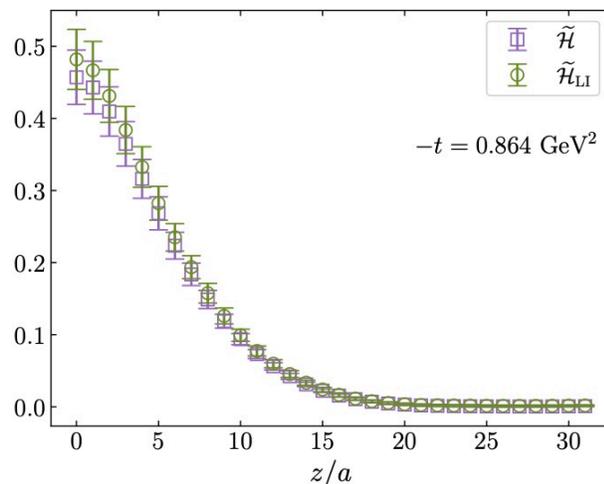
$$P_z = 1.937 \text{ GeV},$$

$$\xi = -0.33,$$

$$-t = \{0.864, 2.748\} \text{ GeV}^2$$



$$\downarrow \widetilde{H}_{LI}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_1 + \frac{z \cdot \Delta}{z \cdot P} A_3 \downarrow$$

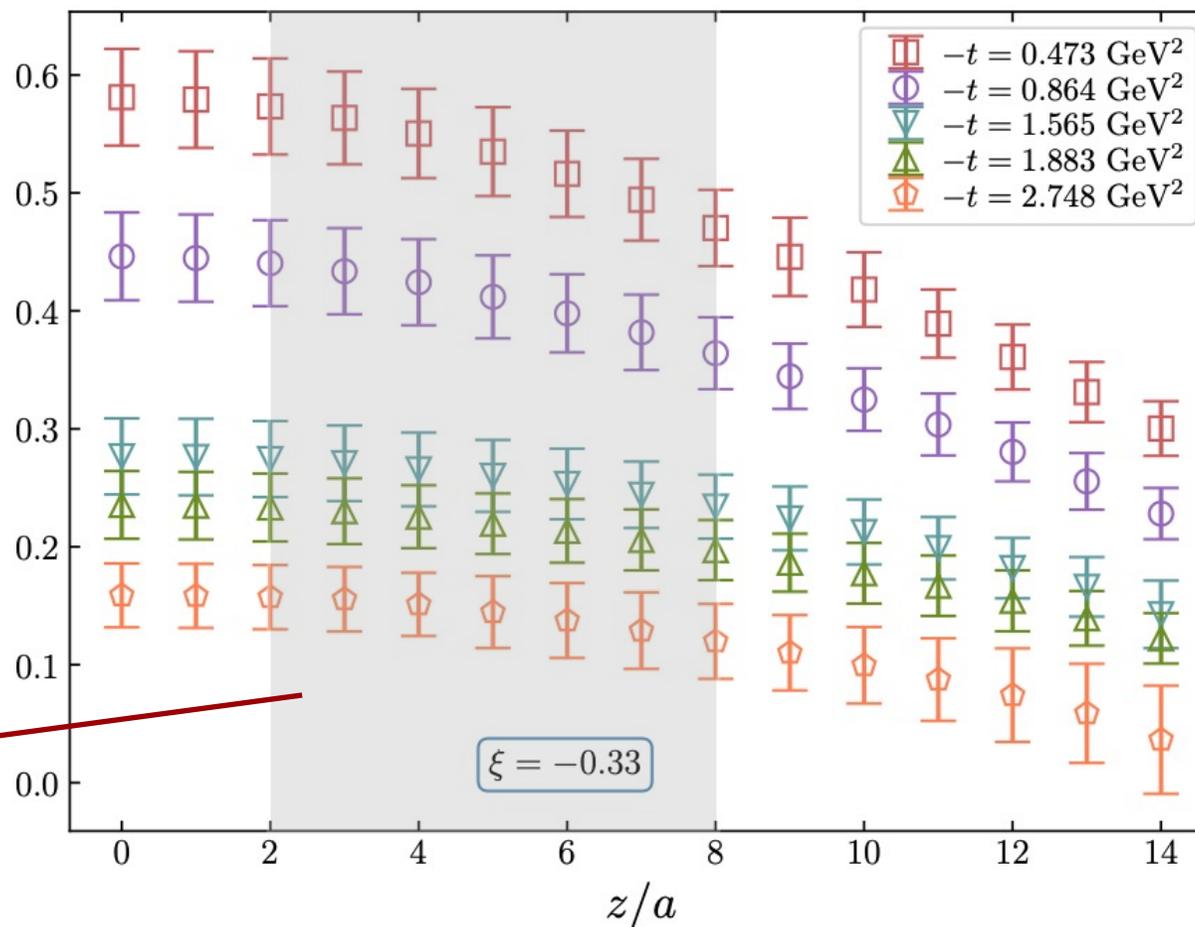


- Original definition by γ^0
- $$\widetilde{H}(z, P_z, \xi, t) = \frac{1}{P^0} M^0(z, P, \Delta)$$
- Adopted it in this work.

Renormalized matrix elements

$P_z = 1.937 \text{ GeV},$
 $\xi = -0.33$

$$\widetilde{H}^R(z, P_z, \xi, t)$$



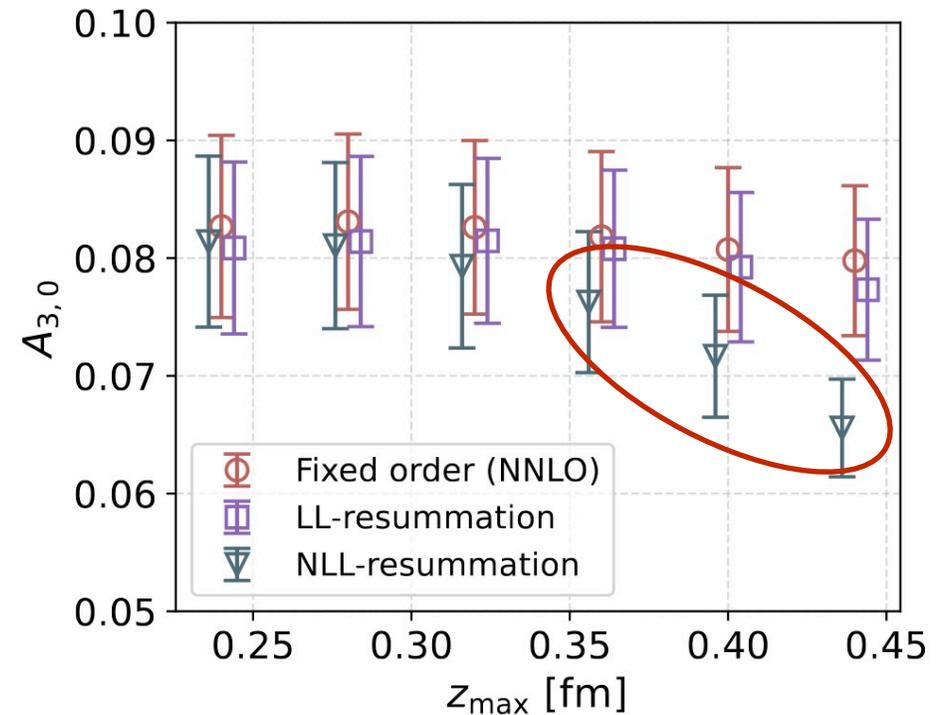
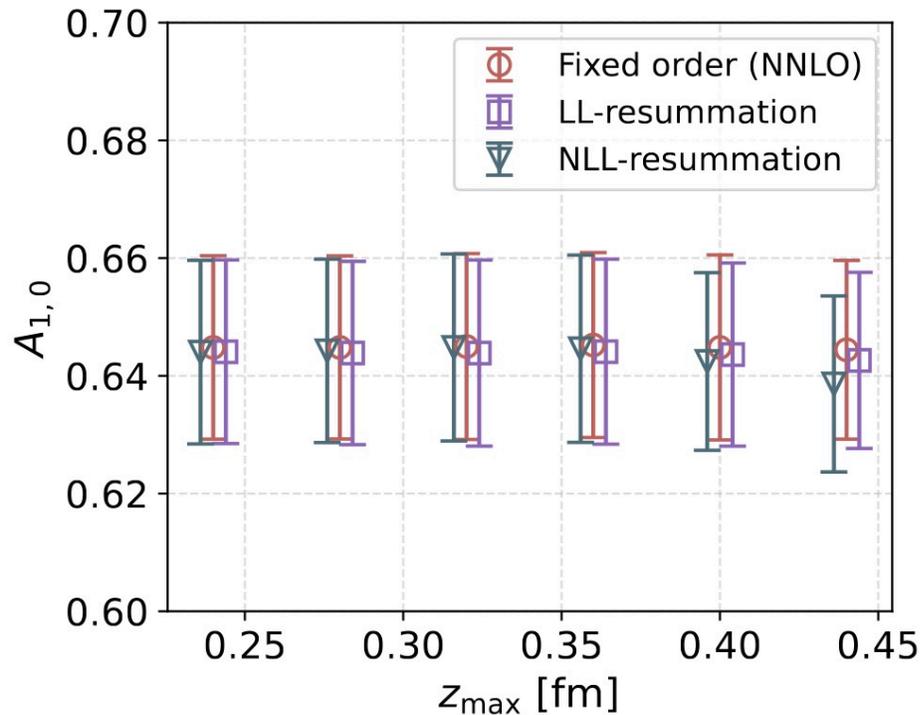
The Fitting region in this work:

$z = \{2a, 8a\} = \{0.08, 0.32\} \text{ fm}$

Why we choose this fitting region?

$$P_z = 1.937 \text{ GeV},$$
$$\xi = 0,$$
$$-t = 0.446 \text{ GeV}^2$$

- Fixed $z_{\min} = 0.08 \text{ fm}$, then varied z_{\max} .
 - Considering both **LL** and **NLL resummation** effects ($\mu_0 = (2\kappa e^{-\gamma_E})/z$, $\kappa = \sqrt{2}$).
 - $A_{3,0}$ becomes unstable when $z_{\max} > 0.32 \text{ fm}$.
- H.T. Ding et al., JHEP 02, 056 (2025)



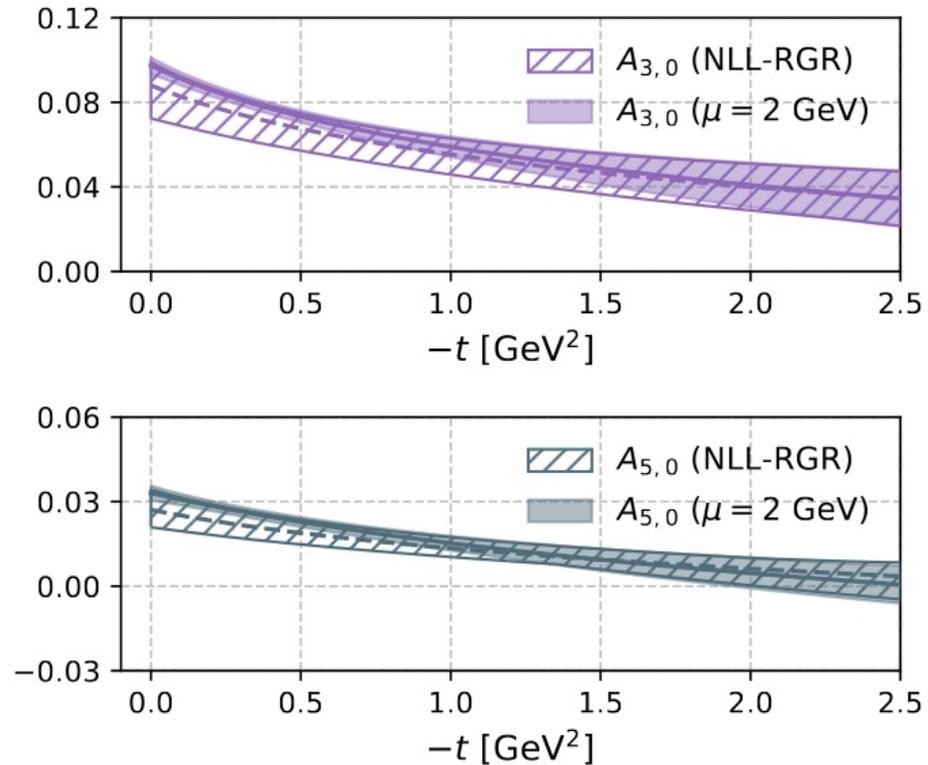
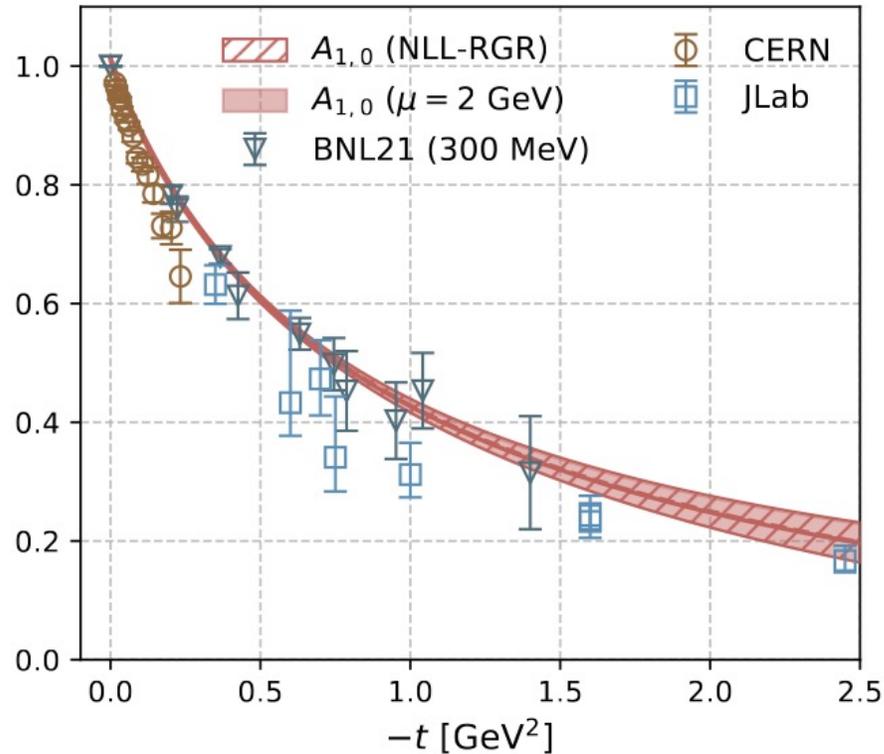
Numerical results for GPD moments

- Extracting the moments through a **joint fit** of matrix elements over multiple ξ and $-t$,

$$\begin{aligned}
 \widetilde{H}^R(z, P_z, \xi, t) &= \sum_{n=1}^{\infty} \frac{(-izP_z)^{n-1}}{(n-1)!} \sum_{k=1}^n H_k(\xi, t, \mu^2) \xi^{n-k} c_{n,n-k}(z^2 \mu^2) \\
 &= H_1(\xi, t) c_{1,0} + \frac{(-izP_z)^2}{2!} (H_1(\xi, t) \xi^2 c_{3,2} + H_3(\xi, t) c_{3,0}) + \frac{(-izP_z)^4}{4!} (H_1(\xi, t) \xi^4 c_{5,4} + H_3(\xi, t) \xi^2 c_{5,2} + H_5(\xi, t) c_{5,0}) \\
 &= A_{1,0}(t) c_{1,0} + \frac{(-izP_z)^2}{2!} \left[A_{1,0}(t) \xi^2 c_{3,2} + \underbrace{(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2)}_{H_3} c_{3,0} \right] + \frac{(-izP_z)^4}{4!} \left[A_{1,0}(t) \xi^4 c_{5,4} + \right. \\
 &\quad \left. \underbrace{(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2) \xi^2 c_{5,2} + (A_{5,0}(t) + A_{5,2}(t)(2\xi)^2 + A_{5,4}(t)(2\xi)^4)}_{H_5} c_{5,0} \right]
 \end{aligned}$$

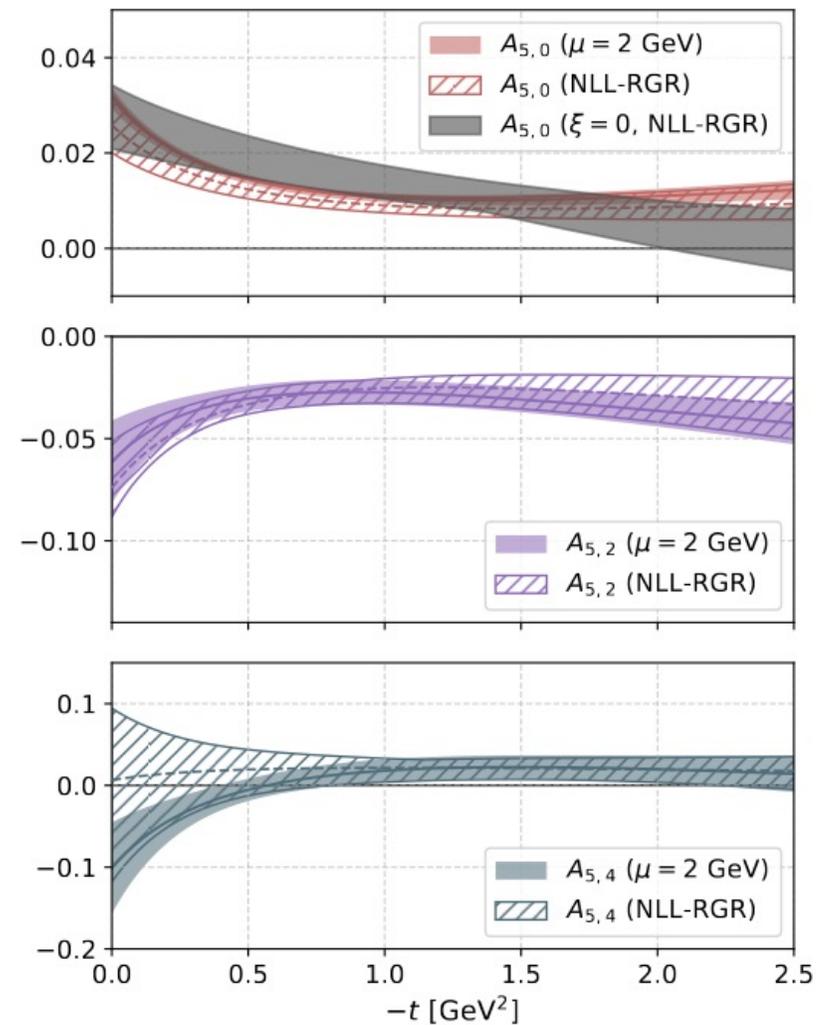
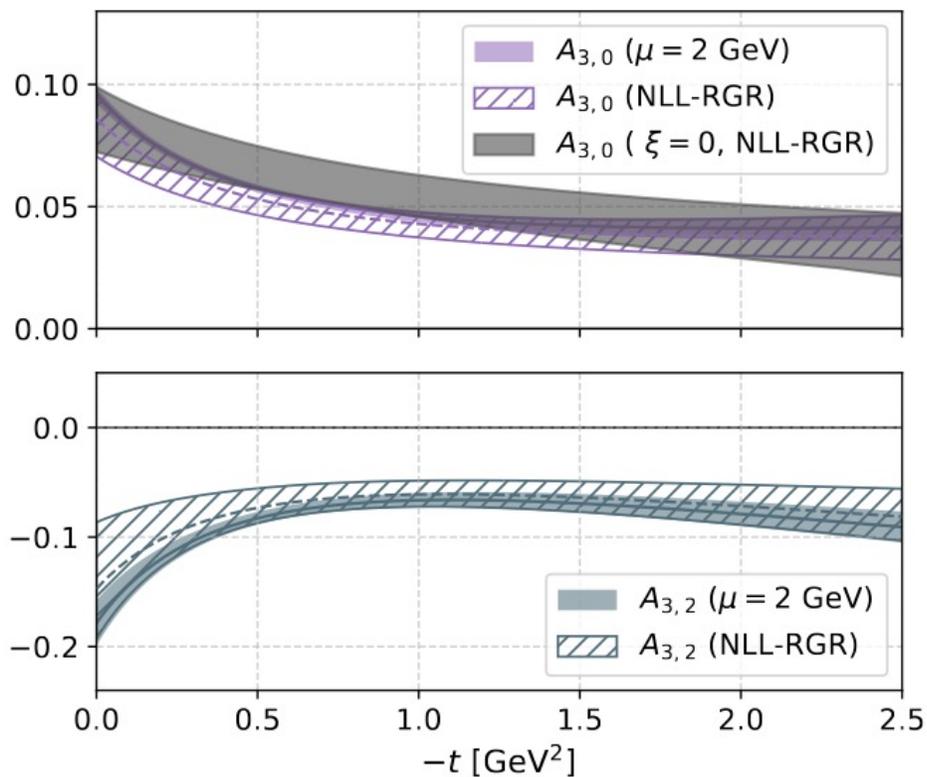
$$A_{n,k}(t) = \sum_{l=0}^2 a_{n,k,l} \mathbf{z}^l, \quad \mathbf{z}(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}$$

GFFs $A_{1,0}$, $A_{3,0}$ and $A_{5,0}$ ($\xi = 0$)



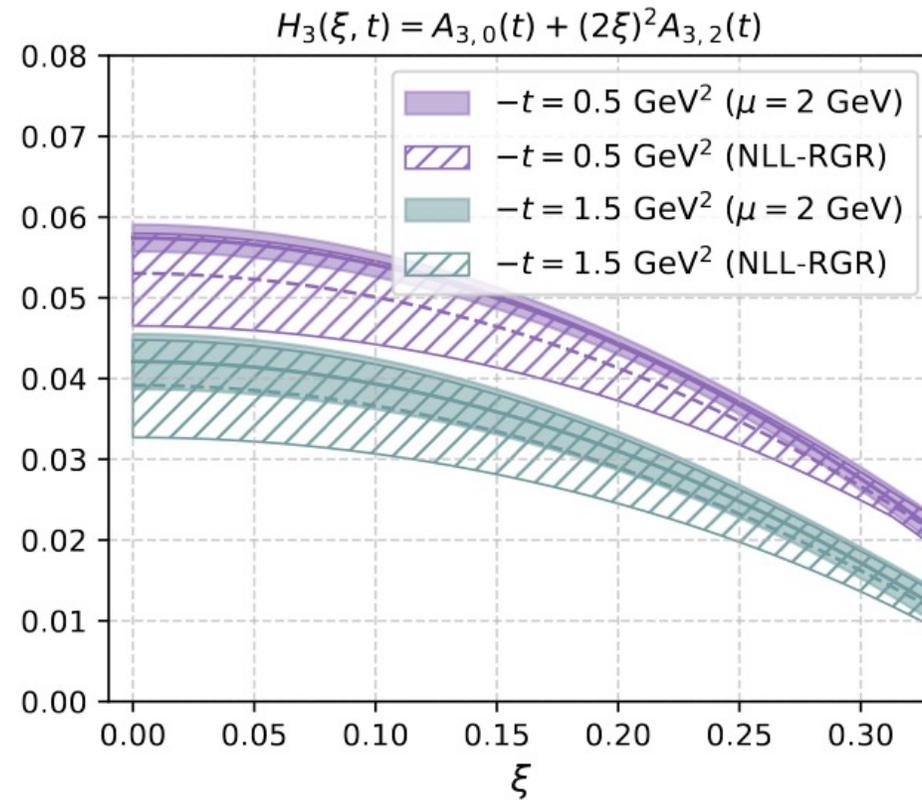
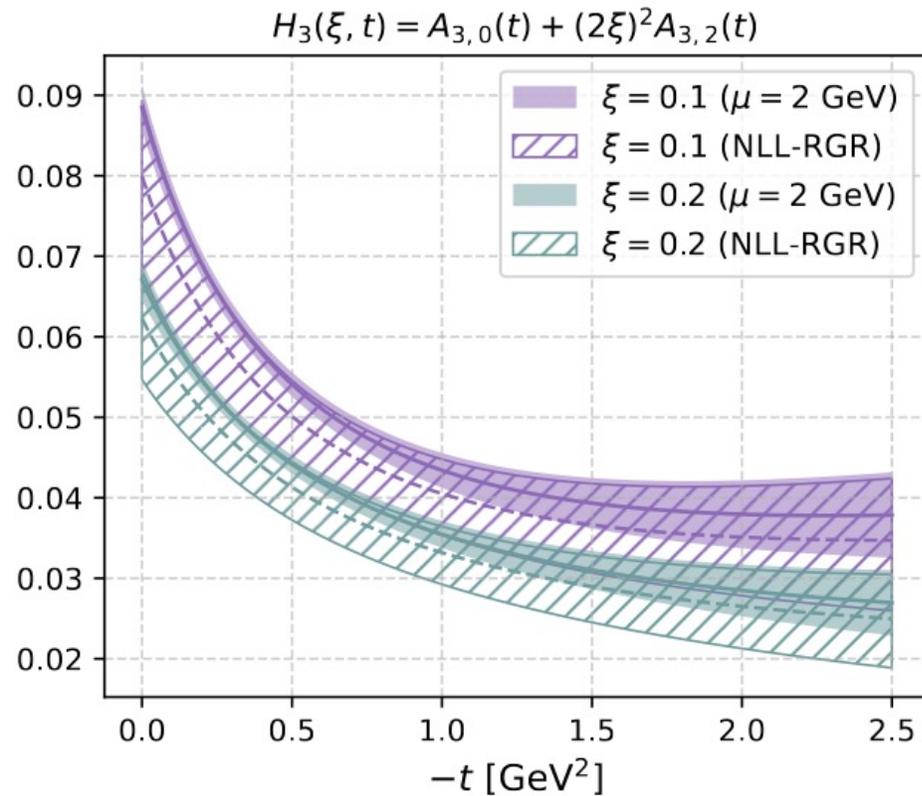
- The GFFs are extracted from zero-skewness data.
- Agree with experimental and BNL21 results. X. Gao et al., *Phys.Rev.D* 104 (2021) 11, 114515
- Dashed filled bands show NLL-resummed results (center value $\kappa = \sqrt{2}$), and uncertainties from varying κ between 1.2 and 2.

GFFs $A_{3,i}$ and $A_{5,i}$ ($\xi \neq 0$)



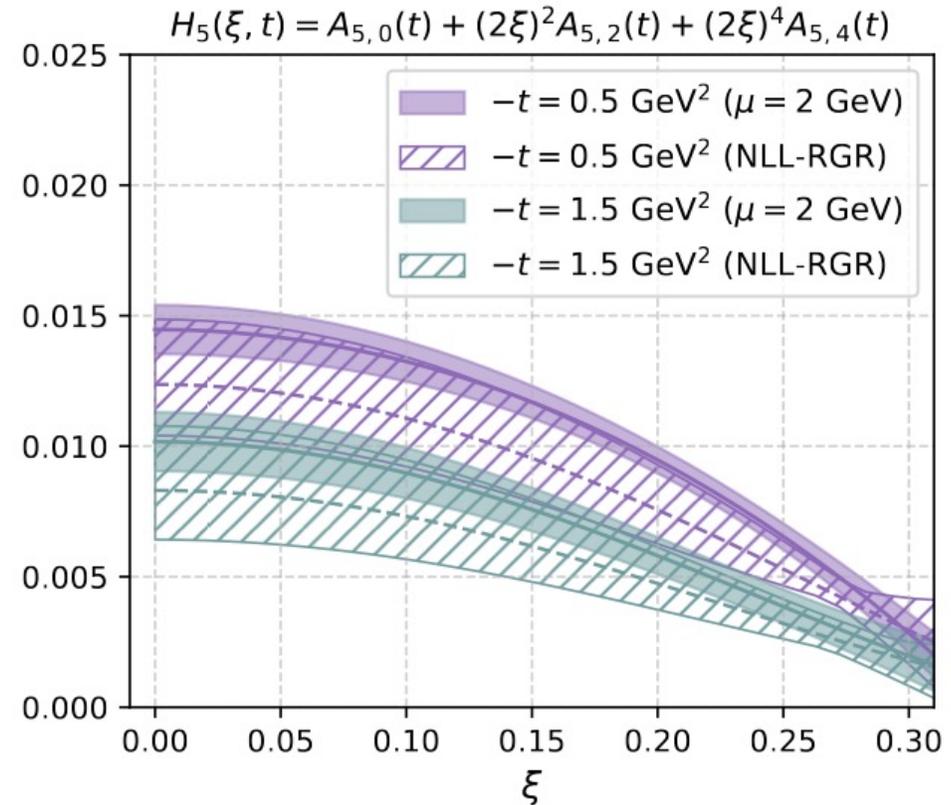
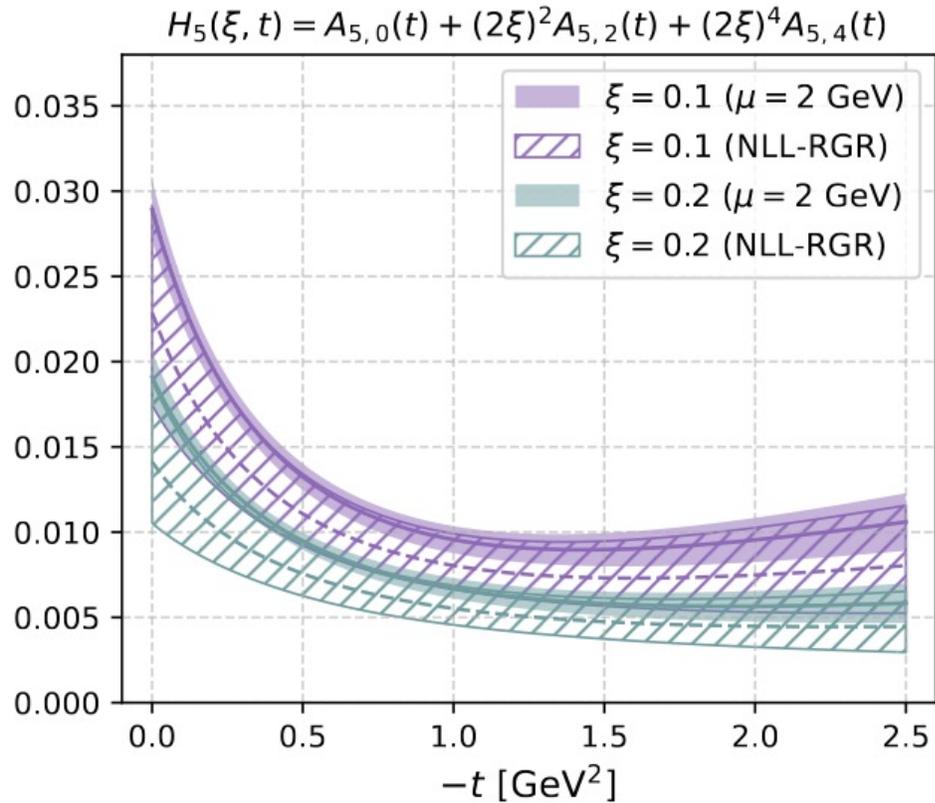
- The GFFs are obtained from a combined fit to both $\xi = 0$ and $\xi \neq 0$.
- $A_{3,2}$, $A_{5,2}$ are negative, indicating that the moments **decrease** as ξ **increases**.
- Dashed bands: NLL-resummed ($\kappa = \sqrt{2}$ is center value, with κ variation from 1.2 to 2.)

Moments of Pion GPDs ($\xi \neq 0$)



- For fixed $\xi = \{0.1, 0.2\}$, the third moment H_3 decrease as $-t$ increases.
- For fixed $-t = \{0.5, 1.5\}$ GeV², the third moment H_3 decrease as ξ increases. **Opposite trend for nucleon case!!**
- Dashed bands: NLL-resummed ($\kappa = \sqrt{2}$ is center value, with κ variation from 1.2 to 2.)

Moments of Pion GPDs ($\xi \neq 0$)



- For fixed $\xi = \{0.1, 0.2\}$, the fifth moment H_5 decrease as $-t$ increases.
- For fixed $-t = \{0.5, 1.5\}$ GeV², the fifth moment H_5 decrease as ξ increases.
- Dashed bands: NLL-resummed ($\kappa = \sqrt{2}$ is center value, with κ variation from 1.2 to 2.)

Summary and outlook

- Pion GPDs offer essential information on the pion's 3D structure, and lattice calculation can provide theoretical support complementary to experimental studies..
- First lattice QCD determination of the first three odd Mellin moments of pion GPDs **at nonzero skewness**:
 - Combined zero- and nonzero-skewness ensembles, giving the first quantitative input on the (ξ, t) **dependence** of pion GPD moments.
 - Implemented **NLL resummation** to reduce perturbative matching uncertainties.
- Follow-up:
 - Extraction of **pion GPDs at nonzero skewness**.
 - **D-term extraction** from the imaginary part of matrix elements.
 - Incorporate **higher statistics** and **physical pion masses**



Thank you for listening !

Speaker: Fei Yao (fyao@bnl.gov)



$$\int_{-1}^1 dx x^{n-1} \begin{pmatrix} H^{u-d} \\ E^{u-d} \end{pmatrix}(x, \xi, t) = \sum_{k=0, \text{ even}}^{n-1} \begin{pmatrix} A_{n,k}(t) \\ B_{n,k}(t) \end{pmatrix} \xi^{k \pm \text{mod}(n+1,2)} \xi^n (C_n(t)).$$

- Solution (γ_0, γ_i do not commute)

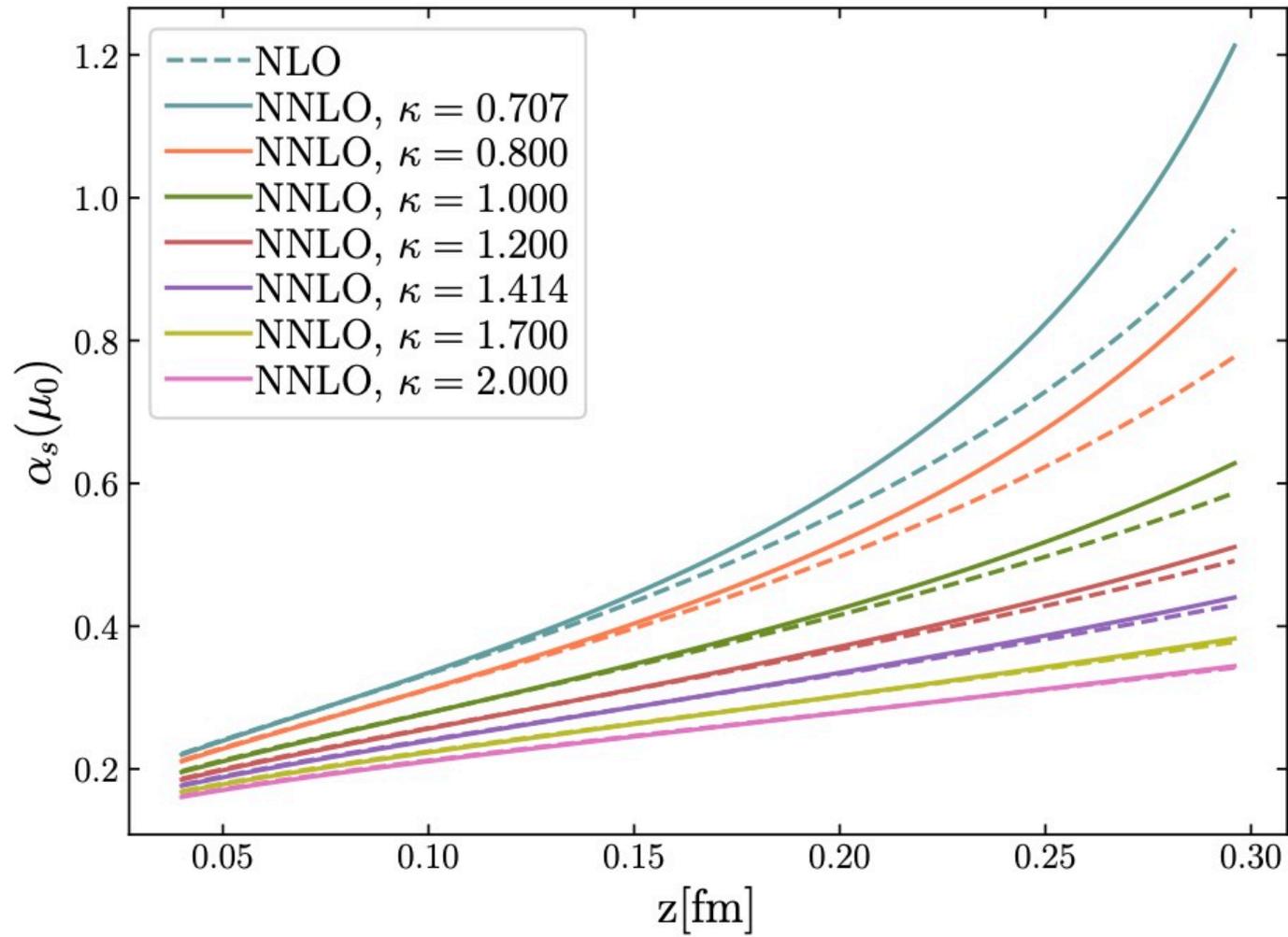
$$C(z^2 \mu^2) = \mathcal{P} \exp \left[2 \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \gamma(\alpha) \right] C(z^2 \mu_0^2),$$

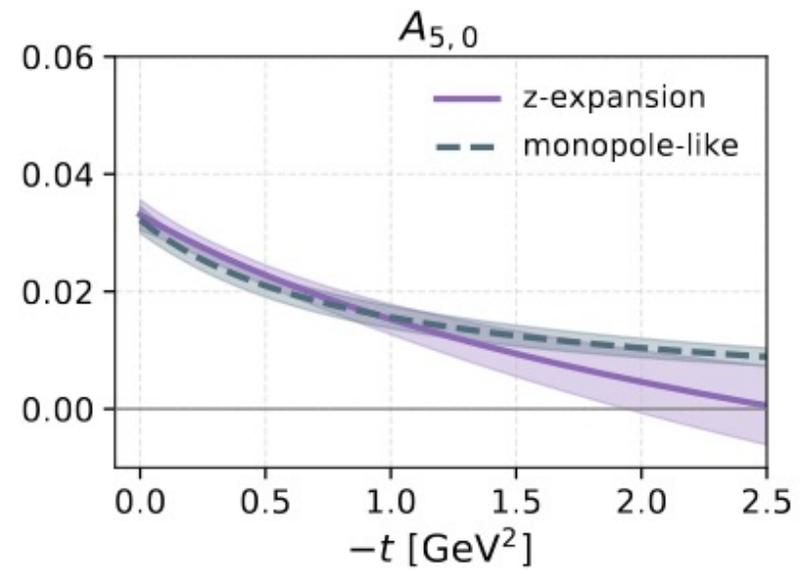
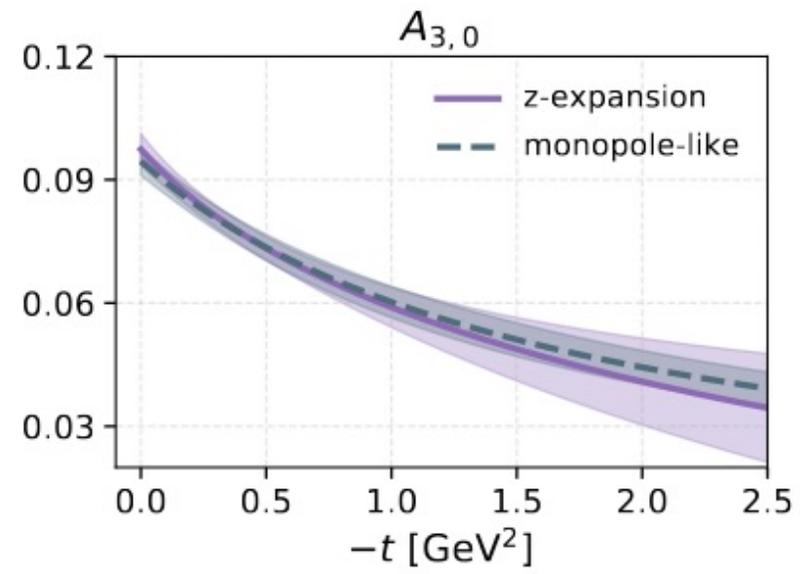
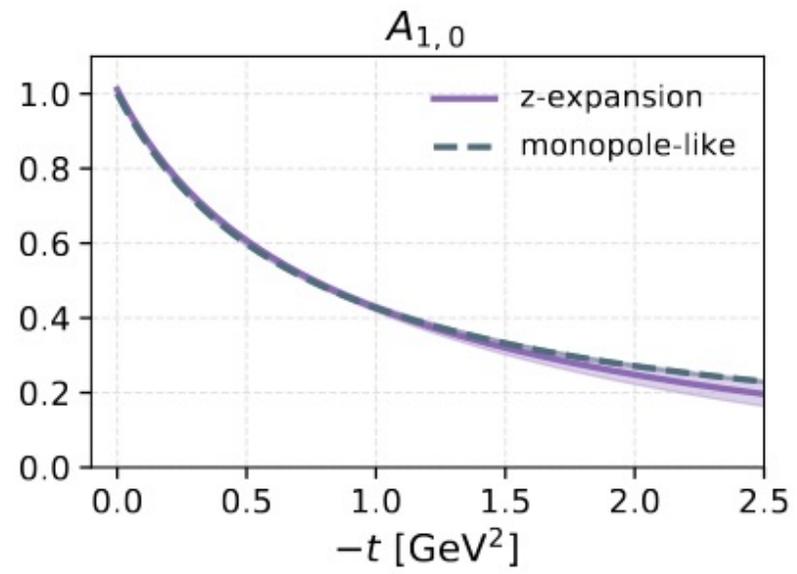
- LL-resummation

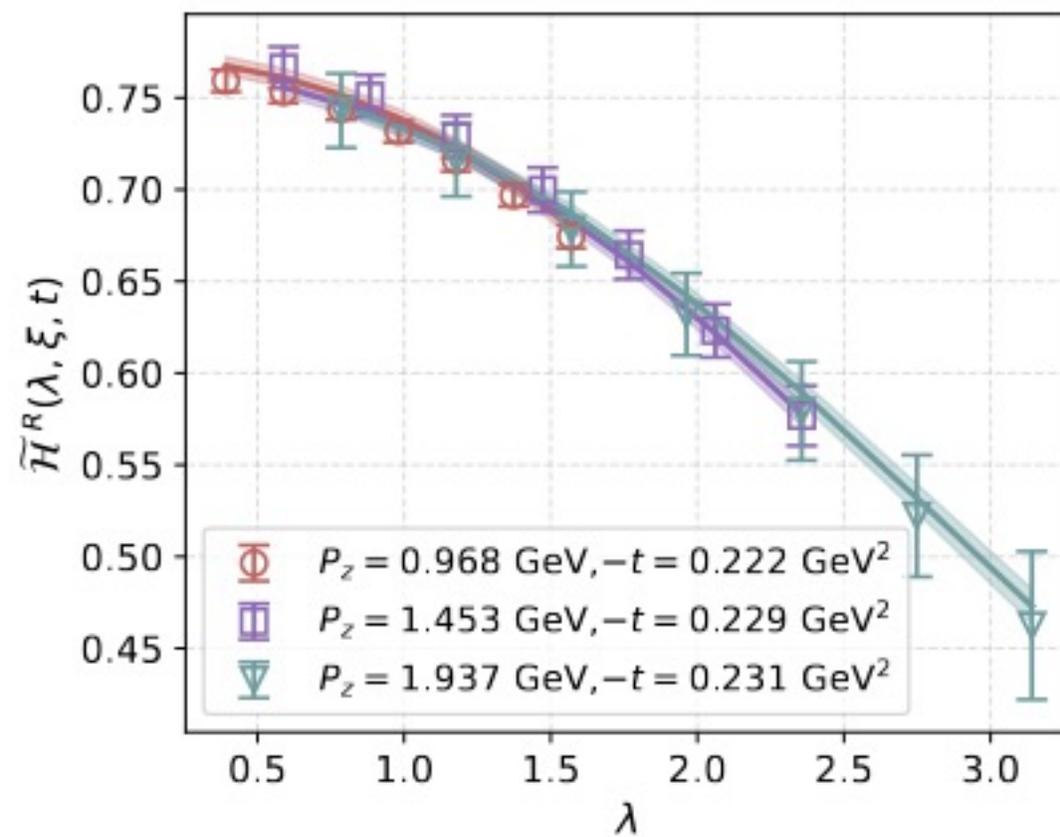
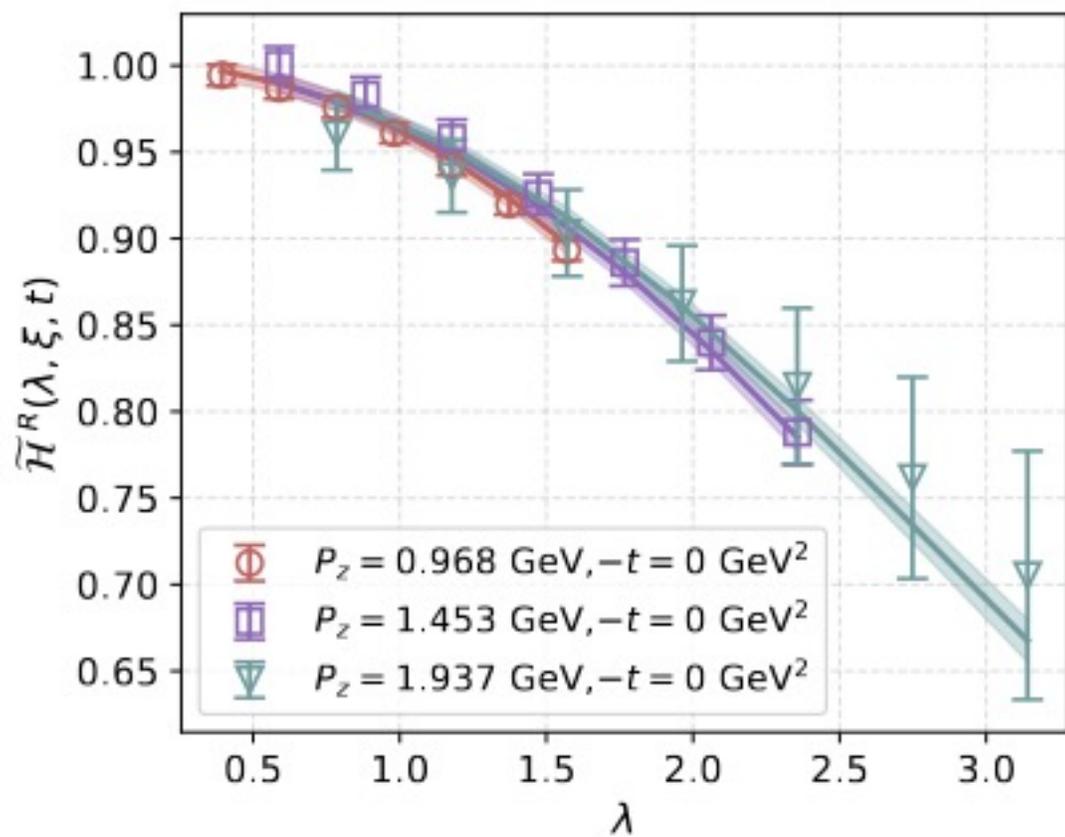
$$c_{i,j}(\alpha_s(\mu), z^2 \mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2 \mu_0^2) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{\frac{\gamma_0}{\beta_0}},$$

- NLL-resummation

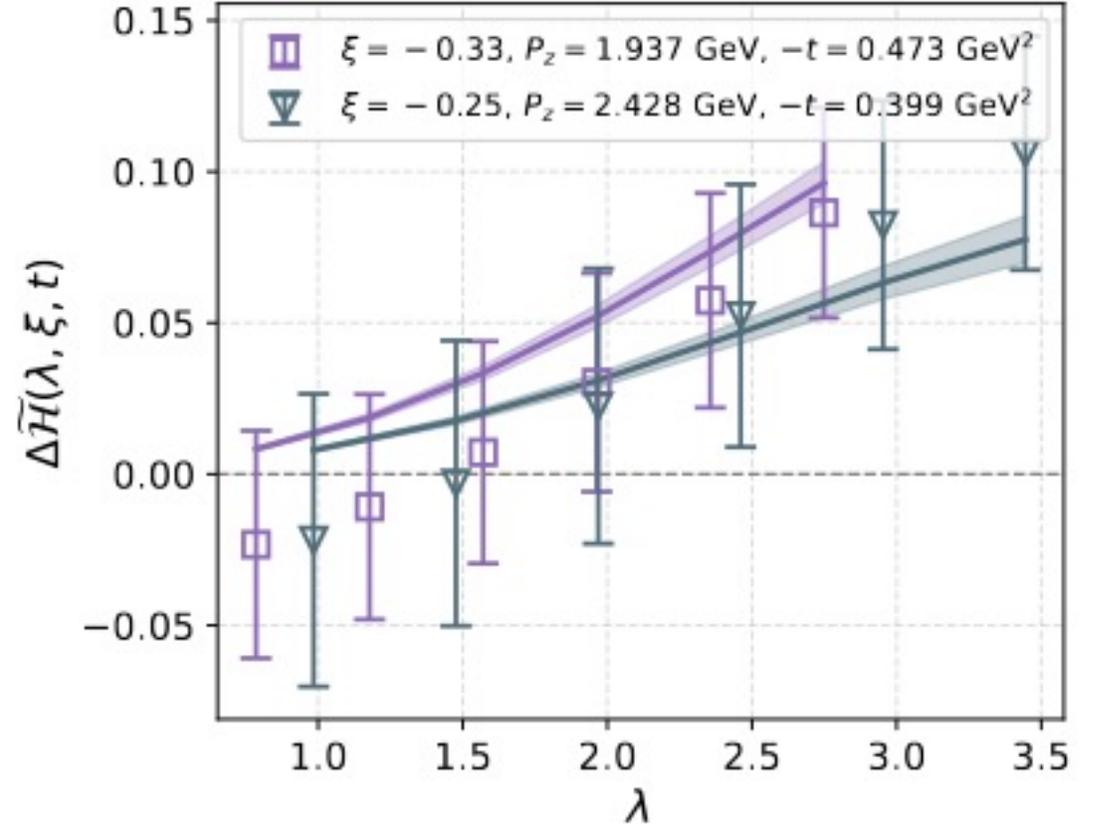
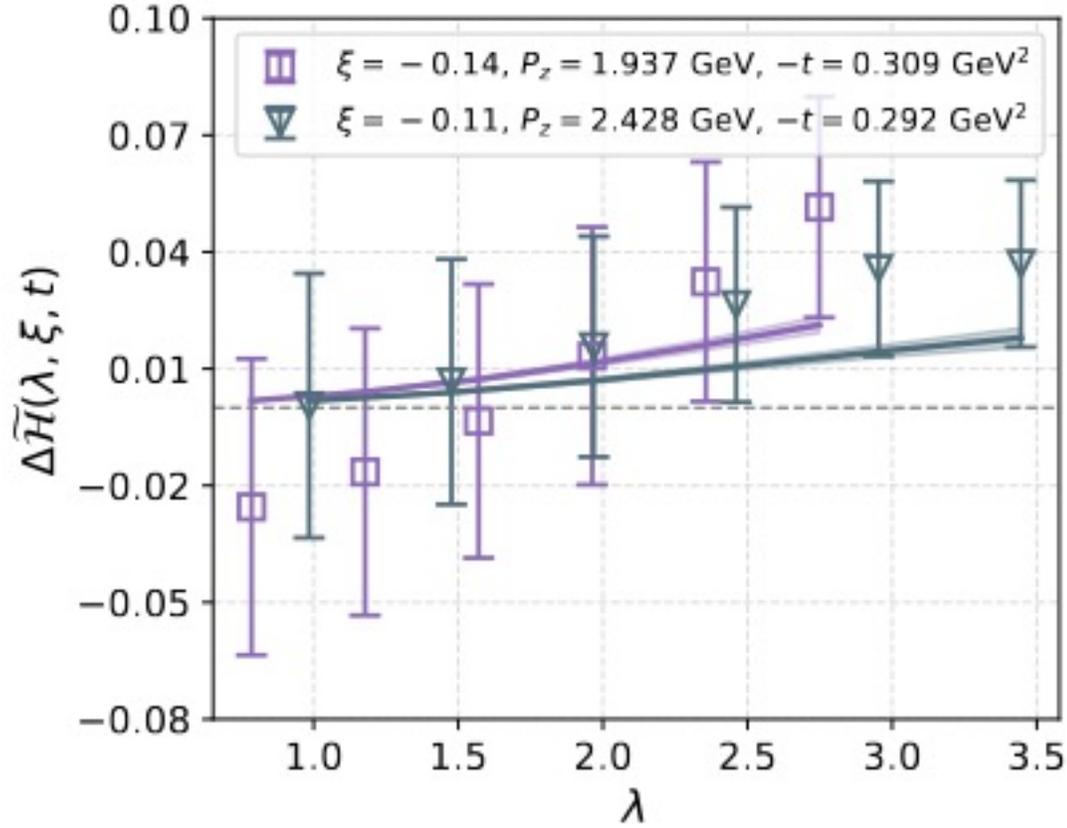
$$c_{i,j}(\alpha_s(\mu), z^2 \mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2 \mu_0^2) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{\frac{\gamma_0}{\beta_0}} \exp \left\{ \frac{(\beta_0 \gamma_1 - \beta_1 \gamma_0)}{\beta_0 \beta_1} \ln \left(\frac{\alpha_s(\mu_0) \beta_1 + (4\pi) \beta_0}{\alpha_s(\mu) \beta_1 + (4\pi) \beta_0} \right) \right\}.$$







$\chi^2/\text{d.o.f.} = 0.9$



$\chi^2/\text{d.o.f.} = 1.59$