Numerical Precision for LaMET Calculations

Rui Zhang

Massachusetts Institute of Technology

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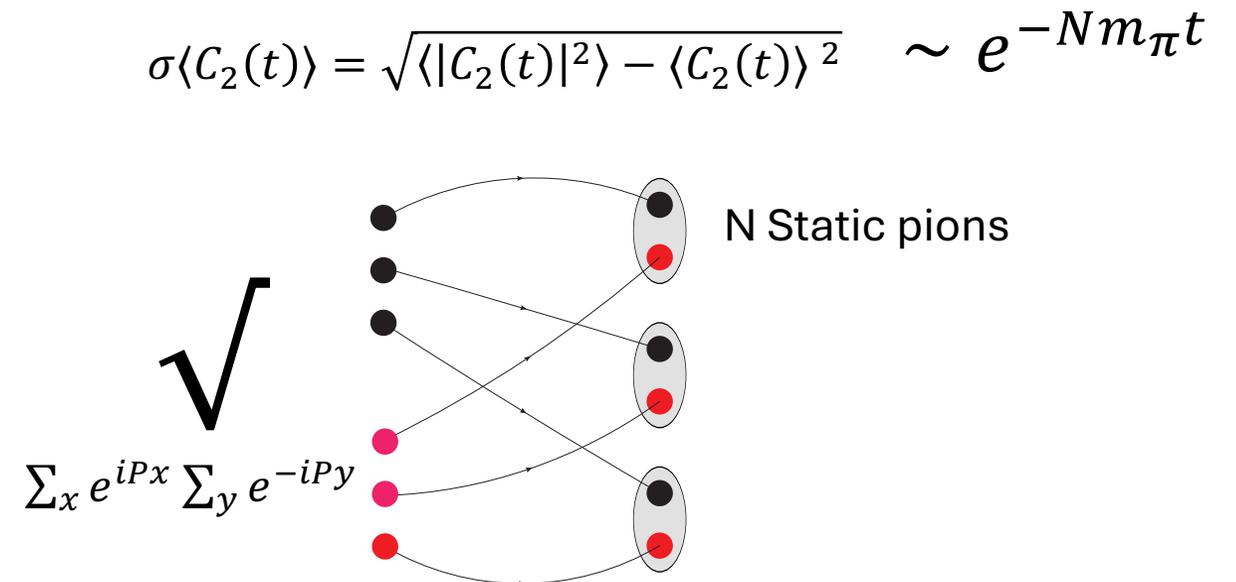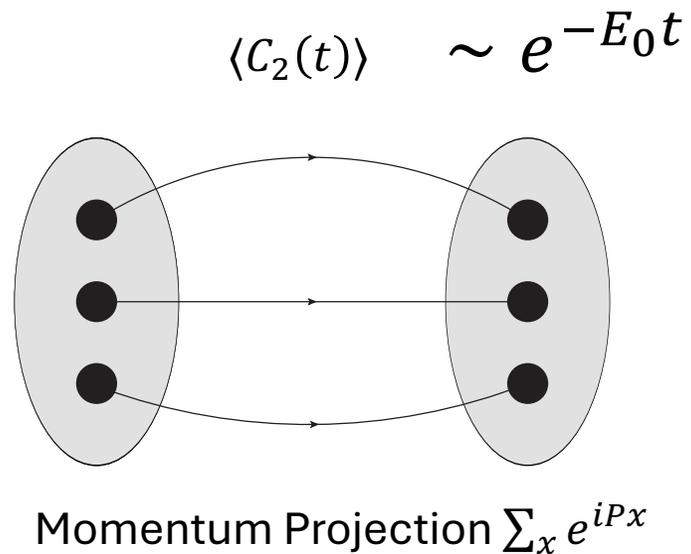
Stony Brook University, NY, 10/08/2025

arxiv:2501.00729

QUARK-GLUON TOMOGRAPHY COLLABORATION

PHENOMENOLOGY · QCD · LATTICE QCD · THEORY

Worsening Precision at Large Momentum

- Data quality depends on signal and fluctuation of the correlators

- Signal of data on lattice

$$\langle C_2(t) \rangle \quad \sim e^{-E_0 t}$$

Momentum Projection $\sum_x e^{iPx}$

- Statistical Fluctuations (Noise)

$$\sigma\langle C_2(t) \rangle = \sqrt{\langle |C_2(t)|^2 \rangle - \langle C_2(t) \rangle^2} \quad \sim e^{-Nm_\pi t}$$

N Static pions

$\sum_x e^{iPx} \sum_y e^{-iPy}$

Data quality $\equiv$ Signal-to-Noise ratio (SNR) $\sim e^{-(E_0 - Nm_\pi)t}$ decays exponentially
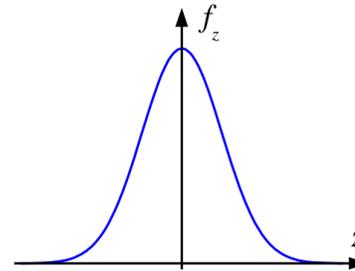
How to improve the precision: an example

The first leap: momentum smearing

- Improve signal
- Suppress Noise
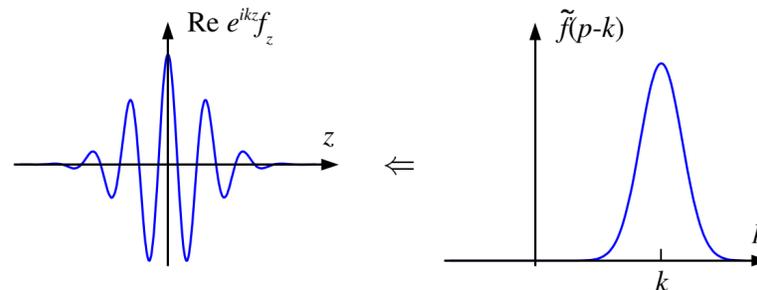
Conventional Smearing

position space

Exponential suppressed
overlap with boosted hadron
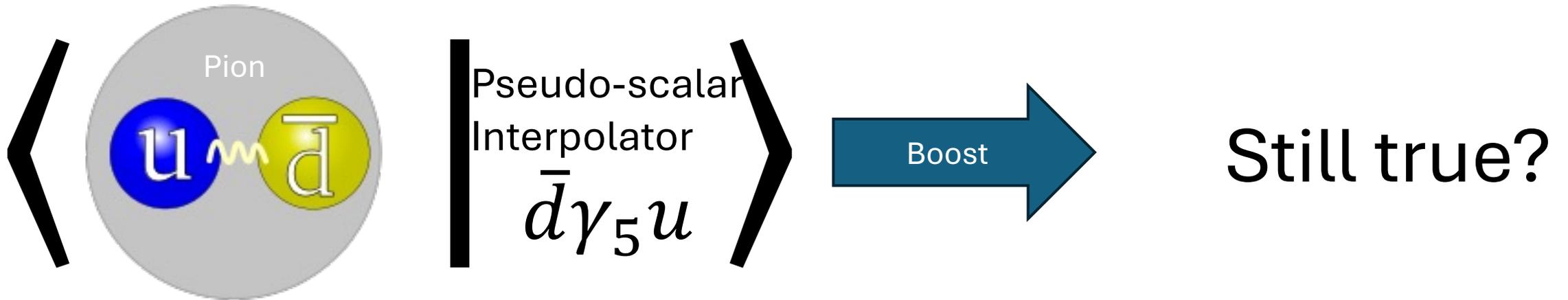
$O(1)$

Momentum Smearing

$O(1)$

Exponential suppressed
overlap with static pion

Signal-to-Noise ratio (Precision) is exponentially enhanced!

A New Perspective: spin structure

Largest overlap on lattice

$$\left\langle \; \text{Pion} \; \left| \begin{array}{c} \text{Pseudo-scalar} \\ \text{Interpolator} \\ \bar{d}\gamma_5 u \end{array} \right. \right\rangle$$

Boost →

Still true?

No!

Near the lightcone, the plus-component of spinors are most important!

Spinors on the lightcone

$$\gamma_\pm \equiv (\gamma_t \pm i\gamma_z)/\sqrt{2}, \qquad \psi_\pm = \gamma_\mp \gamma_\pm \psi/\sqrt{2}$$

- $\psi = \begin{pmatrix} \uparrow_L \\ \downarrow_L \\ \uparrow_R \\ \downarrow_R \end{pmatrix} = \psi_+ + \psi_- = \begin{pmatrix} 0 \\ \downarrow_L \\ \uparrow_R \\ 0 \end{pmatrix} + \begin{pmatrix} \uparrow_L \\ 0 \\ 0 \\ \downarrow_R \end{pmatrix}$ in chiral basis
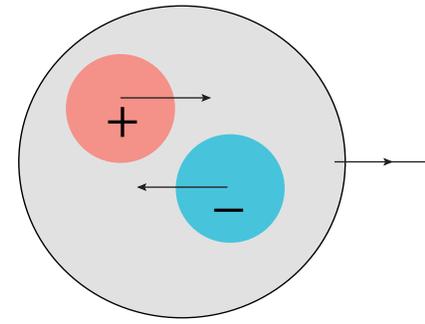
Traditional

Leading-twist

- In moving frame, $\psi_+ \propto \sqrt{E + P_z}$, $\psi_- \propto \sqrt{E - P_z}$

- Constructing the interpolator with $\psi_+^\dagger$ and ψ_+ provides the largest amplitude
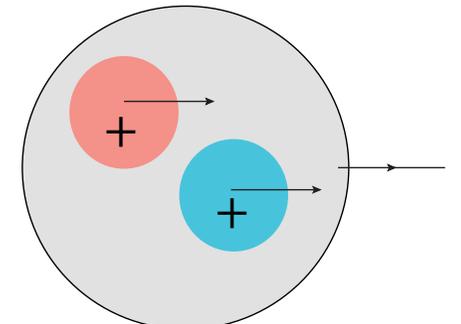
  Ji, Ma, Yuan, EPJC(2004)

- Traditional interpolator: $\bar{\psi}\gamma_5\psi = \dfrac{\left(\psi_+^\dagger \gamma_t \gamma_5 \psi_- + \psi_-^\dagger \gamma_t \gamma_5 \psi_+\right)}{2} \propto M$

- Pion leading-twist wave function: $\psi_+^\dagger \gamma_5 \psi_+ = \sqrt{2}\bar{\psi}\gamma_+\gamma_5\psi \propto E + P_z$
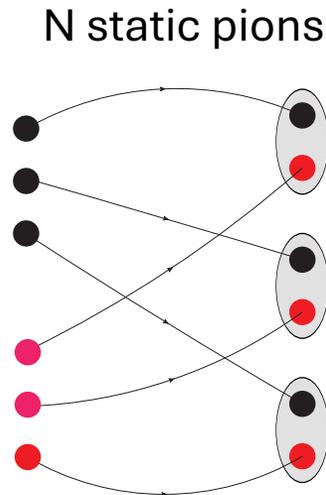
Lepage, Brodsky, PLB(1979)
Efremov, Radyushkin, PLB(1980)

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Noise

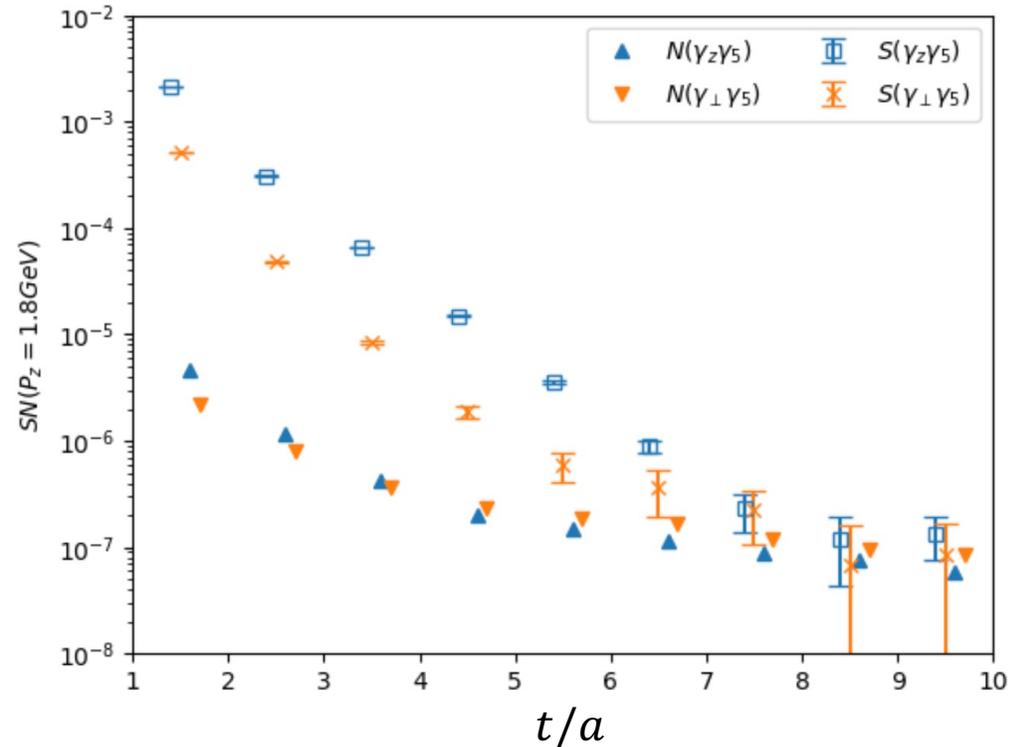- What about noise? $\quad \text{Var}(C_{2\text{pt}}) = \left\langle \Re[C_{2\text{pt}}]^2 \right\rangle - \left\langle \Re[C_{2\text{pt}}] \right\rangle^2 = \frac{1}{2} \left\langle C_{2\text{pt}}^{\dagger} C_{2\text{pt}} \right\rangle + \cdots$

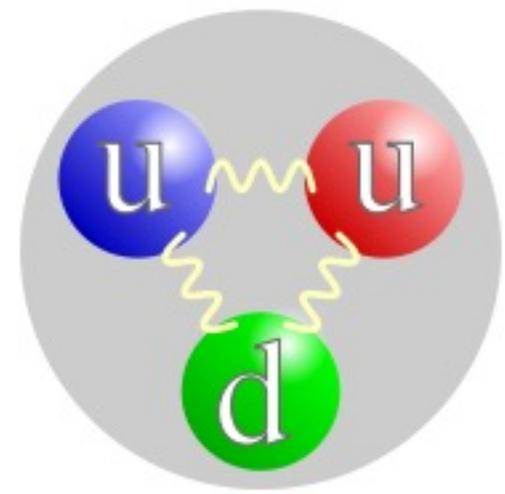- The asymptotic contribution is from static multi-pion states
 - No kinematic enhancement in noise

N static pions

The SNR indeed gets enhanced!

Interpolators for baryons

- Traditional interpolator: $\epsilon_{abc}(d_a^T C \gamma_5 u_b)u_c$

- Constructing the interpolator with only plus components

- Diquark: $\epsilon_{abc}d_a^T C \gamma_5 u_b \rightarrow \epsilon_{abc}d_a^T C \gamma_5 \gamma_\mu u_b$ and $\epsilon_{abc}d_a^T C \gamma_\mu u_b$

- Free quark: $u_c \rightarrow \gamma_\mu u_c$ (already been used in the parity projection $1 \pm \gamma_t$)

- $\epsilon_{abc}(d_a^T C \gamma_5 u_b)u_c$ provides largest component at rest, but when boosted, the following two are largest:
  - $\epsilon_{abc}(d_a^T C \gamma_5 \gamma_\mu u_b)\gamma_t u_c$
  - $\epsilon_{abc}(d_a^T C \gamma_\mu u_b)\gamma_t u_c$

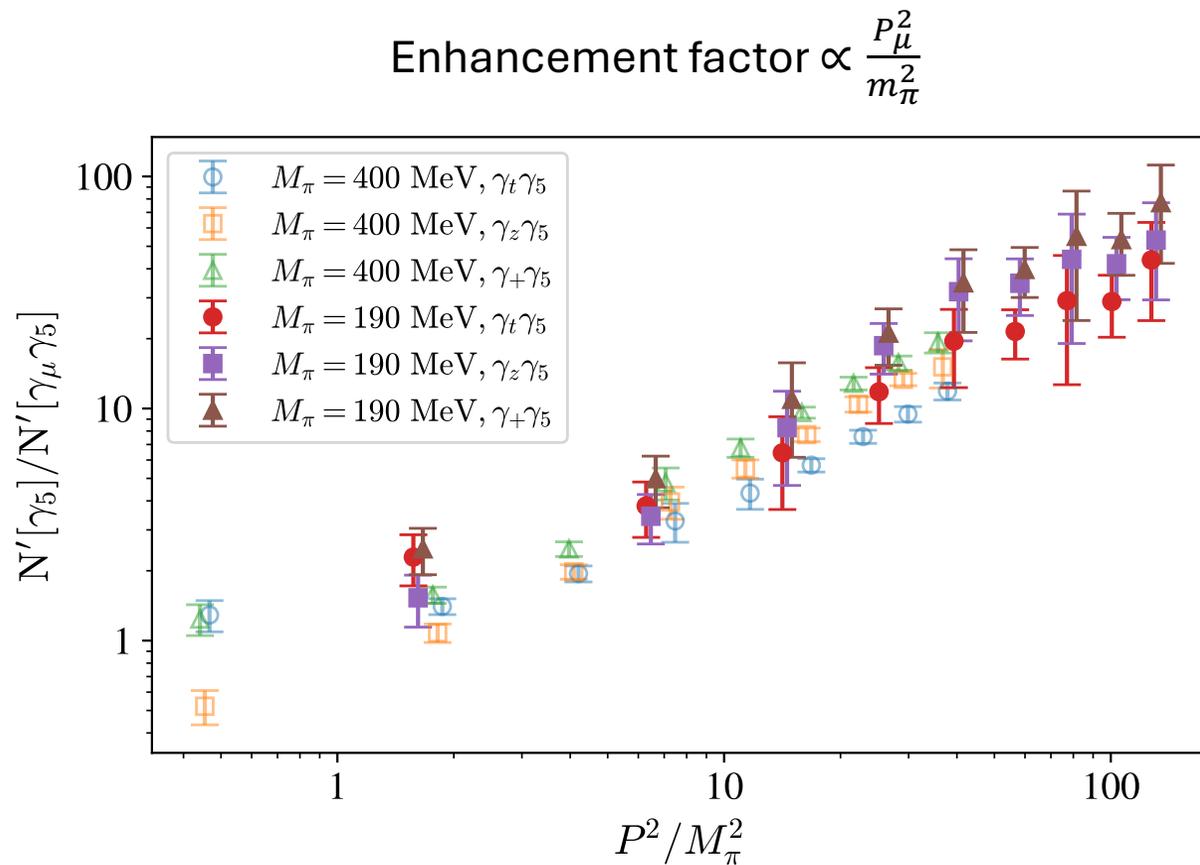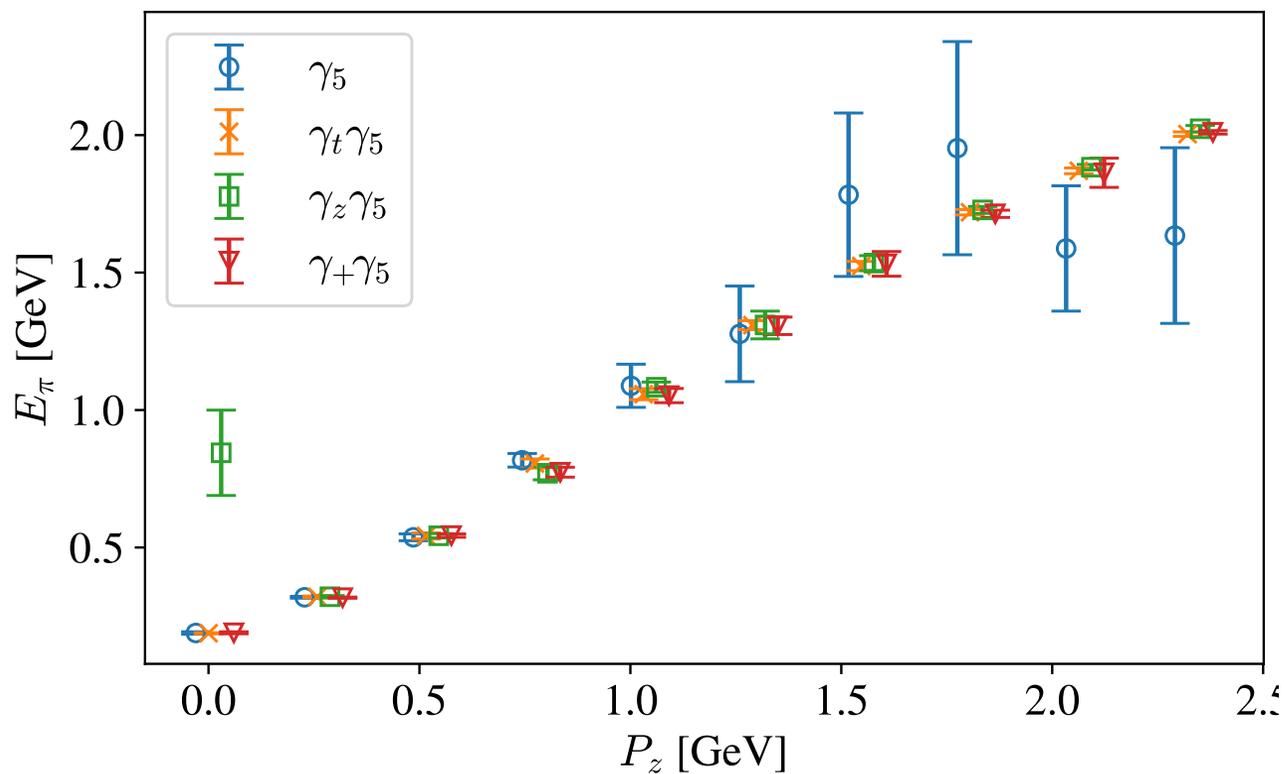# Numerical Tests

- Lattice Spacing: $a = 0.15$ fm
- Volume: $L^3 \times T = 32^3 \times 48$
- Action: clover-on-HISQ
- Pion Masses:
  - $m_\pi^{sea} = 135$ MeV
  - $m_\pi^{val} = 190$ MeV
- Smearing:
  - 2 Steps HYP
  - $k = 1.55$ GeV
- Momentum: $P = [0,2.3] GeV$
- Stat: 334 cfgs
- Sources: 64 on each cfg
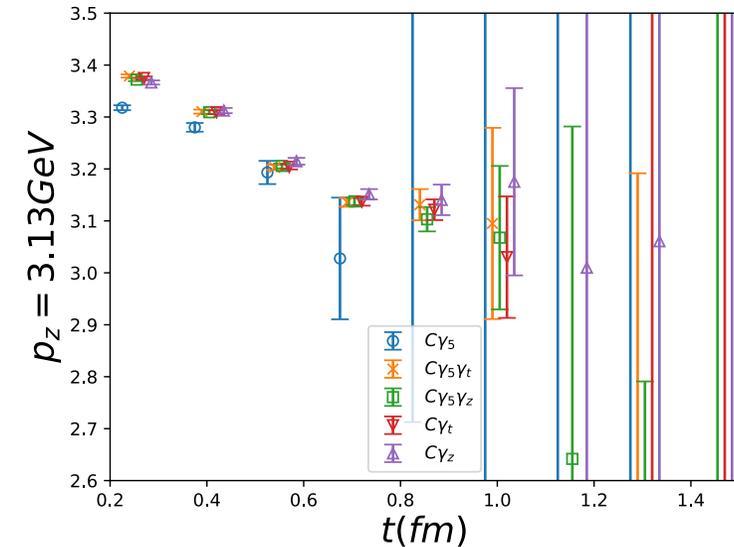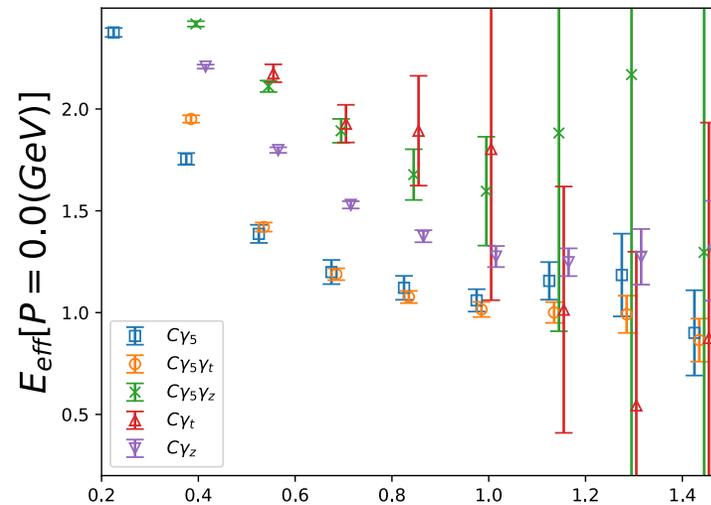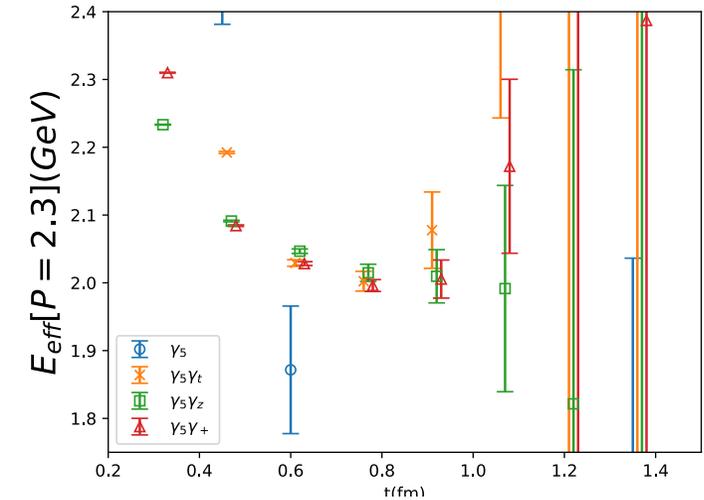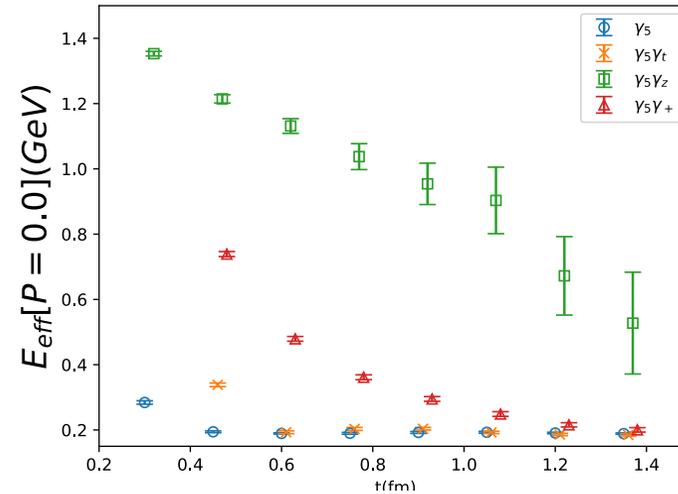
Quantitative analysis of the enhancement

- Extract the energy spectrum:

Enhancement factor $\propto \dfrac{P_\mu^2}{m_\pi^2}$

Larger momentum = Larger improvement

Better improvement for lighter pion masses

Suppress pion excited states at smaller P_z

- $\bar{q}\gamma_z\gamma_5 q$ at small P_z: smaller contamination from spin-0 states, but larger contamination from spin-1 states

- The same spin-1 states exist in $\bar{q}\gamma_\perp\gamma_5 q$ with smaller strength

- $\langle\bar{q}\gamma_z\gamma_5 q|\bar{q}\gamma_z\gamma_5 q\rangle - \langle\bar{q}\gamma_\perp\gamma_5 q|\bar{q}\gamma_\perp\gamma_5 q\rangle$ suppresses the spin-1 states

Before subtraction:
Large contamination

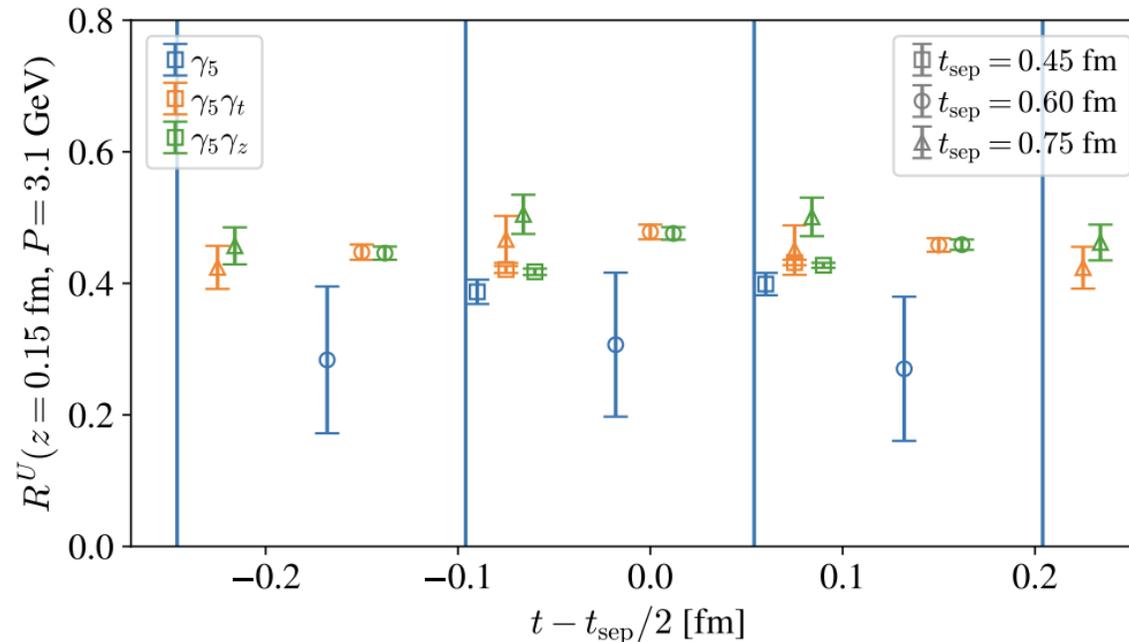After subtraction:
Small contamination
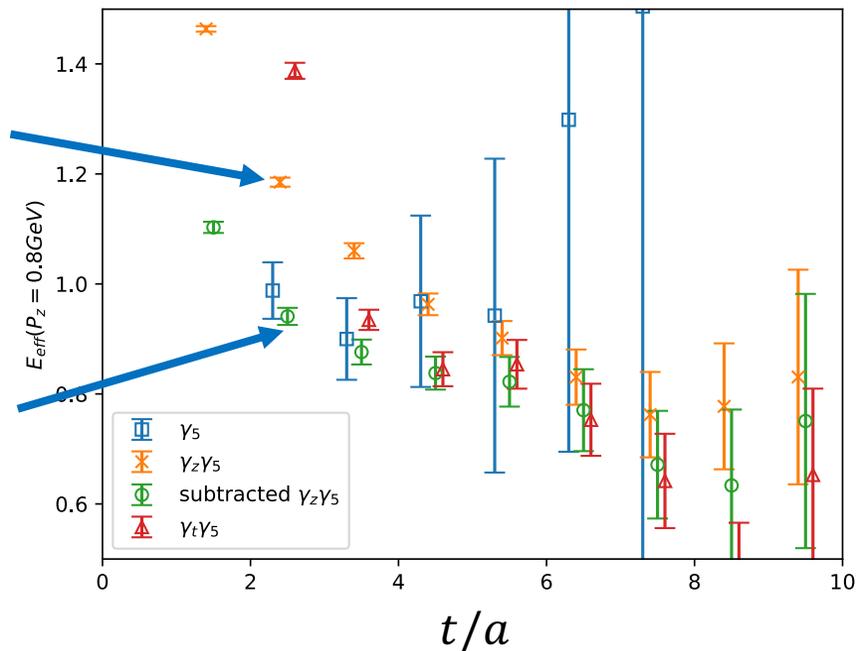
Same improvement in 3pt correlators

Extraction of matrix elements:

$$C_2(t) = \sum |c_n|^2 e^{-E_n t}, \qquad C_3(t, t_{sep}) = \sum c_m^* c_n \langle m|O|n \rangle e^{-E_m(t_{sep}-t)} e^{-E_n t}$$

$$\langle 0|O|0 \rangle \approx R(t, t_{sep}) \equiv \frac{C_3(t, t_{sep})}{C_2(t_{sep})}$$

- SNR is enhanced by a factor of 10 at $P_z > 3 GeV$ for $t_{sep} \geq 0.6$ fm

One more exciting fact:

All-plus-component interpolators have the best signal:

Only 6 components of the fermion at source/sink are necessary:
- Half Inversions
- Half Smearings
- Less-Than-Half Contractions

Reduce your total cost by at least 50% with even better precision

Outlook

- The new interpolators open a new door to calculations of hadron structure at very large momentum.

- Improve the precision of calculating parton physics

- Enable us to access form factors at very large Q^2

- Improve the the calculation of $\pi - \pi$ scattering with large invariant mass by a factor of P_z^4/m_π^4

- Improve the study of heavy meson decay to pion

- More studies on excited state contaminations are needed especially for nucleons