# Probing hadron structure with TMD observables

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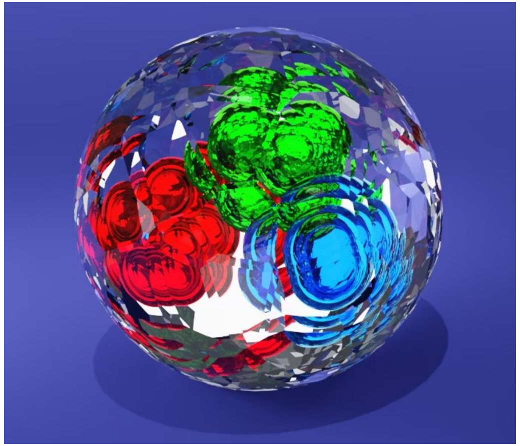
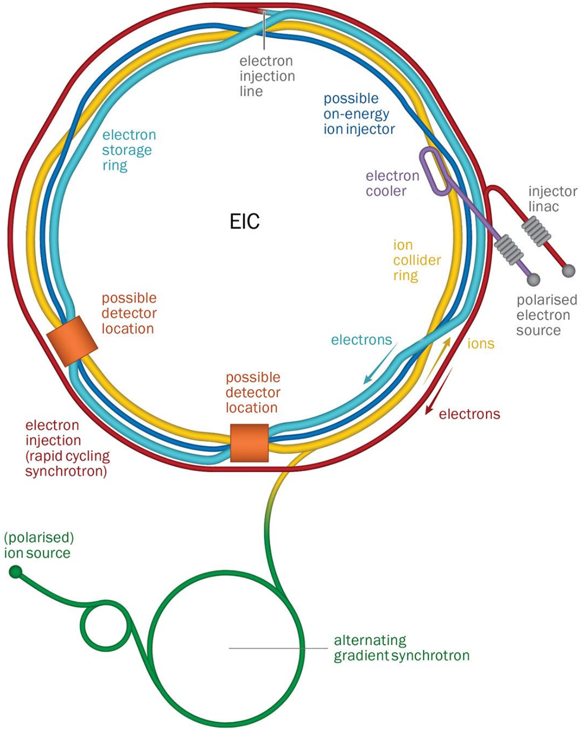
CFNS Seminar, July 2025

# Hadron structure & formation

- Confinement: extraction of information is indirect
- High energy collision: send in a probe and measure the outcome

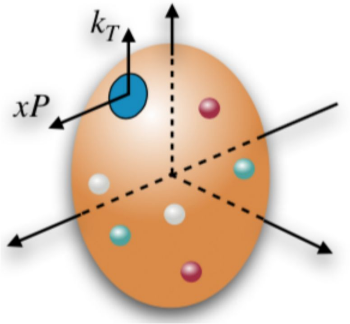




# 3D structure in momentum space

TMD (transverse momentum dependent) distributions:

- longitudinal momentum fraction
- transverse momentum $k_T$




**TMD Handbook**

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration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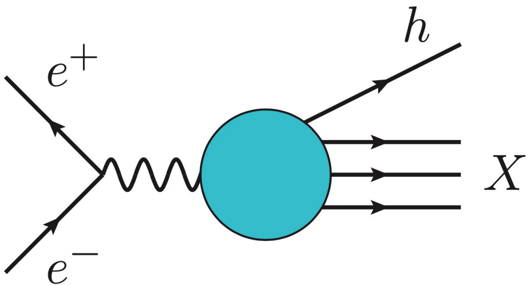
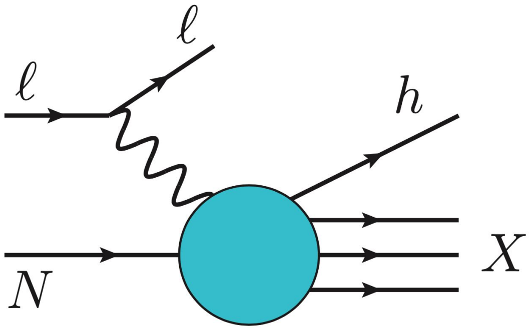
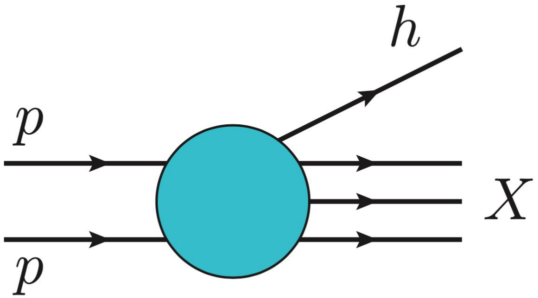
# Processes to extract TMDs

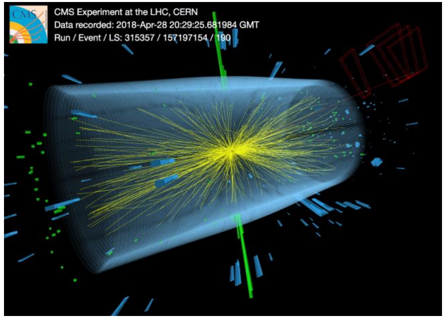
- Standard processes: $e^+e^-$, SIDIS, $pp$ collisions

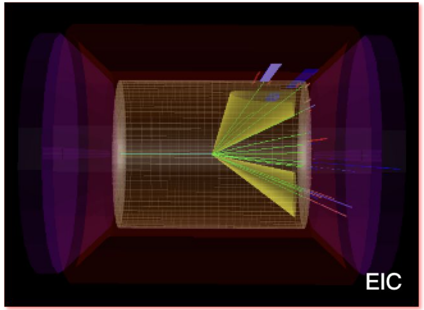


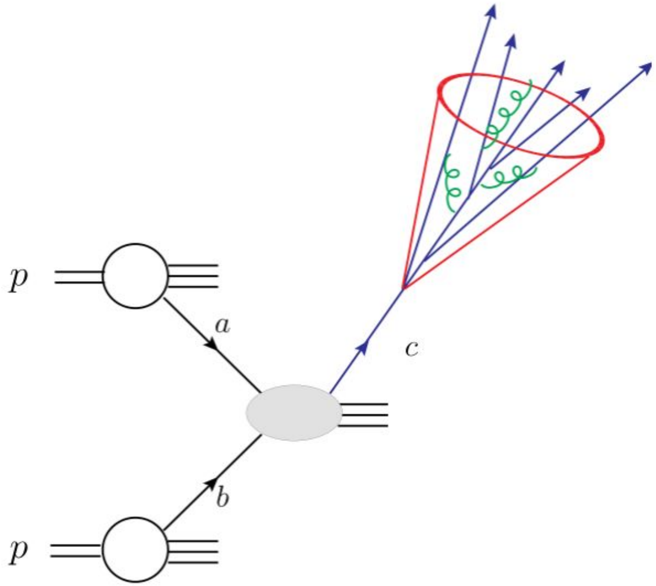

- Using jets for 3D imaging



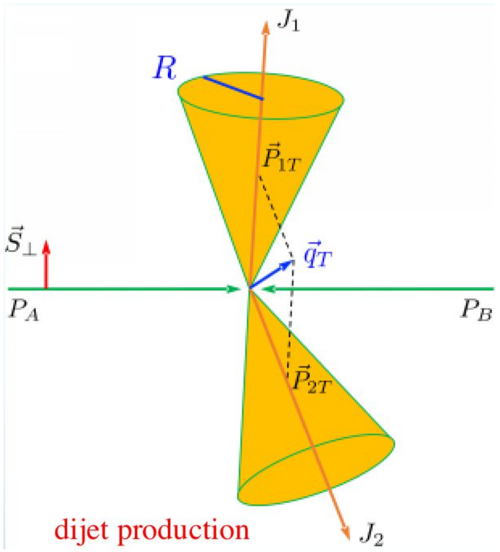
# Two types of jet production

**Single inclusive**: only care about a single jet




**Exclusive**: a fixed number of final state jets

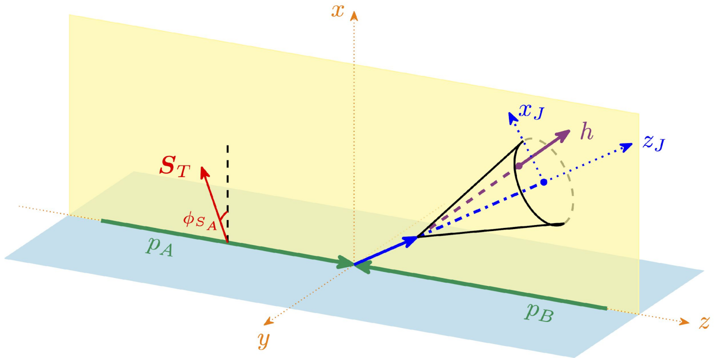
(back-to-back dijet/$Z$+jet)



# Single inclusive jet production

- Collinear PDFs: only one scale $p_T$ is measured.
- TMD FFs: when hadron transverse momentum distribution is measured.

Kang, Ringer & Vitev: 16; Dai, Kim & Leibovich: 16; Kaufmann, Mukherjee & Vogelsang: 15




$$\frac{d\sigma^{pp\to \mathrm{jet}(h)+X}}{dp_T\, d\eta\, dz_h} \propto f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{D}_1^{h/c}(z, z_h, p_T R, \mu),$$

$$\frac{d\sigma^{pp\to \mathrm{jet}(h)+X}}{dp_T\, d\eta\, dz_h\, d^2\boldsymbol{j}_\perp} \propto f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{G}_1^{h/c}(z, z_h, \boldsymbol{j}_\perp, p_T R, \mu, \zeta_J),$$

Collinear and TMD fragmenting jet functions (FJFs)

- $z_h$: large momentum fraction of hadron v.s. jet
- $\boldsymbol{j}_\perp$: hadron transverse momentum w.r.t jet axis

# Relation between **TMD** FFs and **TMD** FJFs

## If you measure both $z_h$ and $\boldsymbol{j}_\perp$

**Leading Quark TMDFFs**

Hadron Spin · Quark Spin

| | | Quark Polarization | | |
|---|---|---|---|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Unpolarized (or Spin 0) Hadrons | | $D_1$ = Unpolarized | | $H_1^\perp$ = Collins |
| Polarized Hadrons | L | | $G_1$ = Helicity | $H_{1L}^\perp$ = |
| Polarized Hadrons | T | $D_{1T}^\perp$ = Polarizing FF | $G_{1T}^\perp$ = | $H_1$ = Transversity<br>$H_{1T}^\perp$ = |

TMD handbook: [2304.03302](2304.03302)

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Hadron polarization | U | $\mathcal{D}_1$ = | | $\mathcal{H}_1^\perp$ = |
| Hadron polarization | L | | $\mathcal{G}_{1L}$ = | $\mathcal{H}_{1L}^\perp$ = |
| Hadron polarization | T | $\mathcal{D}_{1T}^\perp$ = | $\mathcal{G}_{1T}$ = | $\mathcal{H}_1$ =<br>$\mathcal{H}_{1T}^\perp$ = |

Kang, Xing, Zhao and Zhou: [2311.00672](2311.00672)

How do we connect them?

# TMD FJFs

If you measure both $z_h$ and $\boldsymbol{j}_\perp$

$$\mathcal{D}_1(z, z_h, \boldsymbol{j}_\perp) = \hat{H}^U_{c\to i}(z) \int \frac{b\,\mathrm{d}b}{(2\pi)^2} J_0\left(\frac{j_\perp b}{z_h}\right) \widetilde{D}_1^{h/i}(z_h, \boldsymbol{b})$$

Hard splitting & jet formation

Hadron fragmentation & soft correction.

Standard TMD FFs

ALL coefficients have been computed in our work!

Kang, Xing, Zhao and Zhou: [2311.00672](2311.00672)

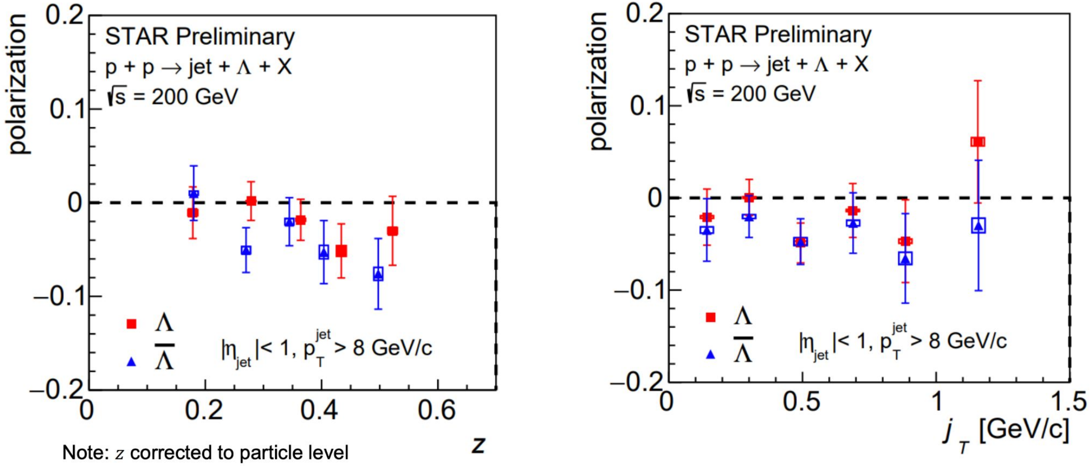
# $\Lambda$-baryon polarization

STAR, 2023

- Large transverse polarization found for $\Lambda$ produced in unpolarized hadron scattering

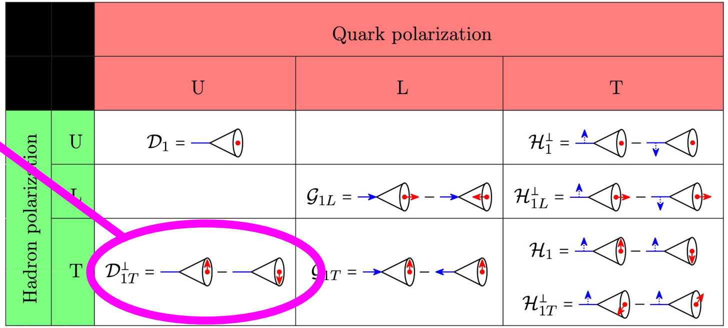
- STAR recent measurement: test of universality of $\Lambda$ polarized FFs

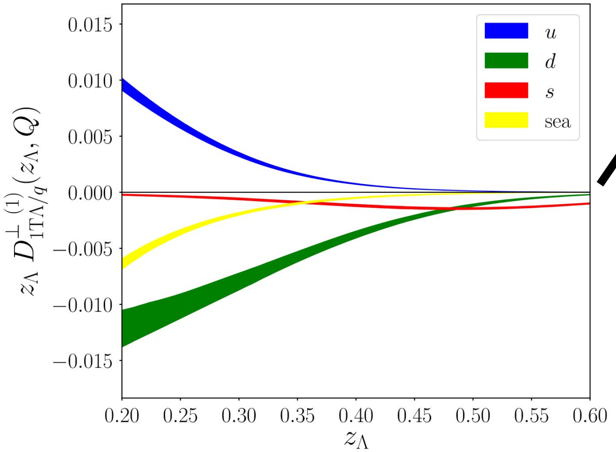
Polarization as a function of $z$ and $j_T$



# $\Lambda$-baryon polarization

$$\mathcal{D}_{1T}^{\perp}(z, z_h, \boldsymbol{j}_\perp) = \hat{H}^{U}_{c\to i}(z) \int \frac{b^2\,\mathrm{d}b}{(2\pi)^2} \frac{z_\Lambda^2 m_\Lambda^2}{j_\perp} J_1\left(\frac{j_\perp b}{z_\Lambda}\right) \widetilde{D}_1^{\perp,(1)}(z_h, \boldsymbol{b})$$



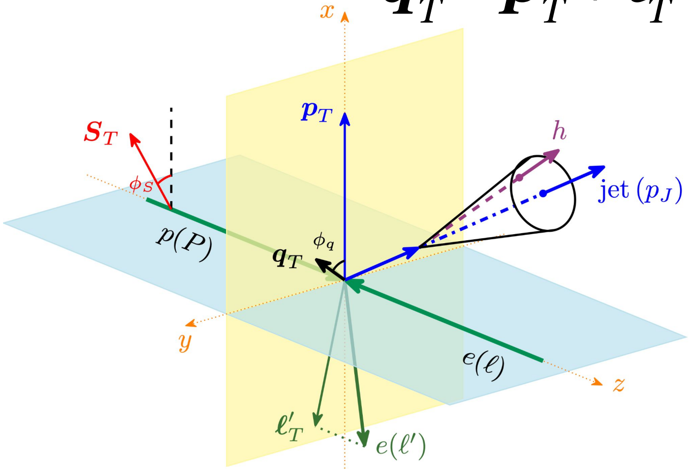

currently in preparation

Extracted from BELLE data 2003.04828

# Exclusive jet production

- Momentum imbalance $\boldsymbol{q}_T$: sensitive to initial-state TMD distributions
- Hadron $\boldsymbol{j}_\perp$: sensitive to TMD FFs

$\boldsymbol{q}_T = \boldsymbol{p}_T + \boldsymbol{l}_T$



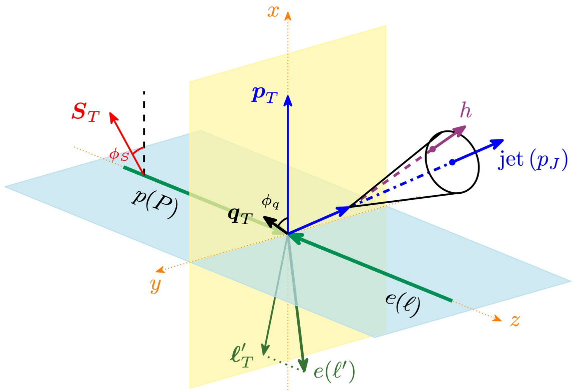

$$\frac{d\sigma_{pp}}{d\mathcal{PS}} = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}} \tilde{f}_a^{q/p}(x_a, b)\tilde{f}_b^{q/p}(x_b, b)$$
$$\times \widetilde{S}_{n\bar{n}n_J}(\boldsymbol{b})\widetilde{S}^{cs}_{n_J}(\boldsymbol{b}, R) H_{ab\to cZ}(p_T, m_Z) J_c(p_{JT}R)$$

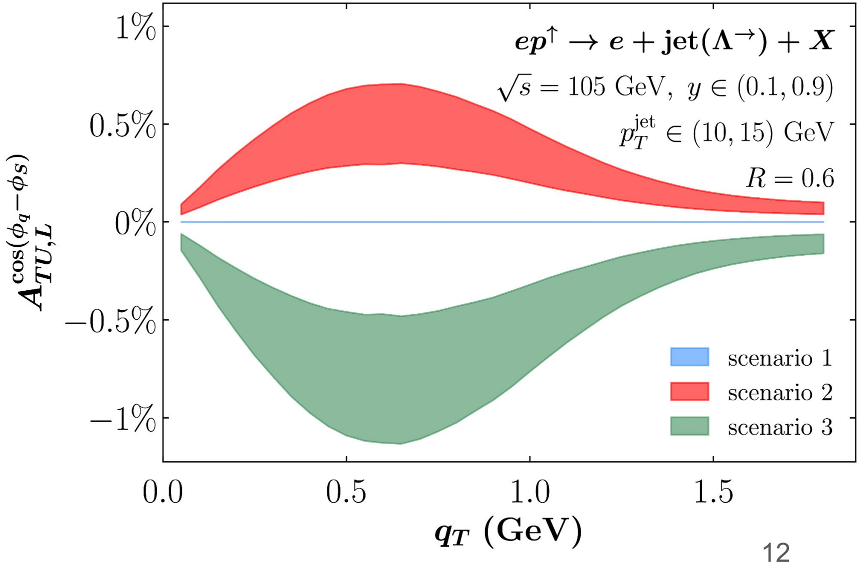
Kang, Lee, Shao & Zhao: 2106.15624

# Exclusive jet production: $ep \to e + \text{jet}(h)$

- worm-gear function
- longitudinally polarized FFs

$$A_{TU,L}(z_h, j_\perp) \equiv \frac{\hat{\sigma}_0 H \mathscr{G}_{1L} \widetilde{g}_{1T} \overline{S}_{\text{global}} \overline{S}_{\text{cs}}}{\hat{\sigma}_0 H \mathscr{D}_1 \widetilde{f}_1 \overline{S}_{\text{global}} \overline{S}_{\text{cs}}}$$

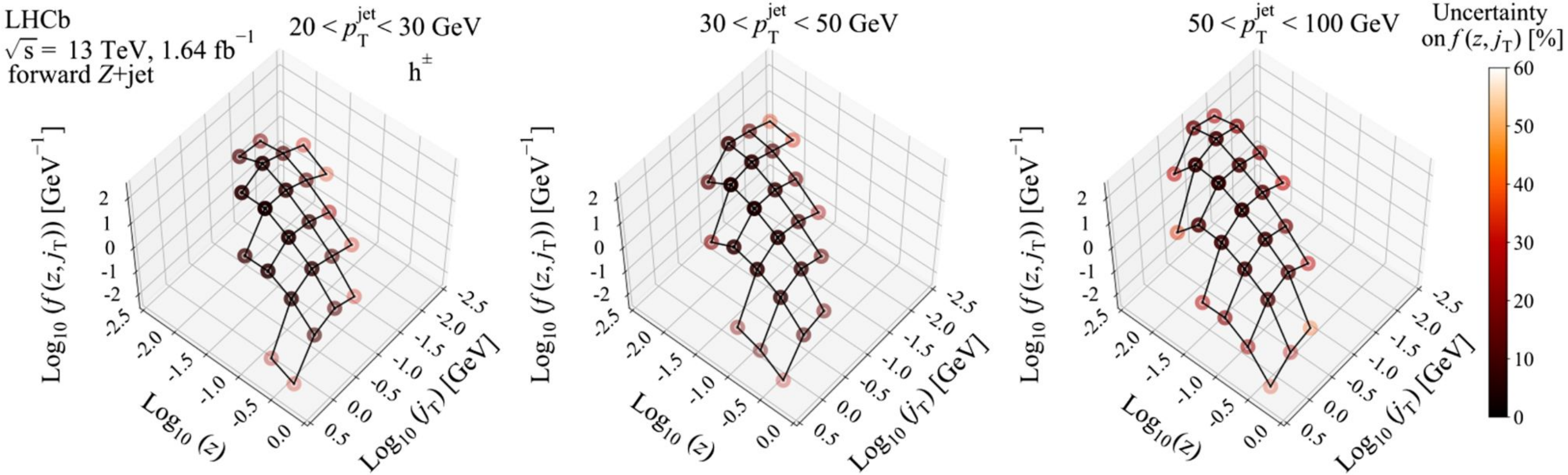



Kang, Xing, Zhao and Zhou: 2311.00672

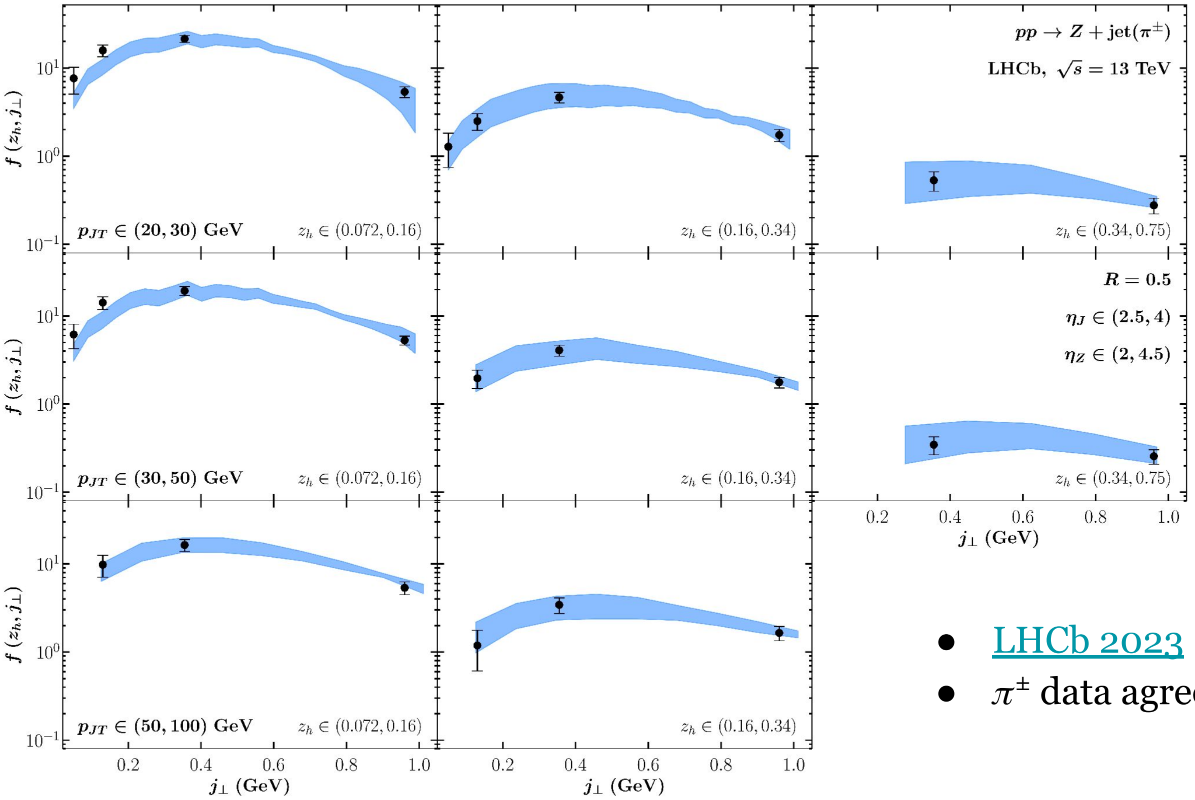
# Exclusive jet production: $pp \to Z + \text{jet}(h)$

- Recent measurement by LHCb (2208.11691)
- First time differential in both $z_h$ and $\boldsymbol{j}_\perp$ (proposed in 1906.07187)



# Exclusive jet production: $pp \to Z + \text{jet}(\pi^{\pm})$

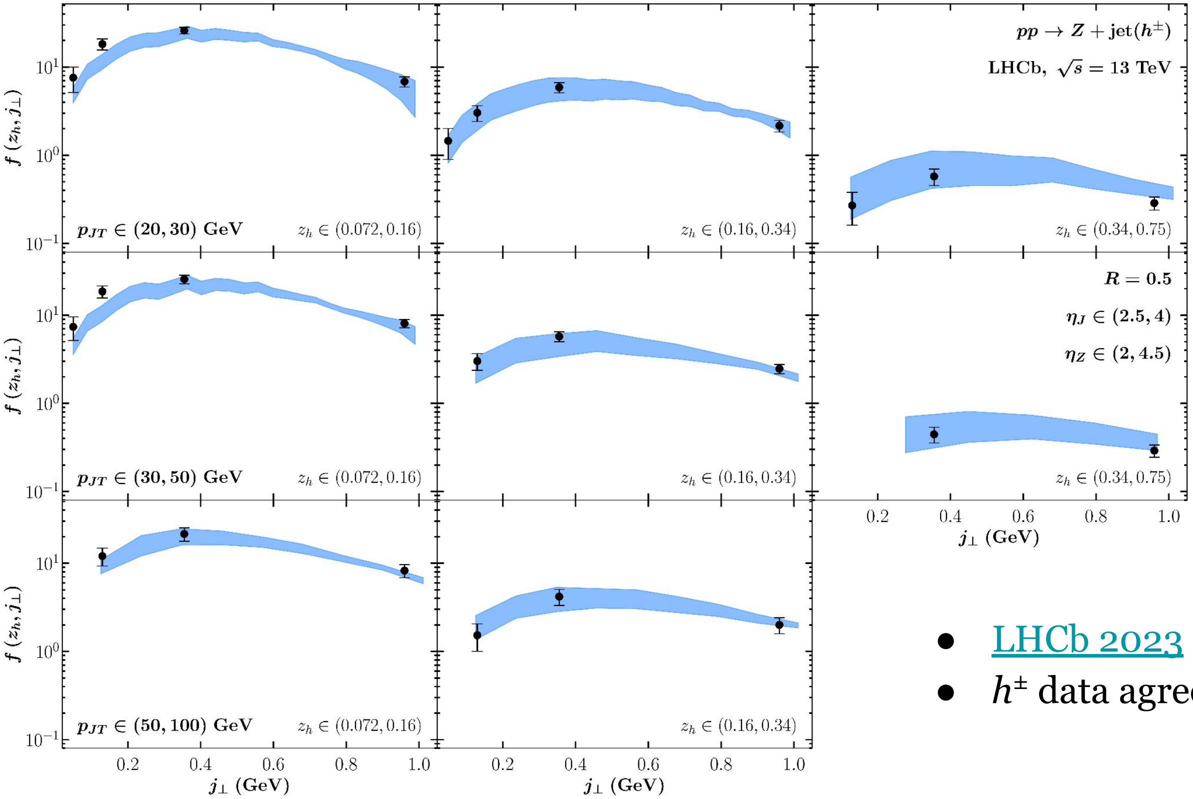
$\pi^{\pm}$




- LHCb 2023
- $\pi^{\pm}$ data agree very well ✓

# Exclusive jet production: $pp \to Z + \text{jet}(h^{\pm})$

$h^{\pm}$




- LHCb 2023
- $h^{\pm}$ data agree very well ✓

# Exclusive jet production: $p\text{Pb} \to Z + \text{jet}(h)$

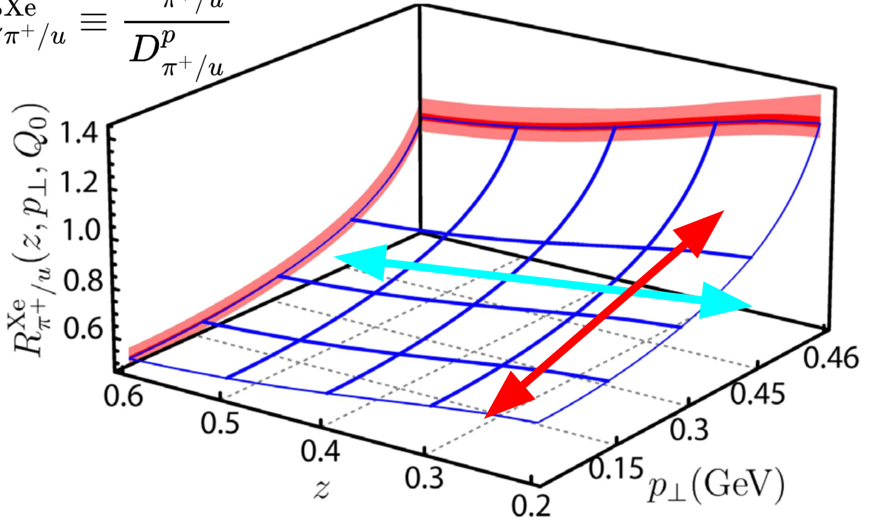
- Nuclear TMD modification extracted in [Alrashed, Anderle, Kang, Terry and Xing, 2107.12401]

- Fitted for TMD PDFs & TMD FFs

$$S_{\mathrm{NP}}^{q/A,f}(b, Q_0, \sqrt{\zeta_a}) = \frac{g_2}{2} \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{\sqrt{\zeta_a}}{Q_0}\right) + g_1^{q/A} b^2 ,$$

$$g_1^{q/A} = g_1 + a_N L, \quad L = A^{1/3} - 1$$

$$R_{\pi^+/u}^{\mathrm{Xe}} \equiv \frac{D_{\pi^+/u}^{\mathrm{Xe}}}{D_{\pi^+/u}^{p}}$$




- **Broadening of transverse momentum distribution**
- **Behaviour driven by collinear FFs**

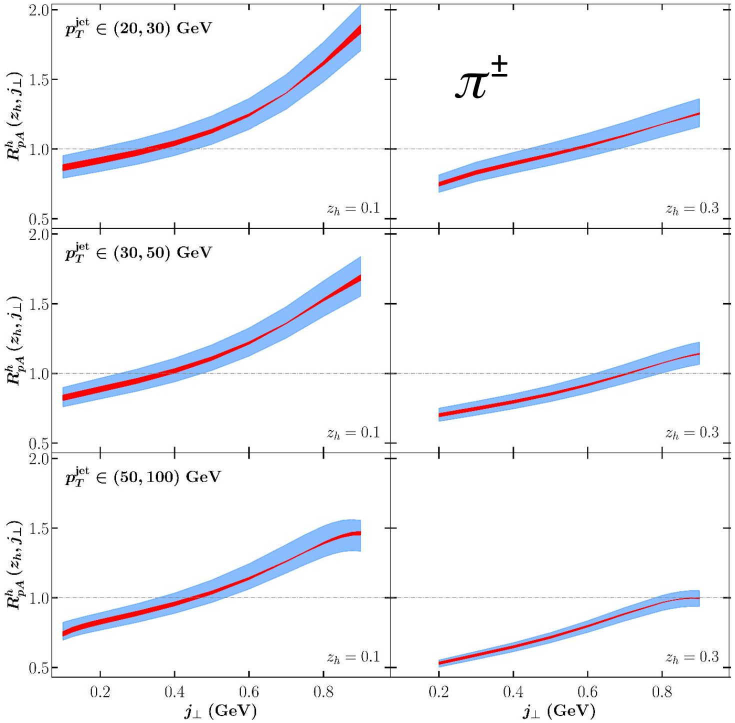
# Exclusive jet production: $p\text{Pb} \to Z + \text{jet}(h)$

- Nuclear TMD modification extracted in [Alrashed, Anderle, Kang, Terry and Xing, 2107.12401]

- The reaction is $p\text{Pb} \to Z + \text{jet}(\pi^{\pm})$

- LHC is interested in the observable and is planning to measure it

$$S_{\text{NP}}^{q/A,f}(b, Q_0, \sqrt{\zeta_a}) = \frac{g_2}{2} \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{\sqrt{\zeta_a}}{Q_0}\right) + g_1^{q/A} b^2 \,,$$

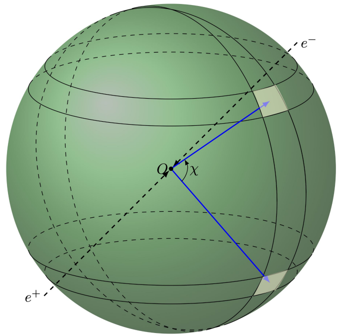


# Transverse energy-energy correlator (TEEC)

# Energy-energy correlator (EEC)

- Two-point energy correlator
- One of the first infrared safe event shape in QCD
  - Basham, Brown, Ellis & Love: 1978, 1979
- Probe jet substructure when $\chi \to 0$
- In $e^+e^-$ annihilation:

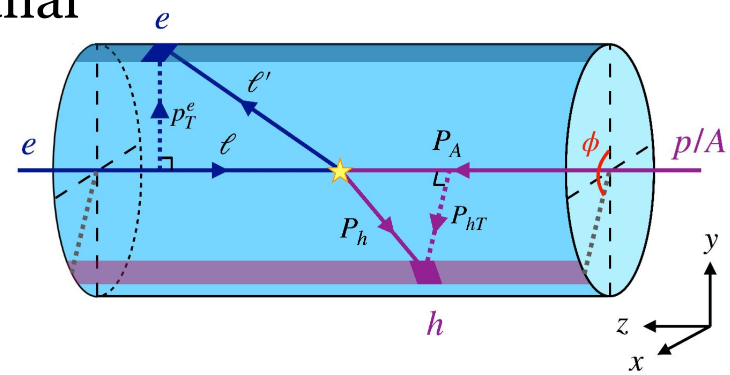
$$\frac{d\Sigma_{e^+e^-}}{d\cos\chi} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(\cos(\theta_{ij}) - \cos(\chi)\right)$$




Moult and Zhu, 1801.02627

# Transverse EEC (TEEC) at EIC

- Two-point energy correlator in azimuthal plane
- Generalization of energy-energy correlator
  - Ali, Pietarinen & Stirling (1984)
- In $ep \to e + h$:

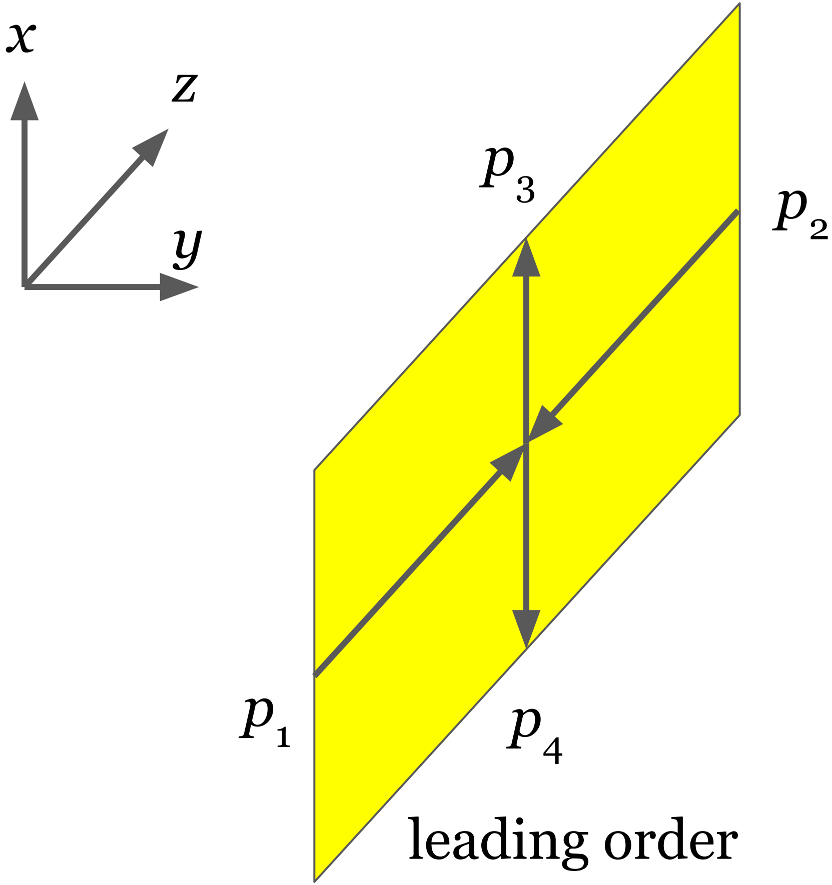



$$\mathbf{TEEC} = \sum_h \int d\sigma_{\mathrm{DIS}} \frac{E_{T,l} E_{T,h}}{E_{T,l} \sum_i E_{T,i}} \delta\left(\tau - \frac{1+\cos\phi}{2}\right)$$

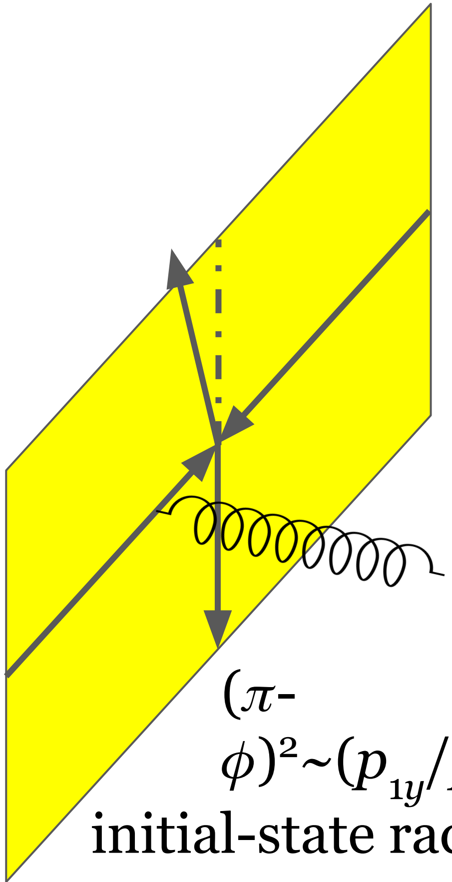
- sum over final state hadrons

- weighted cross section
- soft radiation does not contribute at LO

# TEEC: LO and NLO corrections



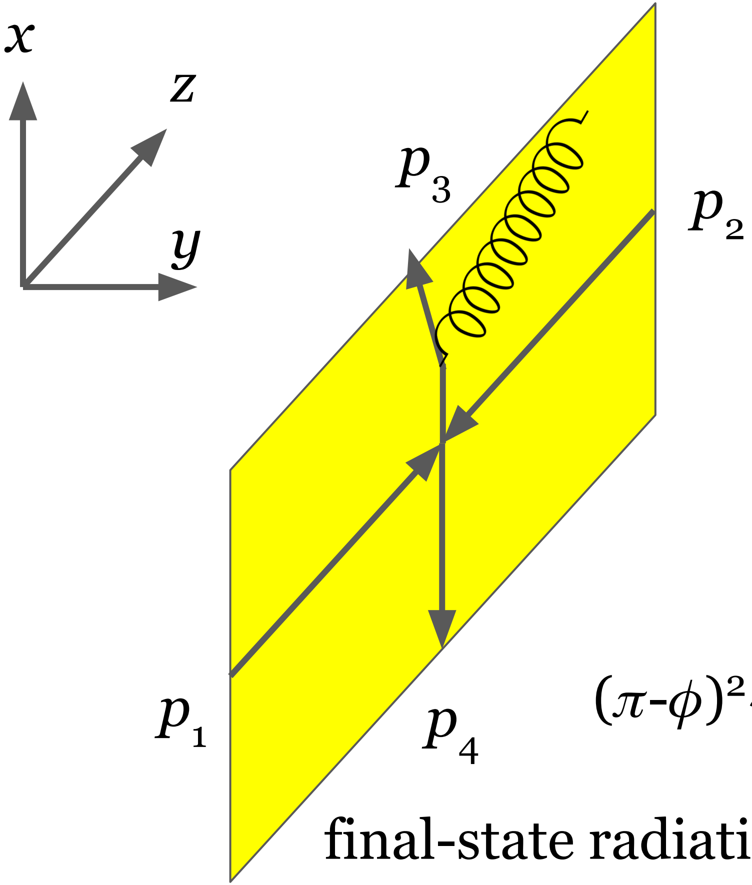

leading order



$(\pi-\phi)^2 \sim (p_{1y}/p_T)^2$

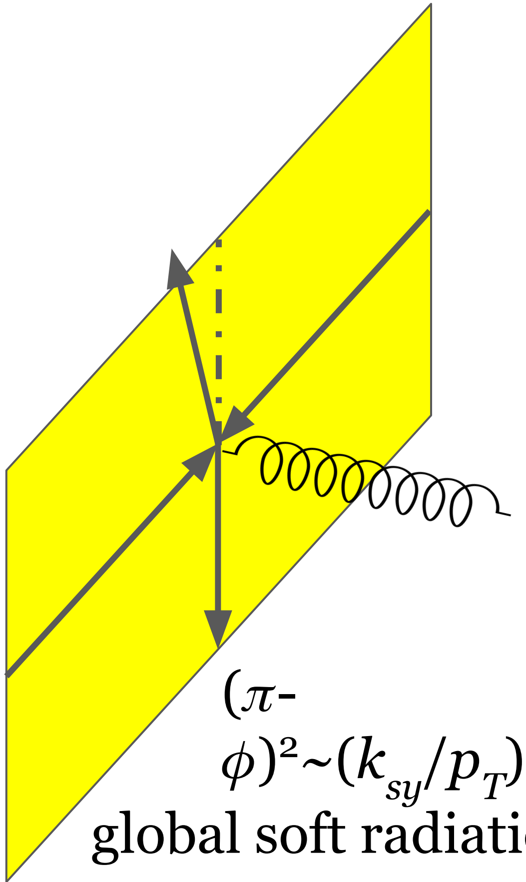
initial-state radiation

# TEEC: LO and NLO corrections




$(\pi\text{-}\phi)^2 \sim (p_{3y}/\xi_3 p_T)^2$

final-state radiation



$(\pi\text{-}\phi)^2 \sim (k_{sy}/p_T)^2$

global soft radiation

# TEEC: $ep \to e + h$

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos\left(2b\sqrt{\tau}p_T^e\right) f_q(x,b,\mu,\zeta) J_q\left(b,\mu,\hat{\zeta}\right)$$

- TMD PDFs

$$\tau \equiv \frac{1+\cos\phi}{2}$$

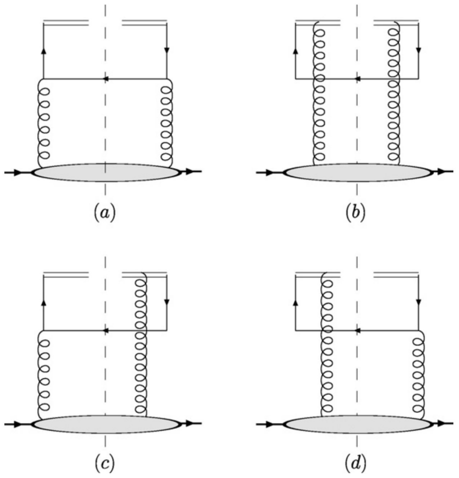
- TEEC jet function

$$J_q\left(b,\mu,\hat{\zeta}\right) \equiv \sum_h \int_0^1 \mathrm{d}z\, z \widetilde{D}_{1,h/q}\left(z,b,\mu,\hat{\zeta}\right)$$

# TEEC - $ep \to e + h$

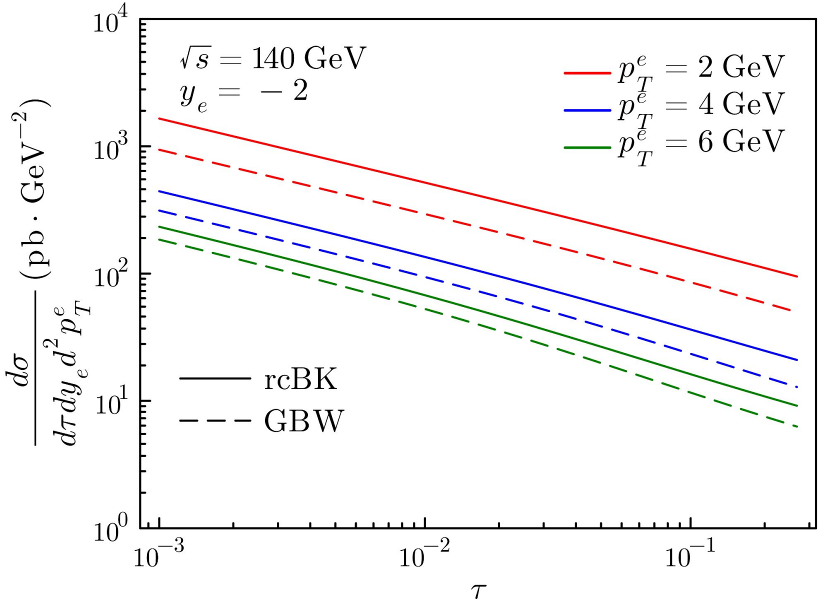
Marquet, Xiao & Yuan: [0906.1454](0906.1454)

- Quark TMD PDFs expanded in terms of small-$x$ gluon dipoles
- Allows us to probe small-$x$ gluon saturation physics
- We further combined TMD evolution + small-$x$ evolution (BK)




$$xf_q(x, b, \mu_{b_*}, \mu_{b_*}^2) = \frac{N_c S_\perp}{8\pi^4} \int \mathrm{d}\epsilon_f^2\, \mathrm{d}^2\boldsymbol{r}\, \frac{(\boldsymbol{b}+\boldsymbol{r})\cdot\boldsymbol{r}}{|\boldsymbol{b}+\boldsymbol{r}||\boldsymbol{r}|} \epsilon_f^2 K_1(\epsilon_f|\boldsymbol{b}+\boldsymbol{r}|) K_1(\epsilon_f|\boldsymbol{r}|) \times \left[1 + \mathcal{S}_x(|\boldsymbol{b}|) - \mathcal{S}_x(|\boldsymbol{b}+\boldsymbol{r}|) - \mathcal{S}_x(|\boldsymbol{r}|)\right]$$

# Probe small $x$ PDFs with TEEC



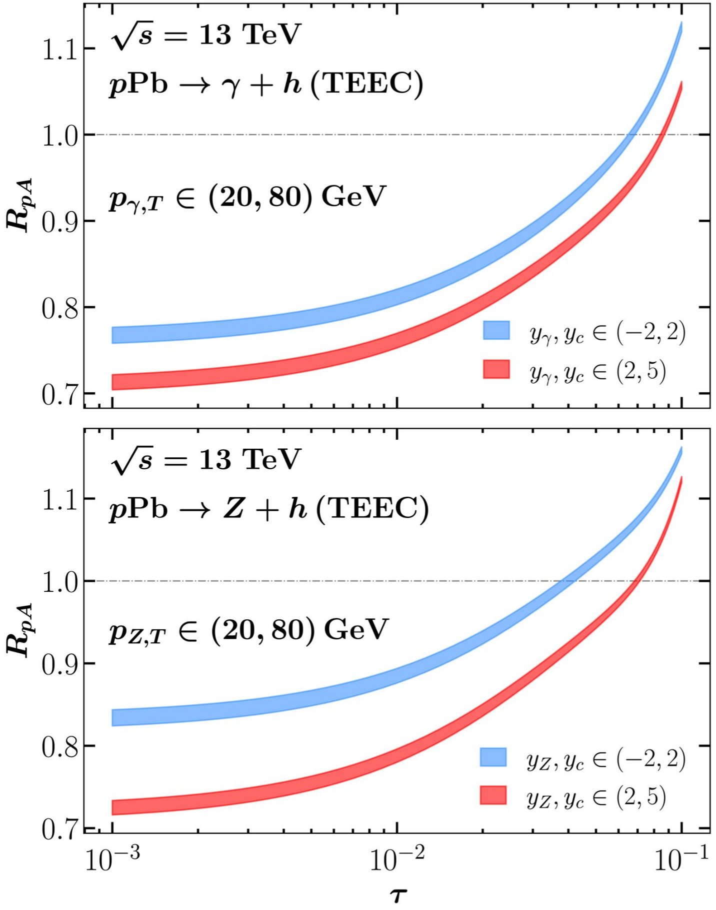

- Can both describe inclusive DIS
- Significant difference in TEEC

Kang, Penttala, Zhao & Zhou: [2311.17142]

# TEEC: $p\mathrm{Pb} \rightarrow Z + h$

$$R_{pA} \equiv \frac{1}{A} \frac{\mathrm{d}\Sigma^{pA}}{\mathrm{d}\tau\, \mathrm{d}y_V\, \mathrm{d}p_{V,T}} \Bigg/ \frac{\mathrm{d}\Sigma^{pp}}{\mathrm{d}\tau\, \mathrm{d}y_V\, \mathrm{d}p_{V,T}}$$

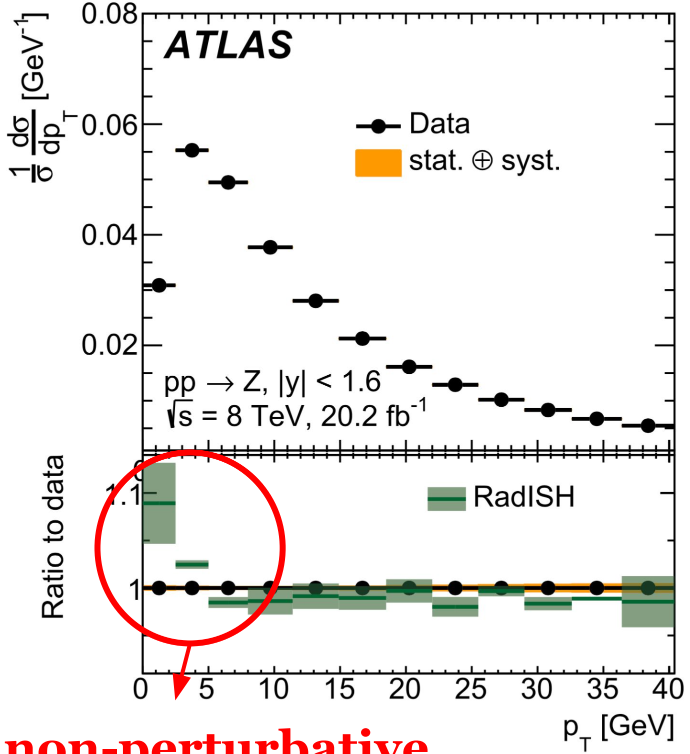
- Nuclear broadening effect of transverse momentum
- Different rapidity bins probe different kinematic regions




Kang, Lee, Penttala, Zhao & Zhou: 2410.02747

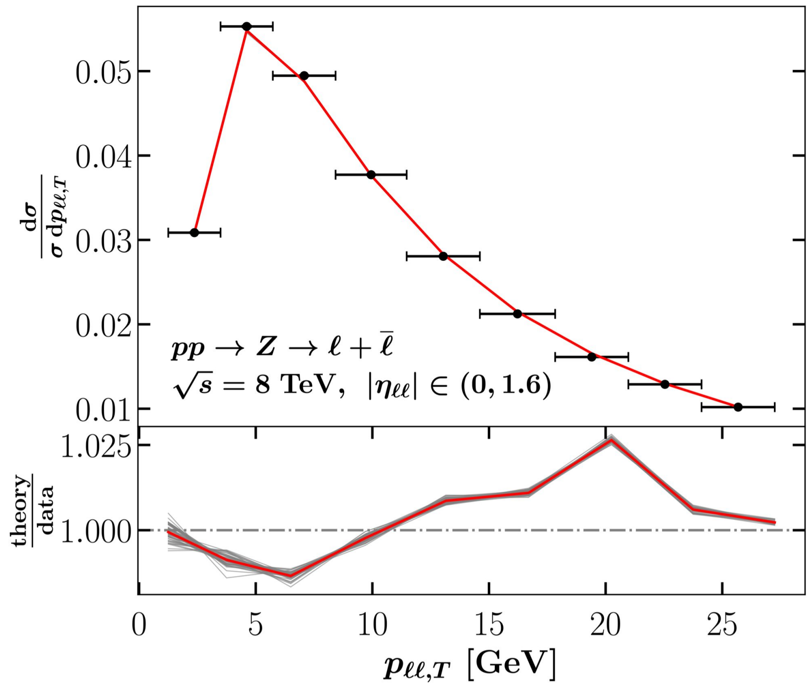
# Currently working on...

ATLAS: 2309.09318



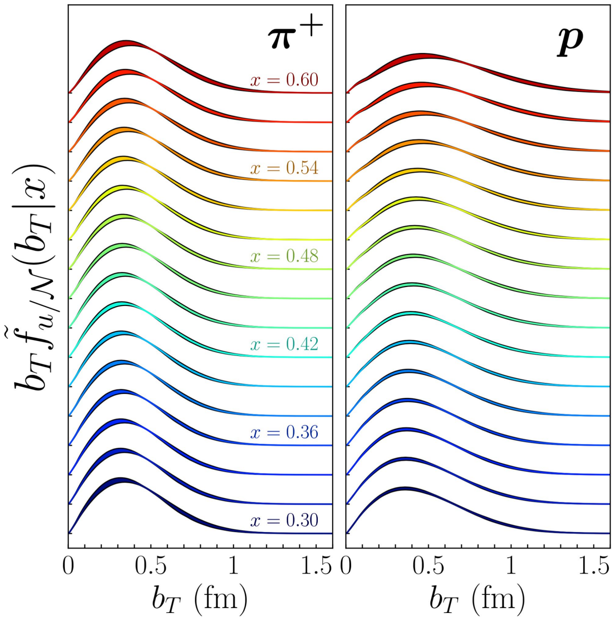

**No non-perturbative modelling**

**Implement NP kernel**



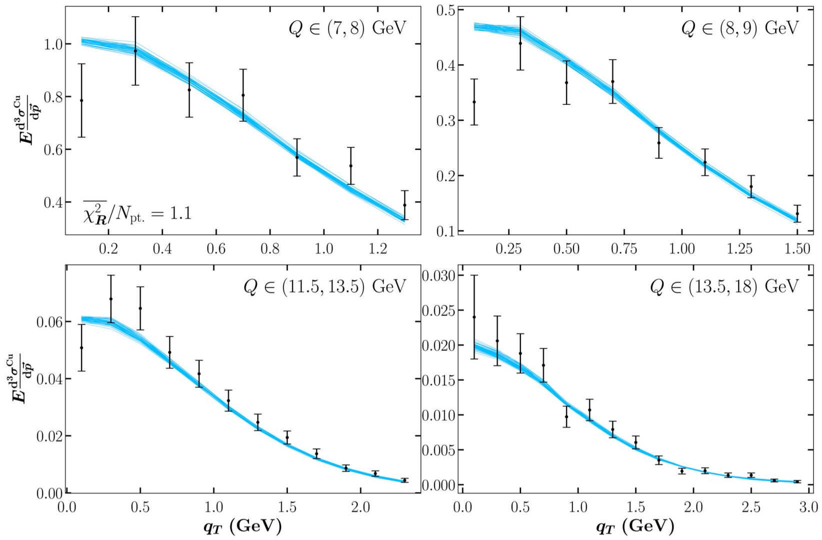
# Currently working on...

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$




JAM: 2302.01192

$$\tilde{f}_{q/\mathcal{N}(A)}(x,b_T;\mu_Q,Q^2) = (C\otimes f)_{q/\mathcal{N}(A)}(x;b_*)$$
$$\times \exp\left\{-g_{q/\mathcal{N}(A)}(x,b_T) - g_K(b_T)\ln\frac{Q}{Q_0} - S(b_*,Q,\mu_Q)\right\}$$




$$\frac{1}{\sqrt{N_{\rm rep.}}}(\sigma_{\rm corr.} - \sigma_{\rm vacc.})$$

# Summary

- We established relations between fragmenting **jet** functions and **standard** fragmentation functions in all possible polarizations
- We use them to describe experimental data at RHIC and LHC
  - Kaon FFs can be further constrained from LHCb data
  - $\Lambda$ polarization in jet from STAR can be described by our formalism
- Nuclear TMD corrections can also be studied with our formalism
- Transverse energy-energy correlators offer new opportunities in probing TMD physics

# Thank you for your attention!