Accuracy and Efficiency of Integration Methods used in Castro Code

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Introduction

Our application is the double detonation sub-Chandrasekhar model for type Ia supernovae, occurring when a carbon-oxygen white dwarf accumulates sufficient helium to ignite at the surface. This detonation then triggers a second detonation in the carbon core. Another application is X-ray bursts arising from unstable thermonuclear burning of accreted fuel on the surface of neutron stars.

Modeling astrophysical reacting flow can be challenging due to the disparate timescales of burning and hydrodynamics. Implicit methods are frequently applied to such stiff systems, meaning the solution varies slowly, but nearby solutions rapidly change, requiring small time-steps for accuracy. Implicit methods can be described as an ordinary differential equation, $\dot{y} = f(t, y(t))$, with the general solution $y^{n+1} = y^n + \Delta t f(t^{n+1}, y^{n+1})$. A Jacobian matrix, $J = \frac{\partial f}{\partial y'}$, must be calculated and stored at each time-step, along with linear algebra operations, all of which are computationally expensive. This expense motivates exploring more economical explicit methods. The general solution is $y^{n+1} = y^n + \Delta t f(t^n, y^n)$, which updates the system using only the current and possibly previous values of the dependent variables. These methods use less memory but can be unstable.

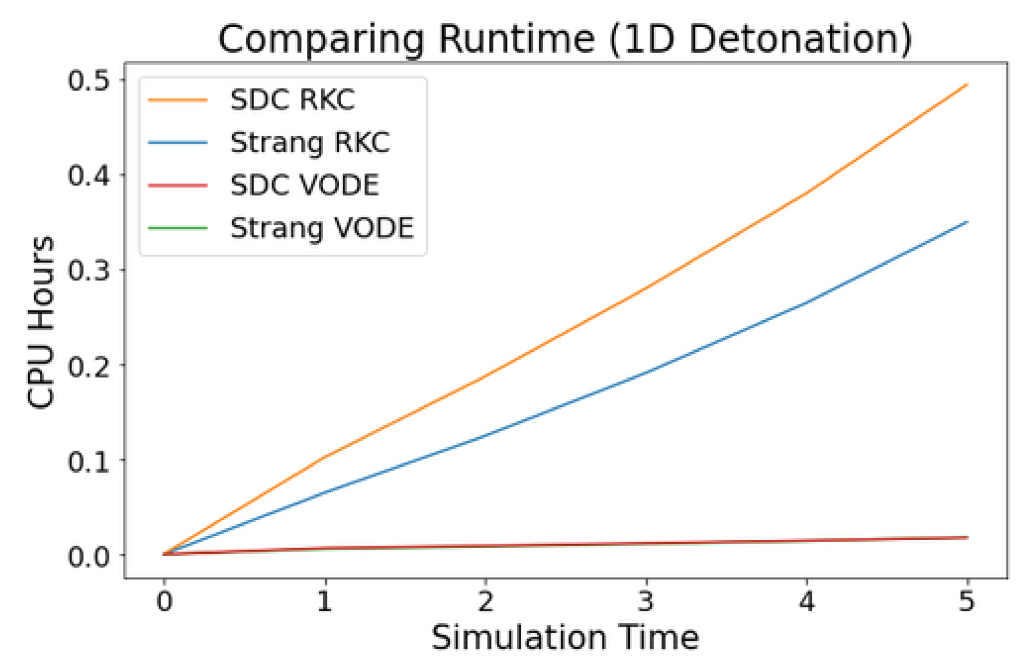
The goal of this project is to investigate how accurate and efficient the explicit Runge-Kutta-Chebyshev (RKC) method is compared to the implicit VODE method. We also investigate Strang and spectral deferred correction (SDC) coupling between the hydrodynamics and reactions.

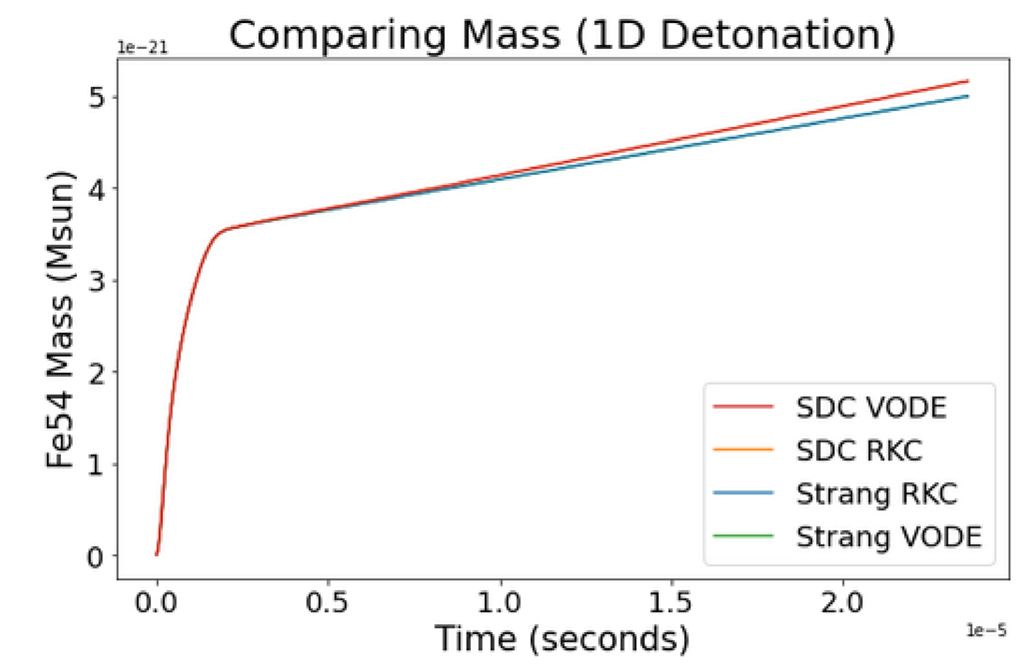
Methods

- Castro
  - Solves equations of hydrodynamics on adaptive grid
  - Support for a general equation of state (EOS), nuclear reaction networks, rotation, radiation, and full self-gravity
  - Used for exploring Type Ia supernovae and X-ray bursts
  - Open/freely available: https://github.com/AMReX-Astro/Castro
- Microphysics
  - Collection of microphysics routines for stellar explosions
  - Common routines include: EOS, integration, interfaces, networks, neutrinos, screening.
  - Open/freely available: https://github.com/AMReX-Astro/Microphysics
- National Energy Research Scientific Computing Center
  - Simulations ran on the Perlmutter supercomputer
  - 10,000 node hours on GPU, 3,000 node hours on CPU

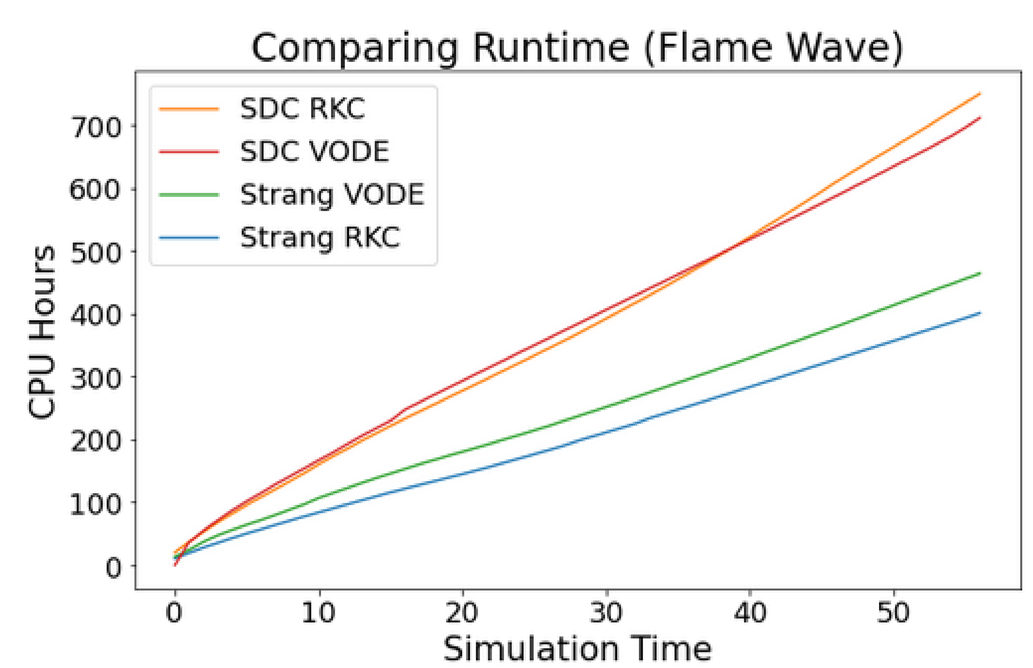
Results

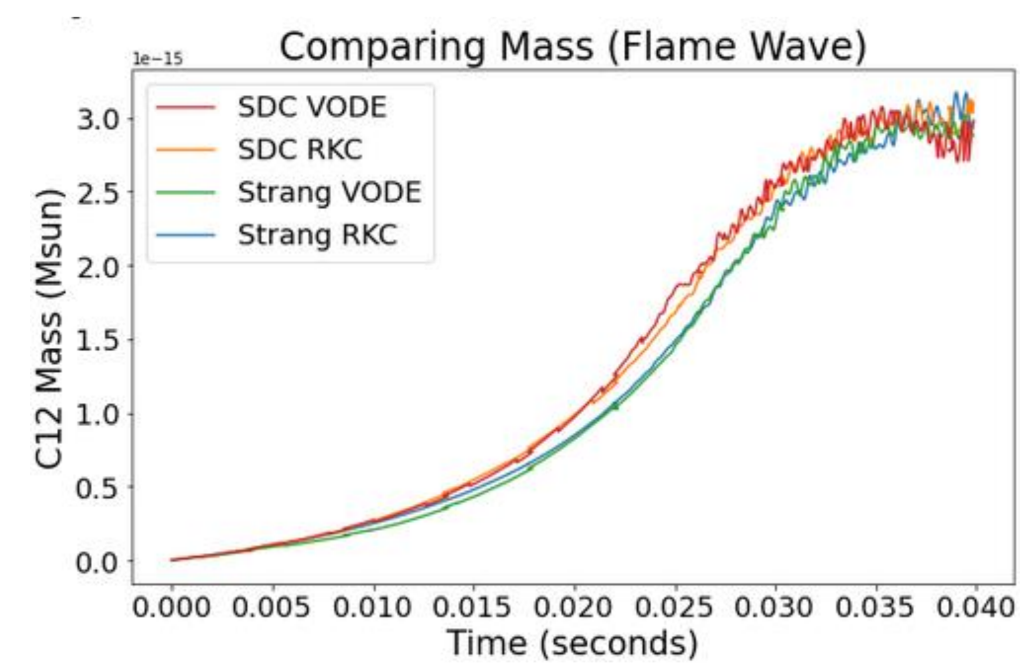
Problem: Simple 1D carbon detonation to nuclear statistical equilibrium



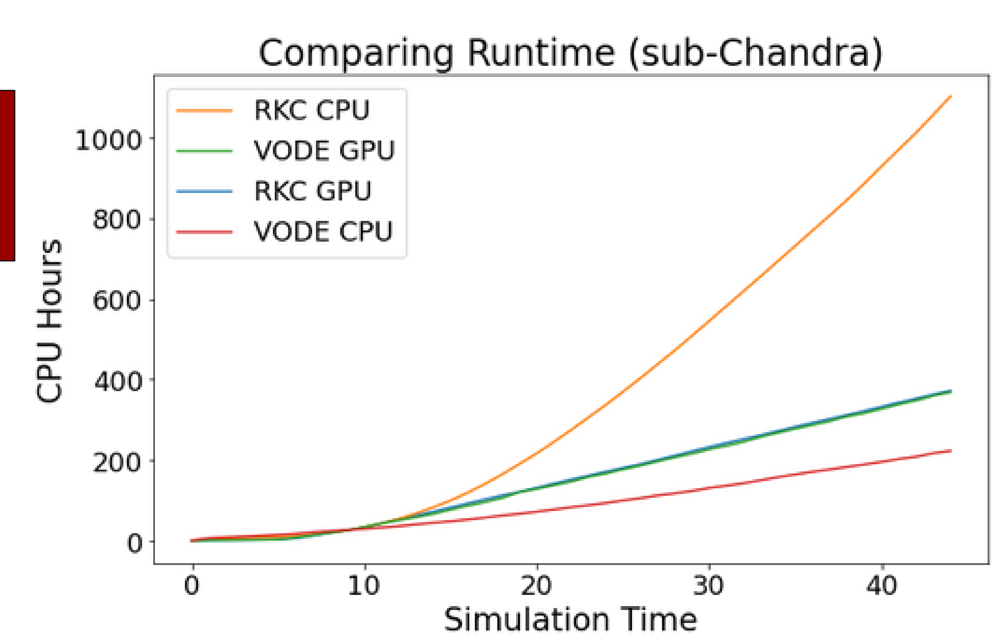


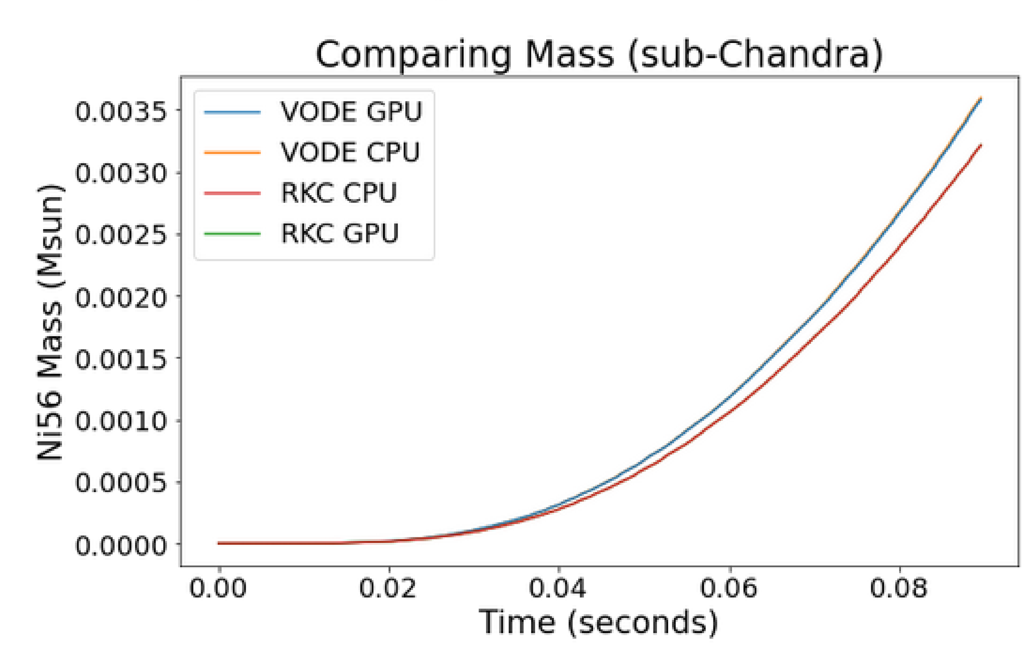
Problem: Flame spreading across the surface of a neutron star in X-ray burst

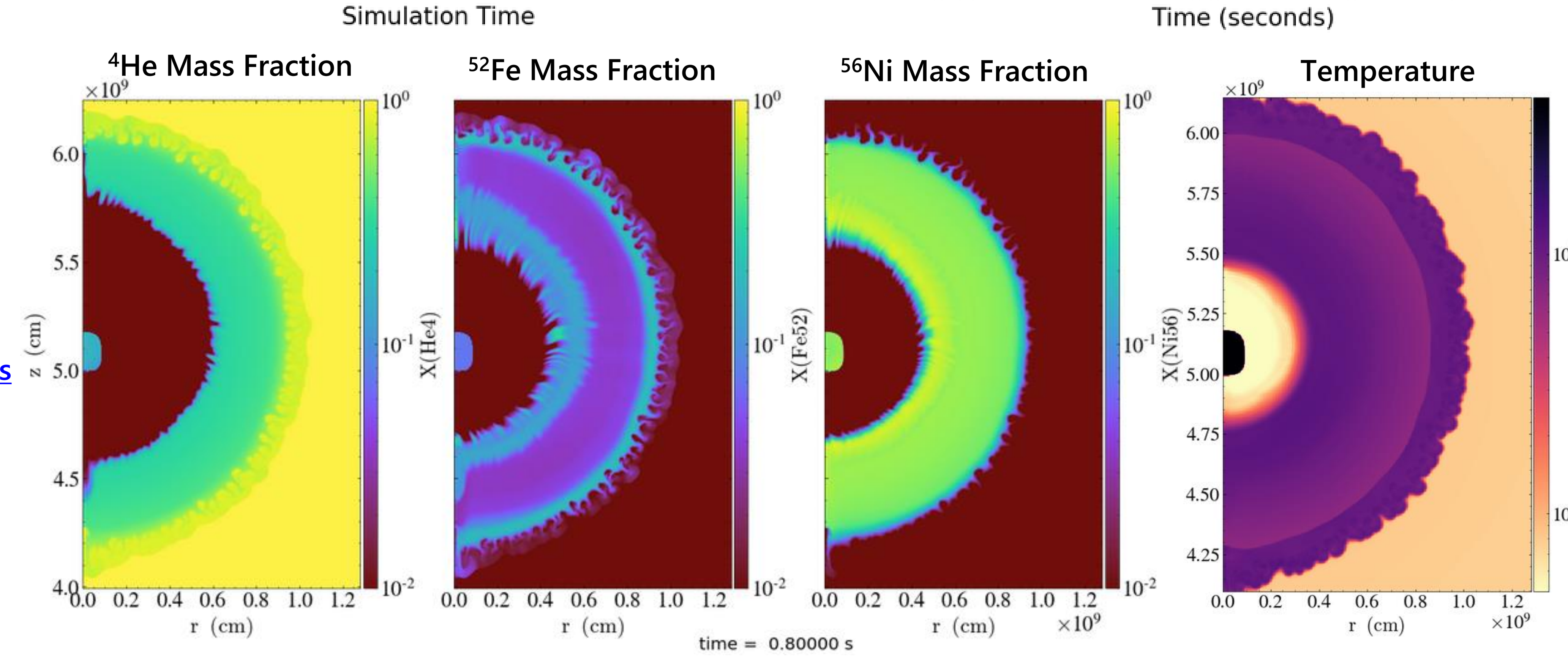




Problem: Sub-Chandra double detonation by accreting helium rich material







Summary

Conclusion

- 1D Carbon Detonation
  - RKC runs significantly slower with both the Strang and SDC time integration methods than VODE.
  - RKC remains as accurate as VODE with both Strang and SDC.
- Flame Wave X-Ray Burst
  - RKC has a greater efficiency than VODE using Strang, and nearly the same efficiency with SDC.
  - RKC is as accurate as VODE using both the Strang and SDC method.
- Sub-Chandra Double Detonation
  - RKC struggles to model the system past any of the detonations.
  - RKC appears to have similar accuracy and efficiency when running on GPUs, however, it crashes at detonation.

Future work

Choosing the ideal integration method for a problem can effect the accuracy of the solution, as well as the computational cost to solve the problem. This is the motivation behind the pursuit of publishing these findings in a formal paper. I plan to continue to work on writing this paper going into the Fall semester. This work will be beneficial for future studies to know what integration method to use for their system.

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References

[1] Fink M. et al. 2007, A&A, 476, Double-detonation supernovae of sub-Chandrasekhar mass white dwarfs
[2] Zingale M., An Introduction to Computational Astrophysical Hydrodynamics
[3] Timmes F. X., 1999, ApJS, 124, Integration of Nuclear Reaction Networks for Stellar Hydrodynamics
[4] Zingale M. et al. 2022, ApJ, 936, An Improved Method for Coupling Hydrodynamics with Astrophyscial Reaction Networks