Small x behavior in QCD from maximal entanglement and conformal invariance

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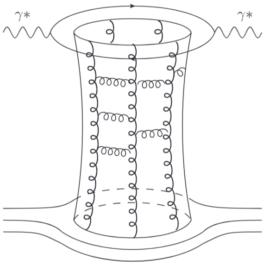
DIS at small Bjorken x

High energy QCD: scattering amplitudes described by exchange of gluons between virtual quark-antiquark pair

$\gamma^\star$ emitted by scattered lepton splits into $q\bar{q}$ pair. Reggeized gluons are exchanged between $q\bar{q}$ pair and hadron.



Lepton-hadron scattering amplitude given by sum of Feynman diagrams. Leading log: ladder gluon exchange diagrams dominate. BFKL (Balitsky, Fadin, Kuraev, and Lipatov) tried to re-sum these diagrams.

Our claim: All gluons contribute

Bjorken x

- Bjorken x measures fraction of nucleon's momentum carried by a struck parton.
- Large $x \sim 1$: parton carries nearly all of nucleon's momentum.
- Small $x \ll 1$: parton carries tiny fraction of momentum.

Bjorken Scaling:

- High energies and fixed x, structure functions become approximately independent of momentum transfer Q^2.
- Scaling behavior suggests that partons behave as quasi-free particles at short distances (asymptotic freedom).

BFKL Pomeron

Small x behavior of DIS structure functions related to high-energy (Regge) asymptotics of scattering amplitude. First perturbative contribution from two reggeized gluons [BFKL pomeron] leads to following asymptotic small x behavior:

$$xG(x) \sim \frac{1}{x^{\Delta}}, \quad \Delta = \frac{g^2 N_c^2 \log 2}{8\pi^2}$$

Applicability of perturbative result depends on scale Q^2. Contribution from exchange of multiple reggeized gluons also becomes important at small x. Lipatov's effective theory is dual to a spin chain.

Describe contribution of all of reggeized gluons to the scattering amplitude through dynamics of Lipatov's spin chain.

Gluon structure function

Gluon structure function $G(x)$ describes momentum distribution of gluons inside hadron as function of x.

Meaning: $G(x)$ tells us how momentum of proton is distributed among the gluons.

Why is it important? Plays central role in deep inelastic scattering (DIS) and in parton distribution functions (PDFs). At small x (high energies), gluons dominate structure of proton. Accurate knowledge of gluon structure function crucial for predictions at colliders like LHC.

How is it determined? Indirectly, from QCD fits to experimental data, mainly from DIS (e.g., at HERA).

Lipatov's spin chain and LNS

- BFKL equation describing high-energy QCD scattering in the Regge limit captured by nearest neighbor spin chain Hamiltonian $H_L = \sum_{k=1}^{L} H_{k,k+1}$.
- Hamiltonian exhibits $SU(2)$ symmetry and maps to the XXX Heisenberg model which is integrable (exact solutions via Bethe Ansatz methods).
- XXX model with negative spin $s = -1$ is equivalent to quantum lattice non-linear Schrödinger model.
- Bethe equations for lattice NLS model, at $\kappa = 1$, and $\Delta = 2$

$$(-1)^L \left(\frac{\lambda_k - i}{\lambda_k + i}\right)^L = \prod_{j \neq k}^{N} \frac{\lambda_k - \lambda_j + i}{\lambda_k - \lambda_j - i}$$

The correspondence between lattice NLS and XXX

- In order to apply CFT: finite size correction to the ground state energy in thermodynamic limit $N, L \to \infty$:

$$E = L\varepsilon - \frac{\pi v}{6L}$$

v is Fermi velocity, describing spread of color waves along chain of interacting gluons. The finite size correction shows that the central charge of Virasoro algebra

$$c = 1$$

- For the gluon spin chain the original photon is a quench. The entropy increase logarithmically in time:

$$S(t) = \frac{c}{3} \ln t + \text{constant}$$

D. Kharzeev, V. Korepin, H. Kun, Z. Kun; Phys.Rev. D 105, 014002

Entanglement entropy evolution

- The entanglement entropy scales logarithmically with time:

$$S_{\mathrm{BFKL}}(t) = \frac{c}{3}\ln\frac{t}{\tau} = \frac{1}{3}\ln(mt)$$

  Here the m is the mass of the hadron.

- Kharzeev and Levin (PRD 95, 11, 114008 and PRD 104, L031503): PDFs defined in terms of EE between spatial region probed by DIS and the rest of proton. For very small x: proton maximally entangled state $\Rightarrow S = \ln N$ (relation between average number N of color-singlet dipoles in the proton wave function and the entropy of the produced hadronic state S).
- At small x, multiplicity of dipoles is given by the gluon structure function, $N = xG(x, Q^2)$.

Ioffe Time

Ioffe time characterizes coherence length of quark-gluon fluctuations in a fast-moving nucleon.

- In the target rest frame: corresponds to the lifetime of the virtual photon fluctuation into a quark-antiquark pair.
- It provides natural measure of longitudinal distance over which interaction occurs.
- At small x, Ioffe time becomes large, reflecting the long coherence length of small-x partons.

Putting it all together...

Average number of color-singlet dipoles at Ioffe time $[t = 1/xm]$ can be expressed in terms of the Gluon Structure Function:

$$N = e^{S_{\text{BFKL}}}_{mt=1/x} = xG(x)$$

For central charge $c = 1$ that we have found for Lipatov's spin chain, gluon structure function grows at small x as

$$xG(x) \sim \frac{1}{x^{1/3}}$$

Can be compared to experiment.

HERA data (I. Abt, A.M. Cooper-Sarkar, B. Foster, V. Myronenko, K. Wichmann, M. Wing; Phys. Rev. D 96, 014001 (2017)): show reasonable agreement with $1/3$.

More accelerators will be build ... EIC.