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Overview on energy-momentum tensor form factors, mass and mechanical radii

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Outline

- introduction: EMT form factors, definition, notation, overview
- how to access EMT form factors in experiment
- interpretation, 2D and 3D Fourier transforms
- mass and mechanical radius
- illustration in models
- conclusions & outlook

supported by
• NSF grant no. 2412625
• DOE QGT Topical Collaboration

introduction

getting started $\longrightarrow$ **rest**

definition of EMT operator

- different EMT definitions exist, e.g.
- Noether current of space time translations $\rightarrow$ canonical EMT
- Belinfante EMT, add total derivatives $\rightarrow$ symmetric EMT

- Einstein equation $R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R + \Lambda g^{\mu\nu} = 8\pi G_N\,T^{\mu\nu}$
- use gravity to define
- $\hat{T}_{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\,\frac{\delta S_{\text{grav}}}{\delta g^{\mu\nu}(x)}\bigg|_{g^{\mu\nu}(x)\to\ \text{flat metric}}$
- often **EMT form factors** = **gravitational form factors** (loosely speaking)

- important detail: gravity would not distinguish quarks from gluons, couples to total EMT
- in QCD $\hat{T}_{\mu\nu} = \sum_q \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$ with $\hat{T}^q_{\mu\nu}$ and $\hat{T}^g_{\mu\nu}$ each gauge invariant **(!)** (Ji 1995)
- allows us to discuss quark **and** gluon EMT form factors
- important for mass, spin decomposition (Ji 1996)

definition nucleon EMT form factors I (Ji 1996)

$$\langle N'|\hat{T}^a_{\mu\nu}|N\rangle = \bar{u}(p')\Bigg[\, A^a(t,\mu^2)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M} + \overline{C}^a(t,\mu^2)\,M\,g_{\mu\nu} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\Bigg]u(p)$$

- conserved current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu} \quad (a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, etc, $\sum_a \overline{C}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $B(0) = 0$ $\Leftrightarrow$ total anomalous gravitomagnetic moment vanishes *

  D-term $\Leftrightarrow$ $D(0) \equiv \boldsymbol{D}$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* equivalent to: total nucleon spin $J^q + J^g = \frac{1}{2}$ is due to quarks + gluons (via Gordon identity)

definition nucleon EMT form factors II (Ji 1996)

$$\langle N'|\hat{T}^{a}_{\mu\nu}|N\rangle = \bar{u}(p')\Big[A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M} + J^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + \overline{C}^a(t,\mu^2)\,M\,g_{\mu\nu} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\Big]u(p)$$

- conserved current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ $(a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, etc, $\sum_a \overline{C}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $J(0) = \frac{1}{2}$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon spin *

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
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notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d^q_1(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M^q_2(t)$

* equivalent to: nucleons total anomalous gravitomagnetic moment vanishes (Gordon identity)

how we learn about hadrons

$|N\rangle$ = **strong**ly–interacting particle. Probe it with other forces! (easier approach)

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$Q,\ \mu$
weak:	PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$g_A,\ g_p$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$A(t),\ B(t),\ D(t)$	$\longrightarrow$	$M,\ J,\ D$

$t = 0$ *global properties:*

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40) \times 10^{-19}\text{C} \qquad \text{in particle data book}\\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N\\
g_A &= 1.2694(28)\\
g_p &= 8.06(0.55)\\
M &= 938.272013(23)\,\text{MeV}\\
J &= \tfrac{1}{2}\\
\boldsymbol{D} &= \text{? least known global property, not (yet) in particle data book}
\end{aligned}
$$

$t \neq 0$ *access to radii!*

what we know about EMT form factors

$A(0)$

- equal to 1
- $A^q(0) = \int dx\, x f_1^q(x)$, $\sum_q A^q(0) = 0.60$ and $A^q(0) = 0.40$, $\mu^2 = 4\,\text{GeV}^2$ NNPDF, EPJC (2022)

$B(0)$ (or $J(0)$)

- equal to 0 (or $\frac{1}{2}$)
- spin decomposition, Ji sum rule, not yet known

$\bar{C}^a(t)$

- non-zero only for subsystems, twist-4
- zero for total system

$D(0)$

- not known even at $t = 0$; $D = D(0) = 0$ in free field theory for spin-$\frac{1}{2}$ ($D = 1$ for spin-0)
- chiral limit $D = -1$ for pions, $\boldsymbol{D < -0.95}$ for nucleon Gegelia, Polyakov (2021)

t-dependence

- needed for radii, little is known
- models, lattice QCD, dispersion relations, χPT, pioneering insights from experiment

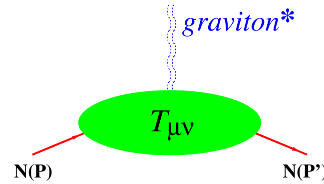
reviews

- Polyakov, PS, Int.J.Mod.Phys.A 33 (2018)
- Burkert et al, Rev.Mod.Phys. 95 (2023)

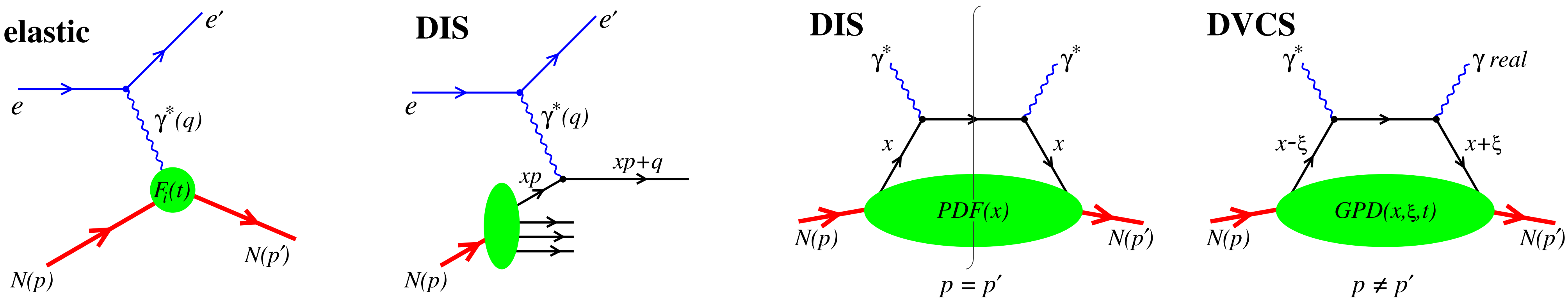
how to access in processes

EMT form factors in experiment

- **scattering due to graviton exchange impractical**
  unless energies $\sim M_{\text{Planck}}$ Kobzarev, Okun 1962
- **hard-exclusive reactions $\rightarrow$ GPDs**



what is a generalized parton distribution function (GPD)?



amplitude
$\Delta = p - p',\ t = \Delta^2$
$F_1(t),\ F_2(t)$
$G_E(t) = F_1(t) + F_2(t)\frac{t}{4M^2}$
$G_M(t) = F_1(t) + F_2(t)$

amplitude
$Q^2 = -q^2$
$x_B = \frac{Q^2}{2p\cdot q}$
$x = x_B$
$f_1^q(x)$

cross section
$Q^2 = -q^2$
$x_B = \frac{Q^2}{2p\cdot q}$
$x = x_B$
$f_1^q(x)$

amplitude
$t,\ \xi$
$\xi = \frac{x_B}{2-x_B}$
$x =$ integr. variable
$H^q(x,\xi,t),\ E^q(x,\xi,t)$

- **properties**

$\lim_{\xi,t\to 0} H^q(x,\xi,t) = f_1^q(x)$

$$\int dx\, H^q(x,\xi,t) = F_1^q(t) \qquad \int dx\, x H^q(x,\xi,t) = A^q(t) + \xi^2 D^q(t)$$
$$\int dx\, E^q(x,\xi,t) = F_2^q(t) \qquad \int dx\, x E^q(x,\xi,t) = B^q(t) - \xi^2 D^q(t)$$

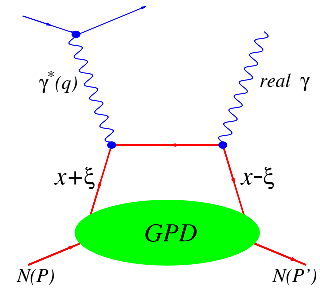
- **encyclopedia article** *"Parton Distribution Functions and their Generalizations"*,
  Lorcé, Metz, Pasquini, PS [2507.12664]

processes

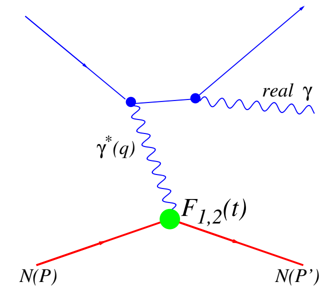
- **em hard exclusive reactions** Müller et al; Ji; Radyushkin; Collins, Frankfurt, Strikman (1990s)
  complex Compton form factors ~~deconvolution!~~ (DDVCS)

$$\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] \boldsymbol{H_q(x,\xi,t)}$$

$$\xi = \frac{x_B}{2 - x_B},\ x_B = \frac{Q^2}{2P\cdot q},\ -t \ll Q^2$$



DVCS



Bethe–Heitler

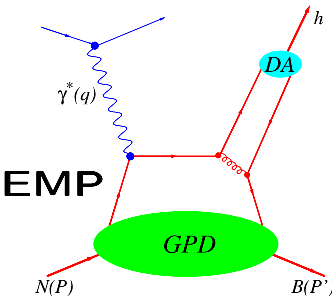
- **polynomiality**

$$\int \mathrm{d}x\ x\, \boldsymbol{H^q(x,\xi,t)} = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$$

$$\int \mathrm{d}x\ x\, \boldsymbol{E^q(x,\xi,t)} = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$$

Ji 1996

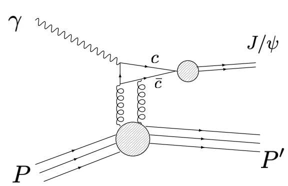
HEMP

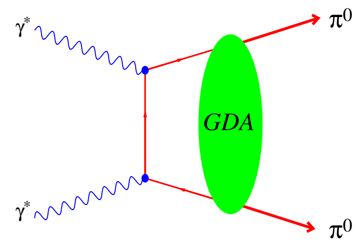


- **fixed-t dispersion relation** Anikin, Teryaev; Diehl, Ivanov 2007

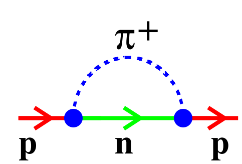
$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + ⨍_0^1 \frac{d\xi'}{\pi}\left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right]\mathrm{Im}\mathcal{H}(\xi',t), \qquad \lim_{\mu^2\to\infty} \Delta(t) = 5\sum_q e_q^2 \boldsymbol{D^q(t,\mu^2)}$$

- **gluon EMT form factors**
  threshold J/ψ production
  Kharzeev 1995, 2021



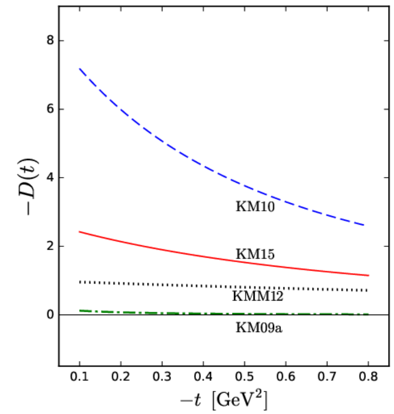


- **generalized distribution amplitudes**
  analogons of GPDs in the crossed channel
  opportunity to study short-lived hadrons like e.g. π^0
  Müller et al 1994; Diehl, Gousset, Pire, Teryaev 1998

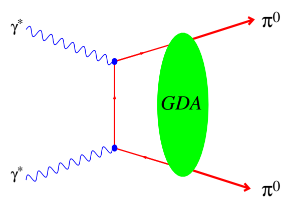


- **Sullivan process**
  hard-exclusive reaction on pion Hatta, Schoenleber 2025

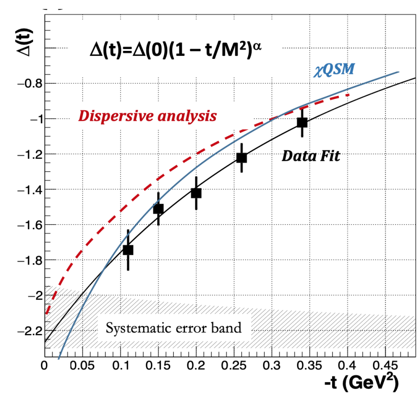
selected phenomenological results



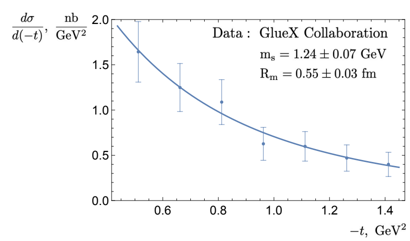
- DVCS parametrizations Kumerički, Müller et al 2009-2015; stat. uncert. 2012 $\sim 50\,\%$, 2015 $\sim 20\,\%$; syst. unestimated GUMP1.0 Guo, Aslan, Ji, Santiago PRL 2025



- $A^q(t)$, $D^q(t)$ of π^0 for $t > 0$ $\gamma\gamma^* \to \pi^0\pi^0$ Belle, PRD, 2016 generalized parton amplitudes (analytic continuation of GPDs) $D^{u+d}_{\pi^0} \approx -0.7$, $\langle Q^2 \rangle = 16.6$ GeV^2 Kumano, Song, Teryaev 2018



- fixed-t dispersion relation JLab DVCS data
  $\mathcal{I}m\,\mathcal{H}$ $\rightsquigarrow$ BSA Girod et al PRL 100 (2008) 162002
  $\mathcal{R}e\,\mathcal{H}$ $\rightsquigarrow$ $\sigma_{\rm unp}$ Jo et al PRL 115 (2015) 212003
  $D^{u+d} = -1.63 \pm 0.11 \pm 0.26 \pm \ldots$ at $\langle Q^2 \rangle = 1.5\,$GeV^2
  Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)



- threshold J/ψ photo-production
  Kharzeev 1995, 2021; Hatta et al 2018
  Guo et al 2021, Duran et al Nature (2023)

Gluex PRL 123, 072001 (2019).

D-term remains least known global property, but now something is known

Why are EMT form factors interesting?

- proton mass decomposition $\rightsquigarrow$ $A^a(0)$, $\overline{C}^a(0)$ $\rightsquigarrow$ several workshops
  Ji 1995, Hatta et al 2018, Tanaka 2019, Metz et al, Rodini et al 2020, Lorcé et al 2021, Ji 2021, ...

- proton spin decomposition $\rightsquigarrow$ $J^a(0)$ $\rightsquigarrow$ tens of workshops
  Ji 1996, many more works, Leader & Lorcé 2014, many works, Ji, Yuan, Zhao 2021, ...

- D-term $\rightsquigarrow$ D $\rightsquigarrow$ so far 1 workshop, 1 in preparation
  Polyakov, Weiss 1999, interest increasing

- fascinating interpretation
  M. V. Polyakov (2003)

- new radii!!!

interpretation

observables $\longrightarrow$ **interpretation** $\longrightarrow$ **radii**

recall 3D interpretation of electric form factor

Hofstadter 1961

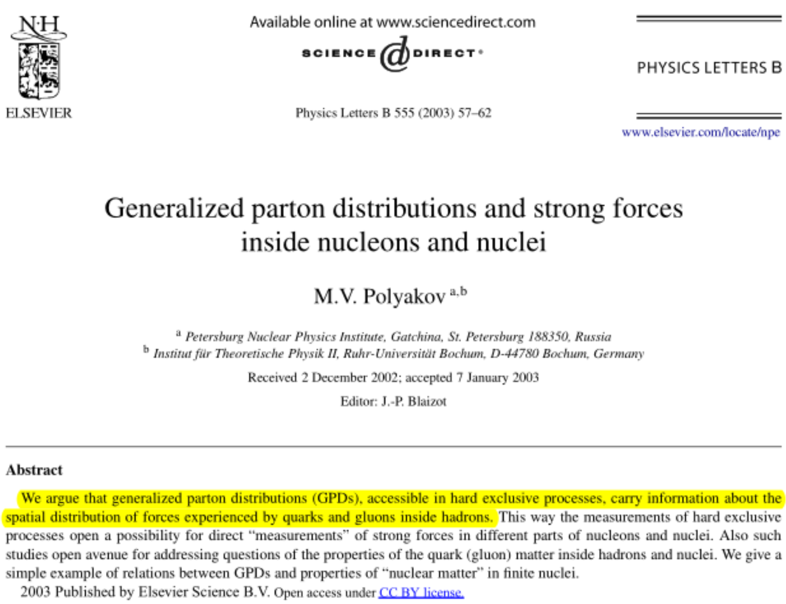
Breit frame $\Delta^\mu = (0, \vec{\Delta})$, $t = -\vec{\Delta}^2$

$$G_E(t) = \int d^3r\, e^{i\vec{\Delta}\cdot\vec{r}} \varrho(r) = Q + \frac{1}{6}\langle r^2_{\rm ch}\rangle t + \mathcal{O}(t^2)$$

charge radius $\langle r^2_{\rm ch}\rangle = \dfrac{\int d^3r\, r^2 \varrho(r)}{\int d^3r\, \varrho(r)} = \dfrac{6\, G'_E(0)}{G_E(0)}$ for proton, nuclei

3D interpretation of EMT form factors

Maxim Polyakov



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Generalized parton distributions and strong forces inside nucleons and nuclei

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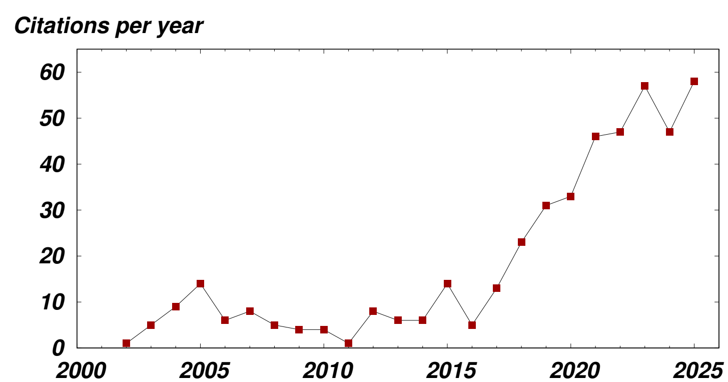
Abstract

We argue that generalized parton distributions (GPDs), accessible in hard exclusive processes, carry information about the spatial distribution of forces experienced by quarks and gluons inside hadrons. This way the measurements of hard exclusive processes open a possibility for direct "measurements" of strong forces in different parts of nucleons and nuclei. Also such studies open avenue for addressing questions of the properties of the quark (gluon) matter inside hadrons and nuclei. We give a simple example of relations between GPDs and properties of "nuclear matter" in finite nuclei.



interest increasing! ☺

interpretation of EMT form factors

M.V.Polyakov, PLB 555 (2003)

- **define static 3D EMT**

  in the Breit frame $\Delta^\mu = (0, \vec{\Delta})$

$$T_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{-i\vec{\Delta}\vec{r}} \langle P'|\hat{T}_{\mu\nu}|P\rangle$$

$$\int d^3r\, T_{00}(\vec{r}) = M \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5} M \int d^3r \left(r^i r^j - \frac{r^2}{3}\delta^{ij}\right) T_{ij}(\vec{r}) \equiv D \qquad \textbf{new!}$$

- consider the nucleon (or nucleus) as continuous medium

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij} \qquad \textbf{stress tensor}$$

$s(r)$ related to distribution of *shear forces*
$p(r)$ distribution of *pressure* inside hadron $\Big\} \longrightarrow$ **"mechanical properties"**

grain of salt with 3D interpretation

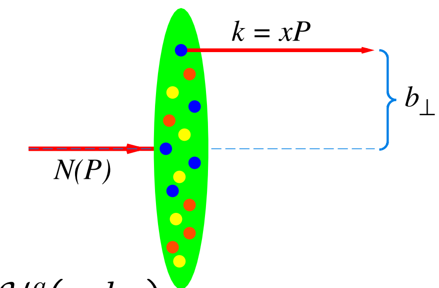
- interpretation of quantum mechanics $\longrightarrow$ since 100 years
  Max Born, 25. June 1926 (footnote added in proof: $|\psi|^2 \propto$ probability), ...

- 3D interpretation of em form factors $\longrightarrow$ since 70 years
  M. K. Rose, PR73, 279 (1948), Yennie, Lévy, Ravenhall, Rev.Mod.Phys. 29, 144 (publ. 1/1/1957)

- interpretation of $D(t)$ $\rightarrow$ both (i) quantum aspect (proton!) & (ii) 3D aspect (pressure)
  review of discussions Lorcé, PS, Acta Phys.Polon.B 56 (2025) (DPP Memorial Volume)

- **2D partonic probability densities**

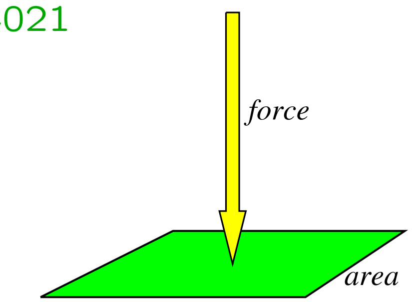
  exact 2D **densities** $\rightarrow$ Matthias Burkardt (2000)
  infinite momentum frame (light-front quantization)
  2D EMT $\rightarrow$ Lorcé, Moutarde, Trawiński (2019); Freese, Miller (2021)
  3D partonic densities do not exist! Miller, PRC112 (2025) 045204
  forces between particles, pressure Ji, Yang NPB (2026)



$\mathcal{H}^q(x, b_\perp) =$
$\int d^2\Delta_\perp e^{i\vec{b}_\perp\cdot\Delta_\perp} H^q(x, 0, \Delta_\perp)$

- **3D distribution interpretation**

  3D Fourier transforms $\rightarrow$ correlations functions (no partonic densities)
  phase-space perspective $\rightarrow$ quasi-probabilistic interpretation Lorcé PRL 125 (2020) 232002
  for heavy targets recoil corrections negligible (nuclei) Polyakov, PLB 555 (2003) 57
  for nucleon rigorously justified in large N_c limit Goeke et al, PRD75 (2007) 094021
  cannot be expressed through light-front wave functions Miller (2025)
  but 3D is needed for pressure interpretation



- **we need to be aware**

  same reservations as for electromagnetic form factors, let's continue

radii

interpretation $\longrightarrow$ **radii**

mass radius

- energy density $T_{00}(r) \geq 0$ positive definite (unlike charge density, $\langle r^2_{\rm ch}\rangle_{\rm neut} = -(0.11\,{\rm fm})^2$)
- $\langle r_E^2\rangle = \dfrac{\int d^3r\, r^2 T_{00}(r)}{\int d^3r\, T_{00}(r)} = \dfrac{6A'(0)}{A(0)} - \dfrac{3D}{2M^2}$ appropriate proxy for particle size
- proton and neutron have different charges + different charge radii but nearly the same masses. Do they have the same $\langle r_E^2\rangle$...?

trace radius

- sometimes also called the "mass radius" (variant)
- radius associated with the slope of the form factor of $\langle N'|\hat{\boldsymbol{T}}^{\boldsymbol{\mu}}{}_{\boldsymbol{\mu}}|N\rangle$
- in QED, QCD → trace anomaly $\hat{T}^{\mu}_{\mu} \equiv \dfrac{\beta(g)}{2g}\, F^{\mu\nu}F_{\mu\nu} + (1+\gamma_m)\sum_q m_q\bar{\psi}_q\psi_q$
- $\langle N'|\hat{T}^{\mu}{}_{\mu}|N\rangle = \bar{u}(p')\Big[\,M F_{\rm tr}(t)\,\Big]u(p) \quad \to \lim_{t\to 0}\langle N'|\hat{T}^{\mu}{}_{\mu}|N\rangle = 2M^2$
- $F_{\rm tr}(t) = A(t) + \dfrac{t\,B(t)}{4M^2} + \dfrac{3\,t\,D(t)}{4M^2}$
- $\langle r^2_{\rm tr}\rangle = \dfrac{6F'_{\rm tr}(0)}{F_{\rm tr}(0)} = \dfrac{6A'(0)}{A(0)} - \dfrac{9D}{2M^2}$
  Kharzeev, PRD 104, 054015 (2021)

mechanical radius

- **EMT conservation** $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$ $\Rightarrow$ $\frac{2}{3}s'(r) + \frac{2}{r}s(r) + p'(r) = 0$

  $\hookrightarrow$ necessary condition for stability $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

  $$\boldsymbol{D} = -\frac{16\pi}{15} M \int_0^\infty \mathrm{d}r\, r^4 s(r) = 4\pi M \int_0^\infty \mathrm{d}r\, \boldsymbol{r^4\, p(r)} \rightarrow \text{internal forces}$$

- **eigenvalues of stress tensor**

  symmetric 3×3 matrix $\rightarrow$ diagonalize Polyakov, PS, Int.J.Mod.Phys.A 33 (2018)

  $\frac{2}{3}s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)

  $-\frac{1}{3}s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$, degenerate for spin 0 and $\frac{1}{2}$)

- **stability of a mechanical system**

  $\Leftrightarrow$ normal force directed towards outside

  $\Leftrightarrow$ $T^{ij} e_r^j\, dA = \underbrace{[\frac{2}{3}s(r) + p(r)]}_{>0} e_r^i\, dA$ $\Rightarrow$ $\boldsymbol{D < 0}$ Perevalova et al (2016)

  $\rightarrow$ in fact, this is how neutron star radii are determined!

- $\frac{2}{3}s(r) + p(r) > 0$ is positive definite. We can use it to define a radius!

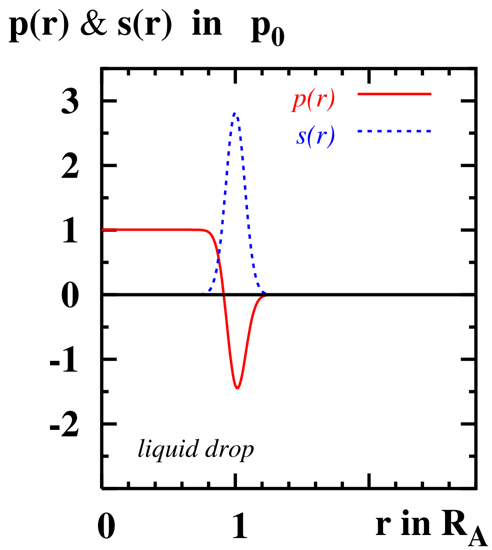
$$\langle \boldsymbol{r^2} \rangle_{\rm mech} = \frac{\int \boldsymbol{d^3r\; r^2[\frac{2}{3}s(r) + p(r)]}}{\int \boldsymbol{d^3r\; [\frac{2}{3}s(r) + p(r)]}} = \frac{\boldsymbol{6D(0)}}{\int_{-\infty}^{0} \boldsymbol{\mathrm{d}t\, D(t)}} \quad \text{vs} \quad \langle r^2_{\rm ch} \rangle = \frac{6\, G_E'(0)}{G_E(0)} \quad \text{"anti-derivative"}$$

illustration

how do these mechanical properties look like?

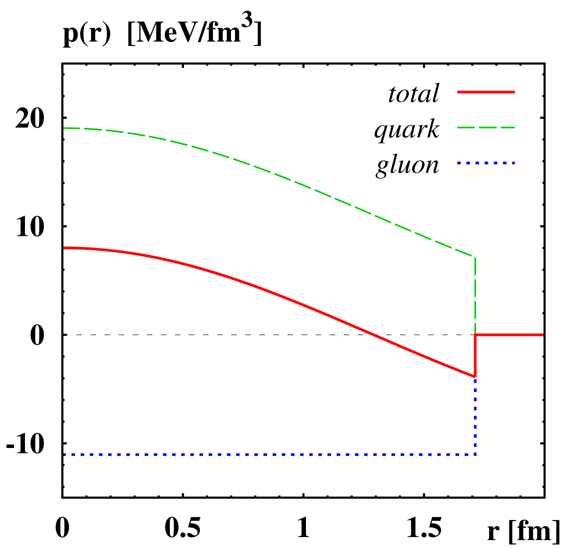
nuclei (liquid drop model) Polyakov (2003)

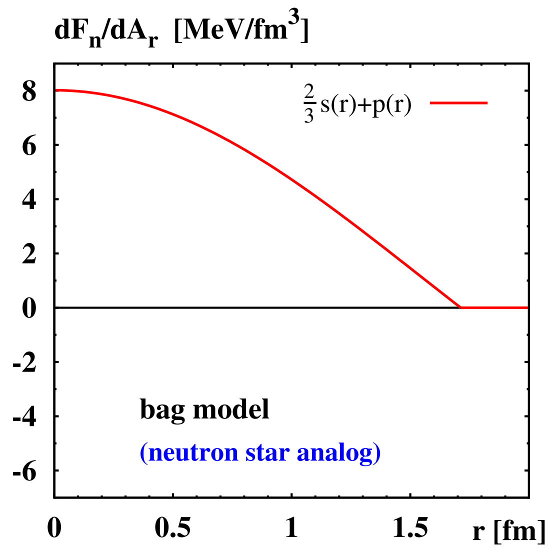
- $p(r) = p_0\,\Theta(R_A - r) - \frac{1}{3}p_0 R_A\,\delta(r - R_A)$, $\quad s(r) = \gamma\,\delta(r - R_A)$
  $R_A = R_0\,A^{1/3}$, $m_A = m_0\,A$, surface tension $\gamma = \frac{1}{2}p_0 R_A$
  D-term $\boldsymbol{D = -\frac{4\pi}{3}\,m_A\,\gamma\,R_A^4 \approx -0.2\,A^{7/3}}$



- mechanical radius in nuclei
  intuitive result for large nucleus
  $\frac{2}{3}\,s(r) + p(r) = p_0\,\Theta(R_A - r) \;\; \rightarrow \;\; \langle r^2 \rangle_{\rm mech} = \frac{3}{5}\,R_A^2$

bag model Neubelt et al (2020)





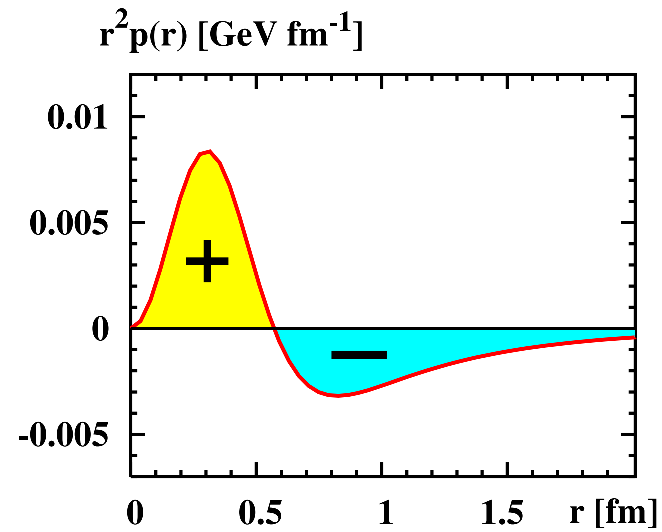
von Laue condition $\int_0^R r^2 p(r) = 0$ ✓

chiral quark soliton model of nucleon Goeke et al, PRD75 (2007) 094021

- $p(0) = 0.23\,\mathrm{GeV/fm^3} \approx 4 \times 10^{34}\,\mathrm{N/m^2}$
  $\gtrsim$ 10-100×(pressure in center of neutron star)

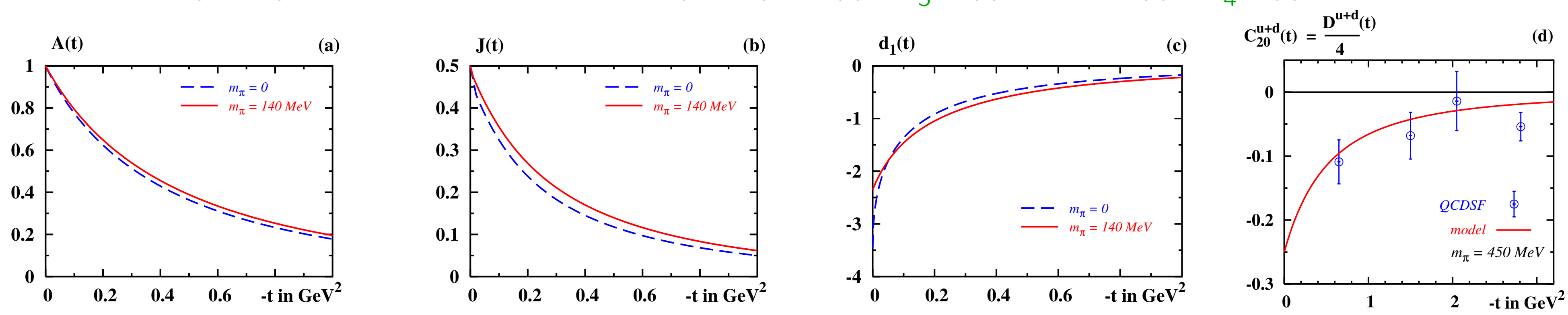
- $p(r) = 0$ at $r = 0.57\,\mathrm{fm}$ change of sign
  $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$✓

- $p(r) = \left(\frac{3g_A^2}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r in chiral limit $m_\pi \to 0$



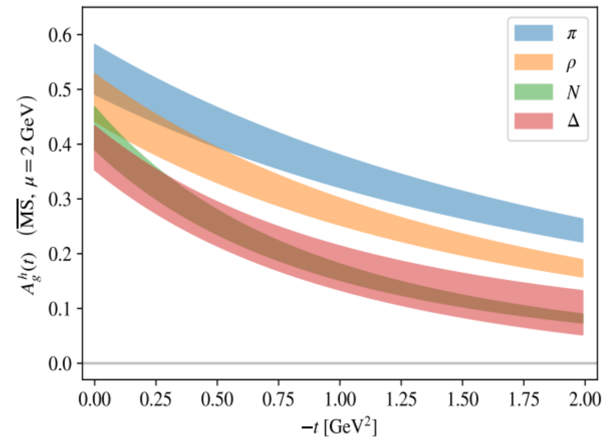
- proton $\langle \boldsymbol{r^2} \rangle_{\mathrm{mech}} \approx \mathbf{0.75}\, \langle \boldsymbol{r^2_{\mathrm{ch}}} \rangle$
  and finite in chiral limit (charge radius not)

- **form factors**
  Goeke et al (2007) vs lattice Göckeler et al (2004); $d_1(t) = \frac{4}{5} D(t)$ and $C_{20}(t) = \frac{1}{4} D(t)$



lattice QCD

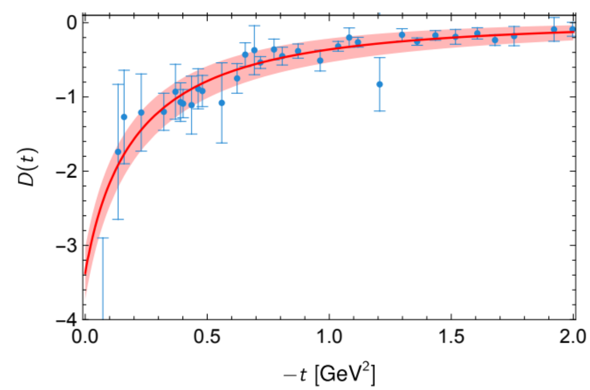
Pefkou, Hackett, Shanahan PRD105 (2022) 054509
$A^g(t)$ for different hadrons
→ distributions of glue
→ and radii!!!



data-driven dispersive approach

Cao, Guo, Li and Yao, Nature Commun. (2025)
incorporates analyticity, unitarity, and sum rules
model-independent with controlled accuracy

$D(t)$ vs lattice data at $m_\pi = 170\,\text{MeV}$ from
Hackett, Pefkou, Shanahan, GPRL 132 (2024)



D-term and radii
$\langle r^2_\Theta \rangle \equiv \langle r^2_{\rm tr} \rangle$

D-term	$-3.38^{+0.34}_{-0.35}$
$\sqrt{\langle r^2_\Theta \rangle}$ [fm]	$0.97^{+0.03}_{-0.03}$
$\sqrt{\langle r^2_{\rm Mass} \rangle}$ [fm]	$0.70^{+0.03}_{-0.04}$
$\sqrt{\langle r^2_{\rm Mech} \rangle}$ [fm]	$0.72^{+0.09}_{-0.08}$
$\sqrt{\langle r^2_J \rangle}$ [fm]	$0.70^{+0.02}_{-0.02}$

pioneering dispersive study
B. Pasquini, M. V. Polyakov and M. Vanderhaeghen
Phys. Lett. B **739**, 133-138 (2014)

conclusions & outlook

conclusions

- **EMT form factors** $\rightarrow$ important: mass, spin, D-term!
- $A(t)$ and $B(t)$ hard to measure (cannot invert CFF)
- $D(t)$ more direct access from fixed-t dispersion relation in DVCS
- meson production including $J\psi$ at threshold and important other processes
- 3D interpretation exact in mean field approach, large N_c QCD (pressure is 3D)
- can define new radii: **mass radius, trace radius, mechanical radius**

outlook

- promise new insights into hadron structure!
- this was only an overview!
- more on Friday!
- talks by

  Daniele Binosi (EMT form factors of π, K, N from Schwinger methods)
  Zein-Eddine Meziani (proton EMT form factors from JLab experiments)
  Dima Kharzeev (proton mass radius from J/ψ-production)
  Yoshitaka Hatta (nuclear mass radii, pressure and D-term)
  Kiminad Mamo (EMT ffs & radii from holographic QCD)

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Thank you!

Support slides

2D vs 3D interpretation

- **3D distribution** not exact, "relativistic corrections" for $r \lesssim \lambda_{\text{Compt}} = \frac{\hbar}{mc}$
  2D densities exact partonic probability densities

  known since earliest days:
  - Yennie, Levy, Ravenhall, Rev. Mod. Phys. 29 (1957) 144
  - Sachs, Physical Review 126 (1962) 2256
  - Belitsky, Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
  - X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
  - G. Miller, PRC80 (2009) 045210 (toy model, very dramatic effect)
  - Lorcé, PRL 125 (2020) 232002 quasi-probabilistic phase-space average á la Wigner
  - Jaffe, arXiv:2010.15887 (not possible to measure spatial dependence of nucleon matrix elements)
  - Freese, Miller, 2102.01683 (expectation value of a local operator within spatially-localized state)
  - Epelbaum, Gegelia, Lange, Meißner, Polyakov, arXiv:2201.02565

- **2D densities** = **partonic probability densities** (unitarity)
  must (and better be) exact! → M. Burkardt (2000)
  apply to any particle (including the light pion)

  3D distributions = **mechanical response functions**
  correlation functions ($\neq$ probabilities!)
  if corrections "reasonably small" → we do not need to worry

  relative correction for $\langle r_E^2\rangle = \int d^3r\, r^2 T_{00}(r)/m$ is $\delta_{\text{rel}} = 1/(2m^2R^2)$ Hudson, PS PRD (2007)

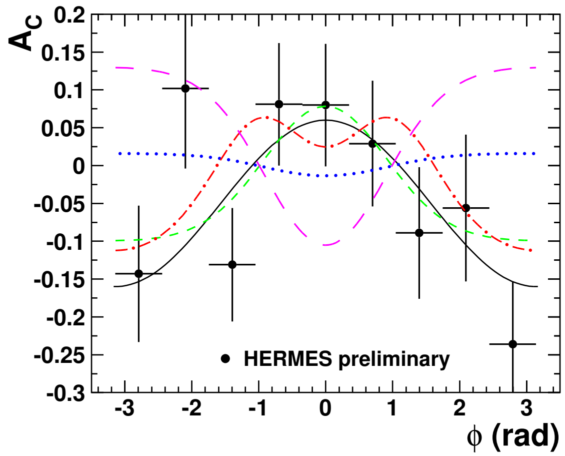
  numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{{}^4\text{He}}_{5\times10^{-4}}$, $\underbrace{{}^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{{}^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{{}^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{{}^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{{}^{208}\text{Pb}}_{2\times10^{-8}}$

- for nucleon 3D description strictly justified in large-N_c limit
  S. Coleman: "$1/N_c$ only small parameter in QCD at all energies" (in Aspects of Symmetry)

- **important: nucleon mass → heavy, quark mass → anything**

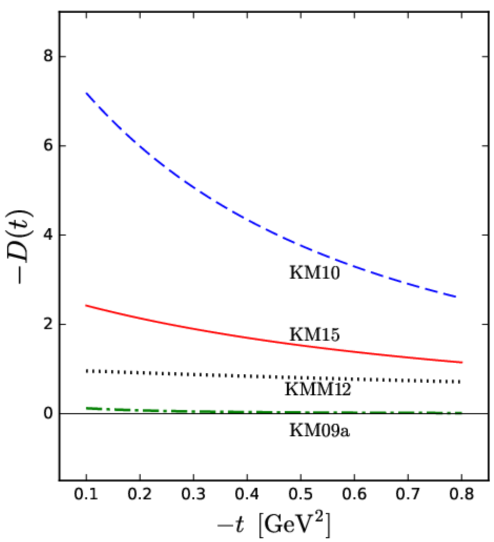
Results from experiment & phenomenology

- HERMES proceeding NPA711, 171 (2002) (model-dependent)



beam charge asymmetry (DVCS e^+ vs e^-)
dotted line: VGG model without D-term (ruled out)
dashed line: VGG model + positive D-term (ruled out)
dashed-dotted: VGG model + **negative** D-term (yeah!)
(cf. Belitsky, Müller, Kirchner, NPB629 (2002) 323)

- fits by Kresimir Kumerički, Dieter Müller et al: $D < 0$ needed! (model-independent)



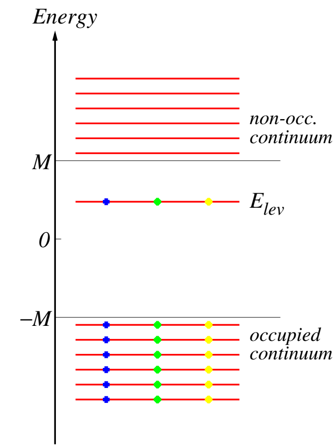
DVCS parametrizations from:
Kumerički, Müller, NPB 841 (2010) 1
Kumerički, Müller, Murray, Phys. Part. Nucl. 45 (2012) 723
Kumerički, Müller, EPJ Web Conf. 112 (2016) 01012
Fig. 9 in ECT* workshop proceeding 1712.04198
statistical uncertainty of D in KMM12: $\sim 50\,\%$,
statistical uncertainty of D in KM15: $\sim 20\,\%$.
unestimated systematic uncertainty
K.Kumerički private communication

more on chiral quark-soliton model

- $\mathcal{L}_{\rm eff} = \overline{\Psi}\,(i\,\partial\!\!\!/ - M\,U^{\gamma_5})\Psi,\quad U = \exp(i\tau^a\pi^a/f_\pi)$
  Diakonov, Petrov, Pobylitsa, NPB 306, 809 (1988)

  solve in large-N_c limit, where $U(x) \to U(\vec{x})$ static mean field
  Witten NPB 160, 57 (1979)

  Hamiltonian $H = -i\gamma^0\gamma^i\nabla^i + \gamma^0 M\,U^{\gamma_5}$ with $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$
  spectrum discrete level and continua

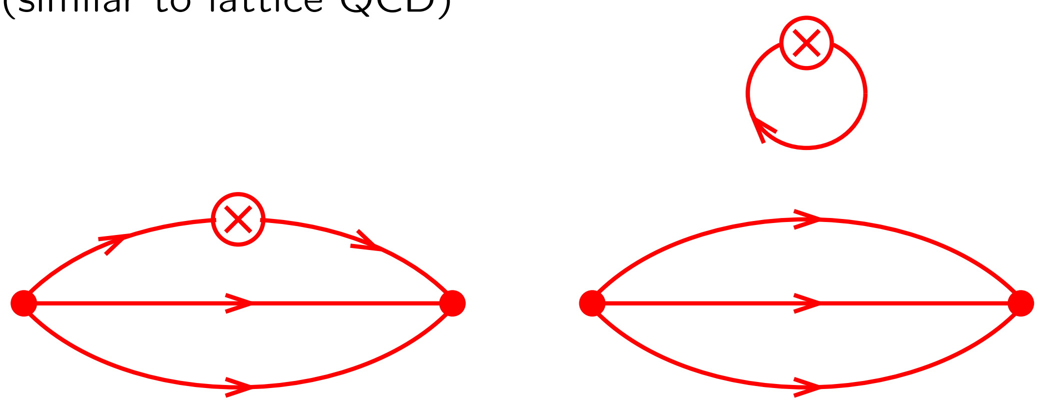


$$\langle N'|\hat{T}_{\mu\nu}|N\rangle = \lim_{T\to\infty} \frac{\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\; J_N(\tfrac{T}{2})\;\hat{T}_{\mu\nu}\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}{\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\; J_N(\tfrac{T}{2})\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}$$

$$= 2M_N \int d^3x\; e^{i(\vec{p}\,'-\vec{p})\vec{x}}\,N_c \sum_{n,\rm occ} \overline{\Phi}_n(\vec{x})(\tfrac{1}{2}i\gamma^\mu\partial^\nu + \tfrac{1}{2}i\gamma^\nu\partial^\mu)\Phi_n(\vec{x}) + \ldots$$

$$= 2M_N \int d^3r\; e^{i\vec{\Delta}\,\vec{r}}\,T_{\mu\nu}(\vec{r}) + \mathcal{O}(1/N_c^2) \quad \to$$ **3D spatial densities!**

connected & disconnected diagrams
(similar to lattice QCD)



$\sum_n \overline{\Phi}_n \ldots \Phi_n \neq$ nucleon wave function(!)
but Greens function (quark field propagator in background mean field)

$$G_{ij}(x,y) = i\langle 0|T\{\psi_i(y)\bar{\psi}_j(x)\}|0\rangle$$
$$= \langle y|\frac{1}{i\partial\!\!\!/ - MU^{\gamma_5}}|x\rangle$$
$$= i\Theta(x^0-y^0)\sum_{\rm non} e^{-iE_n(x^0-y^0)}\Phi_{n,i}(\vec{x})\bar{\Phi}_{n,j}(\vec{y})$$
$$-i\Theta(y^0-x^0)\sum_{\rm occ} e^{-iE_n(x^0-y^0)}\Phi_{n,i}(\vec{x})\bar{\Phi}_{n,j}(\vec{y})$$

- We compute a three-point function in effective theory (not related to 2D light front densities) matrix element in momentum space = F.T.(3D distribution) in mean field approach

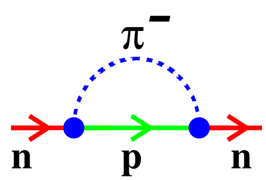
- **response functions!** Response to what? Insertion = stress tensor $\rightarrow$ mechanical properties response to ripples in metric, variations of g^{ik} $\Leftrightarrow$ virial theorem

- soliton field time-independent in leading order of $N_c \rightarrow \infty$.
  do we miss dynamics, quantum fluctuations, loop corrections? $\rightarrow$ We miss nothing!
  model reproduces the leading non-analytic terms of spontaneous chiral symmetry breaking!

$$M(m_\pi) = M(0) + b_{M1}\, m_\pi^2 + b_{M2}\, m_\pi^3 + \dots \text{ with } b_{M2} = -\,k\,\frac{3g_A^2}{32\pi f_\pi^2}$$

$$D(m_\pi) = D(0) + b_{D1}\, m_\pi + \dots \text{ with } b_{D1} = k\,\frac{3g_A^2 M}{16\pi f_\pi^2}$$

$$D(t) = D(0) + b_{D2}\sqrt{-t} + \dots \text{ with } b_{D2} = k\,\frac{9g_A^2 M}{128\pi f_\pi^2}$$

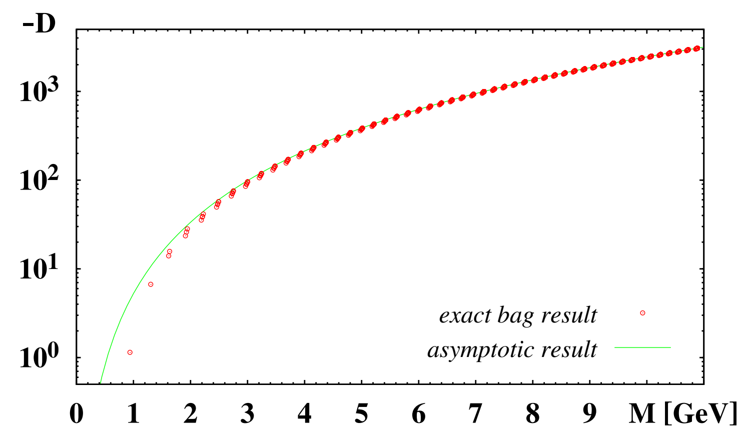


$\sqrt{-t}$ and $m_\pi \propto \sqrt{m_q}$ are non-analytic.
leading non-analytic terms model independent!
technical: $k = 1$ for N_c finite, $k = 3$ for $N_c \rightarrow \infty$

- concern: $N_c = 3$ not large. True. But large enough!
  $\frac{1}{N_c}$ the only small parameter of QCD at all energies.
  Witten 1979, Dashen, Jenkins, Manohar PRD 1994

D-term theory overview, selected results

- **free spin-$\frac{1}{2}$ fermion** $D = 0$ Donoghue et al (2002), Hudson, PS (2018)
- **pions** Goldstone bosons of chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- **scalar particles in** Φ^4 $D = -\frac{1}{3}$ Brean Maynard (2024)
  improvement term, Callan, Coleman, Jackiw (1970)
- **nuclei** (liquid drop model, Walecka model) $D \propto -A^{7/3}$
  Polyakov (2002), Guzey, Siddikov (2006); Liuti, Taneja (2005)
- **nucleon** bag model Ji, Melnitchouk, Song (1997), ...
  chiral quark soliton Goeke et al, PRD75 (2007), ...
- **χPT** Belitsky, Ji (2002), Alharazin, Djukanovic, Gegelia, Polyakov (2020), ...
  bound $D \leq -0.95(9)$ in chiral limit Gegelia, Polyakov (2021)
- **lattice QCD** Göckeler et al, PRL92 (2004), Shanahan, Detmold (2019), ...
  nucleon $D = -(3.87 \pm 0.97)$ Hackett, Pefkou, Shanahan (2024)
- **dispersion relations**
  $D = -2.66$ Pasquini, Polyakov, Vanderhaeghen (2014)
- **excited states** N^{th} excited Q-ball state: $M \propto N^3$ vs
  $D \propto N^8$ Mai, PS (2012); bag model Neubelt et al (2019)
- **reviews** Polyakov, PS, Int.J.Mod.Phys.A 33 (2018)
  Burkert et al, Rev.Mod.Phys. 95 (2023)



of all properties, D-term most sensitive to details of interaction & dynamics!

D-term in theory

- **free spin-0 particle** $D = -1$
  Pagels 1966; Hudson, PS 2017
- **free spin $\frac{1}{2}$ particle** $D = 0$
  Donoghue et al, (2002), Hudson, PS PRD97 (2018) 056003
- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

$$\begin{aligned} D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\ D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\ D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \end{aligned}$$

$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\,(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$
Donoghue, Leutwyler (1991)
estimates: Hudson, PS (2017)

- **nuclei** (liquid drop model, Walecka model) $\boldsymbol{D \approx -0.2 \times A^{7/3}}$ $\rightarrow$ DVCS with nuclei!
  Polyakov (2002),
  Guzey, Siddikov (2006);
  Liuti, Taneja (2005)

$$\begin{aligned} {}^{12}\text{C}: & \quad D = -6.2 \\ {}^{16}\text{O}: & \quad D = -115 \\ {}^{40}\text{Ca}: & \quad D = -1220 \\ {}^{90}\text{Zr}: & \quad D = -6600 \\ {}^{208}\text{Pb}: & \quad D = -39000 \end{aligned}$$

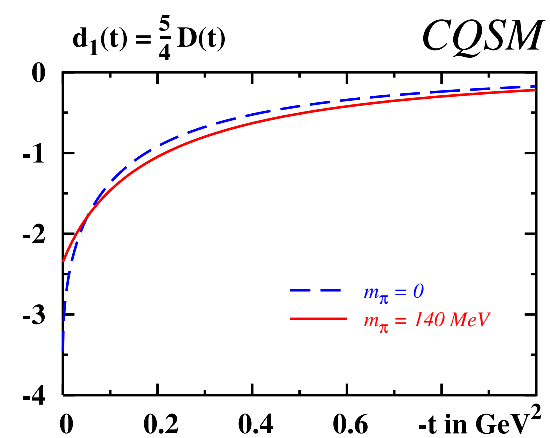
- ***Q*-balls** N^{th} excited Q-ball state: mass $M \propto N^3$ but $\boldsymbol{D \propto N^8}$
  Mai, PS PRD**86**, 096002 (2012)

- **nucleon, bag model** $\boldsymbol{D = -1.15 < 0}$
  Ji, Melnitchouk, Song (1997)

- **chiral quark soliton**
  Goeke et al, PRD75 (2007)

$$d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5\,k\,g_A^2 M}{64\,\pi\,f_\pi^2}\,m_\pi + \dots$$

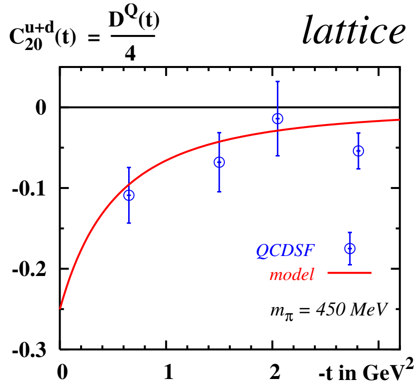
$$\overset{\circ}{d}{}_1'(0) = -\frac{k\,g_A^2\,M}{32\,\pi f_\pi^2\,m_\pi} + \dots \quad k = \begin{cases} 1, & N_c \text{ finite} \\ 3, & N_c \to \infty \end{cases}$$

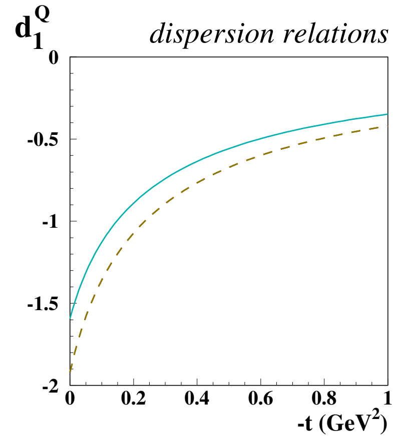


- **χPT**
  Belitsky, Ji (2002), Diehl et al (2006),
  Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023

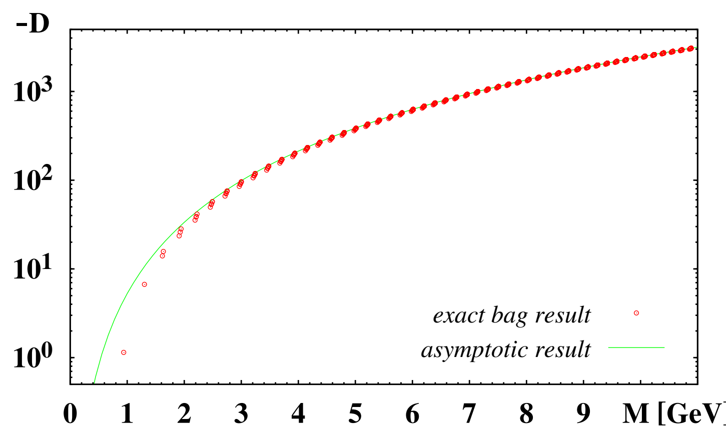
- **non-relativistic limit** $\boldsymbol{D = -N_c^2\,\frac{4\pi^2 - 15}{45}} = -4.89$
  Neubelt et al (2019) (in bag)

- **lattice:** QCDSF
  Göckeler et al, PRL92 (2004)
  $\mu = 2\,\text{GeV}$, $m_\pi = 450\,\text{MeV}$
  disconnected diagrams neglected
  recently:
  $D^g(t) < 0$ with $|D^g(t)| > |D^Q(t)|$
  Shanahan, Detmold, PRD99 (2019)





- **dispersion relations** $d_1^Q(t) = \frac{5}{4}\,D^Q(t)$
  Pasquini, Polyakov, Vanderhaeghen (2014)
  pion PDFs are input, scale $\mu^2 = 4\,\text{GeV}^2$



- **excitated states**

  in bag model Neubelt et al (2019)
  M over 1 order of magnitude
  D over 3 orders of magnitude

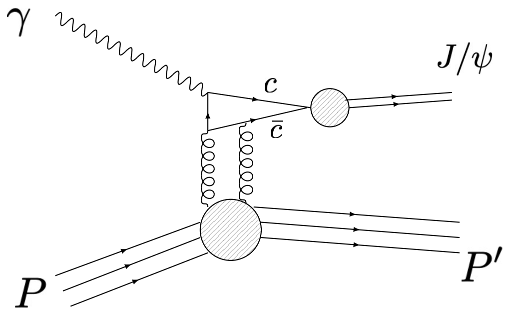
of all properties, D-term most sensitive (parameters, excitations)

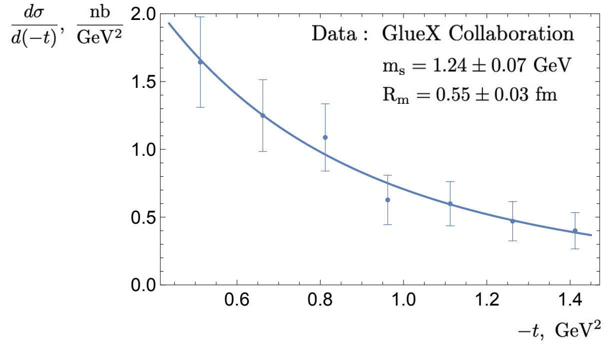
$\Rightarrow$ dynamics!

keep in mind: free spin $\frac{1}{2}$ theory $\rightarrow D = 0$;
i.e. D-term of nucleon due to dynamics!

- **form factor of** $\hat{T}_\mu^{\ \mu} = \frac{\beta(g)}{2g} F^2 + \mathcal{O}(m_q)$ from J/ψ photoproduction at threshold

Hatta 2019, Kharzeev 2021





Gluex PRL 123, 072001 (2019).

$\sqrt{\langle r^2_{\rm trace}\rangle} = 0.55 \pm 0.03$ fm $<$ charge radius ~ 0.84 fm

$\sqrt{\langle r^2_{\rm traceless}\rangle_g} \sim (0.3\text{–}0.35)$ fm of $A^g(t) = A^g(0) + \frac{1}{6}\, t\, \langle r^2_{traceless}\rangle_g + \ldots$ from QCD sum rules

Braun, Górnicki, Mankiewicz, Schäfer, PLB 302, 291 (1993)

explanation:
$\langle r^2_{\rm trace}\rangle_g$ due to one-instanton contributions, vs $\langle r^2_{\rm traceless}\rangle_g$ from instanton-anti-instanton
i.e. suppressed by instanton packing fraction Diakonov, Polyakov, Weiss (1996)

relation to other EMT form factors:

form factor $\langle p'|\hat{T}^\mu{}_\mu|p\rangle = \bar{u}(p')u(p)\, F_{\rm tr}(t)$ where $F_{\rm tr}(t) = 1 + \frac{1}{6}\, t\, \langle r^2_{\rm tr}\rangle + \mathcal{O}(t^2)$

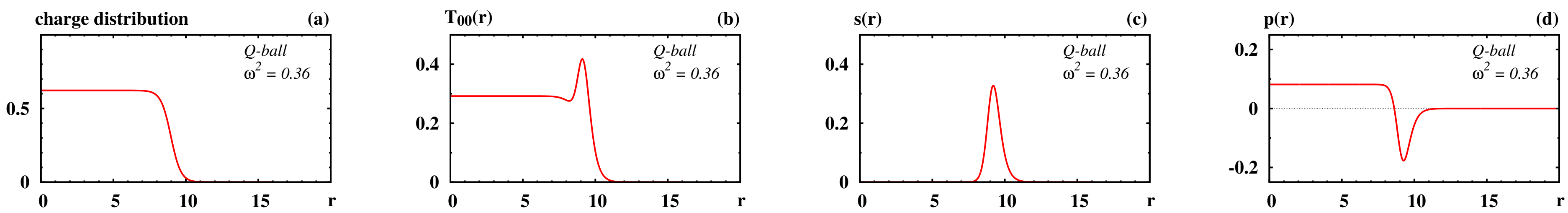
$$F_{\rm tr}(t) = A(t) + \frac{t}{4M^2} B(t) - \frac{3t}{4M^2} D(t) = 1 + t\left(\frac{dA(0)}{dt} - \frac{3D}{4M^2}\right) + \mathcal{O}(t^2)$$

$\langle r^2_{\rm trace}\rangle = 6\, A'(0) - \frac{9D}{2M^2}$ "mass radius"

Q-balls $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$, $V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3$

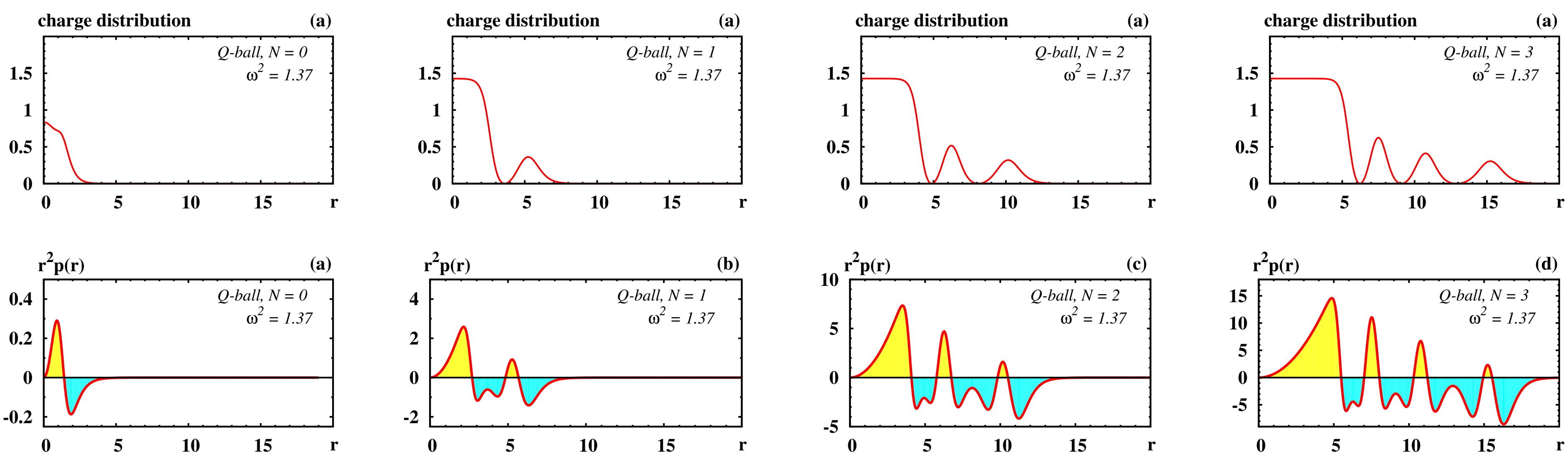
global U(1) symmetry, solution $\Phi(t, \vec{r}) = e^{i\omega t}\,\phi(r)$

- ground state properties for large Q-ball



- excitations: $N = 0$ ground state, $N = 1$ first excited state, etc Volkov, Wohnert 2002; Mai, PS 2012

charge distribution exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros

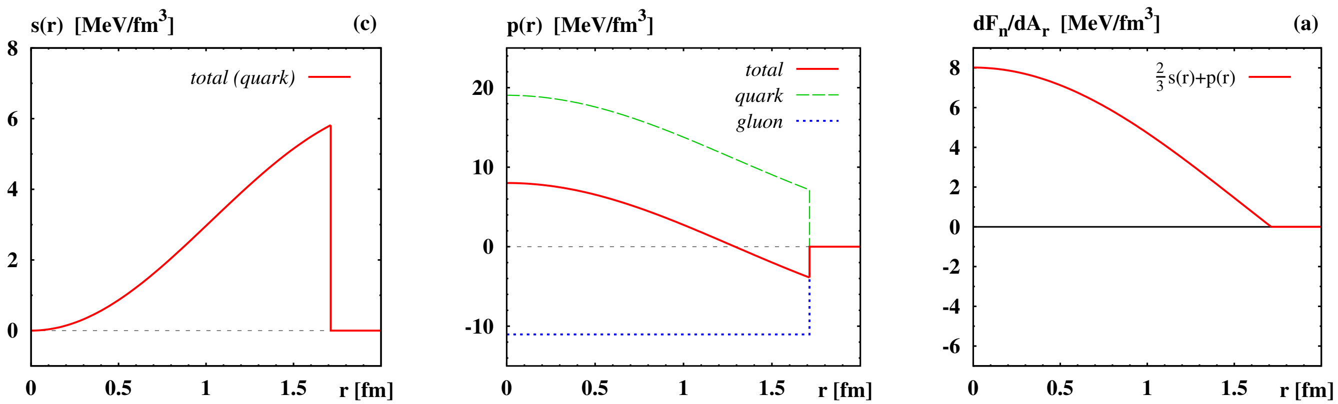


excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

bag model Neubelt, Sampino, Hudson, Tezgin, PS, PRD101 (2020) 034013

- free quarks + boundary condition, formulated in large-N_c
- $T^{\mu\nu}(r) = T^{\mu\nu}_{\text{quarks}}(r) + T^{\mu\nu}_{\text{bag}}(r)$

  $T^{\mu\nu}_{\text{bag}}(r) = B\,\Theta(R-r)\,g^{\mu\nu}$ binding effect ("mimics gluons" Jaffe & Ji 1991)
- all distributions defined with Θ-functions, assume non-zero values at $r = R$



- only exception:
  the normal force = $\frac{2}{3}\,s(r) + p(r) > 0$ for $r < R$, becomes exactly zero at $r = R$
- this is how one determines the radius of a neutron star:
  solve Tolman-Oppenheimer-Volkoff equations with an "equation of state"
  where "radial pressure" $\frac{2}{3}\,s(r) + p(r)$ turns negative, define "end of the system"
- excitated states different pattern than Q-balls:
  $p(r)$ has one node (here 3163th excited state)
  but $D \sim \text{const} \times M^{8/3}$ bag & Q-balls
  deeper reason?