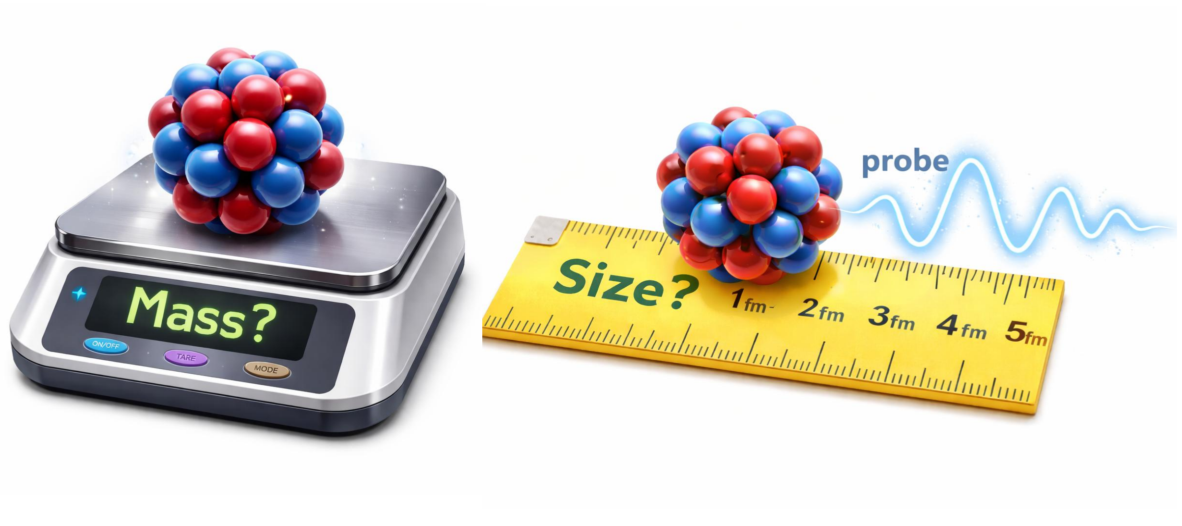
Workshop on NREC 2026

Ab initio charge form factors and radii of light iso-scalar nuclei

April 13–17 2026, Stony Brook Uni.



- Chiral forces & nuclear *ab initio* theory
- No-Core Shell-Model (NCSM)
- Charge-radius challenge
- Charge form factor and radius
- Summary and perspective

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PRC112 (2025) 024317, arXiv2512.15454, arXiv2601.09614

In collaboration with
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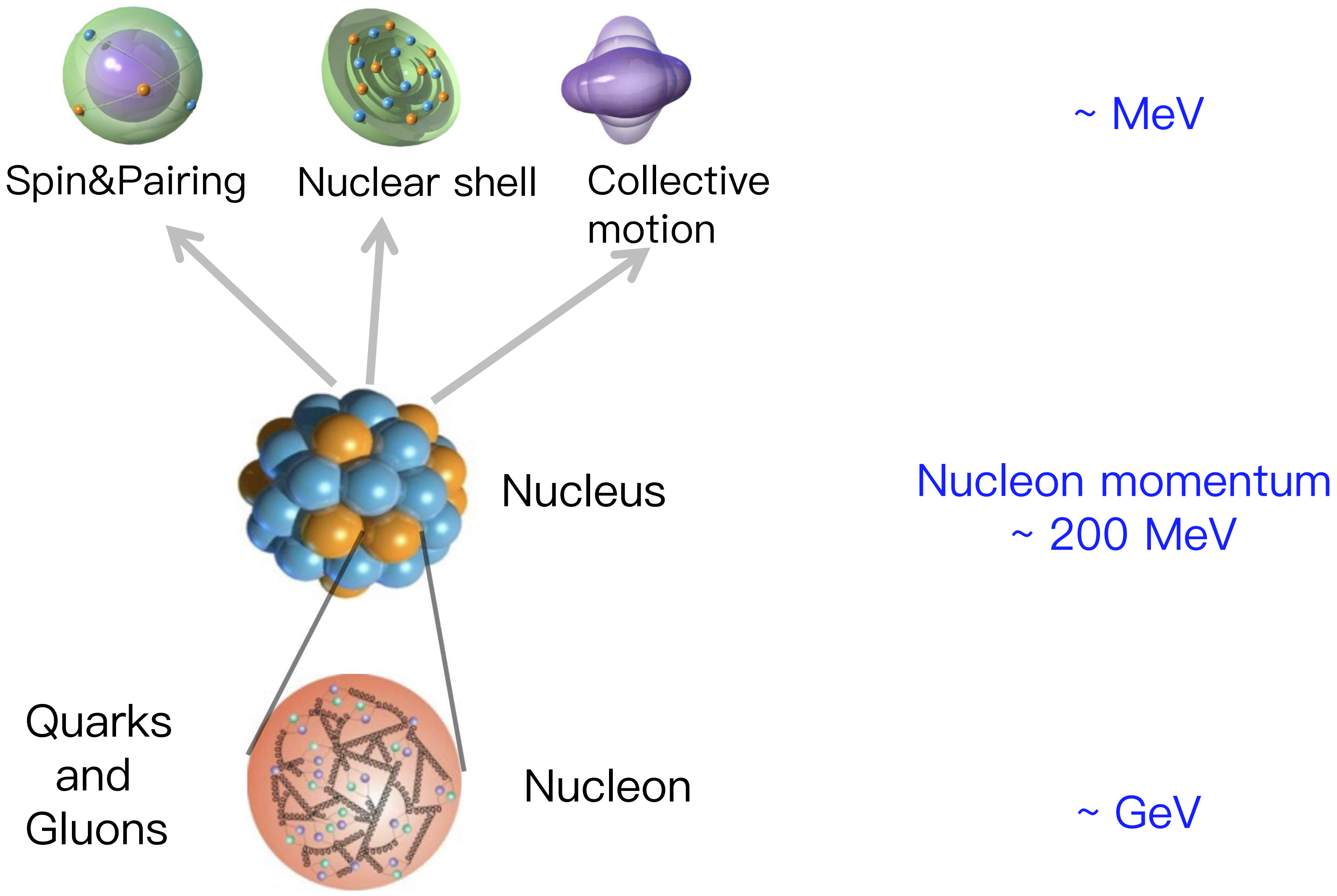
FZJ: Hoai Le, Ulf-G. Meißner, Andreas Nogga

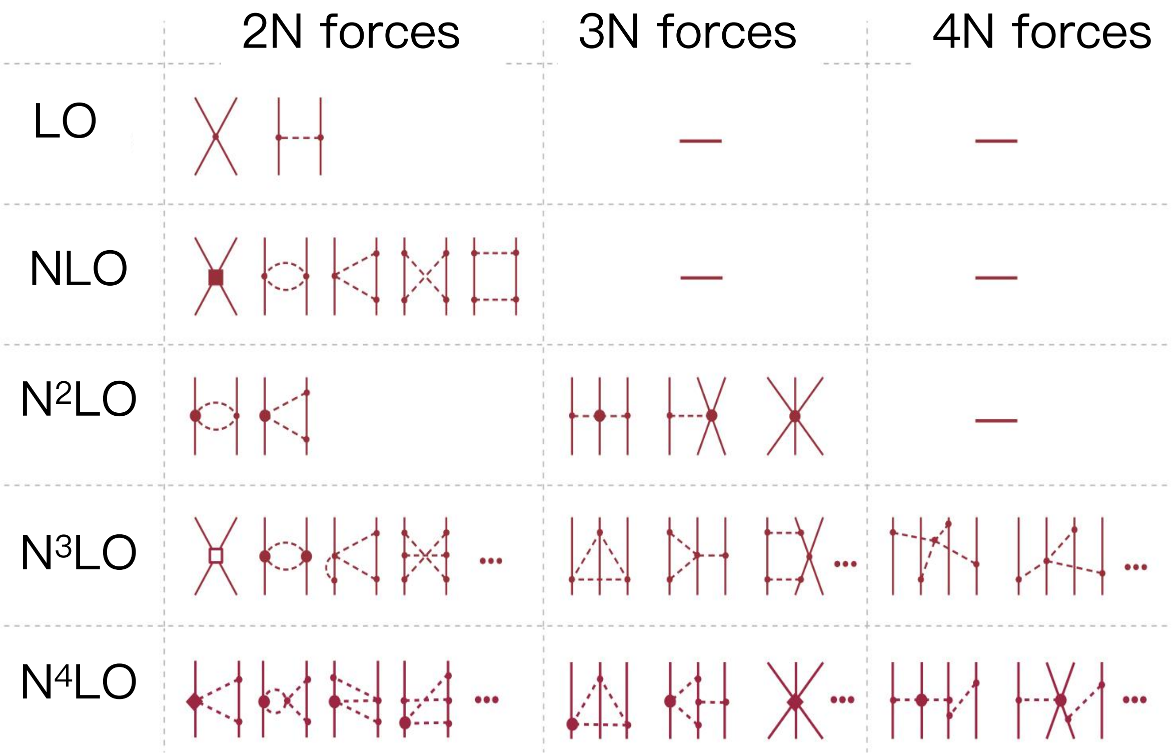
JÜLICH Forschungszentrum

The Emergence of Nuclei from QCD



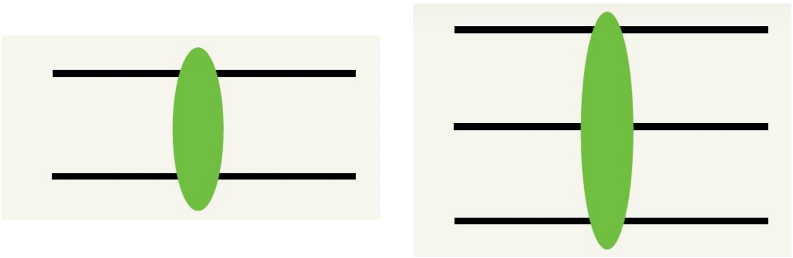
courtesy of M. J. Savage

Chiral effective field theory (EFT)



arXiv:1908.09349

Chiral forces



- Low–energy EFT with nucleons/pions as effective degrees of freedom
- Expansion parameter from separation of scale at low–energies

$$Q/\Lambda_b \sim 1/4$$

- High–energy physics captured by few low–energy coupling constants (LECs)
- Power counting predicts emergence of higher–body operators and controlls uncertainties
- Consistent charge density and current

Nuclear *ab initio* theory

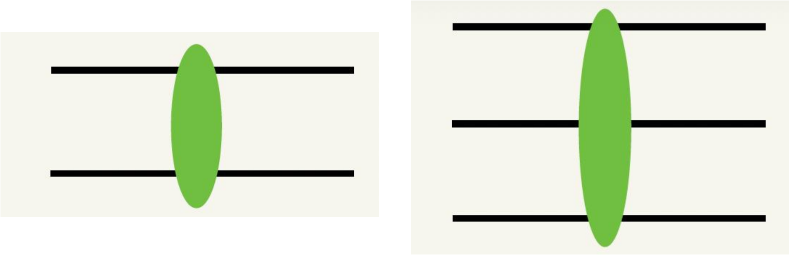
High precision era!

Exp. uncertainty
Nuclear mass ~ keV (isomeric states: eV)
Charge radius ~ 0.01 fm (μ–atom: 10^{-3} fm)

How about the *ab initio* descriptions for the basis properties, e.g., nuclear mass and size?

courtesy of M. J. Savage

Many-Body Methods



Chiral NN and NNN forces

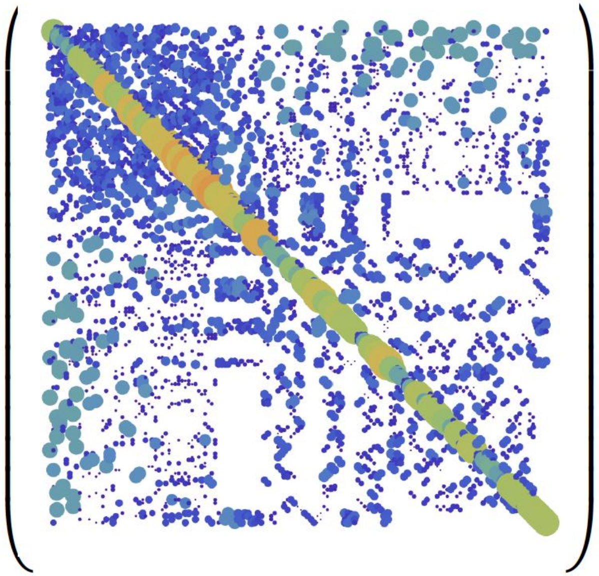
Nuclear *ab initio* theory: A microscopic framework that predicts nuclear observables by solving the many-body systems using chiral (bare) interactions as the only input.

Machleidt, FBS 63-77 (2023)

Nuclear *ab initio* theory: No–Core Shell Model (NCSM)

$$H = T_{\rm rel} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

- NCSM is an *ab initio* Configuration Interaction method that solves the *A*–body Schrödinger equation without a frozen core in the Harmonic Oscillator (HO) basis.



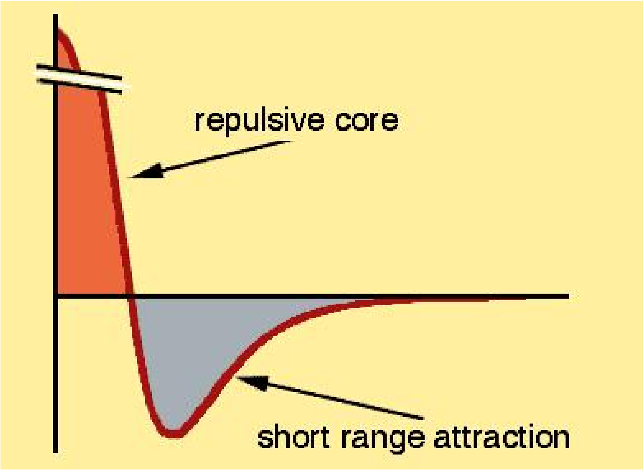
$$\begin{pmatrix} \vdots \\ c_{i'}^{(n)} \\ \vdots \end{pmatrix} = E_n \begin{pmatrix} \vdots \\ c_{i}^{(n)} \\ \vdots \end{pmatrix}$$

Exact solution for given model space.

The "Curse of Dimensionality" and memory wall lead to that NCSM is only for light system.
The HO basis is localized e^{-r^2}, it is excellent for bound states but struggles to describe long–range observable.

Barrett, Navratil, Vary,
Prog. Part. Nucl. Phys. 69, 131 (2013).

Precondition of NCSM: Soften the bare interaction

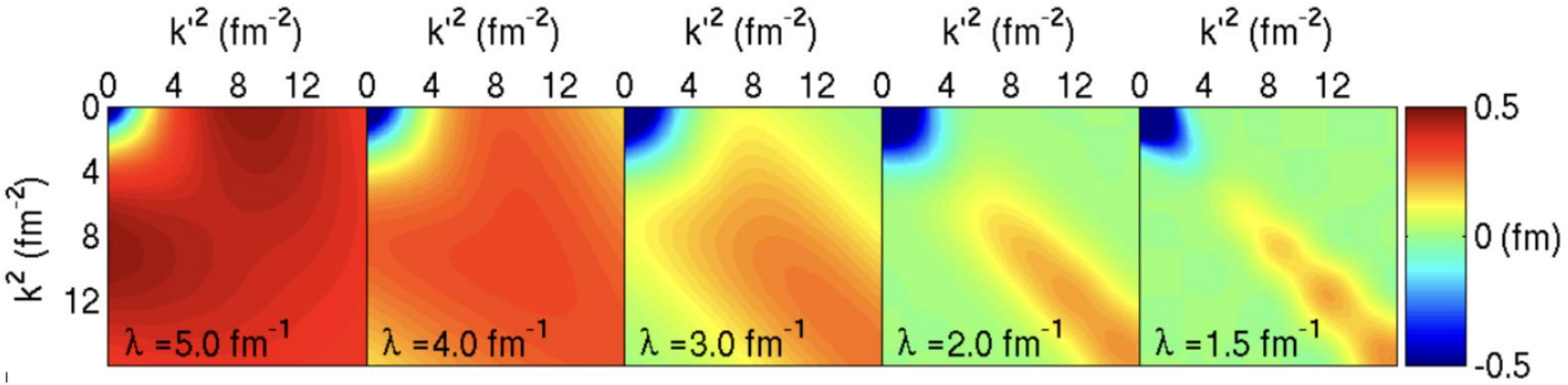


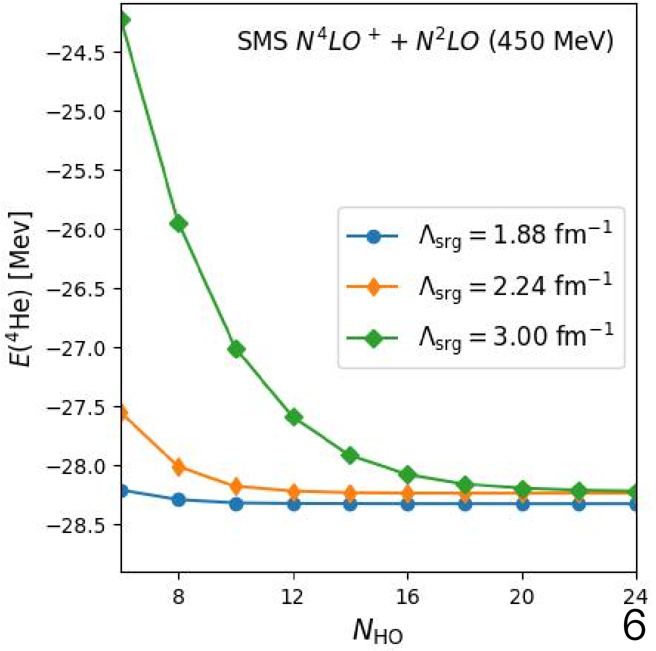
- Short–Range Repulsion: Strong repulsion that couples low– and high–momentum states.
- Convergence Problem: Resolving these fine short–range details requires an unmanageable large HO space.
- Idea: Decouple high– and low–momentum correlations to accelerate convergence while preserving low–energy observable.

The common way to soften the force is **S**imilarity **R**enormalization **G**roup (SRG).

$$H_0 = T + V_0 \quad H_\lambda = U_\lambda H_0 U_\lambda^\dagger \quad \frac{dV_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda], \quad \frac{dU_\lambda}{d\lambda} = \eta_\lambda U_\lambda$$

Bogner, Furnstahl, Perry, PRC 75, 061001 (2007)





Nuclear *ab initio* theory: Mass & radius

Precise and high–order chiral forces → How about the *ab initio* descriptions for the basis properties, nuclear mass and size? How about the accuracy?

Chiral interactions in our studies: SMS forces from the LENPIC Collaboration

- SMS: **S**emilocal **M**omentum–**S**pace regularized Chiral interactions
- NN (N^4LO^+): Perfect description for nucleon–nucleon scattering data

  Reinert *et al.*, PRL 126, 092501 (2021)
- 3N (N^2LO): Nd scattering, ^3H binding energy

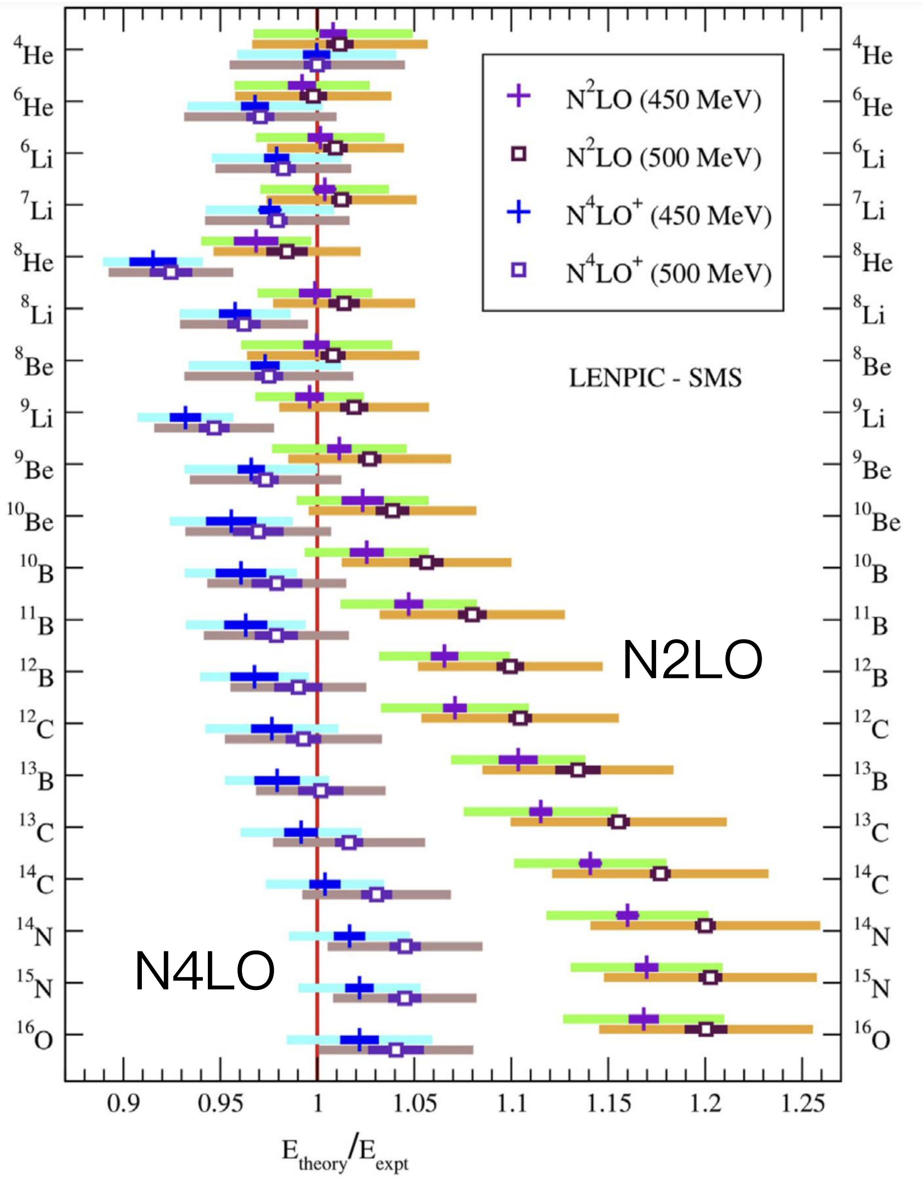
  Epelbaum, Krebs, Reinert, Front Phys 8, 98 (2022)
- 2N Chiral electromagnetic density and current

  → EM process

  Krebs, Epelbaum, Meißner FBS 60, 31 (2019)
  Krebs EPJA 56, 9 (2020) (Review)

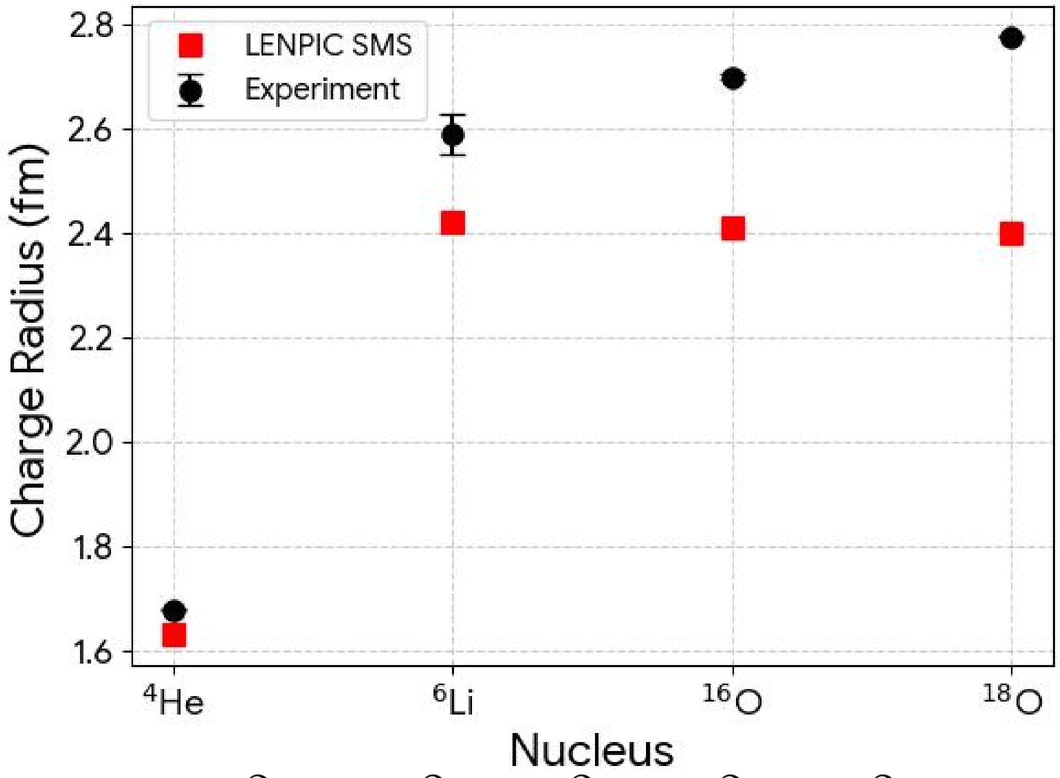
SMS NN(N^4LO^+)+3N(N^2LO) with Jacobi–NCSM (A<9)

Charge radius: Exp. Meets Theory



Maris, Le, Nogga *et al.*, Front Phys. 11, 1098262 (2023)

- ➢ Perfect description of binding energies.
- ➢ Safe for nuclear phenomena at MeV level.



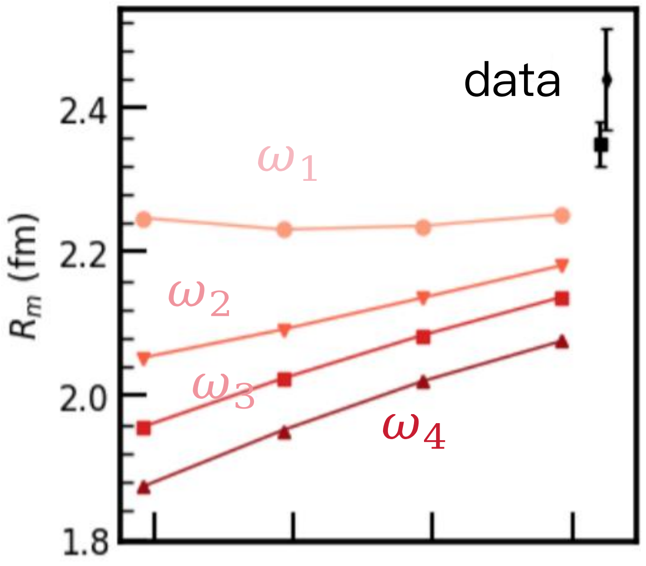
$$R_c^2 = R_p^2 + r_p^2 + r_n^2 + r_{\mathrm{DF}}^2$$

- ➢ Systematic underestimation of charge radii.
- ➢ Also for other chiral interactions, see

Ca: NatPhys12,594(2016), K: NatPhys17,439(2021)
Sc: PRL134,182501(2025), Si: PRL132,162502(2024)
Sn: PRL135,222501(2025), and more ...

What's missing for the study of charge radius in NCSM?

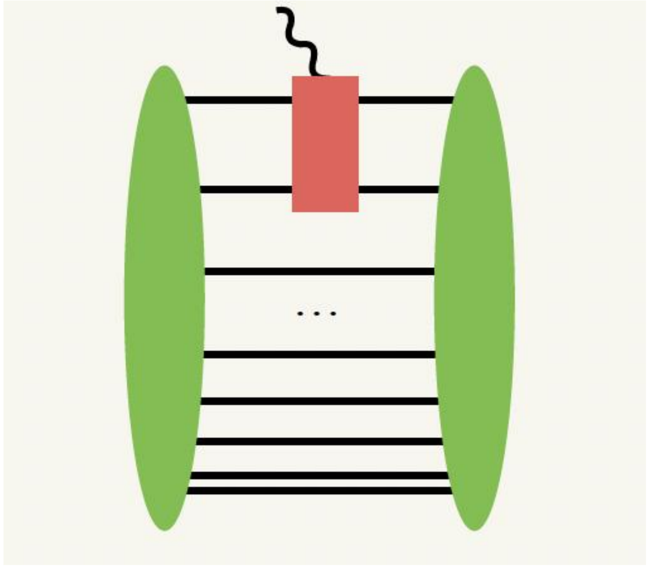
A. Limited basis size



- Long–range operator does not converge with limited basis size.

Barrett, Navratil, Vary,
Prog. Part. Nucl. Phys. 69, 131 (2013).

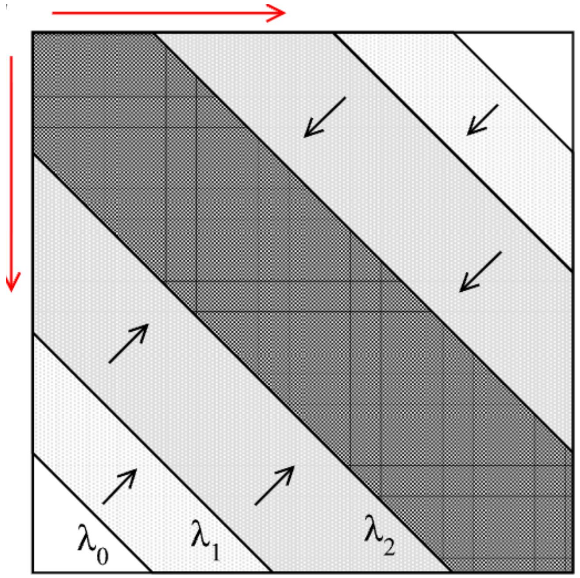
B. Missing two-body contribution



- Consistent regularization
- Unclear for $A \geq 4$ systems

Deuteron：
Filin *et al.*, Phys. Rev. Lett. 124, 082501 (2020)

C. Interaction softening

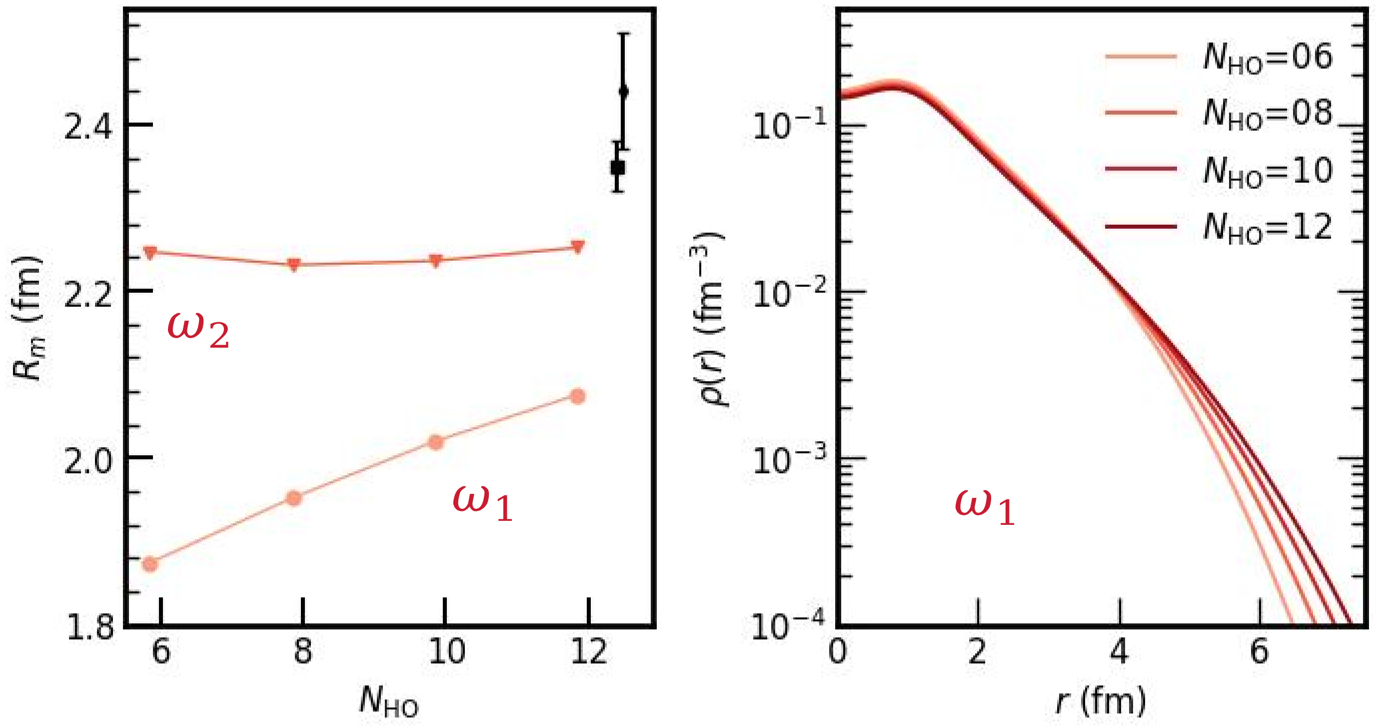


- Small for long–range operators
- Unclear for other observable

Hergert, Phys. Scr. 92, 023002 (2017)
Schuster *et al.*, Phys. Rev. C 90, 011301(R) (2014)

Matter radius: Overcoming the limitation of basis size

- Different convergence patterns.
- Basis space too small to be converged.



$$\rho(r) \sim e^{-\kappa r^2}, \quad r \to \infty$$

SUN, Le, Meißner, Nogga, Phys. Rev. C 112, 024317 (2025)

Matter radius: Overcoming the limitation of basis size

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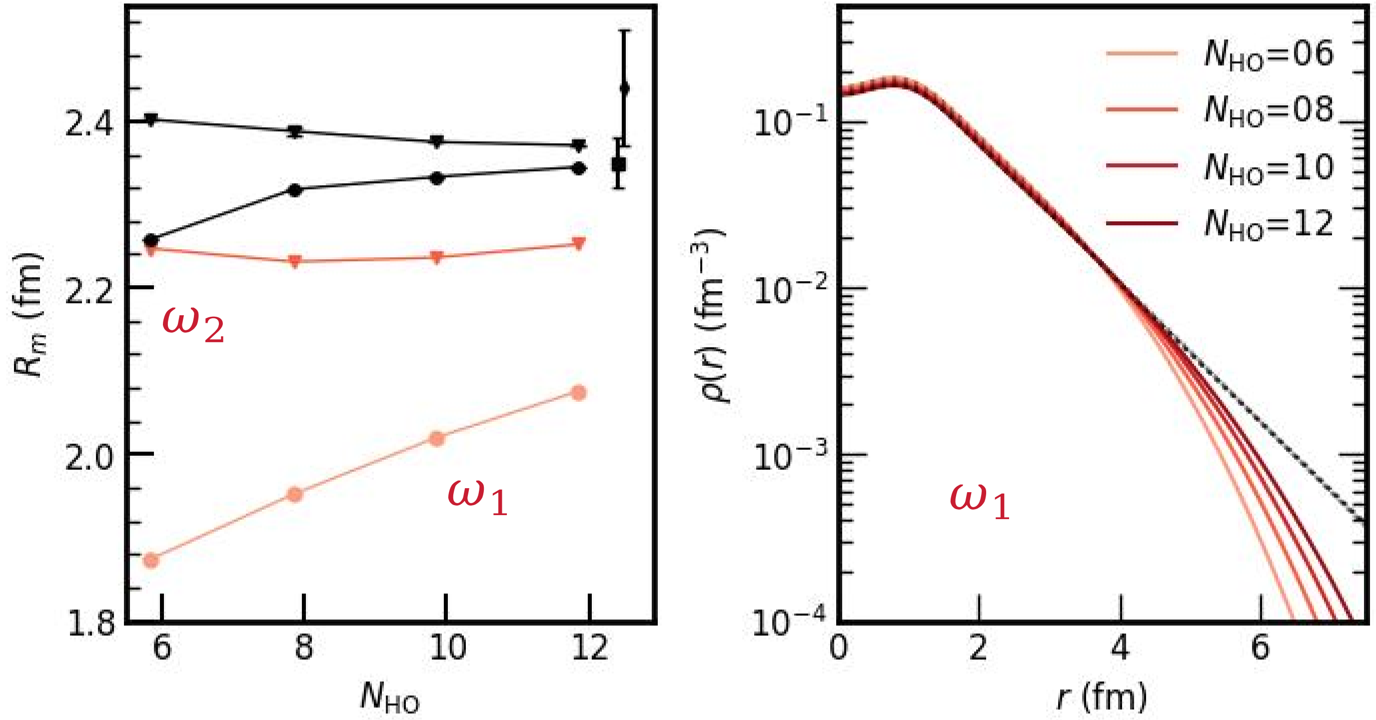


$$\rho(r) \sim e^{-\kappa r^2}, \quad r \to \infty$$

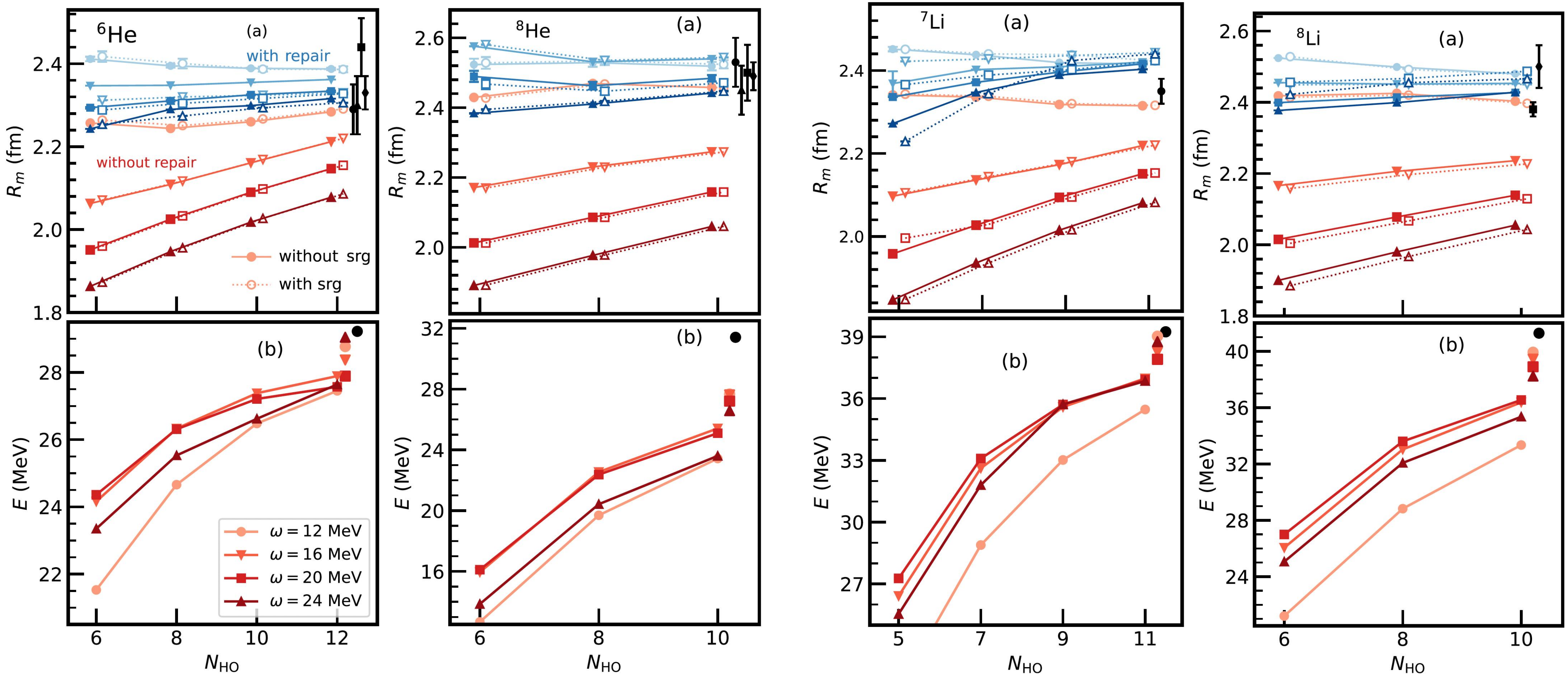
$$\frac{d^2}{dr^2}(r\Psi) - \frac{2m}{\hbar^2}\left(E_i + \frac{q}{r}\right)(r\Psi) = 0, \qquad r \to \infty$$

$$\rho(r) \ \to \ r^{-2\alpha}\, e^{-r/a} \qquad r \to \infty.$$

- After tail correction, radii converge w.r.t. basis size.
- Consistence with experimental matter radius.

SUN, Le, Meißner, Nogga, Phys. Rev. C 112, 024317 (2025)

Matter radius: Overcoming the limitation of basis size

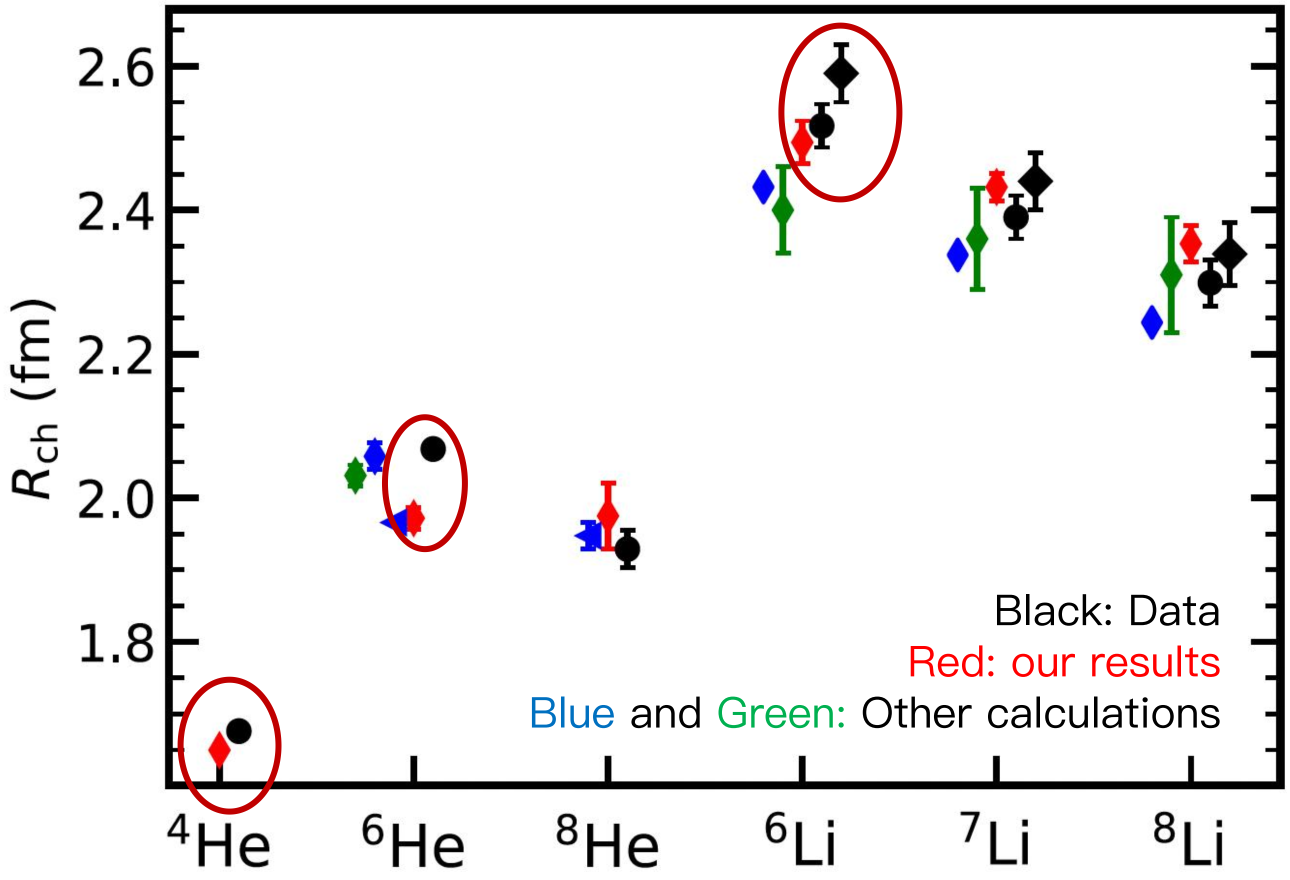


^{6}He: 2n halo, ^{8}He: 4n halo?

Matter radius of light nuclei can be well reproduced, also for halo nuclei.

SUN, Le, Meißner, Nogga, Phys. Rev. C 112, 024317 (2025)

Charge radius: Still underestimation?

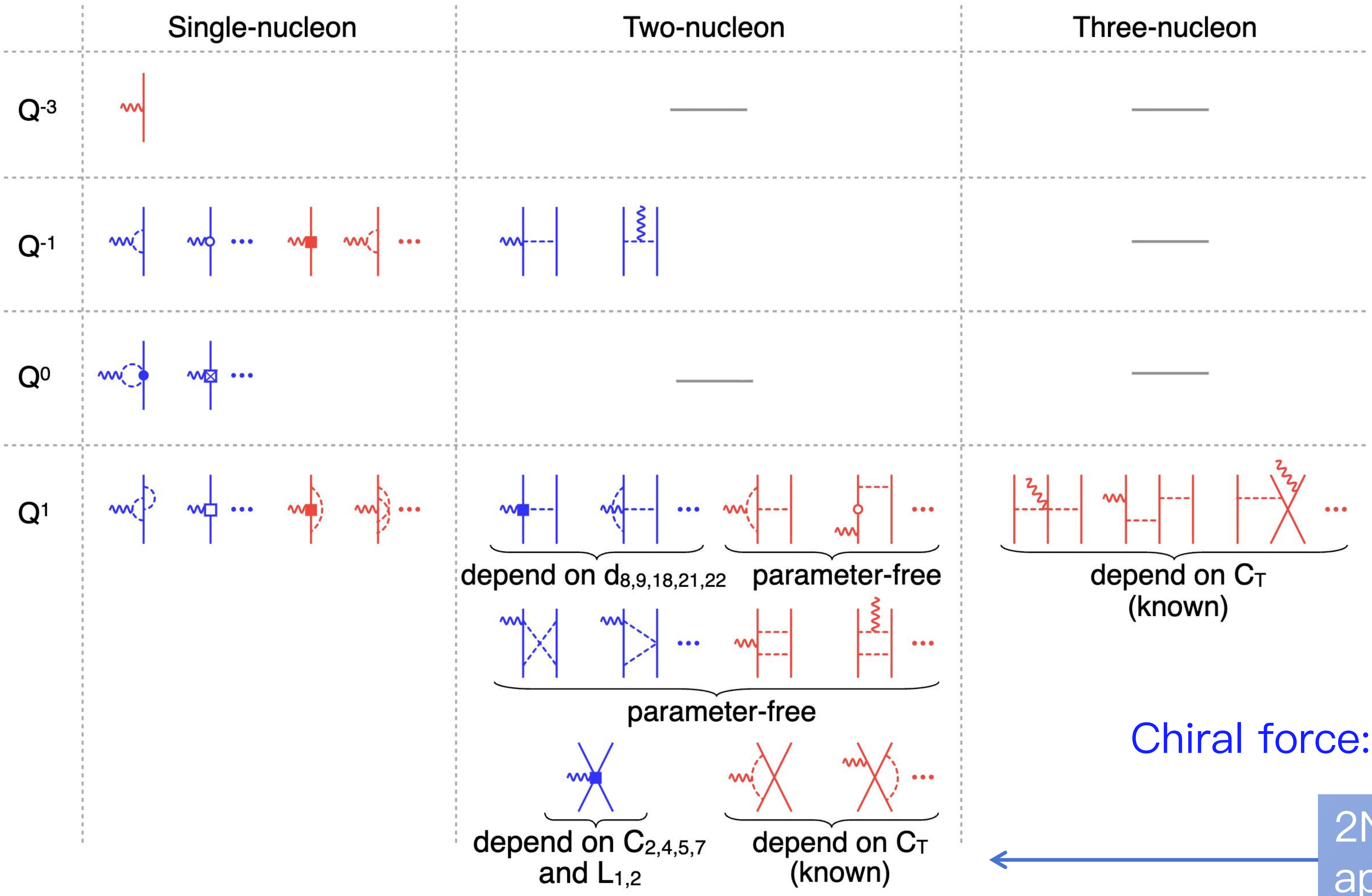


After tail correction, we still slightly underestimate the charge radii.

The contribution from two-body charge density from Chiral EFT.

SUN, Le, Meißner, Nogga, Phys. Rev. C 112, 024317 (2025)

Charge radius as a EM observable: Role of two-body operators



- Meson exchange currents before 2000s
- Chiral EFT allows us a systematic expansion for charge and current operators
- Consistent regularization for the charge and current operators for rigorous predictions
- Iso–scalar operators have been developed.

Chiral force: SMS 2N(N^4LO^+)+3N(N^2LO)

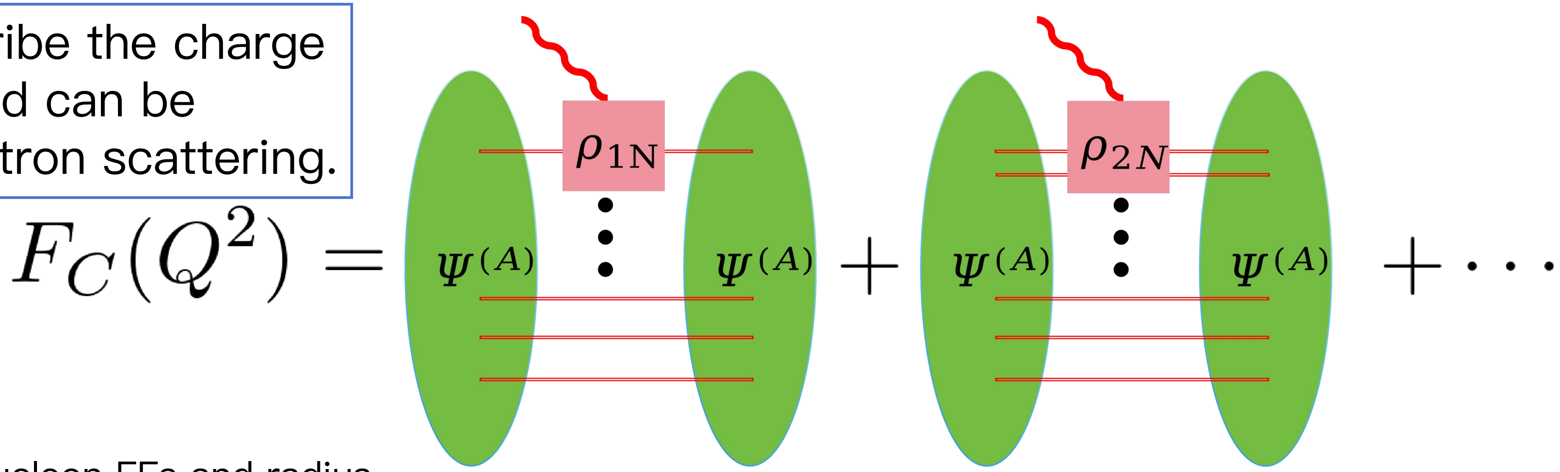
2N operators naturally appear in high Chiral orders.

More details: arXiv:1902.06839, arXiv:1908.09349, PRC103–024313(2021)

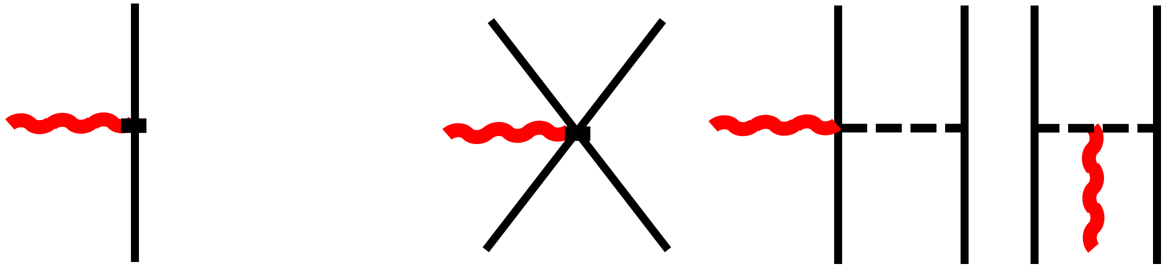
Consistence of the theory requires the contribution from two–body operators.

Charge Form Factors (FFs) and radius

The charge FFs describe the charge spatial distribution and can be measured by the electron scattering.



Nucleon FFs and radius
Lin, Hammer, Meißner PRL128, 052002(2022)
r_p=0.84075(64) fm CODATA2022
r_n^2=−0.0105 fm² PRL 124, 082501 (2021)



Contact term

3 LECs

One−pion−exchange (OPE) terms

Parameter free

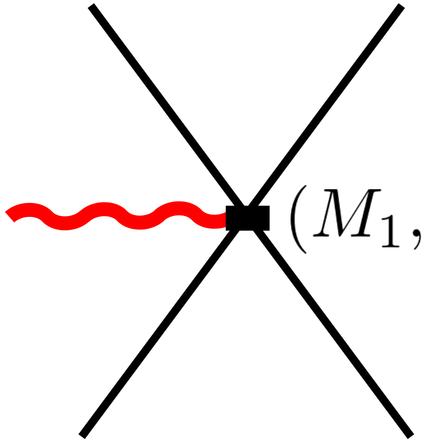
- 1N: Photon interacts with one−nucleon inside the nucleus
- 2N: Photon interacts with nucleon−pair directly (contact term) and by the exchange of pion (OPE term).

LECs in the contact charge operator

GES: Lin,Hammer,Meißner PRL128, 052002(2022)

r_p=0.84075(64) fm CODATA2022

r^2_n=−0.0105 fm^2 PRL 124 (2020) 082501



$$\rho_{\text{Cont}}(\boldsymbol{k}) = 2eG_E^S(\boldsymbol{k}^2)\Big[\, M_1 \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+3}{4}\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\boldsymbol{k}^2 + M_2\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\left((\boldsymbol{k}\cdot\sigma_1)(\boldsymbol{k}\cdot\sigma_2)-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{k}^2\right) + M_3\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\left(\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+3}{4}\right)\boldsymbol{k}^2\Big],$$

Isospin−zero to isospin−zero M_1 and M_2 are fixed by deuteron FFs PRL 124 (2020) 082501

LECs in the contact charge operator

GES: Lin,Hammer,Meißner PRL128, 052002(2022)
r_p=0.84075(64) fm CODATA2022
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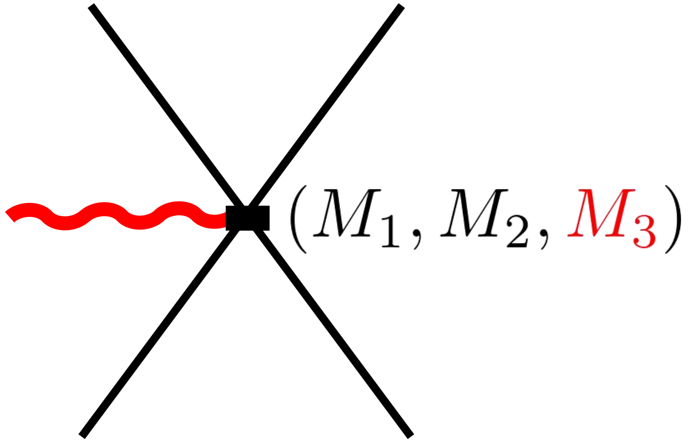
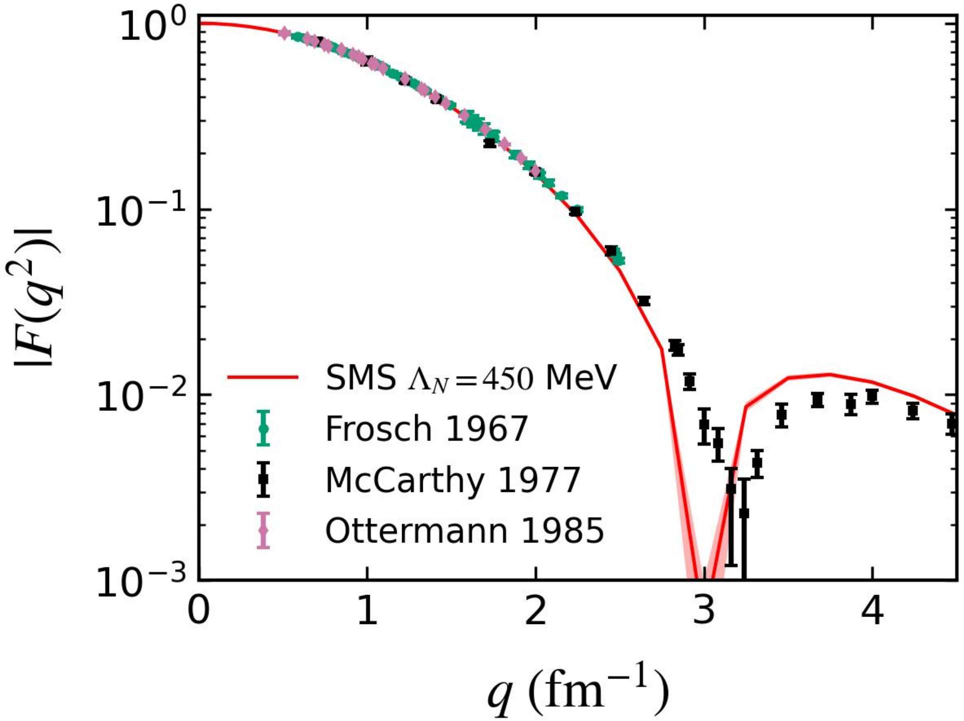


$$\rho_{\text{Cont}}(\boldsymbol{k}) = 2eG_E^S(\boldsymbol{k}^2)\Bigg[M_1 \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+3}{4}\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\boldsymbol{k}^2 + M_2\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\left((\boldsymbol{k}\cdot\sigma_1)(\boldsymbol{k}\cdot\sigma_2)-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{k}^2\right) + M_3\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\left(\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+3}{4}\right)\boldsymbol{k}^2\Bigg],$$

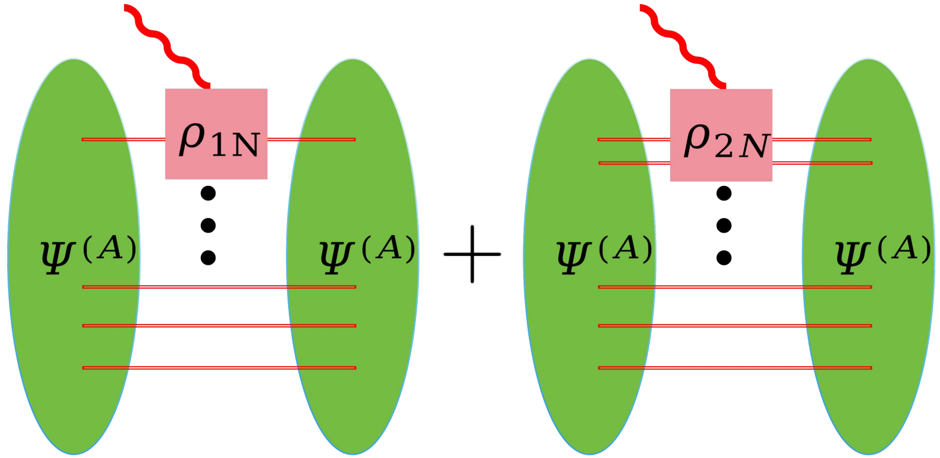
^{4}He charge radius 1.67824(83) fm Nature 589 527–531 (2021) for M_3

SUN *et al.*, arXiv2601.09614



Charge FFs and radius in NCSM

$$F_C(Q^2) =$$



$$+\cdots$$

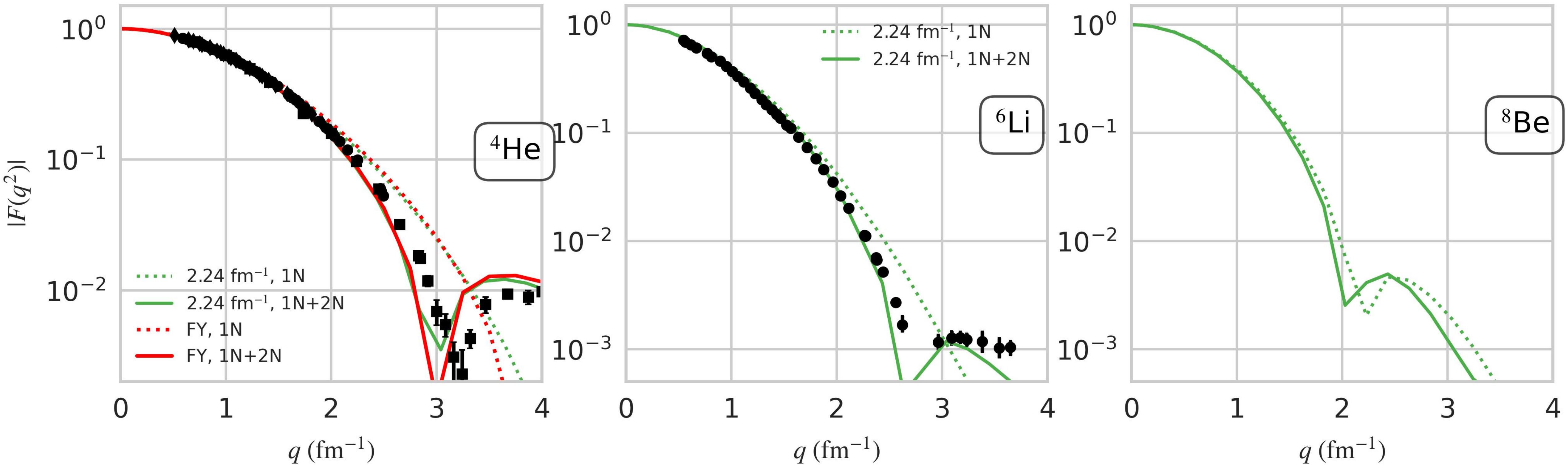
$$\mathrm{Tr}\left[\rho^{2\mathrm{b}}(Q)\mathcal{O}^{2\mathrm{b}}(Q)\right]$$

- Numerical partial wave decomposition for one- and two-body charge density operators to get matrix elements.
- Transition density matrix constructed from nuclear many-body wave functions in NCSM.

$$r_C^2 \equiv -6\,\frac{\partial F_C(Q^2)}{\partial Q^2}\bigg|_{Q=0} = \underbrace{R_p^2 + r_p^2 + r_n^2 + r_{\mathrm{DF}}^2 + r_{\mathrm{SO}}^2}_{1\mathrm{N}} + \underbrace{r_{1\pi}^2 + r_{\mathrm{Cont}}^2}_{2\mathrm{N}}$$

Missing 2N contributions for almost all *ab initio* calculations for charge radius!

Charge Form Factors: 1N + 2N



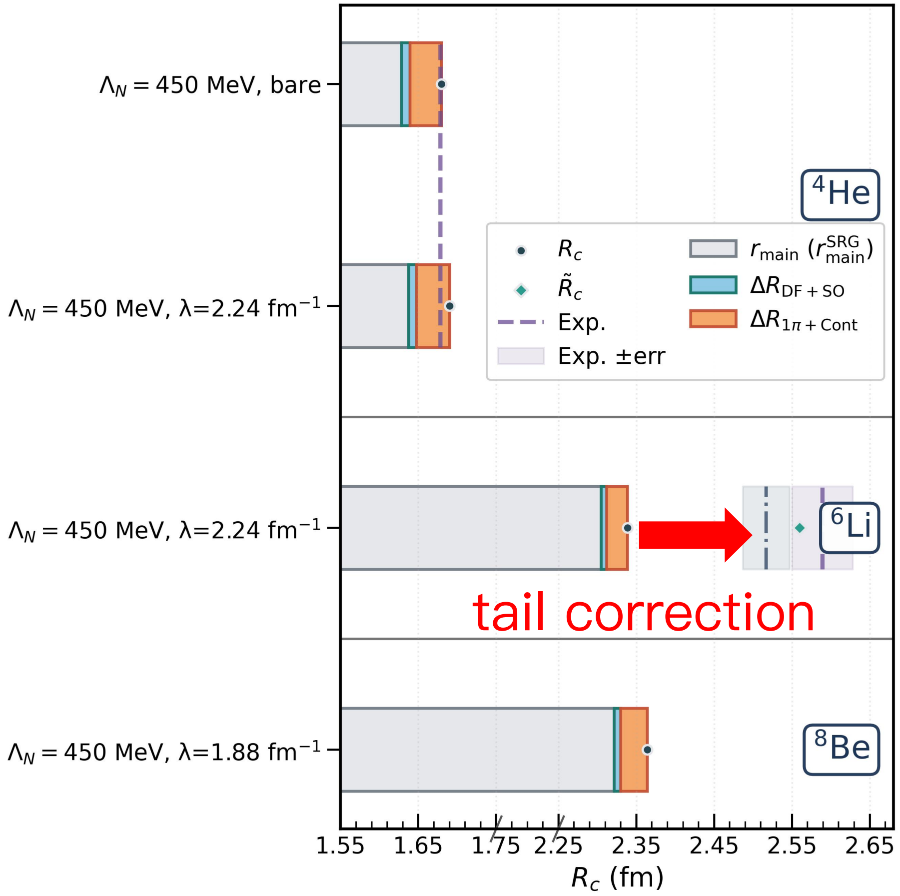
FY: Faddeev–Yakubovsky with the bare SMS interaction

- First *ab initio* FFs calculations for $A > 4$ systems using consistently regularized 2N operator
- 1N cannot produce the data at high momentum transfer and 1N+2N well reproduce the data
- 2N plays an important role in high-momentum part and pisition of diffraction minimum

Charge radius in NCSM: 1N + 2N

$$R_p^2 + r_p^2 + r_n^2 + r_{\mathrm{SO}}^2 + r_{\mathrm{DF}}^2 + r_{\mathrm{Cont}}^2 + r_{1\pi}^2$$

- 2N contribution is about 0.04 fm for *p*–shell nuclei.
- 2N plays an important role in accurate describe the charge radius.
- Lay the foundation for the extension to medium and heavy nuclei using other *ab initio* methods for solving nuclear radius challenge.



Truncation error: $\left(\frac{m_\pi}{\Lambda_b}\right)^3 \times R_c \approx 0.0015$ fm

Ab initio charge form factors and radii of light isoscalar nuclei: Role of the two-body charge density

Xiang-Xiang Sun, Vadim Baru, Arseniy A. Filin, Evgeny Epelbaum, Hermann Krebs Show All(7)

Jan 14, 2026

6 pages

e-Print: 2601.09614 [nucl-th]

Soften the chiral forces with SRG: Effects on charge radius

NCSM: $H_\lambda|\Psi_\lambda\rangle = E|\Psi_\lambda\rangle$, $\langle O\rangle = \langle\Psi_\lambda|O|\Psi_\lambda\rangle$

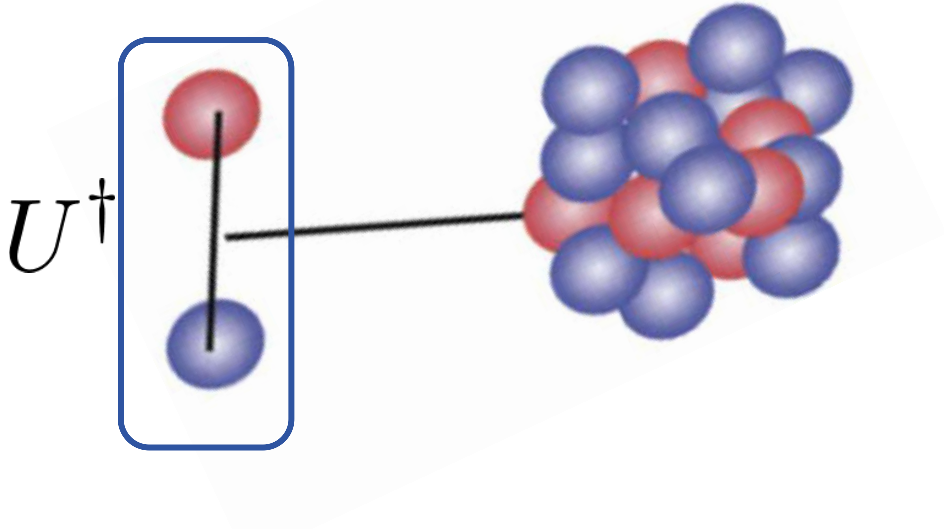
Self consistent evolution of other observable

$$\langle O\rangle \equiv \underline{\langle\Psi_{\lambda=0}|O|\Psi_{\lambda=0}\rangle} = \langle\Psi_\lambda|UOU_\lambda^\dagger|\Psi_\lambda\rangle$$

Our method is to directly calculate

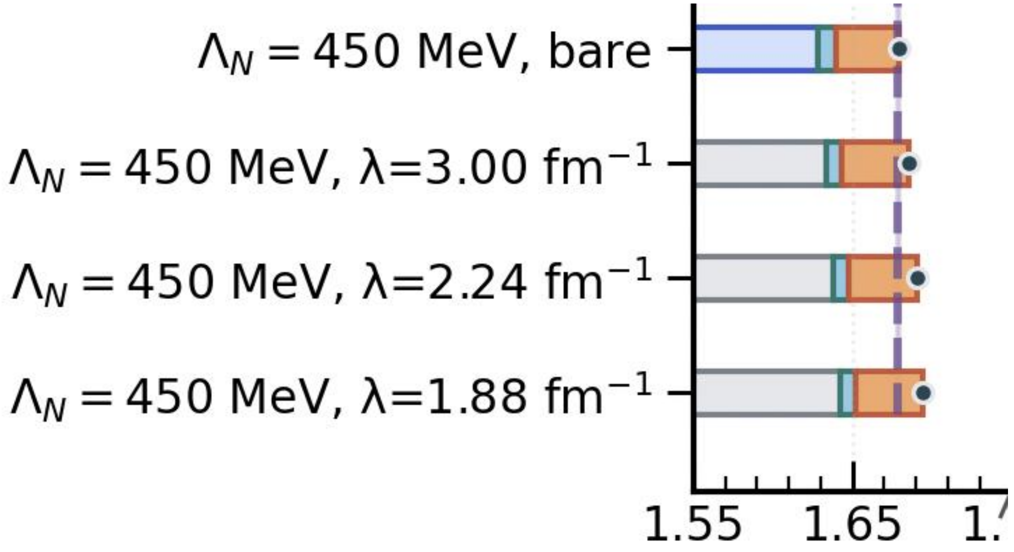
$$|\Psi_{\lambda=0}\rangle = U_\lambda^\dagger|\Psi_\lambda\rangle$$

Truncate U in the two-body level



Exp.: Line

μ−atom: 1.67824(83) fm



Charge radius of ^{4}He

- Ignoring the effects higher than three-body level.
- The SRG transformation influence on radius is only about 0.01 fm.

SUN *et al.*, arXiv2512.15454

Summary on nuclear *ab initio* studies on nuclear radius

A. Limited basis size (overcoming computational limit)

Tail corrections on the nuclear density to get converged matter radii in NCSM

SUN, Le, Meißner, Nogga, Phys. Rev. C 112, 024317 (2025)

B. Two-body contributions (self-consistent treatment of the EM process in Chiral EFT) 0.04 fm

Including consistently regularized charge density is important to high–momentum FFs and to accurately extract charge radius.

SUN *et al.*, arXiv2601.09614

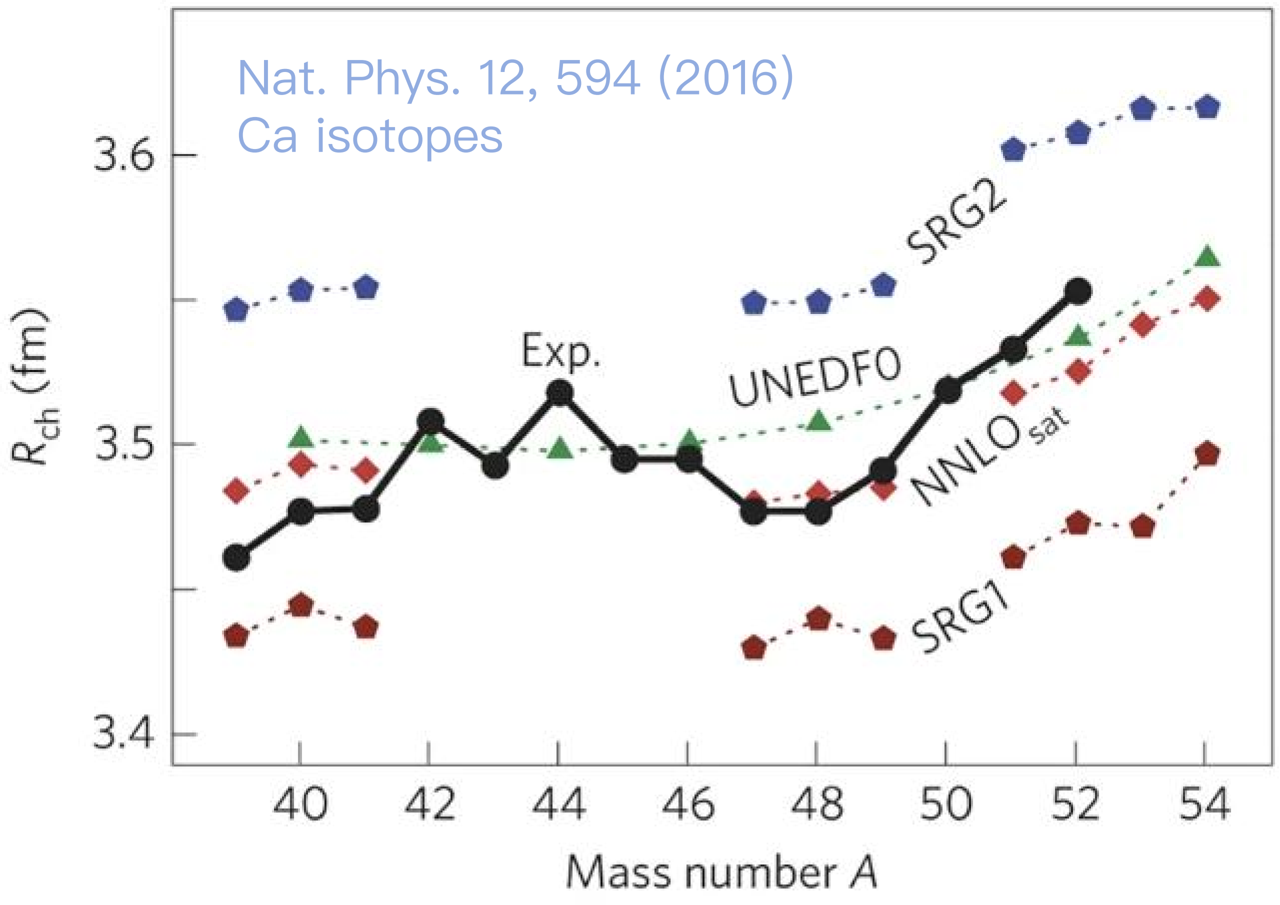
C. Softened forces (restore the influence from SRG transformation) 0.015 fm

- Make unitary transformation on wfs w.r.t. SRG evolved interaction and then calculate observable
- Restore the high momentum information in nuclear many–body wave functions, NN short–range correlations

SUN *et al.*, arXiv2512.15454

A standard framework to study nuclear charge FFs & radius and the extension to medium and heavy nuclei using other *ab initio* methods will provide the insight of nuclear radius challenge!

Perspective on nuclear *ab initio* studies on nuclear radius



- ➢ Iso−vector charge density operators are in progress (Bochum group)
- ➢ Extension to heavier systems (NCSM, IMSRG, …) and 2N effects
- ➢ Systematic of nuclear charge radius
- ➢ Odd−even staggering effects
- ➢ FFs for exotic nuclei

Thank you!

Many thanks to my collaborators：

- Ulf-G. Meißner, Andreas Nogga, Hoai Le, Vadim Baru, Evgeny Epelbaum, Arseniy A. Filin, Hermann Krebs, Daniel Möller, Harald W. Grießhammer, Alex Long, the LENPIC Collaboration