Proton and neutron electromagnetic form factors and radii from Lattice QCD

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Nuclear Radius Extraction Collaboration
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Roadmap of Talk

★ Introduction & Motivation

★ Accessing information on hadron structure from Lattice QCD

★ Form factors from “traditional” methods: local operators

★ Determination of radii from Lattice QCD

★ Other avenues to obtain form factors: non-local operators

★ Synergistic Activities

★ Outlook

Introduction

- ★ Electromagnetic (EM) form factors provide the electric and magnetic distributions within the proton/neutron

- ★ Proton EM form factors known experimentally at high precision

- ★ Neutron EM form factors are lesser-known compared to proton: obtained indirectly through electron-deuteron or electron-helium scattering

- ★ Beyond the light-quark contributions the strange quark contributions can study the virtual particle dynamics in the non-perturbative regime

Why are nucleon EM FFs important for LQCD?

★ Benchmark observables for hadron structure:
- Experimentally accessible and theoretically clean

★ Precision physics:
- Proton charge radius under intense scrutiny for more than a decade;
- Lattice QCD can provide an independent ab initio determination

★ A measure of the maturity of lattice hadron structure:
- Beyond exploratory calculations (continuum-limit, physical-mass, accuracy, etc)

↓

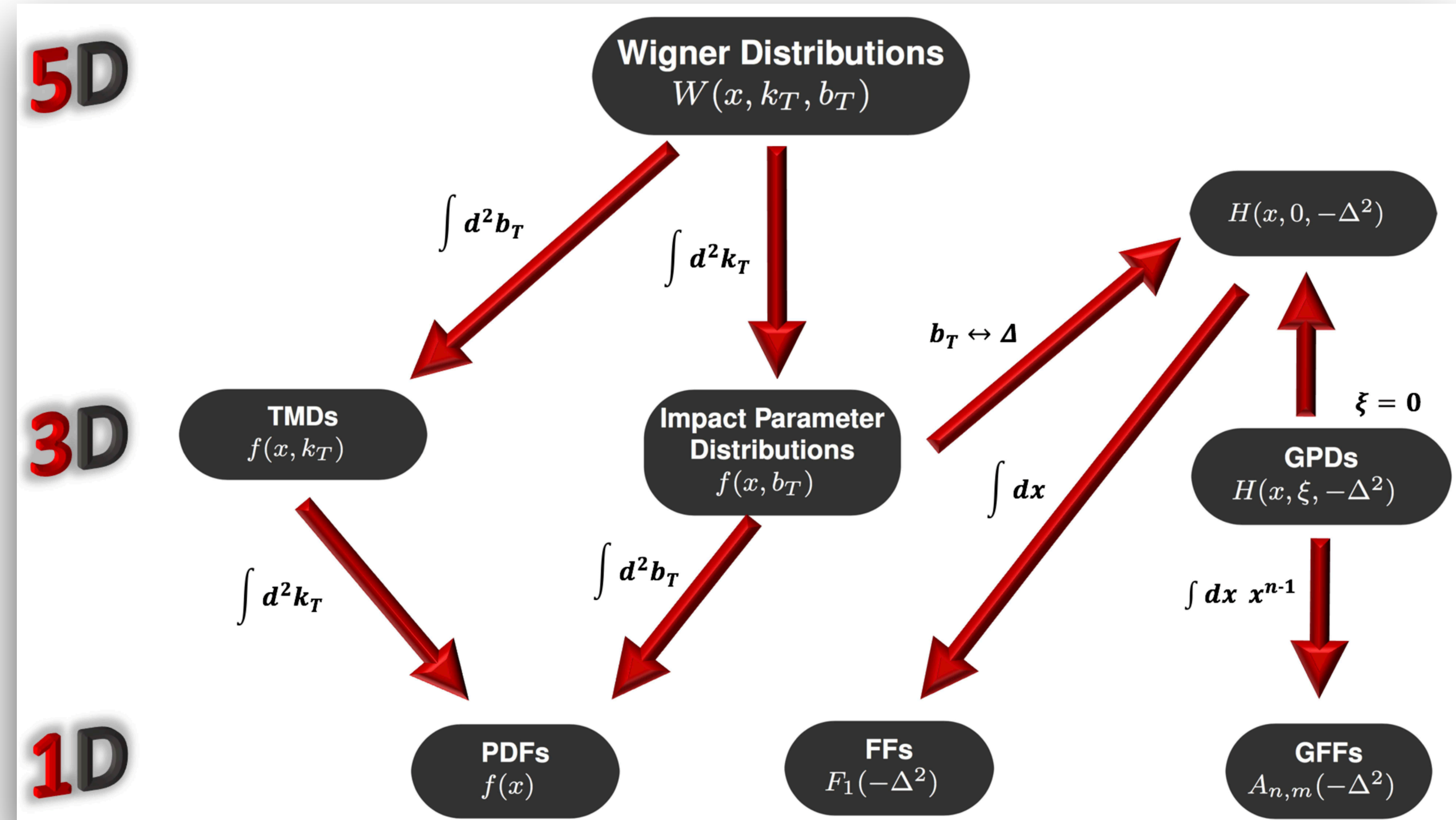
Significant efforts from lattice QCD and new developments

Nucleon Characterization

Wigner distributions

★ Fully characterize partonic structure of hadrons

★ Provide multi-dim images of the parton distributions in phase space



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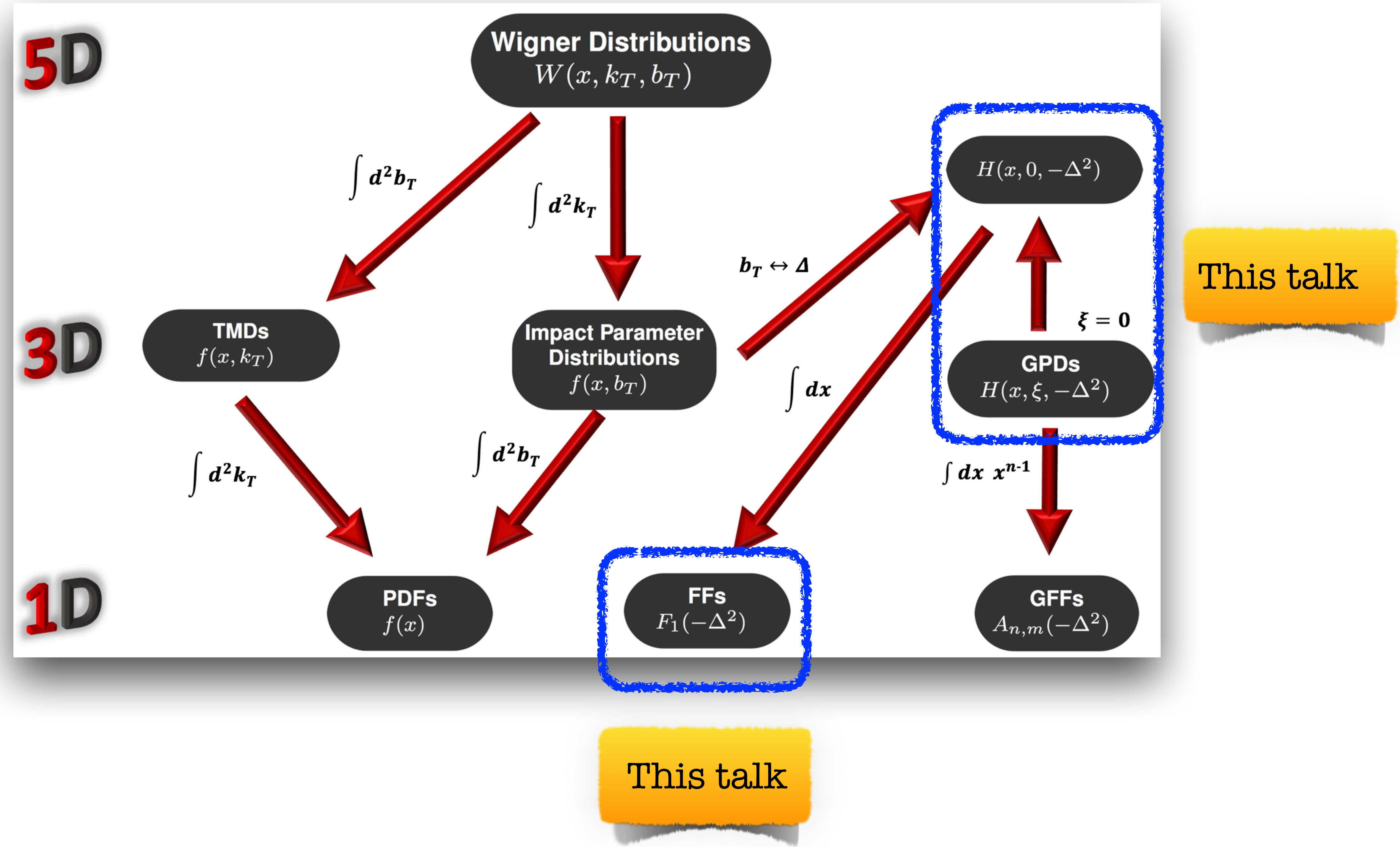
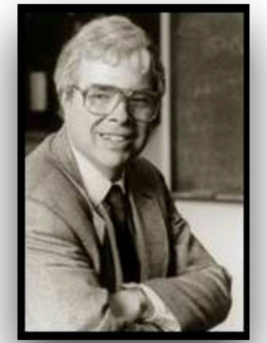




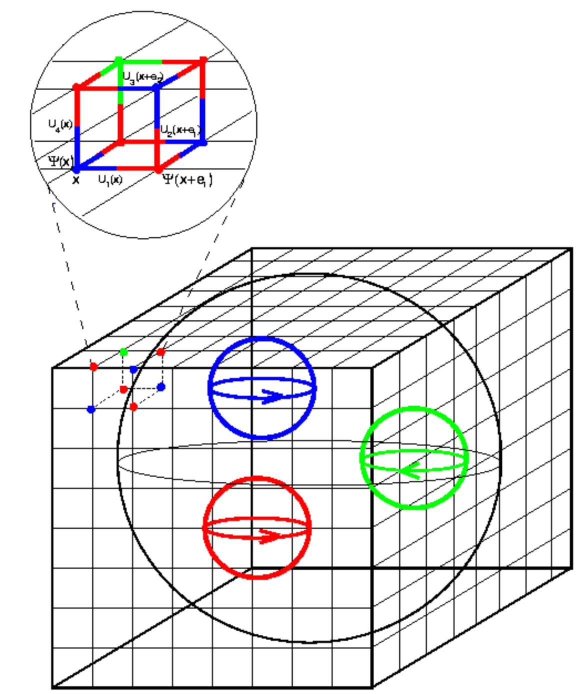
K. Wilson

Lattice formulation of QCD

First principle (ab initial) formulation

M. Creutz

- ★ Space-time discretization on a finite-size 4-D grid



- ★ Serves as a regulator of theory:
  - – UV (hard momentum) cut-off (finite integrals):
    inverse lattice spacing (α^{-1})
    momentum and energy < |π/α|

$$\int_{-\infty}^{\infty} dp \;\rightarrow\; \int_{-\pi/a}^{\pi/a} \frac{dp}{2\pi}$$

  - – IR cut-off (finite number of d.o.f): inverse lattice size ($V^{-1/4}$)

$$\int dpF(p) \;\rightarrow\; \sum_{n}^{N_{\max}} \frac{2\pi}{L} F(p_0 + \frac{2\pi n}{L})$$

- ★ Removal of regulator

$$L \rightarrow \infty\,, \;\; a \rightarrow 0$$

Accessing the partonic structure of hadrons

- ★ Parton model: physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

- ★ PDFs via matrix elements of nonlocal light-cone operators ($-t^2 + \vec{r}^2 = 0$)

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

- ★ Light-cone correlations inaccessible from Euclidean lattices ($\tau^2 + \vec{r}^2 = 0$)

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A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \dots z_{\alpha_n} \left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \dots \overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i\frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\text{ even}} \Big) \Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\text{ even}} \Big] U(P)$$

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- Frame independence
  (multiple values of $-t$ (Q^2) at same comp. cost)
- Statistical uncertainty can be controlled
- contain physical information

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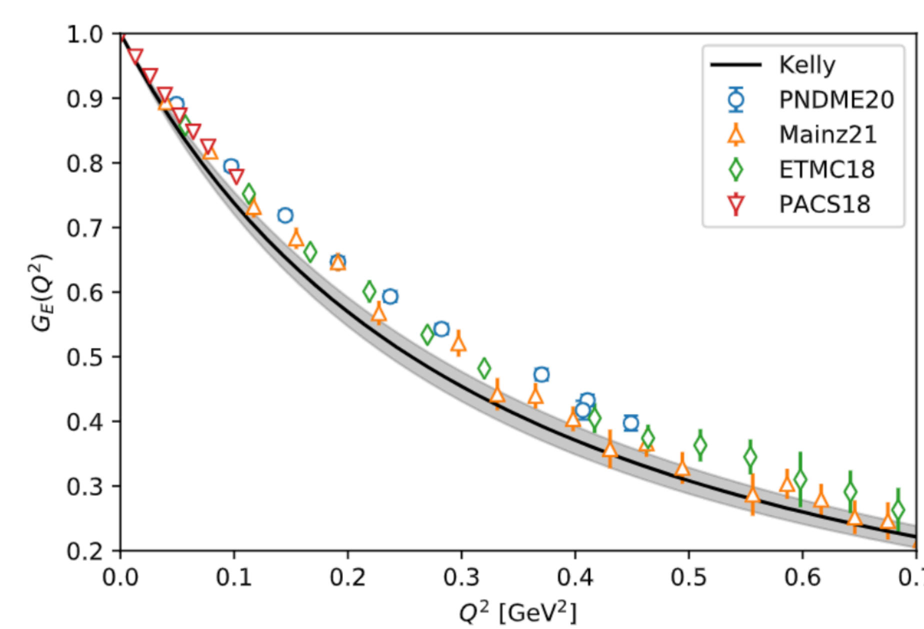
Computationally efficient extraction of Q^2 dependence



“Traditional” calculations of Mellin moments of GPDs

Lattice QCD Calculation

Matrix element:

$$\langle N(p',s')|j_\mu|N(p,s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}}\,\bar{u}_N(p',s')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]u_N(p,s)$$

Form factors:

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Radii:

$$\langle r_X^2\rangle^{\mathrm{f}} = -6\frac{\partial G_X^{\mathrm{f}}(q^2)}{\partial q^2}\Big|_{q^2=0}$$

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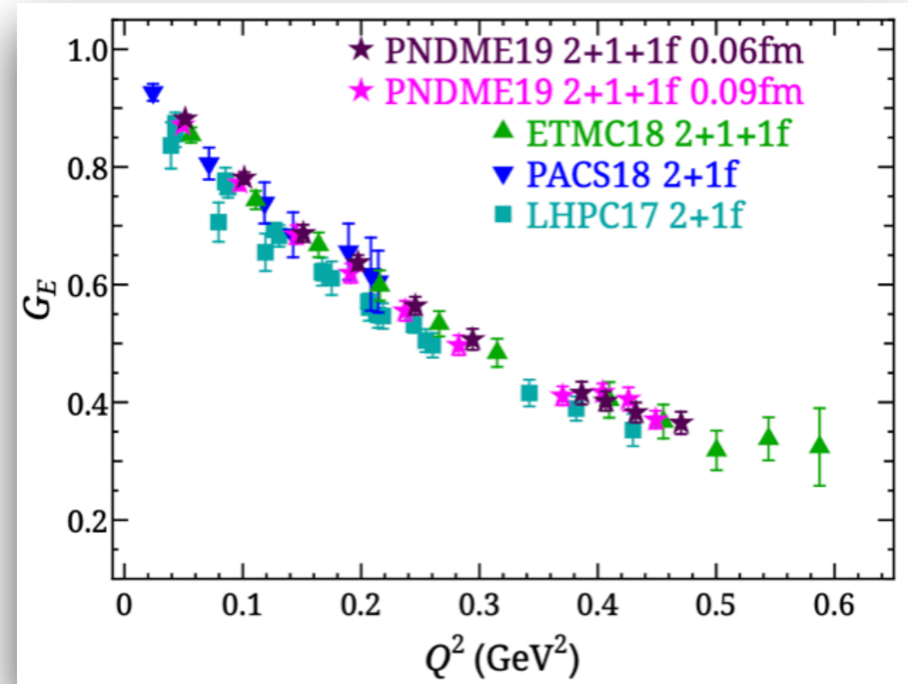
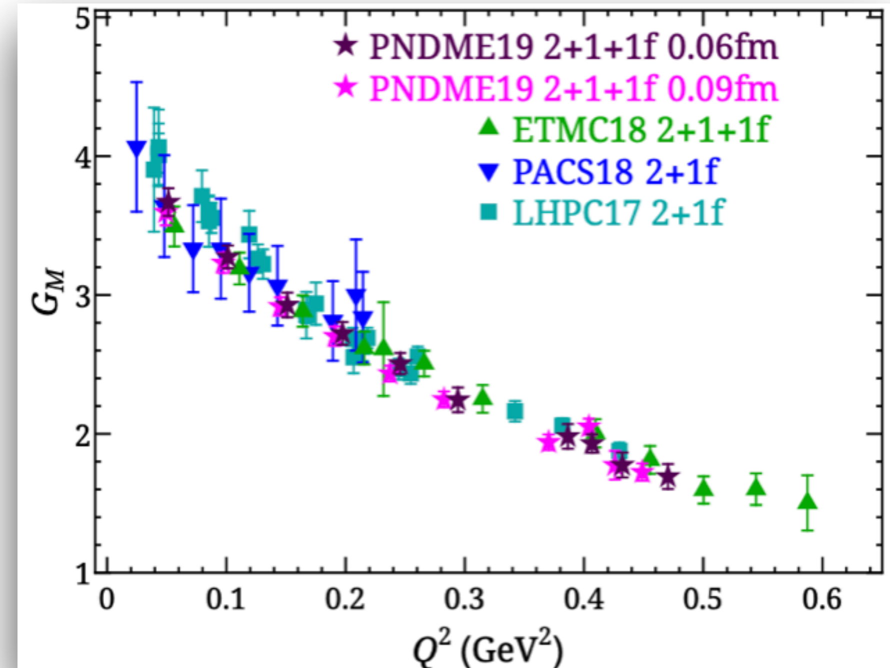
- Dipole fit $\qquad G(Q^2) = \frac{g}{\left(1+\frac{Q^2}{M^2}\right)^2} \qquad \langle r^2\rangle = \frac{12}{M^2}$
- z-expansion $\qquad G(Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k(Q^2) \qquad \langle r^2\rangle = -\frac{3c_1}{2c_0 t_{\mathrm{cut}}}$
- Galster-like fit $\qquad G(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B}\frac{1}{\left(1+\frac{Q^2}{0.71\mathrm{GeV}^2}\right)^2} \qquad \langle r^2\rangle = -\frac{3A}{2m_N^2}$

(Relatively) Recent Developments

[Constantinou et al., Prog.Part.Nucl.Phys. 121 (2021) 103908]





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ETMC'18 (Alexandrou *et al.*, 2019a)	2f & 2+1+1f TM	twisted mass	RI'-MOM	3	0.080, 0.094	130-139 MeV	3.0–4.0
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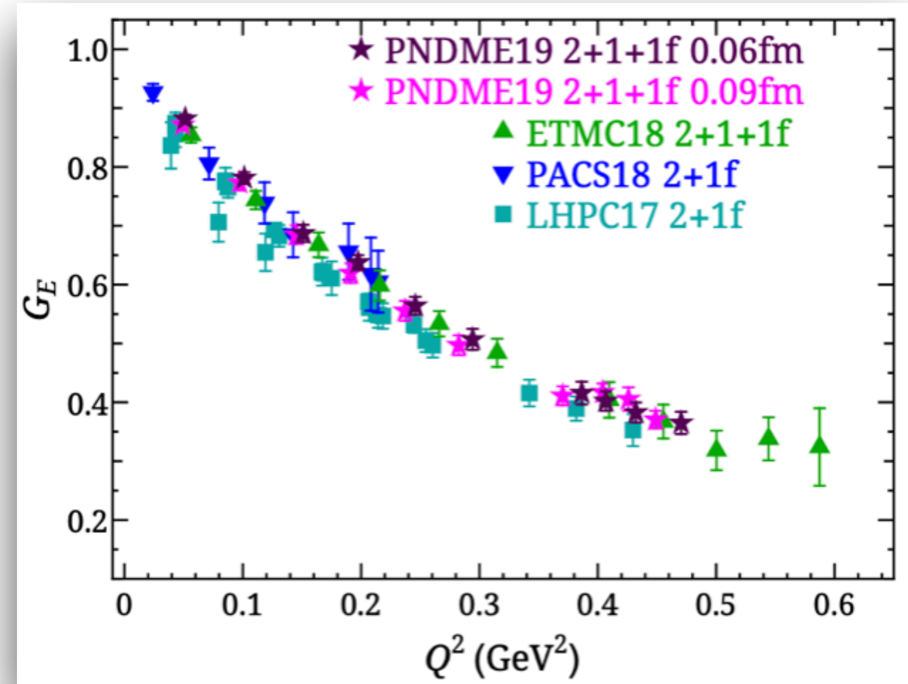

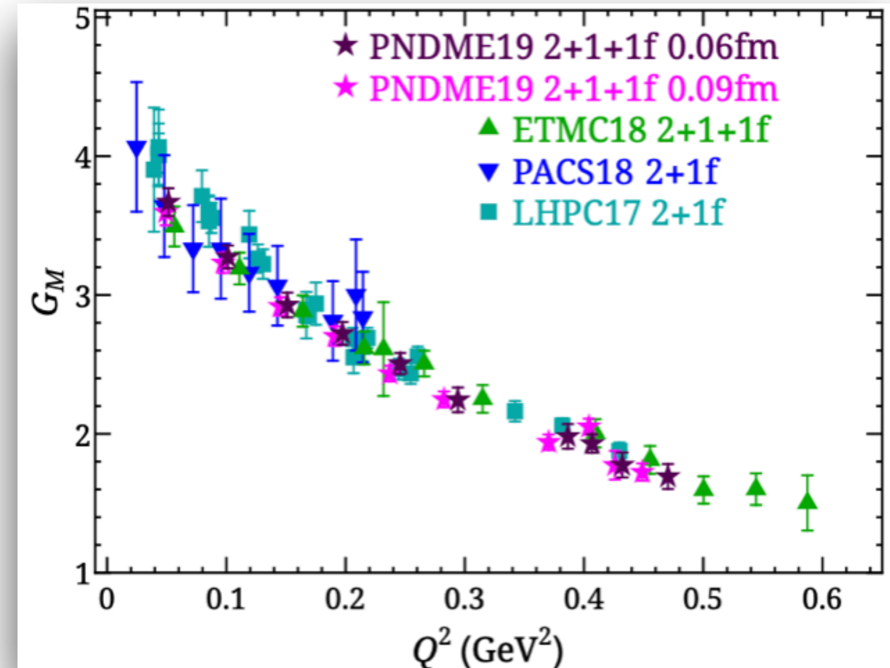

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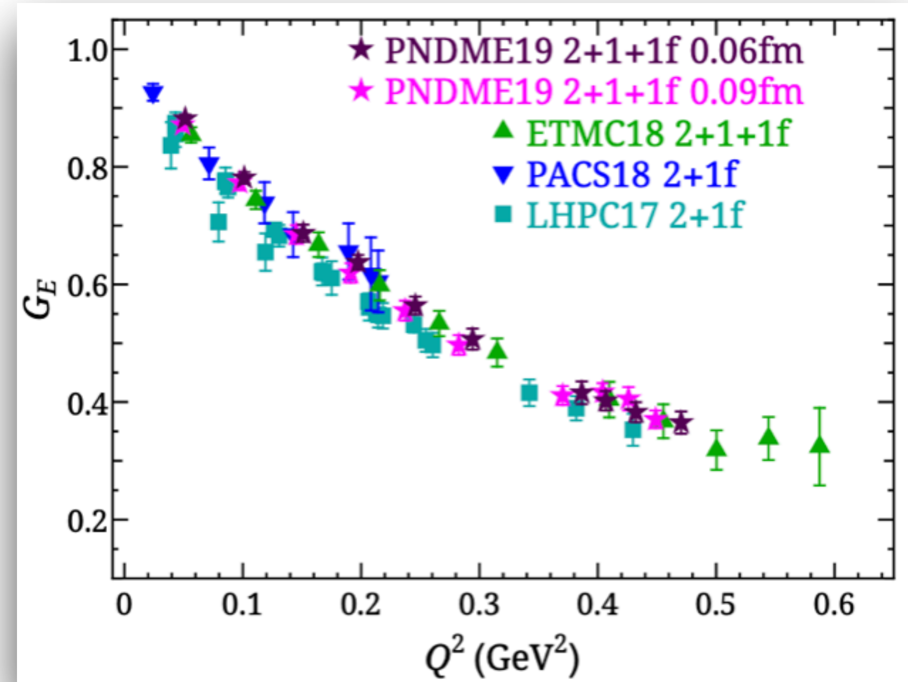

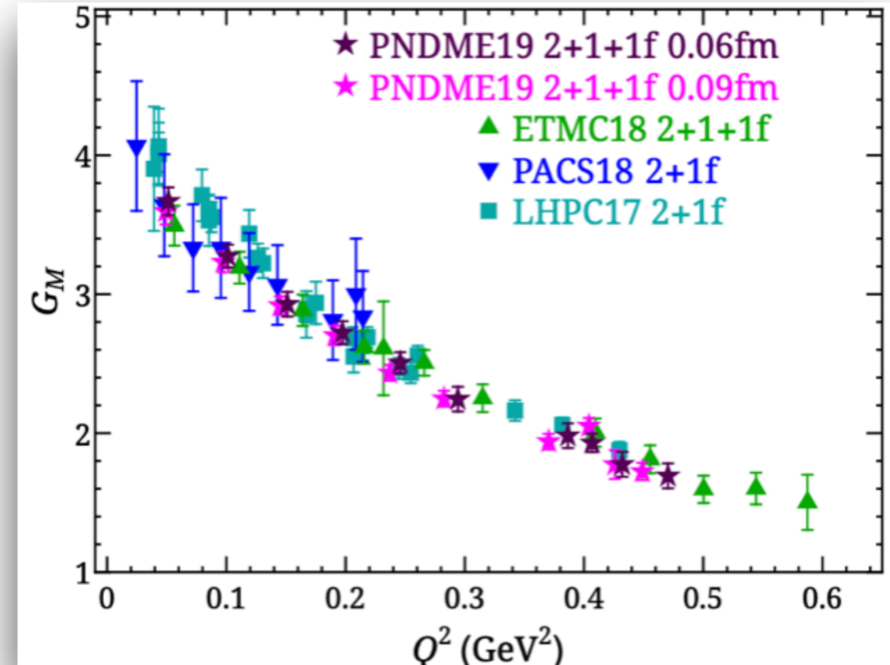

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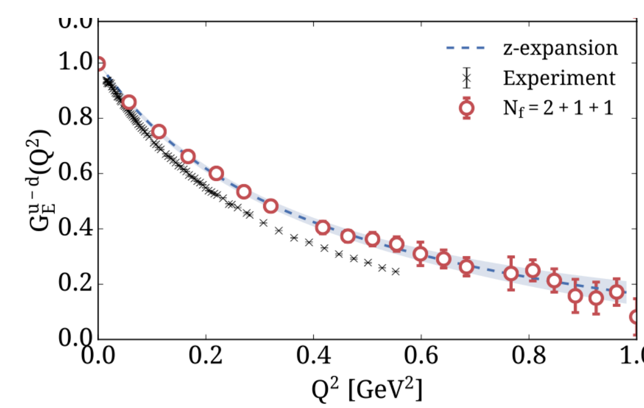

Slope different than experiment

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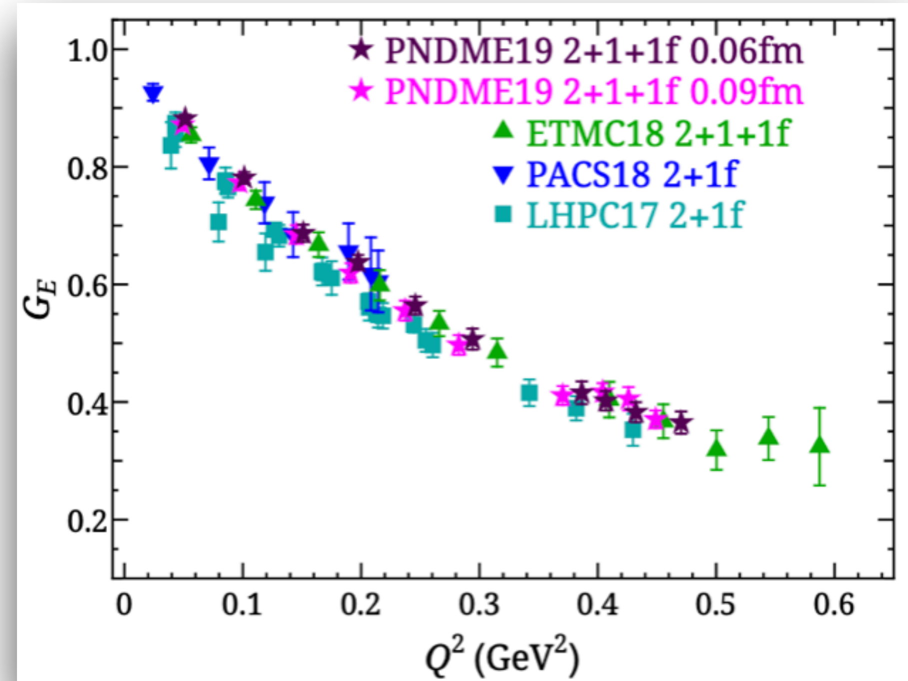

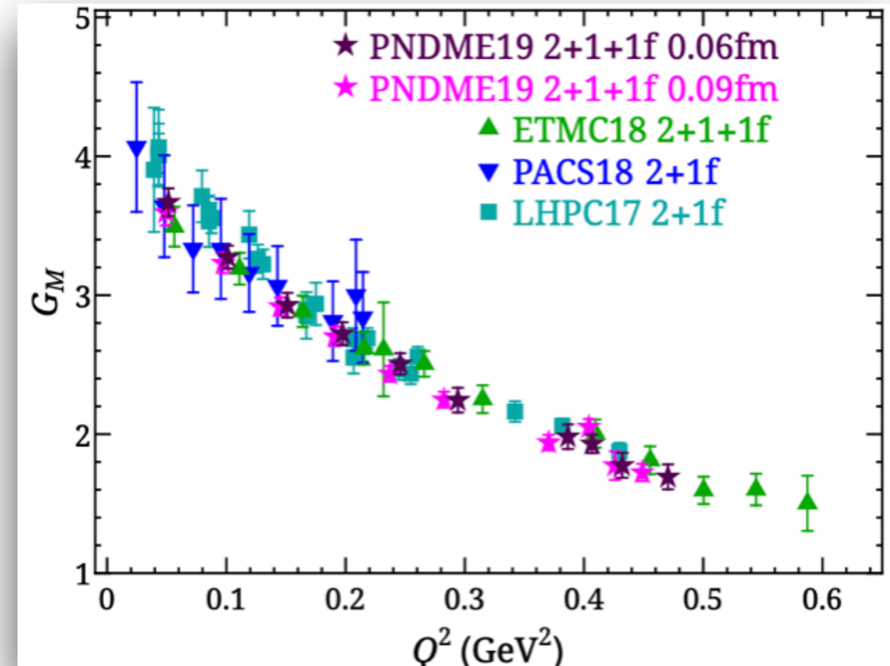

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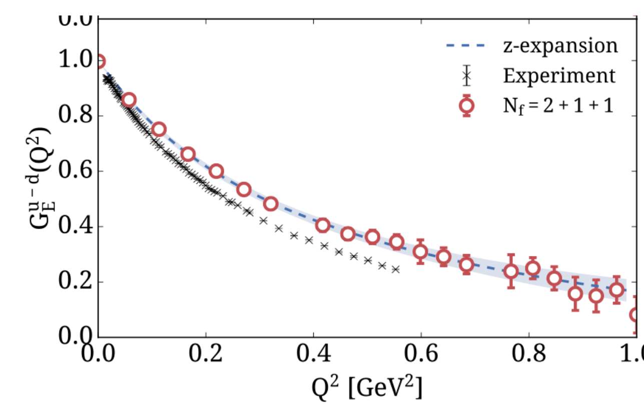

Slope different than experiment

More Recent Developments

★ Percent-level precision achieved

★ Inclusion of disconnected contributions

★ Addressing systematic uncertainties

★ Chiral and continuum extrapolations

References:

Djukanovic et al. Phys.Rev.Lett. 132 (2024) 21

Djukanovic et al. Phys.Rev.D 109 (2024) 9

Djukanovic et al. Phys.Rev.D 110 (2024) 1

Alexandrou et al., arXiv:2507.20910

Alexandrou et al., arXiv:2603.26591

Rodekamp et al, Lattice 2025

Results from the Mainz Group

ID	a [fm]	T/a	L/a	M_π[MeV]	$M_\pi L$	$t_{\rm sep}$[fm]	$N_{\rm cfg}$
H102	0.086	96	32	354	4.96	0.35, 0.43, 0.52, 0.6, 0.69, 0.78, 0.86, 0.95, 1.04, 1.12, 1.21, 1.3, 1.38, 1.47	2005
H105		96	32	280	3.93		1027
C101		96	48	225	4.73		2000
N101		128	48	281	5.91		1596
S400	0.076	128	32	350	4.33	0.31, 0.46, 0.61, 0.76, 0.92, 1.07, 1.22, 1.37, 1.53	2873
N451		128	48	286	5.31		1011
D450		128	64	216	5.35		500
D452		128	64	153	3.79		1000
N203	0.064	128	48	346	5.41	0.26, 0.39, 0.51, 0.64, 0.77, 0.9, 1.03, 1.16, 1.29, 1.41	1543
N200		128	48	281	4.39		1712
D200		128	64	203	4.22		2000
E250		192	96	129	4.04		400
S201		128	32	293	3.05		2093
N302	0.050	128	48	348	4.22	0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.1, 1.2, 1.3, 1.39	2201
J303		192	64	260	4.19		1073
E300		192	64	174	4.21		570

[Djukanovic et al. PRL 132 (2024) 21;
Djukanovic et al. PRD 109 (2024) 9;
Djukanovic et al. PRD 110 (2024) 1]

★ Ensembles allow investigation of systematic uncertainties:

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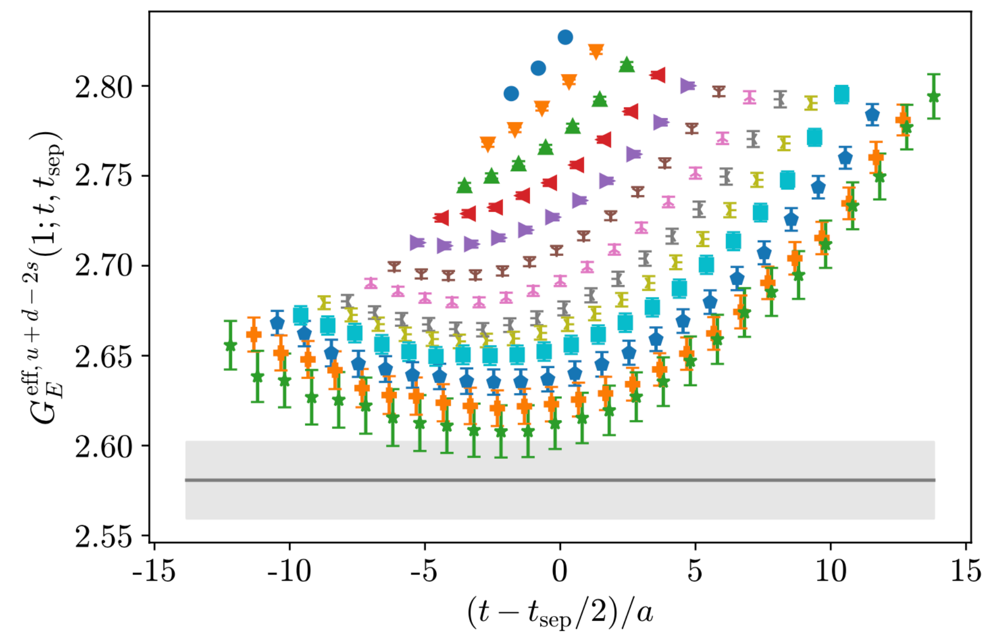
D452		128	64	153	3.79		1000
N203	0.064	128	48	346	5.41	0.26, 0.39, 0.51, 0.64, 0.77, 0.9, 1.03, 1.16, 1.29, 1.41	1543
N200		128	48	281	4.39		1712
D200		128	64	203	4.22		2000
E250		192	96	129	4.04		400
S201		128	32	293	3.05		2093
N302	0.050	128	48	348	4.22	0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.1, 1.2, 1.3, 1.39	2201
J303		192	64	260	4.19		1073
E300		192	64	174	4.21		570

[Djukanovic et al. PRL 132 (2024) 21;
Djukanovic et al. PRD 109 (2024) 9;
Djukanovic et al. PRD 110 (2024) 1]



★ Ensembles allow investigation of systematic uncertainties:

- Excited-state effects;

Results from the Mainz Group

ID	a [fm]	T/a	L/a	M_π[MeV]	$M_\pi L$	t_{sep}[fm]	N_{cfg}
H102	0.086	96	32	354	4.96	0.35, 0.43, 0.52, 0.6, 0.69, 0.78, 0.86, 0.95, 1.04, 1.12, 1.21, 1.3, 1.38, 1.47	2005
H105		96	32	280	3.93		1027
C101		96	48	225	4.73		2000
N101		128	48	281	5.91		1596
S400	0.076	128	32	350	4.33	0.31, 0.46, 0.61, 0.76, 0.92, 1.07, 1.22, 1.37, 1.53	2873
N451		128	48	286	5.31		1011
D450		128	64	216	5.35		500
D452		128	64	153	3.79		1000
N203	0.064	128	48	346	5.41	0.26, 0.39, 0.51, 0.64, 0.77, 0.9, 1.03, 1.16, 1.29, 1.41	1543
N200		128	48	281	4.39		1712
D200		128	64	203	4.22		2000
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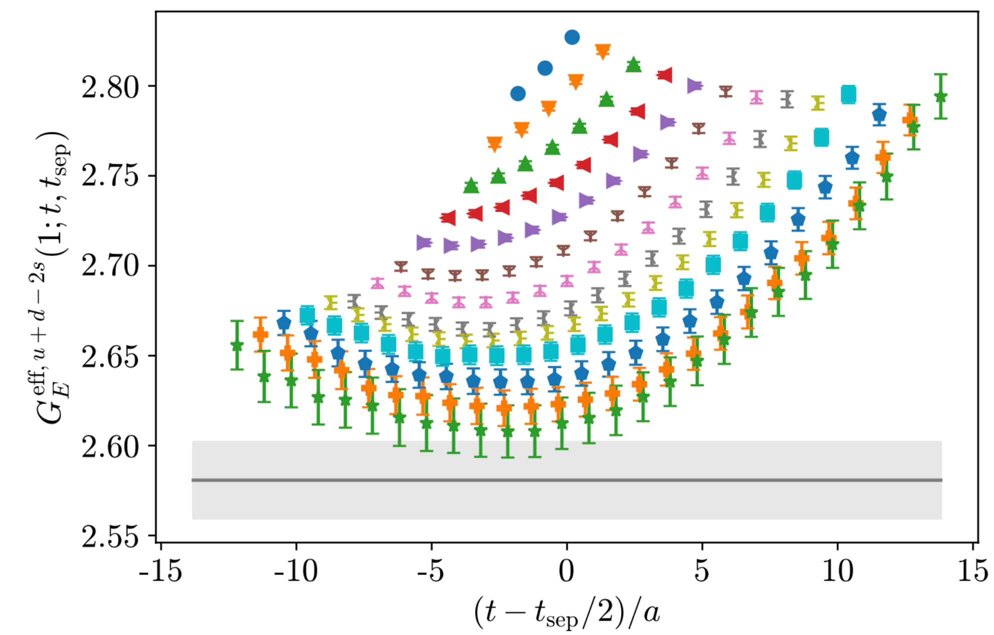
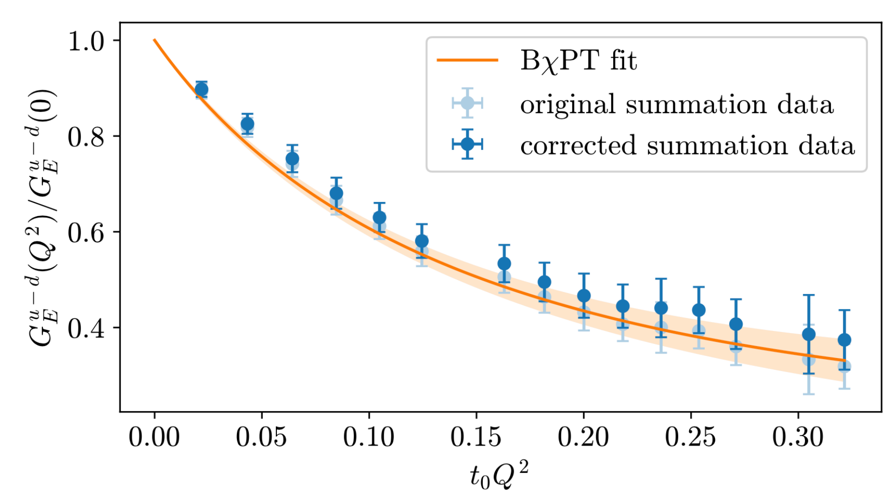


★ Ensembles allow investigation of systematic uncertainties:

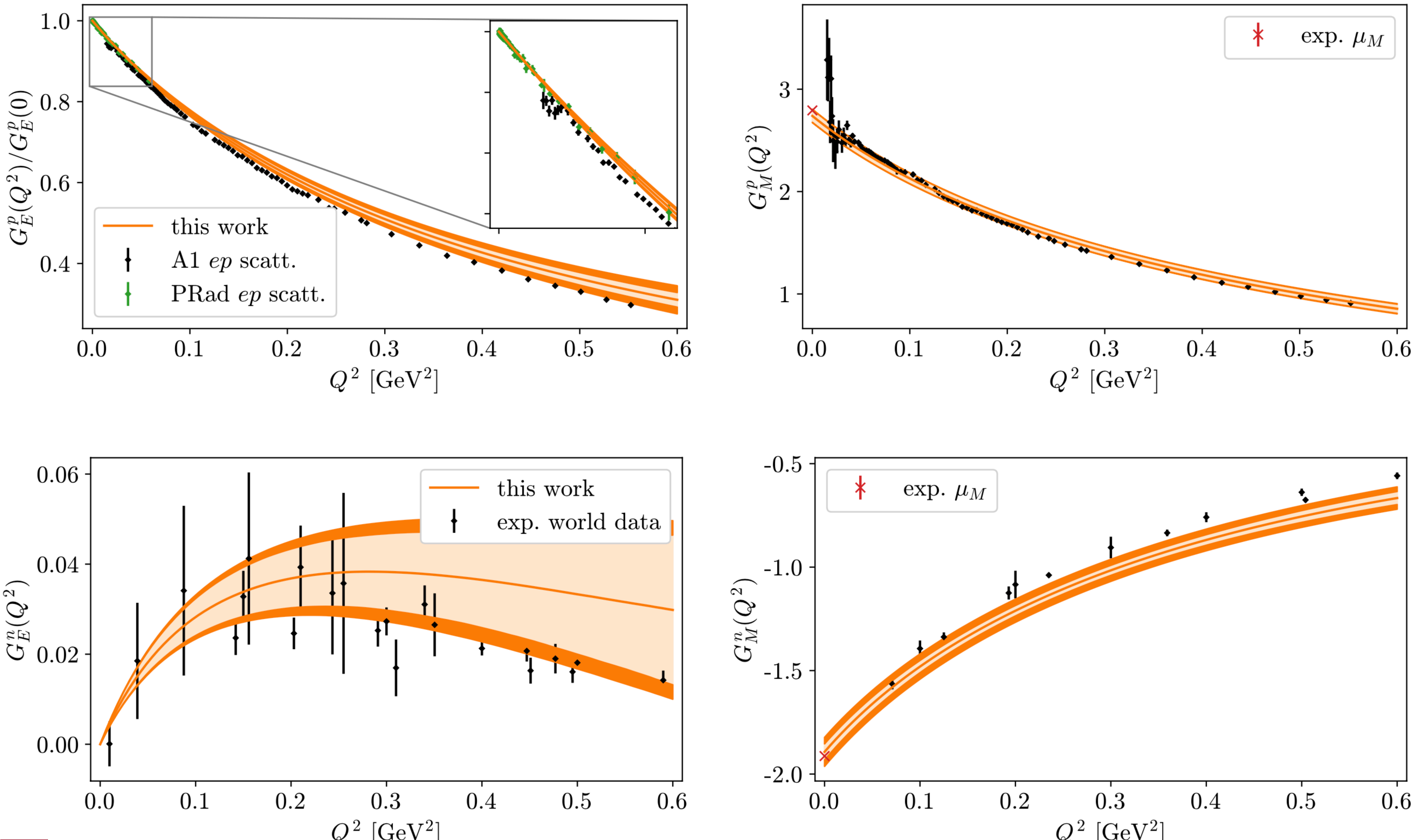
- Excited-state effects;
- Chiral extrapolation;
- Continuum extrapolation;
- Finite-Size extrapolation

$$G_E^{\text{add}}(Q^2) = G_E^{\chi}(Q^2) + G_E^{a} a^2 Q^2 + G_E^{L} t_0 Q^2 e^{-M_\pi L}$$

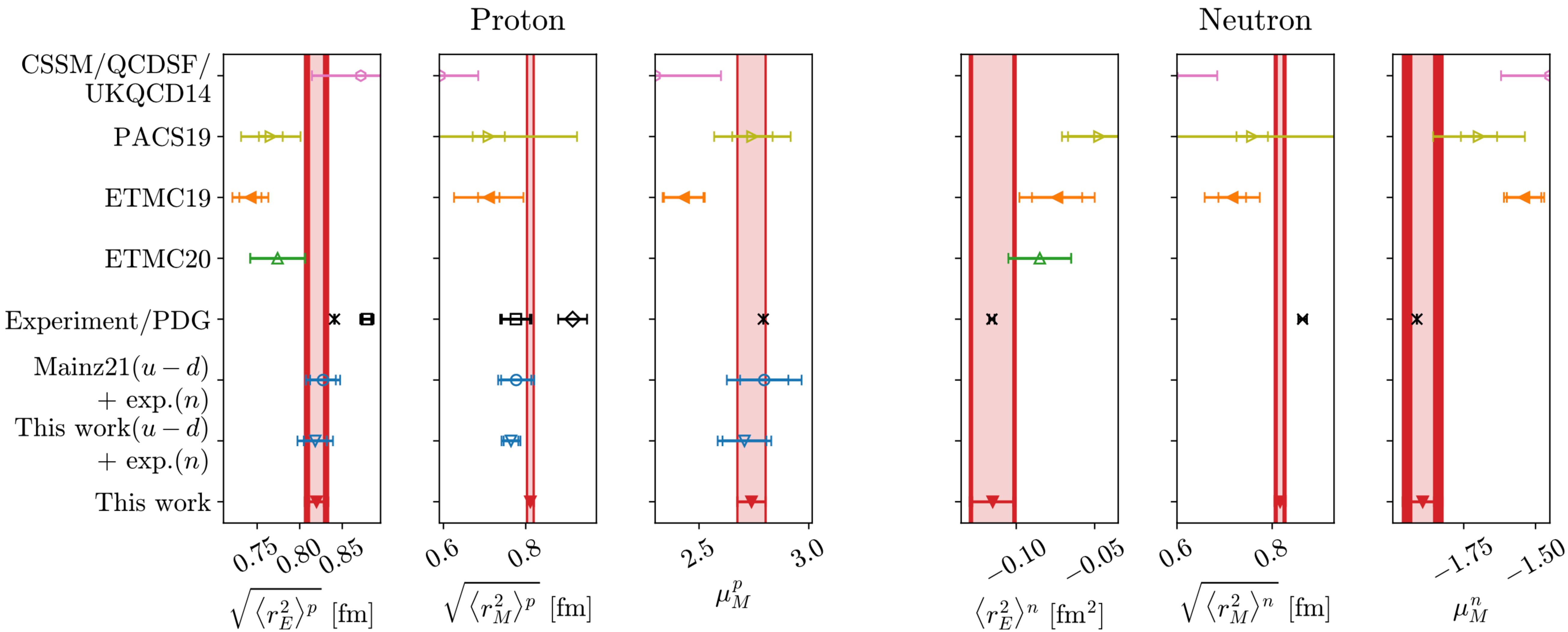


Results from the Mainz Group

Both connected and disconnected quark contributions



Results from the Mainz Group

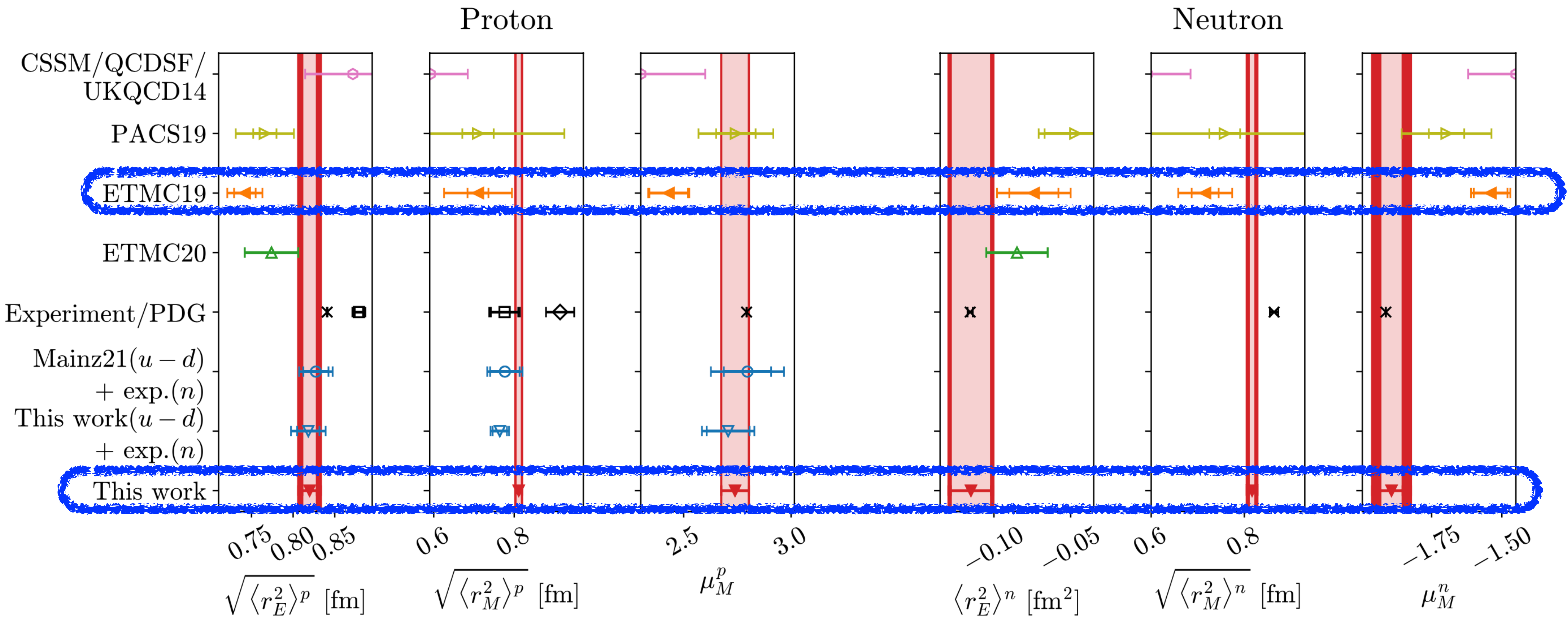


- Electric radius and magnetic moment consistent among lattice results, as well as with experiment

- Some tension in the magnetic radii and the magnetic moments of the proton and neutron

Results from the Mainz Group

Include both connected and disconnected contributions



- Electric radius and magnetic moment consistent among lattice results, as well as with experiment
- Some tension in the magnetic radii and the magnetic moments of the proton and neutron

Results from ETMC

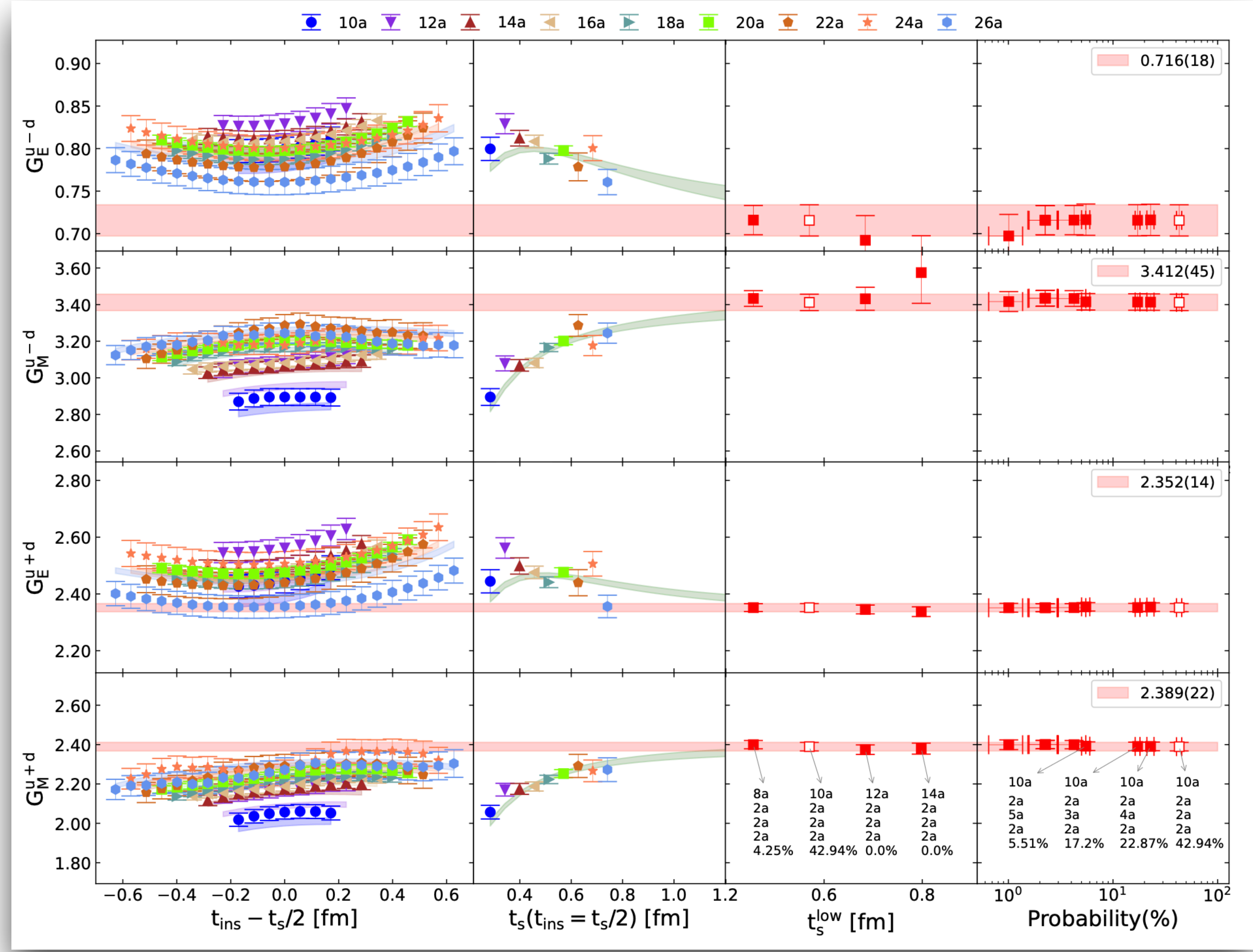
[Alexandrou et al., arXiv:2507.20910; Alexandrou et al., arXiv:2603.26591]

Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	β	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64 (B)	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)	3.62
cC211.060.80 (C)	$80^3 \times 160$	1.836	0.06821(13)	136.7(2)	3.78
cD211.054.96 (D)	$96^3 \times 192$	1.900	0.05692(12)	140.8(2)	3.90
cE211.044.112 (E)	$112^3 \times 224$	1.960	0.04892(11)	136.5(2)	3.79

- ★ Physical point: no chiral extrapolation
- ★ Investigation of excited-states effects,
  continuum extrapolation,
  finite-volume effects
- ★ Strange-quark contributions

Results from ETMC

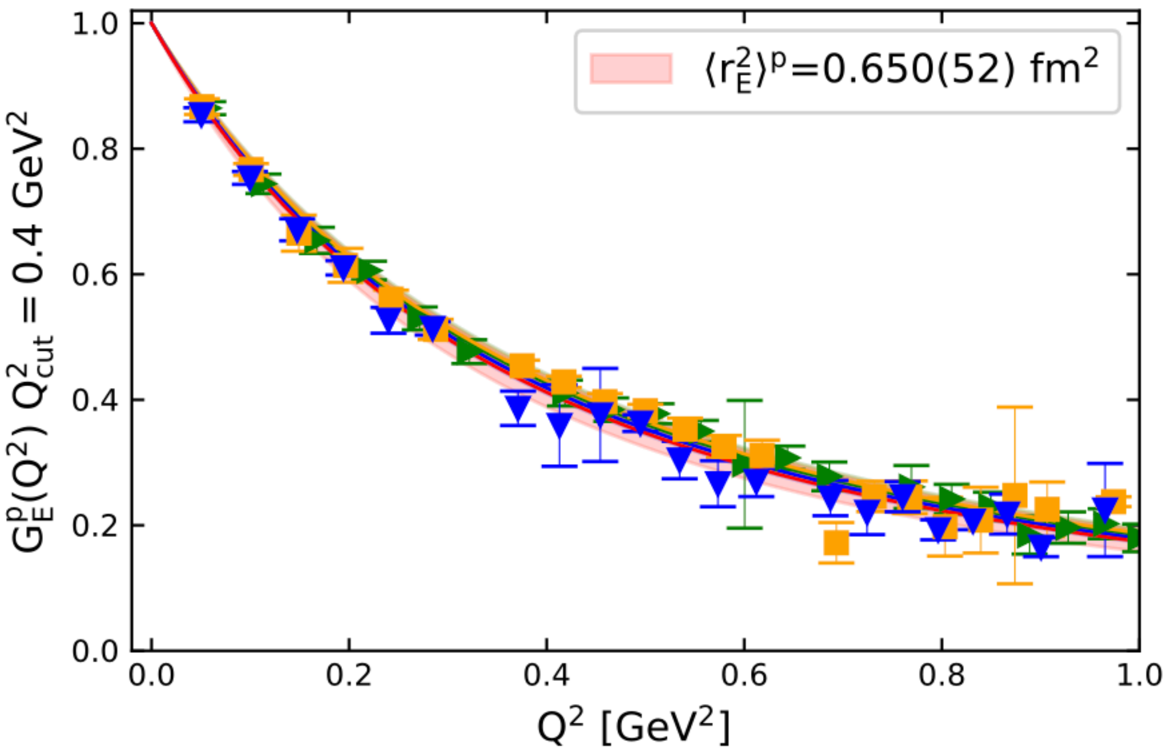
Elaborated investigation of systematic effects: the case of excited states

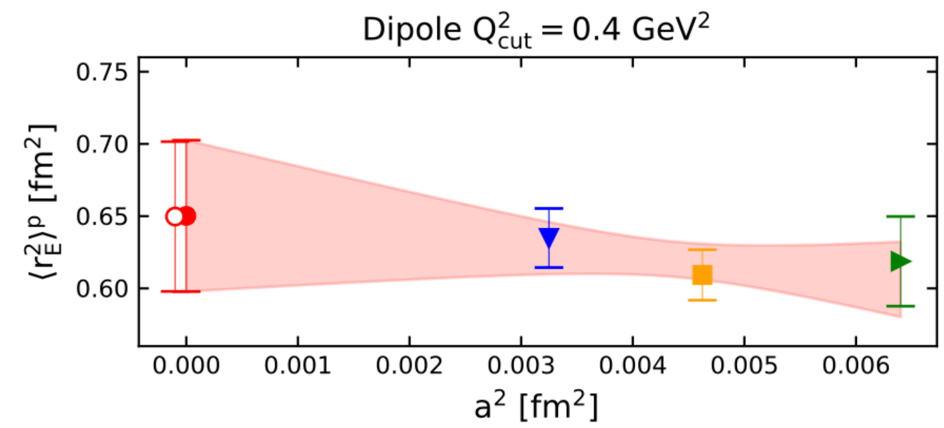


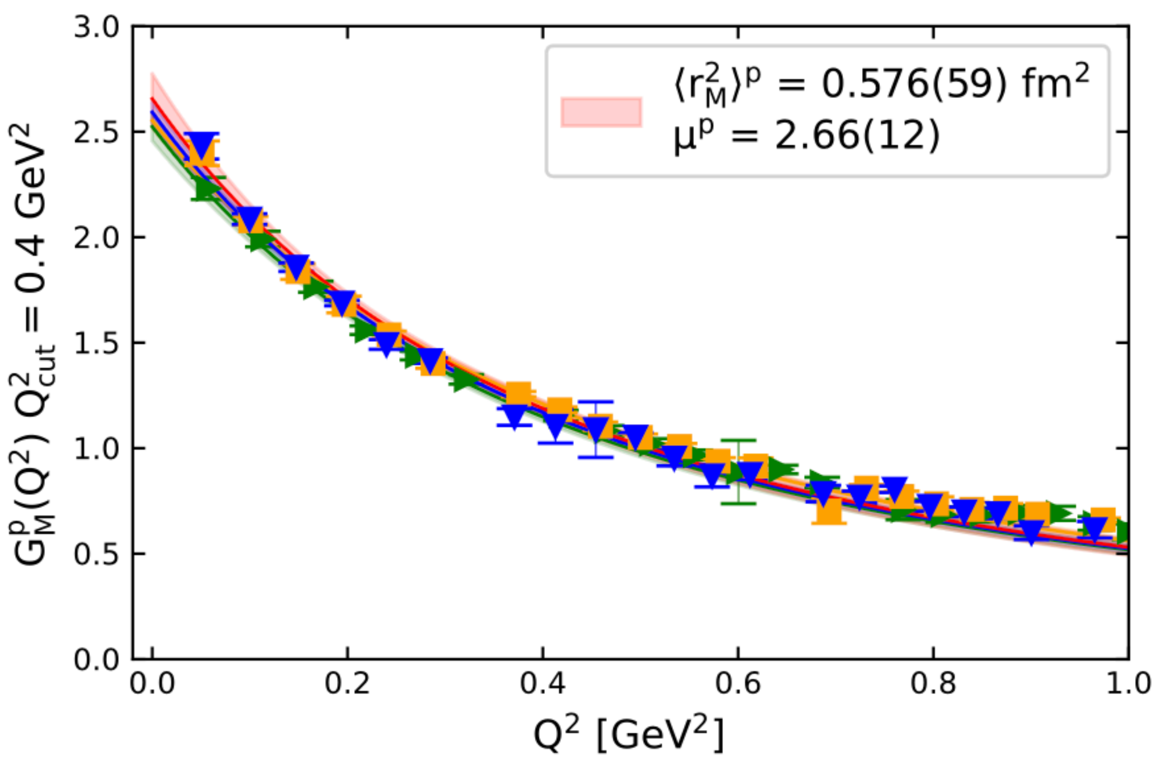
Excited states are important to be eliminated at the precision era

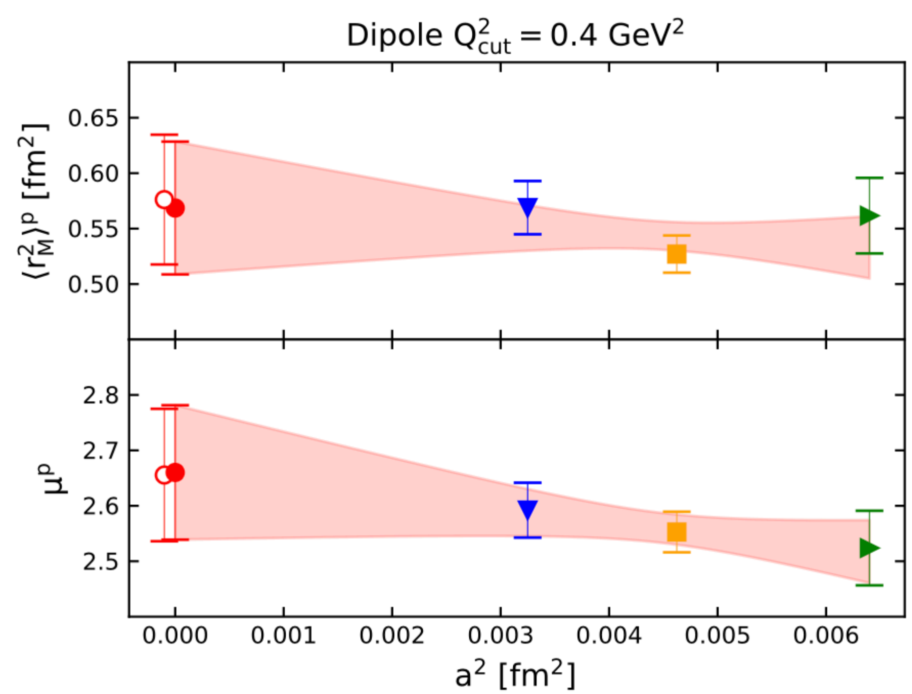
Results from ETMC

Elaborated investigation of systematic effects: the case of finite spacing



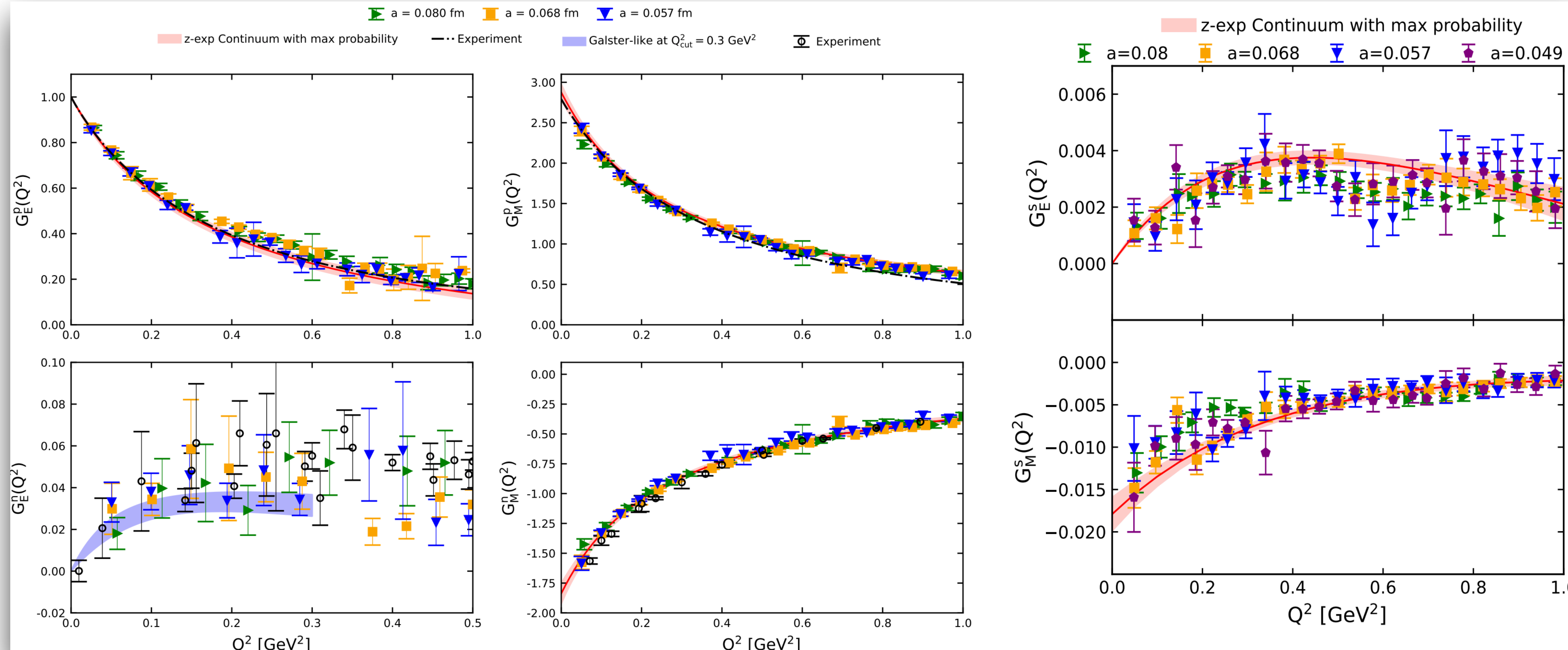






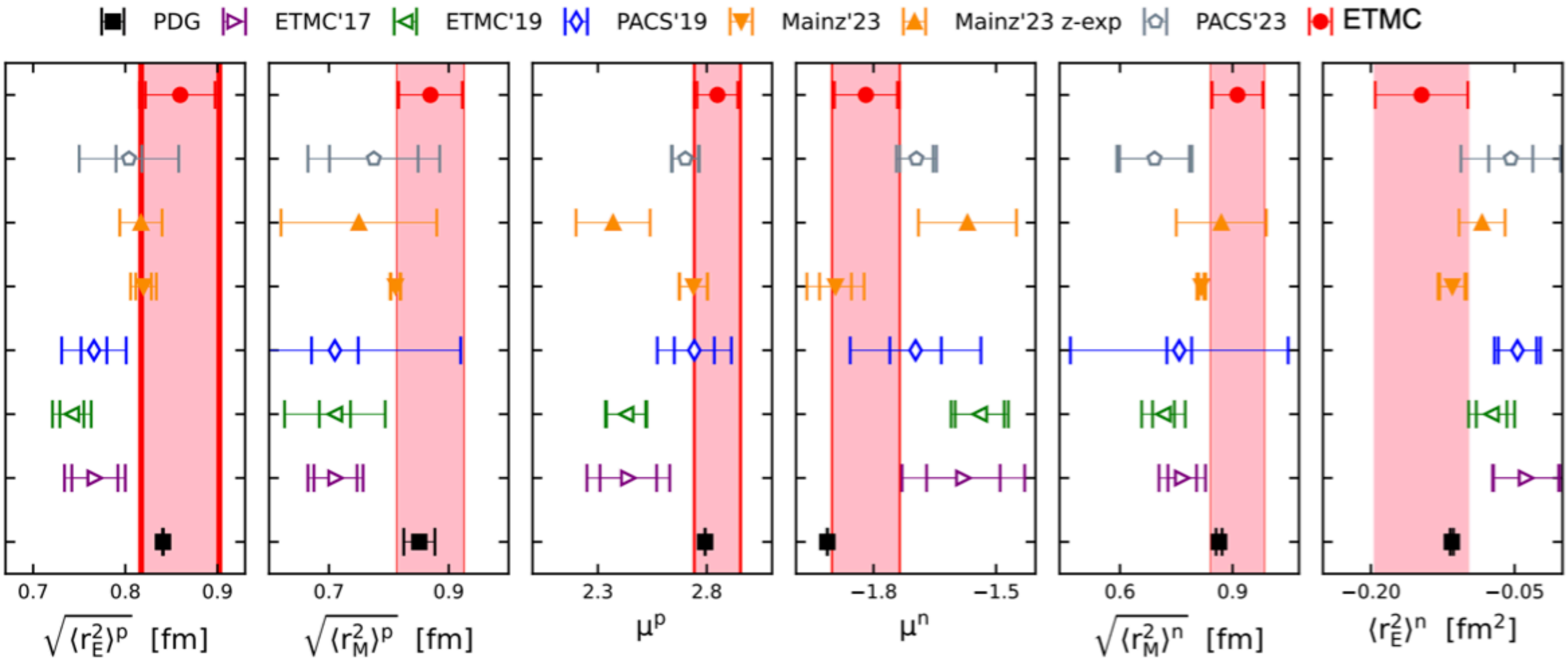
Continuum extrapolation enhances errors

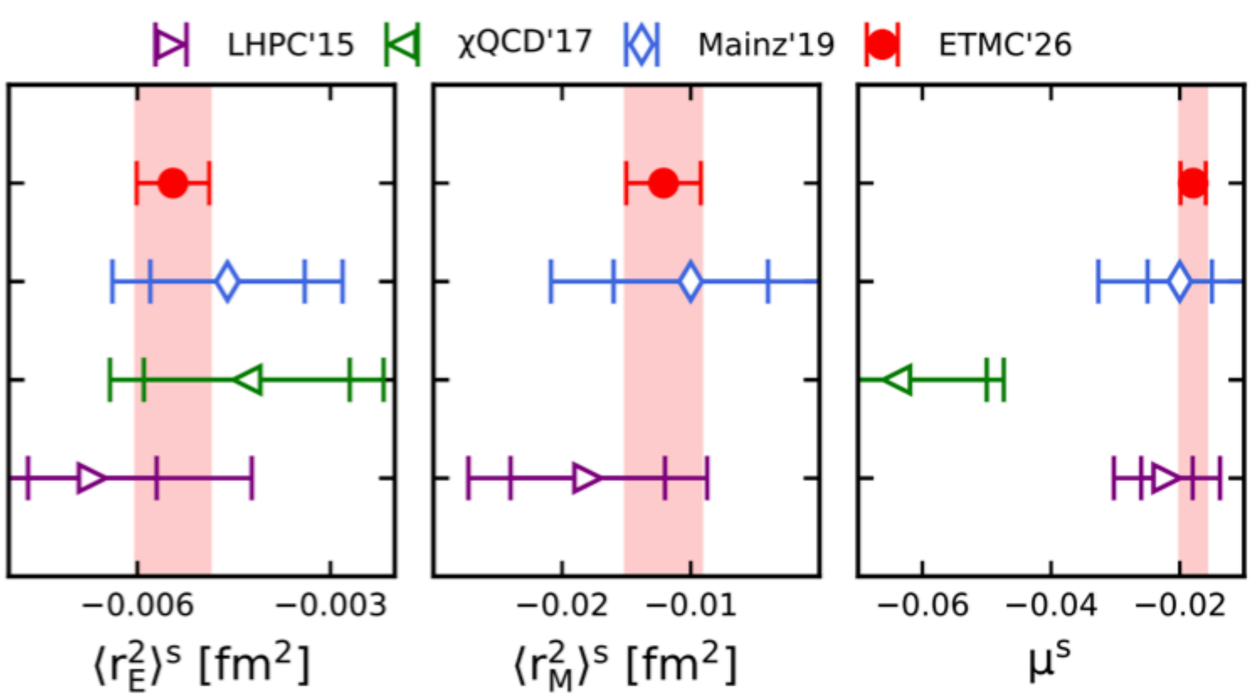
Results from ETMC



- ★ LQCD results for the neutron electric FF are more precise than experimental data
- ★ The strange EM FFs can be reliably determined from LQCD

Summary of radii and magnetic moment



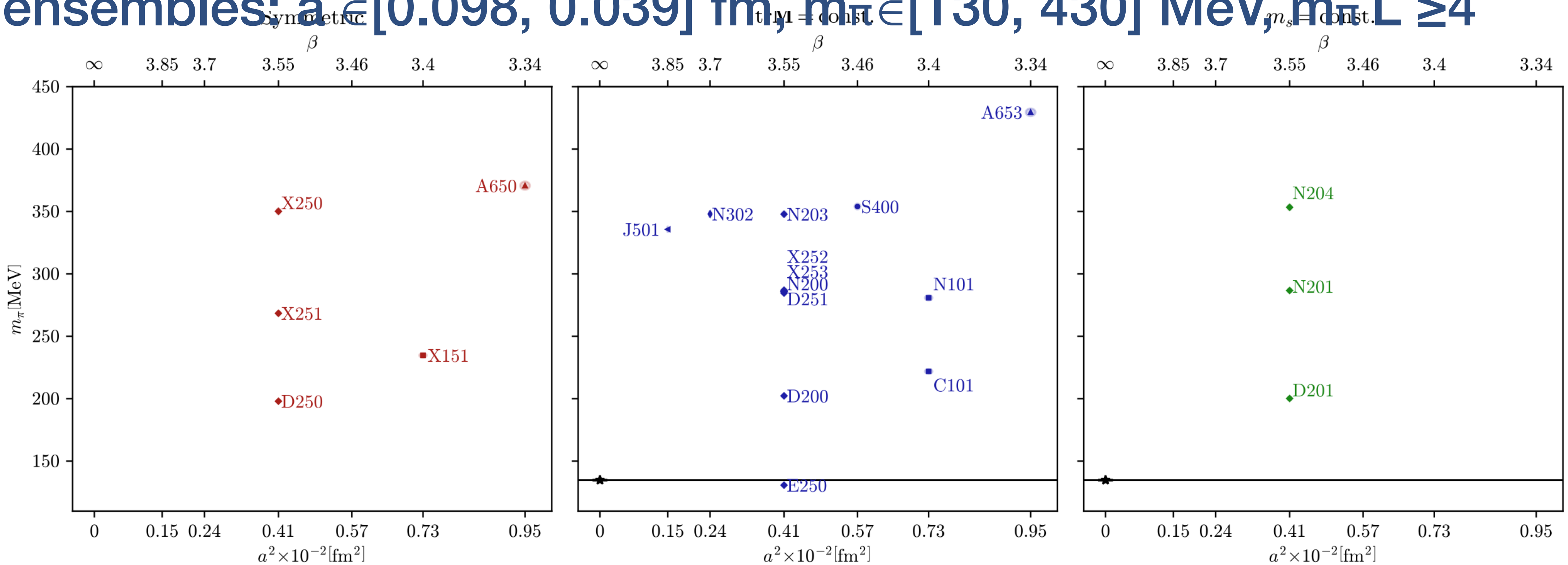


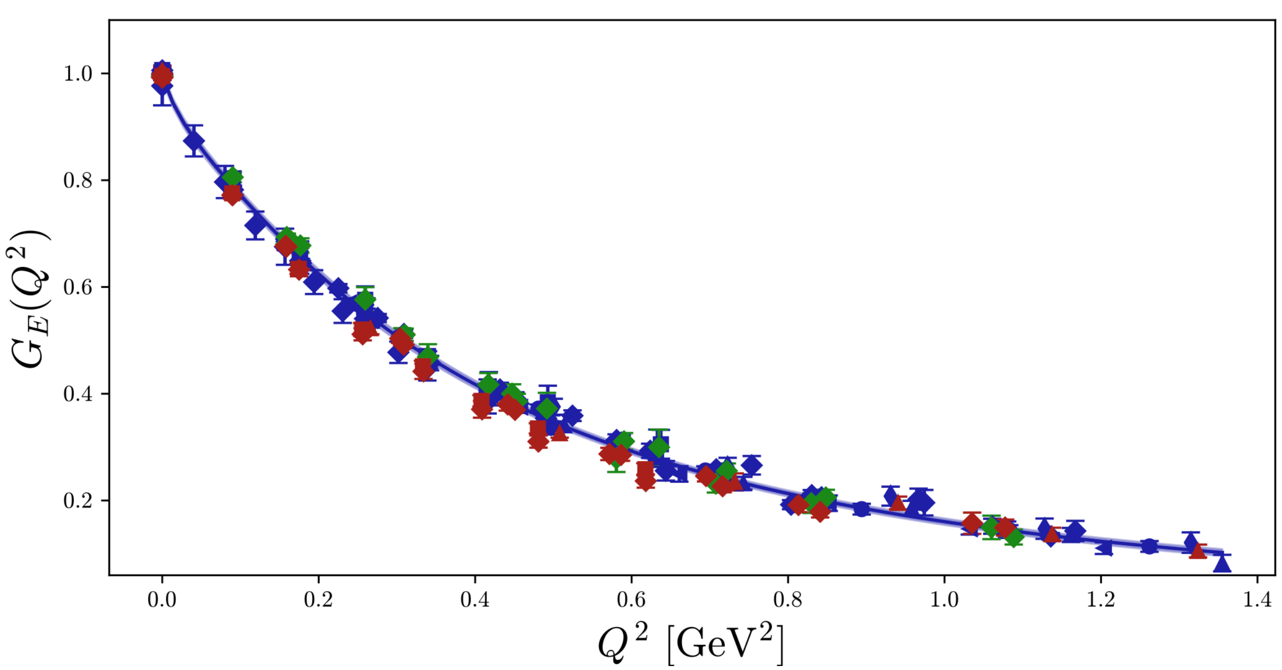
- Recent lattice results with controlled systematics are in agreement with the experimental values.
- Strange-quark contributions tiny but can be extracted

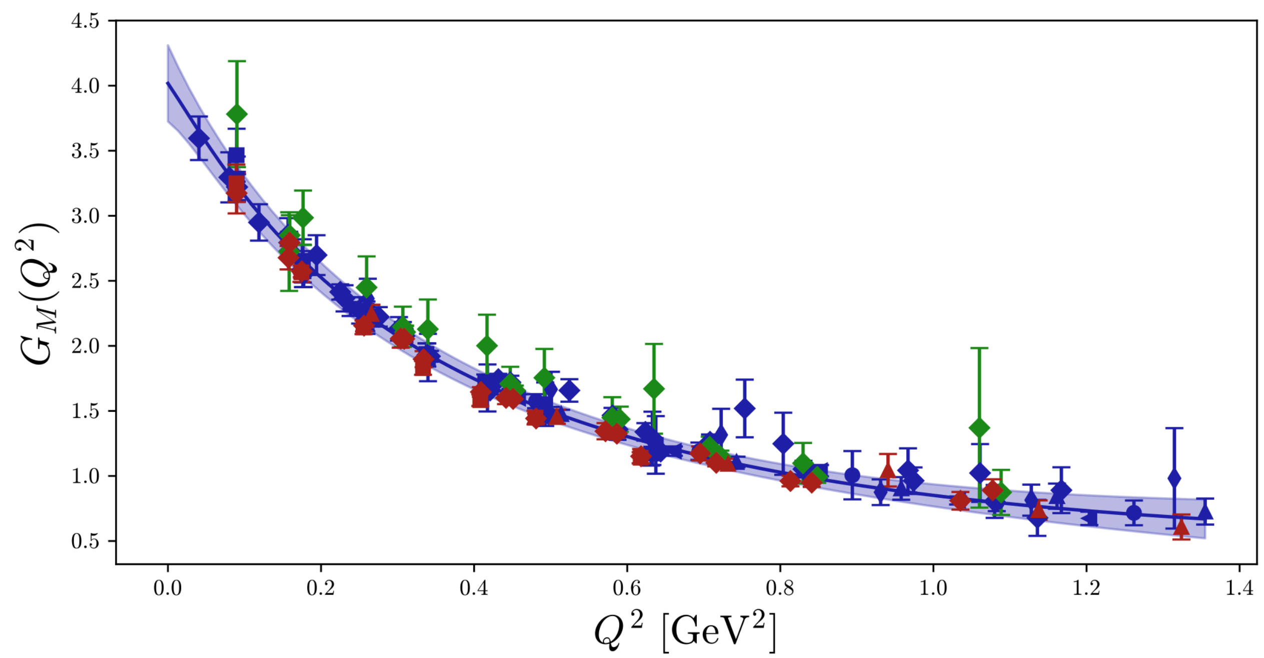
(Preliminary) Results from RQCD

[Rodekamp et al, Lattice 2025]

21 ensembles; a ∈[0.098, 0.039] fm, m_π ∈[130, 430] MeV, $m_\pi L \geq 4$







Further analysis needed to examine systematic effects and extract radii

Alternative avenues to calculate form factors from lattice QCD

Accessing the partonic structure of hadrons

A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big]U(P)$$

- Frame independence (multiple values of $-t$ (Q^2) at same comp. cost)
- Statistical uncertainty can be controlled
- contain physical information

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- Frame independence (multiple values of $-t$ (Q^2) at same comp. cost)
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- contain physical information

- No direct access to x
- skewness independent
- Power-divergent mixing for high Mellin moments (derivatives > 3)
- Signal-to-noise ratio decays with the addition of covariant derivatives
- Number of GFFs increases with order of Mellin moment

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Reconstruction of PDFs/GPDs very challenging

Accessing the partonic structure of hadrons

B. Matrix elements of nonlocal operators (quasi-GPDs, pseudo-GPDs)

$$\langle N(P_f)\,|\,\underline{\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)}\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \mathrm{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \mathrm{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \mathrm{ht}$$

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Calculation challenges

- Standard definition of GPDs in symmetric frame
  separate calculations at each t

- Statistical noise increases with P_3, t
  Projection:
  billions of core-hours for physical point at $P_3 = 3\,\mathrm{GeV}$

Accessing the partonic structure of hadrons

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C. Other methods

See next slide →

Novel Approaches

- ★ Hadronic tensor [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
- Auxiliary scalar quark [U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
- Fictitious heavy quark [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
- Auxiliary scalar quark [V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
- Higher moments [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
- Quasi-distributions (LaMET) [X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
- Compton amplitude and OPE [A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
- Pseudo-distributions [A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
- Good lattice cross sections [Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
- PDFs without Wilson line [Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
- Moments of PDFs of any order [A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

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★ Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248

- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543

- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathscr{W}(z,0)\Psi(0) \,|\, N(P_i)\rangle_\mu$$

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[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

$$\tilde{q}_\Gamma^{\rm GPD}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\, x\, P_3\, z}\, \mathcal{M}(P_f, P_i, z)$$

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathcal{M}(\nu, \xi, t; z_3^2)}{\mathcal{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

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Matching in momentum space
(Large Momentum
Effective Theory)

Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

Well-studied "novel" methods for PDFs/GPDs in LQCD

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[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

Matching resembles factorization:

$$\sigma_{\text{DIS}}(x, Q^2) = \sum_i \left[H^i_{\text{DIS}} \otimes f_i\right](x, Q^2)$$

$$\tilde{q}^{\text{GPD}}_\Gamma(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathcal{M}(P_f, P_i, z)$$

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Light-cone PDFs & GPDs

Quasi-GPDs: contact with light-cone quantities

★ Non-local operators with Wilson line fully renormalizable to all orders

[T. Ishikawa et al., Phys. Rev. D 96, no. 9 (2017) 094019] [X. Ji et al., Phys. Rev. Lett. 120, no. 11 (2018) 112001]

★ Quasi- & light-cone distributions share the same infrared structure

★ Differences in UV region (perturbatively calculable, LaMET)

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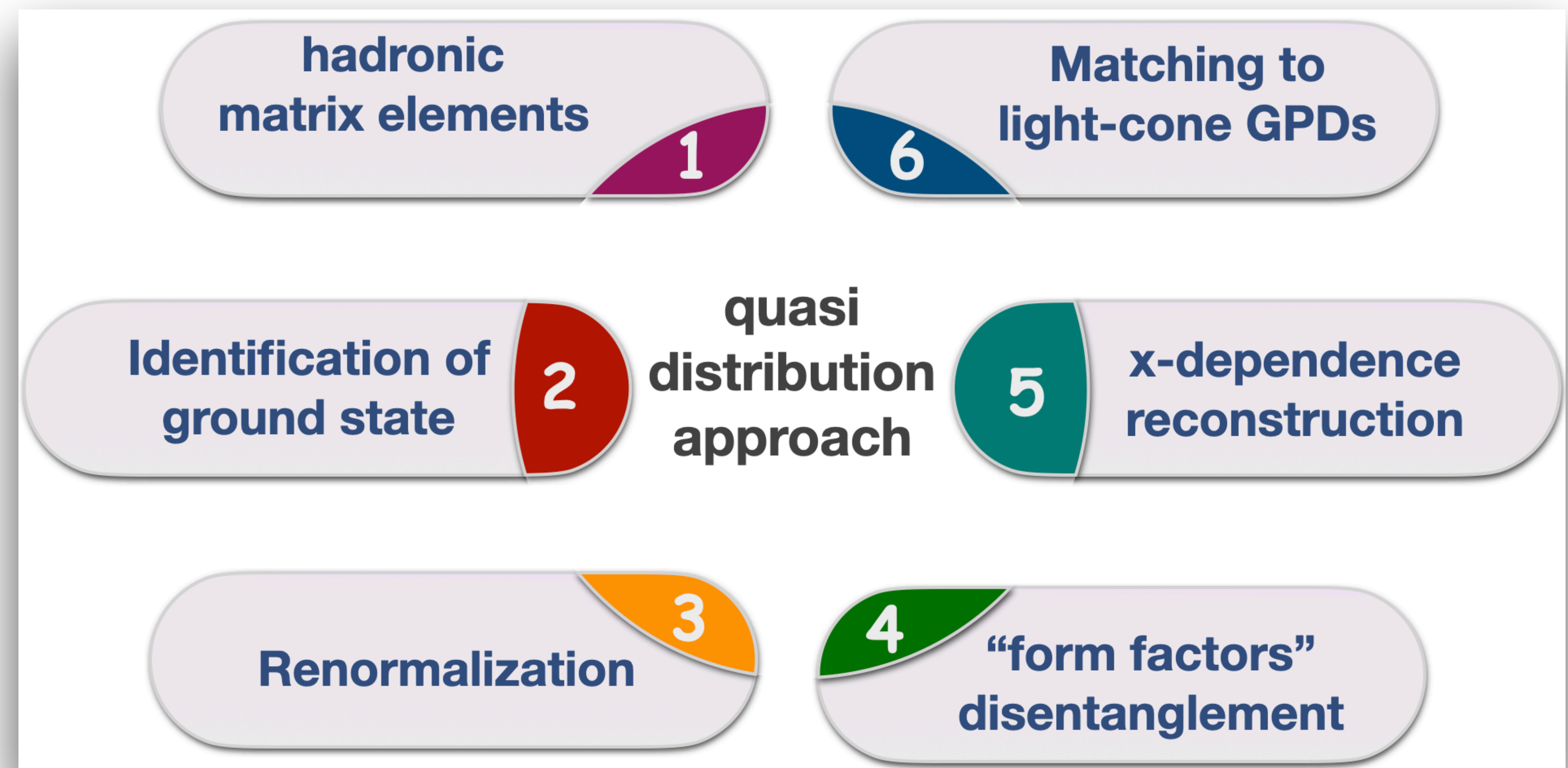
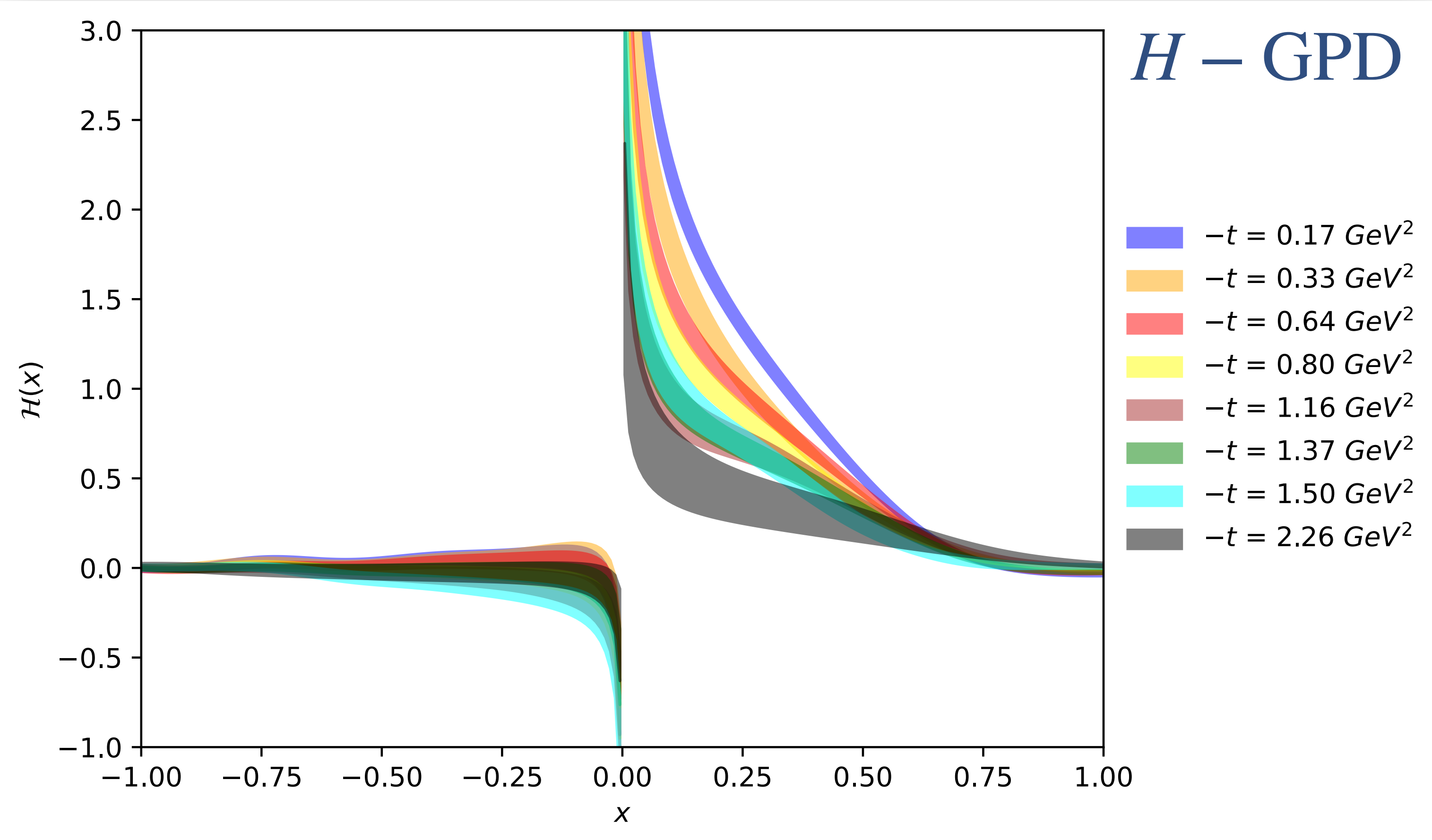
★ Quasi- & light-cone distributions share the same infrared structure

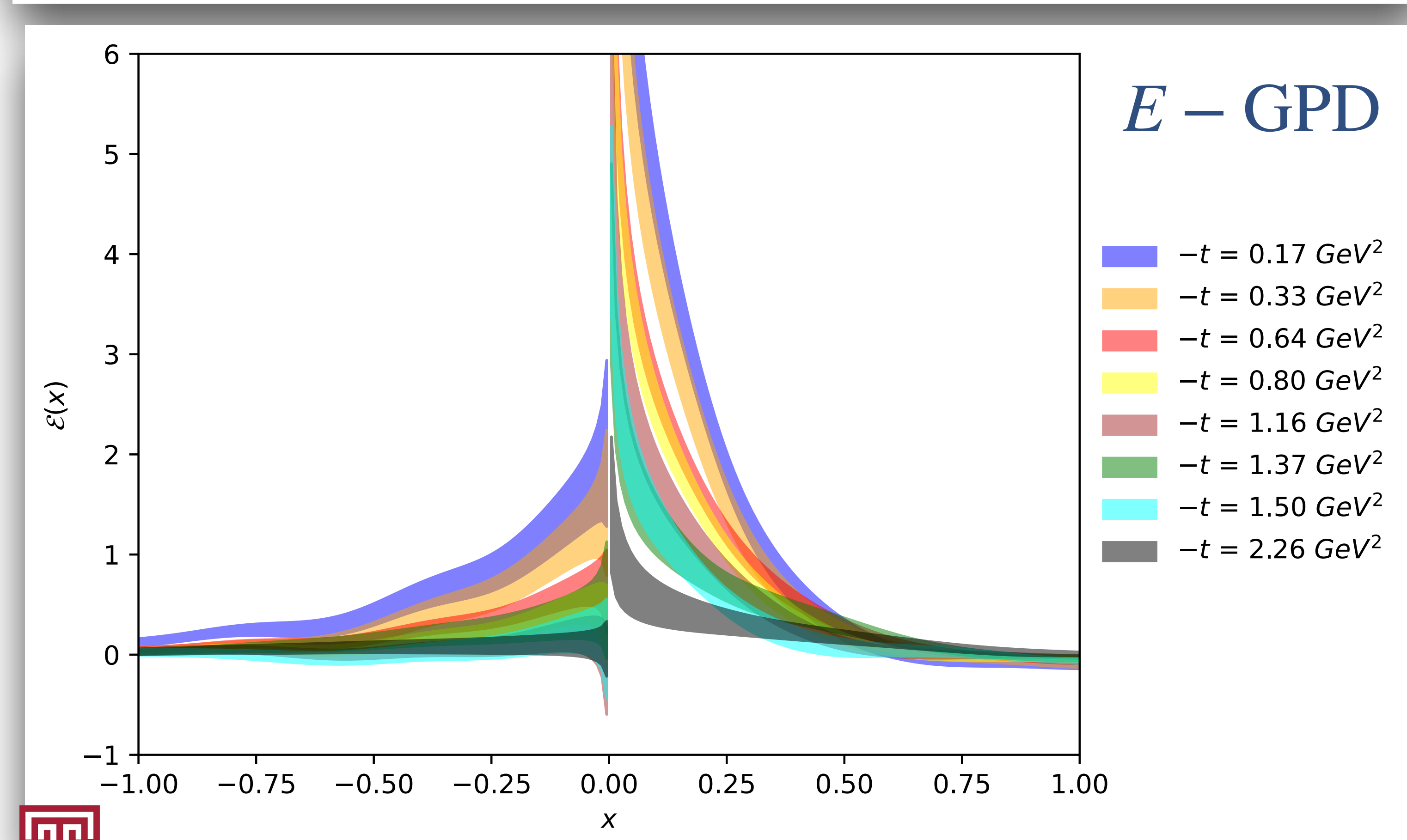
★ Differences in UV region (perturbatively calculable, LaMET)

A number of non-trivial steps

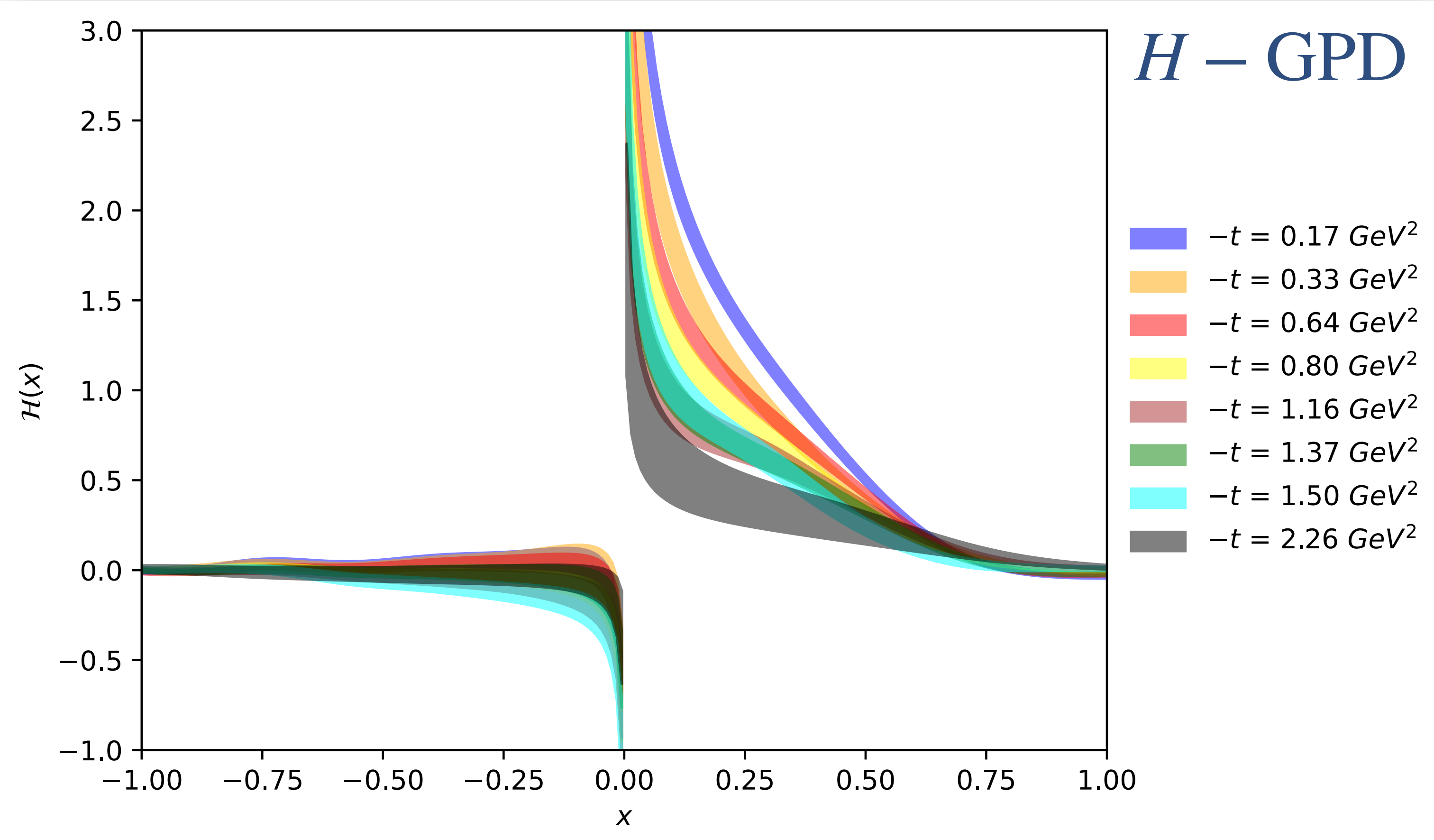


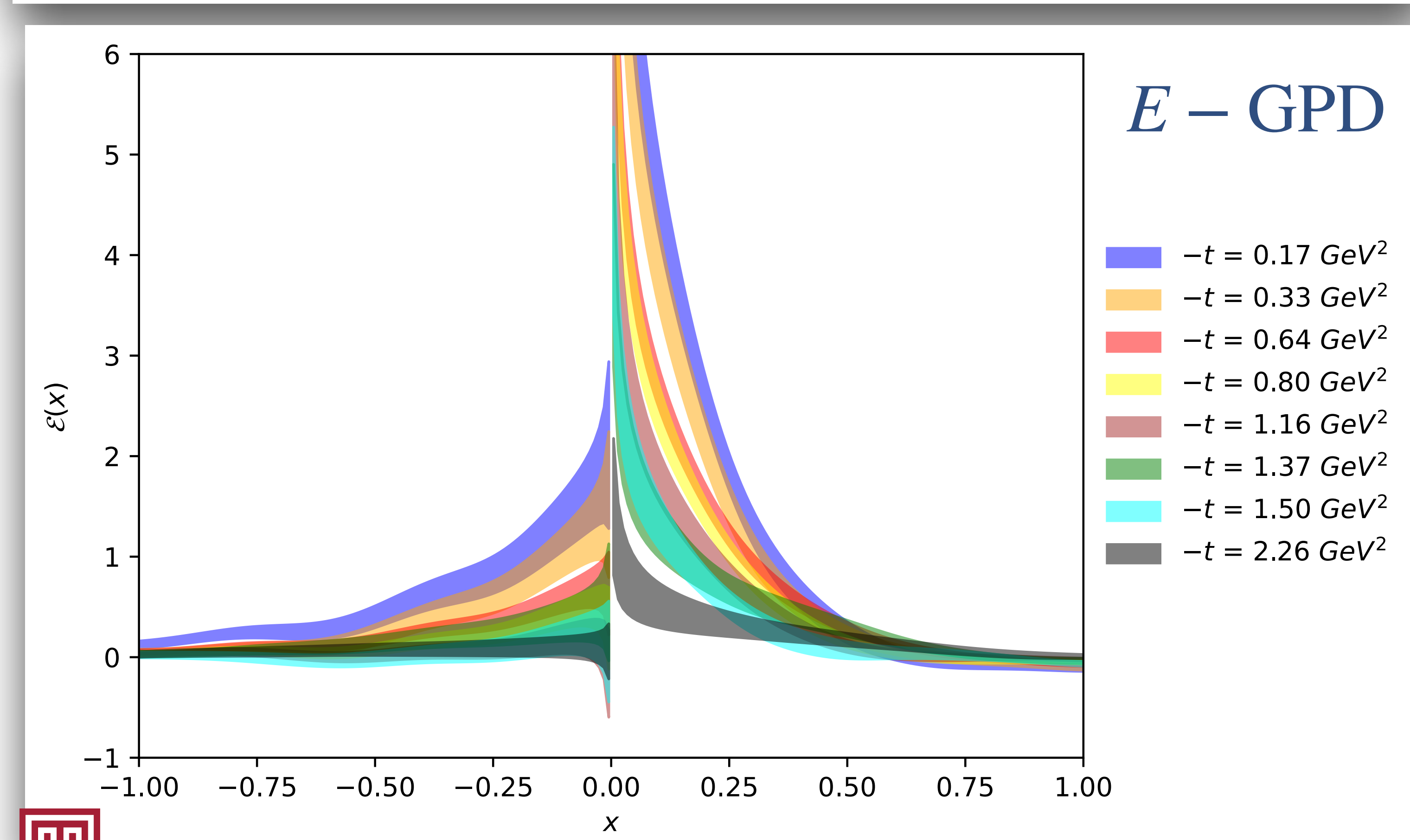
Light-cone GPDs





Light-cone GPDs





Caution:

- ★ +x region: quarks
  -x region: anti-quarks
- ★ anti-quark region susceptible to more systematic uncertainties
- ★ small- and large-x region not reliably extracted
- ★ large -t values unreliable but free

Mellin moments from non-local operators

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu \qquad \mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathscr{M}(\nu, \xi, t; z_3^2)}{\mathscr{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

★ Leading-twist factorization formula

$$\mathscr{M}(z, P, \Delta) \equiv \frac{\mathscr{F}(z, P, \Delta)}{\mathscr{F}(z, P=0, \Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)}{C_0^{\overline{\mathrm{MS}}}(\mu^2 z^2)} \langle x^n \rangle + \mathscr{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$$

★ Avoid power-divergent mixing of multi-derivative operators

★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = \sum_{\substack{i=0\\ \text{even}}}^{n} A_{n+1,i}^q(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = \sum_{\substack{i=0\\ \text{even}}}^{n} B_{n+1,i}^q(t)$$

Mellin moments from non-local operators

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507]

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

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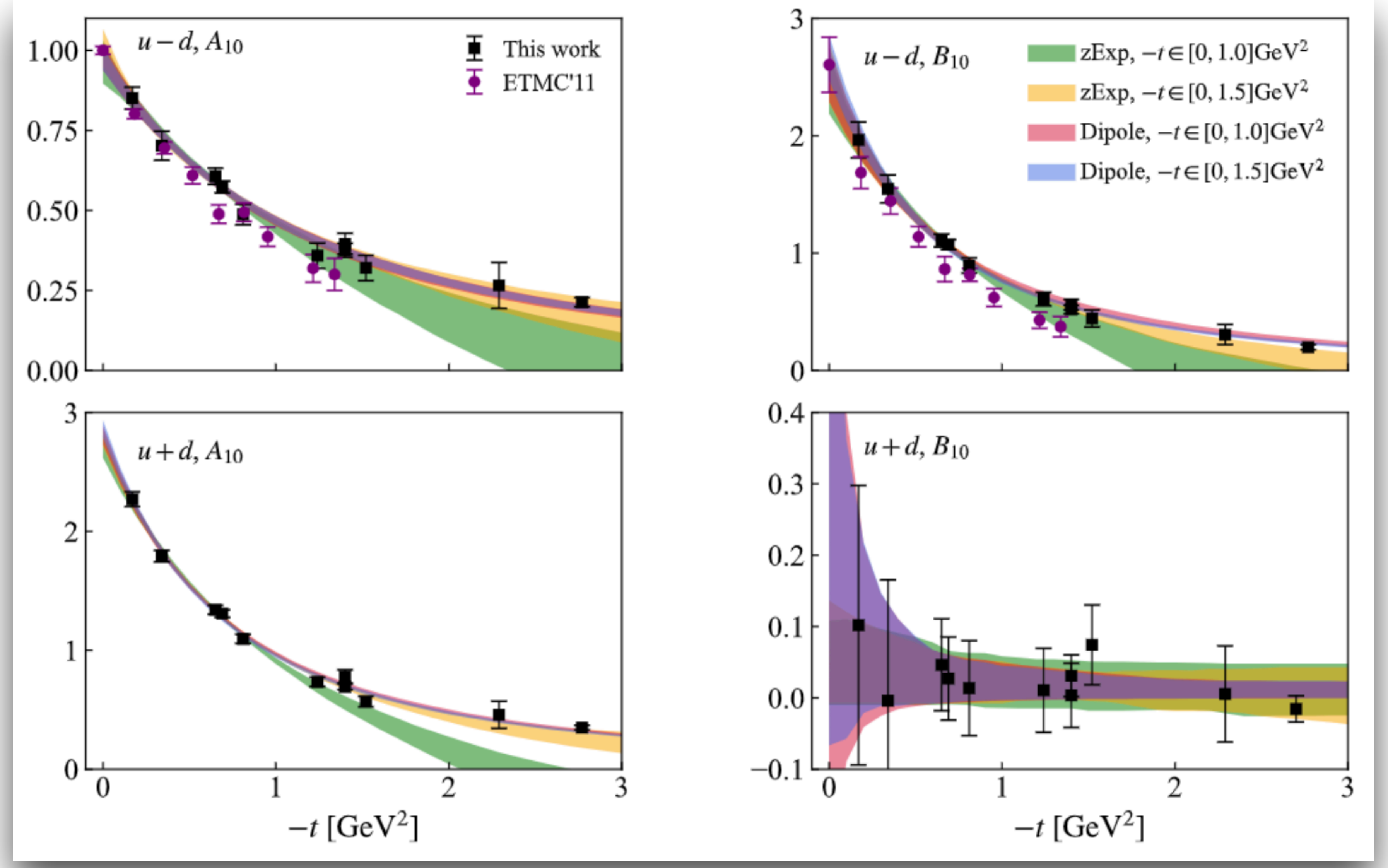

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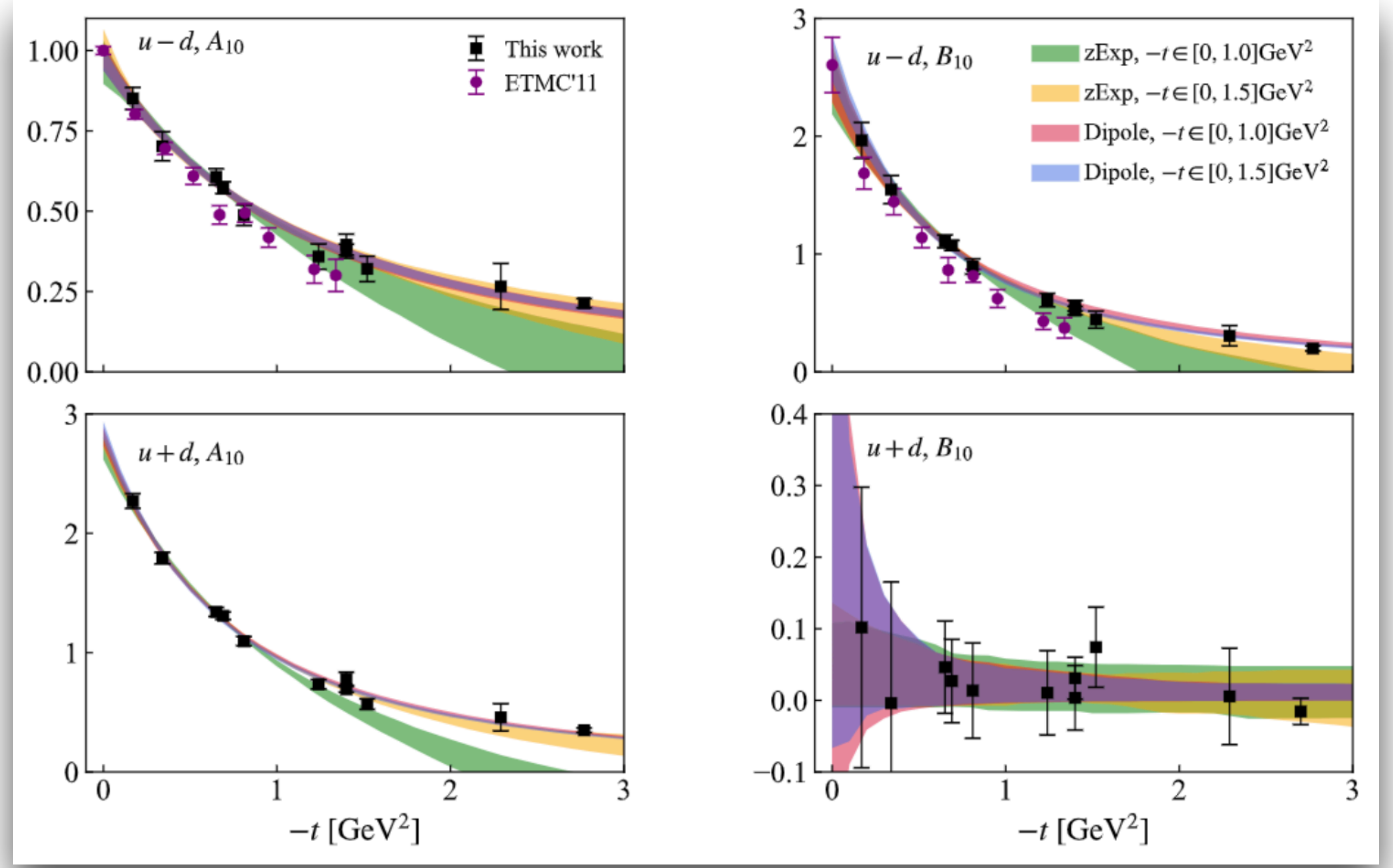
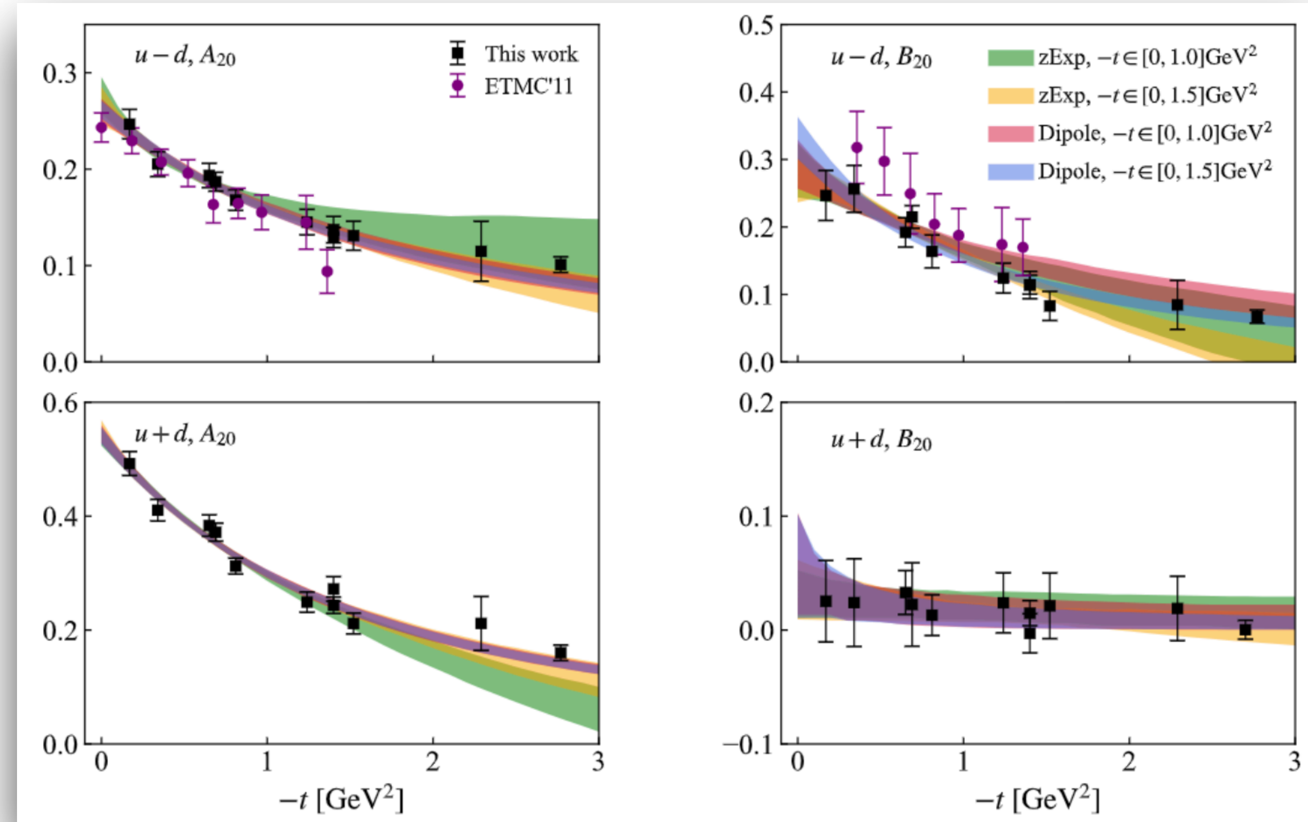
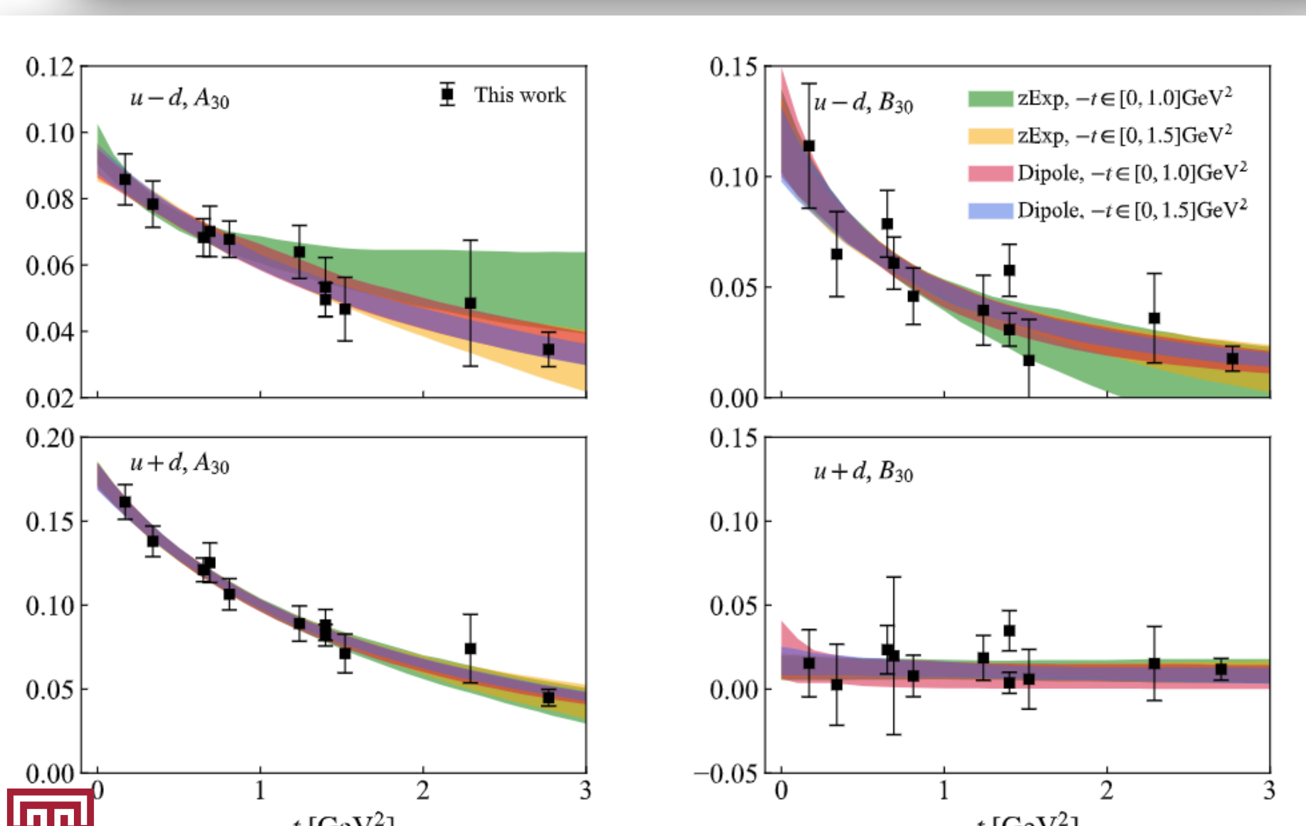


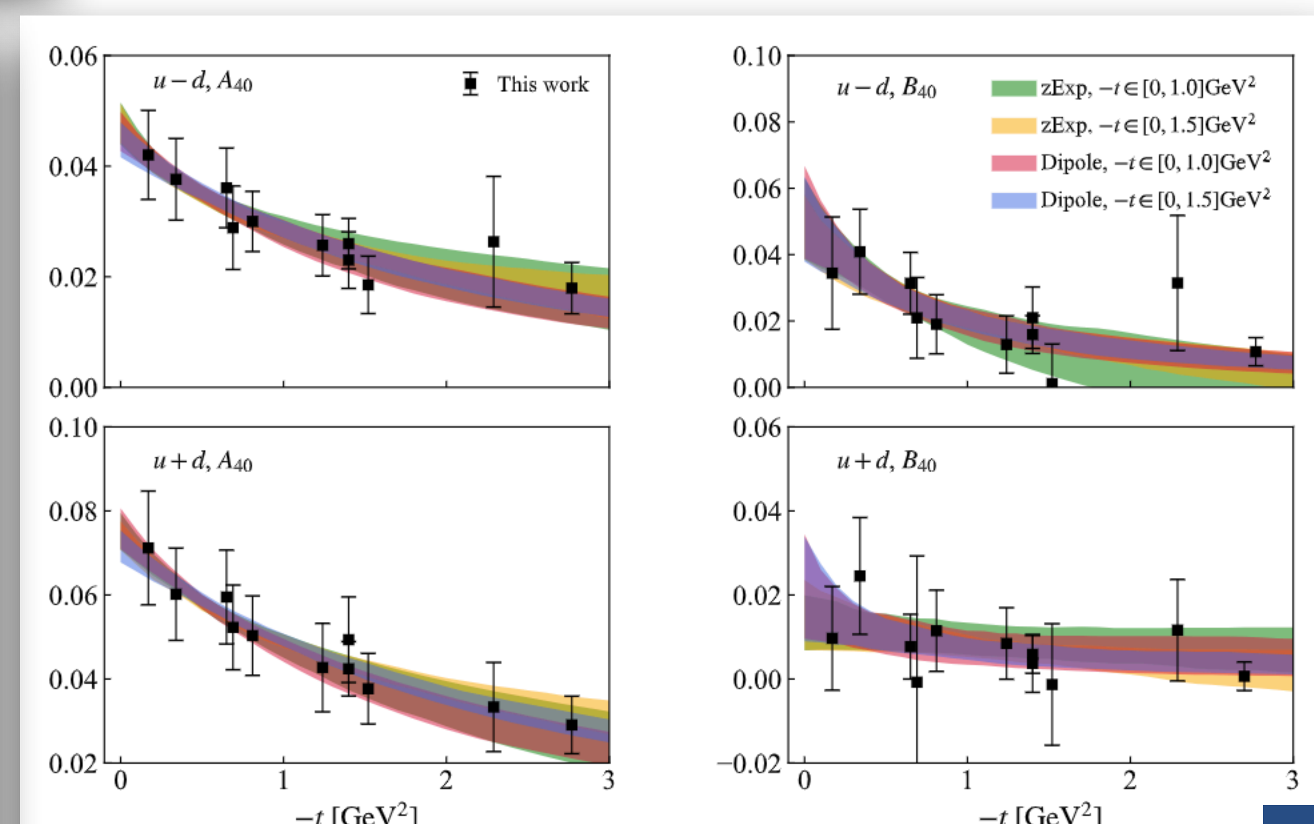
★ Extraction of radii can be obtained from the parameterizations

Mellin moments from non-local operators

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507]







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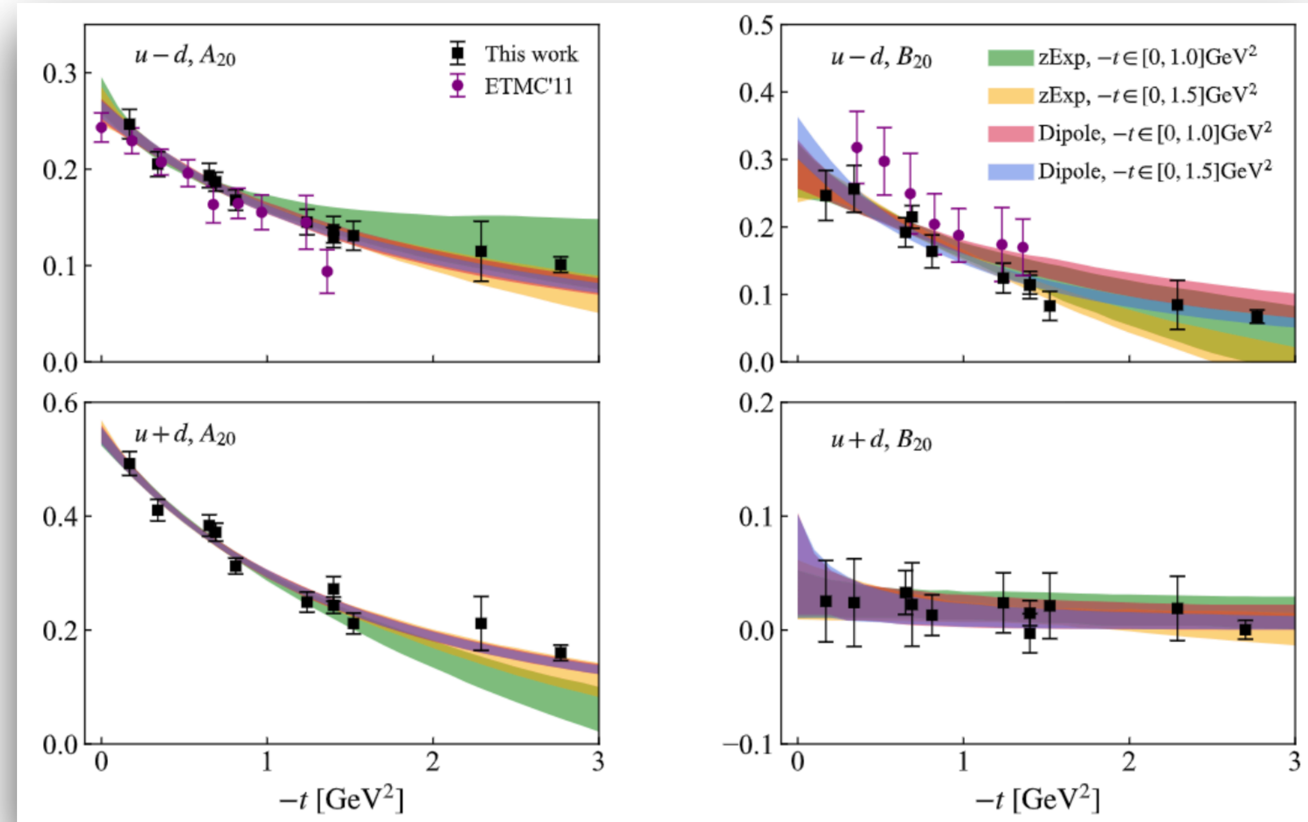
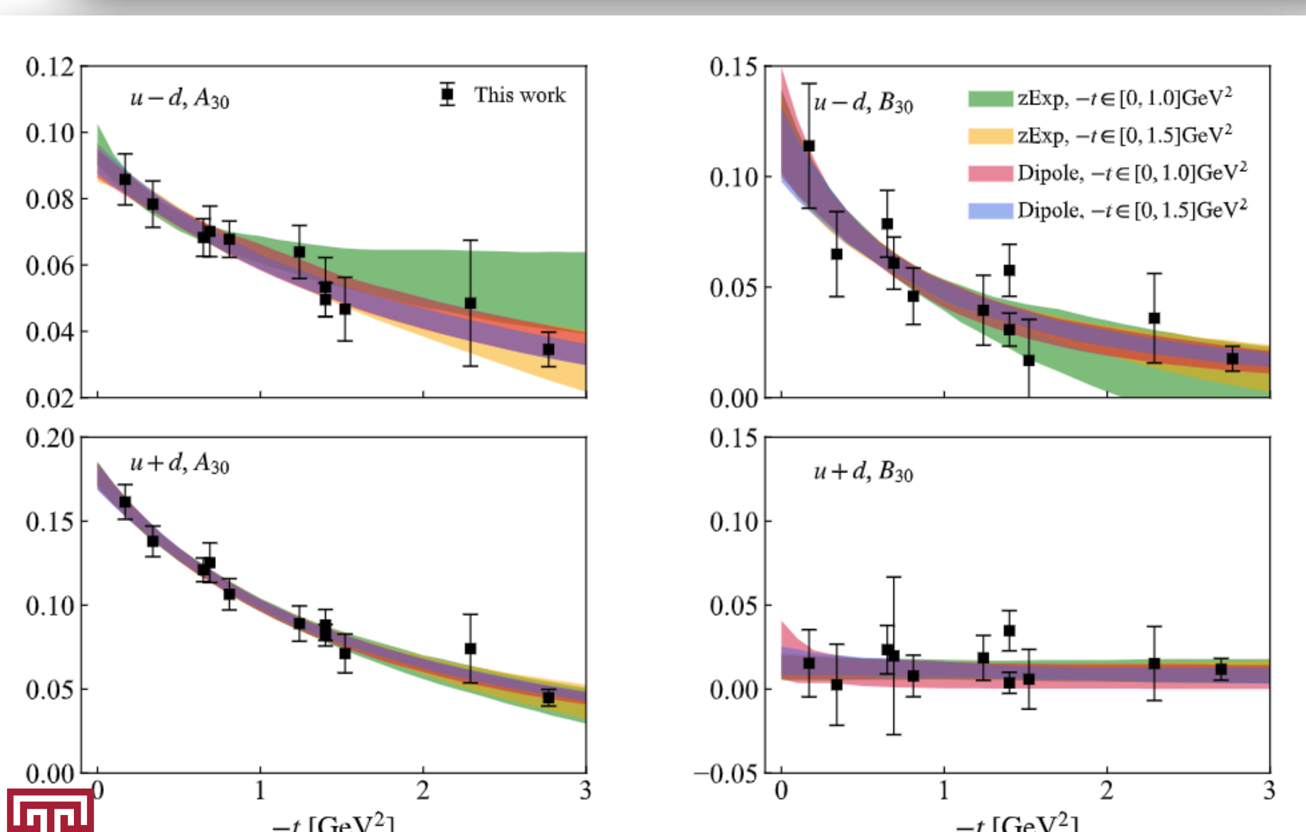
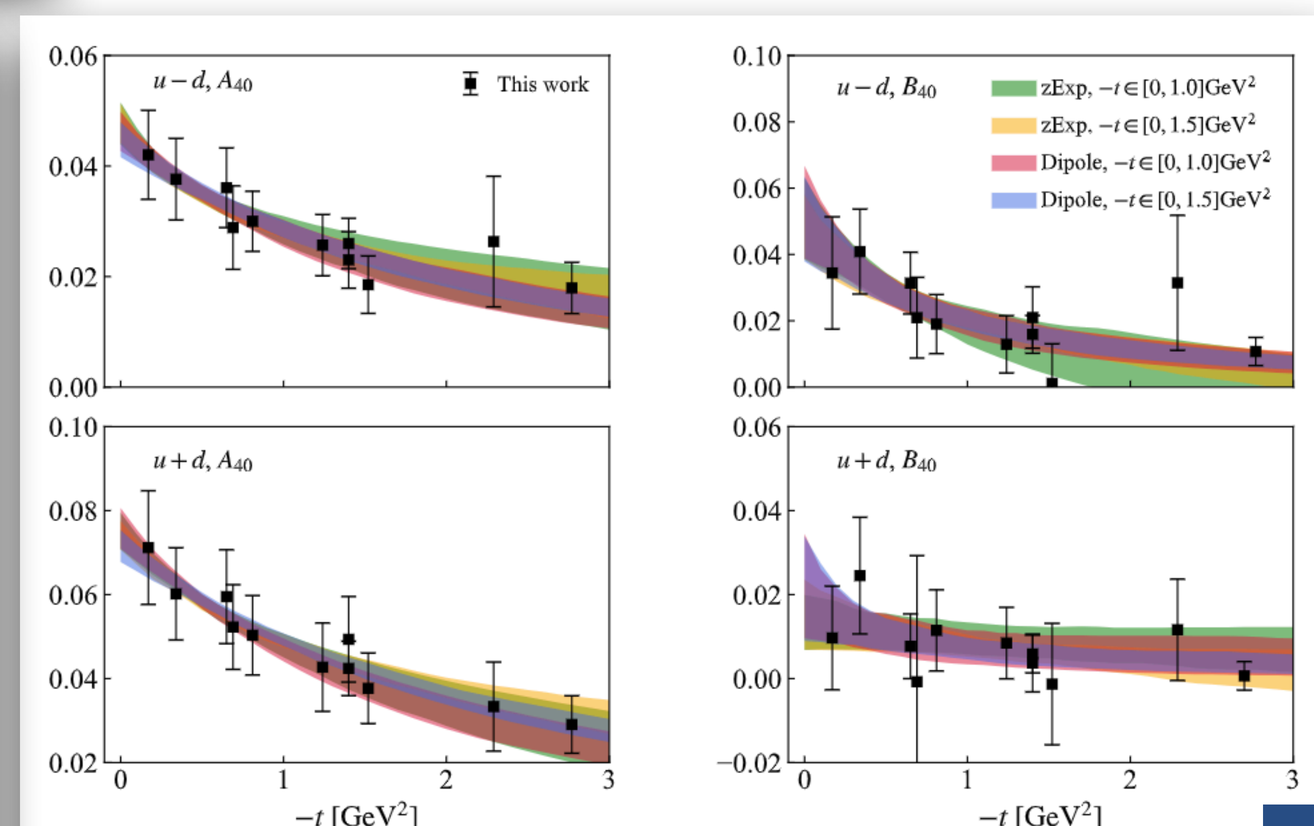


- ★ Access to higher Mellin moments is desirable and achievable

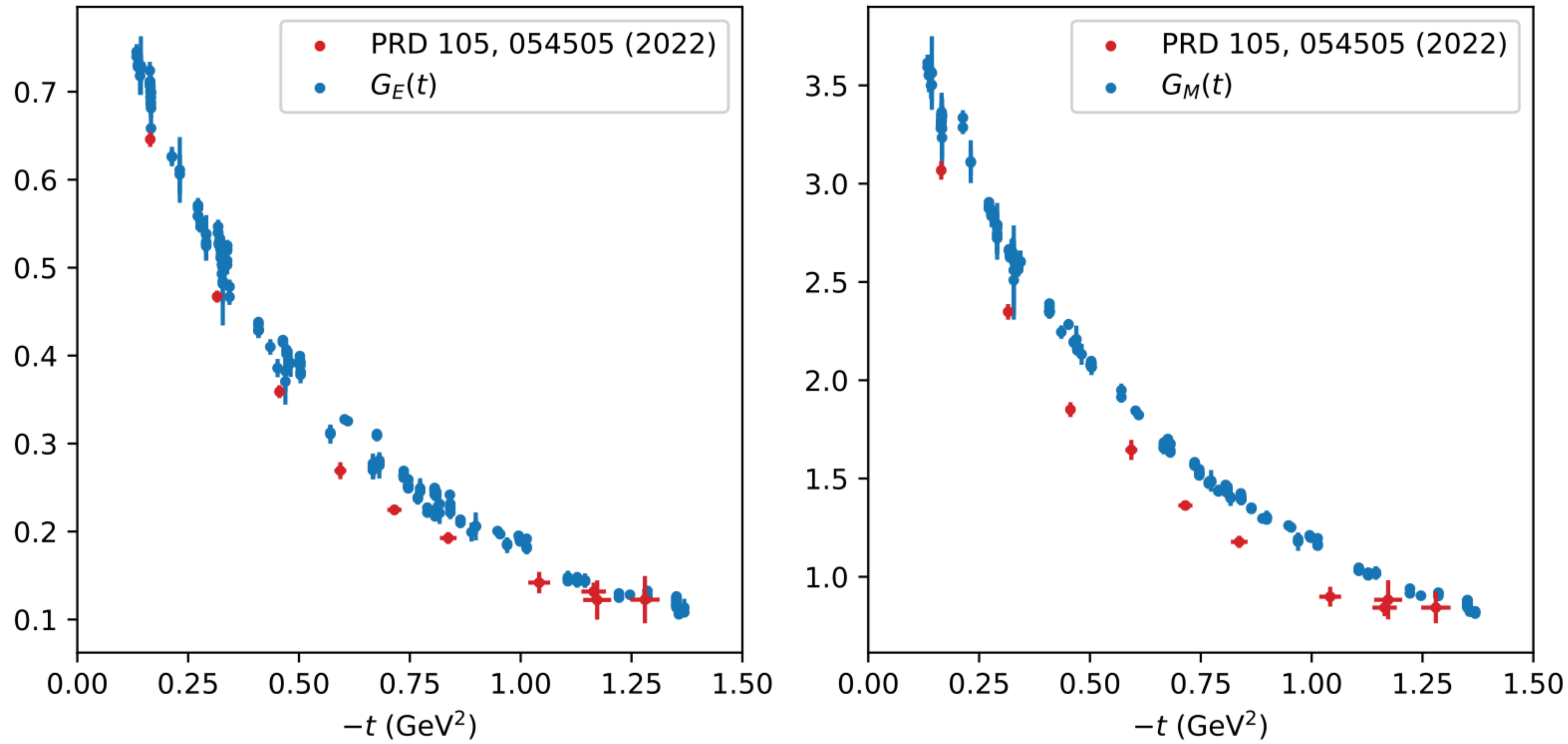




Mellin moments from non-local operators

[H. Dutrieux et al., *JHEP* 08 (2024) 162]

ID	a (fm)	m_π (MeV)	β	$m_\pi L$	$L^3 \times N_T$	$N_{\rm cfg}$	$N_{\rm srcs}$	rk ($\mathcal{D}$)
a094m358	0.094(1)	358(3)	6.3	5.4	$32^3 \times 64$	348	4	64



Mellin moments from non-local operators

[H. Dutrieux et al., *JHEP* 08 (2024) 162]

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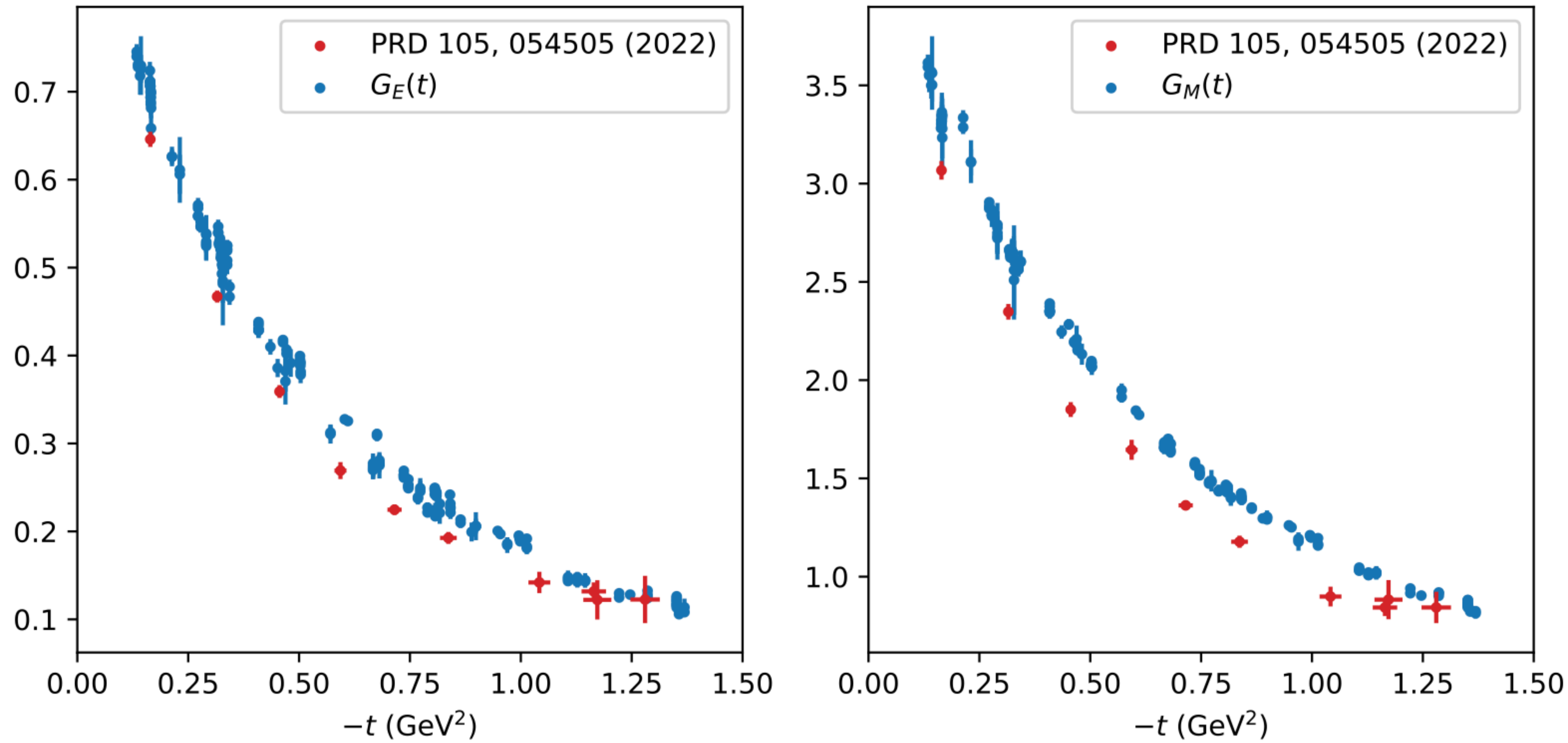


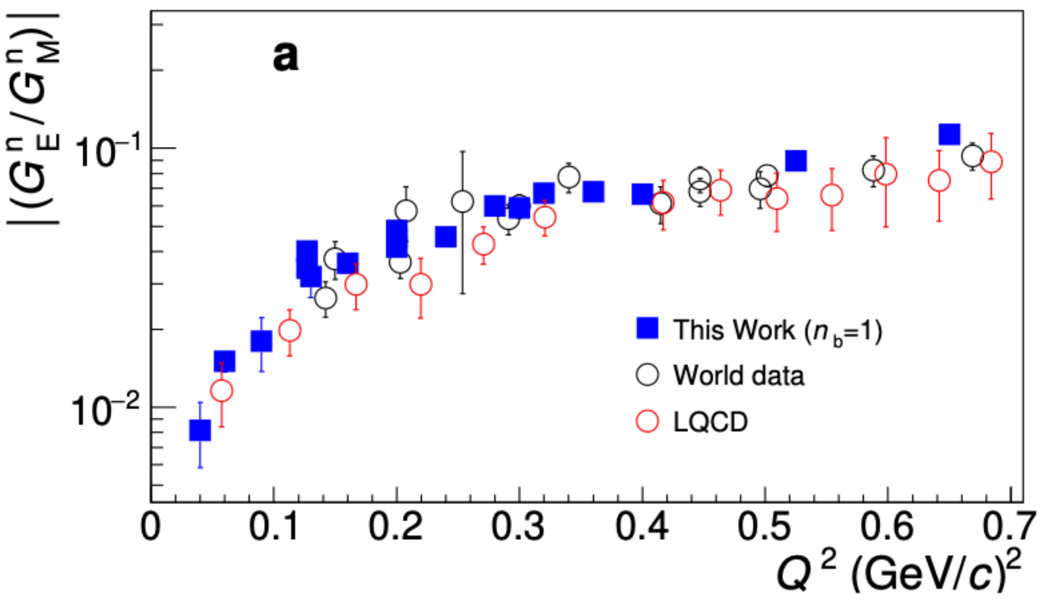
★ Comparison with FFs from local operators reveals the need to study systematic uncertainties

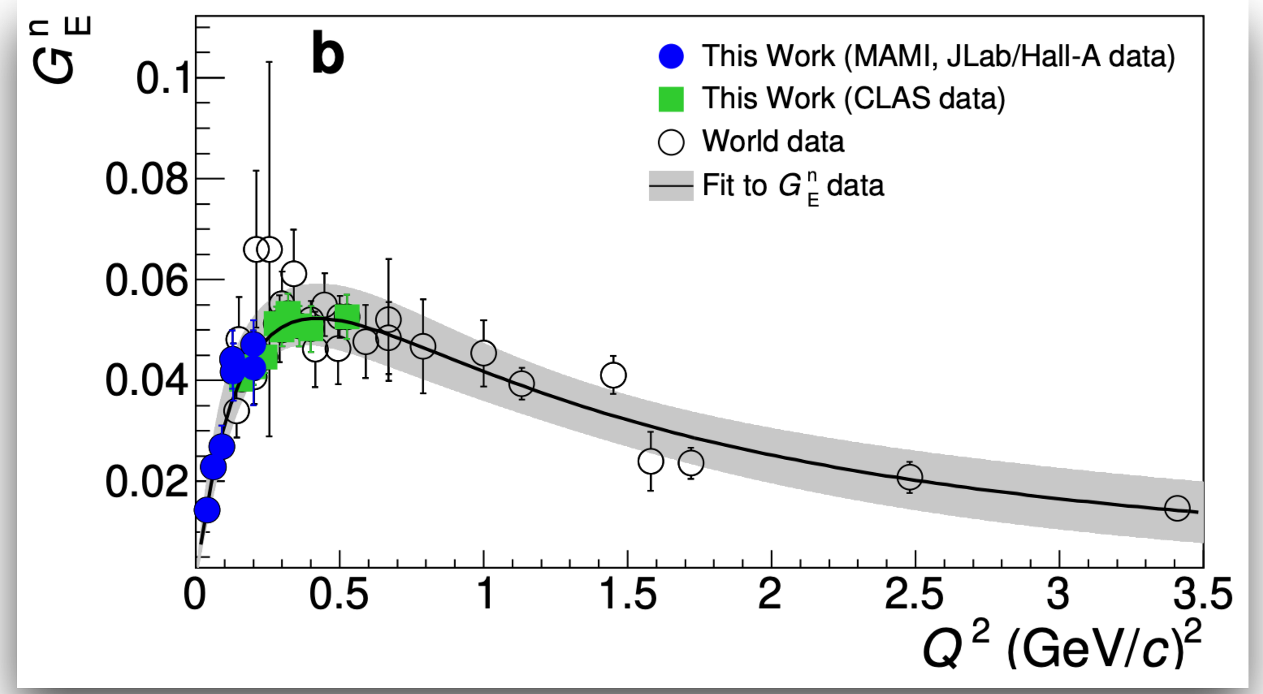
★ (Comparison for different ensembles)

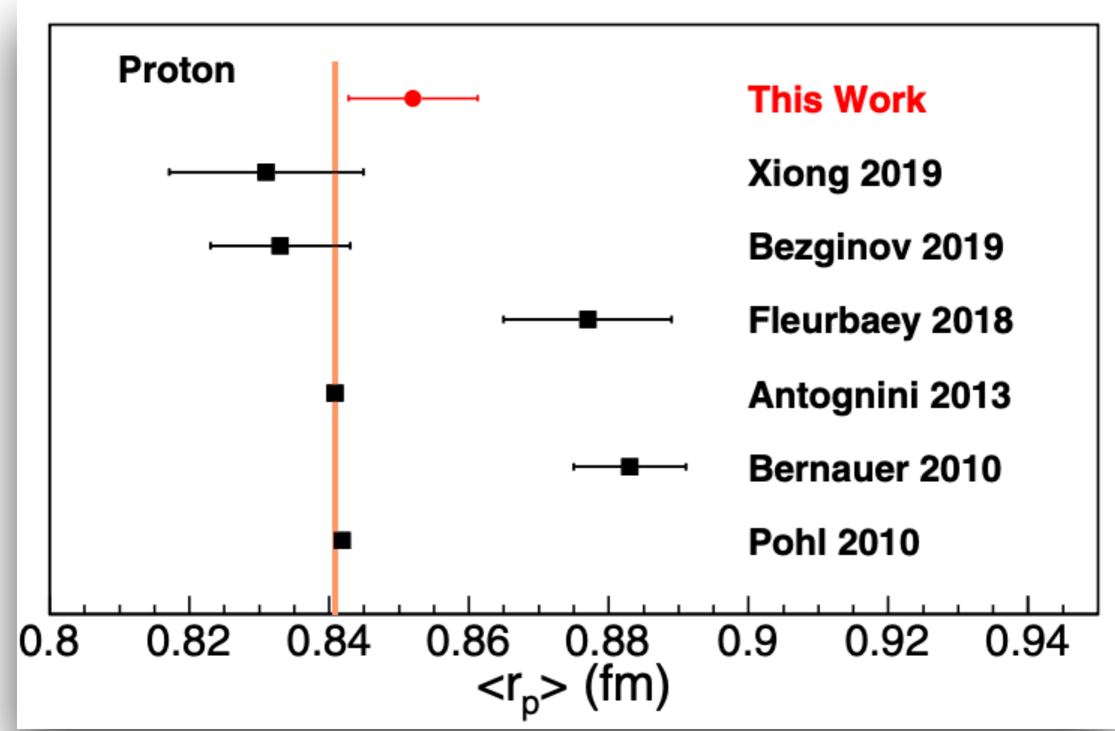
Synergistic activities

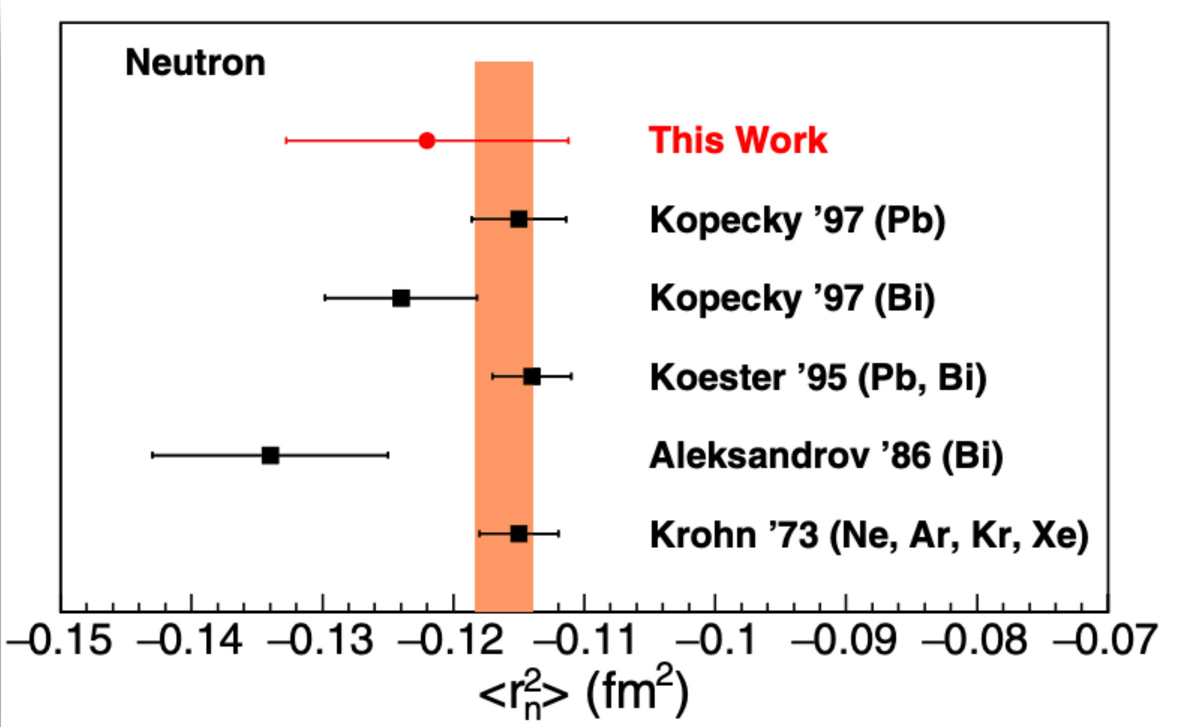
LQCD results with experimental data

[H. Atac et al., Nature Comm. 12, 1759 (2021)] [H. Atac et al., Eur. Phys. J. A 57, 65 (2021)]









Summary

★ Calculations of EM form factors and radii using local operators have been refined

★ Percent level has been achieved with systematic uncertainties addressed.

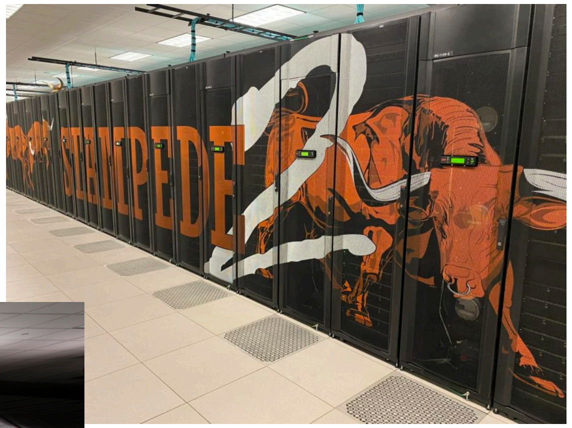
★ Comparison between lattice data from different formulations, as well as with experiments have improved

★ Extraction of EM form factors using non-local operators with boosted hadrons is an alternative avenue.

★ Synergy with global analysis is an exciting prospect

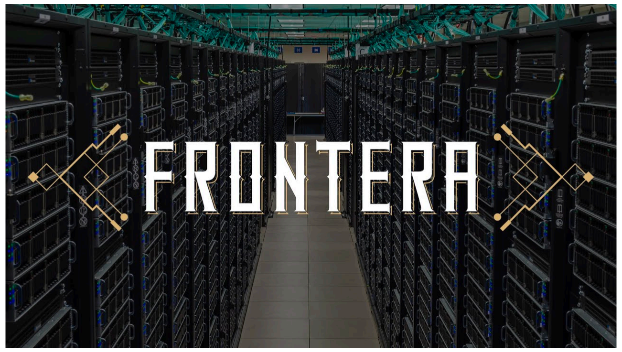
Acknowledgments

Oak Ridge NL



Texas

Oak Ridge NL



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Thank you

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