IN-MEDIUM ELECTROMAGNETIC FORM FACTORS AND CHARGE RADII OF PSEUDOSCALAR AND VECTOR MESONS

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OUTLINE OF THE TALK

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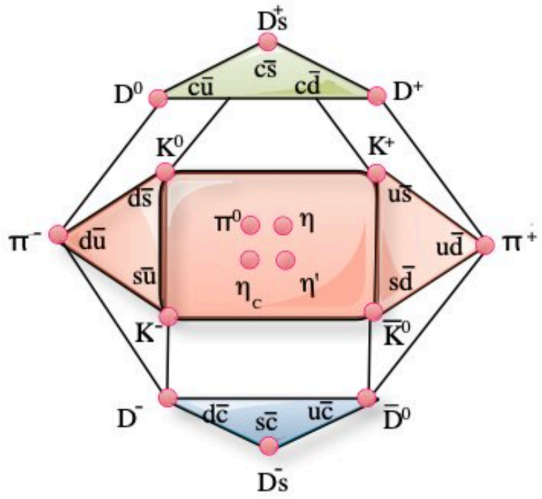
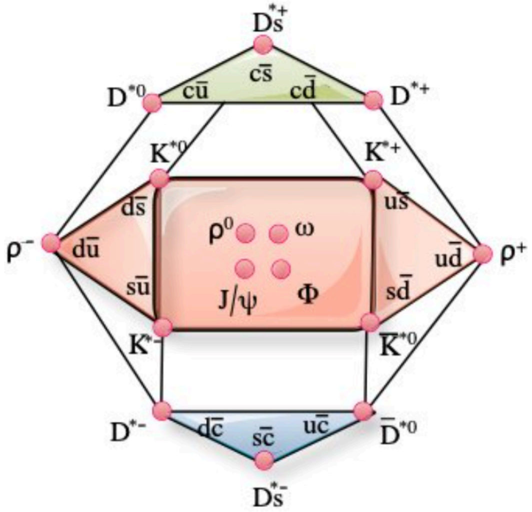
- Motivations
- Pseudoscalar & Vector Mesons
- Electromagnetic Form Factor
- Charge Radii
- Nuclear Medium—Constructed in the quark level (QMC)
- Results and Predictions
- Summary and Outlook





Based on our recent papers:
1. PRD 111 (2025), 074004 or e-Print. 2412.09883
2. PRD 112 (2025), 114030 or e-Print. 2508.20501

Courtesy: Gutierrez-Guerrero, *et al.*, PRD100, 114032 (2019)

Motivations

- Modifications of the hadron structure and properties in a nuclear medium and nuclei has been explored through a variety of perspectives RMP **82**, 2949 (2010); PPNP **96**, 99 (2017); IJMPE **19**, 147 (2010); NPA **741**, 81 (2004); PPNP **97**, 199 (2017); PPNP **112**, 103770 (2020)
- One of the well known examples—The modifications of the structure functions of the bound nucleons in nuclei—European Muon Collaboration (EMC) effect—highlights the influence of the nuclear medium on the internal quark-gluon dynamics in the nucleons PLB **123**, 275 (1983)
- This study will provide us with the important and deepen insights into the complicated hadron dynamics in terms of the quark-gluons in the nuclear medium

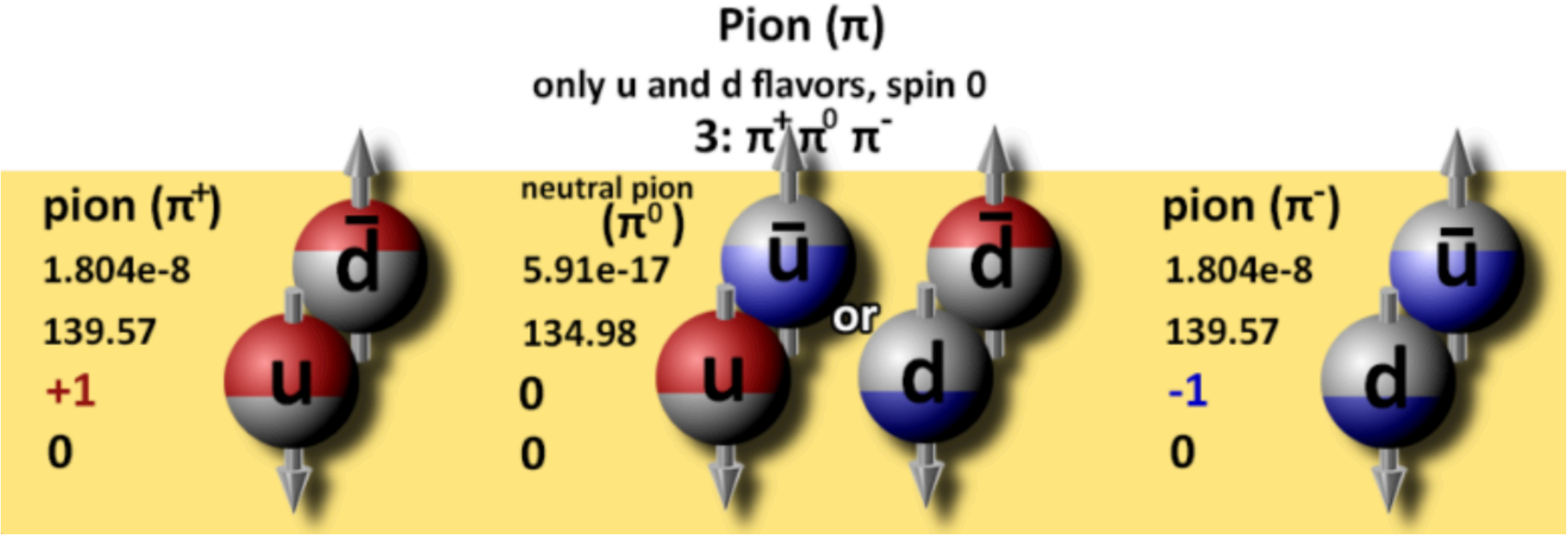
Motivations

- Instead of the EMC effect, different hadron medium modifications have been observed—Mass modifications, width broadening and the increase of the radii in medium

  PLB **860**, 139172 (2025); PRL **98**, 042501 (2007); PRL **91**, 052301 (2003)

- Mass modifications in the nuclear medium are associated with the modifications in the scalar mean field—might be expected to link to the quark chiral condensate—Signaling partial restoration of chiral symmetry
- In this study, we will focus on the nuclear medium modifications of the meson space like EMFFs—connected to ongoing JLAB 12 GeV program and future EIC facility

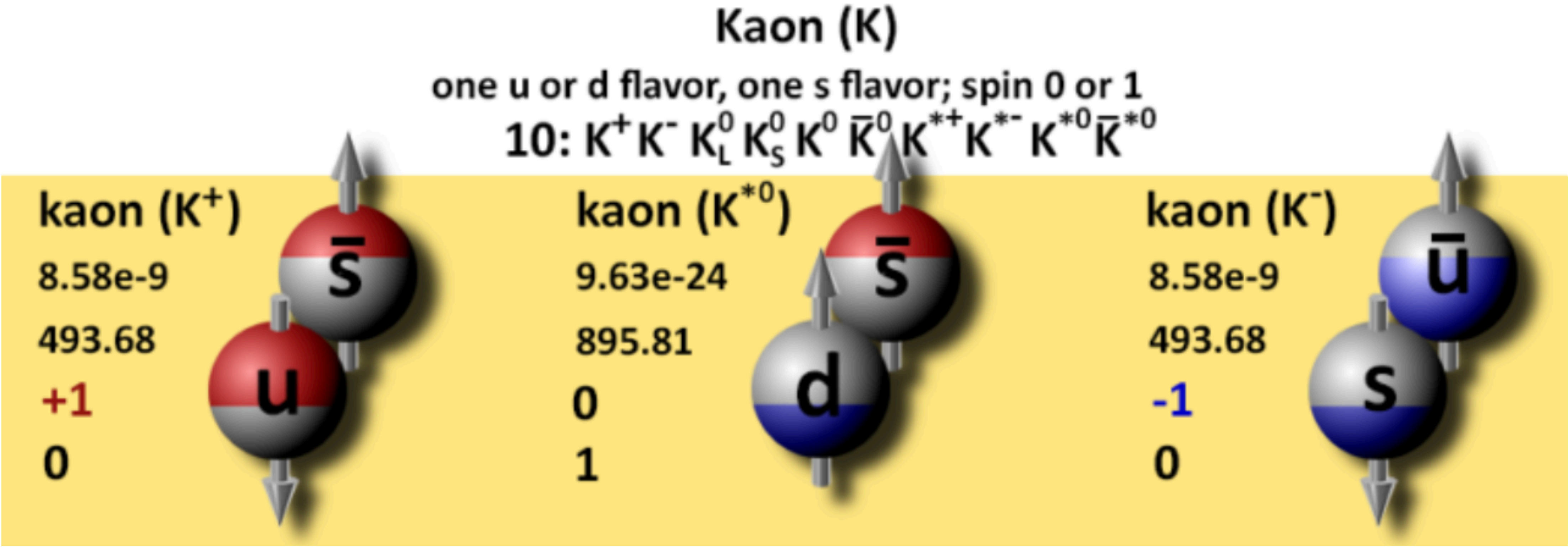
- Pseudoscalars (Goldstone Bosons)—Different quark contents



- Different quark flavor content—different properties

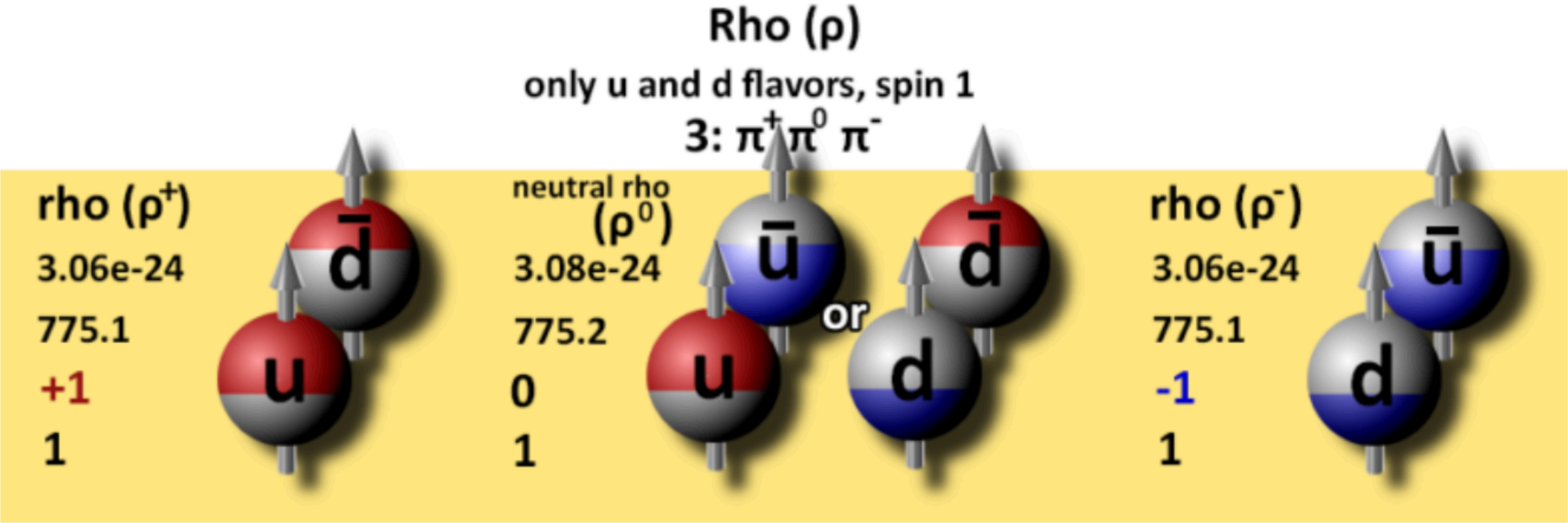
PSEUDOSCALAR &VECTOR MESONS

- Pseudoscalars (Goldstone Bosons)—Different quark contents



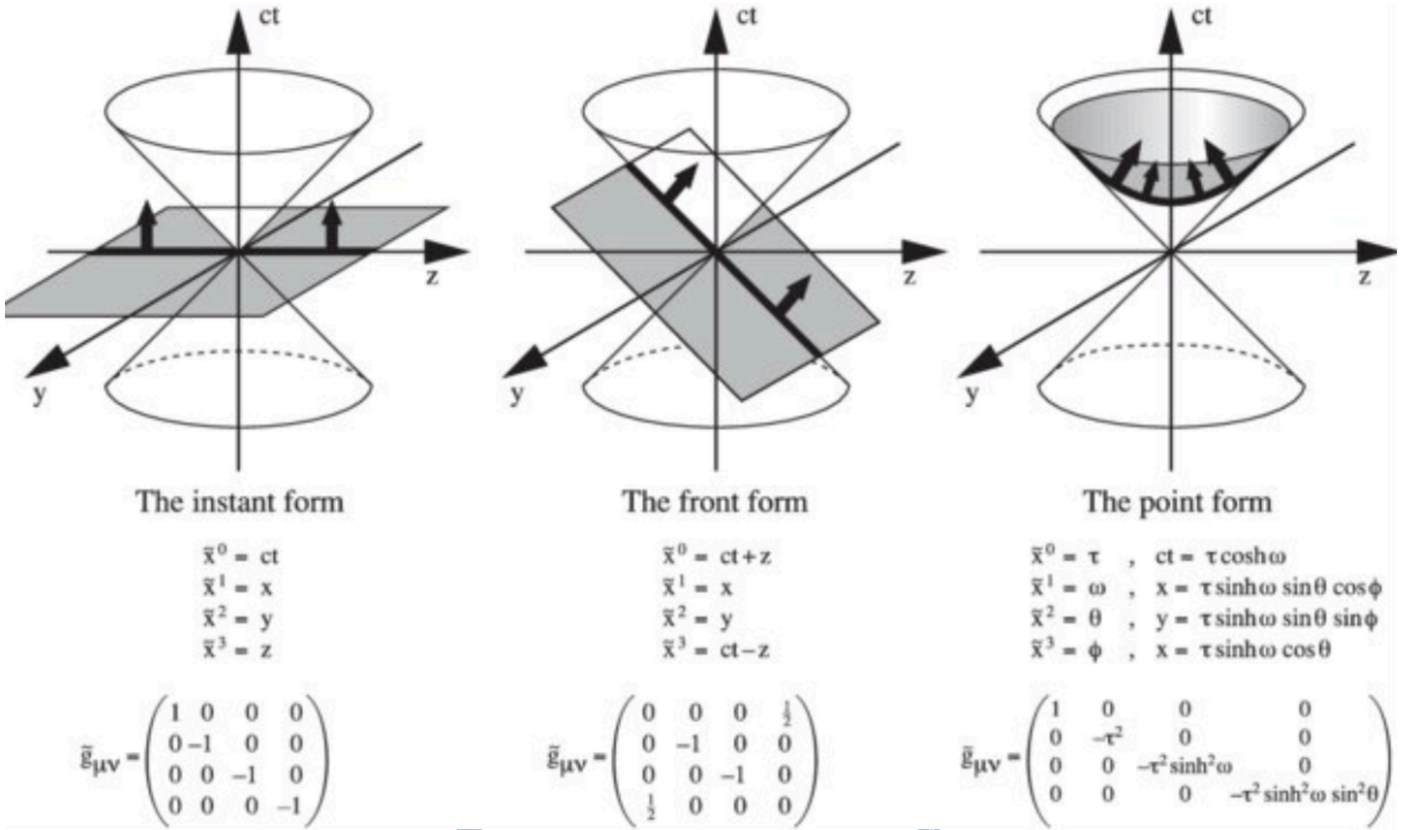
- Different quark flavor content—different properties

- Vector Mesons—Different quark contents



- Different quark flavor content—different properties

- Based on Dirac paper on RMP 21 (1949, he proposed the forms of the relativistic dynamics



Light-Front Quark Model (LFQM)

- In the LFQM, the meson state $|\mathscr{M}\rangle \equiv |\mathscr{M}(P, J, J_z)\rangle$ as a bound state of the constituent quark and antiquark with meson momentum P and the total angular momentum (J, J_z) is defined as

$$|\mathscr{M}\rangle = \int [d^2\mathbf{p}_q][d^2\mathbf{p}_{\bar{q}}]2(2\pi)^3\delta^3(\mathbf{P}-\mathbf{p}_q-\mathbf{p}_{\bar{q}}\sum \Psi^{JJ_z}_{\lambda_q\lambda_{\bar{q}}}(x, \mathbf{k}_\perp)\,|q_{\lambda_q}(p_q)\bar{q}_{\lambda_{\bar{q}}}(p_{\bar{q}})\rangle$$

- The LFWF of the ground state of the pseudo scalar meson in momentum space can be defined as

Spin-orbit wave functions

$$\Psi^{JJ_z}_{\lambda_q\lambda_{\bar{q}}}(x, \boldsymbol{k}_\perp) = \Phi(x, \boldsymbol{k}_\perp)\mathcal{R}^{JJ_z}_{\lambda_q\lambda_{\bar{q}}}(x, \boldsymbol{k}_\perp)$$

Radial wave function

- Matrix elements of the spin-orbit wave function can be derived via the Melosh transformation in the covariant form

$$\mathcal{R}^{00}_{\lambda_q\lambda_{\bar{q}}} = \frac{1}{\sqrt{2}\tilde{M}_0}\bar{u}_{\lambda_q}(p_q)\gamma_5 v_{\lambda_{\bar{q}}}(p_{\bar{q}})$$

$$\tilde{M}_0 \equiv \sqrt{M_0^2 - (m_q - m_{\bar{q}})^2}$$

$$M_0^2 = \frac{\boldsymbol{k}_\perp^2 + m_q^2}{x} + \frac{\boldsymbol{k}_\perp^2 + m_{\bar{q}}^2}{1-x}$$

- The explicit form of the spin-orbit wave functions for pseudo scalar meson is given

$$k^{R(L)} = k_x \pm i k_y \quad \text{and} \quad \mathcal{A} = x m_{\bar{q}} + (1-x) m_q$$

$$\mathcal{R}^{00}_{\lambda_q \lambda_{\bar{q}}}(x, \boldsymbol{k}_\perp) = \frac{1}{\sqrt{2}\sqrt{\mathcal{A}^2 + \boldsymbol{k}_\perp^2}} \begin{pmatrix} k^L & \mathcal{A} \\ -\mathcal{A} & k^R \end{pmatrix}$$

- The spin orbit wave function satisfy the orthonormality conditions

$$\sum_{\lambda_q, \lambda_{\bar{q}}} \left\langle \mathcal{R}^{JJ_z}_{\lambda_q \lambda_{\bar{q}}} \middle| \mathcal{R}^{J'J'_z}_{\lambda_q \lambda_{\bar{q}}} \right\rangle = \delta_{JJ'} \delta_{J_z J'_z}$$

Quark-Meson Coupling (QMC) Model

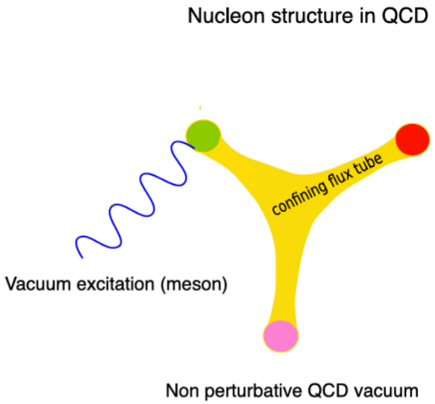
◉ Effective Lagrangian density for the SNM at hadronic level

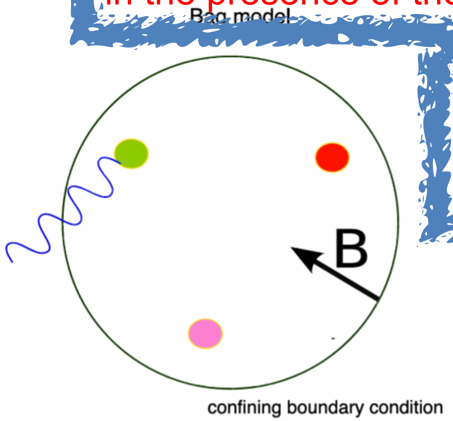
$$\mathcal{L}_{\rm QMC} = \mathcal{L}_{\rm nucleon} + \mathcal{L}_{\rm meson} + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_{\rm nucleon} = \bar{\psi}[i\partial\!\!\!/ - m_N]\psi$$

$$\mathcal{L}_{\rm meson} = \frac{1}{2}(\partial_\mu \hat{\sigma}\partial^\mu \hat{\sigma} - m_\sigma^2 \hat{\sigma}^2) - \frac{1}{2}[\partial_\mu \hat{\omega}_\nu(\partial^\mu \hat{\omega}^\nu - \partial^\nu \hat{\omega}^\mu) - m_\omega^2 \hat{\omega}^\mu \hat{\omega}_\mu]$$

$$\mathcal{L}_{\rm int} = \tilde{g}_{N\sigma}(\hat{\sigma})\bar{\psi}\psi\hat{\sigma} - g_{N\omega}\hat{\omega}^\mu \bar{\psi}\gamma_\mu \psi$$



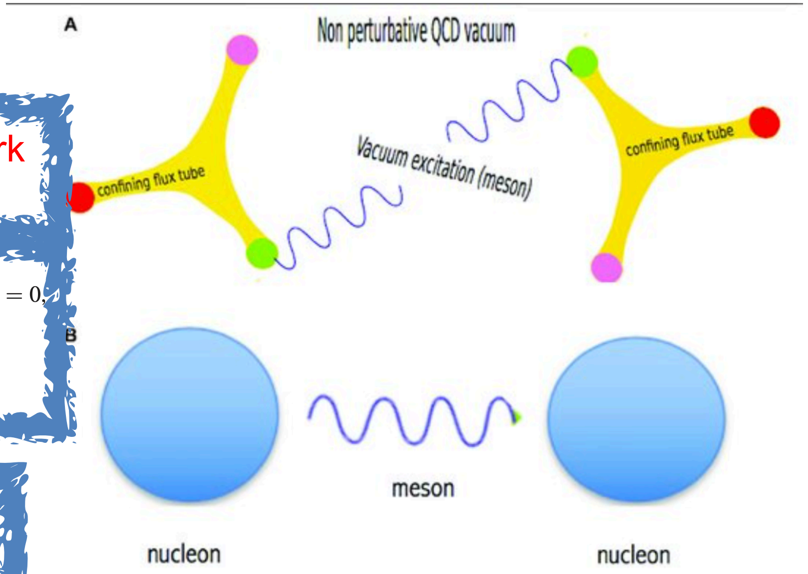


Dirac equations for the quark and antiquark in the presence of the mean field potential

$$[i\partial\!\!\!/ - (m_q - V_{q\sigma}) \mp \gamma^0 V_{q\omega}]\begin{pmatrix}\psi_q(z)\\ \psi_{\bar{q}}(z)\end{pmatrix} = 0,$$

$$[i\partial\!\!\!/ - m_Q]\begin{pmatrix}\psi_Q(z)\\ \psi_{\bar{Q}}(z)\end{pmatrix} = 0,$$

$$m_q^* = m_q - V_{q\sigma}.$$



Quark-Meson Coupling (QMC) Model

◉ Effective nucleon mass in nuclear medium

$$m_N^*(\hat{\sigma}) = m_N - \tilde{g}_{N\sigma}(\hat{\sigma})\hat{\sigma}$$

◉ At nucleon level, the EOM of the meson fields are given by

$$(\Box + m_\sigma^2)\sigma = \left(-\frac{\partial m_N^*(\sigma)}{\partial\sigma}\right)(\bar{\psi}\psi) = \tilde{g}_{N\sigma}(\sigma)\rho_s,$$

$$(\Box + m_\omega^2)\omega = g_{N\omega}(\bar{\psi}\gamma^0\psi) = g_{N\omega}(\psi^\dagger\psi) = g_{N\omega}\rho,$$

◉ The effective nucleon mass appears as

$$-\frac{\partial m_N^*(\sigma)}{\partial\sigma} = \tilde{g}_{N\sigma}(\sigma) = g_{N\sigma}C_N(\sigma),$$

Scalar polarizability—characterizes
the nucleon response yo the external scalar field
If $C_N(\sigma) = 1$—-Pointlike nucleon (no internal)

Quark-Meson Coupling (QMC) Model

- Vector and scalar meson fields are defined as

$$\omega = \frac{g_{N\omega}\rho}{m_\omega^2}, \qquad \sigma = \frac{g_{N\sigma}\rho_s}{m_\sigma^2} C_N(\sigma),$$

- Nuclear baryon density and scalar density are given

$$\rho = \frac{4}{(2\pi)^3}\int d^3\boldsymbol{k}\,\Theta(k_F - k) = \frac{2k_F^3}{3\pi^2} \qquad \rho_s = \frac{4}{(2\pi)^3}\int d^3\boldsymbol{k}\,\Theta(k_F - k)\frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + k^2}}$$

- Total energy per nucleon

$$\frac{E_{\rm tot}}{A} = \frac{1}{\rho}\left[\frac{4}{(2\pi)^3}\int d^3\boldsymbol{k}\,\Theta(k_F - k)\sqrt{m_N^{*2}(\sigma) + k^2} + \frac{1}{2}g_{N\sigma}C_N(\sigma)\sigma\rho_s + \frac{1}{2}g_{N\omega}\omega\rho\right].$$

Quark-Meson Coupling (QMC) Model

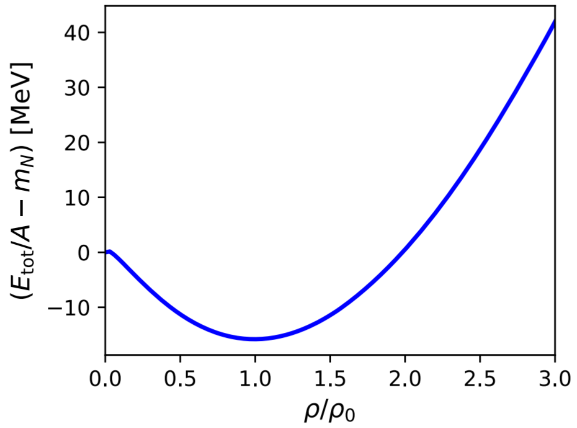
- QMC model parameter are determined by fitting the nuclear matter saturation properties at saturation density $\rho_0 = 0.15\ \mathrm{fm}^{-3}$ and $E_{tot}/A - m_N = -15.5$ MeV

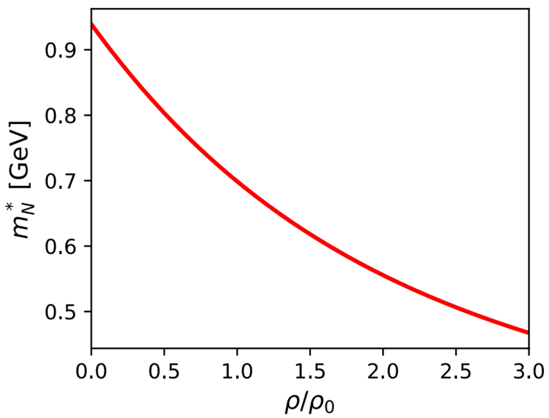
TABLE II. Bag parameters used in this study, fitted to match the nucleon mass and radius in free space.

m_q [MeV]	$B^{1/4}$ [MeV]	Z_N	x_q	$S_N(\sigma = 0)$
220	148	4.327	2.368	0.609

TABLE III. Coupling constants and the incompressibility K obtained in the QMC model. (m_N^* value is at $\rho_0 = 0.15\ \mathrm{fm}^{-3}$).

m_q [MeV]	$g_{N\sigma}^2/4\pi$	$g_{N\omega}^2/4\pi$	m_N^* [MeV]	K [MeV]
220	6.40	7.57	699	321





EMFFs

- In the matrix elements, the EMFFs of the pseudo scalar meson is given

$$\langle P'|j^{\mu}_{em}|P\rangle = (P'^{\mu} + P^{\mu})F_M(q^2),$$

- In the LFQM, the plus component is typically used to compute the EMFFs—kinematical factor $(P'^{+} + P^{+}) = 2P^{+}$
- The EMFFs of the pseudo scalar meson can be decomposed into their quark sector form factors

$$F_M(Q^2) = e_q F^q_M(Q^2, m_q, m_{\bar{q}}) + e_{\bar{q}} F^{\bar{q}}_M(Q^2, m_{\bar{q}}, m_q)$$

EMFFs

- The quark sector form factor can be expressed

$$F_M^q = \int \frac{\mathrm{d}x\mathrm{d}^2\boldsymbol{k}_\perp}{16\pi^3} \frac{\Phi(x,\boldsymbol{k}_\perp)\Phi'(x,\boldsymbol{k}'_\perp)}{2P^+} \times \sum_{\lambda,\lambda',\bar{\lambda}} \mathcal{R}^{00\dagger}_{\lambda'\bar{\lambda}}(x,\boldsymbol{k}'_\perp) \frac{\bar{u}_{\lambda'}(p'_1)}{\sqrt{x}} \gamma^\mu \frac{u_\lambda(p_1)}{\sqrt{x}} \mathcal{R}^{00}_{\lambda\bar{\lambda}}(x,\boldsymbol{k}_\perp)$$

$$F_M^i(Q^2) = \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{2(2\pi)^3} \Phi(x,\boldsymbol{k}_\perp)\Phi'(x,\boldsymbol{k}'_\perp) \times \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{k}'_\perp + \mathcal{A}^2}{\sqrt{\boldsymbol{k}_\perp^2 + \mathcal{A}^2}\sqrt{\boldsymbol{k}'^2_\perp + \mathcal{A}^2}}.$$

- The normalization of EMFFs

$$F_M(0) = e_q F_M^q(0) + e_{\bar{q}} F_M^{\bar{q}}(0),$$

Charge Radii

- Mean square charge radius of the meson

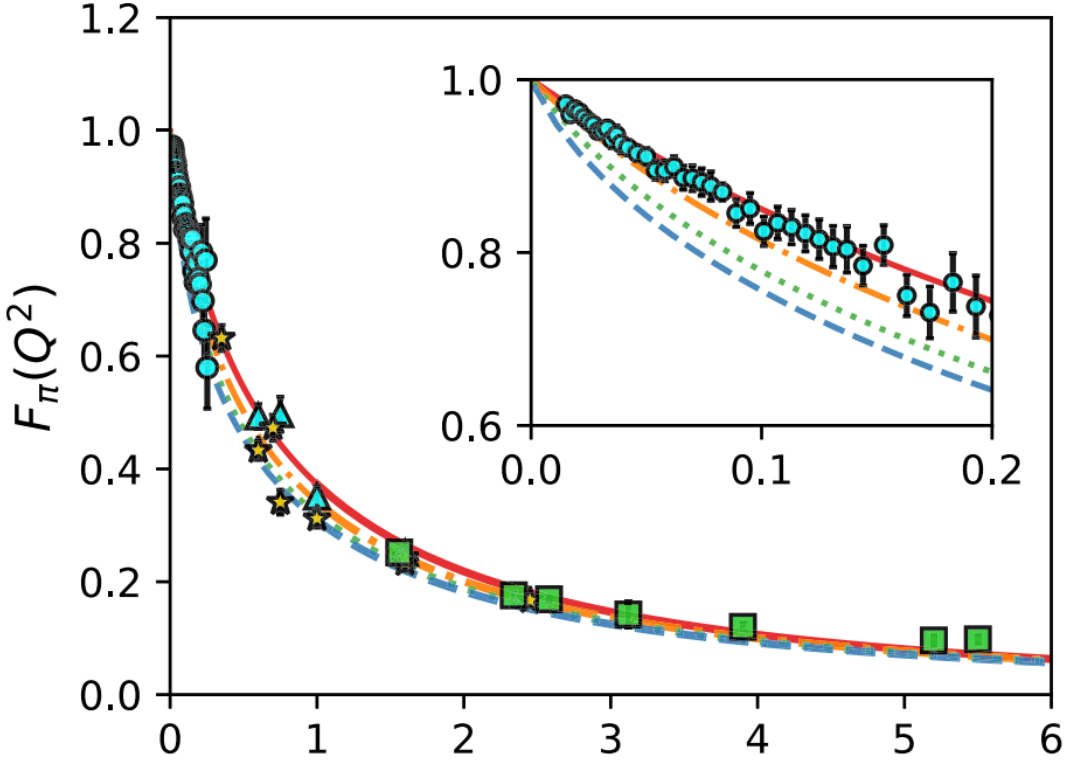
$$\langle r_M^2 \rangle = -6 \frac{\partial F_M(Q^2)}{\partial Q^2}\bigg|_{Q^2 \to 0},$$

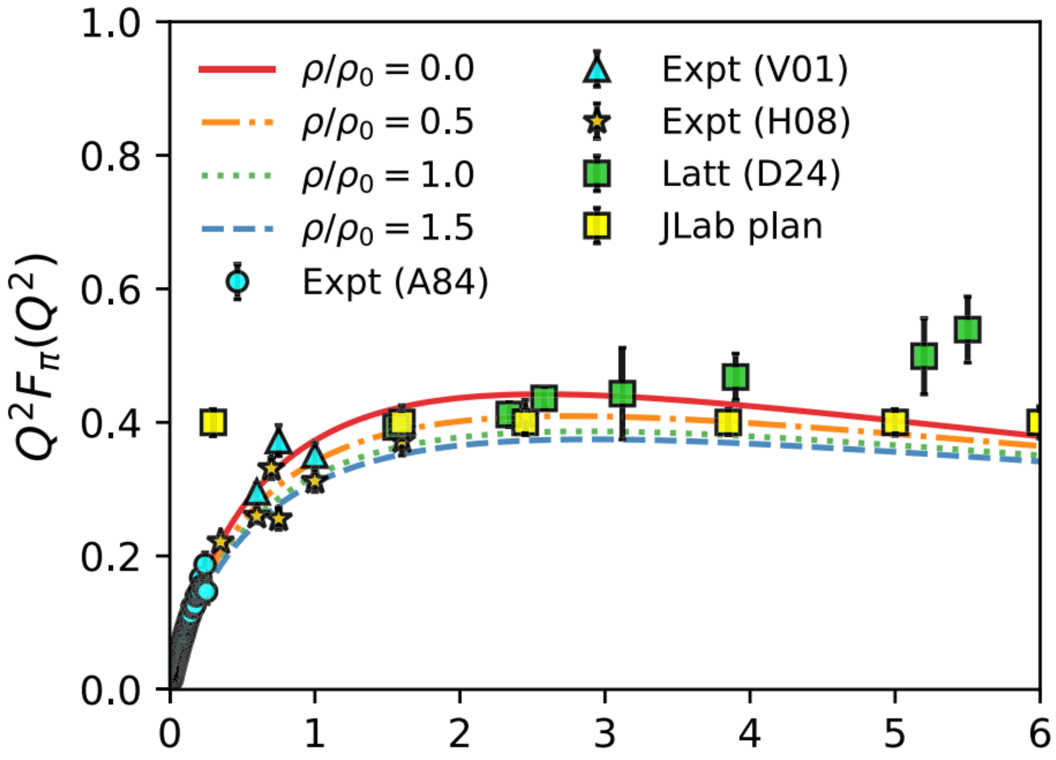
- Charge radius in terms of the quark flavors

$$\langle r_M^2 \rangle = e_q \langle r_{M,q}^2 \rangle + e_{\bar{q}} \langle r_{M,\bar{q}}^2 \rangle.$$

Result and Predictions

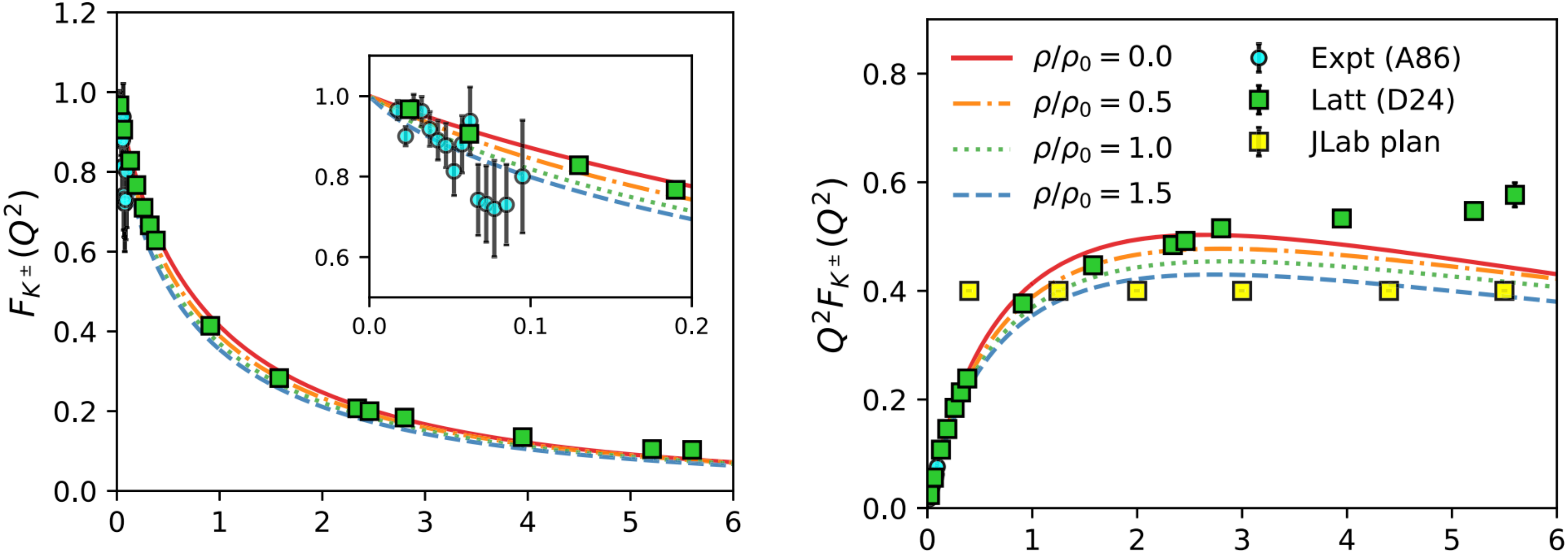
◉ Results for the pion in-medium EMFFs



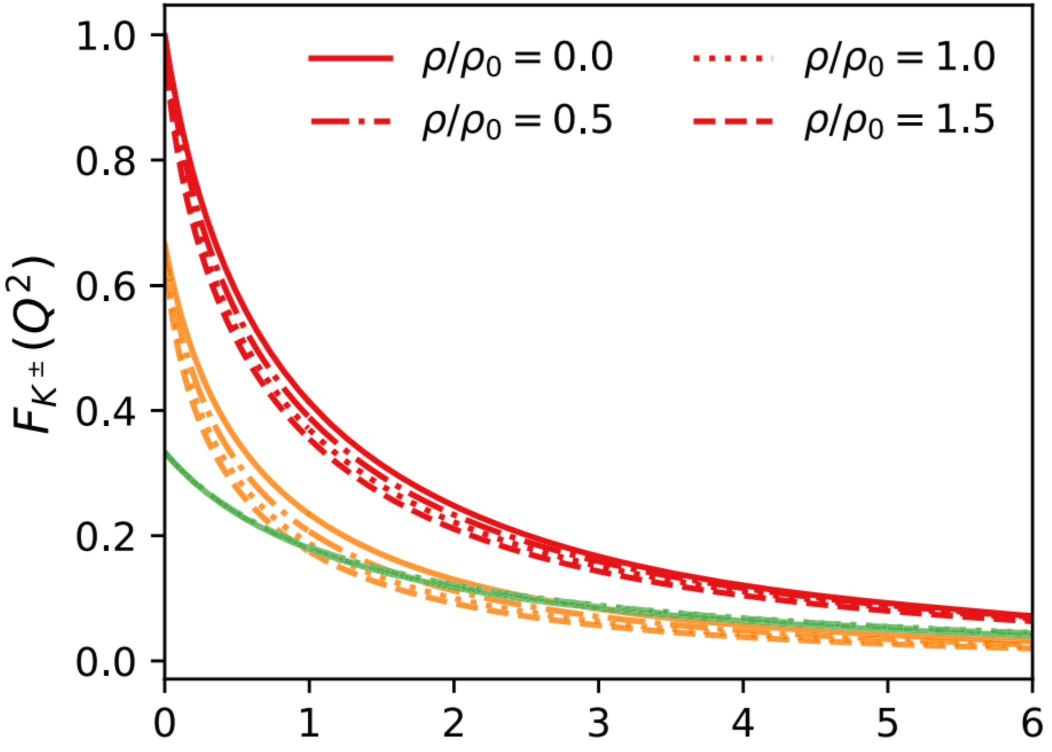


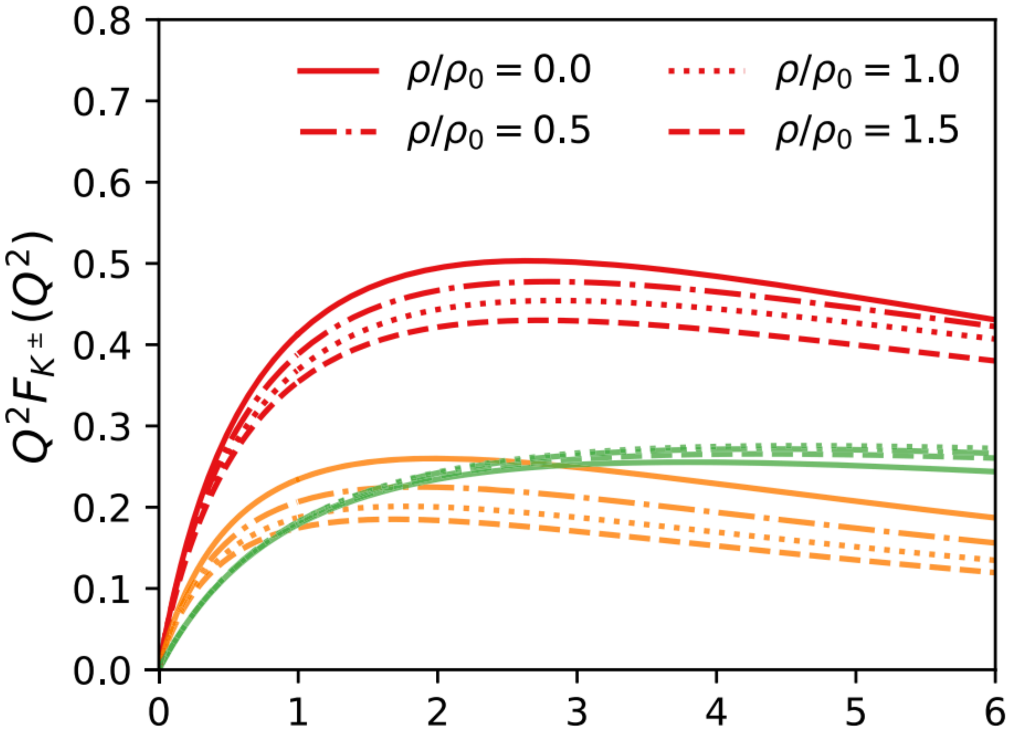
Result and Predictions

◉ Results for the kaon in-medium EMFFs

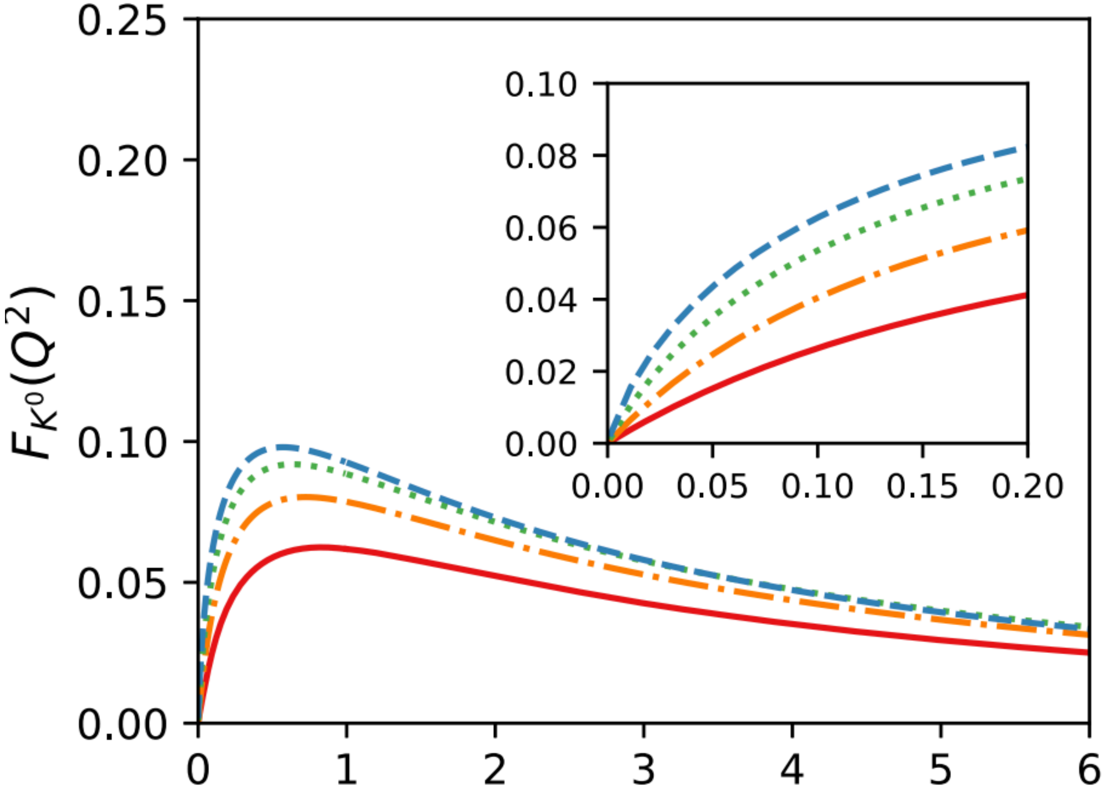


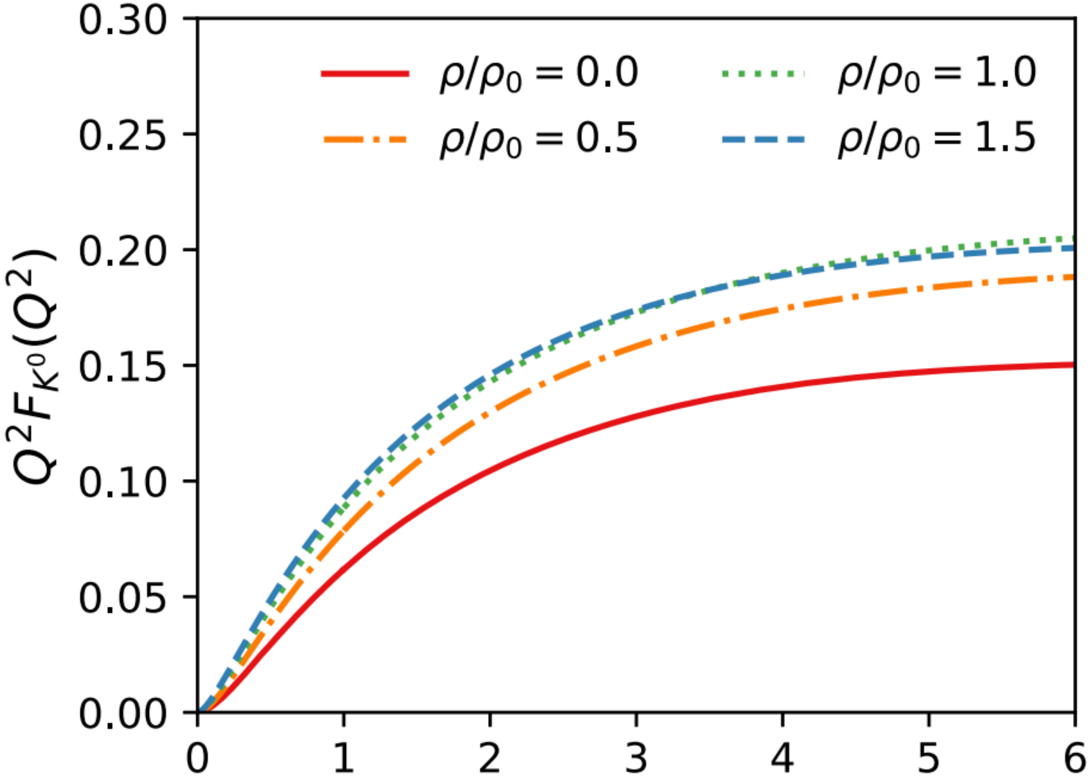
- Results for the kaon in-medium EMFFs with quark sector — up quark (orange) and strange quark (green)



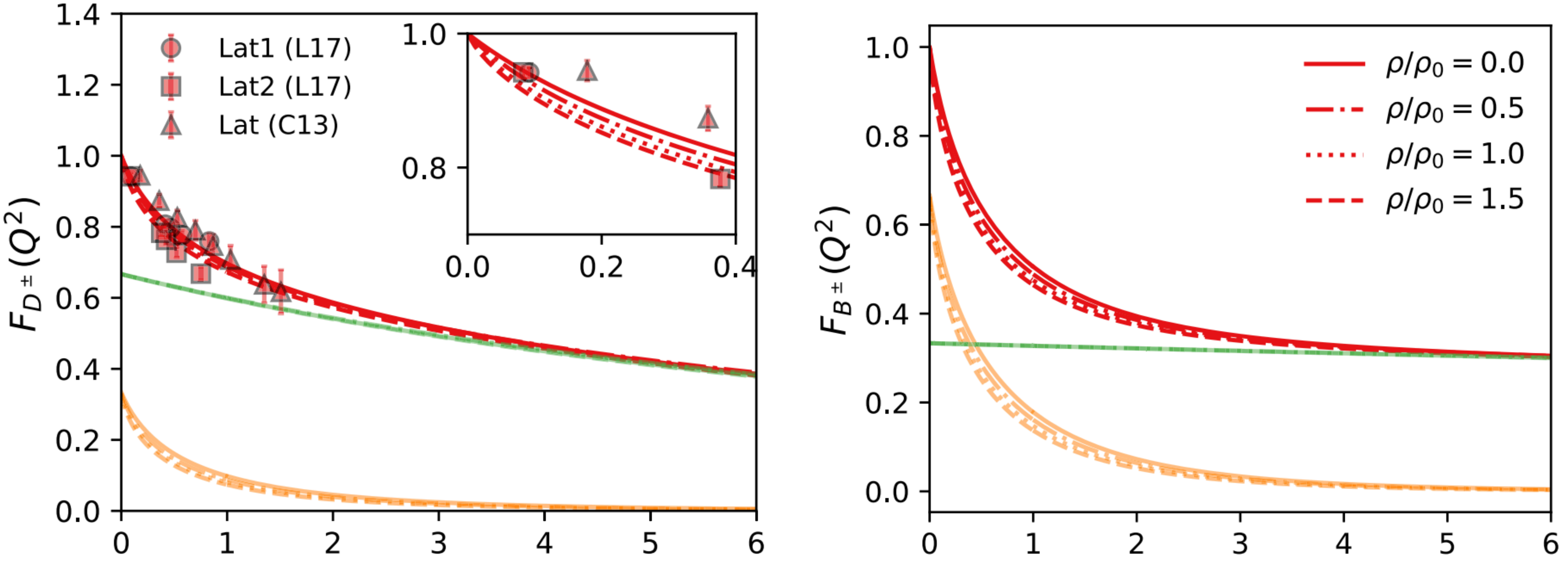


◉ Results for the neutral kaon in-medium EMFFs





◉ Results for the neutral kaon in-medium EMFFs



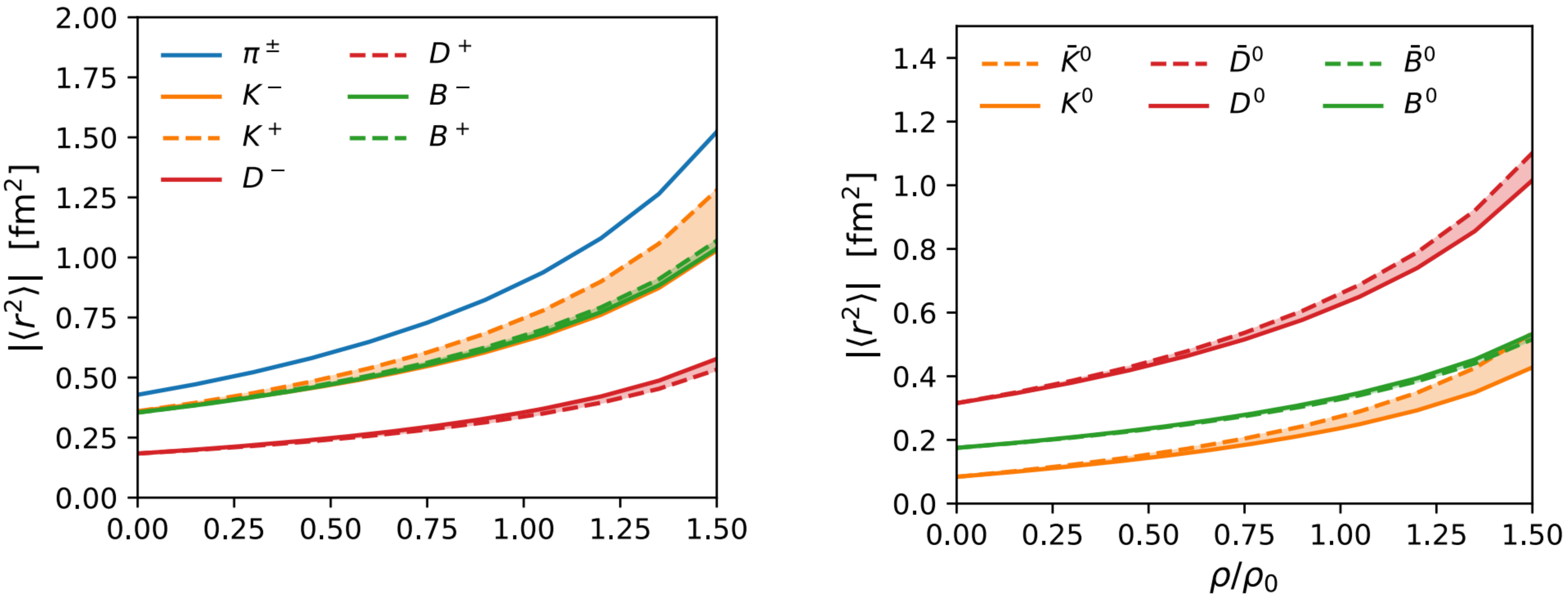
◉ Charge radius

TABLE IV. Absolute value of charge radius $|\langle r^2\rangle|$ of charged and neutral mesons in a nuclear medium in a unit of fm^2. The numerical values represent the averaged radius, modified only by the scalar potential.

ρ/ρ_0	$\pi^\pm$	$K^\pm$	$D^\pm$	$B^\pm$
0.00	0.428	0.360	0.184	0.354
0.25	0.505	0.415	0.211	0.407
0.50	0.602	0.485	0.245	0.474
0.75	0.728	0.576	0.288	0.557
1.00	0.897	0.697	0.346	0.666
1.25	1.136	0.871	0.427	0.817
1.50	1.523	1.154	0.555	1.053
Experiment [78,81]	0.427(10)	0.34(5)	...	...
Lattice [41,44]	0.458(15)	0.380(12)	0.152(26)	...

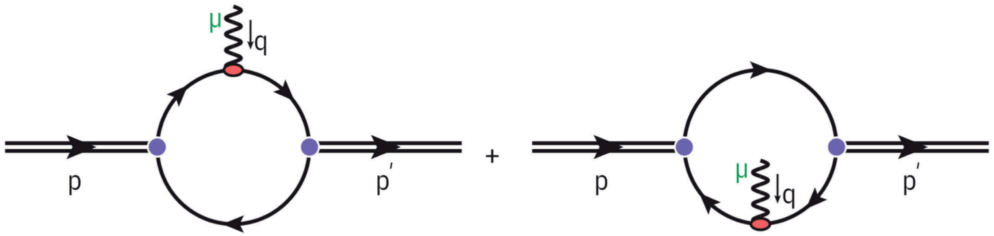
ρ/ρ_0	K^0	D^0	B^0
0.00	0.084	0.315	0.175
0.25	0.113	0.370	0.202
0.50	0.149	0.439	0.235
0.75	0.194	0.526	0.277
1.00	0.255	0.641	0.331
1.25	0.341	0.802	0.406
1.50	0.481	1.058	0.525
Lattice [44]	0.055(10)	...	...

◉ Charge radius for pseudocalar mesons



Result and Predictions

◉ Vector Mesons



$$\mathcal{J}_{\rho}^{\mu,\alpha\beta}(p'^{*},p^{*}) = \left[g^{\alpha\beta}F_1^{*\rho}(Q^2) - \frac{q^{\alpha*}q^{\beta*}}{2m_{\rho}^{*2}}F_2^{*\rho}(Q^2)\right] \times (p'^{*}+p^{*})^{\mu} - [q^{\alpha*}g^{\mu\beta} - q^{\beta*}g^{\mu\alpha}]F_3^{*\rho}(Q^2),$$

$$G_C^*(Q^2) = F_1^{*\rho}(Q^2) + \frac{2}{3}\eta G_Q^*(Q^2),$$

$$G_Q^*(Q^2) = F_1^{*\rho}(Q^2) + (1+\eta)F_2^{*\rho}(Q^2) - F_3^{*\rho}(Q^2),$$

$$G_M^*(Q^2) = F_3^{*\rho}(Q^2),$$

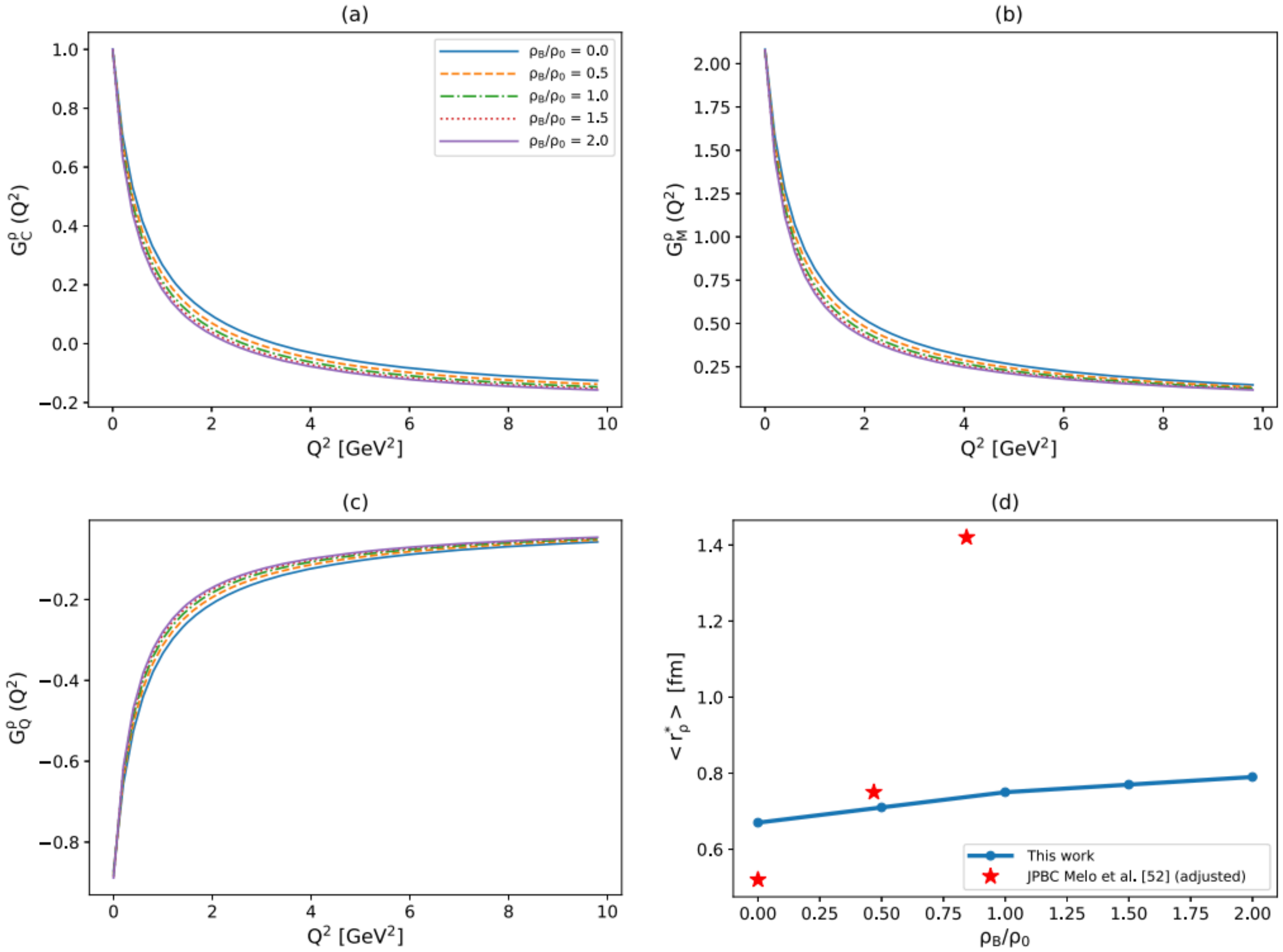
$$\mu_{\rho}^* = G_M^*(Q^2=0)\,\frac{M_N^*}{m_{\rho}^*},$$

$$Q_{\rho}^* = \frac{G_Q^*(Q^2=0)}{m_{\rho}^{*2}},$$

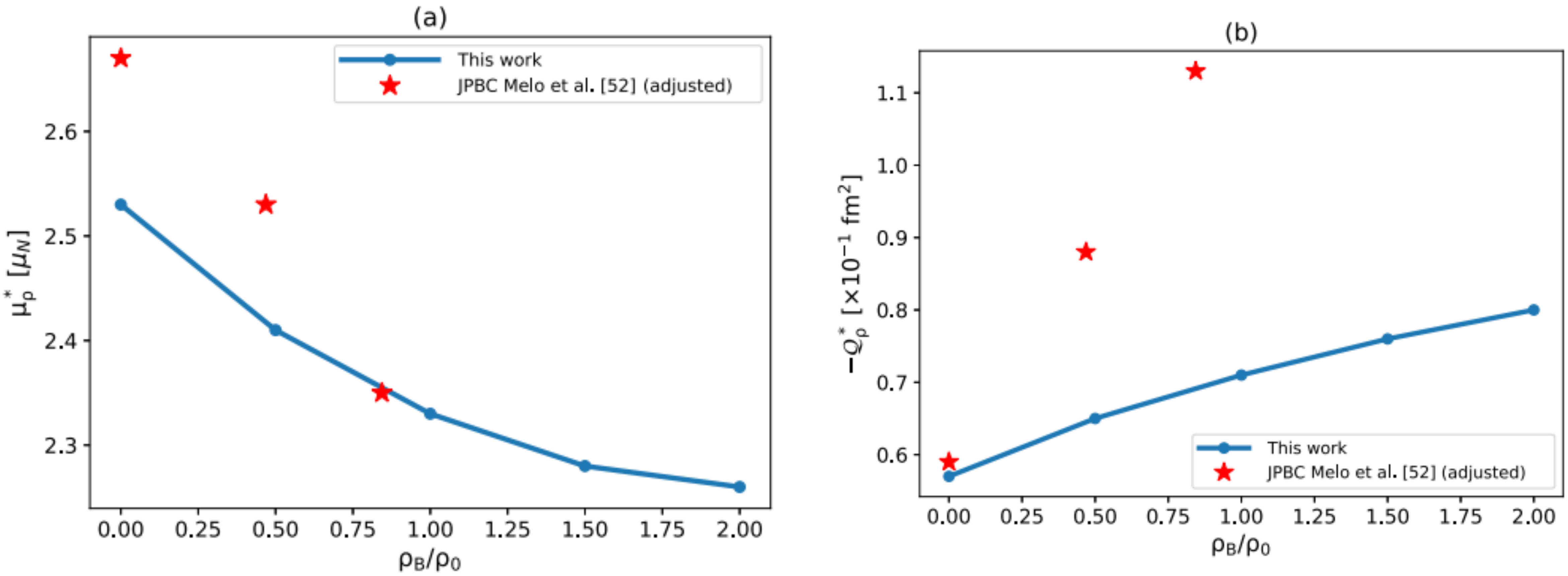
$$\langle r_C^{*2}\rangle = -6\left.\frac{\partial G_C^*(Q^2)}{\partial Q^2}\right|_{Q^2=0},$$

Result and Predictions

◉ Vector Mesons

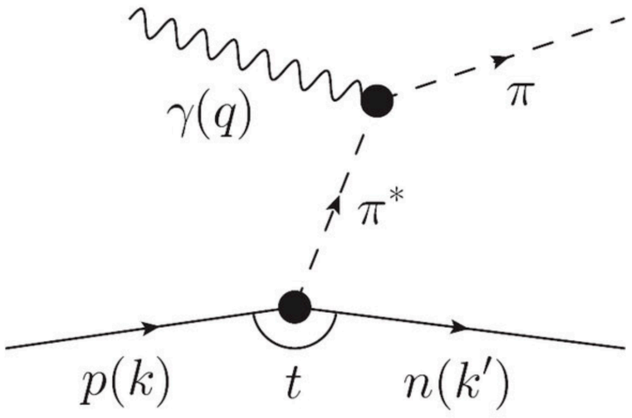


◉ Vector Mesons EMFFs



Summary

- We have present our results on pseudoscalar and vector mesons in the LFQM and QMC models
- We found the modifications of EMMFs meson in nuclear medium, in addition to properties of meson in nuclear medium
- The big question arise is how to extract the in-medium EMFFs in nuclear medium in experiment—Sullivan process



Acknowledgement

Thank you for your attention

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