Non-Perturbative QCD Insights: Meson Form Factors and Charge Radii

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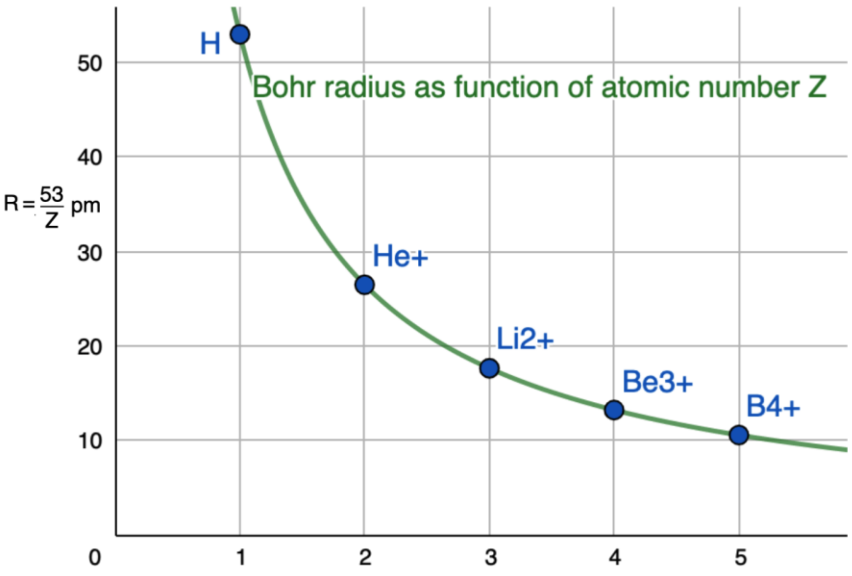
Center for Frontiers in Nuclear Science

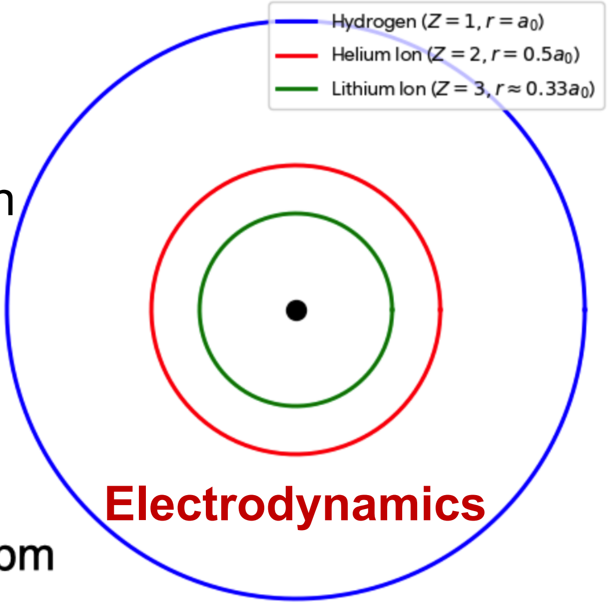
Bohr radii of Hydrogen-like atoms



The Bohr radius depends depend only on constants, whose better determination improves its accuracy.

$$a_0 = \frac{\hbar}{\alpha m_e c}$$

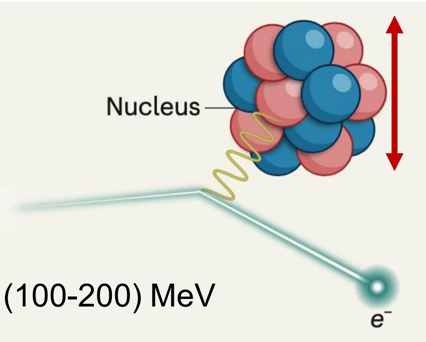
$$a_0 = 52.9177210903(80)\,\text{pm}$$



The **Bohr radius** feeds into many observables such as **precision spectroscopy**, **Lamb shift**, **hyperfine structure**, **atomic polarizability**, providing insights into **quantum electrodynamics**.

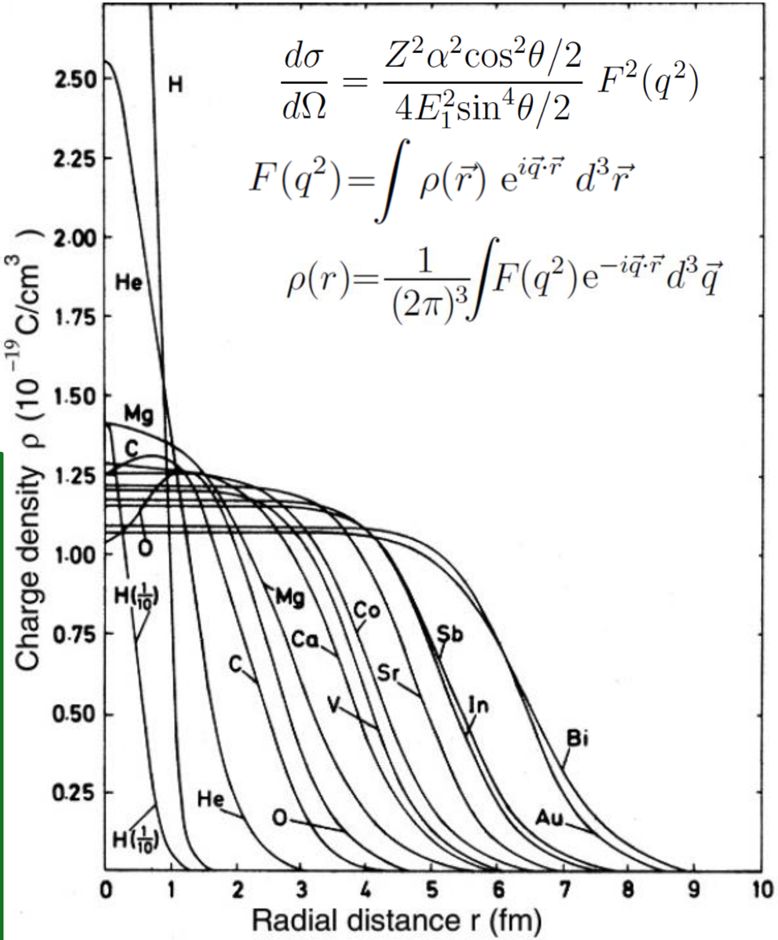
These observables have implications for **nuclear radius extraction** (proton, deuteron, light and heavy nuclei), **proton radius puzzle** and **muonic atom spectroscopy**.

Nuclear charge distribution and radii



QCD footprints

Short range interaction, **large** infrared **coupling**, quark-gluon **dressing**, dynamical chiral **symmetry breaking**, quark-gluon interaction **vertex** and perhaps **confinement**.



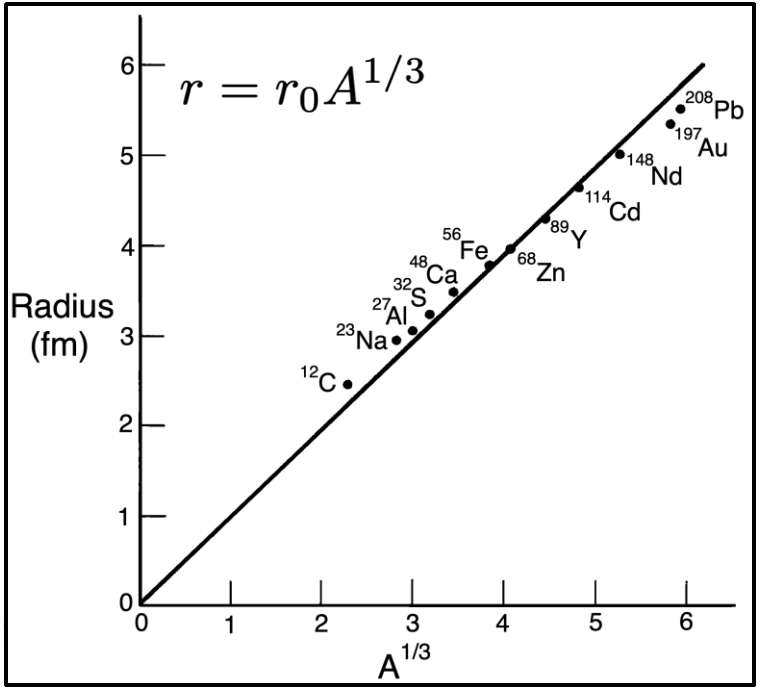
$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2\cos^2\theta/2}{4E_1^2\sin^4\theta/2}\, F^2(q^2)$$

$$F(q^2) = \int \rho(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d^3\vec{r}$$

$$\rho(r) = \frac{1}{(2\pi)^3}\int F(q^2)\, e^{-i\vec{q}\cdot\vec{r}}\, d^3\vec{q}$$

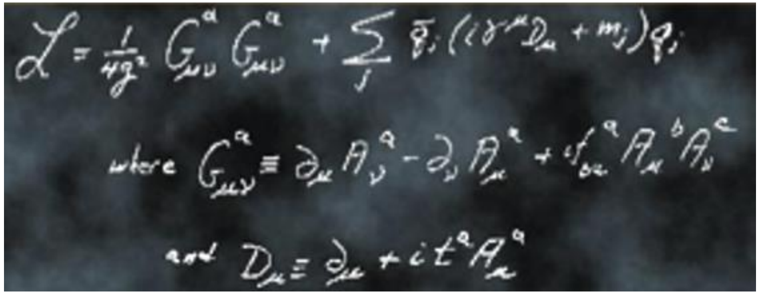
Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
Hofstadter, Rev. Mod. Phys. 28, 214 (1956)

$$\langle r^2 \rangle = -6 \left.\frac{\partial F(q^2)}{\partial q^2}\right|_{q^2=0}$$



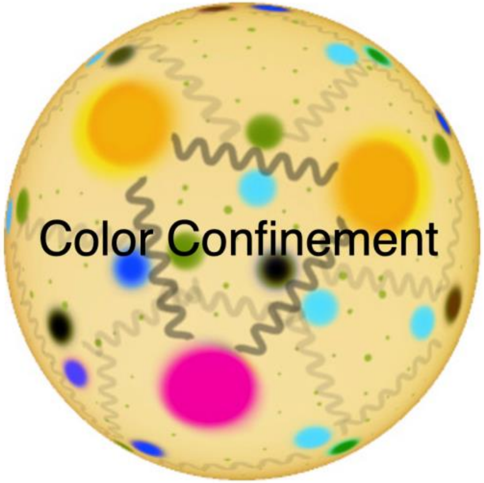
QCD: emergent phenomena and challenges

QCD is characterized by two **emergent** phenomena: **confinement** and **emergent hadron mass**.



- Quarks and gluons do not reach detectors.
- Formation of color-singlet bound states: “**Hadrons**” mesons, baryons, tetraquarks, molecules

- Emergence of hadron masses from **QCD dynamics**

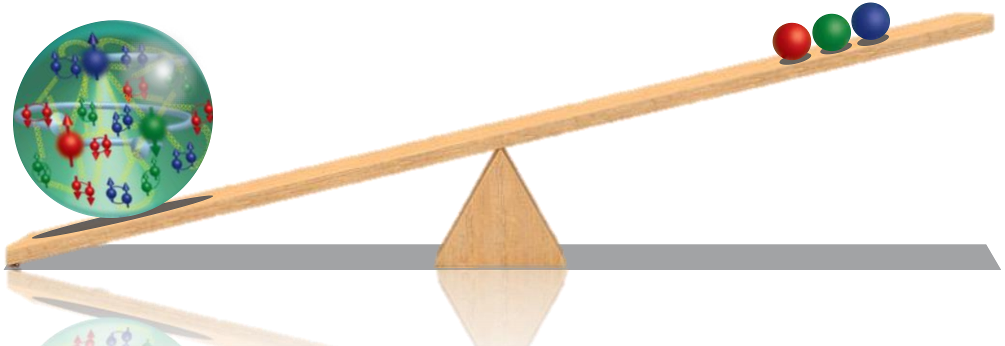


$\sim 2\%$ of M_p

$m_{u,d} \approx 5 - 10$ MeV

$\sim 98\%$ of M_p strong QCD

$M_{p,n} \approx 940$ MeV

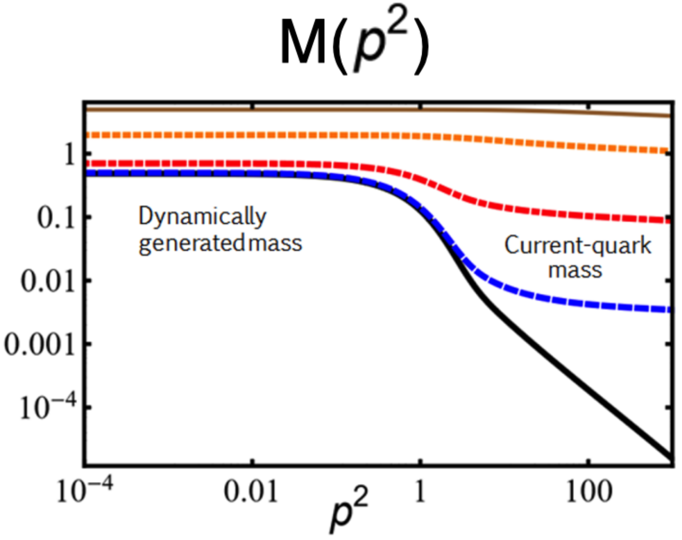


QCD: emergent phenomena and challenges

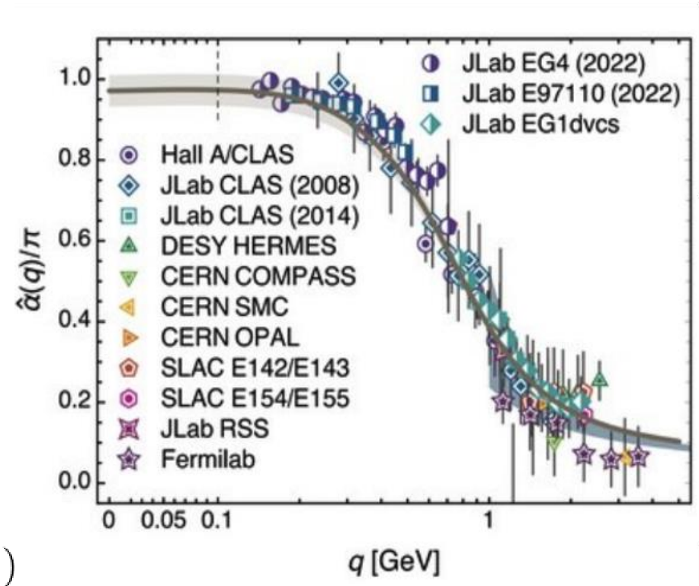
Origins of **confinement** and **dynamical mass generation** can perhaps be traced back to the Green functions of **quarks** and **gluons**.

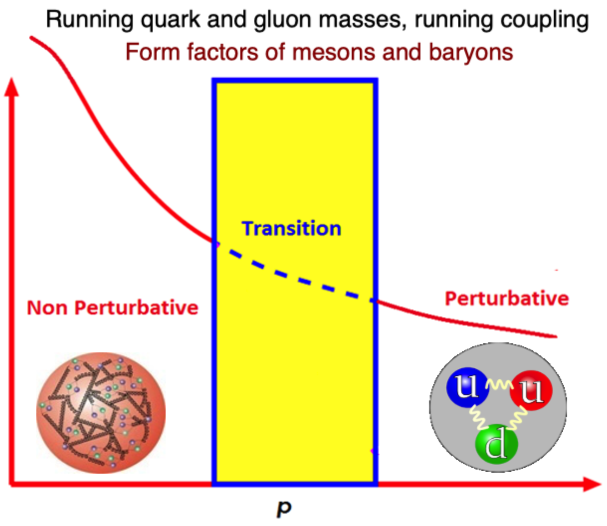
These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to the infrared enhancement of the **strong running coupling**.

The effects of of the pattern of **dynamical mass generation** are traceable in the **Q^2 evolution** of the π and **K** **electromagnetic form factors** explored and planned in the **JLab** and the **EIC**.



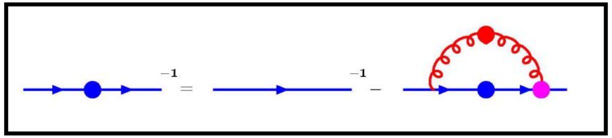
$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + \mathbf{M_f(p^2)})$$



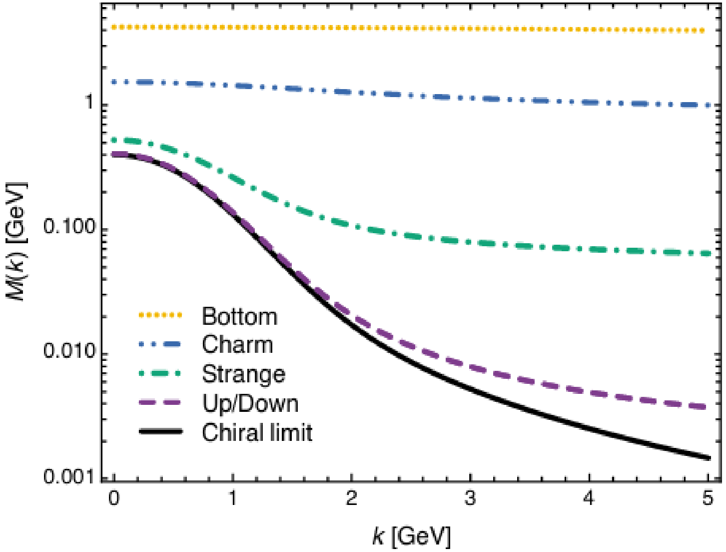


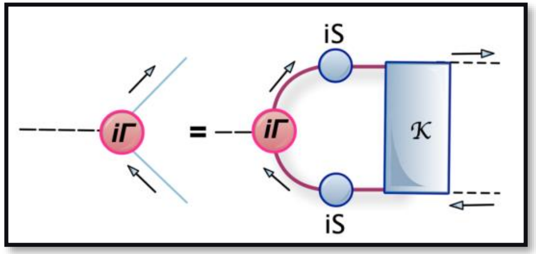
π and K: bound states and NGSW bosons

Pions are simultaneously **bound states** of quark-anti-quark and the **Nambu-Goldstone-Salam-Weinberg bosons** associated with **dynamical chiral symmetry breaking**.



$$\Gamma_\pi(k,P) = \gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P) \right]$$





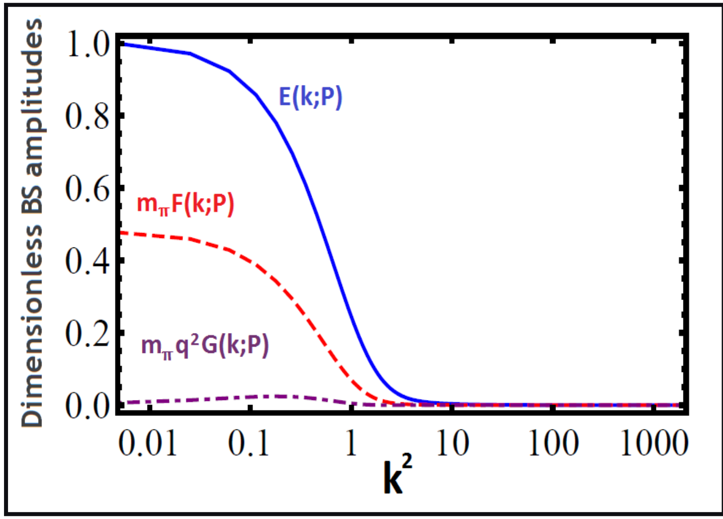
$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$f_\pi E_\pi(k;P=0) = B(p^2)$$

$$F_R(k;0) + 2\,f_\pi F_\pi(k;0) = A(k^2)$$

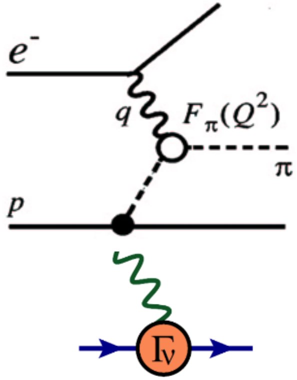
$$G_R(k;0) + 2\,f_\pi G_\pi(k;0) = 2A'(k^2)$$

$$H_R(k;0) + 2\,f_\pi H_\pi(k;0) = 0$$



π and K: probing quarks with photons

In studying the **electromagnetic form factors**, it is the **photon** which probes the **dressed quarks** inside the **bound states**, highlighting the importance of the **quark-photon vertex**.



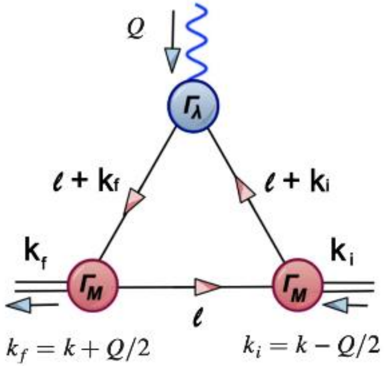
$$\Gamma^L_\mu(p,k,q) = \sum_{i=1}^{4} \lambda_i L^i_\mu(p,k)$$

$$L^1_\mu = \gamma_\mu$$
$$L^2_\mu(p,k) = (\not p + \not k)(p+k)_\mu$$
$$L^3_\mu(p,k) = -(p+k)_\mu$$
$$L^4_\mu(p,k) = -\sigma_{\mu\nu}(p+k)^\nu$$

$$\Gamma^T_\mu(p,k,q) = \sum_{i=1}^{8} \tau_i(p^2,k^2,q^2) T^i_\mu(p,k)$$

$$T^1_\mu = p_\mu (k\cdot q) - k_\mu (p \cdot q)\,,$$
$$T^2_\mu = [p_\mu (k\cdot q) - k_\mu (p\cdot q)](\not p + \not k)\,,$$
$$T^3_\mu = q^2 \gamma_\mu - q_\mu \not q\,,$$
$$T^4_\mu = q^2 [\gamma^\mu (\not k + \not p) - (k+p)^\mu] + 2(k-p)^\mu \sigma_{\nu\lambda} p^\nu k^\lambda\,,$$
$$T^5_\mu = -\sigma_{\mu\nu} q^\nu\,,$$
$$T^6_\mu = \gamma_\mu (p^2 - k^2) + (p+k)_\mu \not q\,,$$
$$T^7_\mu = \tfrac{1}{2}(p^2-k^2)[\gamma_\mu(\not p + \not k) - (p+k)_\mu] - (p+k)_\mu \sigma_{\nu\lambda} p^\nu k^\lambda\,,$$
$$T^8_\mu = \gamma_\mu \sigma_{\nu\lambda} p^\nu k^\lambda - p_\mu \not k + k_\mu \not p\,.$$

Gauge covariance (WTI,TTI, LKFT),
Singularities,
perturbation theory,
Multiplicative **renormalizability**

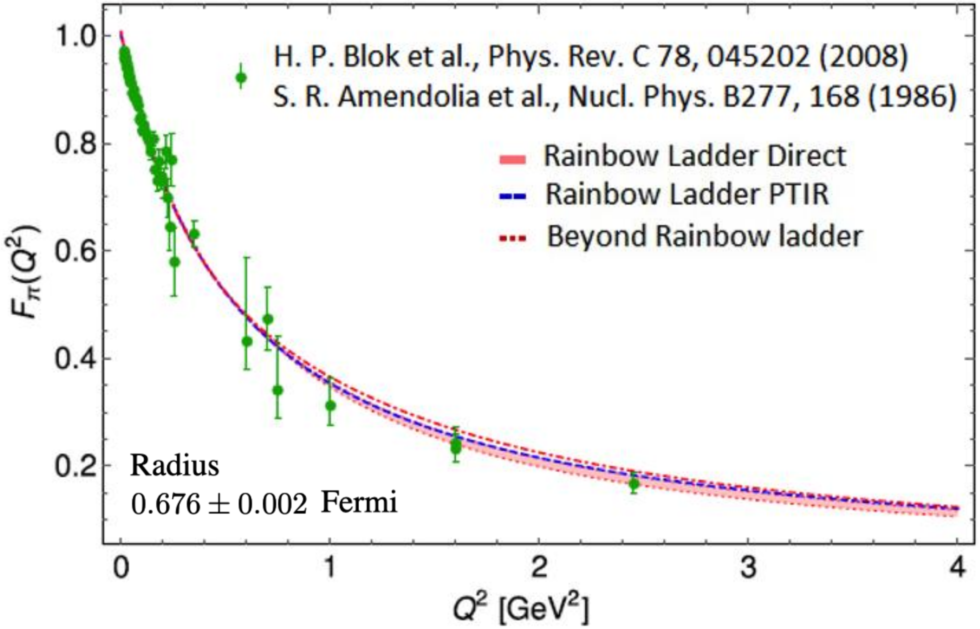


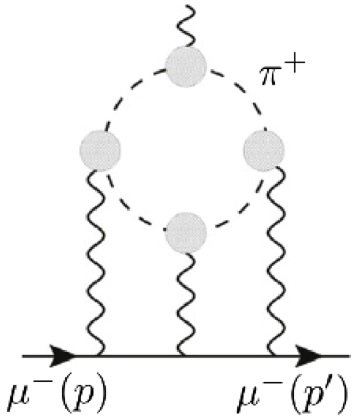
AB, **R. Bermudez**, L. Chang, C.D. Roberts
Phys. Rev. C85, 045205 (2012)

M. Atif, K. Raya, F. Akram, AB, B. Masud,
Phys. Rev. D103 (2021) 5, 054036.

A. Ashraf, J. Aslam, F. Akram, AB,
Phys. Rev. D111 (2025) 3, 034043.

π form factor and charge radius





HIbl scattering contribution to the **muon anomalous magnetic moment**

SDE

$$a_\mu^{\pi^\pm-\text{box}} = -(15.6 \pm 0.2) \times 10^{-11}$$

Dispersive

$$a_\mu^{\pi^\pm-\text{box}} = -(15.9 \pm 0.2) \times 10^{-11}$$

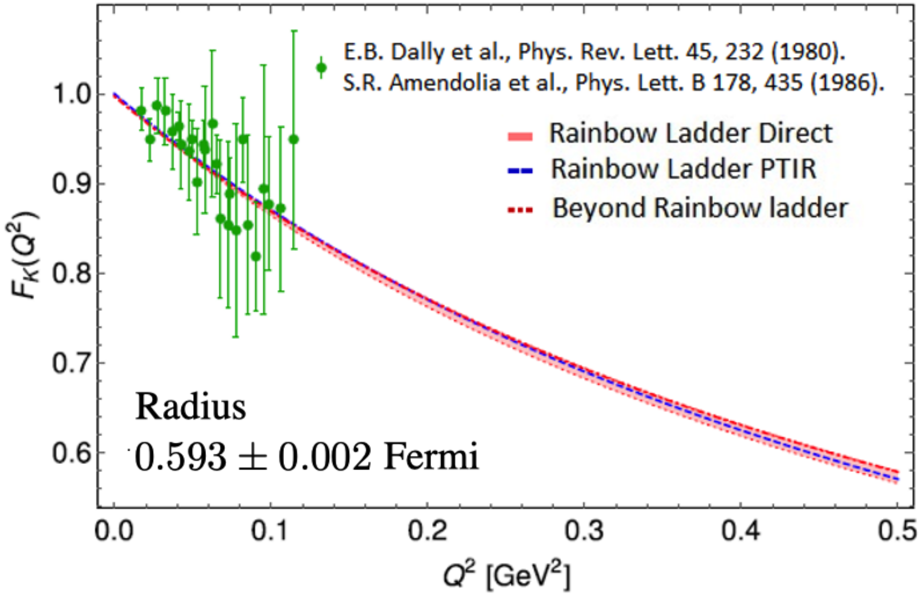
Dispersive: G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP, 04, 161, (2017).

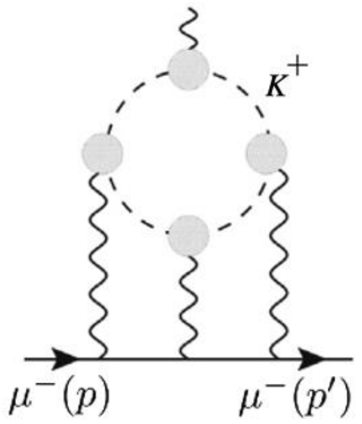
Pion charge radius - Fermis

Framework	Radius
Continuum QCD 2022	0.676 ± 0.002
Lattice QCD 2021	0.648 ± 0.141
PDG 2020	0.659 ± 0.004
Hybrid model 2000	0.66
Light front 2002	0.66

Continuum QCD:
A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D105 7, 074013 (2022).
Lattice QCD: X. Gao et. al., Phys. Rev. D104, 114515 (2021).
PDG 2020: P. A. Zyla et al. (PDG), PTEP 2020, 083C01 (2020).
Hybrid Model: R. J. Lombard, J. Mares, Phys. Lett. B472, 150 (2000).
Light Front: C.-W. Hwang, Eur. Phys. J. C23, 585 (2002).

K form factor and charge radius





HlbI scattering contribution to the muon anomalous magnetic moment

SDE

$$a_\mu^{K^\pm-\text{box}} = -(0.48 \pm 0.02) \times 10^{-11}$$

SDE previous

$$a_\mu^{K^\pm-\text{box}} = -(0.46 \pm 0.02) \times 10^{-11}$$

SDE previous: G. Eichmann, C. S. Fischer, R. Williams. Phys. Rev. D101 5, 054015 (2020).

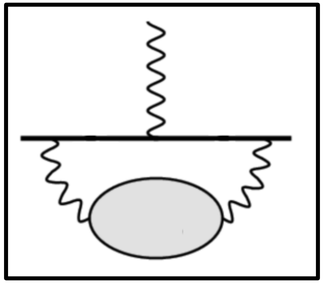
Kaon charge radius - Fermis

Framework	Radius
Continuum QCD 2022	0.593 ± 0.002
Lattice QCD 2018	0.566
PDG 2020	0.560 ± 0.031
Hybrid model 2000	0.65
Light front 2002	0.58

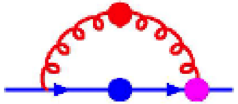
Continuum QCD:
A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D105 7, 074013 (2022).

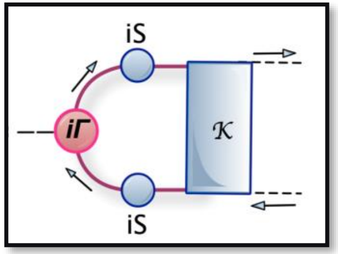
Lattice QCD: C. T. H. Davies, et. al., (HPQCD), PoS LATTICE2018, 298 (2018).
PDG 2020: P. A. Zyla et al. (PDG), PTEP 2020, 083C01 (2020).
Hybrid Model: R. J. Lombard, J. Mares, Phys. Lett. B472, 150 (2000).
Light Front: C.-W. Hwang, Eur. Phys. J. C23, 585 (2002).

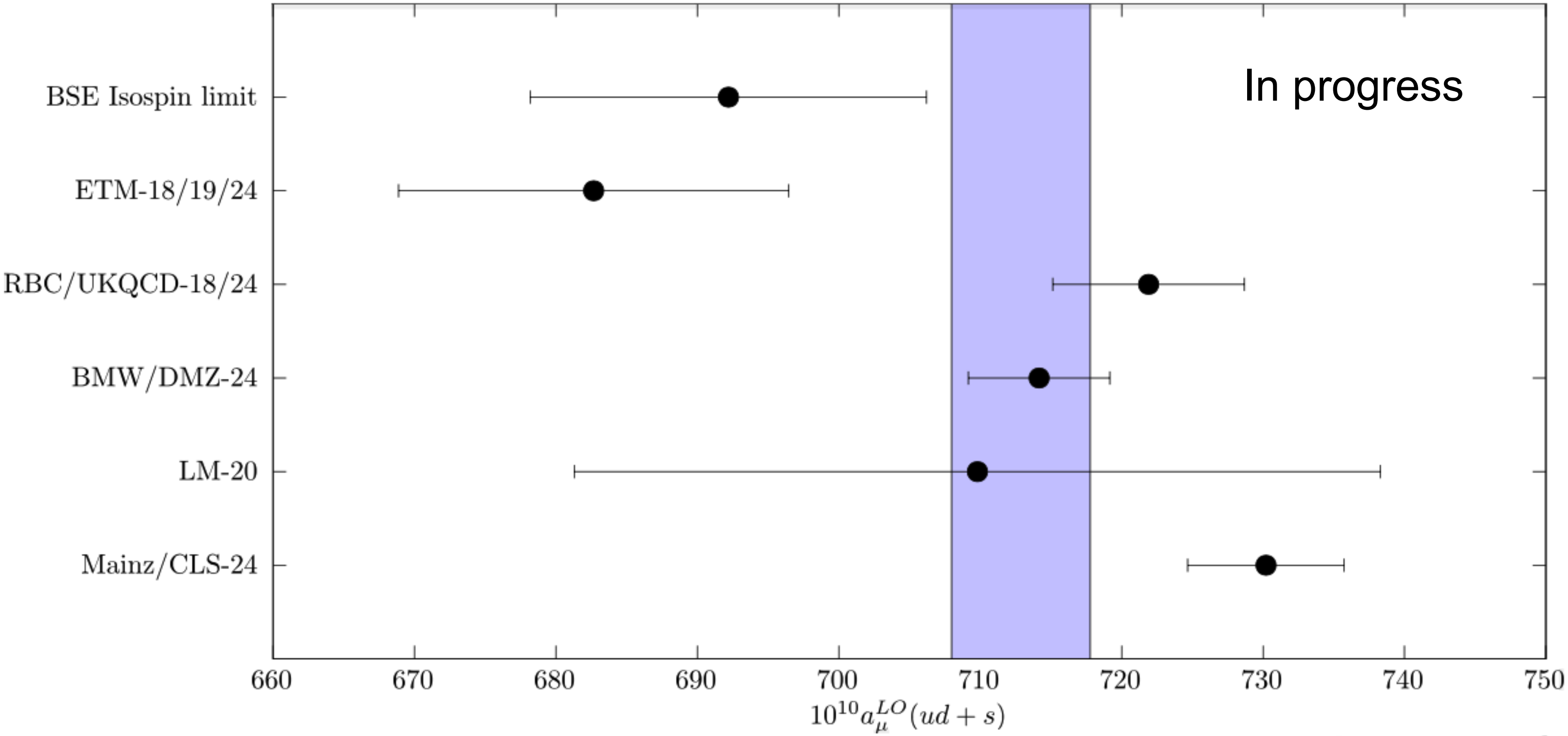
SDE - Hadronic Vacuum Polarization



Continuum QCD:
A. Miramontes,
AB, et. al.
in progress

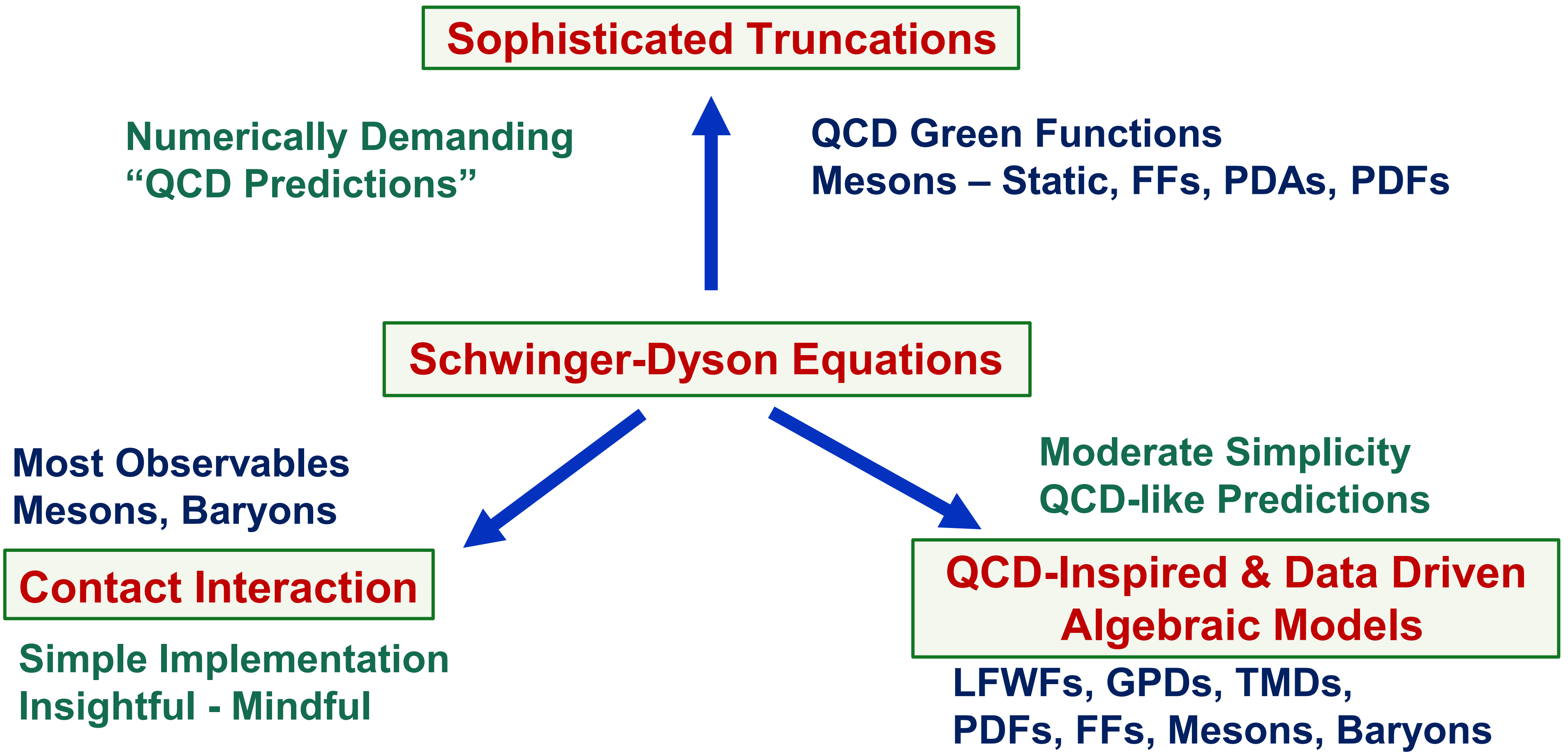






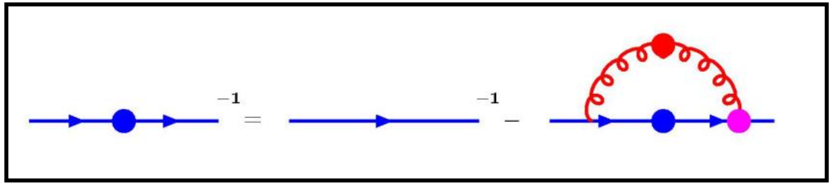
a_μ compared with different lattice calculations for $ud + s$ flavors. The results of this work are compared to those found by the EMT-18/19/24, RBC/UKQCD-18/24, BMW-20 - 24, LM-20 and Mainz/CLS-24 lattice collaborations. The purple band stands for the Muon g-2 Theory Initiative recommended value, resulting from the combination of the lattice computations shown.

Truncating Schwinger-Dyson equations



The contact interaction (CI)

The **SDE** for the **quark propagator** of **flavor f**:



It is known that a **mass scale** is generated for the **gluon** in the **infrared**.

$$g^2 D_{\mu\nu}(k) = 4\pi\hat{\alpha}_{\rm IR}\delta_{\mu\nu} \qquad m_g = 500 \text{ MeV}$$

$$\hat{\alpha}_{\rm IR} = \alpha_{\rm IR}/m_g^2 \qquad M_u = 0.367 \text{ GeV}$$

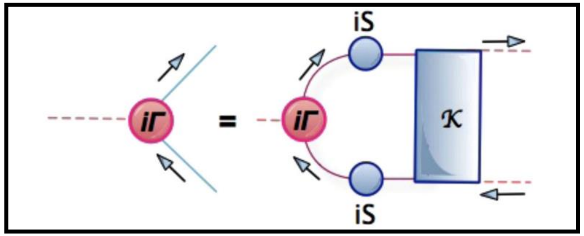
$$\Gamma_\nu(q,p) = \gamma_\nu$$

$$S(q,M_f) \equiv -i\gamma\cdot q\sigma_V(q,M_f) + \sigma_S(q,M_f)$$

$$\sigma_V(q,M_f) = \frac{1}{q^2+M_f^2}, \quad \sigma_S(q,M_f) = M_f\sigma_V(q,M_f)$$

L.X. Gutierrez, AB, et. al., Phys. Rev. C81 (2010) 065202

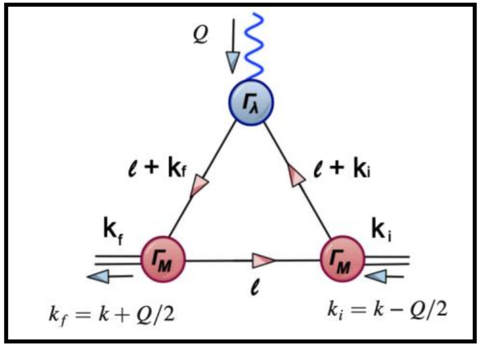
Bethe-Salpeter equation:

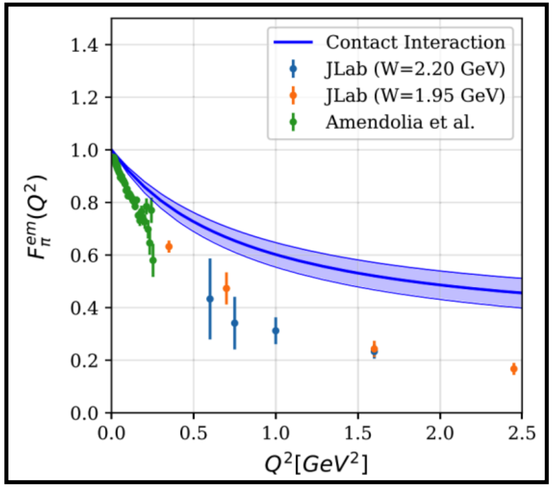


$$\Gamma_{PS}(P) = i\gamma_5 E_{PS}(P) + \frac{1}{2M_R}\gamma_5\gamma\cdot P\, F_{PS}(P)$$

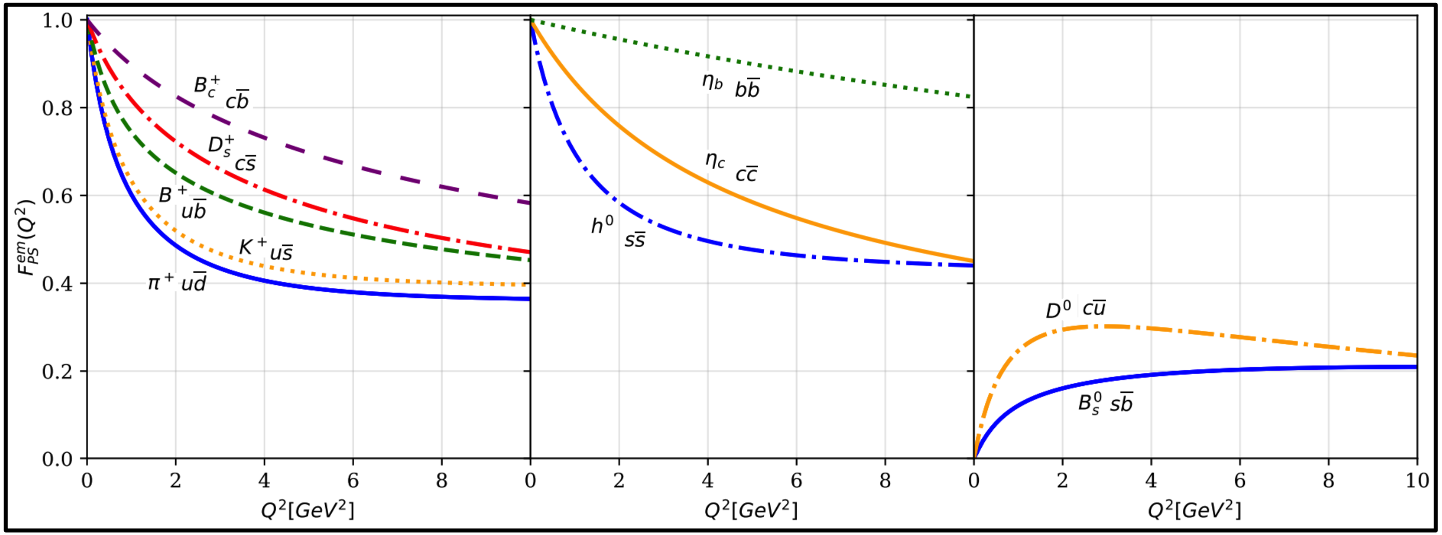
Bethe-Salpeter amplitude

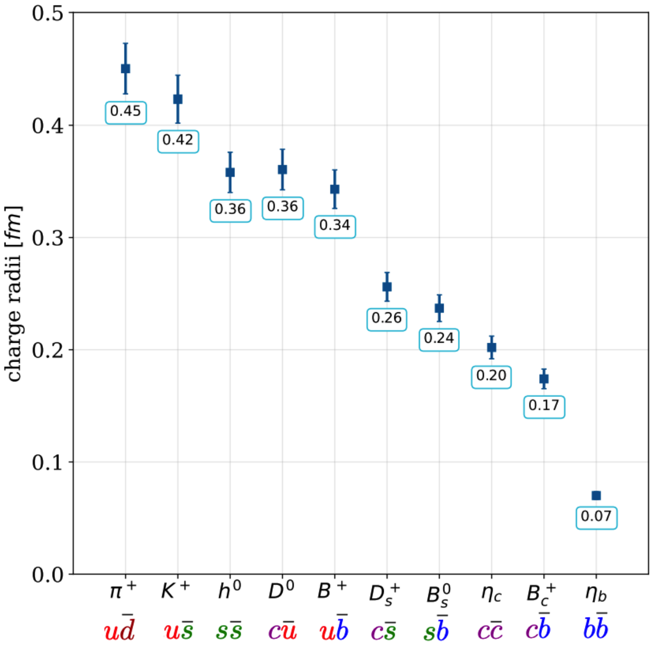
Pion form factor





Pseudoscalar mesons in CI





Electromagnetic form factors of **electrically charged** and **neutral pseudo-scalar mesons** in a **Contact Interaction** model.

Charge radii of **pseudo-scalar mesons** with a permissible variation of the defining **parameters**.

Hierarchy of the **charge radii** of **pseudo-scalar mesons**:

$$r_{u\bar{d}} > r_{u\bar{s}} > r_{c\bar{u}} > r_{u\bar{b}}\,,$$
$$r_{u\bar{s}} > r_{s\bar{s}} > r_{c\bar{s}} > r_{s\bar{b}}\,,$$
$$r_{c\bar{u}} > r_{c\bar{s}} > r_{c\bar{c}} > r_{c\bar{b}}\,,$$
$$r_{u\bar{u}} > r_{s\bar{s}} > r_{c\bar{c}} > r_{b\bar{b}}\,.$$

R.J. Hernández, L.X. Gutiérrez, AB, M.A. Bedolla, I.M. Higuera, Phys. Rev. D 107 (2023) 5, 054002

Truncating Schwinger-Dyson equations

Sophisticated Truncations

Numerically Demanding
"QCD Predictions"

QCD Green Functions
Mesons – Static, FFs, PDAs, PDFs

Schwinger-Dyson Equations

Most Observables
Mesons, Baryons

Contact Interaction

Simple Implementation
Insightful - Mindful

Moderate Simplicity
QCD-like Predictions

QCD-Inspired & Data Driven Algebraic Models

LFWFs, GPDs, TMDs,
PDFs, FFs, Mesons, Baryons

The Algebraic Model (AM)

- It retains the **constant term** from original models, setting it to $\mathbf{M_q}$.
- There is a **term linear** in **w** with the coefficient $\mathbf{(M_h^2 - M_q^2)}$. For same **flavored quarks**, it ceases to contribute by construction.
- There is a **quadratic term $\mathbf{w^2}$** with coefficient $\mathbf{m_M^2}$. The condition

$$|M_{\bar{h}} - M_q| \leq m_{\mathrm{M}} \leq M_{\bar{h}} + M_q$$

- It guarantees the **positivity** of

$$\Lambda^2(w)$$

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}]\Delta(k^2, M^2_{q(\bar{h})})$$

$$\Delta(s,t) = (s+t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w)[\hat{\Delta}(k_w^2, \Lambda_w^2)]^{\nu}$$

$$\hat{\Delta}(s,t) = t\Delta(s,t), \quad k_w = k + (w/2)P$$

$M_{q(\bar{h})}$ is constituent quark mass for a given flavor

n_{M} is a normalization constant

$\rho_{\mathrm{M}}(w)$ is a spectral density

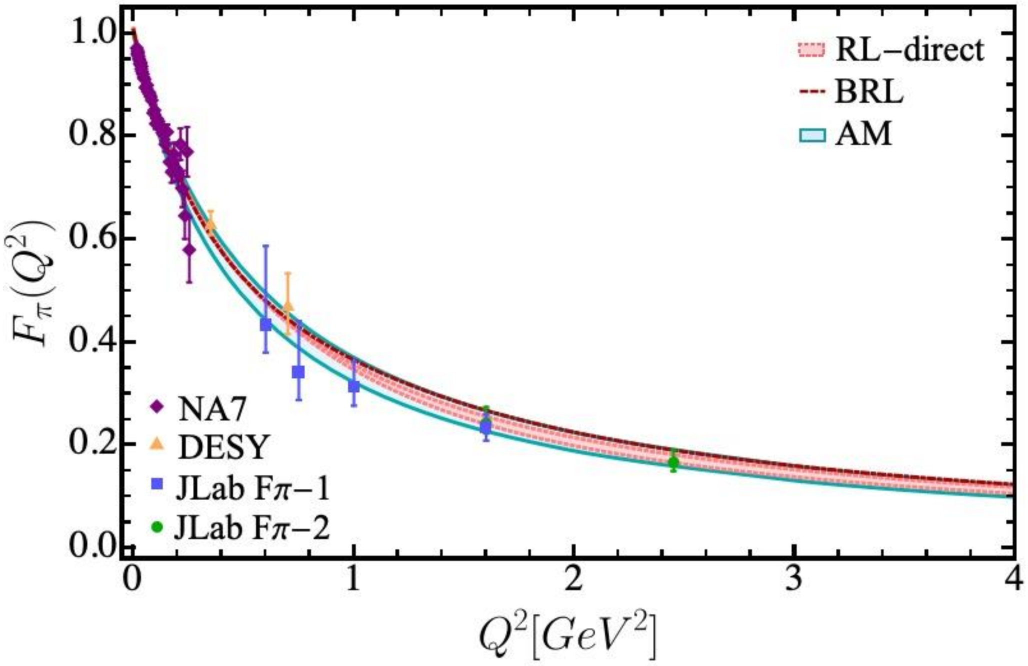
$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\mathrm{M}}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$$

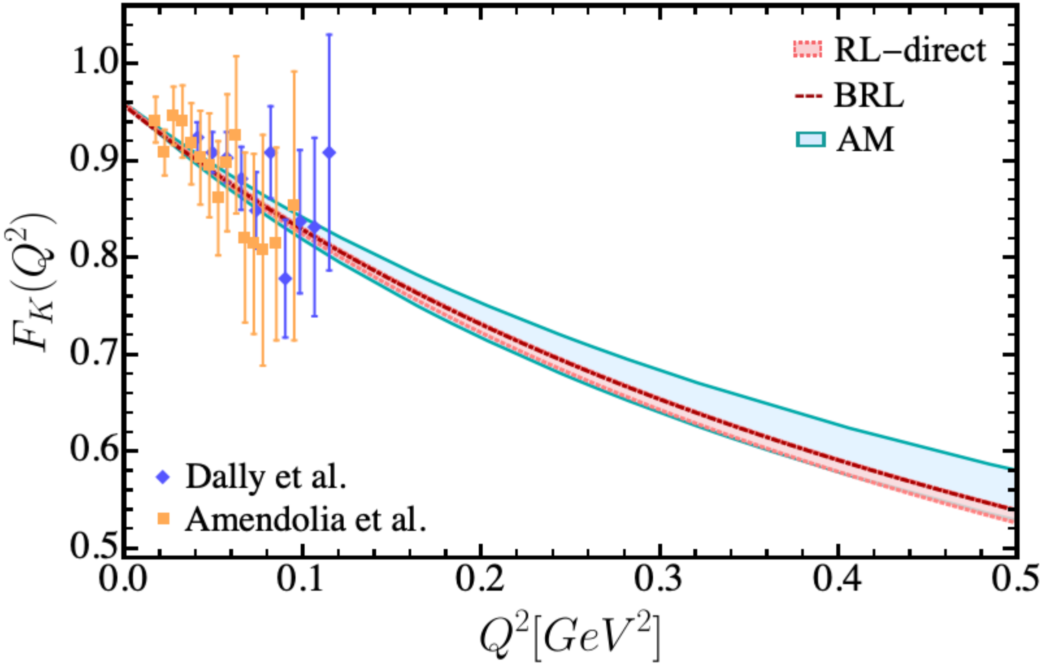
Form Factors and charge radii in algebraic model

Electromagnetic form factors and the **charge radii** are obtained via **generalized parton distributions** or the direct evaluation of the triangle diagram.

A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

I.M. Higuera, R.J. Hernández, K.Raya, A. Bashir, Phys. Rev. D 110 (2024) 3, 034013

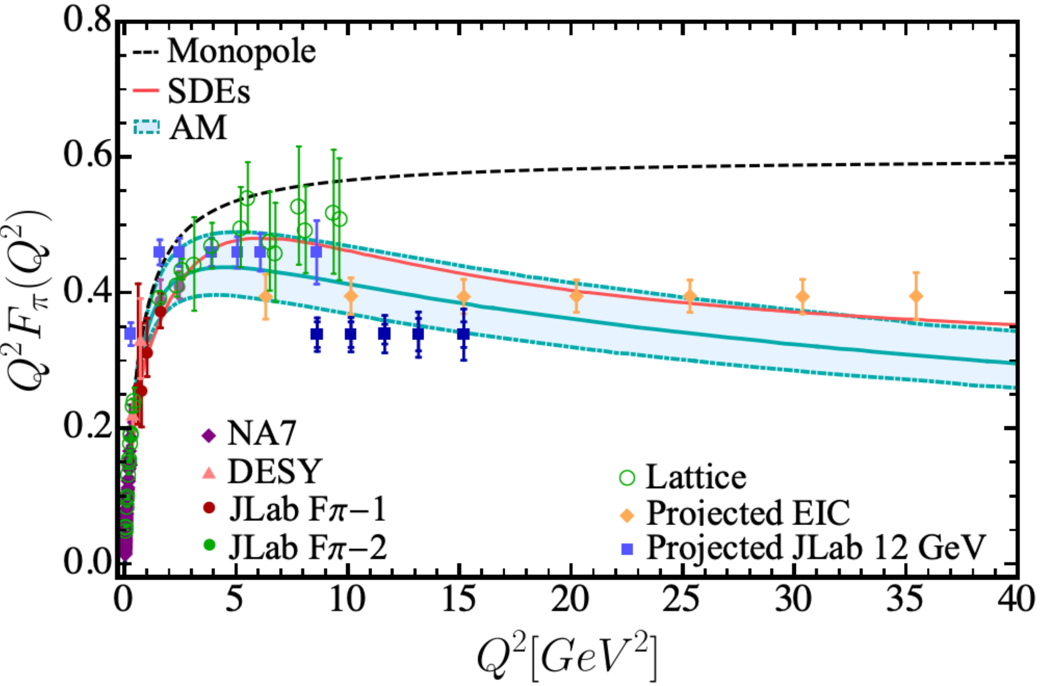


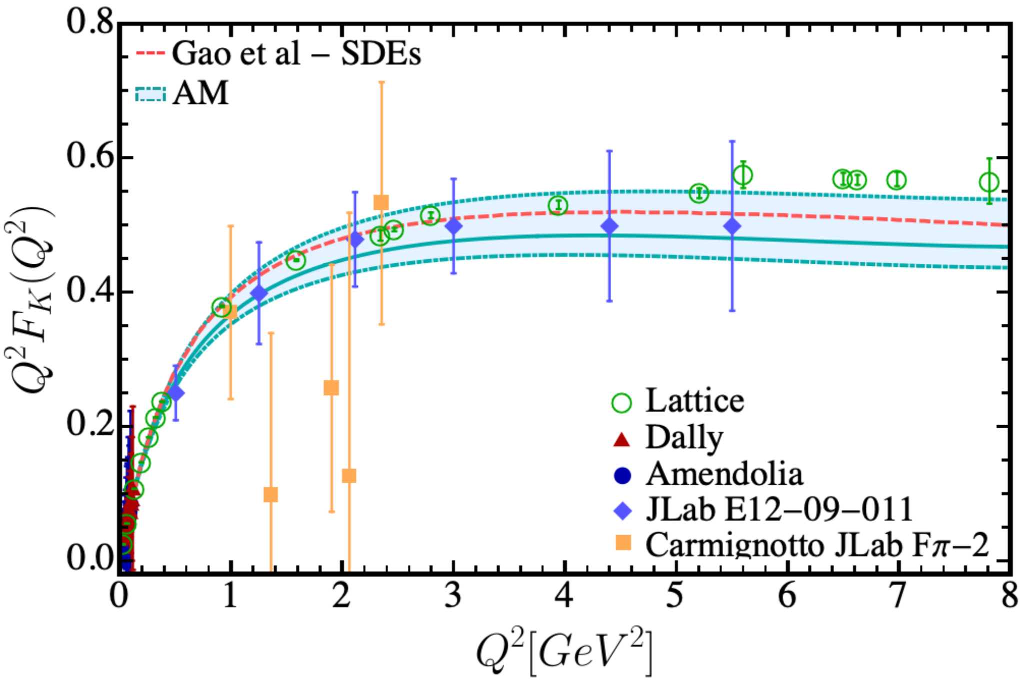


Form Factors and charge radii in algebraic model

We can test the efficacy of the **Algebraic Model** and evaluate the electromagnetic form factor to larger **Q^2** range: **0-40 GeV^2.**

I.M. Higuera, R.J. Hernández, K.Raya, AB, Phys. Rev. D 110 (2024) 3, 034013
H-T Ding, X. Gao, A.D. Hanlon, S. Mukherjee, P. Petreczky, Q, Shi, et. al., Phys. Rev Lett. 133 (2024) 18, 181902

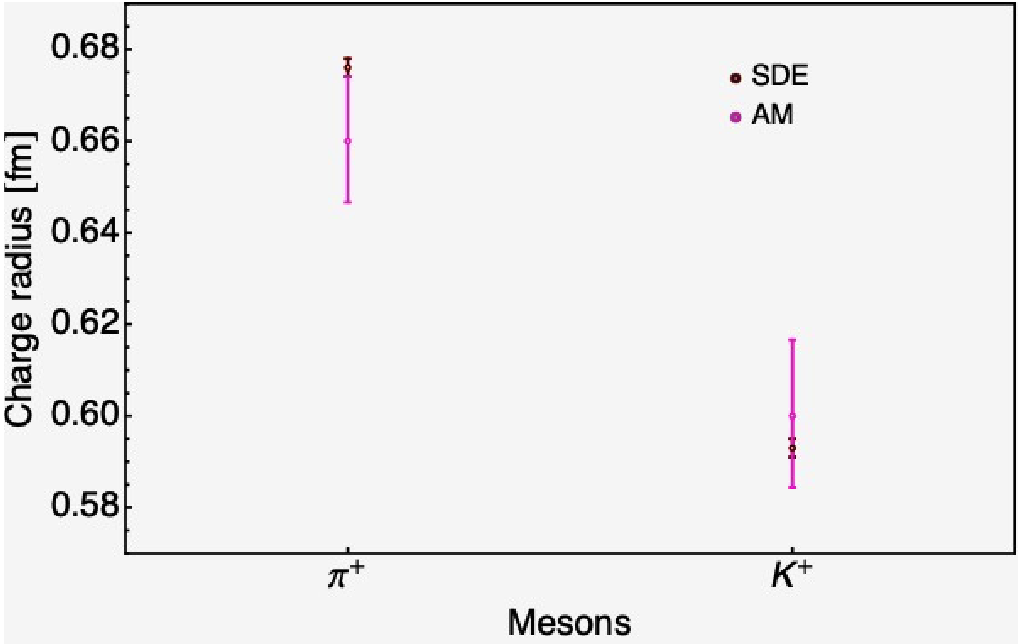


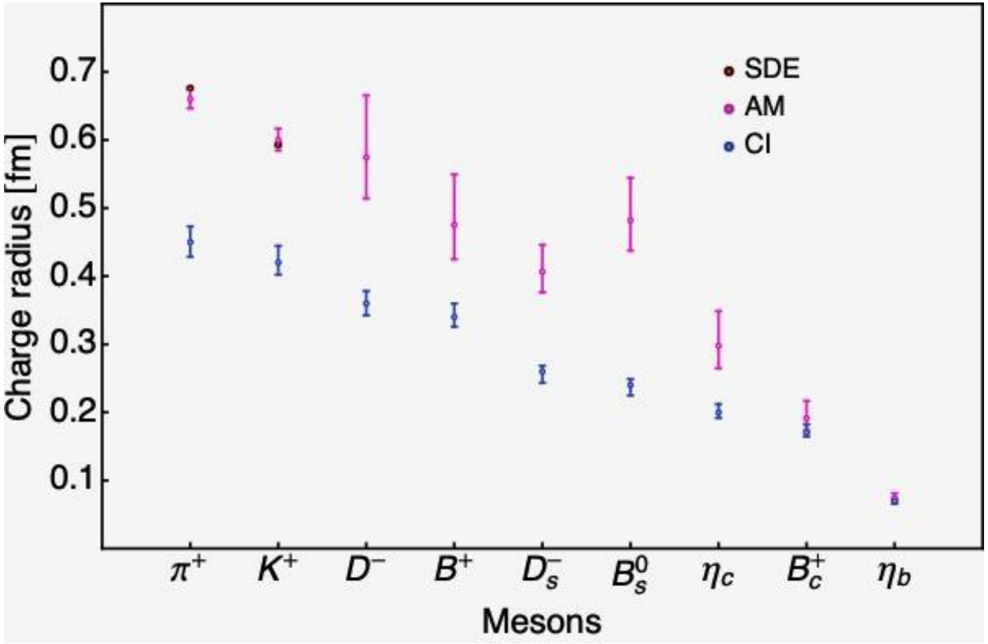


Form Factors and charge radii in algebraic model

We can now compare the **charge radii** for different modelling of the **Schwinger-Dyson** and **Bethe-Salpeter equations**.

A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013
B. Almeida-Zamora, L. Albino, AB, J.J. Cobos-Martínez, J. Segovia,e-Print: 2602.10775 [hep-ph]





Summary and Outlook

- **Electromagnetic form factors** and **charge radii** of **pion** and **kaon**, as well as their box diagram contributions to the g-2 of the muon were studied in **QCD akin truncations** of SDEs.

A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

- A similar study on the **excited pion** and **kaon**, and their **charge radii** has also been complete:

A.S. Miramontes, K. Raya, AB, P. Roig, G. Paredes, Chin. Phys. C49 (2025) 8. 083108

- Through the **contact interaction model**, a detailed analysis of **form factors** and **charge radii** of **pseudo-scalar**, **scalar**, **vector** and **axial-vector** mesons can be found in:

R.J. Hernández-Pinto, L.X. Gutiérrez-Guerrero, AB, M.A. Bedolla, I.M. Higuera-Angulo, Phys. Rev. D 107 (2023) 5, 054002
R.J. Hernández-Pinto, L.X. Gutiérrez-Guerrero, M.A. Bedolla, AB, Phys. Rev. D 110 (2024) 11, 114015
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- **Algebraic model** computations can be found in:

L. Albino, I.M. Higuera-Angulo, K. Raya, AB, Phys. Rev. D 106 (2022) 3, 034003
I.M. Higuera-Angulo, R.J. Hernández-Pinto, K. Raya, AB, Phys. Rev. D 110 (2024) 3, 034013
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Thank you for your attention