Precision Chiral EFT Analysis of Low-energy Lepton-Proton Two-Photon Exchange including Hadronic Effects

Based on:
Goswami et al., 2601.14636 [hep-ph] (2026) , *Goswami et al.*, PRD **111**, 113009 (2025) ,
Das et al., PRD **113**, 033007 (2026) , *Choudhary et al.*, EPJA **60**, 69 (2024) ,
Talukdar et al., PRD **104**, 053001 (2021) , *Talukdar et al.*, PRD **101**, 013008 (2020)

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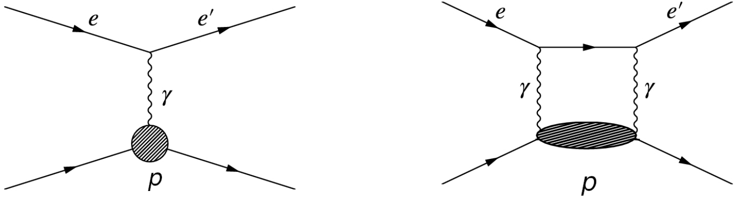
The 2$^{\text{nd}}$ NREC Workshop 2026, Stony Brook University, 13–17 April

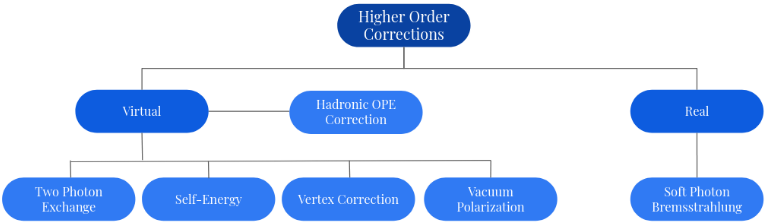
Two-photon Exchange in Lepton-Proton (ℓ-p) Elastic Scattering Process

- TPE potentially resolves fundamental discrepancies of the Low-energy Proton Structure, e.g., **Form factor** & **Charge Radius** Puzzles
- Evaluation of the TPE till date has been quite unsatisfactory $\rightarrow$ Model driven

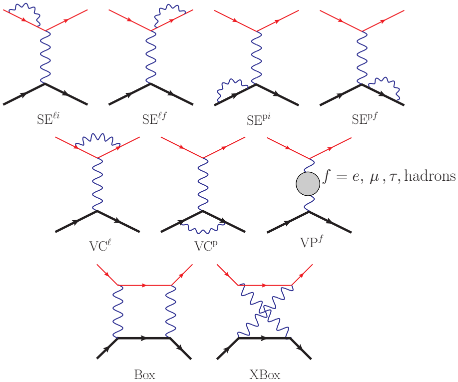


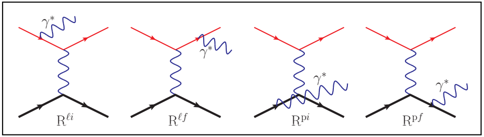
(a) Lepton scattering in the Born approximation.

(b) Two-photon exchange contribution to lepton scattering.



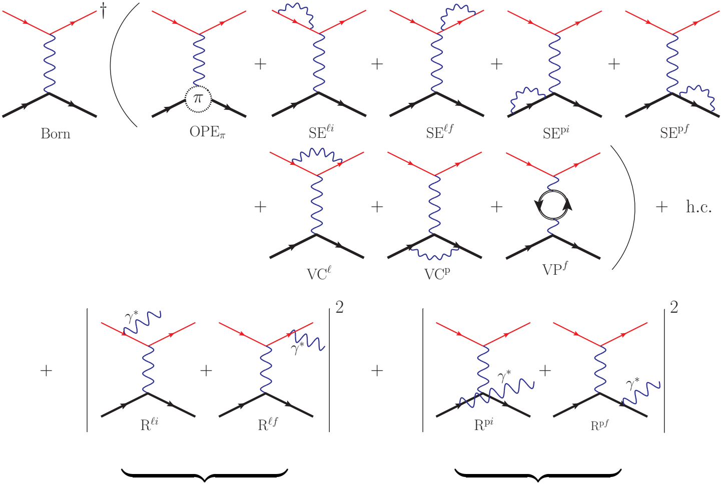
Radiative/QED corrections (up to one-loop) in ℓ-p Scattering





- Include soft-γ^* bremsstrahlung to remove IR-divergent terms from loops
  $\hookrightarrow$ mandated by the well-known **Kinoshita–Lee–Nauenberg (KLN) Theorem**

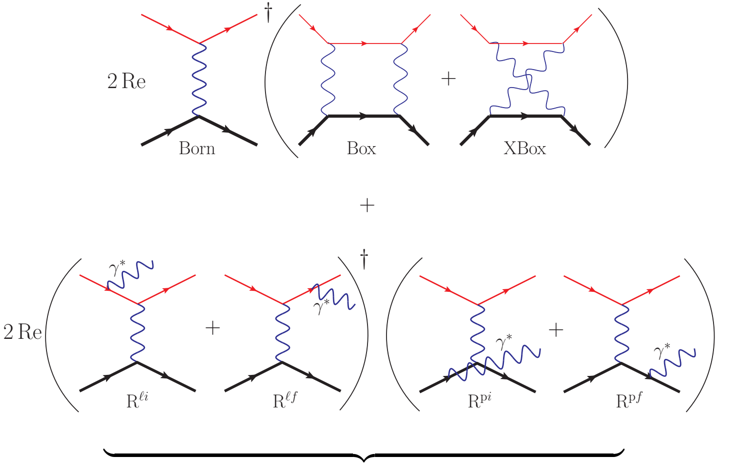
Charge-even contributions (up to one-loop) to ℓ-p cross section



"**Direct**" (ℓ-ℓ) & (p-p) Interference Bremsstrahlung

$$\left[\frac{d\sigma_{\rm el}^{\rm (even)}(Q^2)}{d\Omega_\ell}\right]^{(\mp)} = \left[\frac{d\sigma_{\rm el}(Q^2)}{d\Omega_\ell}\right]_{\rm Born}\left\{1+\overline{\delta^{\rm (even)}_{{\rm OPE}_\pi}}(Q^2)+\overline{\delta^{\rm (even)}_{\rm Virt}}(Q^2)+\overline{\delta^{\rm (even)}_{\rm Brem}}(Q^2)\right\}$$

Charge-odd contributions (up to one-loop) to ℓ-p cross section

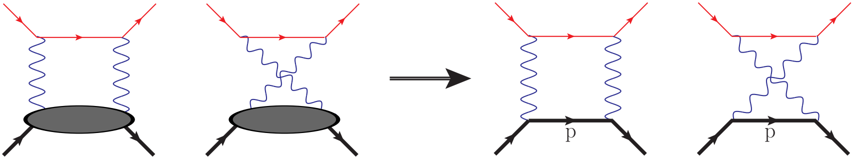


"**Exchange**" (ℓ-p) Interference Bremsstrahlung

$$\left[\frac{\mathrm{d}\sigma_{\rm el}^{\rm (odd)}(Q^2)}{\mathrm{d}\Omega_\ell}\right]^{(\mp)} = \pm\left[\frac{\mathrm{d}\sigma_{\rm el}(Q^2)}{\mathrm{d}\Omega_\ell}\right]_{\rm Born}\left\{\overline{\delta_{\gamma\gamma}^{\rm (TPE)}}(Q^2) + \overline{\delta_{\rm Brem}^{\rm (odd)}}(Q^2)\right\}$$

Low-$Q^2 \lesssim 0.1$ (GeV/c)^2 higher-order corrs. to $\left[\frac{d\sigma_{el}}{d\Omega_\ell}\right]_{\rm Born}$ (crude estimates)

1. Two kinds of higher-order corrections: (i) *Hadronic (QCD)*, and (ii) *Radiative (QED)*
2. Hadronic and radiative corrections are of the same order of magnitude ($\sim 30 - 40\%$ Born)
3. *Soft Bremsstrahlung* & *Vacuum Polarization* corrections constitute the most dominant radiative corrections ($\sim 25\%$ Born)
4. *Vertex* and *Multi-γ Exchange* constitute smaller radiative corrections ($\sim 10 - 15\%$ Born)
5. Largest uncertainties ($\sim 1 - 2\%$ Born) stemming from Two-Photon Exchange (TPE) corrections, which resolve bulk of the existing discrepancies (e.g., form factor and radius)
6. **Note:** At very low-energies only elastic proton TPE intermediate state dominantly contributes to elastic cross section with insignificant contributions ($\lesssim 0.5\%$ Born) from inelastic nucleon intermediate states ($\Delta(1232)$, N^*, ..., etc. resonances)
$\hookrightarrow$ In low-energy scattering analysis replace TPE blobs with elastic proton propagators!



- Simultaneous (unpolarized) scattering of electrons ($e^\mp$) and muons ($\mu^\mp$) with protons at unprecedented low-Q^2 range:

$$\left[0.0016 \lesssim Q^2 \lesssim 0.082_e(0.079_\mu)\right]\ \mathrm{GeV}^2/c^2$$

- **Accuracy:** Measure elastic cross-section below 1% precision (**Target** $\sim$ 0.1%)
- Plans at an accurate extraction of TPE *via charge asymmetry* observable

$$\mathcal{A}_{\ell\mp}(Q^2) = \frac{\left[\frac{d\sigma_{\rm el}(Q^2)}{d\Omega_\ell}\right]^{(-)} - \left[\frac{d\sigma_{\rm el}(Q^2)}{d\Omega_\ell}\right]^{(+)}}{\left[\frac{d\sigma_{\rm el}(Q^2)}{d\Omega_\ell}\right]^{(-)} + \left[\frac{d\sigma_{\rm el}(Q^2)}{d\Omega_\ell}\right]^{(+)}} = \frac{\boxed{\overline{\delta^{(\rm TPE)}_{\gamma\gamma}}(Q^2)} + \overline{\delta^{(\rm odd)}_{\rm Brem}}(Q^2)}{1 + \overline{\delta^{(\rm even)}_{{\rm OPE}_\pi}}(Q^2) + \overline{\delta^{(\rm even)}_{\gamma-\rm loop}}(Q^2) + \overline{\delta^{(\rm even)}_{\rm Brem}}(Q^2)}$$

- IR-finite parts of TPE and odd Brem. corrections must be first isolated

$$\overline{\delta^{(\rm TPE)}_{\gamma\gamma}}(Q^2) = \frac{2{\rm Re}\sum_{spins}\left(\mathcal{M}^*_{\rm Born}\,\mathcal{M}_{\rm TPE}\right)}{\sum_{spins}|\mathcal{M}_{\rm Born}|^2} - \mathrm{IR}^{(\rm odd)}_{\rm box}(Q^2)$$

$$\overline{\delta^{(\rm odd)}_{\rm Brem}}(Q^2) = \frac{2{\rm Re}\sum_{spins}\left(\mathcal{M}^{(\ell i)}_{\rm Brem} + \mathcal{M}^{(\ell f)}_{\rm Brem}\right)^\dagger\left(\mathcal{M}^{(pi)}_{\rm Brem} + \mathcal{M}^{(pf)}_{\rm Brem}\right)}{\sum_{spins}|\mathcal{M}_{\rm Born}|^2} - \mathrm{IR}^{(\rm odd)}_{\rm Brem}(Q^2)$$

$$\boxed{\mathrm{IR}^{(\rm odd)}_{\rm box}(Q^2) = -\mathrm{IR}^{(\rm odd)}_{\rm Brem}(Q^2)}$$

Framework: $SU_f(2)$ Heavy Baryon Chiral PT (HBχPT)

- Low-energy Effective Field Theory of QCD + QED
- **Model-independent** with **gauge invariance** naturally incorporated
- Light D.O.F.s, e.g., γ, $e^{\mp}$, $\mu^{\mp}$, $\pi^{0,\pm}$, are treated relativistically
- Heavy D.O.F.s like nucleons & baryons, are treated non-relativistically
- **Hard Scale:** All UV physics above $\gtrsim \Lambda_H$ are *integrated out*

chiral symmetry breaking scale: $\Lambda_H \equiv M_N \sim \Lambda_\chi \sim 4\pi f_\pi \sim 1\ \mathrm{GeV/c}$

- **Power-counting scheme:** Ensures control over systematic uncertainties
  ↪ Simultaneous perturbative expansion of Feynman amplitudes in powers of Q/M_N in addition to the standard chiral expansion in powers of $Q/\Lambda_\chi \sim m_\pi/\Lambda_\chi$
  ↪ Allows to include dominant proton's **recoil effects** at low-Q^2
  ↪ **Chiral Dimension:** $\mathcal{M}^{(\nu)}_{\pi N} \sim \mathcal{O}(1/M^\nu_N)\,;\quad \nu = 0, 1, 2, 3, \cdots$ (N$^\nu$LO)

$$\nu = 2L_\pi + \sum_{n=1}(2n-2)V^{(p^{2n})}_{\pi\pi} + \sum_{\aleph=1}^{\infty}(\aleph-1)V^{(p^{\aleph})}_{\pi N} + \sum_{\alpha=0}^{\infty}\alpha\Delta_N^{(1/M^\alpha)}$$

Effective Lagrangian → TPE cross section upto NNLO [i.e., $\mathcal{O}(\alpha^3/M_N^2)$]

- Operators constructed by low-energy symmetries (e.g., P, C, T, χ, ...)

$$\mathcal{L}_{\ell\pi N\gamma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_\ell\left(i\gamma^\mu D_\mu - m_\ell\right)\psi_\ell + \boxed{\mathcal{L}^{\rm eff}_{\pi N}}, \qquad \text{where}$$

$$\mathcal{L}^{\rm eff}_{\pi N} = \left[\mathscr{L}^{(p^2)}_{\pi\pi} + \cdots\right]^{\rm (Rel)}_{\chi\rm PT} + \left[\sum_{\aleph=1}^{3}\mathcal{L}^{(p^\aleph)}_{\pi N} + \cdots\right]^{\rm (Non-Rel)}_{\rm HB\chi PT}$$

- For LO [i.e., $\mathcal{O}(\alpha^3)$] & NLO [i.e., $\mathcal{O}(\alpha^3/M_N)$] cross section analyses:
  ↪ Involves phenomenological **Low-energy Constants** (LECs) → $g_A, c_i, \kappa_{s,v}, \cdots$

$$\mathcal{L}^{(p^1)}_{\pi N} = \overline{N}\Big[i(v\cdot D) + g_A(u\cdot S)\Big]N \quad , \quad N = (\mathrm{p}\ \mathrm{n})^{\rm T} \ \ \& \ \ \mathrm{v} = (1, \mathbf{0})$$

Bernard, Kaiser, Meißner, Fettes, Stininger, ...

$$\mathcal{L}^{(p^2)}_{\pi N} = \overline{N}\Bigg[\frac{(v\cdot D)^2 - D\cdot D}{2M_N} - \frac{ig_A}{2M_N}\{S\cdot D, v\cdot u\} + c_1\mathrm{Tr}(\chi_+) + \left(c_2 - \frac{g_A^2}{8M_N}\right)(v\cdot u)^2$$
$$+ c_3 u\cdot u + \left(c_4 + \frac{1}{4M_N}\right)[S^\mu, S^\nu]u_\mu u_\nu + c_5\mathrm{Tr}\left\{\chi_+ - \frac{1}{2}\mathrm{Tr}\chi_+\right\}$$
$$- \frac{i}{4M_N}[S^\mu, S^\nu]\left\{(1+\kappa_v)f^+_{\mu\nu} + \frac{1}{2}(\kappa_s - \kappa_v)\mathrm{Tr}(f^+_{\mu\nu})\right\}\Bigg]N$$

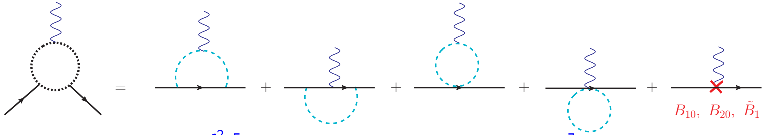
Non-linear Pion fields: $u_\mu = i\sqrt{U^\dagger}\,\nabla_\mu\left(U\sqrt{U^\dagger}\right)$ $\qquad U(x) = \sqrt{1 - \frac{\vec{\pi}^{\,2}(x)}{f_\pi^2}} + \frac{i\,\vec{\tau}\cdot\vec{\pi}(x)}{f_\pi}$

↪ Up to NLO chiral order ($\nu = 0, 1$) pions does not appear explicitly: $U(x) \to 1$
↪ Proton behaves effectively "point-like"

Effective Lagrangian → TPE cross section upto NNLO [i.e., $\mathcal{O}(\alpha^3/M_N^2)$]

- For NNLO cross section analysis:
  ↪ Involves pion-loops and counter-terms that renormalize the pγp vertices
  ↪ Generate the *Sachs form factors* $G^p_{E,M}$ and rms radii $\sqrt{\langle r^2_{E,M}\rangle}$

$$\Gamma^{(\mathrm{p}\gamma\mathrm{p})}_\mu(Q^2) = G^p_E(Q^2)v_\mu + \frac{1}{M_N}\,[S_\mu, S_\nu]\,Q^\nu\,G^p_M(Q^2) \;\; ; \;\; \langle r^2_{E,M}\rangle = -\,6\,\frac{\mathrm{d}G^p_{E,M}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

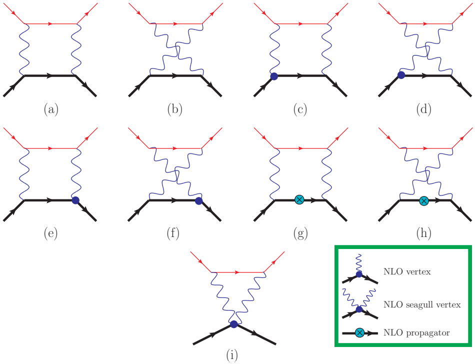


$$\mathscr{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\left[\mathrm{Tr}\left(\nabla_\mu U \nabla^\mu U^\dagger\right) + \mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]$$

Bernard, Kaiser, Meißner, Fettes, Stininger, ...

$$\mathcal{L}^{(p^3)}_{\pi N} = \frac{1}{(4\pi f_\pi)^2}\,\overline{N}\left[\left(B_{10} - \pi^2 f_\pi^2\,\frac{1+2\kappa_v}{M_N^2}\right)\left[D^\mu, f^{(v)}_{\mu\nu}\right] v^\nu + B_{20}\left[\mathrm{Tr}(\chi_+)\, iv\cdot D + \mathrm{h.c.}\right]\right.$$
$$\left. + \left(\tilde{B}_1 - \pi^2 f_\pi^2\,\frac{1+2\kappa_s}{M_N^2}\right)\left[D^\mu, v^{(s)}_{\mu\nu}\right] v^\nu + \cdots\right] N$$
$$- \frac{i}{4M_N^2}\,\overline{N}\left[(v\cdot D)^3 - \frac{1}{2}\left(D^2(v\cdot D) - (v\cdot D^\dagger)D^{\dagger 2}\right)\right] N + \mathcal{O}\left(\frac{1}{M_N^3}\right)$$

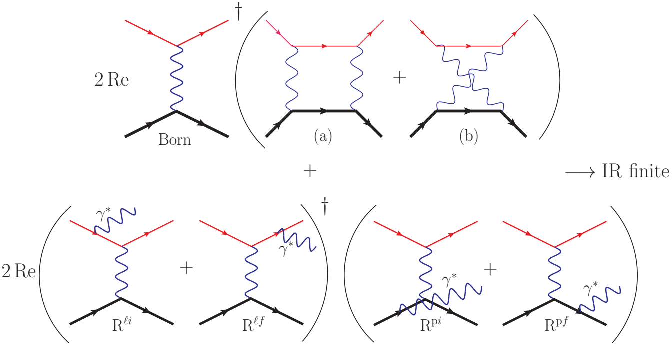
TPE diagrams @LO+NLO ($\nu = 0\,\&\,1$) in HBχPT and IR-divergence



- Contributions to cross section from LO diagrams (a) & (b) are **IR-divergent**
- Contributions to cross section from NLO diagrams (c) - (i) are all **finite**
- $\mathcal{O}(1/M_N)$ IR-divergences isolated using **Dim. Regularization**, with $\varepsilon = (4-D)/2 < 0$

  $\hookrightarrow$ IR-divergence cancels @ $\mathcal{O}(M_N^0)$, since $(E_\ell - E'_\ell) \sim (p_\ell - p'_\ell) \sim \mathcal{O}(M_N^{-1})$

$$\mathbf{IR}^{(\rm odd)}_{\rm box}(Q^2) = \frac{\alpha}{\pi}\left\{\frac{1}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m_\ell^2}\right)\right\}\left\{\frac{E_\ell}{p_\ell}\ln\sqrt{\frac{E_\ell + p_\ell}{E_\ell - p_\ell}} - \frac{E'_\ell}{p'_\ell}\ln\sqrt{\frac{E'_\ell + p'_\ell}{E'_\ell - p'_\ell}}\right\} \sim \mathcal{O}\left(\frac{1}{M_N}\right)$$



- $\mathcal{O}(1/M_N)$ IR-divergences stem from LO charge-odd Bremsstrahlung diagrams only

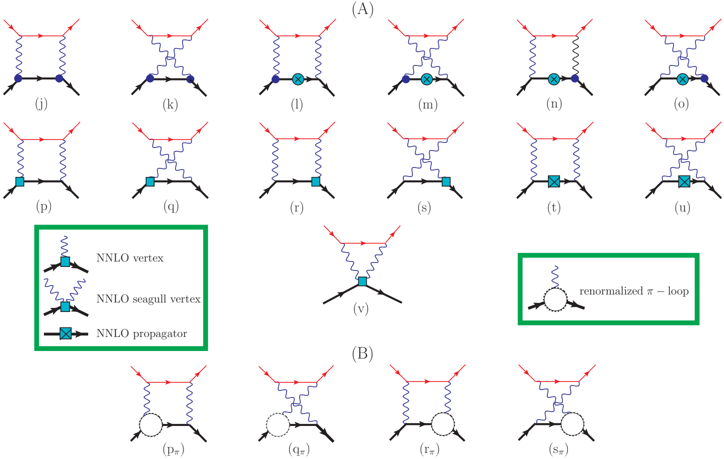
  ↪ IR-divergence cancels @ $\mathcal{O}(M_N^0)$, since $(E_\ell - E'_\ell) \sim (p_\ell - p'_\ell) \sim \mathcal{O}(M_N^{-1})$

$$\mathbf{IR}^{(\mathrm{odd})}_{\mathrm{Brem}}(Q^2) = -\frac{\alpha}{\pi}\left\{\frac{1}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m_\ell^2}\right)\right\}\left\{\frac{E_\ell}{p_\ell}\ln\sqrt{\frac{E_\ell + p_\ell}{E_\ell - p_\ell}} - \frac{E'_\ell}{p'_\ell}\ln\sqrt{\frac{E'_\ell + p'_\ell}{E'_\ell - p'_\ell}}\right\} \sim \mathcal{O}\left(\frac{1}{M_N}\right)$$

$$\boxed{\overline{\delta^{(\mathrm{odd})}_{\mathrm{TPE+Brem}}}(Q^2) = \delta^{(\mathrm{TPE})}_{\gamma\gamma}(Q^2) + \delta^{(\mathrm{odd})}_{\mathrm{Brem}}(Q^2) \longrightarrow \text{IR-finite result @LO+NLO}}$$

TPE diagrams @NNLO ($\nu = 2$) in HBχPT with 1-loop Approximation

- Includes dominant class of diagrams with proton's form factor contributions, e.g., $(p + p_\pi)$ - $(s + s_\pi)$

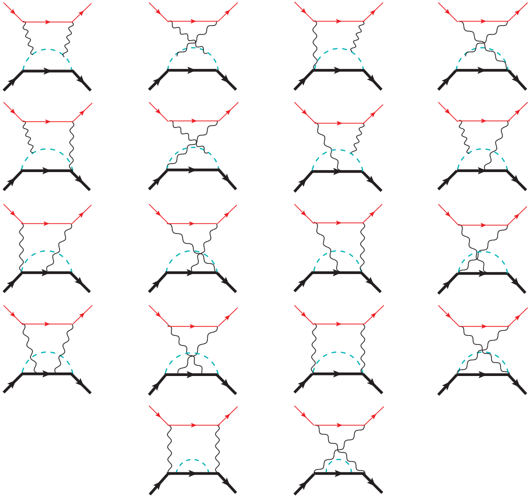


- Non-relativistic proton propagator up to NNLO [i.e., $\mathcal{O}(1/M_N^2)$]:

$$iS^{(\mathrm{NR})}_{(\mathrm{p})}(k) = \left[\frac{i}{v\cdot k + i0_+}\right]_{\mathrm{LO}} + \frac{i}{2M_N}\left[1 - \frac{k^2}{(v\cdot k + i0_+)^2}\right]_{\mathrm{NLO}} + \frac{i}{4M_N^2}\left[\frac{(v\cdot k)^3 - k^2(v\cdot k)}{(v\cdot k + i0_+)^2}\right]_{\mathrm{NNLO}} + \mathcal{O}\left(\frac{1}{M_N^3}\right)$$

Genuine 2-loop TPE diagrams @NNLO ($\nu = 2$) were not considered

- Naively 2-loop amplitudes are kinematically suppressed compared to 1-loop amplitudes
- Challenging to evaluate analytically with overlapping UV divergences → ongoing work



TPE results up to NNLO in HBχPT for ℓ-p scattering relevant to MUSE

Lepton momentum (p_ℓ) in MeV/c	115	153	210
$\lvert Q^2 \rvert$ in (GeV/c)^2 for Electron			
Angle $\theta = 20^\circ$	0.0016	0.0028	0.0052
Angle $\theta = 100^\circ$	0.027	0.046	0.082
$\lvert Q^2 \rvert$ in (GeV/c)^2 for Muon			
Angle $\theta = 20^\circ$	0.0016	0.0028	0.0052
Angle $\theta = 100^\circ$	0.026	0.045	0.080

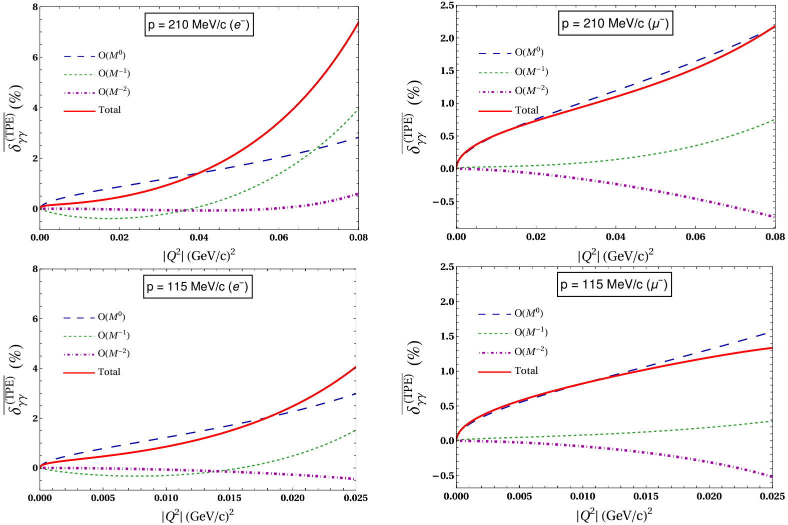
- **AIM:** TPE contribution to the charge-odd diff. cross section up to NNLO[i.e., $\mathcal{O}(\alpha^3/M_N^2)$]:

$$\left[\frac{\mathrm{d}\sigma_{\rm el}(Q^2)}{\mathrm{d}\Omega_\ell}\right]^{(\mp)}_{\rm TPE} = \pm\frac{\alpha^2 p_\ell'}{p_\ell Q^2}\left(1-\frac{Q^2}{4M^2}\right)\left[1+\frac{4E_\ell E_\ell'}{Q^2}\right] \times\left\{\overline{\delta^{\rm TPE(0)}_{\gamma\gamma}}(Q^2)+\overline{\delta^{\rm TPE(1)}_{\gamma\gamma}}(Q^2)+\underbrace{\overline{\delta^{\rm TPE(2)}_{\gamma\gamma}}(Q^2)}_{\rm Preliminary}\right\}+\mathcal{O}\left(\frac{\alpha^3}{M_N^3}\right)$$

$$\boxed{\overline{\delta^{\rm TPE(\nu)}_{\gamma\gamma}}(Q^2)\sim\mathcal{O}\left(\frac{\alpha}{M_N^\nu}\right)}$$

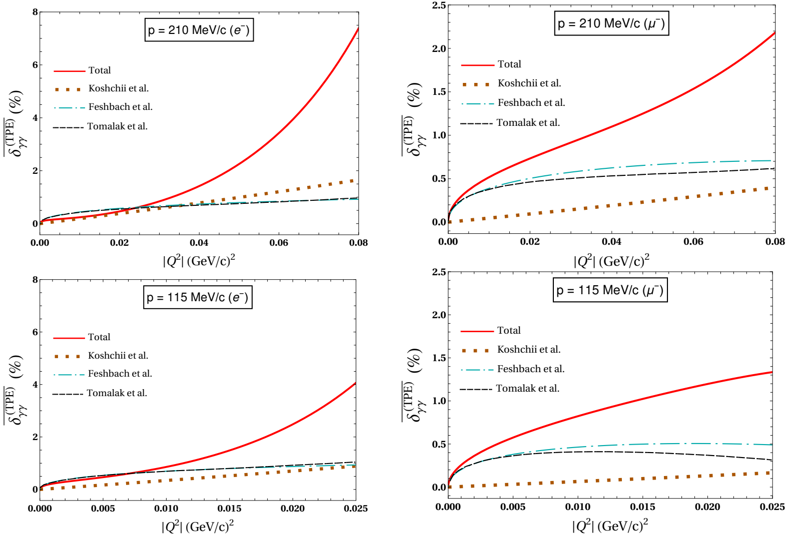
- NNLO corrections rely on rms proton's charge radius $\overline{r_p} = 0.855$ fm (free parameter):
  CREMA (PSI): $r_p = 0.84087(39)$ fm *Antognini, et al*, 2013, *Pohl et al.* 2013
  A1 (MAMI): $r_p = 0.87 \pm (0.014)\mathrm{stat.} \pm (0.024)\mathrm{syst.} \pm (0.003)_{\mathrm{mod.}}$ fm *Mihovilovic et al.* 2021

Numerical TPE results for e^--p and μ^--p scattering up to NNLO



- Large cancellations between box (a) and crossed-box (b) diagrams especially at LO
- $I(Q|1,1,1,0) \propto \ln^2\left(\dfrac{Q^2}{m_\ell^2}\right)$ from (e) & (f) NLO graphs lead to high-Q^2 enhancement for e-p scattering

Comparison of TPE corrections for e^--p and μ^--p scattering



Tomalak and Vanderhaeghen (2016): Dispersion Relations

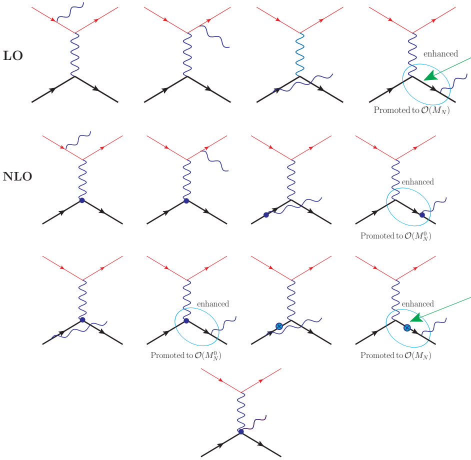
McKinley and Feshbach (1948): Non-relativistic potential scattering of leptons using 2^{nd} Born approximation

Koshchii and Afanasev (2017): QED+hadronic model $\longrightarrow$ diagrammatic, with elastic proton intermediate state invoking Soft Photon Approximation

Charge-odd Soft-γ^* Bremsstrahlung diagrams @LO+NLO ($\nu = 0\,\&\,1$)

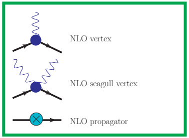
AIM: Determine $d\sigma^{(\mathrm{odd})}_{\mathrm{elastic}}$ up to NLO [i.e., $\mathcal{O}(\alpha^3/M_N)$]

- Consider only charge-odd products of $\ell-$p "exchange" interference bremsstrahlung amplitudes
- LO diagrams contribute to IR-divergent cross section while NLO diagrams lead to finite results
- Graphs only with final-state proton propagaotors are chirally enhanced



- $v\cdot Q = -\frac{Q^2}{2M_N}$ where $Q^2 < 0$

$$iS_p^{(LO)}(Q) = \frac{i}{v\cdot Q + i0_+} \sim \frac{i}{\left(-\frac{Q^2}{2M_N}\right)} \sim \mathcal{O}(M_N)$$

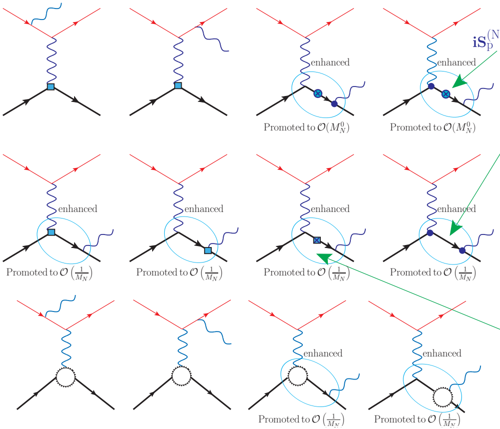


$$iS_p^{(NLO)}(Q) = \frac{i}{2M_N}\left[1 - \frac{Q^2}{(v\cdot Q + i0_+)^2}\right] \sim \frac{i}{2M_N}\frac{1}{\left(\frac{Q^4}{4M_N^2}\right)} \sim \mathcal{O}(M_N)$$

Charge-odd Soft-γ^* Brem. @LO+NLO (including $\nu = 2$ topologies)

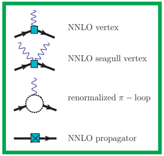
AIM: Determine $d\sigma^{(\text{odd})}_{\text{elastic}}$ up to NLO [i.e., $\mathcal{O}(\alpha^3/M_N)$]

- Consider only charge-odd products of $\ell - p$ "exchange" interference bremsstrahlung amplitudes
- NNLO (chiral order $\nu = 2$) diagrams should naively scale as $\mathcal{O}(1/M_N^2)$, instead some are enhanced and promoted to lower ($\nu = 0, 1$) chiral orders



$$iS_p^{(NLO)}(Q) = \frac{i}{2M_N}\left[1 - \frac{Q^2}{(v\cdot Q + i0_+)^2}\right] \sim \frac{i}{2M_N}\frac{1}{\left(\frac{Q^4}{4M_N^2}\right)} \sim \mathcal{O}(M_N)$$

$$iS_p^{(LO)}(Q) = \frac{i}{v\cdot Q + i0_+} \sim \frac{i}{\left(-\frac{Q^2}{2M_N}\right)} \sim \mathcal{O}(M_N)$$

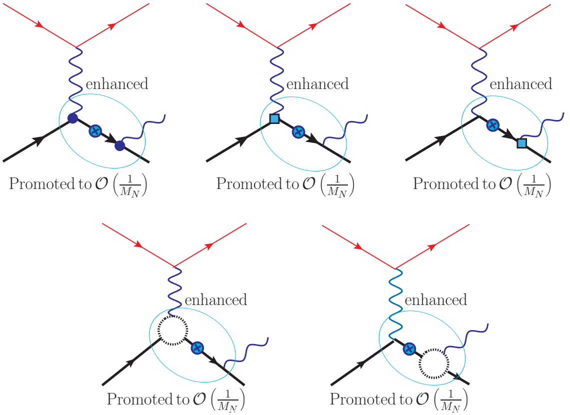


$$iS_p^{(NNLO)}(Q) = \frac{i}{4M_N^2}\left[\frac{(v\cdot Q)^3 - Q^2(v\cdot Q)}{(v\cdot Q + i0_+)^2}\right] \sim \mathcal{O}\left(\frac{1}{M_N}\right)$$

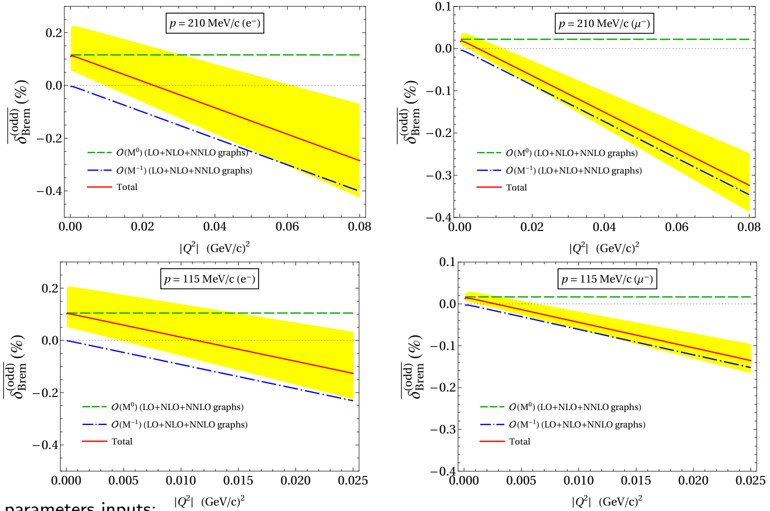
Charge-odd Soft-γ^* Brem. @LO+NLO (including $\nu = 3$ topologies)

AIM: Determine $d\sigma^{(\text{odd})}_{\text{elastic}}$ up to NLO [i.e., $\mathcal{O}(\alpha^3/M_N)$]

- Consider only charge-odd products of $\ell - \mathrm{p}$ "exchange" interference bremsstrahlung amplitudes
- N^{3}LO (chiral order $\nu = 3$) diagrams should naively scale as $\mathcal{O}(1/M_N^3)$, instead some get enhanced and promoted to $\nu = 1$ chiral order



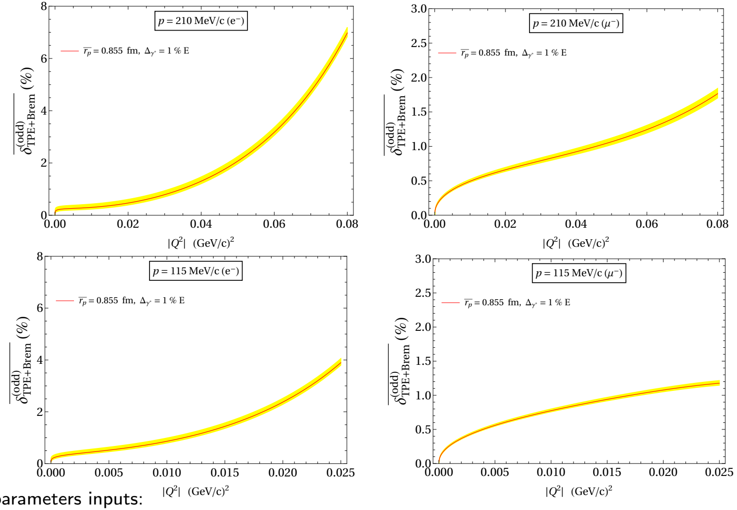
Numerical Charge-odd Brem. results for e^--p and μ^--p up to NLO



Two free parameters inputs:

- **Detector threshold:** $\Delta_{\gamma^*} \approx 1\%$ of E_ℓ (central) & $0.5\% \, E_\ell \lesssim \Delta_{\gamma^*} \lesssim 2\% \, E_\ell$ (yellow band)
- Proton's charge radius $\overline{r_p} = 0.855$ fm (central) & Error band (not visible):
  CREMA (PSI): $r_p = 0.84087(39)$ fm *Antognini, et al,* (2013), *Pohl et al.* (2013)
  A1 (MAMI): $r_p = 0.87 \pm (0.014)\mathrm{stat.} \pm (0.024)\mathrm{syst.} \pm (0.003)_{\mathrm{mod.}}$ fm *Mihovilovic et al.* (2021)

Total charge-odd (TPE + Brem.) radiative corrs. ≈ "Physical" TPE results



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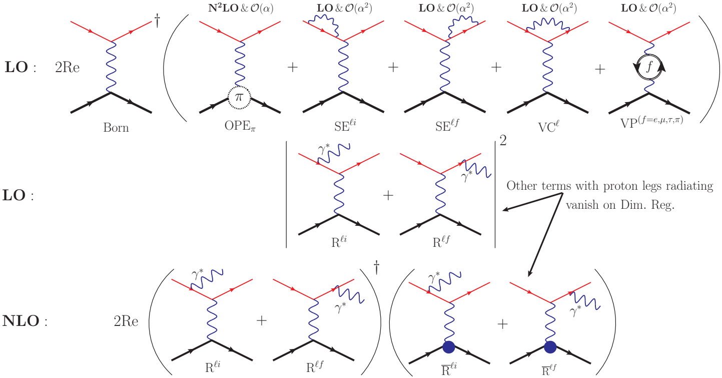
Charge-even contributions (up to one-loop) in ℓ-p Scattering in HBχPT

AIM: Determine $d\sigma^{(\text{even})}_{\text{elastic}}$ [i.e., $\mathcal{O}(\alpha^2/M_N^2)_{\text{OPE}_\pi} + \mathcal{O}(\alpha^3/M_N)_\gamma$]



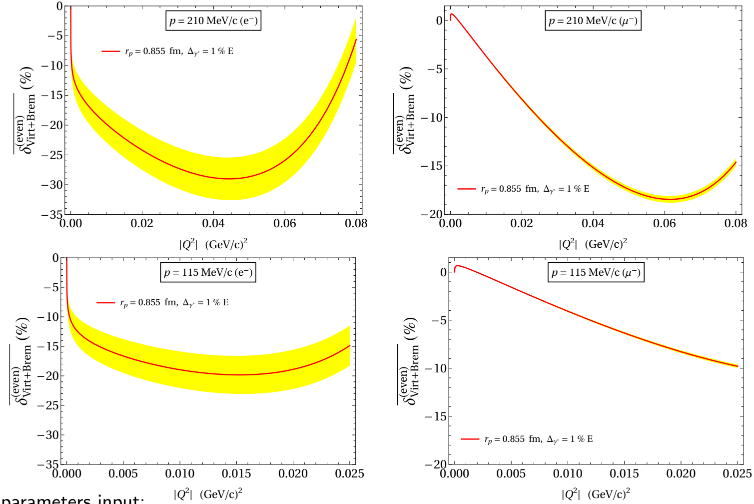
- IR-divergent terms arise ONLY at the LO

$$\mathbf{IR}^{(\text{even})}_{\gamma-\text{loop}}(Q^2) = -\mathbf{IR}^{(\text{even})}_{\text{Brem}}(Q^2) = \frac{\alpha}{\pi}\left\{\frac{1}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m_\ell^2}\right)\right\}\left\{\frac{\nu_\ell^2+1}{2\nu_\ell}\ln\left[\frac{\nu_\ell+1}{\nu_\ell-1}\right] - 1\right\}\;;\quad \nu_\ell = \sqrt{1-\frac{4m_\ell^2}{Q^2}}$$

$$\boxed{\bar{\delta}^{(\text{even})}_{\text{Virt+Brem}}(Q^2) = \delta^{(\text{even})}_{\text{Virt}}(Q^2) + \delta^{(\text{even})}_{\text{Brem}}(Q^2) \longrightarrow \text{IR-finite result}}$$

- **NOTE:** No chirally enhanced charge-even bremsstrahlung contributions

Total charge-even (Virtual + Brem.) radiative corrections



Two free parameters input:

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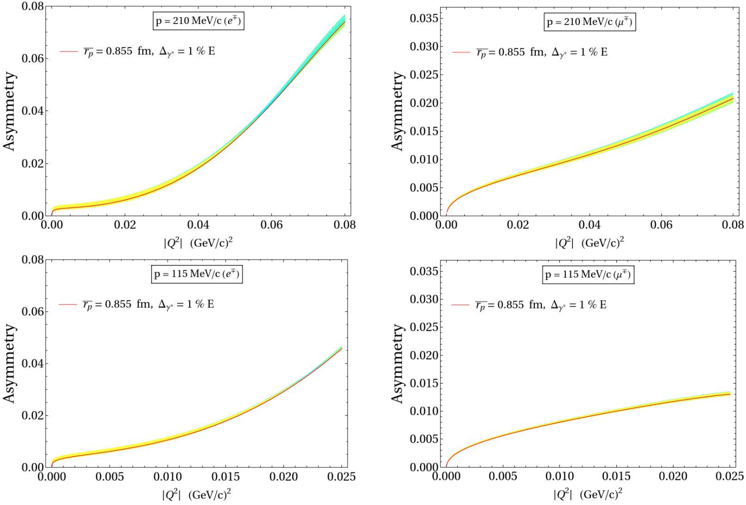


$$\mathcal{A}_{\ell^{\mp}}(Q^2) = \frac{\overline{\delta^{(\mathrm{TPE})}_{\gamma\gamma}}(Q^2) + \overline{\delta^{(\mathrm{odd})}_{\mathrm{Brem}}}(Q^2)}{1 + \overline{\delta^{(\mathrm{even})}_{\mathrm{OPE}_\pi}}(Q^2) + \overline{\delta^{(\mathrm{even})}_{\gamma-\mathrm{loop}}}(Q^2) + \overline{\delta^{(\mathrm{even})}_{\mathrm{Brem}}}(Q^2)}$$

- **TPE Results in HBχPT up to NNLO:**
  ↪ Large cancellations are manifest at LO between box and crossed diagrams
  ↪ NLO corrections are logarithmically enhanced in the high-Q^2 region
  ↪ NNLO contributions are tiny with dominant contributions arising from proton's form factors/radii

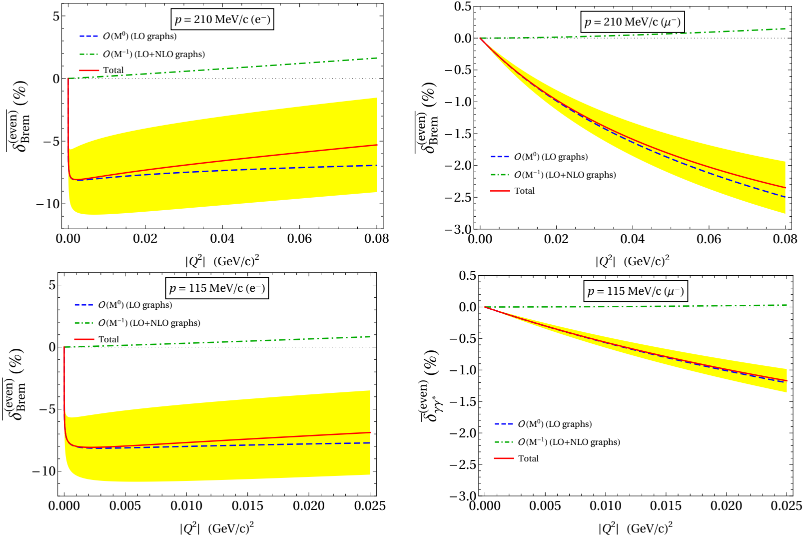
- **Soft-γ^* Bremsstrahlung Results in HBχPT up to NLO:**
  ↪ Charge-odd contributions lead to chiral enhancement of amplitudes with final-state proton propagators
  ↪ There is no chiral enhancement in charge-even contributions

- **Charge Asymmetry Results:**
  Charge-odd bremsstrahlung contributions are small enough not to preclude an accurate ($\lesssim$ 1% level) extraction of the TPE from charge-asymmetry measurements at MUSE

THANK YOU
for your attention

Back-up Slides

Charge-even Soft-γ^* Brem. corrections for e^--p and μ^--p up to NLO



Free parameters input:

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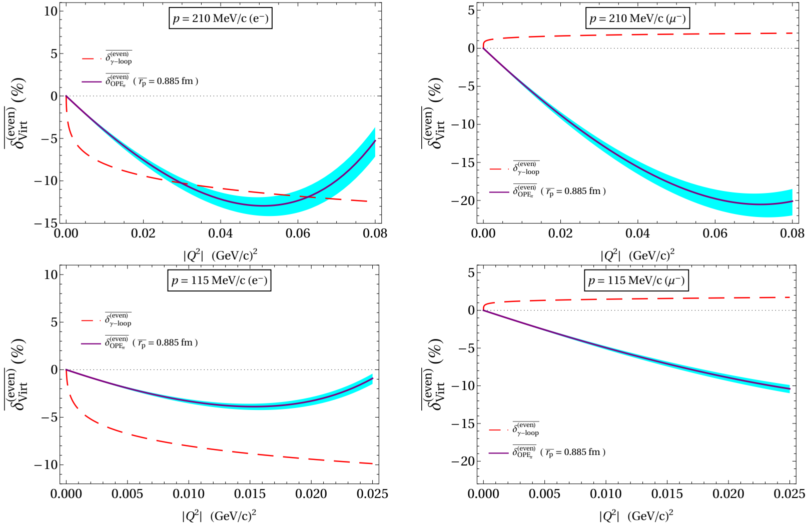

- NNLO corrections rely on proton's rms charge radius $\overline{r_p} = 0.855$ fm (free parameter):

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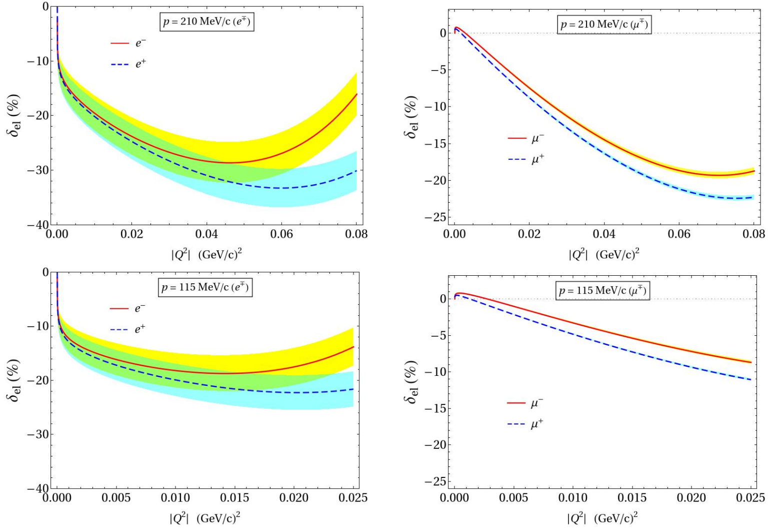
Total corrections (up to one-loop) for $e^{\mp}$-p and $\mu^{\mp}$-p Scattering



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