Instrumental uncertainties in radiative corrections for the MUSE experiment

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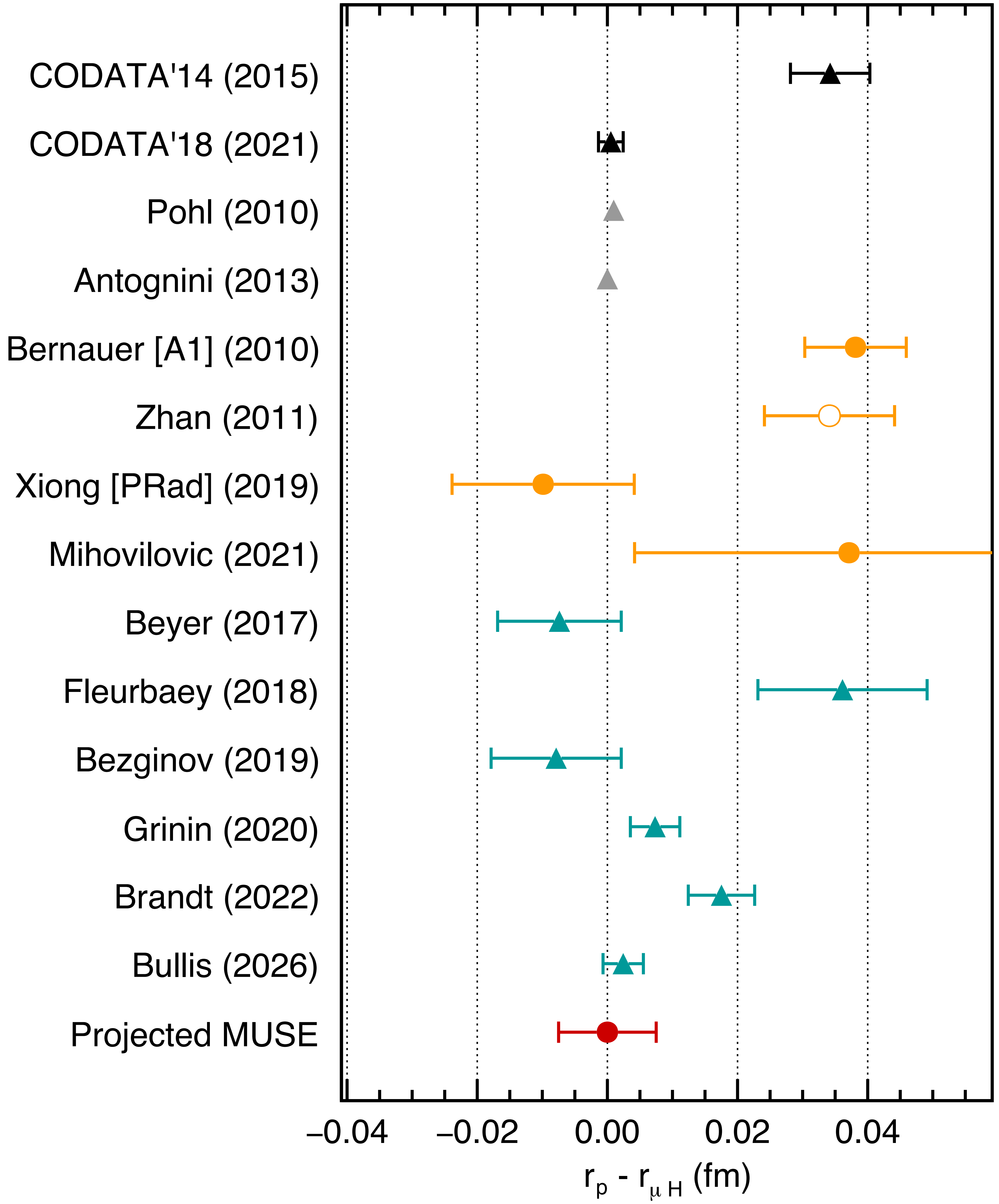
for the MUSE Collaboration

Workshop on NREC 2026 (the second workshop on Nuclear Radius Extraction Collaboration)
Stony Brook University, NY, April 13 — 17, 2026

Supported in parts by the U.S. National Science Foundation: NSF PHY-2111050. The MUSE experiment is supported by the U.S. Department of Energy, the U.S. National Science Foundation, the Paul Scherrer Institute, and the US-Israel Binational Science Foundation.

MUSE and The Proton-Radius Puzzle



Possible explanations of the proton charge-radius puzzle:

Radiative Correction Effects

Two Photon Exchange (TPE)

Physics Beyond Standard Model (Violation of Lepton Universality)

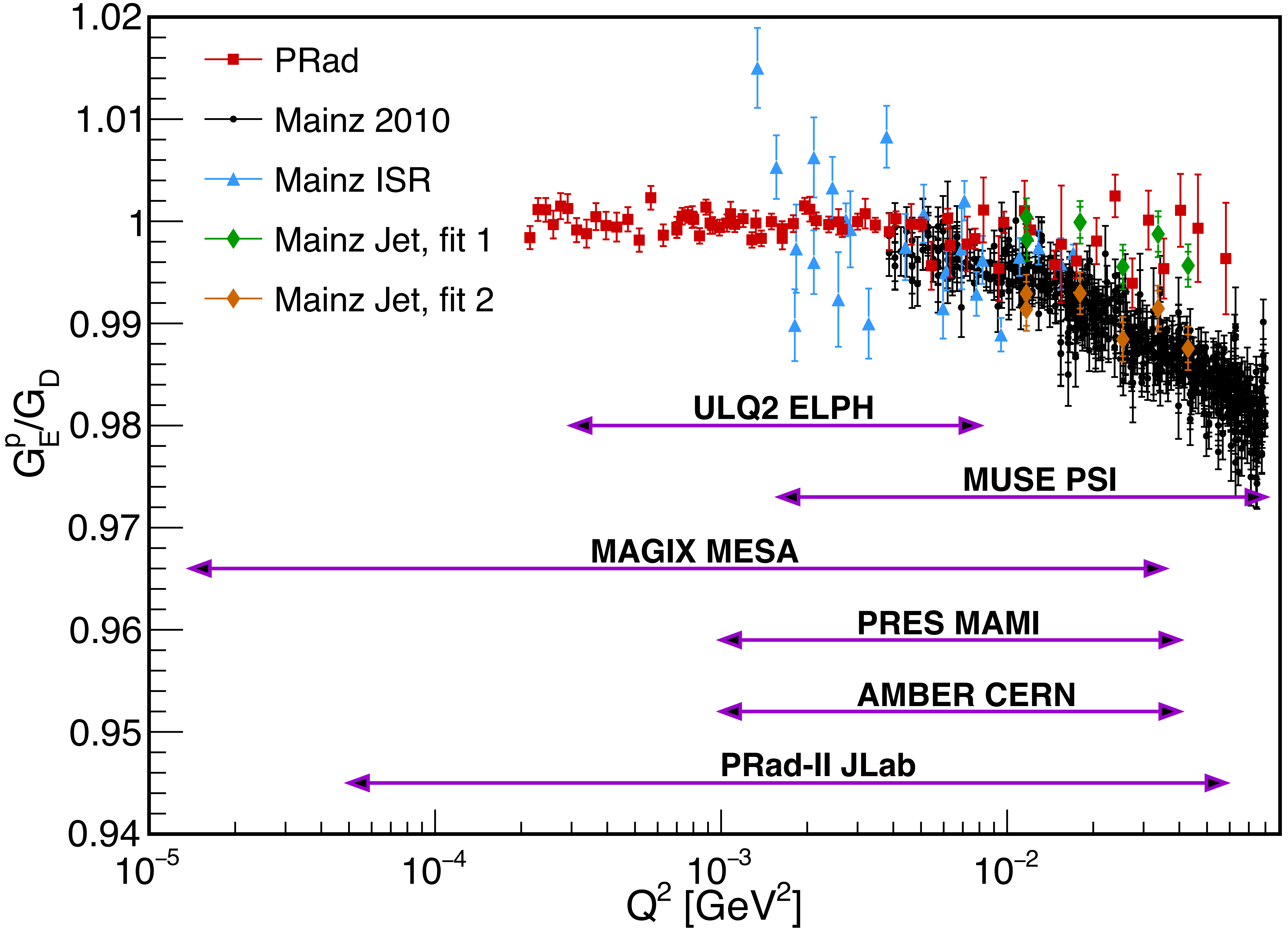
Inconsistent electron-scattering data

Inconsistent hydrogen-spectroscopy data

MUSE

$$e^{\pm}p \rightarrow e^{\pm}p$$
$$\mu^{\pm}p \rightarrow \mu^{\pm}p$$

MUSE is the only low-Q^2 scattering experiment using $e^\pm$ and $\mu^\pm$ beams



Beam	e⁻	e⁺	μ⁻	μ⁺	
PRad	✓				completed
Mainz 2010	✓				
Mainz ISR	✓				
Mainz Jet	✓				
MUSE PSI	✓	✓	✓	✓	running
PRad-II JLab	✓				
ULQ2 ELPH	✓				
AMBER CERN			✓	✓	future
MAGIX MESA	✓				
PRES MAMI	✓				unlikely

W. Xiong and C. Peng, ``Proton Electric Charge Radius from Lepton Scattering,'' Universe **9**, no.4, 182 (2023), doi:10.3390/universe9040182, [arXiv:2302.13818 [nucl-ex]].

MUSE at the secondary πM1 beam line at PSI

Q^2 = 0.002 to 0.08 GeV^2

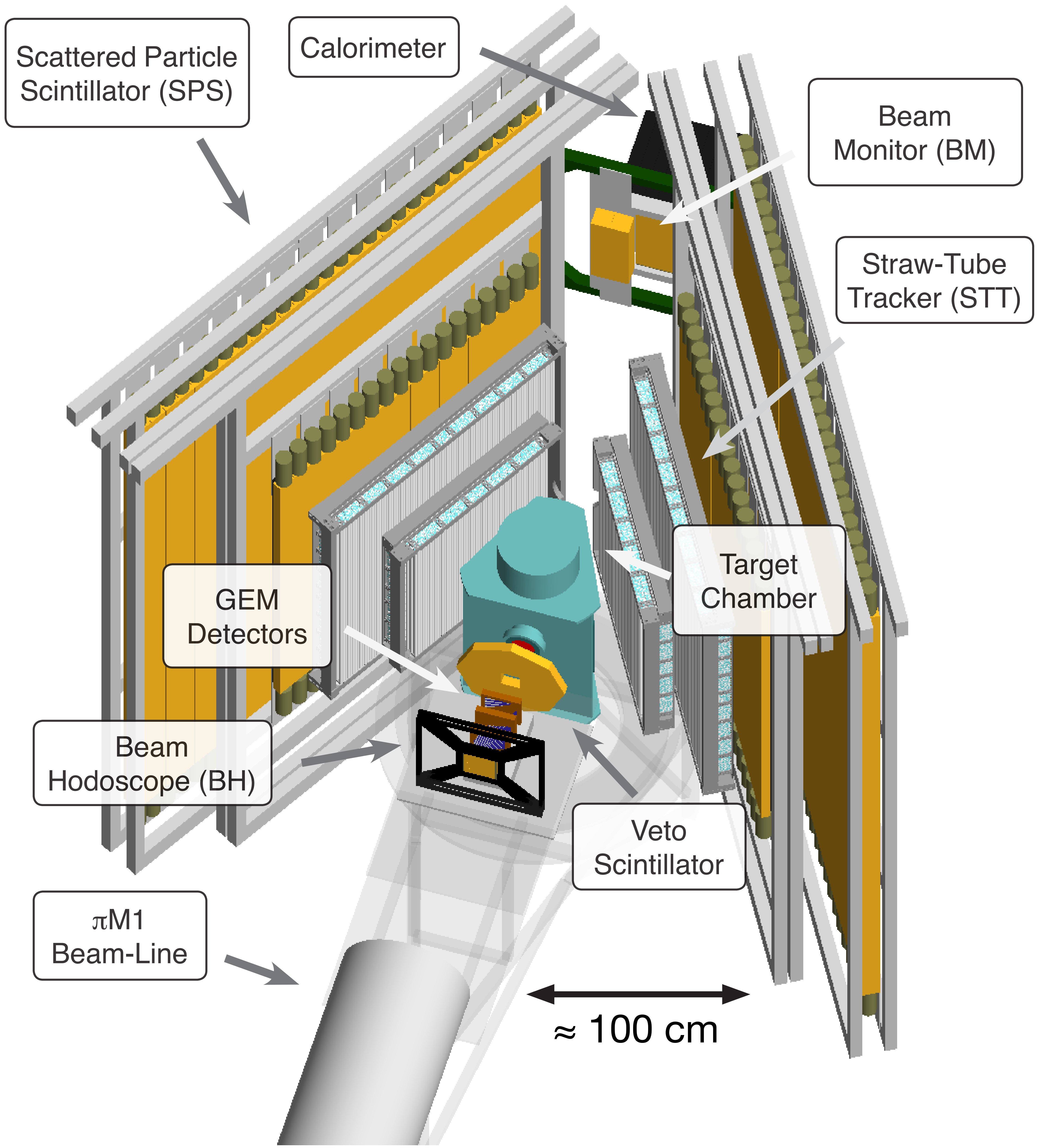
Beam

50 MHz RF (19.75 ns bunch separation)
e, μ, π beams with large emittance
Momenta: 115, 160, 210 MeV/c

Beam line detectors

Scattered particle detectors

The MUon Scattering Experiment at PSI (MUSE), MUSE Technical Design Report, arXiv:1709.09753 [physics.ins-det]
E. Cline et al. (MUSE Collaboration), Phys. Rev. C 105 (2022) 5, 055201.

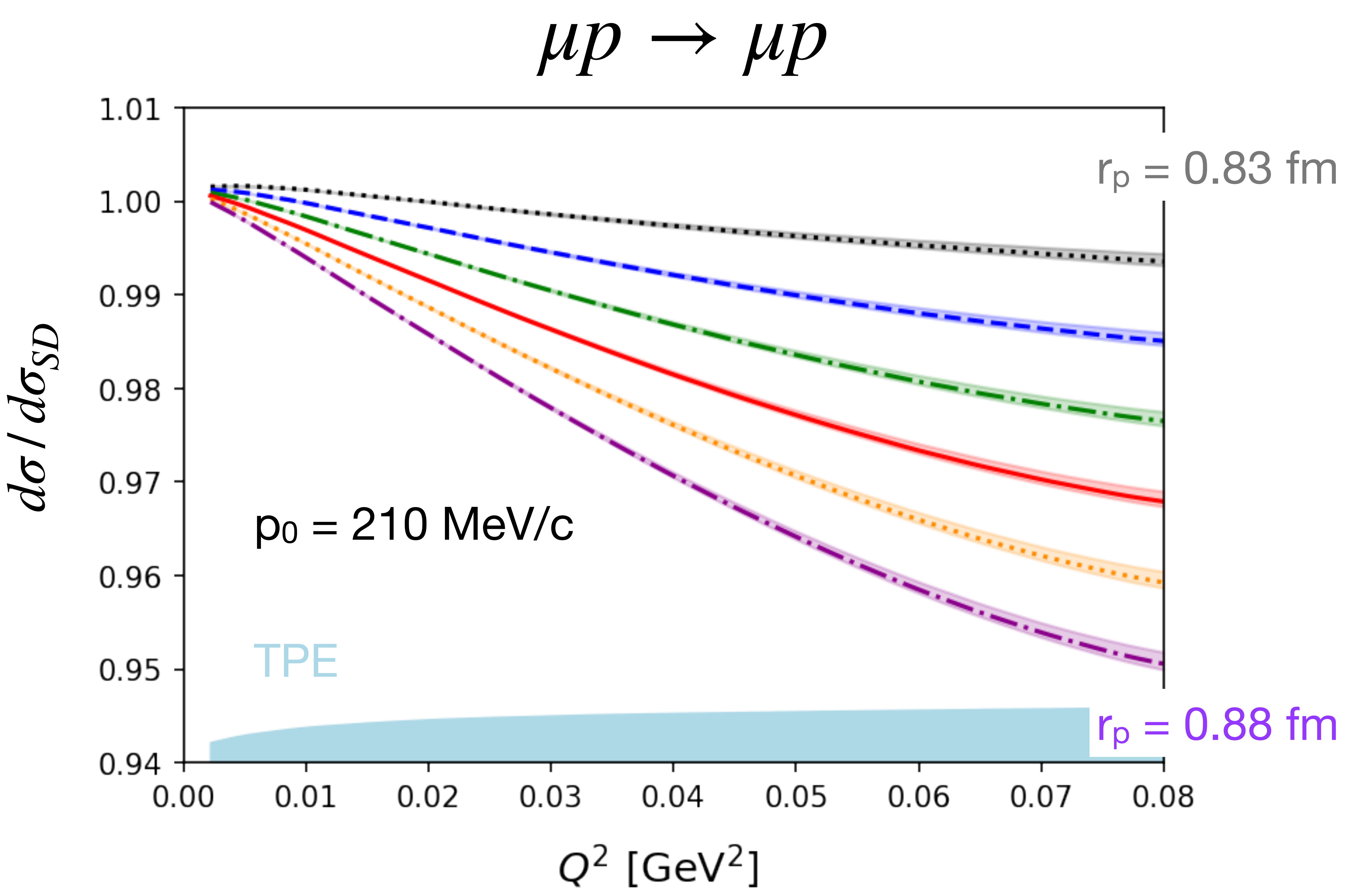


Predicted sensitivity of the μp cross section to the proton charge radius

A 0.01 fm change in radius corresponds to about 0.9 % change in cross section.

Radiative corrections are the largest contributor to systematic uncertainties in MUSE and depend on detector thresholds and ISR.

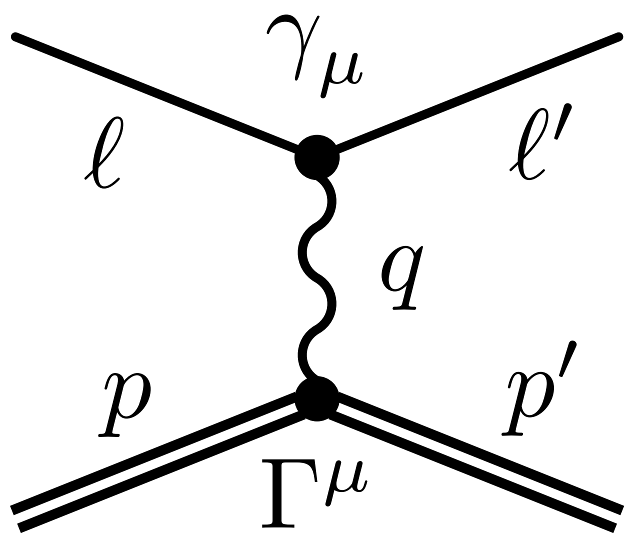
⇒ Controlling radiative corrections is essential for sub-percent precision.



Dispersively improved chiral effective field theory:
F. Gil-DomÍnguez, J.M. Alarcón and C. Weiss, Phys. Rev. D 108, no.7, 074026 (2023)

Experimental lepton-proton cross section $\ell^{\pm}p \to \ell'^{\pm}p'\gamma$

First Born approximation



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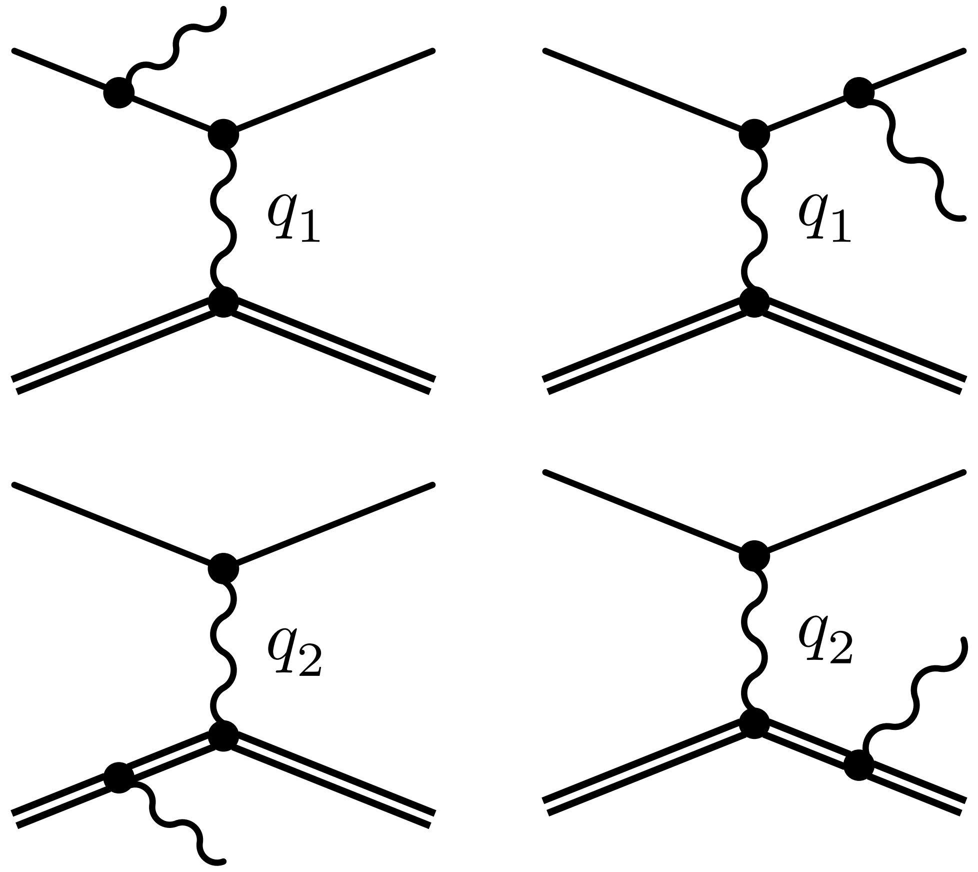
First-order bremsstrahlung processes



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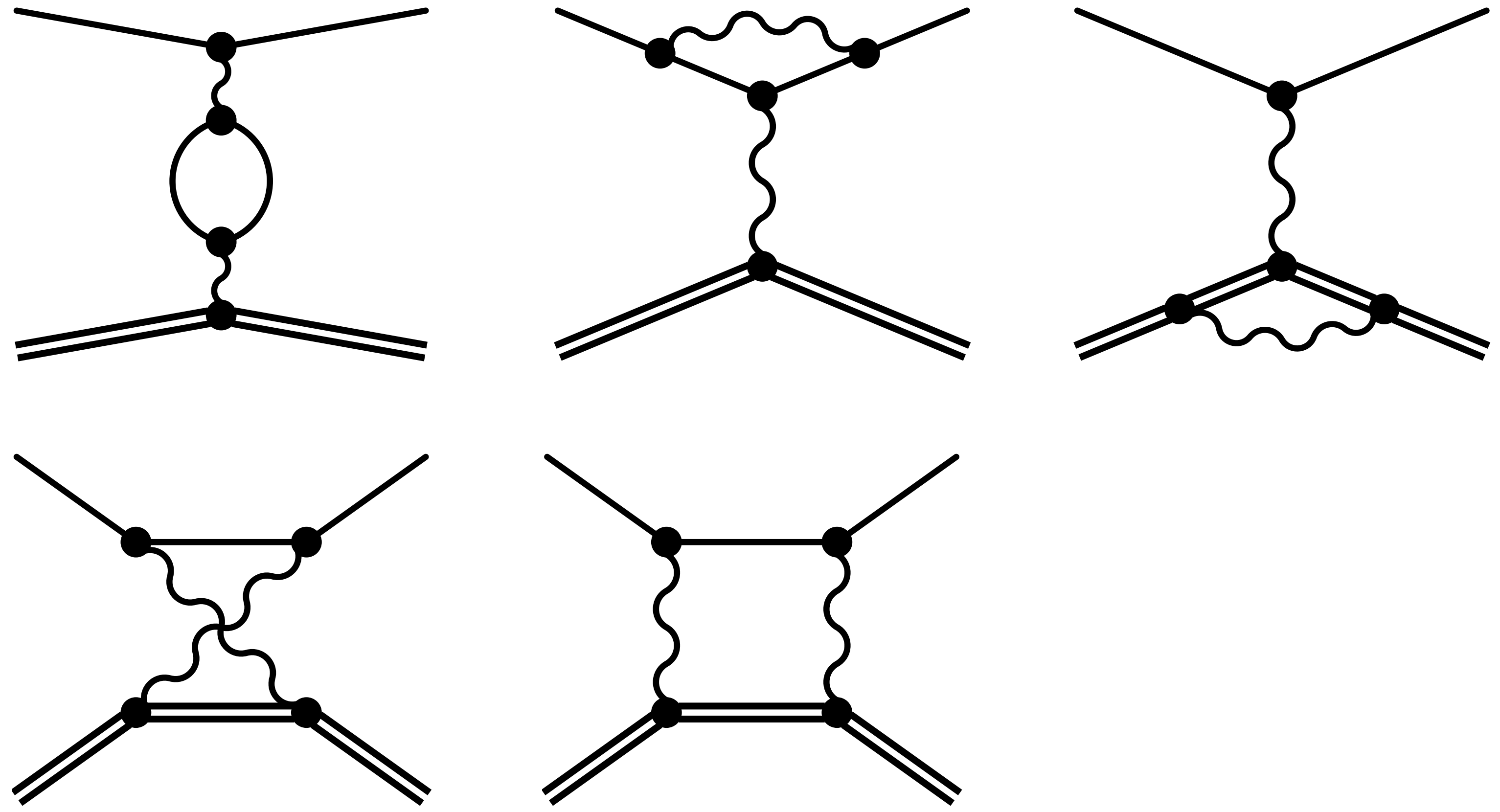
Virtual-photon corrections



+ …

Vacuum polarization
Lepton/proton vertex corrections
Two-photon exchange corrections

Born cross section:

- is needed to determine form factors and charge radius,
- can be obtained from σ^{exp} with radiative corrections.

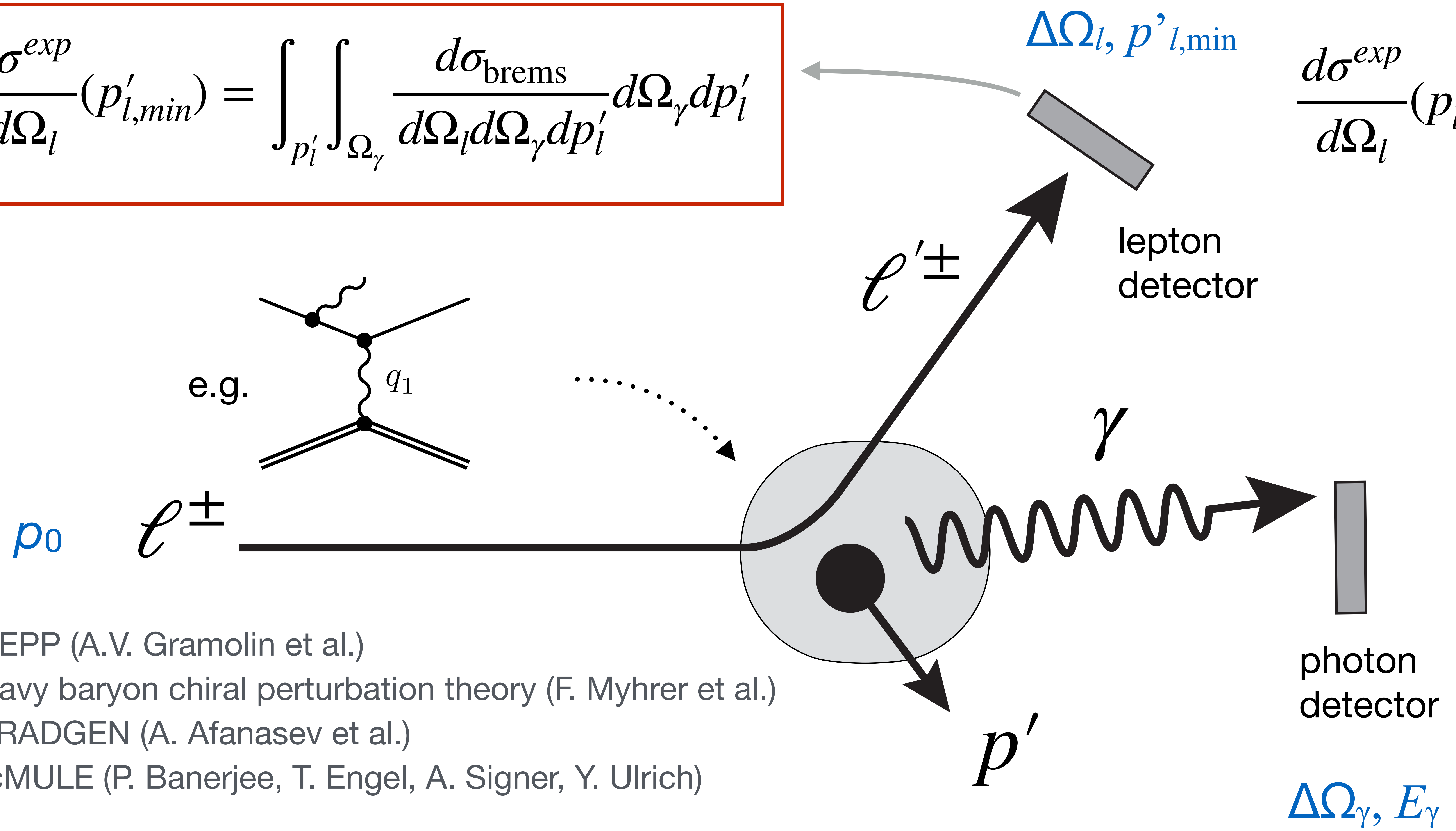
ESEPP includes emission of **hard radiated photon**, beyond soft-photon approximation and the **mass of lepton**.

A.V. Gramolin et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 115001 (28pp)

Experimental cross section depends on kinematic setting and acceptance

Experimental bremsstrahlung cross-section

$$\frac{d\sigma^{exp}}{d\Omega_l}(p'_{l,min}) = \int_{p'_l}\int_{\Omega_\gamma} \frac{d\sigma_{\text{brems}}}{d\Omega_l d\Omega_\gamma dp'_l} d\Omega_\gamma dp'_l$$



Born cross section

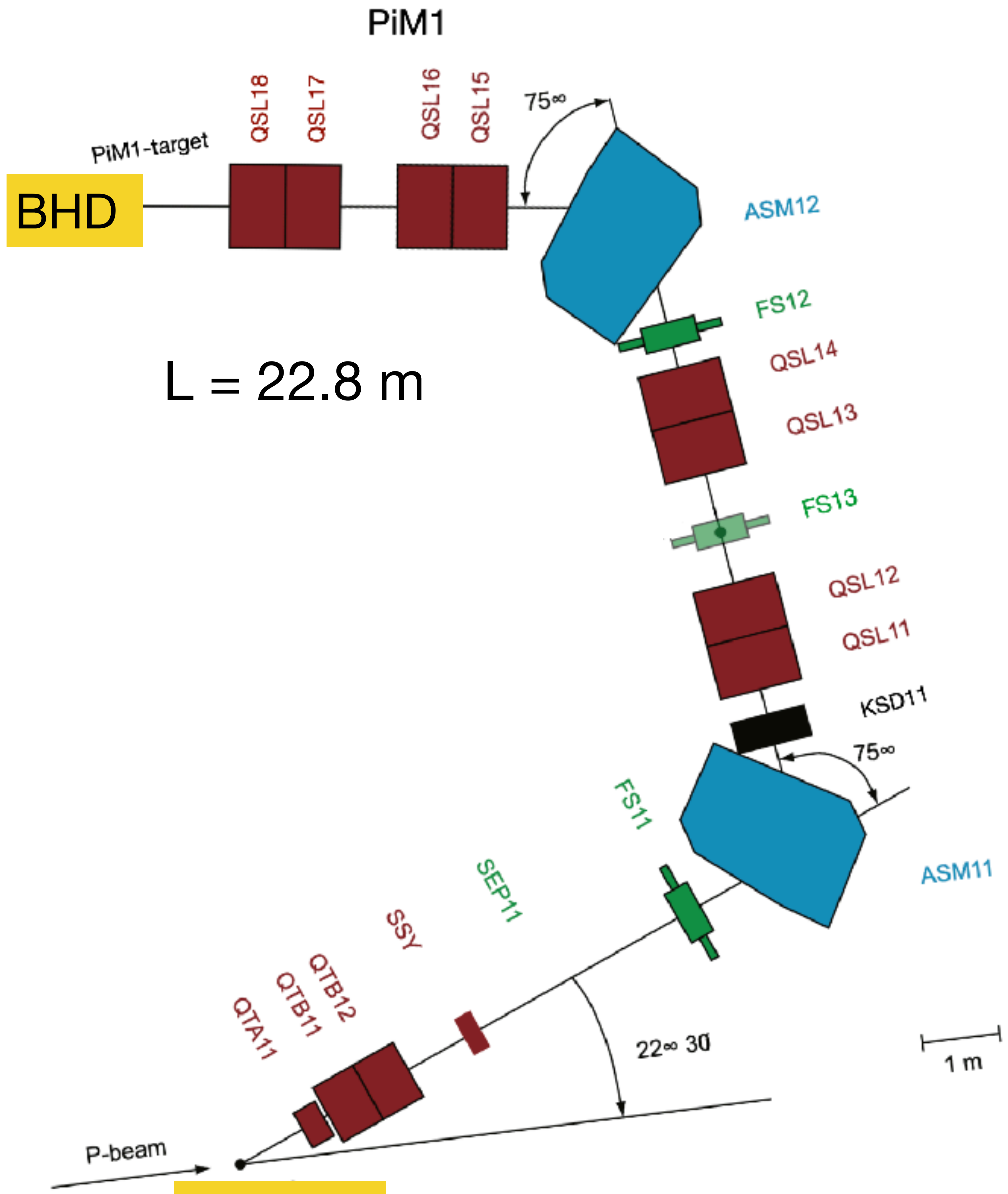
$$\frac{d\sigma^{exp}}{d\Omega_l}(p'_{l,min}) = \frac{d\sigma_0}{d\Omega_l}\left[1 + \delta(p'_{l,min})\right]$$

Calculated radiative corrections

$$\delta = \frac{d\sigma^{MC}}{d\Omega_l} \Big/ \frac{d\sigma_0}{d\Omega_l} - 1$$

ESEPP (A.V. Gramolin et al.)
Heavy baryon chiral perturbation theory (F. Myhrer et al.)
ELRADGEN (A. Afanasev et al.)
McMULE (P. Banerjee, T. Engel, A. Signer, Y. Ulrich)
…

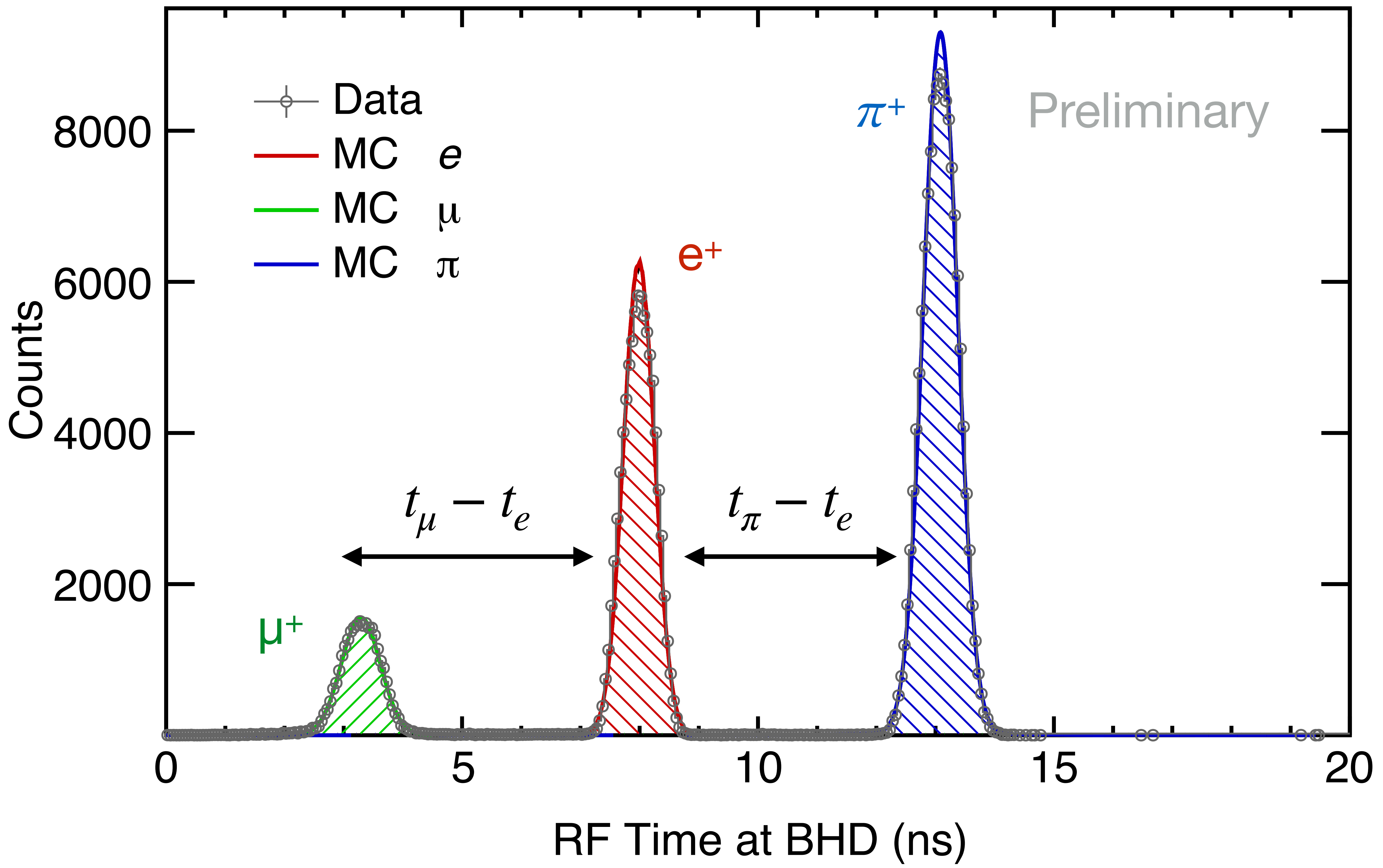
Beam-particle identification and beam-momentum determinations from RF time differences $t_\mu - t_e$ and $t_\pi - t_e$



L = 22.8 m

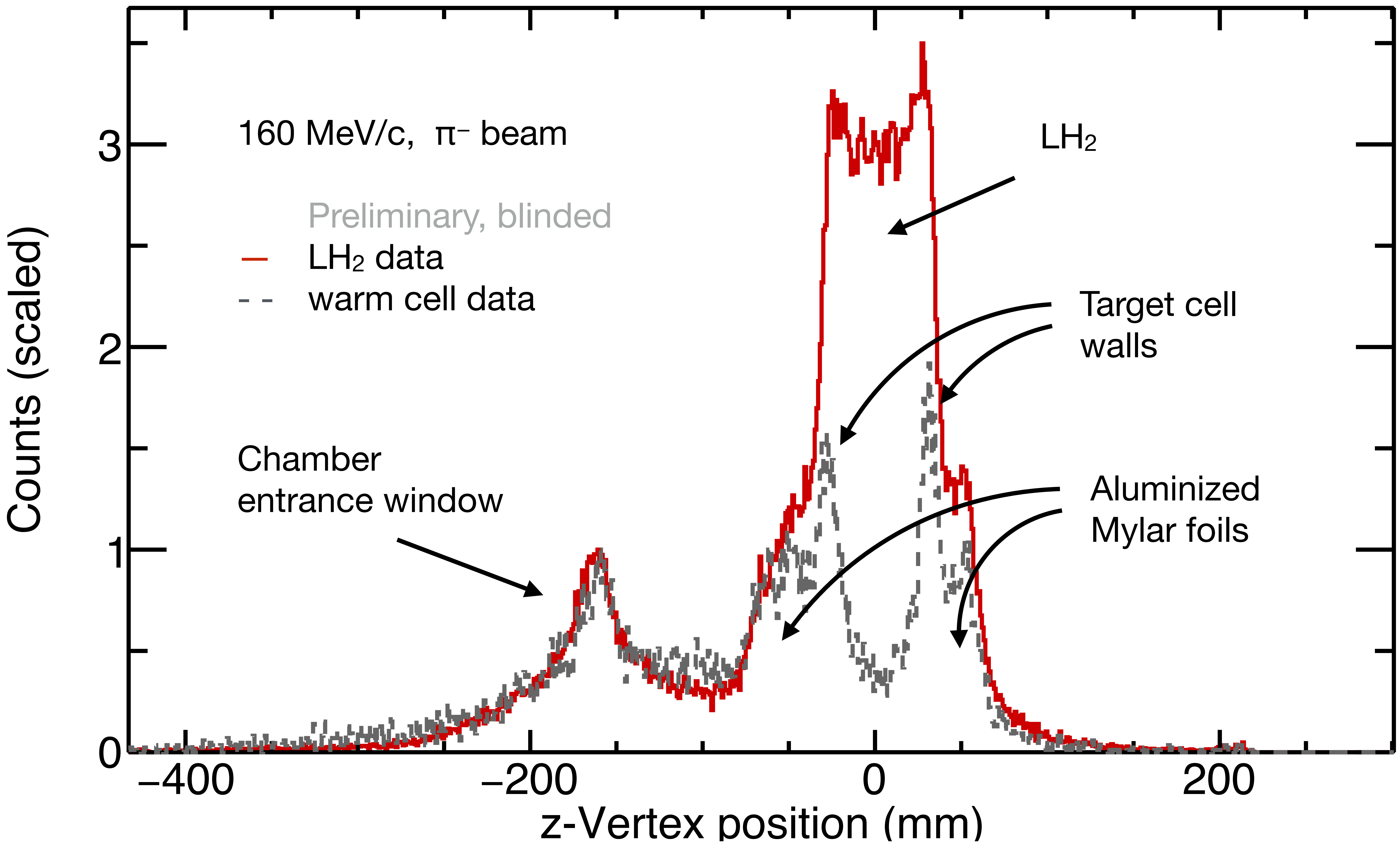
(production target)

Comparing simulation and data gives *relative* shifts to the electron reference: Δt_x

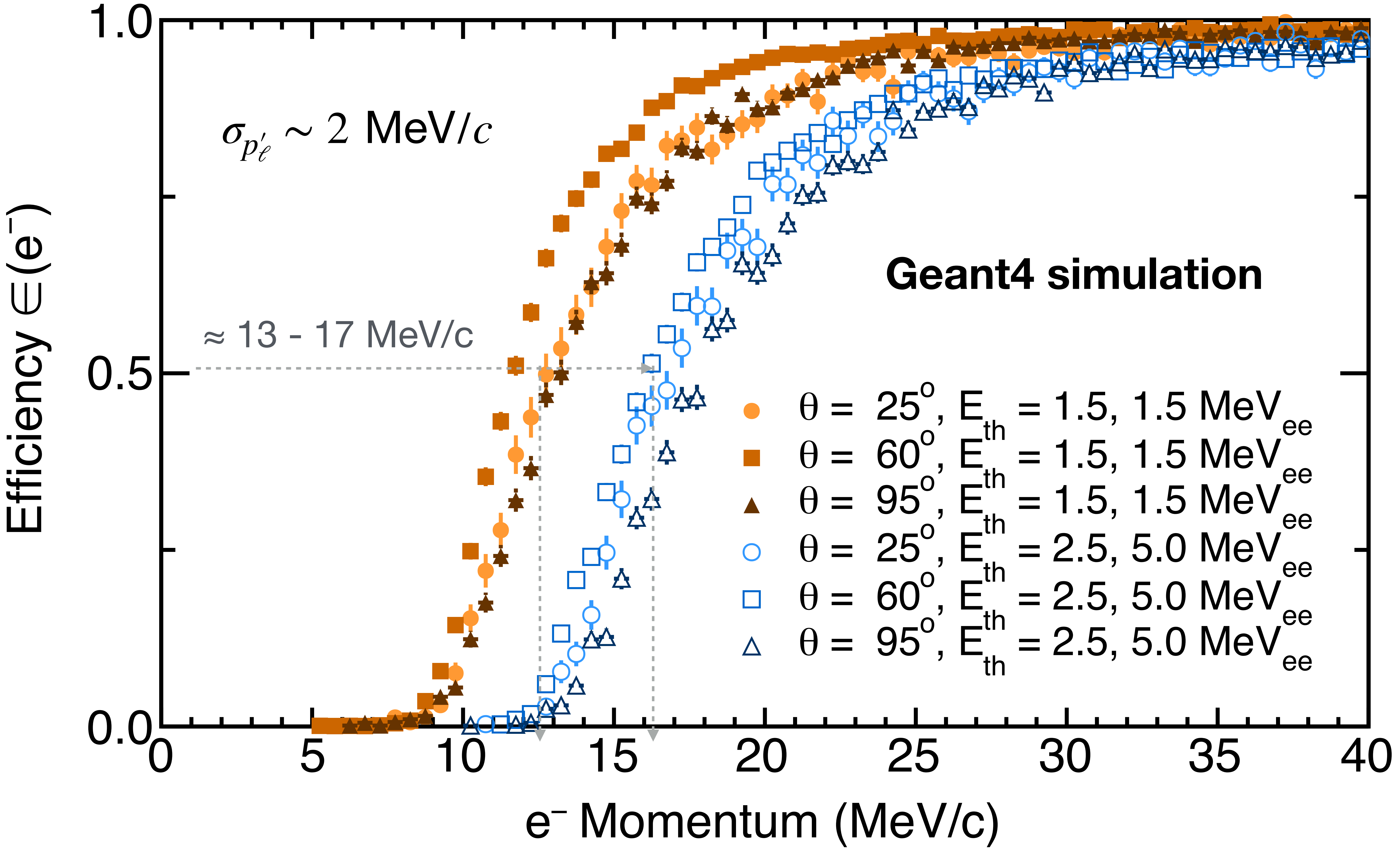


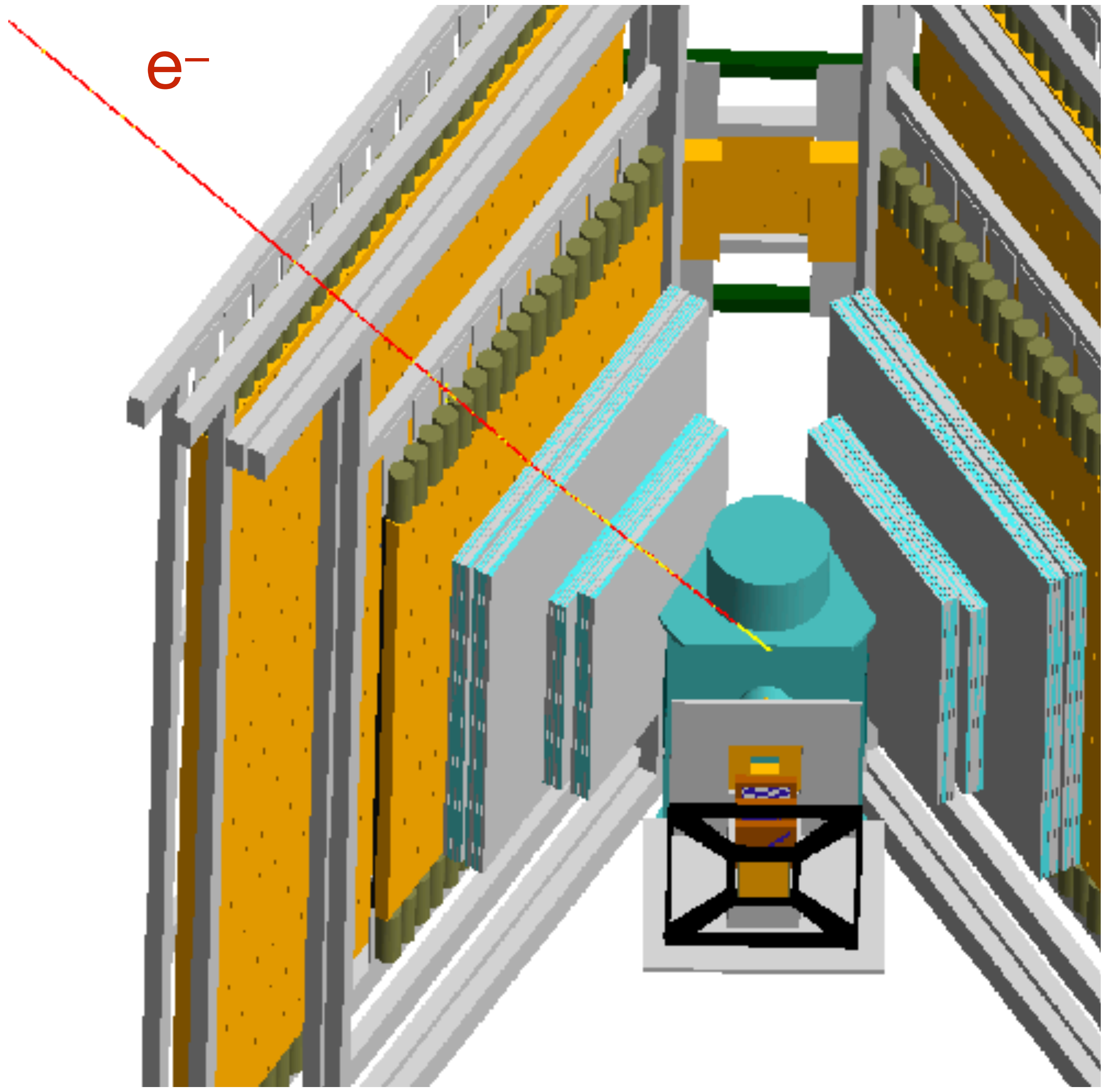
p = (160.1 ± 0.1) MeV/c

Reconstructed z-vertex from incident- (GEM) and scattered- (STT) particle tracks



Scattered lepton-momentum threshold is primarily determined by the SPS detector

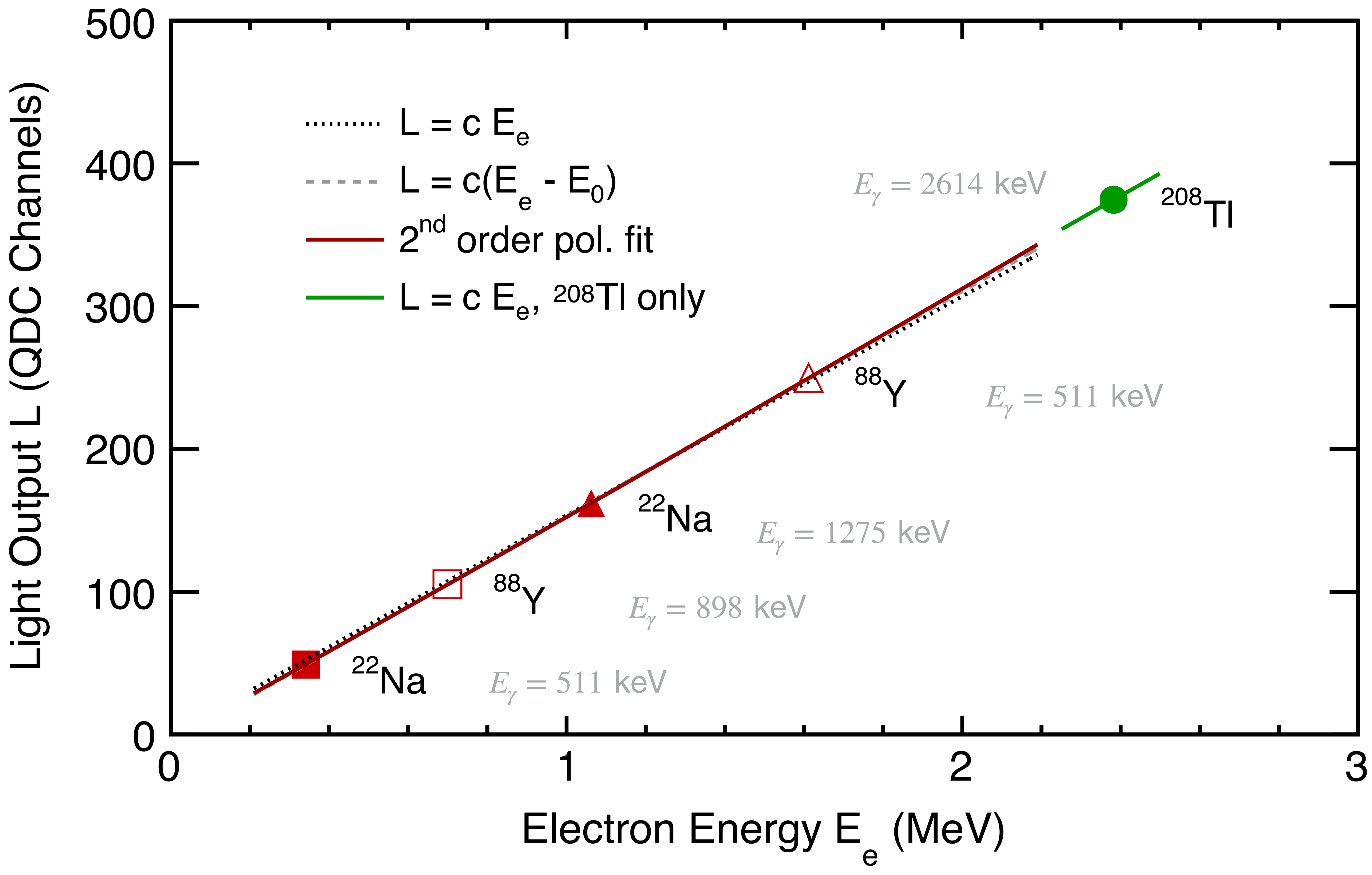


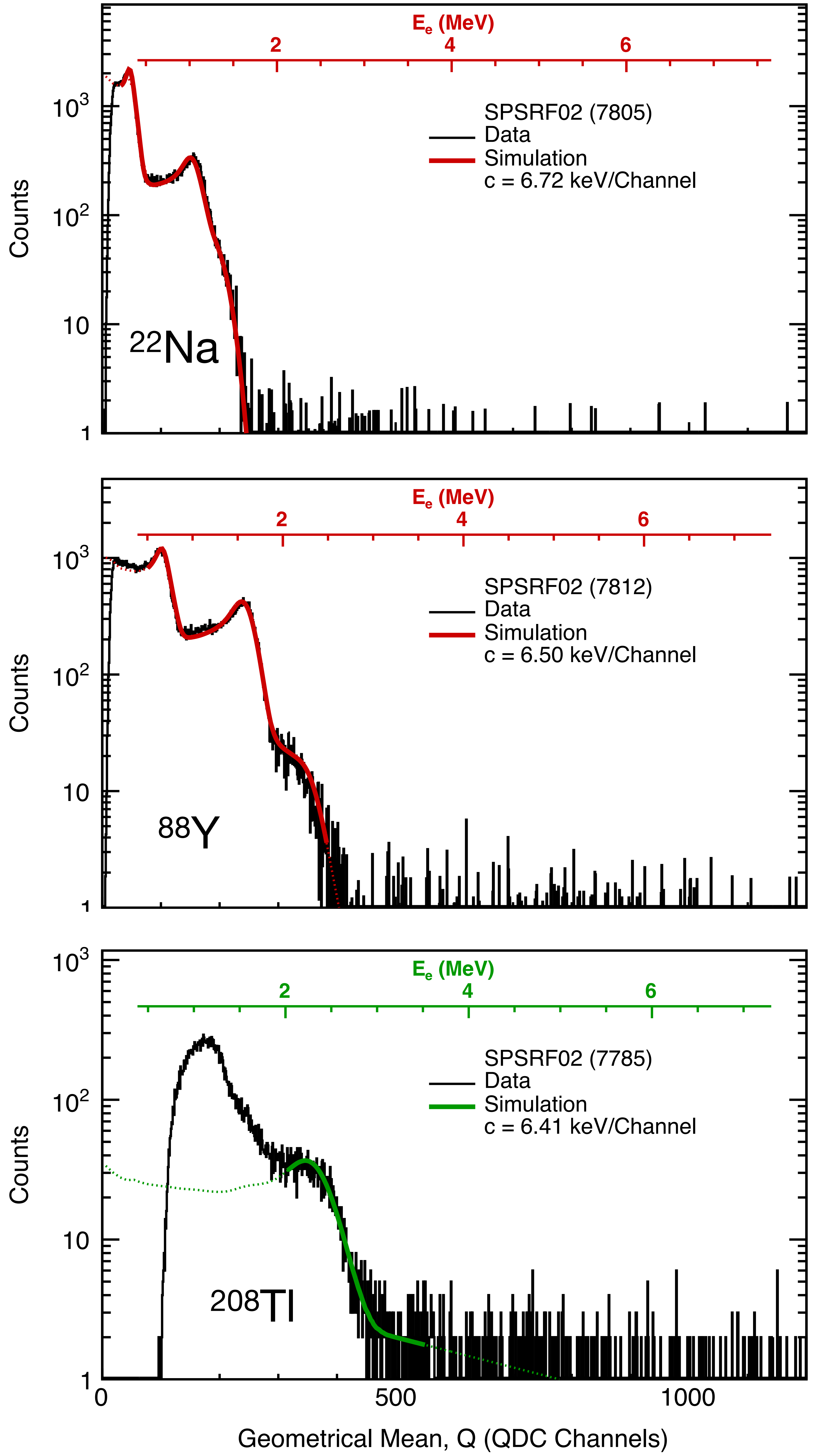


- Function of the SPS discriminator **thresholds** in the front and rear walls
- Function of the lepton **scattering angle**

L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

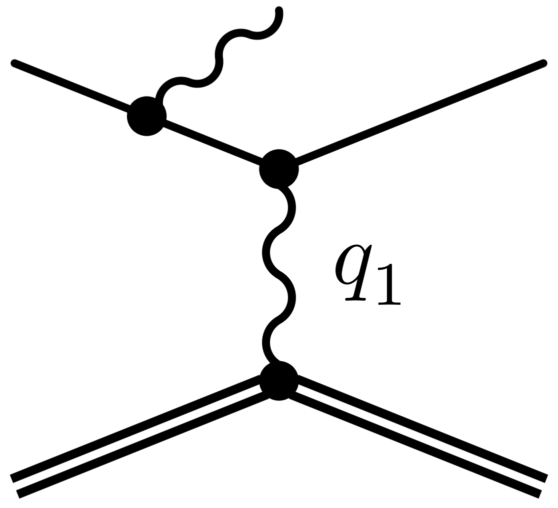
High-precision calibration of the SPS detector essential for determining the detector threshold



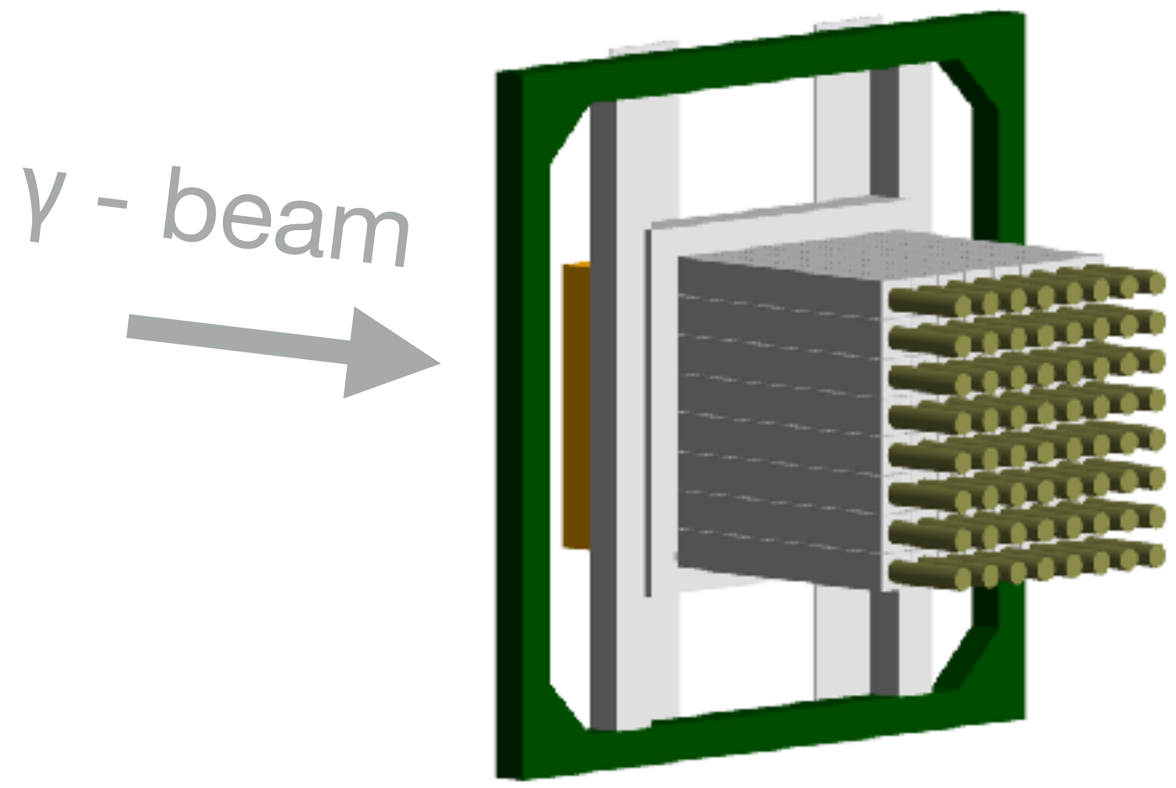


Calibrations with room background ^{208}Tl agree with source-data (^{22}Na and ^{88}Y) results to better than 1% at the anticipated SPS threshold of 2 MeV.

Radiative effects in *ep* and *μp* scattering in MUSE kinematics

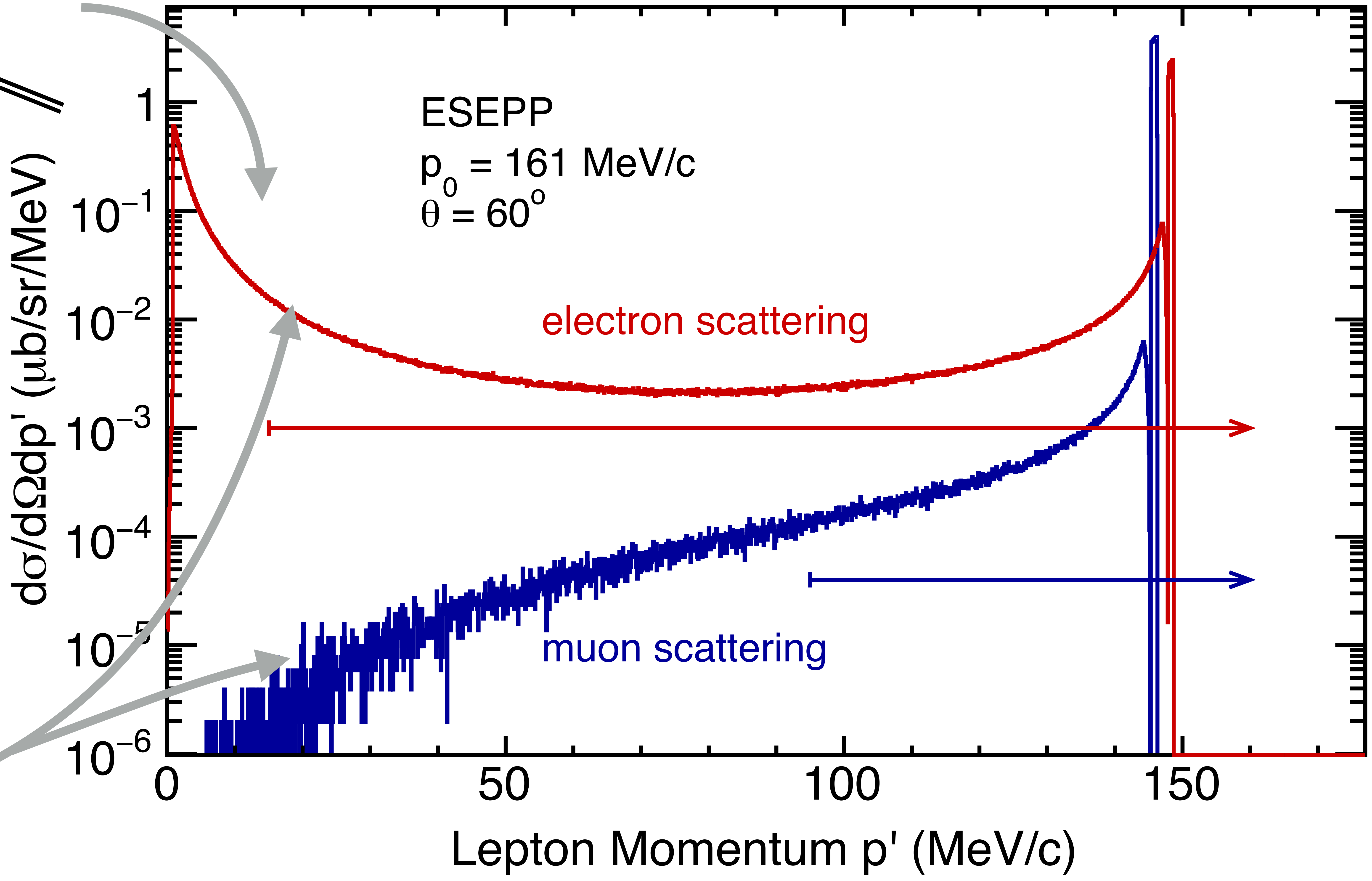


Initial-state radiation detectable by the downstream calorimeter



64 lead-glass crystals (4 cm x 4 cm x 30 cm)

MUSE will integrate over a large momentum range.

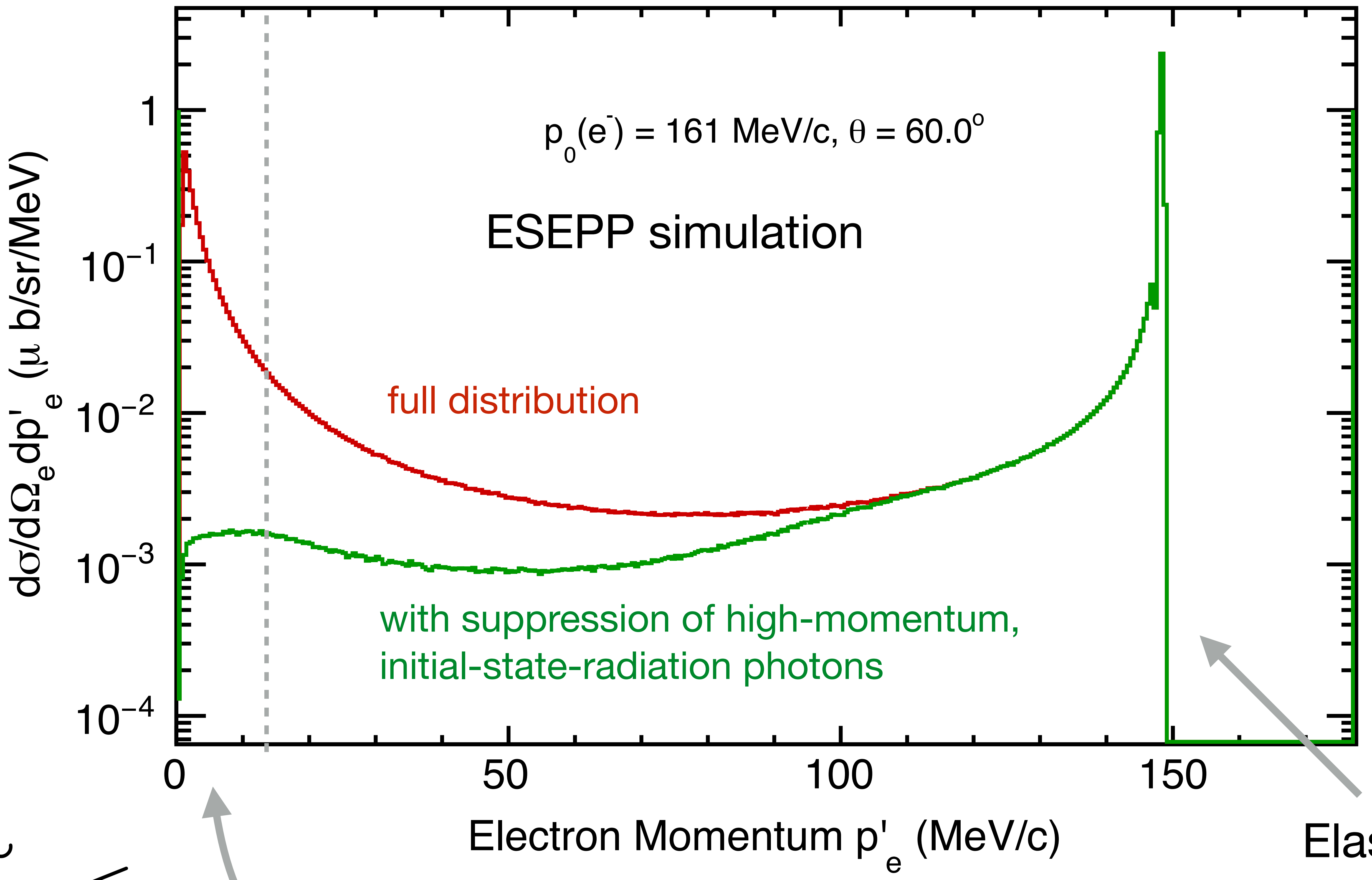


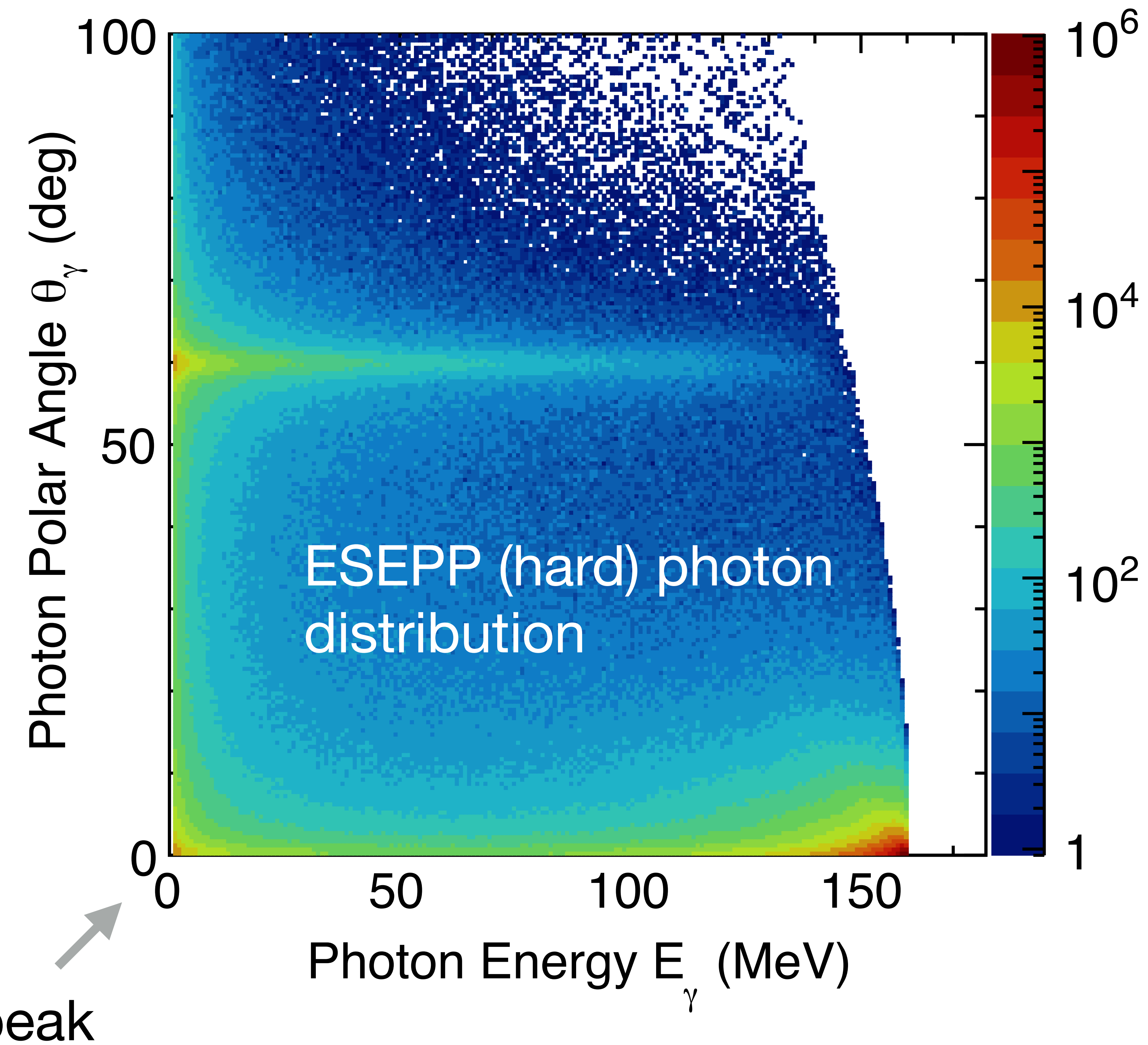
Very different radiative effects in ep and μp scattering

L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

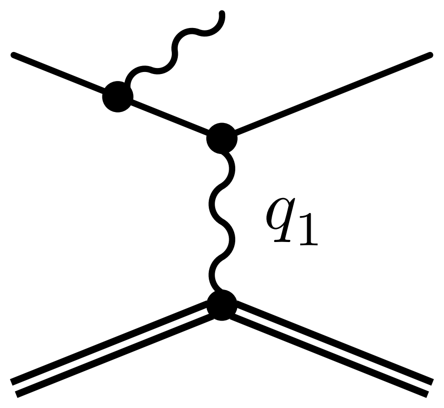
$ep \rightarrow e'p\gamma$ photon distribution

MUSE will integrate over a large momentum range





Elastic peak



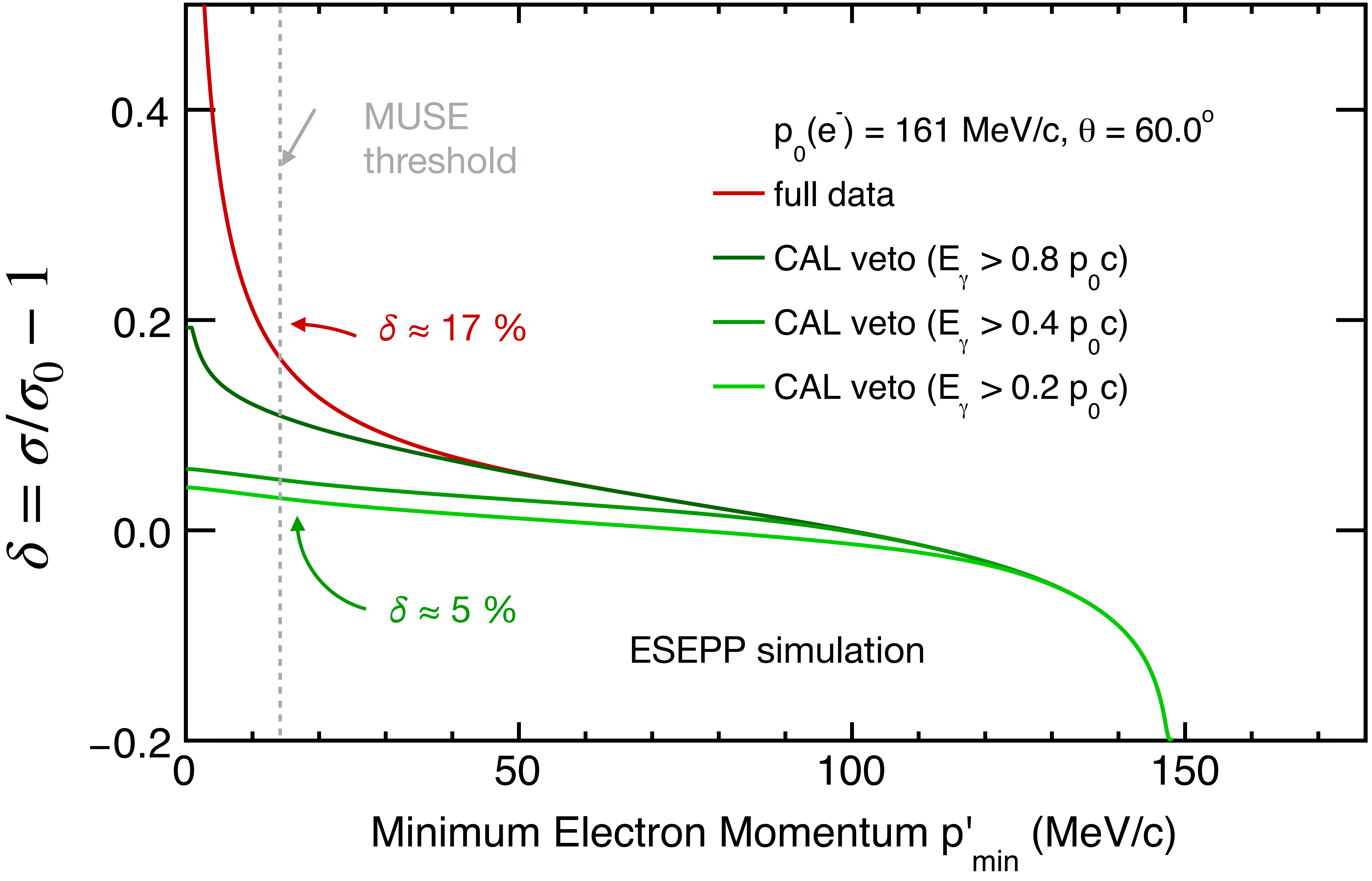
If the incident lepton loses energy due to the emission of a hard photon, then the probability for this lepton to be scattered by the proton increases.

The downstream photon calorimeter will veto events with hard initial-state radiation

Rapidly changing radiative corrections for small p'_{min}.

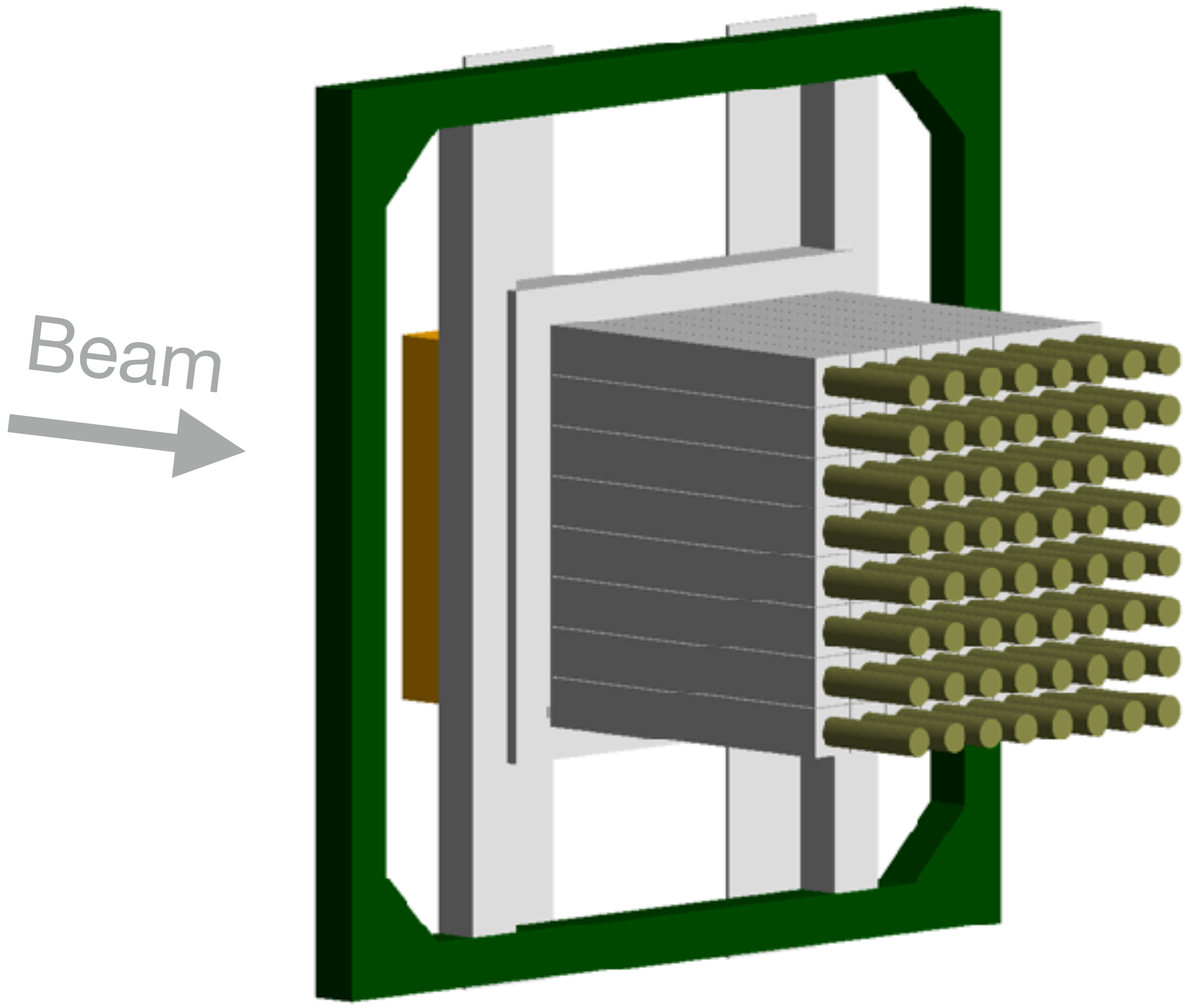
(> 1% change / MeV/c)

CAL veto on initial-state radiation reduces radiative corrections and p'_{min} dependence, reducing uncertainty.



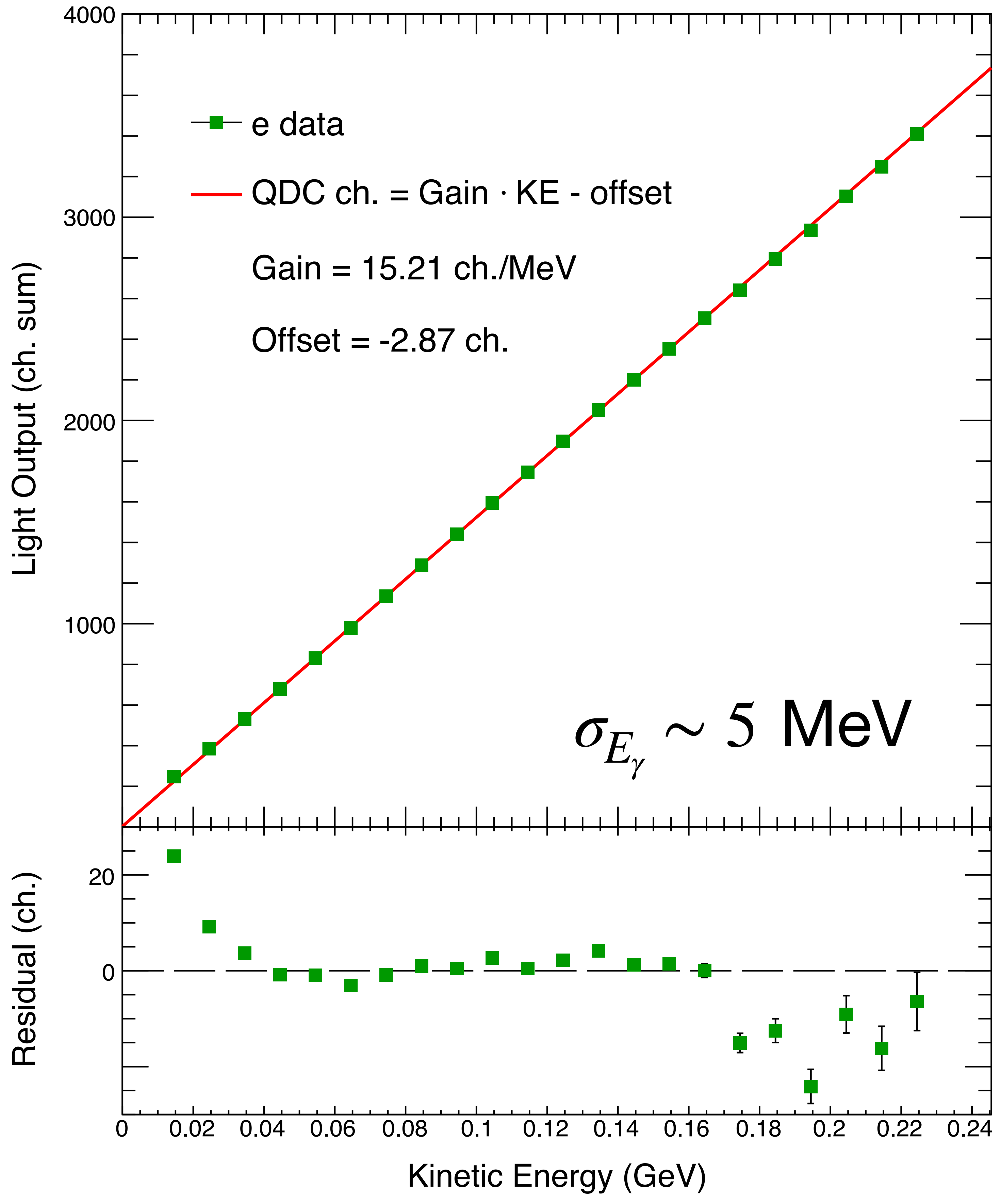
L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

Photon calorimeter will detect initial-state radiation and is calibrated with electron-beam data

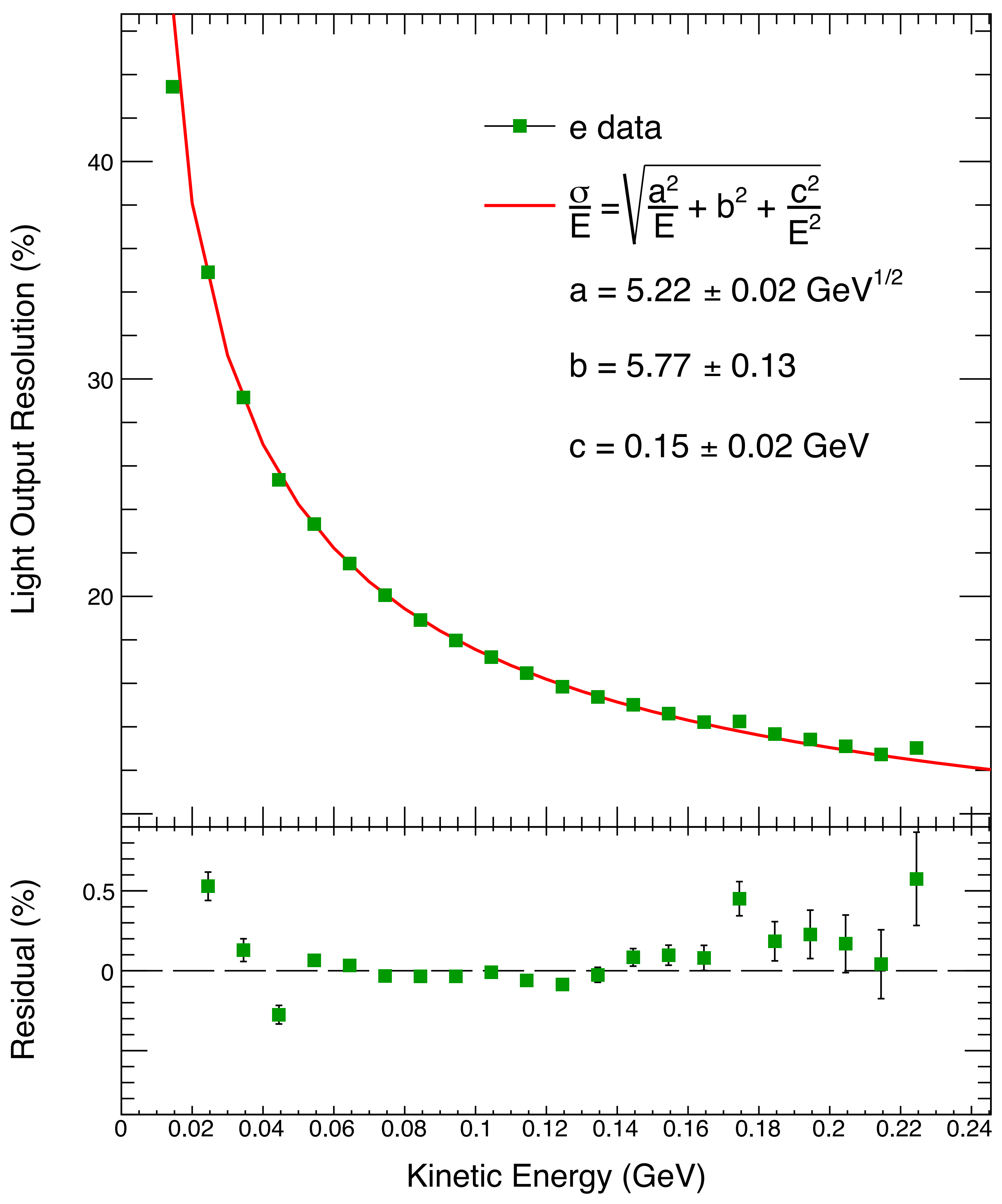


64 lead-glass crystals
(4 cm x 4 cm x 30 cm)

Light output

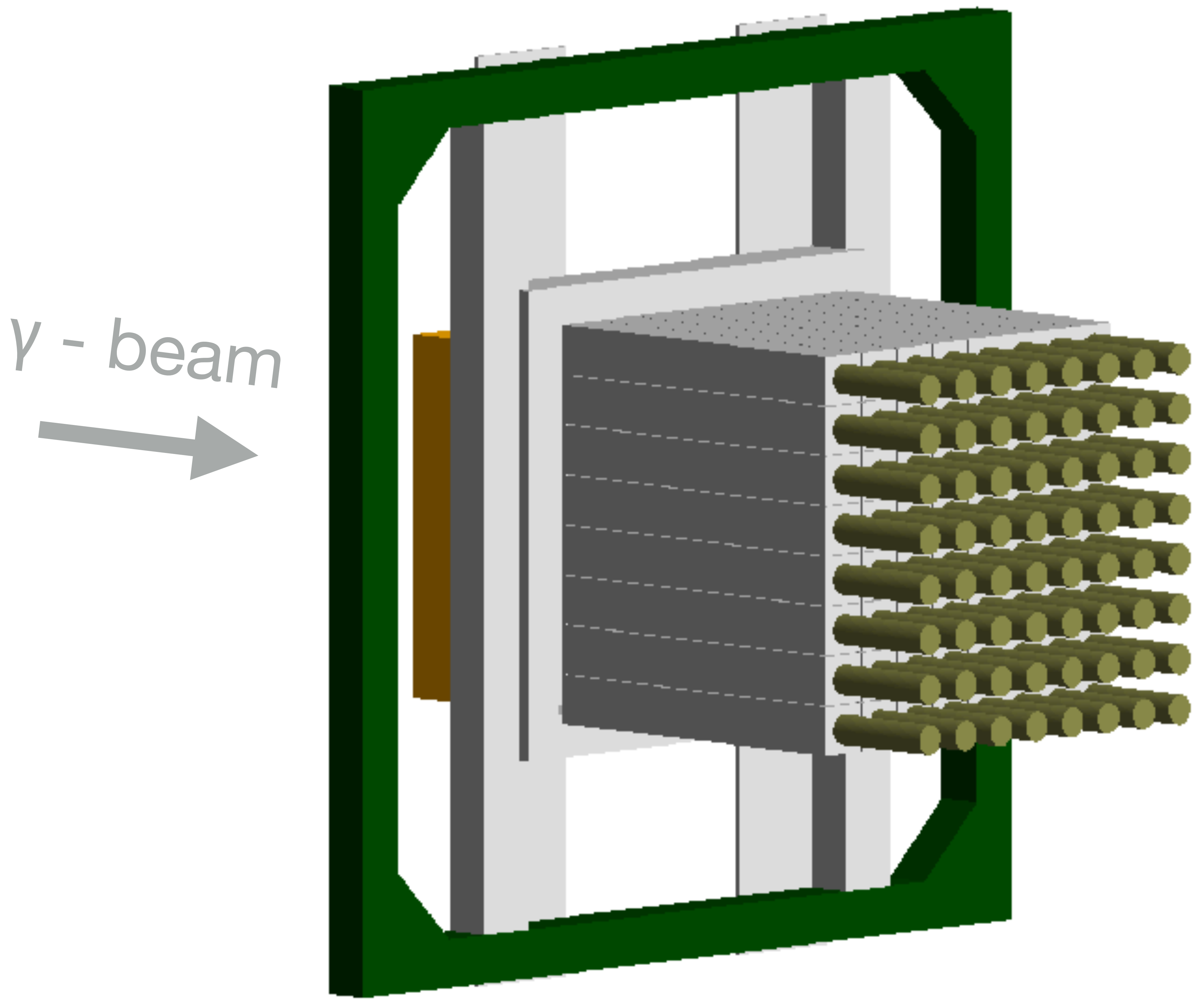


Light output resolution



W. Lin et al. (MUSE Collaboration), NIM A 1080, 170754 (2025)

The downstream photon calorimeter will veto events with hard initial-state radiation



64 lead-glass crystals
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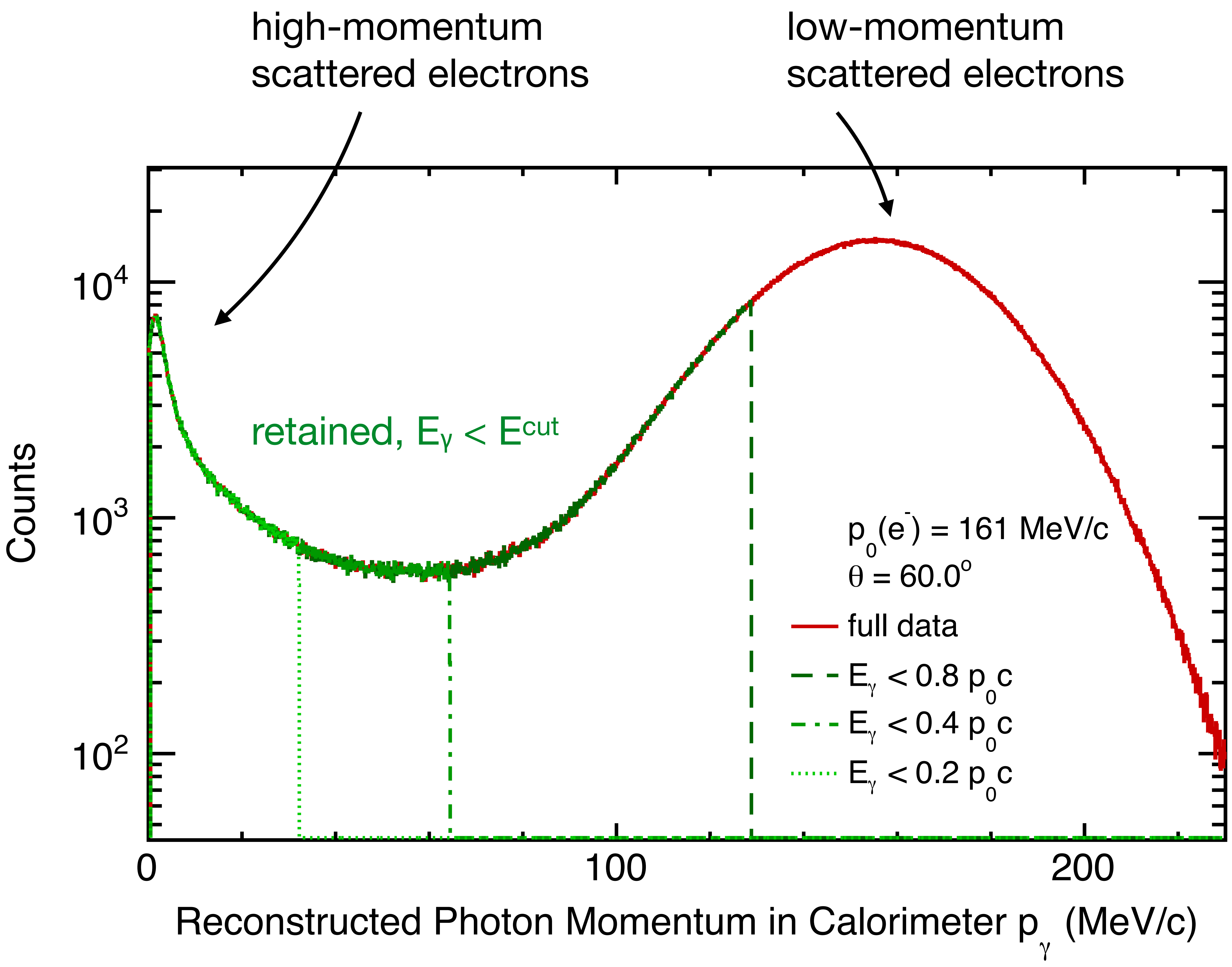
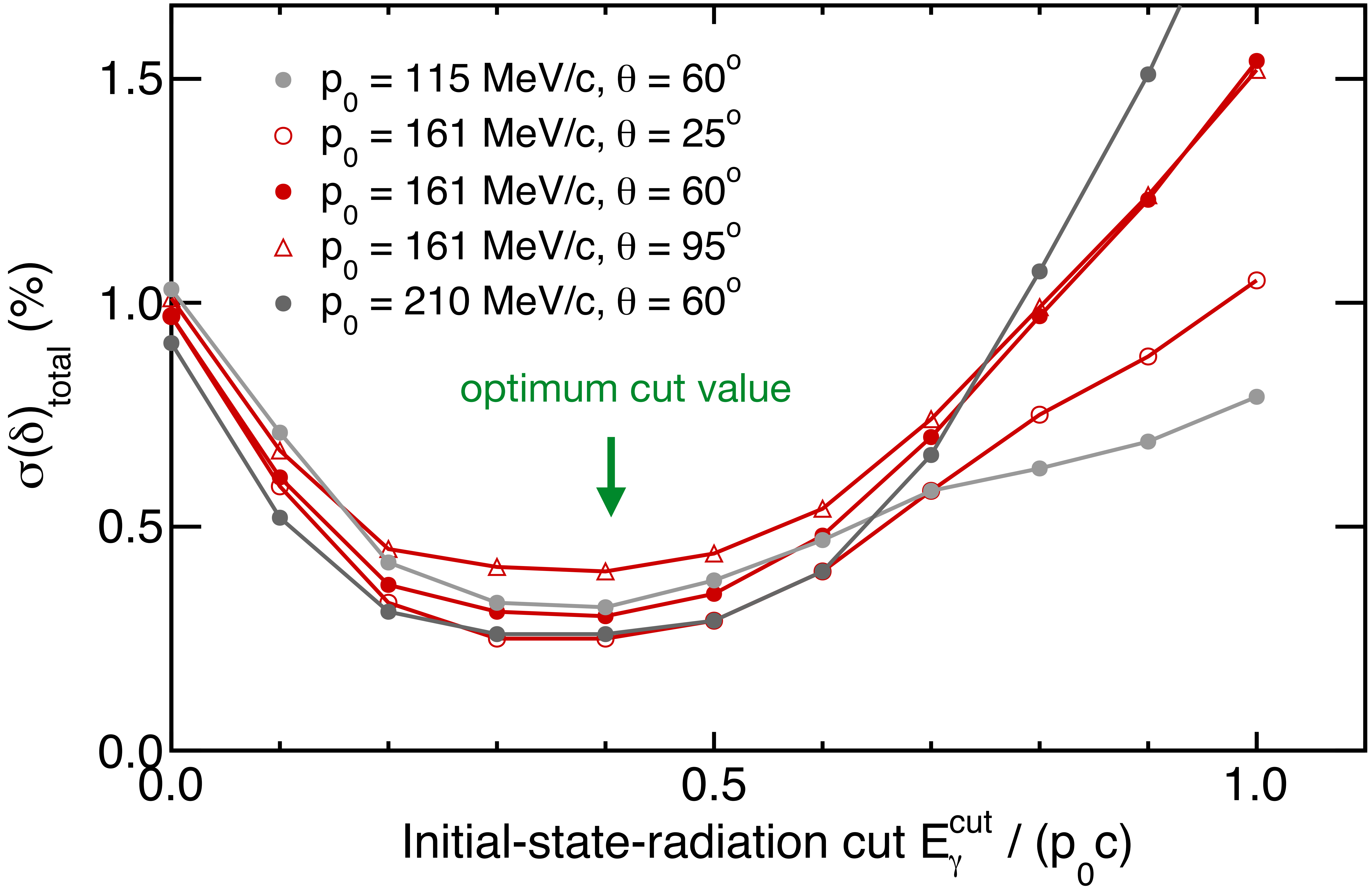


L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

There is an optimal cut balancing photon resolution and lepton momentum resolution

uncertainty in the radiative corrections

uncertainty in the **photon-momentum** determination dominates



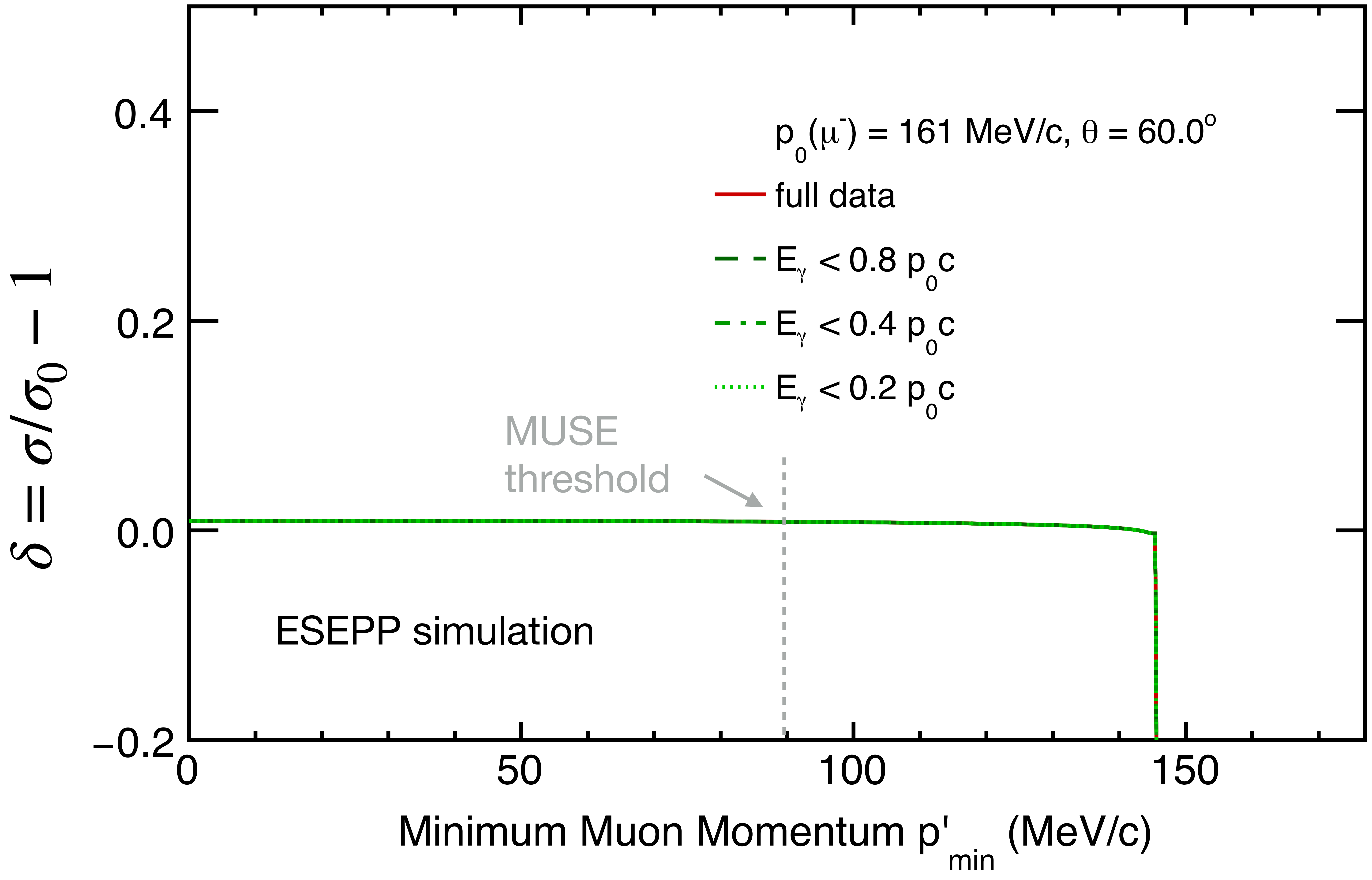
uncertainty in the minimum **electron-momentum** determination dominates

L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

Radiative corrections for $\mu p \rightarrow \mu' p \gamma$ are small and well determined

Compared with electron scattering, the radiative corrections for muon scattering are:

- small, $\delta \leq 1$ %,
- (almost) independent of the muon detection threshold,
- independent of any photon veto.



L. Li et al., Eur. Phys. J. A 60:8 (2024)

Instrumental uncertainties in the radiative corrections of the elastic scattering cross sections

- The preliminary estimates of the total instrumental uncertainties in the radiative corrections for **electrons** are 0.2% - 0.5%.*

$\sigma_\delta(e^-)$	115 MeV/c			161 MeV/c			210 MeV/c		
	20°	60°	100°	20°	60°	100°	20°	60°	100°
p'_{min}	0.05%	0.18%	0.30%	0.03%	0.16%	0.31%	0.02%	0.13%	0.31%
θ	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.03%	0.01%
p_0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
E_γ	0.32%	0.33%	0.33%	0.25%	0.25%	0.26%	0.20%	0.22%	0.22%
Total	**0.32%**	**0.38%**	**0.45%**	**0.25%**	**0.30%**	**0.40%**	**0.20%**	**0.26%**	**0.38%**

angle-dependent uncertainty, relevant for radius extraction, $\lesssim$ 0.3%

angle-independent uncertainty, **not** relevant for radius extraction

- The preliminary estimates of the total instrumental uncertainties in the radiative corrections for **muons** are smaller than 0.01%.*

L. Li et al. (MUSE Collaboration), Eur. Phys. J. A 60:8 (2024)

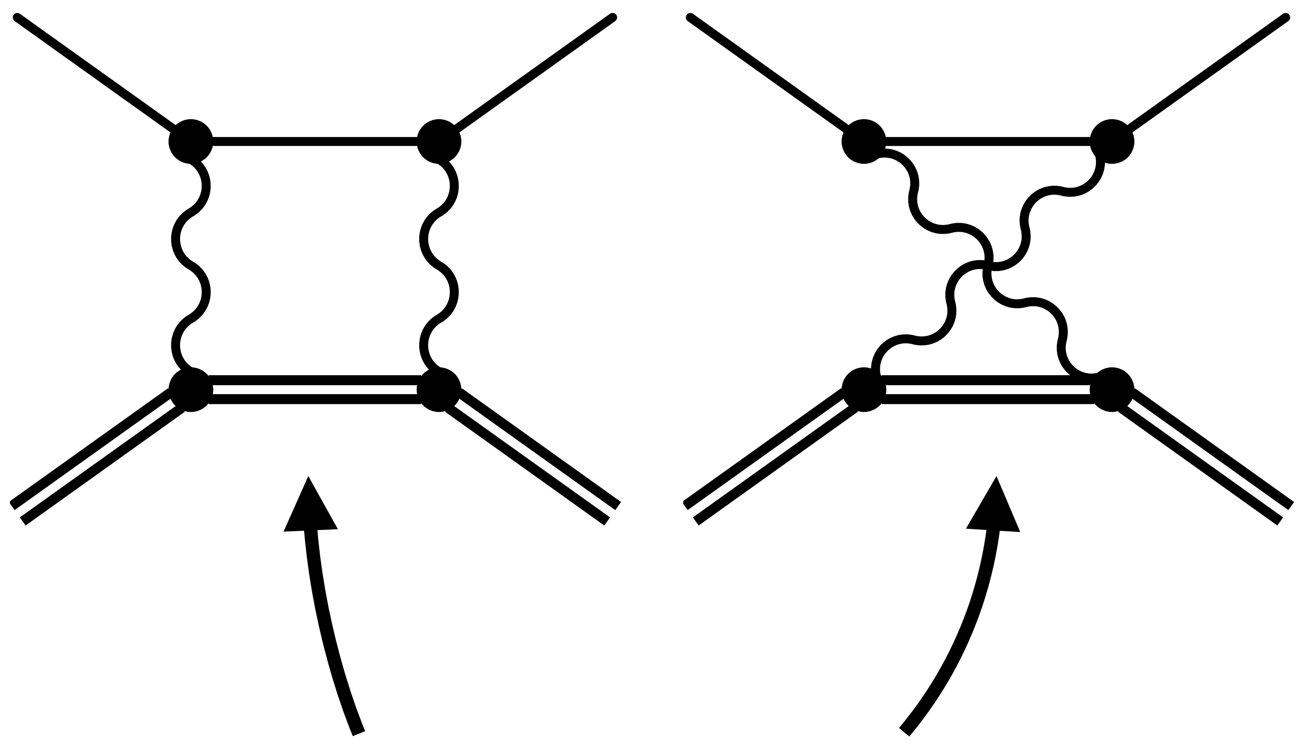
* Not including model uncertainties.

In the ℓ+/ℓ- cross-section ratio, many charge-even effects cancel in the ratio, isolating the TPE contribution

TPE correction $\delta_{2\gamma}$ at leading order

$$\sigma^{\pm} = \sigma_{1\gamma}(1 \pm \delta_{2\gamma})$$

$$\frac{\sigma^{+}}{\sigma^{-}} \approx 1 + 2\delta_{2\gamma}$$

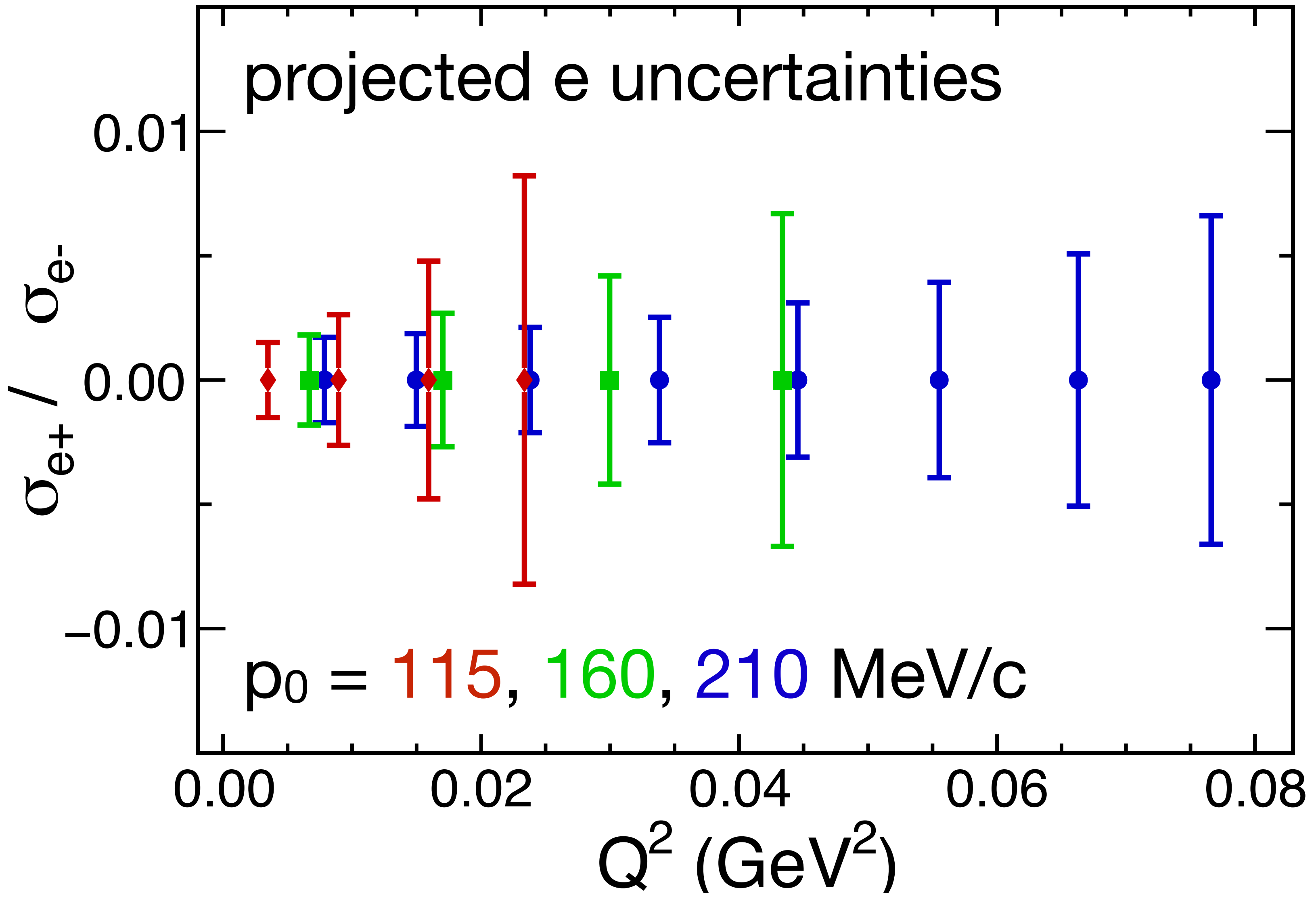


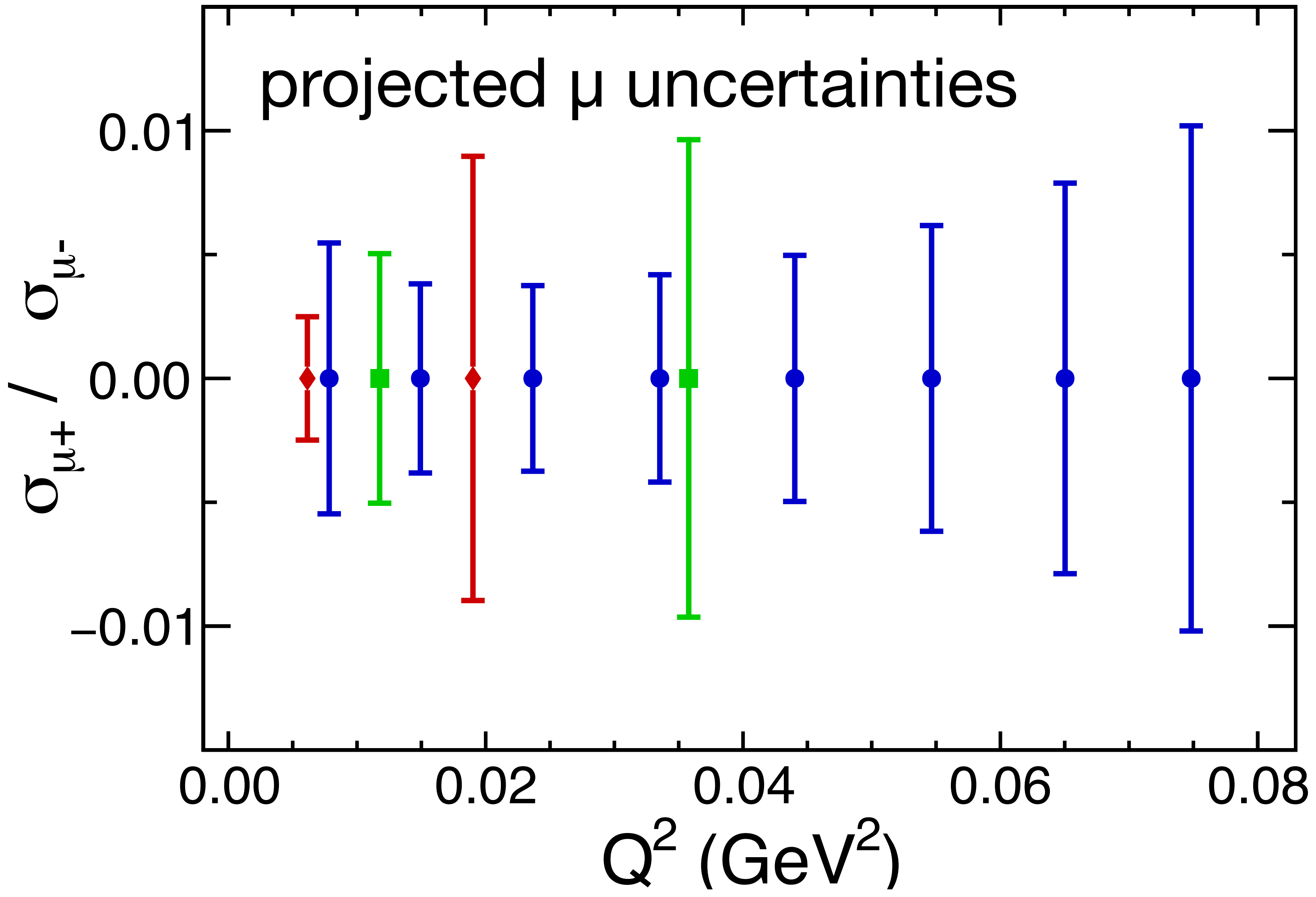
Polarizability effects ($\propto m_{\ell}^{4}$) are part of the inelastic TPE contribution

MUSE TDR, arXiv:1709.09753 [physics.ins-det].

Projected **statistical** uncertainties are < 1%.

Many **systematic** contributions cancel in the cross section ratio, and the uncertainties are small 0.1% — need to be well controlled.

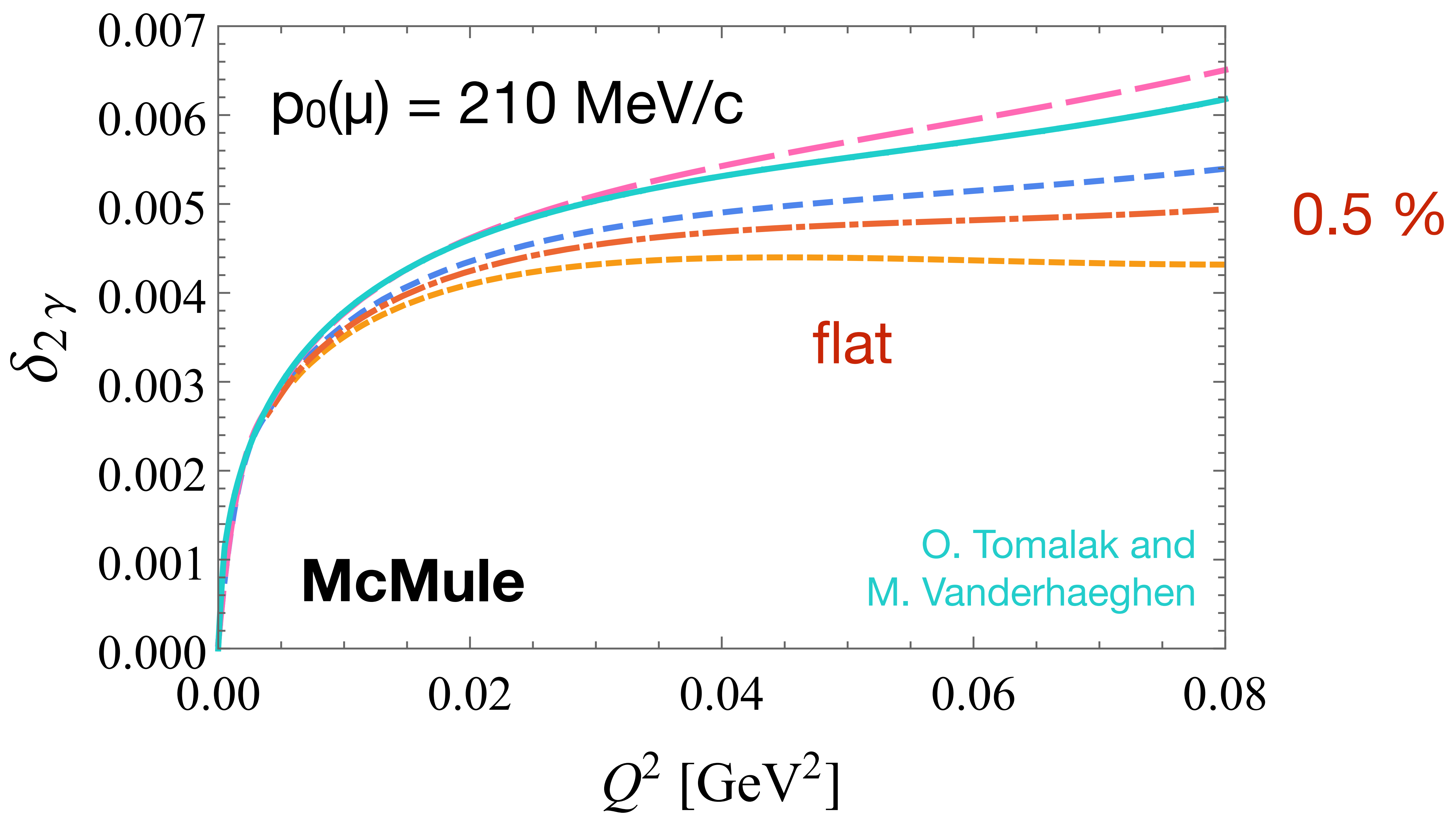


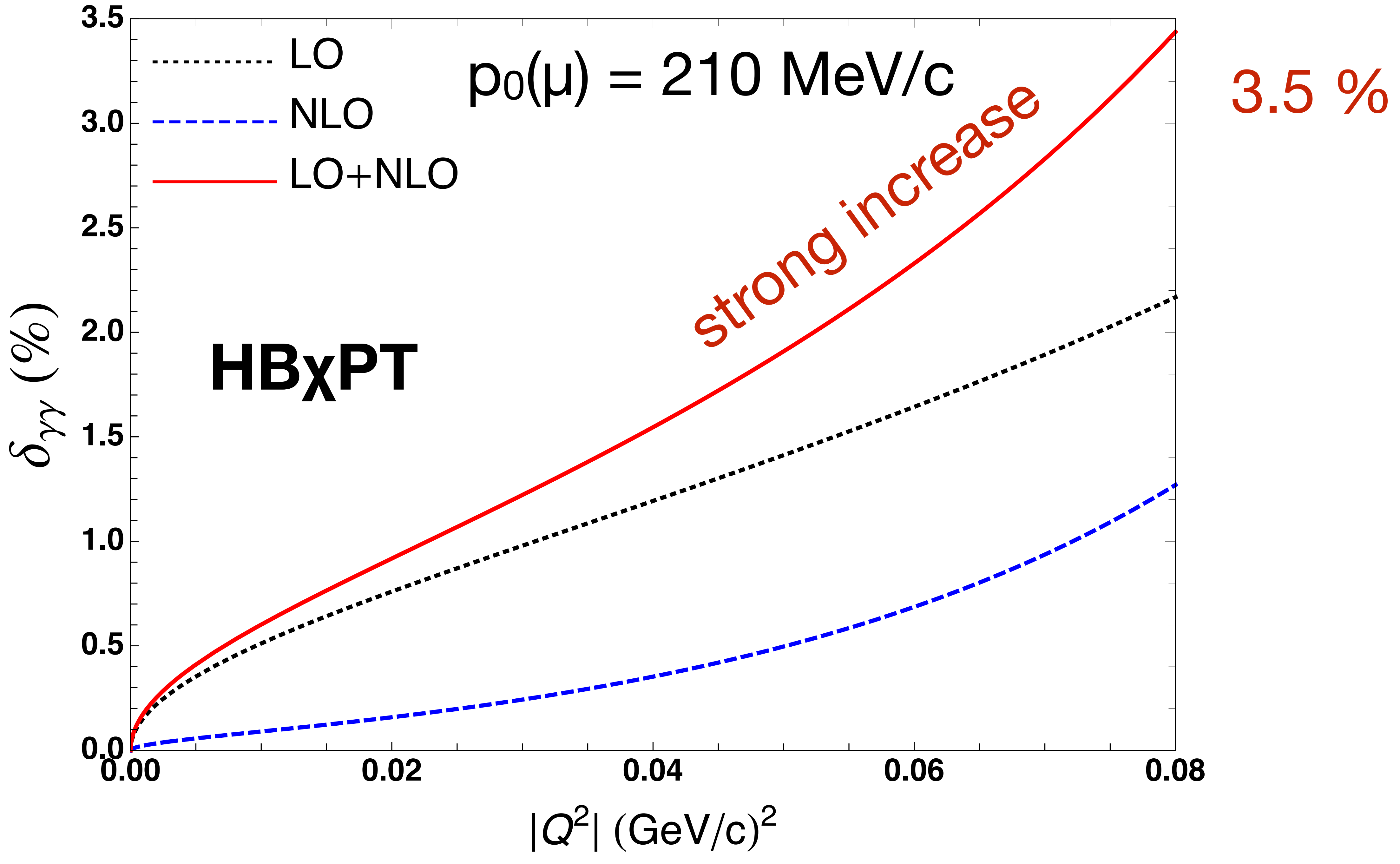


MUSE covers wide ε range, at small values of Q^2 and enables discrimination between models.

Calculations for MUSE differ significantly in their predictions for TPE corrections

Examples for TPE corrections to μ*p*





Oleksandr Tomalak, Few-Body Systems, 59, 87 (2018)
T. Engel, et al., Eur. Phys. J. A 59, 253 (2023) - McMule
P. Choudhary, et al., Eur. Phys. J. A 60, 69 (2024) - HBχPT

TPE corrections to *ep* are typically larger than to μ*p*

Summary and Outlook

MUSE measures $\mu^{\pm}p$ and $e^{\pm}p$ scattering cross sections and will directly compare μp and ep interactions, extract the proton charge radius, and study two-photon exchange effects.

- Radiative corrections dominate systematics
- Instrumental control (SPS detector threshold + ISR veto) reduces them
- Expected sub-percent precision enables charge radius extraction and studies of TPE effects