

# TPE studies in elastic $e-p$ scattering: past, present and future

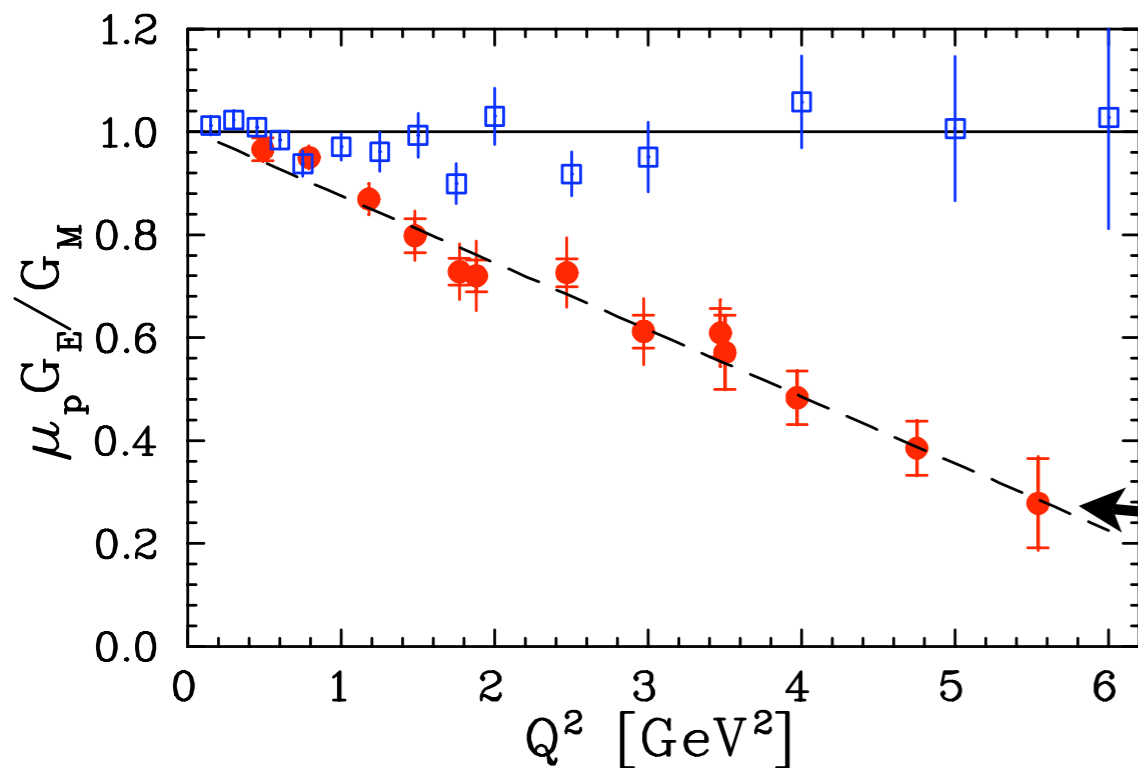
Peter Blunden (University of Manitoba)

Second NREC Workshop

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Sergiy Kondratyuk, Nathan Hall, Jaseer Ahmed, Kyle Shiells

# Proton $G_E/G_M$ Ratio



Rosenbluth (Longitudinal-Transverse)  
Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→  $G_E$  from slope in  $\varepsilon$  plot

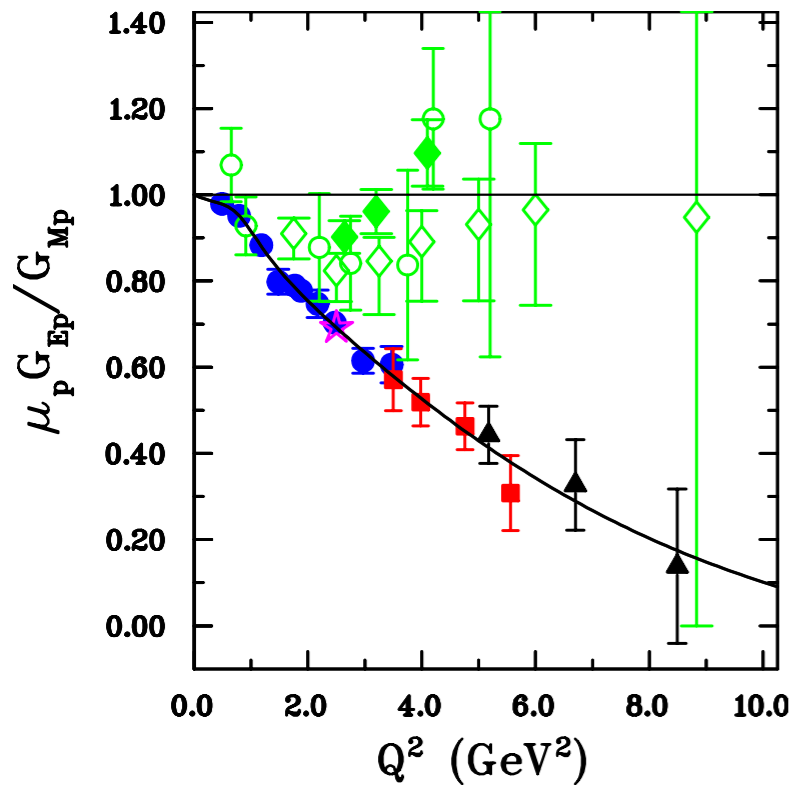
→ suppressed at large  $Q^2$

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→  $P_{T,L}$  recoil proton  
polarization in  $\vec{e} p \rightarrow e \vec{p}$

# Assuming OPE

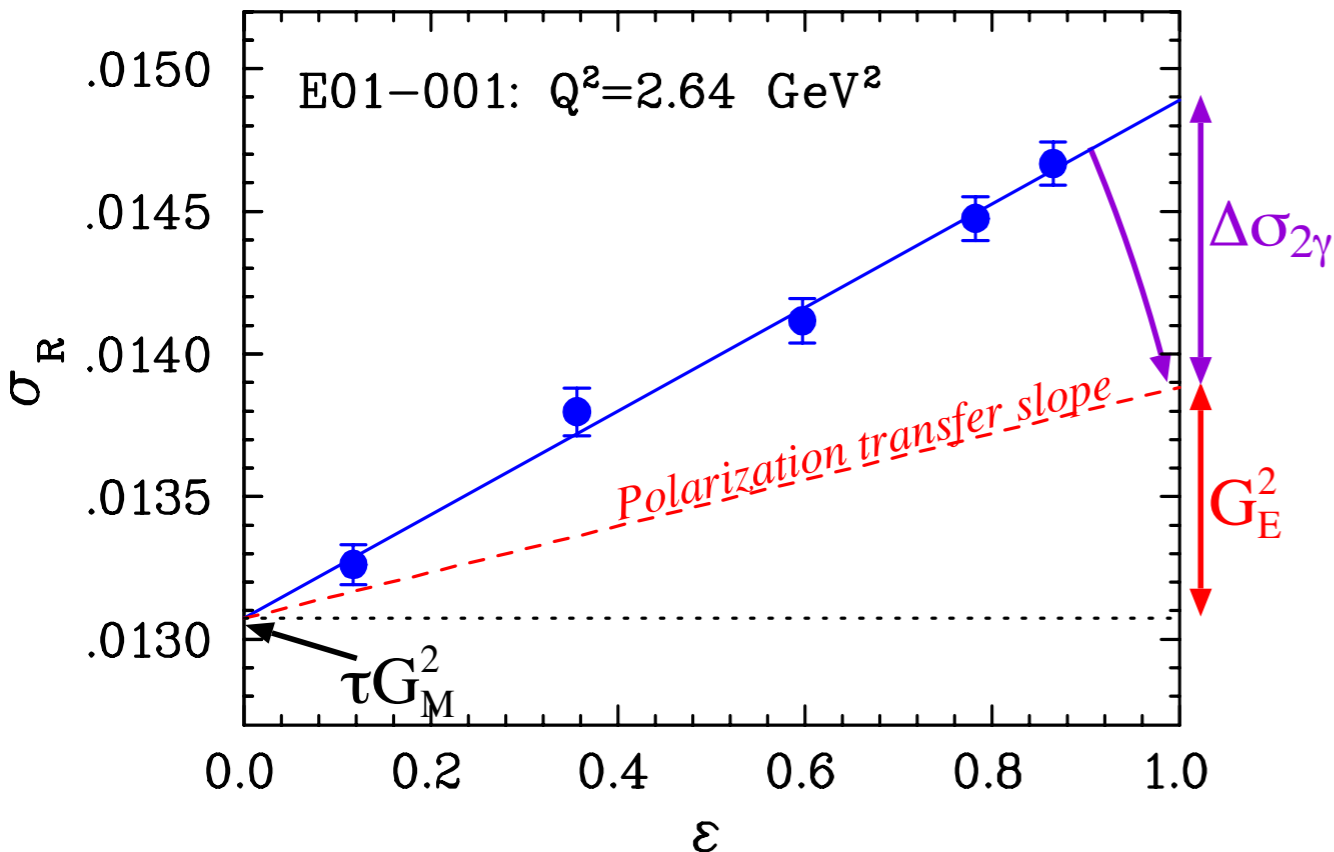


Rosenbluth

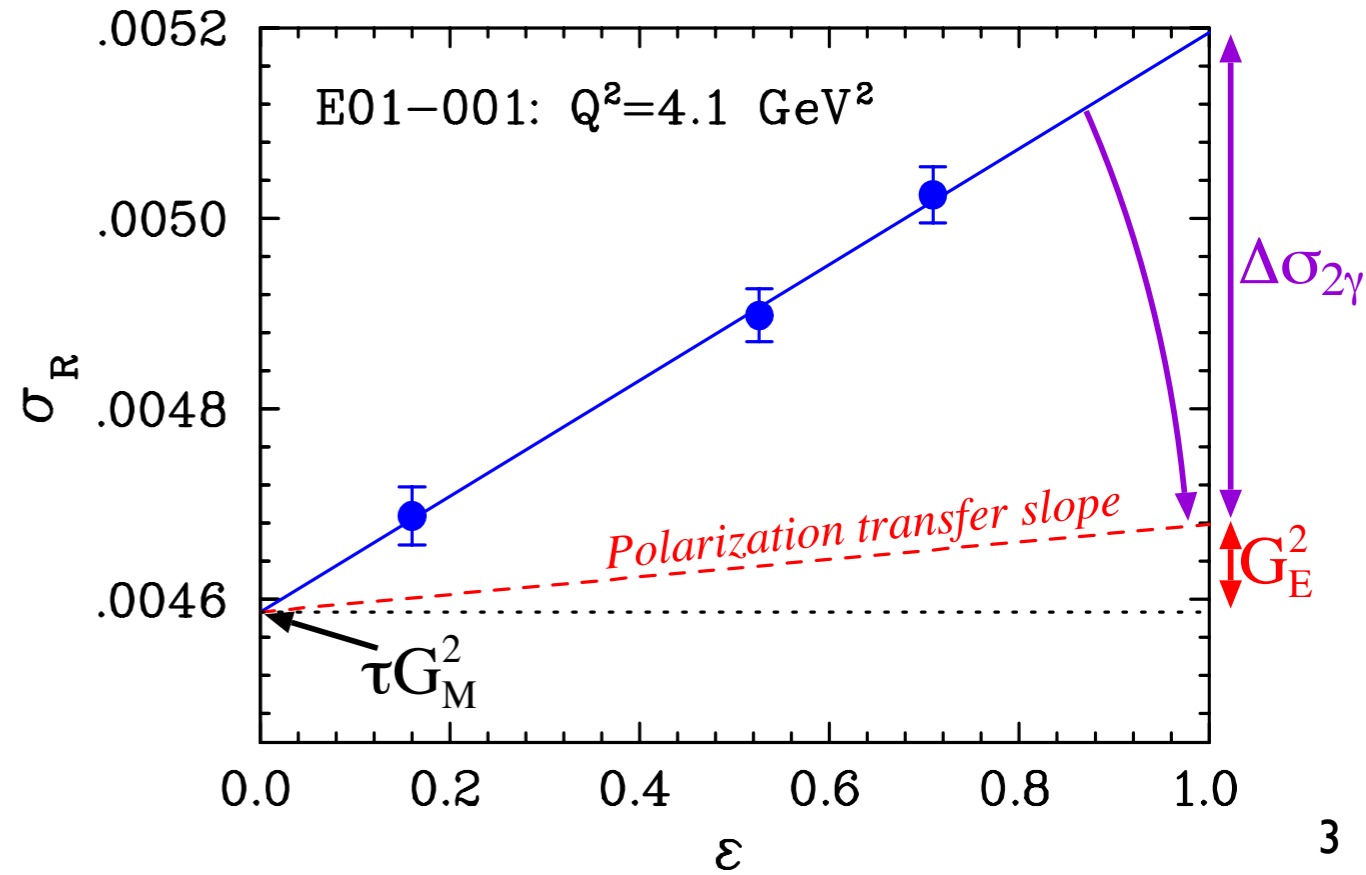
Polarization transfer

- Requires a correction with a **positive** slope vs  $\varepsilon$
- Any radiative correction must vanish as  $\varepsilon \rightarrow 1$  (unitarity)
- Does **not** mean the correction is inherently large

about 50% TPE



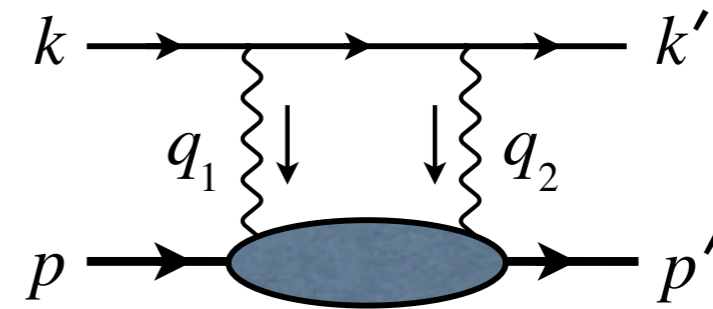
about 80% TPE



# Pointlike target (e.g. $e^- \mu^+$ )

$$\delta_{\gamma\gamma} = -\frac{2\alpha}{\pi} \ln \eta \ln \frac{Q^2}{\lambda^2} + \delta_{\text{hard}}$$

- IR divergence from soft virtual photons ( $q_1 = 0$  or  $q_2 = 0$ )
- Cancelled by  $e-\mu$  soft real photon interference term (bremsstrahlung)

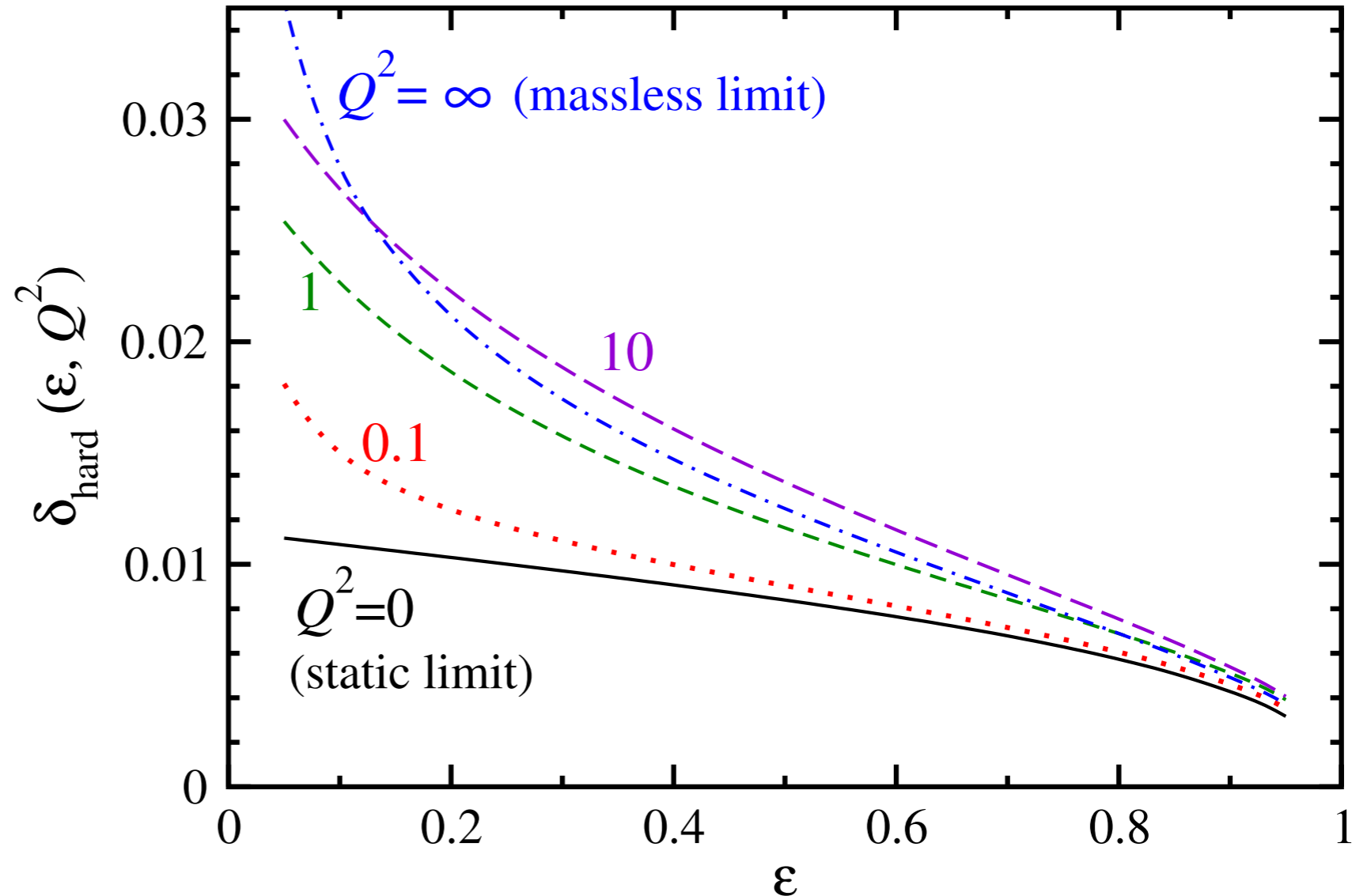


Static limit (Feshbach):

$$Q^2 \rightarrow 0 \text{ or } M \rightarrow \infty$$

Massless limit (quarks):

$$Q^2 \rightarrow \infty \text{ or } M \rightarrow 0$$



How to go from negative slope to required positive slope?

# TPE in DIS kinematics examined in the 1970's

## Difference Between $e^+$ and $e^-$ Deep-Inelastic Scattering\*

Paul M. Fishbane

*Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790  
and Physics Department,† University of Virginia, Charlottesville, Virginia 22901*

R. L. Kingsley

*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England  
(Received 28 June 1973)*

We show that the existence of pointlike constituents within the nucleon makes it plausible that scale-breaking effects due to higher-order electromagnetic corrections in deep-inelastic scattering will be of order  $\alpha \ln^2(-q^2/\mu^2)$  in the region  $s_{ep} \gg 1 \text{ (GeV)}^2$  and large electron scattering angles. Additionally we are led to conclude that, when the final electron energy is finite in the laboratory frame, the difference of electron and positron deep-inelastic scattering is of order  $\alpha \ln^2(-q^2/\mu^2)$  rather than of order  $\alpha$ . We discuss the possible measurement of this difference and what we may learn from such a measurement.

Suggest that

$$\sim \alpha \ln^2 (Q^2 / \mu^2)$$

for  $s \gg 1 \text{ GeV}^2$

and large angles

## Comment on “Difference between $e^+$ and $e^-$ deep-inelastic scattering”\*

G. T. Bodwin† and C. D. Stockham

*Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853*

*(Received 10 February 1975)*

On the basis of a parton model, Fishbane and Kingsley have investigated the difference between  $e^-p$  and  $e^+p$  deep-inelastic scattering in the backward direction and found it to be of relative order  $\alpha \ln^2(Q^2/\mu^2)$ , where  $\mu$  is a typical hadronic mass. We have reexamined this problem in the more general kinematic region  $E_1 \gg E_3$  ( $E_1$  and  $E_3$  are the initial and final electron energies, respectively) and have taken care to distinguish between hard- and soft-photon radiative corrections. The noninfrared part of the virtual photon correction has been found to be of relative order  $\alpha \ln^2(E_1/E_3)$ . It is hard to conceive of a situation where such a contribution to the cross section would be observable.

Showed that

$$\sim \alpha \ln^2 (E_1 / E_3)$$

for hard photons

# TPE and TBE: a brief overview

The first era: **Review: Arrington, Blunden & Melnitchouk, PPNP 66 (2011) 782**

## 2003-2008

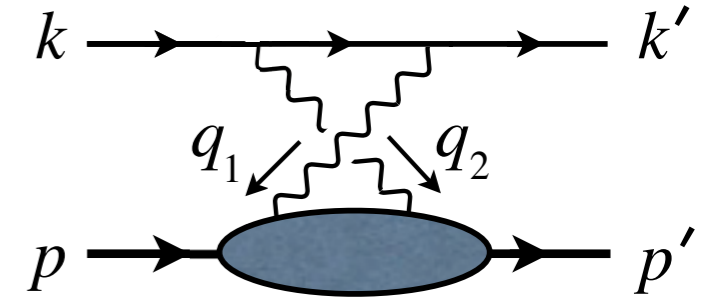
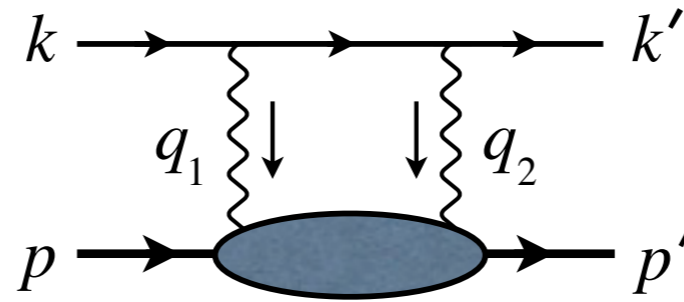
- Suggest neglected two-photon exchange (TPE) as a way to resolve discrepancies
  - First calculations by Blunden *et al.* (2003) using hadronic models were promising
  - Quark model approach by Afanasev *et al.* (2004)
  - At first a rather unwelcome discovery (reaction mechanism more complicated)
- Experiments look for indirect evidence of TPE (non-linear  $\varepsilon$ -dependence, global parameter fits to data, etc.)
- Single Spin Asymmetry (SSA): inelastic beam normal ( $B_n$ ) (measured in PVES experiments) and target normal ( $A_n$ ) measure absorptive (imaginary) part of TPE

# Hadronic and Partonic Approaches (2003-2008)

## Low to moderate $Q^2$ :

hadronic:  $N + \Delta + N^*$  etc.

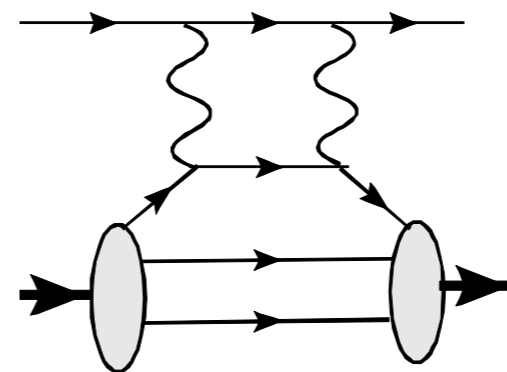
- as  $Q^2$  increases more and more parameters



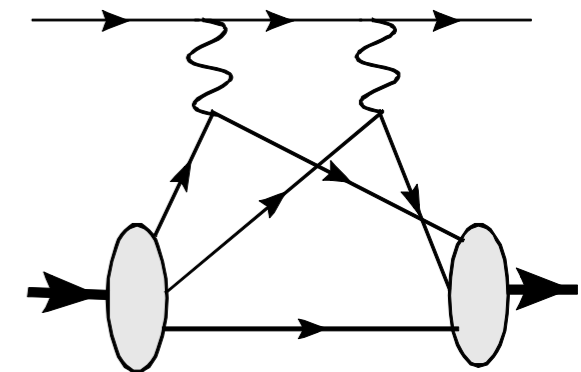
PGB, Melnitchouk, & Tjon, PRL **91**, 142304 (2003)  
Borisyyuk & Kobushkin, PRC **78**, 025208 (2008)

## Moderate to high $Q^2$ :

- GPD approach: assumption of hard photon interaction with 1 active quark (handbag)
  - Embed in nucleon using Generalized Parton Distributions
  - Valid in kinematic range  $|s, t, u| \gg M^2$
  - Ambiguity in how to exclude soft photon terms
- pQCD: later work indicates two active quarks dominate (Borisyyuk & Kobushkin, Kivel & Vanderhaeghen)



“handbag”

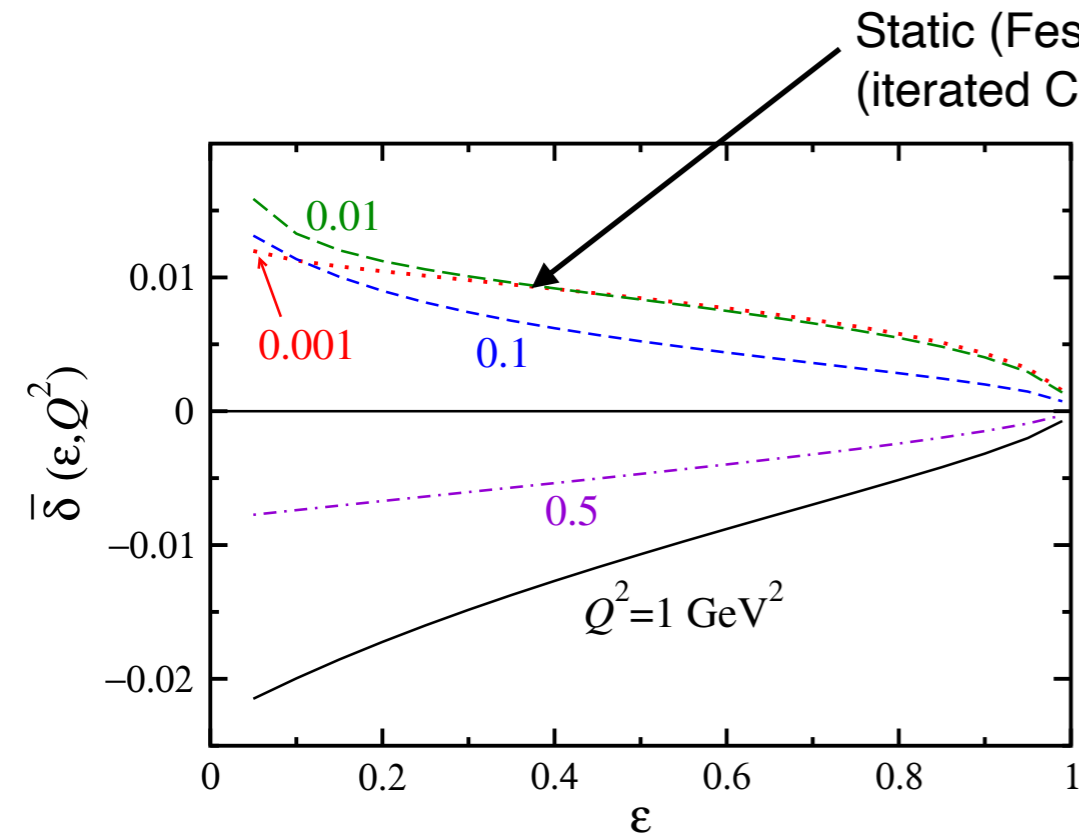


“cat's ears”

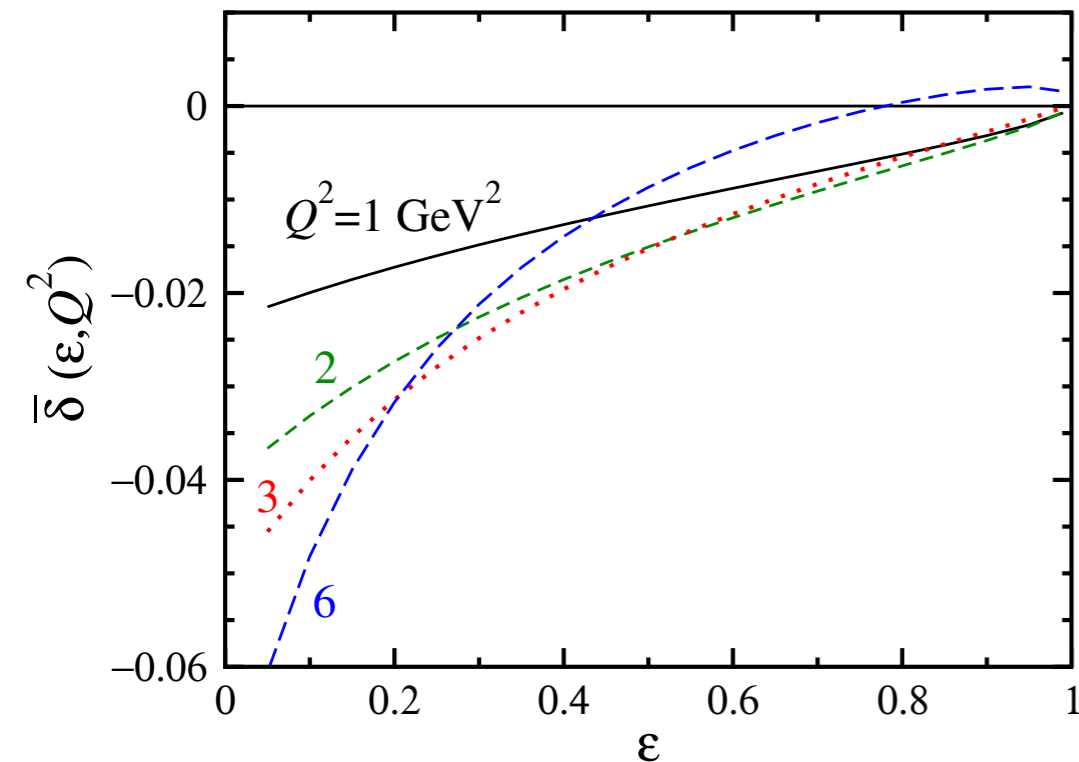
Not included in original GPD model

Afanasev *et al.*, PRD **72**, 013008 (2005)

# Nucleon (elastic) intermediate state



- changes sign at  $Q^2 \approx 0.4 \text{ GeV}^2$
- agrees with static (Feshbach) limit for point particle (**no form factors in loop** and  $Q^2 \rightarrow 0$ )
- $G_E$  dominates in loop integral at low  $Q^2$



- positive slope, vanishes as  $\epsilon \rightarrow 1$
- nonlinearity grows with  $Q^2$
- $G_M$  dominates in loop integral
- right order of magnitude and sign to explain  $G_E/G_M$  ratio
- essentially a form factor effect (not two hard photons)

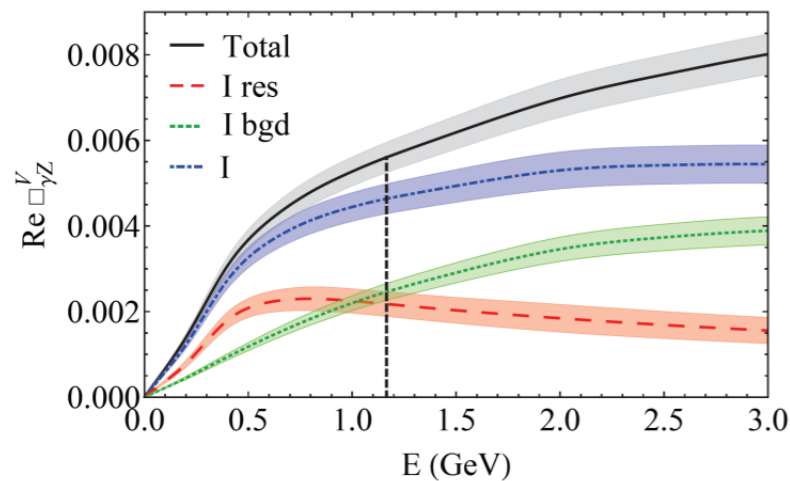
# The second era: Review: Afanasev, Blunden, Hasell & Raue, PPNP **95** (2017) 245

~2008-2015

- Theorists turn their attention to electroweak interaction, the Qweak parity-violating  $e^-p$  scattering experiment, and the effect of two-boson exchange ( $\gamma Z$ )
- Introduce dispersion relations (valid in the forward angle limit) based on structure functions and isospin rotation to reduce model-dependence (Gorchtein, 2009)

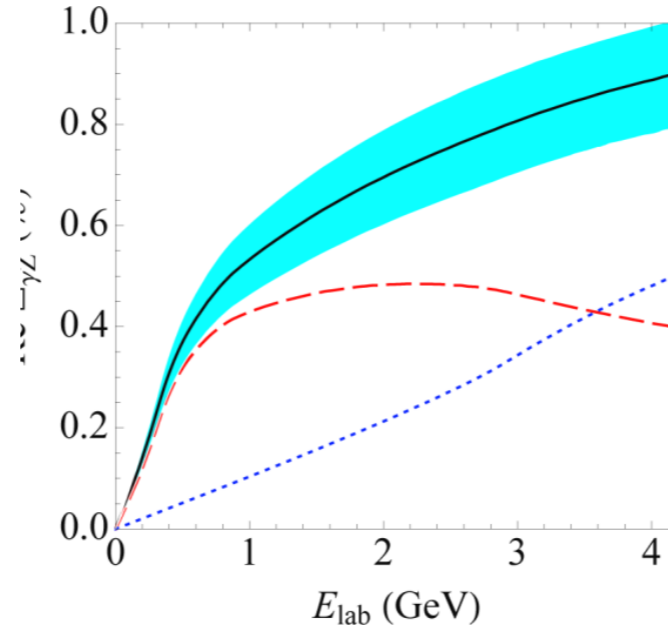
Hall *et al.*

PRD 88, 013011 (2013)



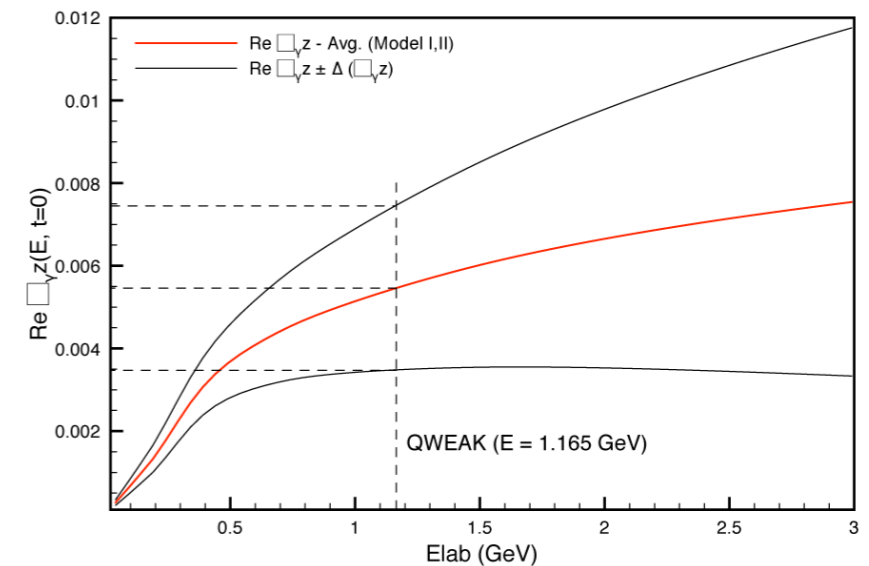
Carlson and Rislow

PRD 83, 113007 (2011)



Gorchtein *et al.*

PRC 84, 015502 (2011)



**Qweak energy:**

$$(5.6 \pm 0.36) \times 10^{-3}$$

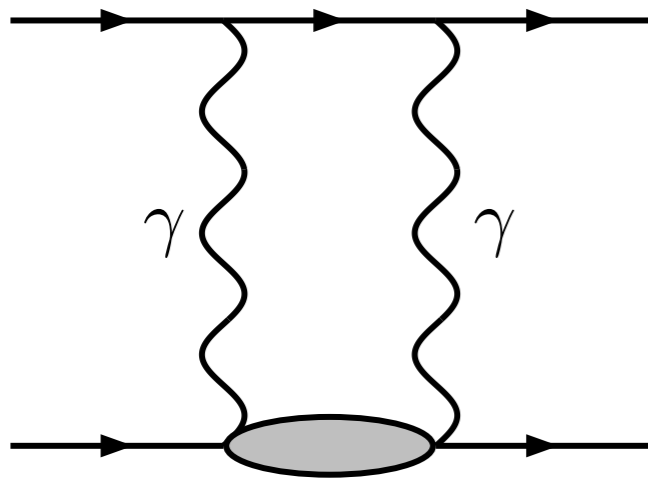
$$\text{Re} \square_{\gamma Z}^V(E = 1.165 \text{ GeV})$$

$$(5.7 \pm 0.9) \times 10^{-3}$$

**8% correction!**

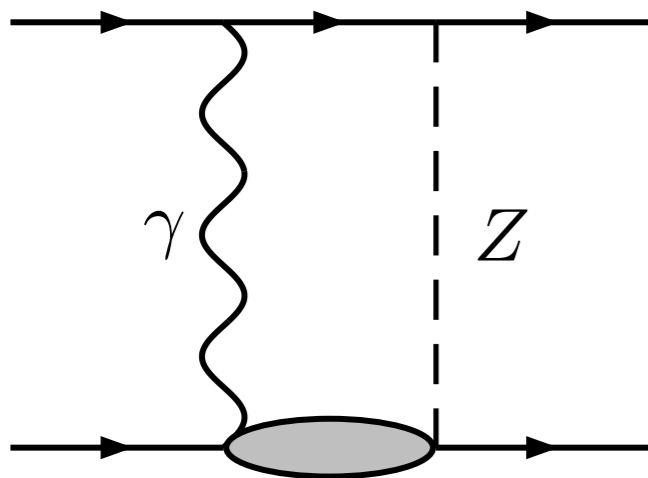
$$(5.4 \pm 2.0) \times 10^{-3}$$

# Electromagnetic and electroweak boxes



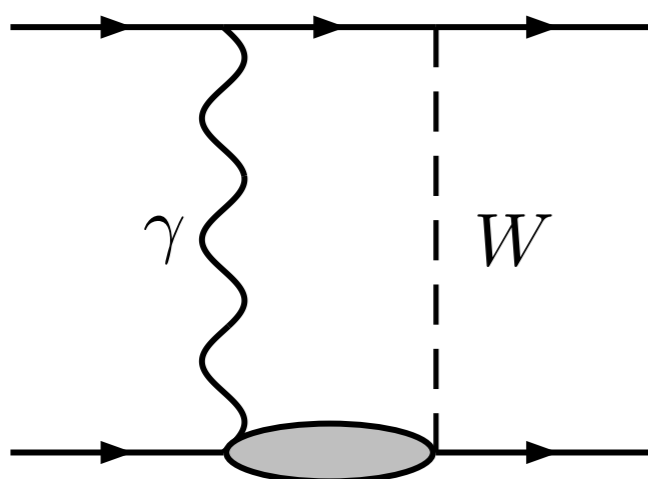
## Long-range

- Forward limit  $Q^2 \rightarrow 0$  (muonic H, charge radius)
  - Use total photonuclear cross section data as input (Brown, 1970; Gorchtein, 2014)



## Short + long-range

- Forward limit  $Q^2 \rightarrow 0$  (atomic PV,  $Q_{\text{weak}}$ )
  - Use isospin rotated cross section data as input
  - Use PDF description in DIS region (short-range)
  - Vector part is largest source of theoretical uncertainty at finite energy



## Short + long-range

- Forward limit  $Q^2 \rightarrow 0$  ( $\beta$  decay, CKM unitarity)
  - One of the largest sources of theoretical uncertainty in CKM unitarity tests using  $V_{ud}$
  - Chen *et al.*, 2018; Shiells & PGB, 2021

## The second era: Review: Afanasev, Blunden, Hasell & Raue, PPNP **95** (2017) 245

### ~2008-2015

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- Introduce dispersion relations (valid in the forward angle limit) based on structure functions and isospin rotation to reduce model-dependence

### ~2011-2014

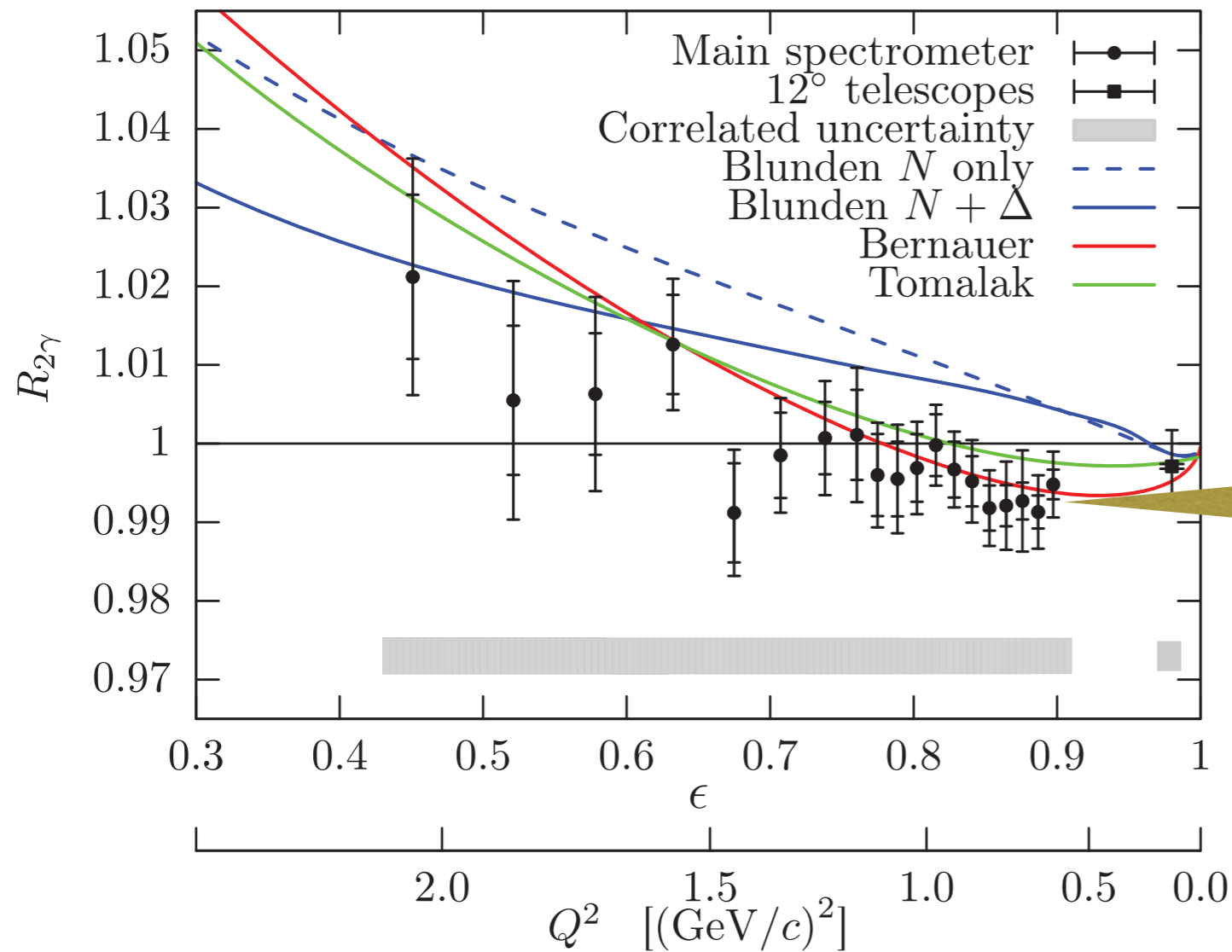
- Muonic hydrogen experiments at PSI measure a proton radius that is  $7\sigma$  away from known results using  $e^-p$  scattering
- High precision  $e^-p$  scattering experiments at very low  $Q^2$  give results confirming the discrepancy (somewhat controversial due to extrapolations to low  $Q^2$ )
- TPE effects play an important role in both muonic hydrogen and  $e^-p$  scattering

### ~2014-2017

- Results of three  $e^-p$  vs  $e^+p$  experiments are reported
- Theorists use dispersive techniques to reanalyze TPE, with a view to reduce model-dependence and make the calculations more robust

# TPE effect on ratio of $e^+p$ to $e^-p$ cross sections

## OLYMPUS (2017)



Suggestive, but would like to see measurements extended to higher  $Q^2$

## ~2018-present

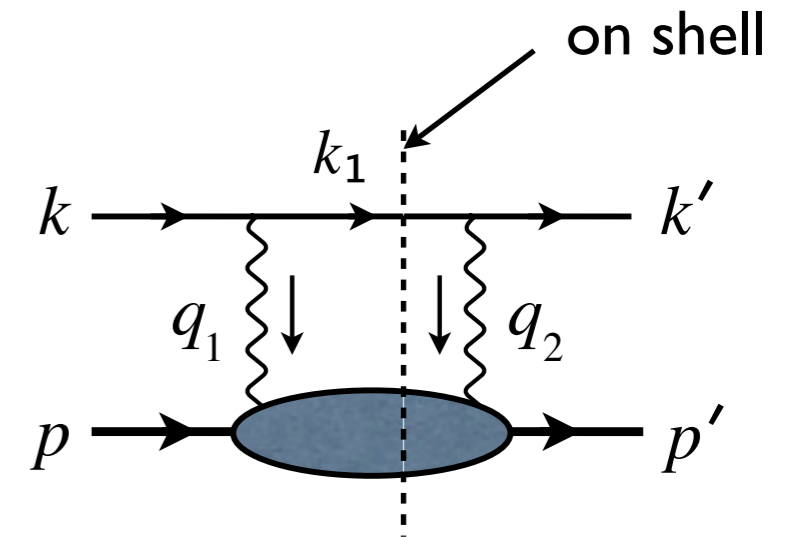
- The two-boson exchange ( $\gamma Z$ ) program is extended to ( $\gamma W$ ), using dispersion relations based on structure function data to reduce model-dependence
- Much additional experimental and theoretical work on proton radius (H, muonic-H,  $e^- p$  scattering)
- Improved treatment of other radiative corrections (*e.g.* McMule)
- Theorists introduce new models and techniques to analyze TPE:
  - Dispersive approach with VVCS amplitudes
  - Pion electroproduction amplitudes using MAID parametrization
  - Resonant intermediate states using CLAS helicity amplitudes
  - Effective field theory formalism
  - Heavy baryon chiral perturbation theory
  - QED-NRQED EFT
  - Lattice calculations

# Dispersive method

Unitarity  $\rightarrow$

$$\text{Im}\langle f|\mathcal{M}|i\rangle \sim \sum_n \int dW_n dQ_1^2 dQ_2^2 \langle f|\mathcal{M}^*|n\rangle \langle n|\mathcal{M}|i\rangle$$

- Imaginary part determined by unitarity
- Real part determined from dispersion relations
- Uses only on-shell form factors (or helicity amplitudes), directly fit to data



- Resonance intermediate states:

$$\Delta(1232)3/2^+, N(1440)1/2^+, N(1520)3/2^-, N(1535)1/2^-, \\ \Delta(1620)1/2^-, N(1650)1/2^-, \Delta(1700)3/2^-, N(1710)1/2^+, N(1720)3/2^+$$

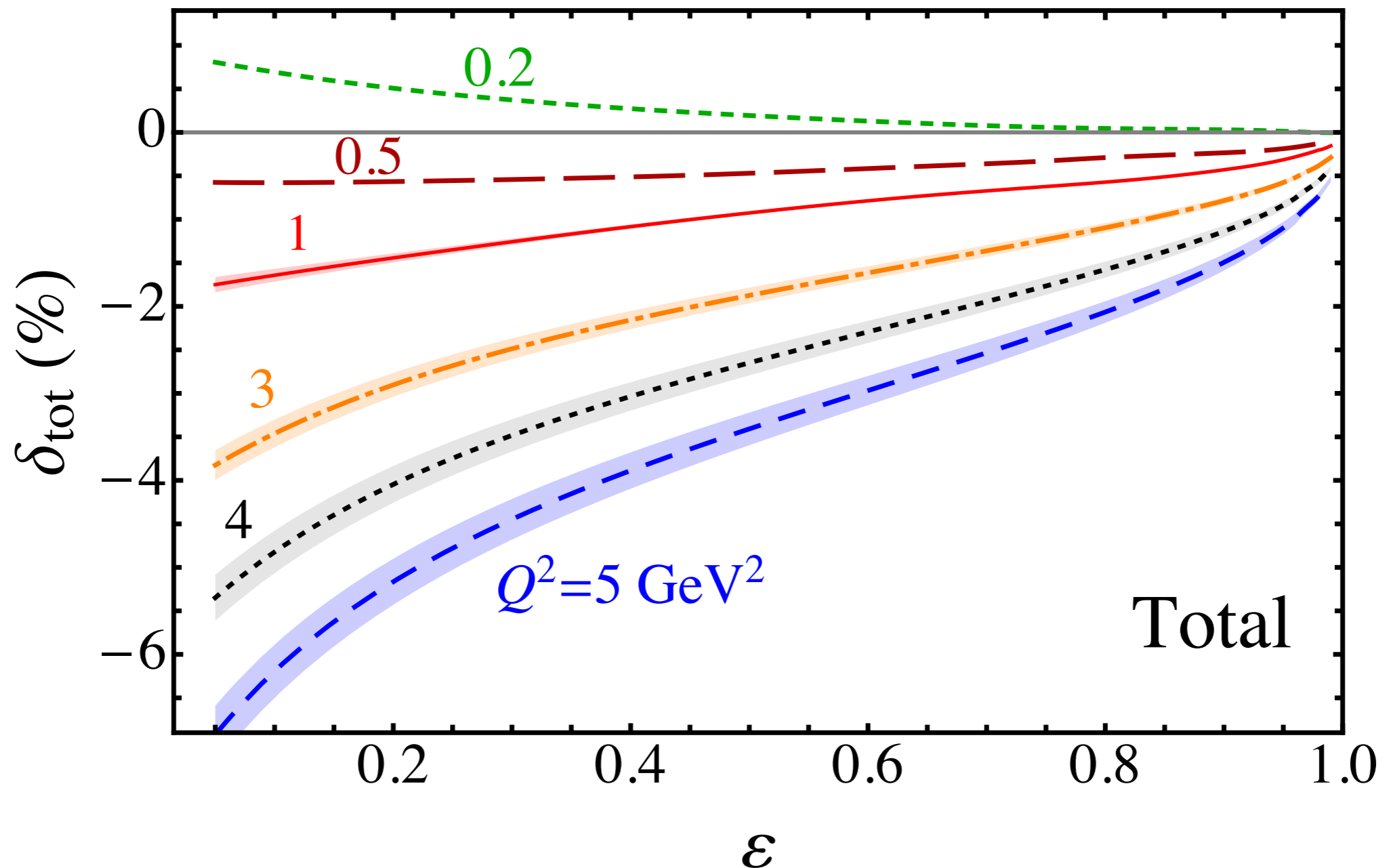
- Use CLAS exclusive meson electroproduction data for helicity amplitudes

$$A_{1/2}, A_{3/2}, S_{1/2}$$

- Include Breit-Wigner lineshape

# TPE corrections to cross section (CLAS resonances)

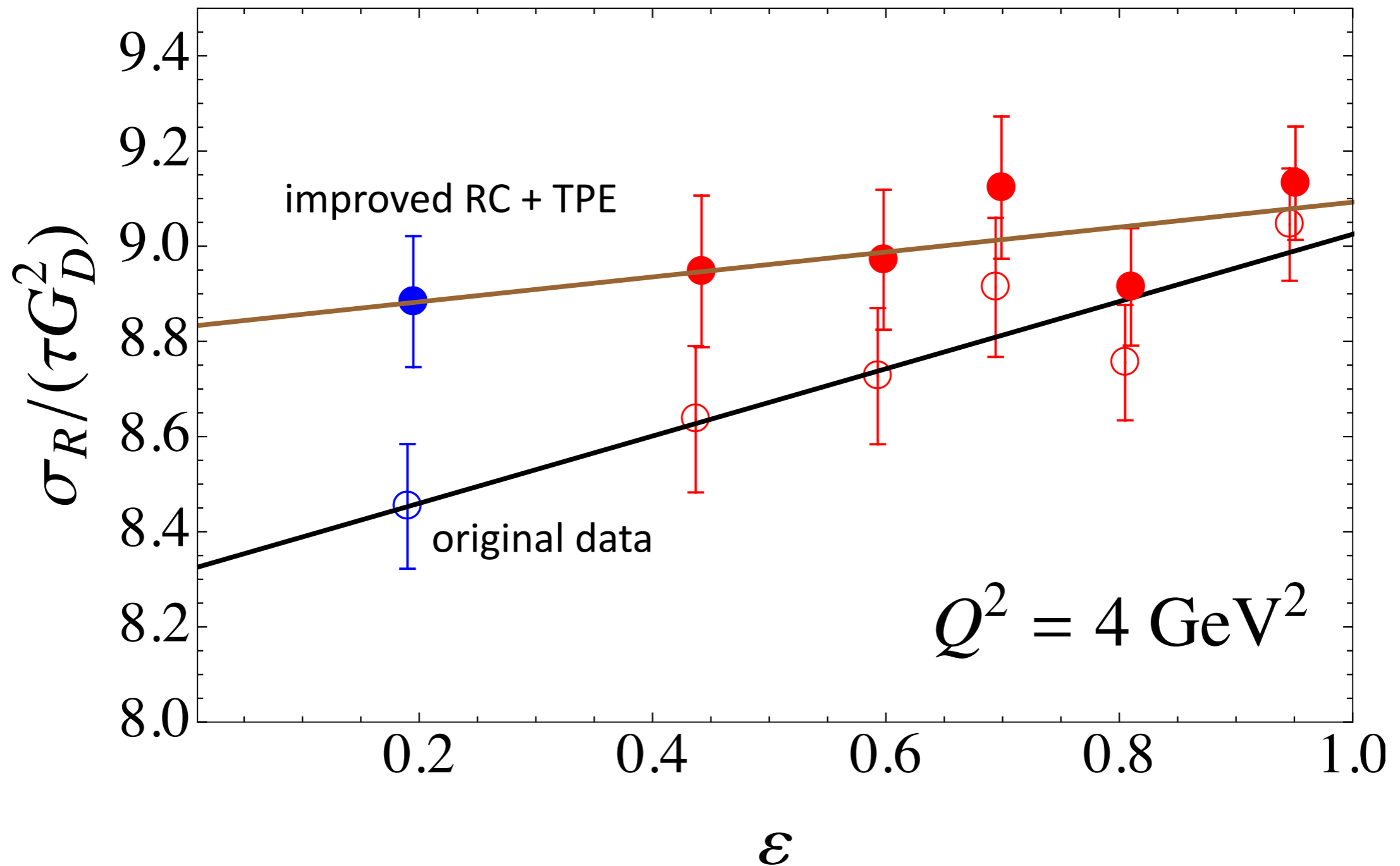
Ahmed, Blunden & Melnitchouk, PRC102, 045205 (2020)



- Linear over mid-range of  $\varepsilon$  values, but curves towards endpoints
- Presence of TPE allows for smaller contribution of  $G_E^2$  to

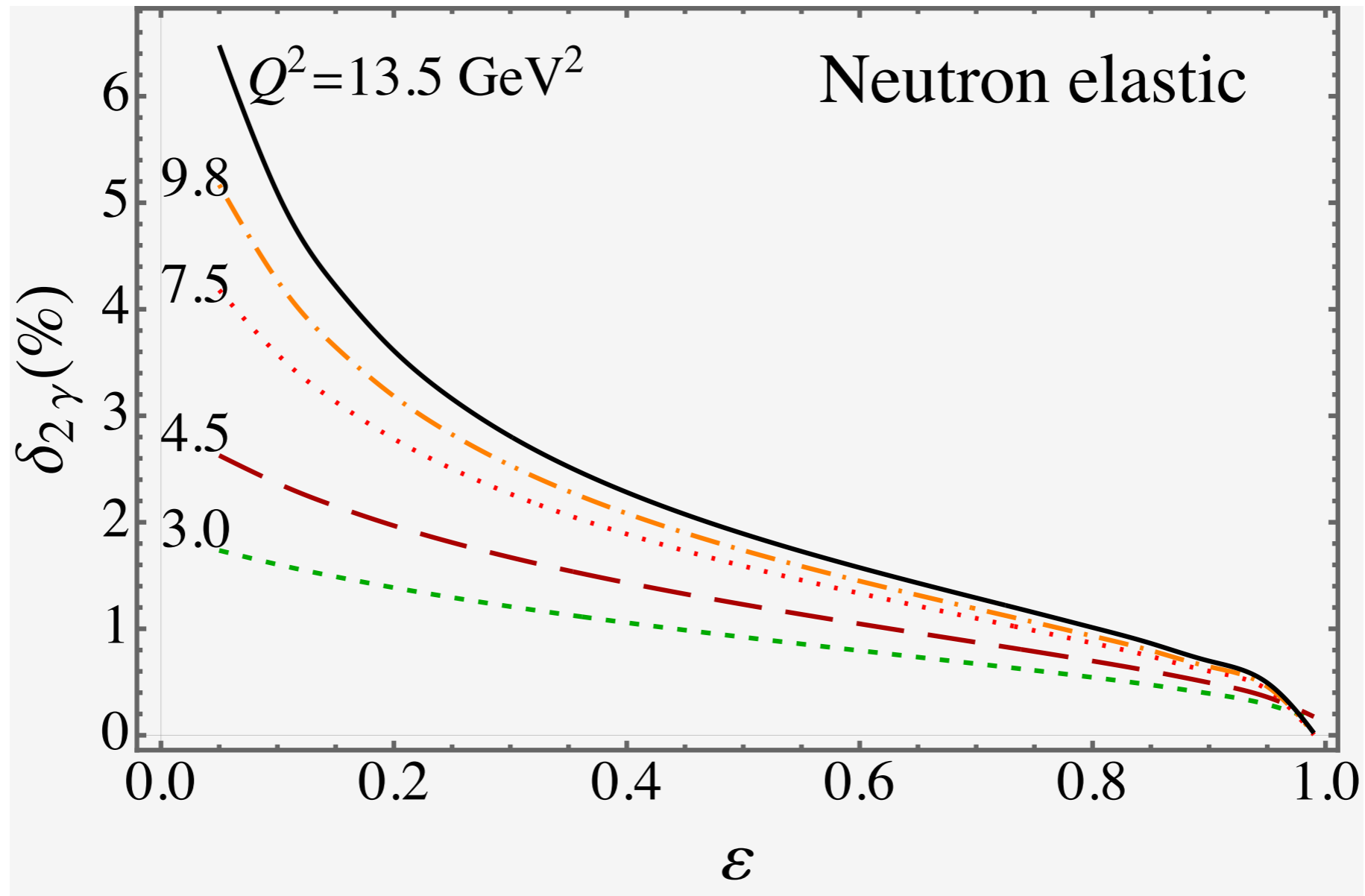
$$\frac{1}{\tau} \sigma_{\text{red}} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

$$\frac{1}{\tau} \sigma_{\text{red}} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

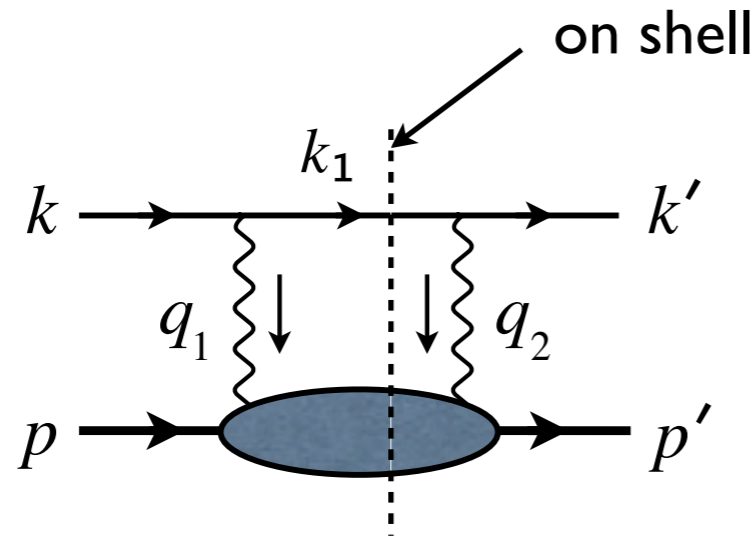


- Example from Andivahis data set at  $Q^2 = 4 \text{ GeV}^2$
- Uses improved RC + our TPE
- No evidence of non-linearity

# Prediction for electron-neutron elastic scattering



# Direct measurements of Im part



This is all in the physical region.

**Beam normal spin asymmetry:** additional dependence on spin-flip FFs

$$B_n = -\frac{m_e}{M} \sqrt{2\epsilon(1-\epsilon)(1+\tau)} \frac{1}{\sigma_R} \text{Im} \{ -G_M G'_a + G_E F'_4 + \nu F_1 F'_5 \}$$

**Target normal spin asymmetry:** same generalized FFs as cross section

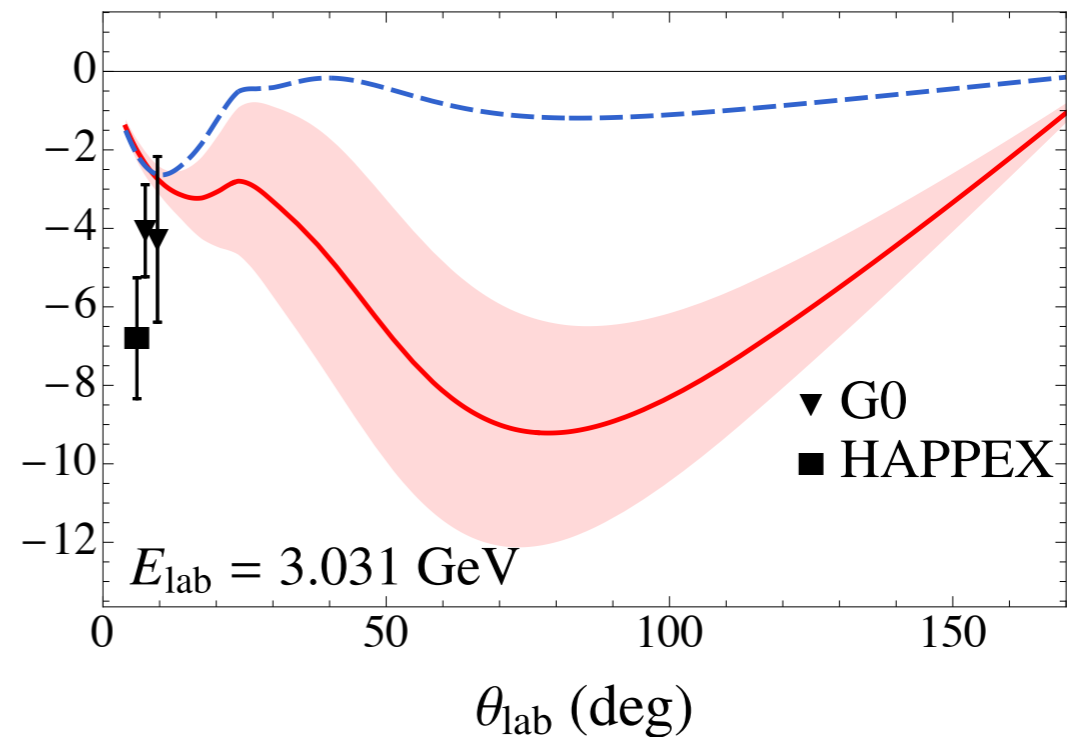
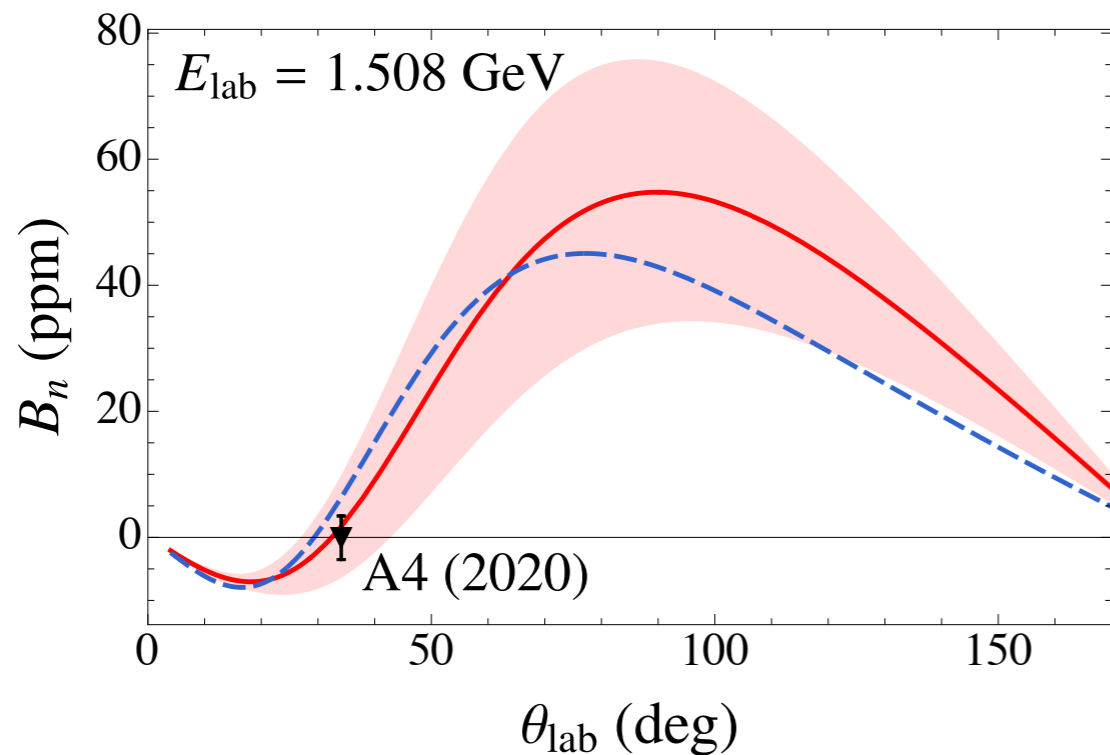
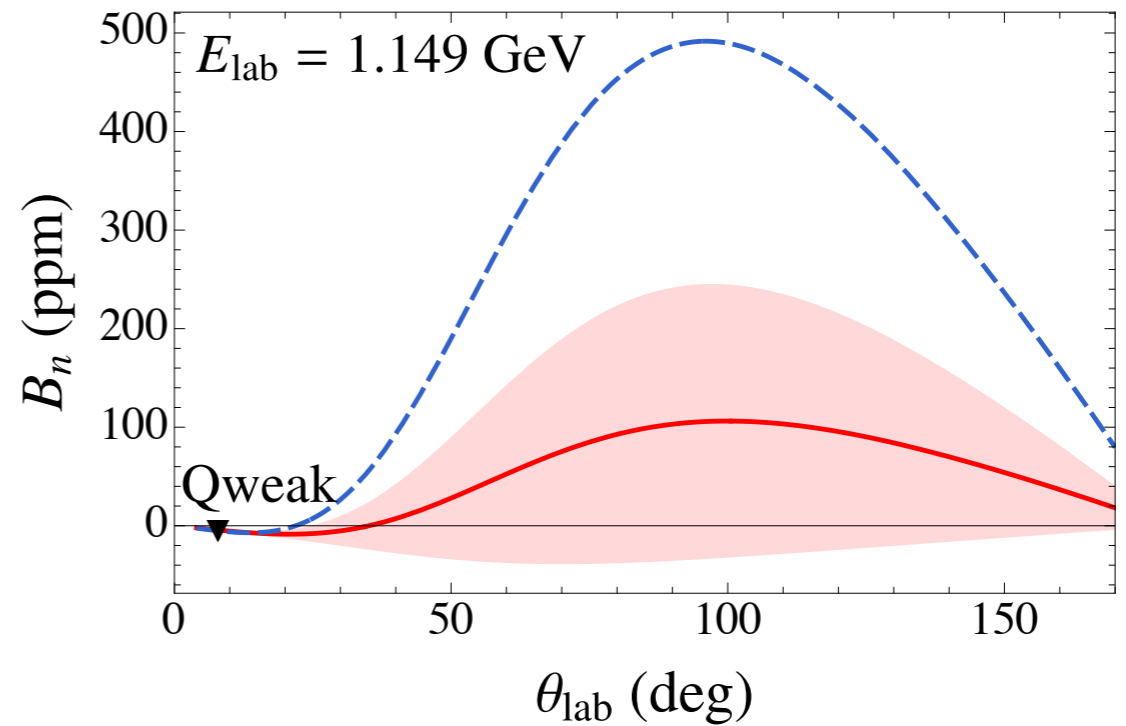
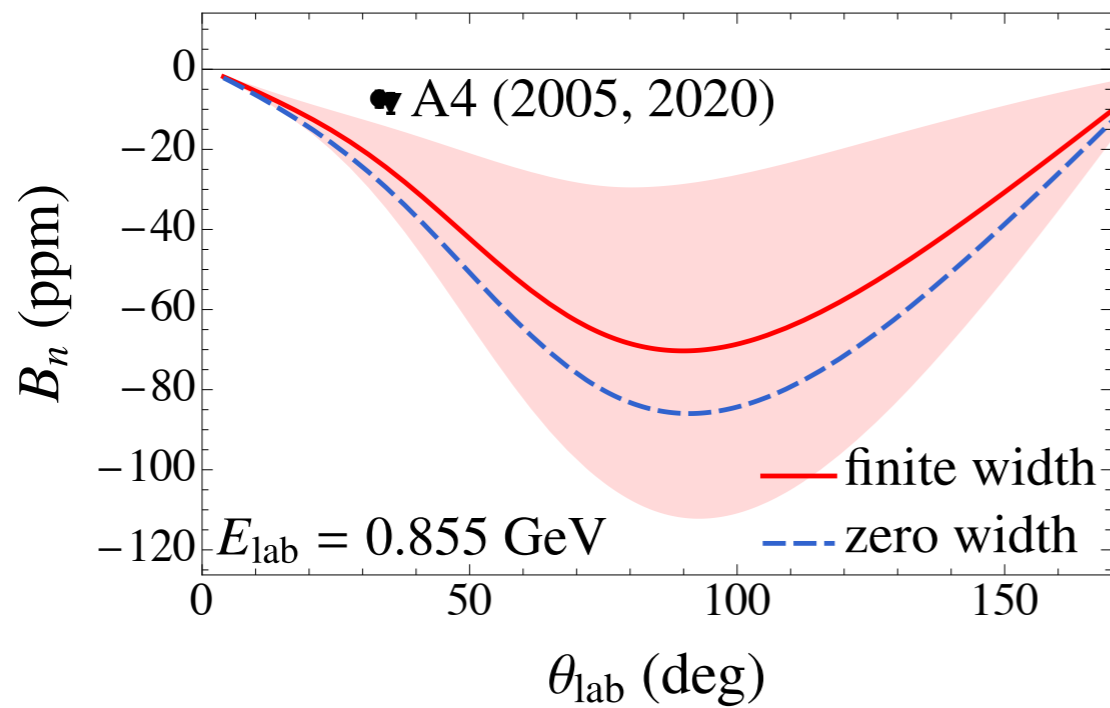
$$A_n = \sqrt{2\epsilon(1+\epsilon)\tau} \frac{1}{\sigma_R} \text{Im} \left\{ -G_M (F'_1 - \tau F'_2) + G_E \left( F'_1 + F'_2 + \frac{1+\tau}{\nu} G'_a \right) \right\}$$

Probe different aspects of TPE!

**Compare with TPE correction to cross section:**

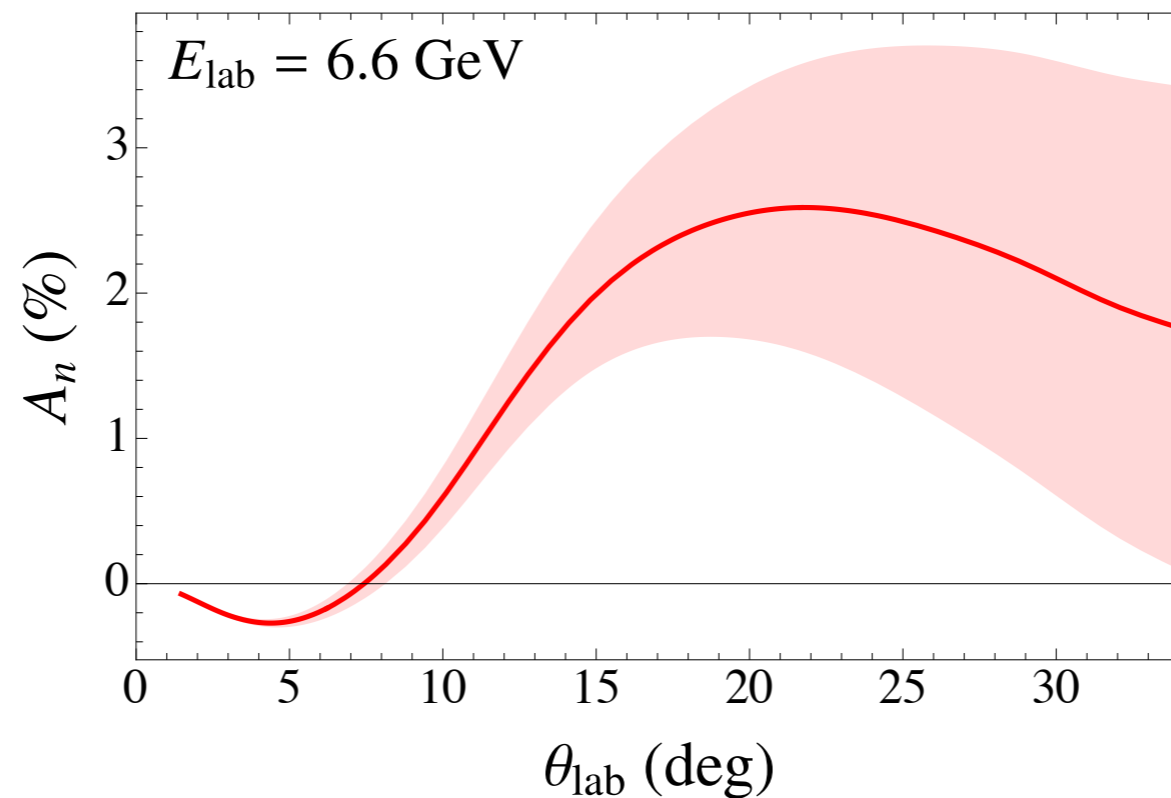
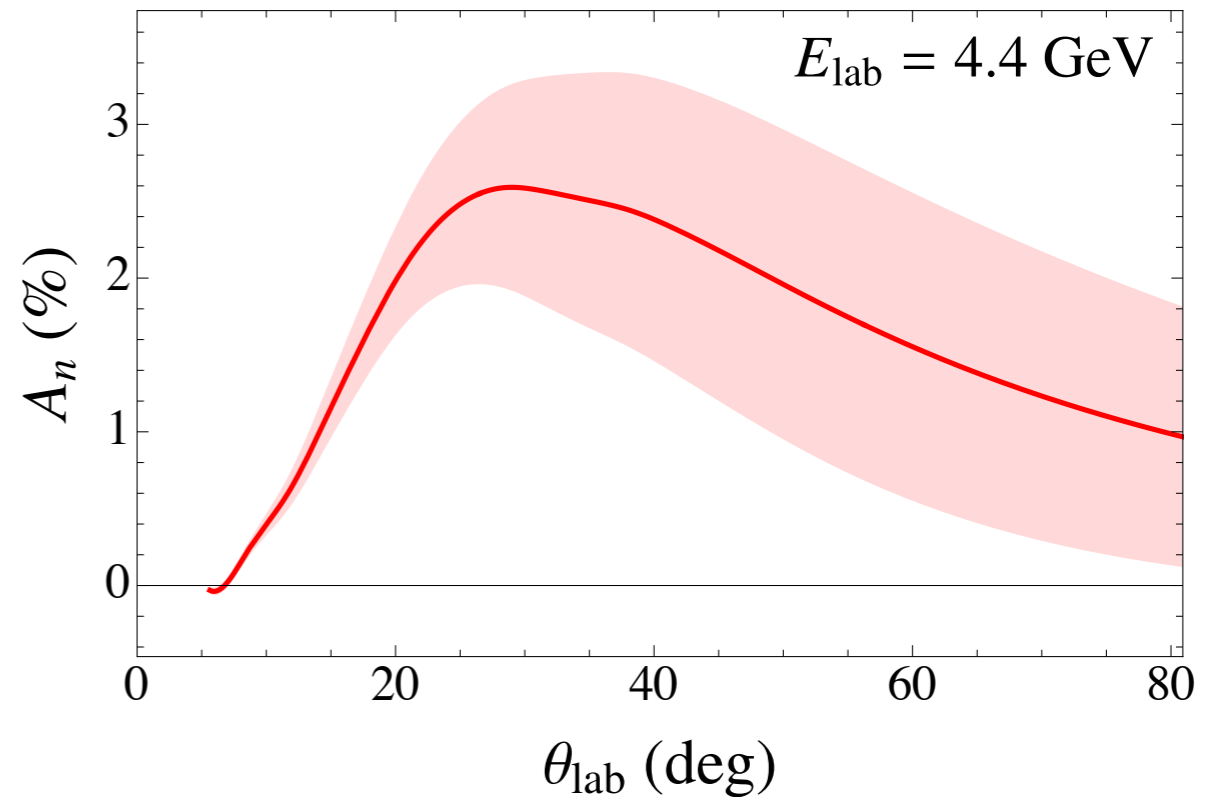
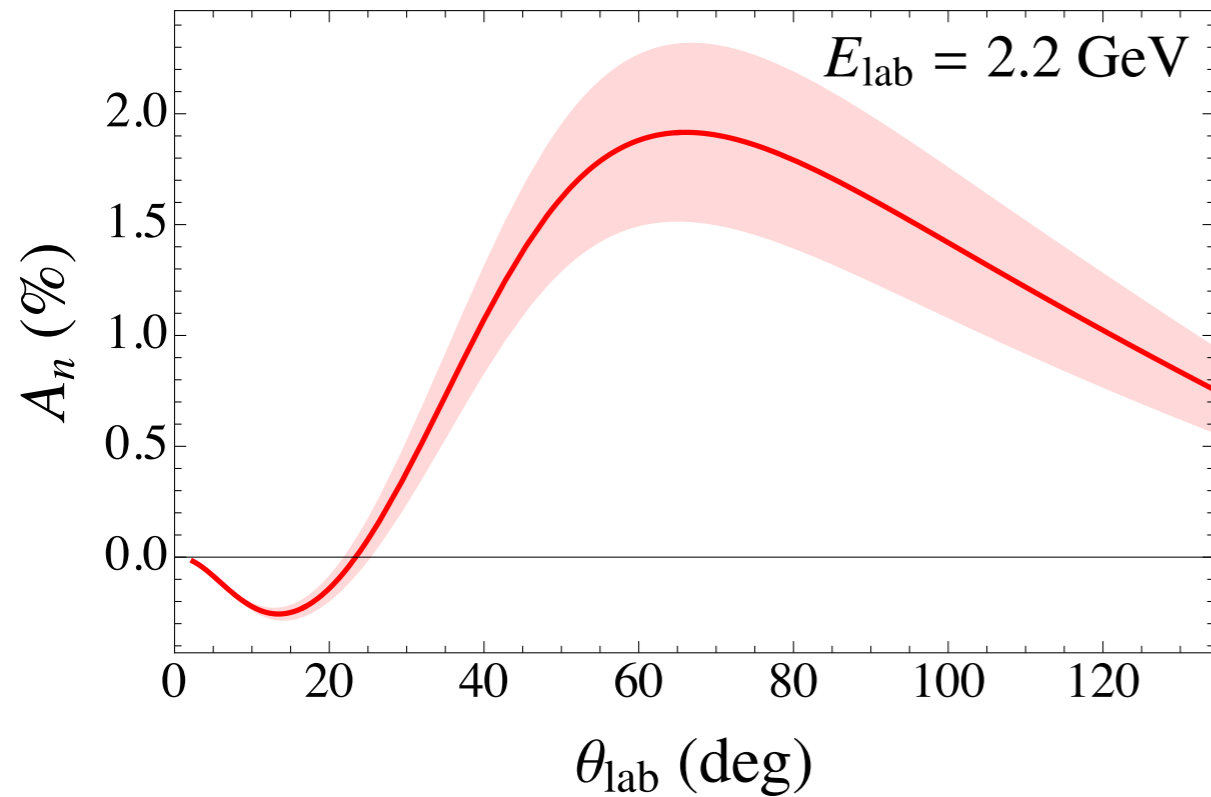
$$\delta_{\gamma\gamma} = \frac{1}{\sigma_R} \text{Re} \{ \epsilon G_E (F'_1 - \tau F'_2) + G_M (\tau (F'_1 + F'_2) + \nu(1-\epsilon) G'_a) \}$$

# Beam normal SSA: fixed $E$ vs $\theta$



- Tends to overshoot data at low energies and forward angles
- Tends to undershoot data at high energies and forward angles

# Target normal SSA @ JLab energies



Potential 2% asymmetry in angular range 20-60° for  $E_{\text{lab}} = 2.2, 4.4,$  and 6.6 GeV.

# TPE: Four key experiments

1. The ratio  $e^+p/e^-p$  in the region  $Q^2 = 3 - 5 \text{ GeV}^2$

This is a definitive experiment, and should find an enhancement of between 6 and 10%. A negative result would be shocking!

2. MUSE  $\mu^\pm p$

A complementary probe of TPE that could shed light on the OLYMPUS result at large  $\varepsilon$ .

3.  $e - n$  scattering in the region  $Q^2 = 1 - 5 \text{ GeV}^2$

Different models should give unambiguous predictions.

4. Target normal SSA  $A_n$  for  $E_{\text{lab}} = 2.2, 4.4, \text{ and } 6.6 \text{ GeV}$

Direct probe of Im part of TPE generalized form factors relevant to cross section TPE correction.

Potential 2% asymmetry in angular range  $20^\circ - 60^\circ$ .