NREC 2026 13th–17th April 2026

Make QED great again

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17th April 2026

QED and the mule

McMule

Monte Carlo for MUons and other LEptons

https://mule-tools.gitlab.io

P. Banerjee, G. Billis, A. Coutinho, Y. Fang, S, Gündogdu
F. Hagelstein, S. Kollatzsch, T. Oruç, D. Radic, M. Rocco,
M. Ronchi, V. Sharkovska, A. Signer, Y. Ulrich, J. Wilson

QED and the mule

⇒ a framework for fully-differential higher-order QED calculations of scattering processes

- fixed-order NNLO QED corrections available/planned for

$\ell \to \ell' \nu\bar{\nu}$	$\ell \to \ell' \nu\bar{\nu}\gamma$	$\ell \to \ell' \nu\bar{\nu}(e^+e^-)$
$e^\pm\mu \to e^\pm\mu$	$e^\pm e^\pm \to e^\pm e^\pm$	$\ell\, p \to \ell\, p$
$e^+e^- \to \gamma^*$	$e^+e^- \to \gamma\,\gamma$	$e^+e^- \to \mu^+\mu^-\gamma$

- also beyond pure QED: $e^+\, e^- \to \pi^\pm\, \pi^\pm\, \gamma$ and EW for PV $e^\pm e^\pm \to e^\pm e^\pm$.
- full NNLO!! (not just the easy parts, "no" approximations)

  $\Rightarrow$ NNLO $2 \to 2$ $\subset$ NLO $2 \to 3$ $\subset$ LO $2 \to 4$

- fully differential $\Rightarrow$ Monte Carlo integrator

QED and the mule

McMule talk @NREC 2024 contained 3 core messages (not necessarily new)

- # 1 QED has seen huge progress, also using input from QCD (LHC community)
- # 2 NNLO QED corrections can be larger than TPE
- # 3 For TPE, always do virtual + real

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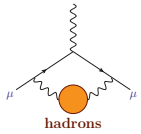
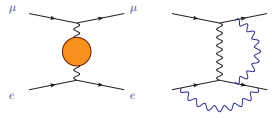
triggered by MUonE



from



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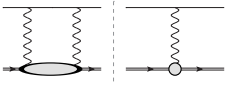
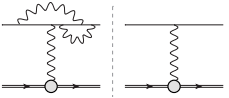
never do  without 

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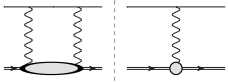
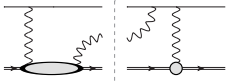
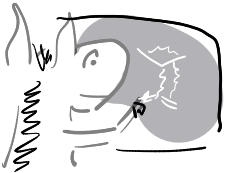
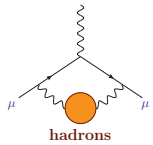
never do  without 

Hard-core QED

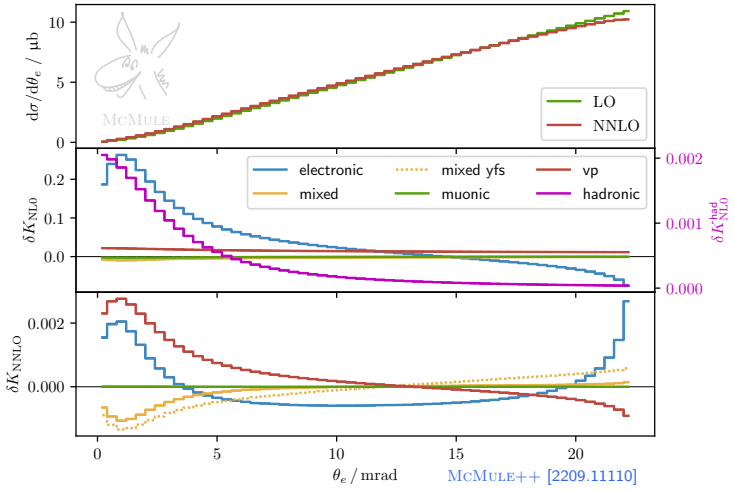


$\mu^\pm e^- \to \mu^\pm e^-$ @10ppm: MUonE



measure a 0.1% effect to 1%
(need to go beyond NNLO)

no elasticity-band cut



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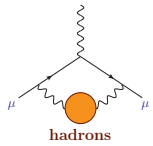
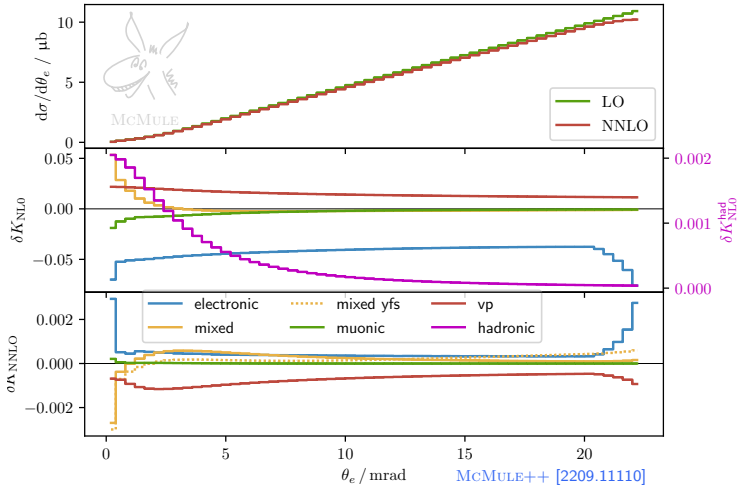
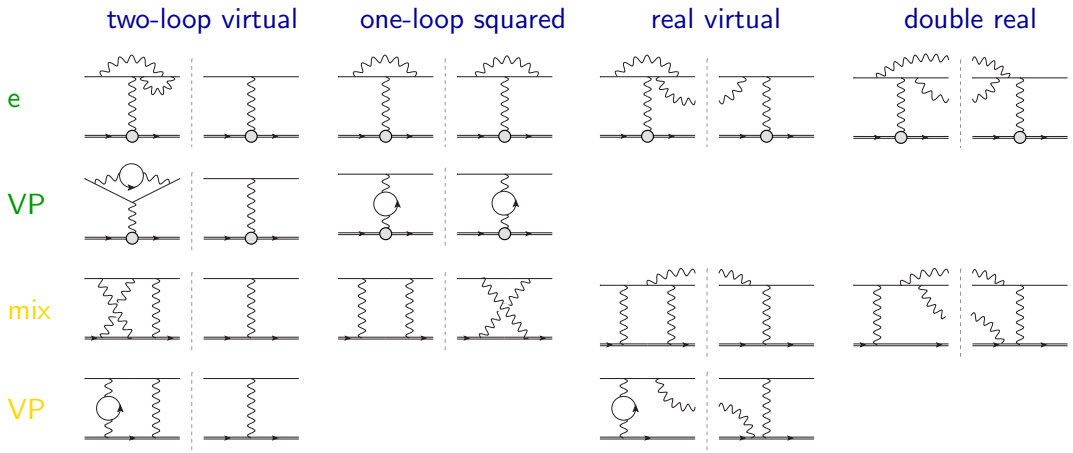


measure a 0.1% effect to 1%
(need to go beyond NNLO)

with elasticity-band cut





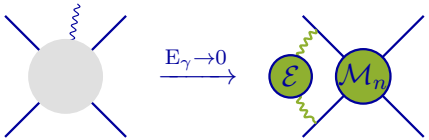
also NNLO corrections on lower line

Many people are involved, directly McMule AND indirectly beyond McMule!

- many diagrams $\rightarrow$ automate generation of diagrams, algebra, reduction to master integrals $\Rightarrow$ amplitude $\sim$ 60 Mb

  [Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano]
- MUonE two-loop integrals with $m_e = 0$ expressed in terms of generalised polylogs $\rightarrow$ develop Fortran tool for fast numerical evaluation [handyG]
- include effects of $m_e \neq 0$ in mixed NNLO approximately (massification) error $\alpha^3 m_e^2/\{s,t\}$ [Penin; Becher, Melnikov; Engel, Gnendiger, AS, Ulrich]
- delicate numerics for one-loop (up to pentagon) diagrams $\rightarrow$ use [OpenLoops]
- delicate numerics in phase space integration $\rightarrow$ use next-to-soft approach, extension of LBK theorem beyond NLO [Engel, AS, Ulrich]

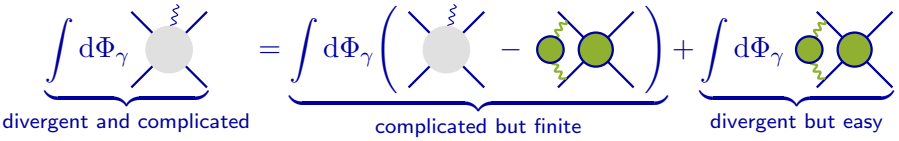
$m \neq 0$ $\Rightarrow$ only soft singularities use dim reg for IR singularities



$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\,\mathcal{M}_n^{(\ell)} \;+\; \mathcal{O}(E_\gamma^{-1})$$

$$\text{eikonal } \mathcal{E} = \sum_{i,j} \frac{p_i \cdot p_j}{(p_\gamma \cdot p_i)\;(p_\gamma \cdot p_j)} \sim \mathcal{O}(E_\gamma^{-2})$$

$\Rightarrow$ subtraction scheme (FKS$^\ell$) at any order in QED [1909.10244; T.Engel, AS, Y.Ulrich]



subtraction scheme

we do not write $\sigma_n^{(1)} = \sigma_n^{(v)}(\lambda) + \sigma_n^{(s)}(\lambda,\omega) + \sigma_{n+1}^{(h)}(\omega)$ photon mass λ, resolution ω

we do write $\sigma_n^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$ at NLO

$\sigma_n^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$ at NNLO

no approximation at all

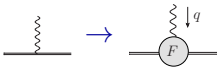
$$\sigma_n^{(1)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \left(\underbrace{\mathcal{M}_n^{(1)}}_{1/\epsilon} + \underbrace{\hat{\mathcal{E}}(\xi_c)}_{1/\epsilon} \mathcal{M}_n^{(0)} \right) = \int \mathrm{d}\Phi_n^{d=4} \underbrace{\mathcal{M}_n^{(1)f}(\xi_c)}_{\text{finite}}$$

$$\sigma_{n+1}^{(1)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1} \right)_c \left(\xi_1 \, \mathcal{M}_{n+1}^{(0)f} \right)$$

ξ_c dependence cancels between the two/three terms (implementation/stability check)

Including hadrons

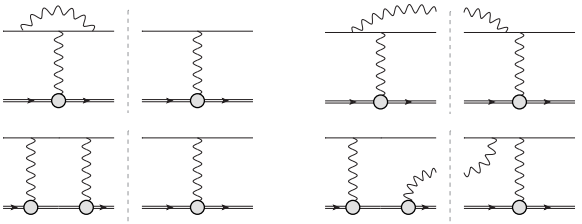
From $e^\pm \mu \to e^\pm \mu$ to $\ell\, p \to \ell\, p$: as a simplistic model



virtual real

electron $q_\ell^4\, F^2$

TPE $\supset$ mixed $q_\ell^3\, F^3$



In [2307.16831], McMule for MUSE: simple dipole for elastic TPE

For 3PE and proton-line corrections: pointlike approximation (rough estimate)

AMBER, $E = 100\,\text{GeV}$ "old"

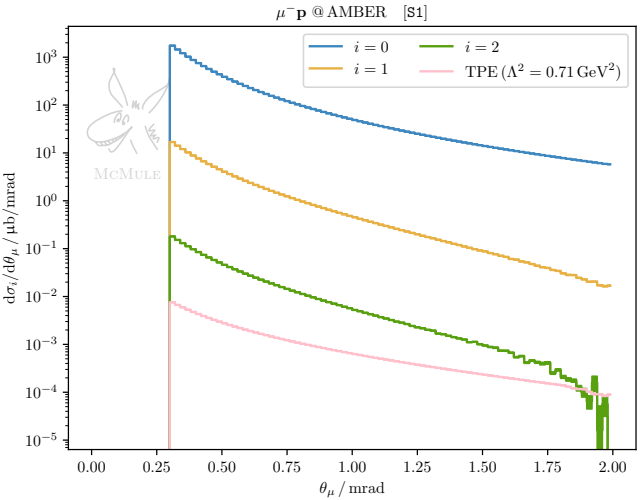
$\mu^- p \to \mu^- p$
with $E_{\text{in}} = 100\,\text{GeV}$

$0.3\,\text{mrad} \leq \theta_\ell \leq 2\,\text{mrad}$

$E_\mu > 99\,\text{GeV}$

our TPE is questionable

recall message # 2



MAGIX $E = 105\,\text{MeV}$ "old"

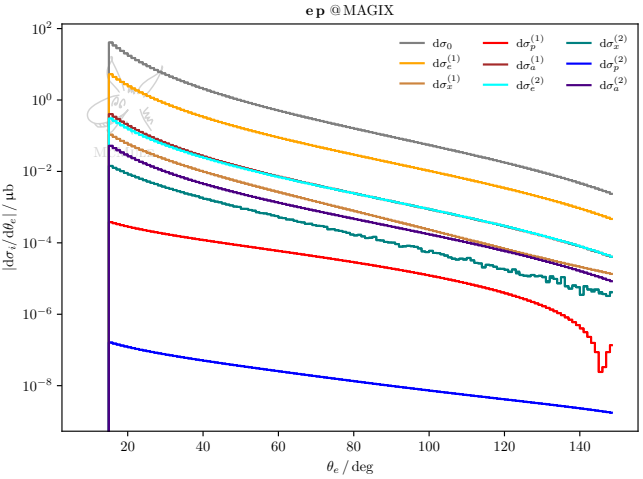
$e^- p \to e^- p$
with $E_{\text{in}}(e^-) = 105\,\text{MeV}$

$15^\circ \leq \theta_e \leq 150^\circ$

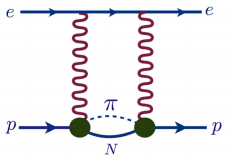
$73.5\,\text{MeV} \leq p_e \leq 105\,\text{MeV}$

$E_{\gamma\text{tot}} < 1\,\text{MeV}$ (!)

recall message # 2



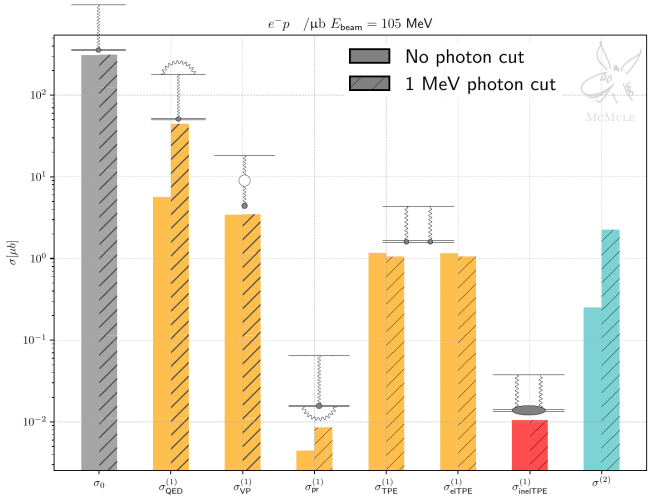
plan: include state-of-the-art TPE models



[Tomalak, Pasquini, Vanderhaeghen, 1708.03303]

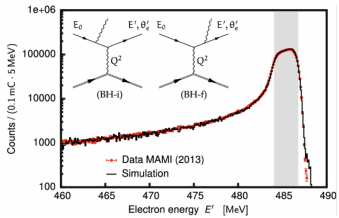
so far 'only' virtual part
Ooops, recall message # 3

⇒ work on real counterpart



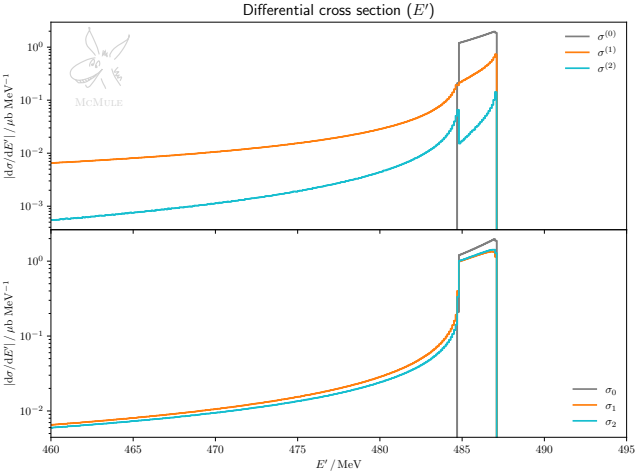
full NLO for $e^- p \to e^- p \gamma$
($\supset$ NNLO for $e^- p \to e^- p$)

$E = 495\,\text{MeV}$

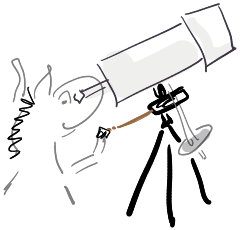


[Mihovilovic et al. 1905.11182]

10% correction in the tail



outlook



- implementation of 'best' available TPE
- think about other targets than proton
- think about polarisation

  PVES (MOLLER) [2507.17652]
- combine fixed-order approach with parton shower / YFS soft resummation
- go to NNLO for $2 \to 3$ processes
- and, yes, of course, go to NNNLO for $2 \to 2$

the end