Extracting nuclear charge radii systematics from spectroscopy of highly charged ions

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Summary

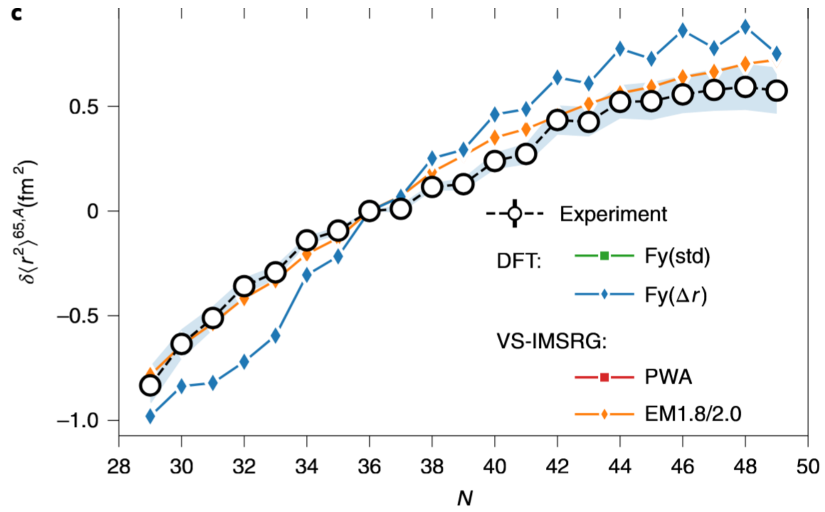
- Nuclear charge radius surface
- Highly charged ion (HCI) based charge radii measurements
  - Why HCIs?
  - Challenges: nuclear model dependence, interpretation of results
- Investigating nuclear structure with charge radius trends
- Conclusions and outlook

Isotopic Structure Studies

Experimental radius data guides + benchmarks nuclear structure theory.

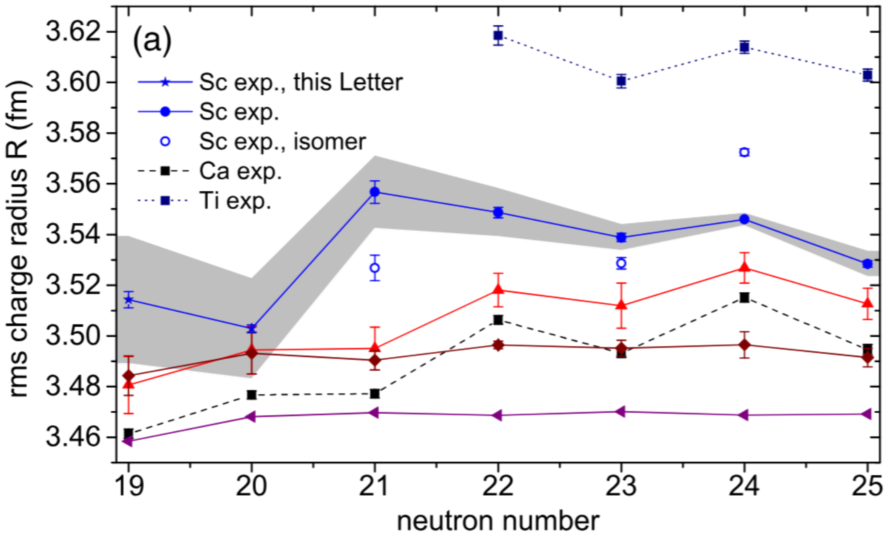
Odd-even staggering in Cu

R.P. de Groote Nat. Comm. 16, 620 (2020)



Kink in Sc charge radii

K. König PRL 131, 102501 (2023)



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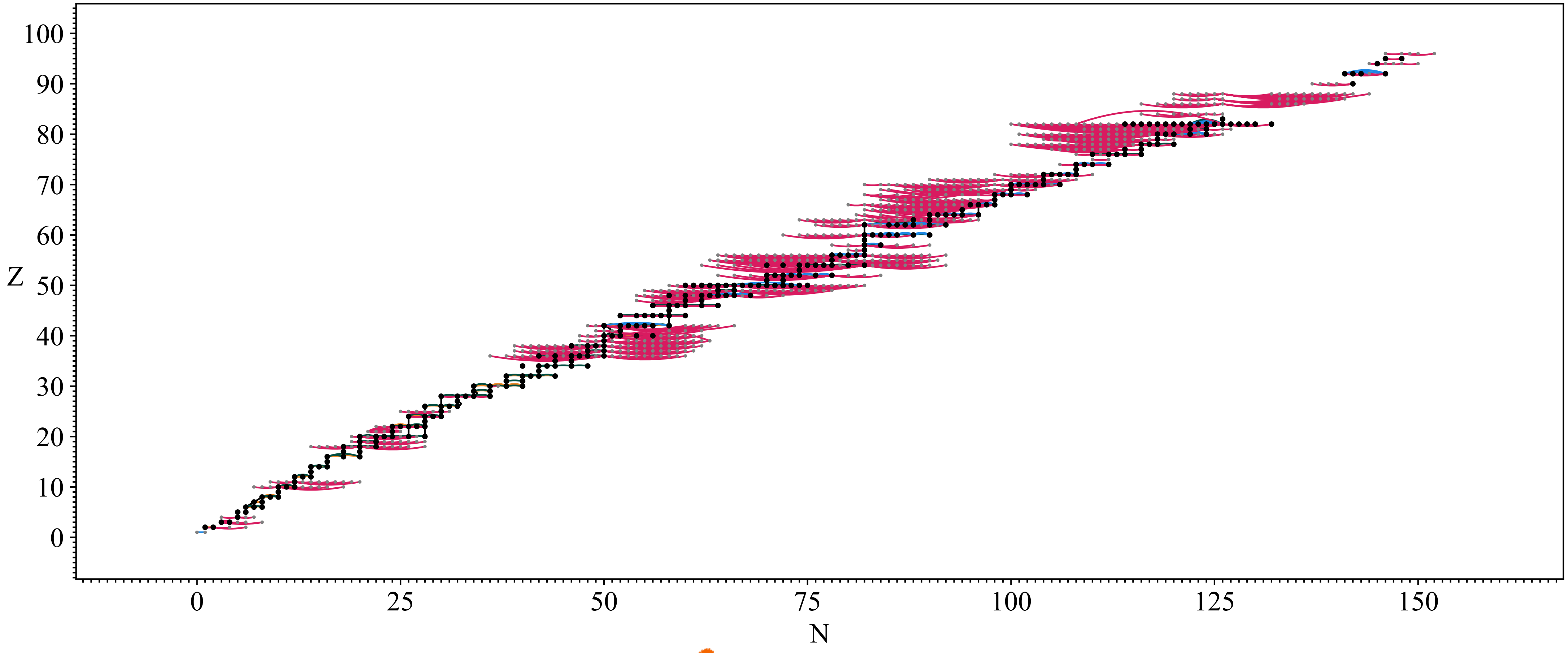
Nuclear Surface is 2 Dimensional



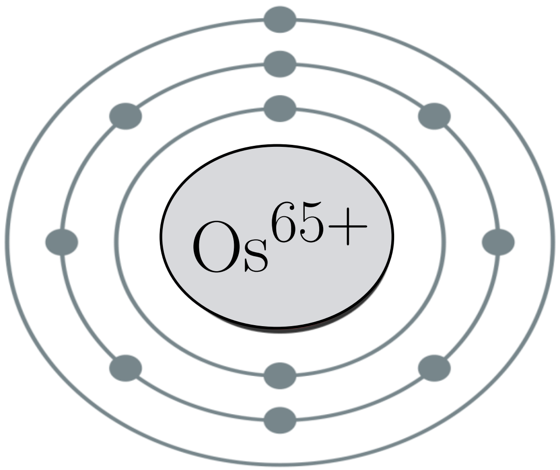
Use Cases of Inter-Element Measurements

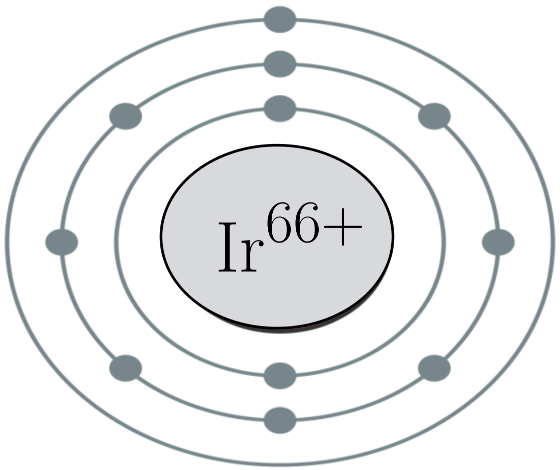
Element A ΔR	Element B ΔR	
Low	Low	▪ Testing consistency of absolute measurements
Low	High	▪ Lowering absolute nuclear charge radius uncertainties through connection.
High	High	▪ Extract differential trends in nuclear charge radii

Challenge: How to Study Isotonic Chains

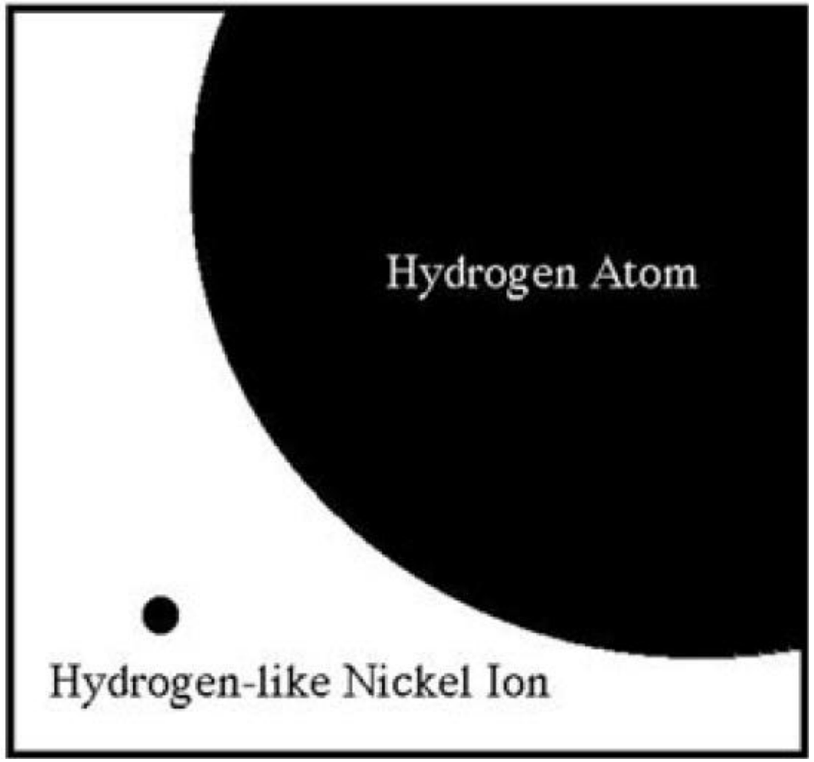
- Isotonic (same N, different Z) chains are challenging to study directly because you have to separate the change in radius from the change in electronic structure.

- What if we forced the electronic structure to be the same?





Highly Charged Ion (HCI) Radius Measurements

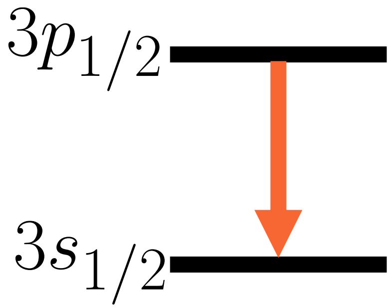


J.D. Gillaspy J. Phys. B. 34, R93 (2001)

$$S \equiv \frac{\partial E}{\partial R} \qquad S \propto Z^4$$

- Simple charge states (Na-like, Li-like) to reduce theoretical uncertainties
- Choose transitions that are strongly sensitive to nuclear radius such as the Na-like D_1 ($3p_{1/2}$-$3s_{1/2}$) transition.

Na-like D_1



HCI Radius Measurements: Experiment

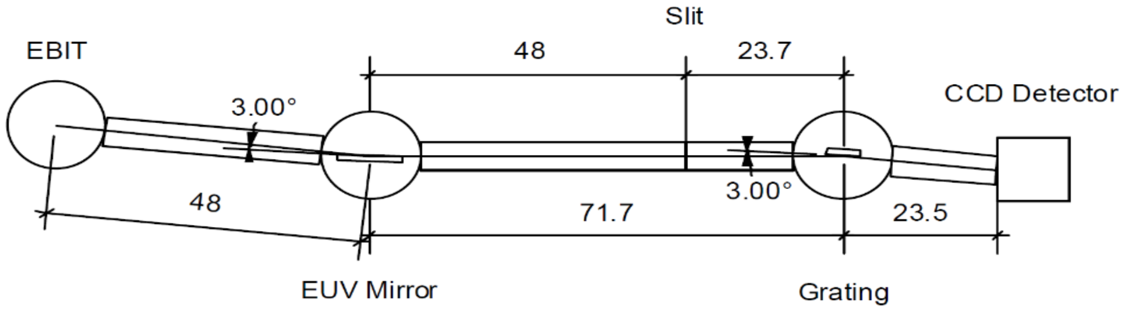
- Creation and excitation of HCIs are done in an EBIT
- Electron beam (E≈15 keV) repeatedly ionizes and radially traps ions
- Drift tubes provide axial trapping

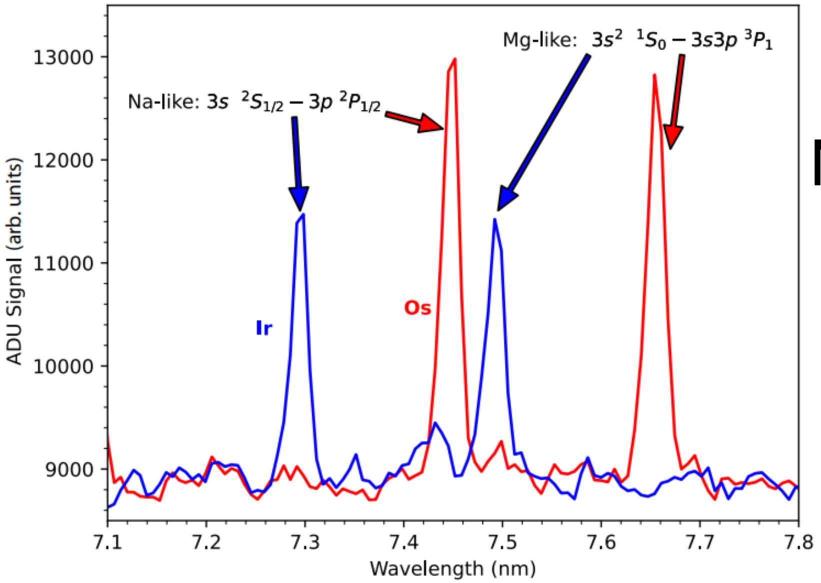
https://www.nist.gov/video/electron-beam-ion-trap

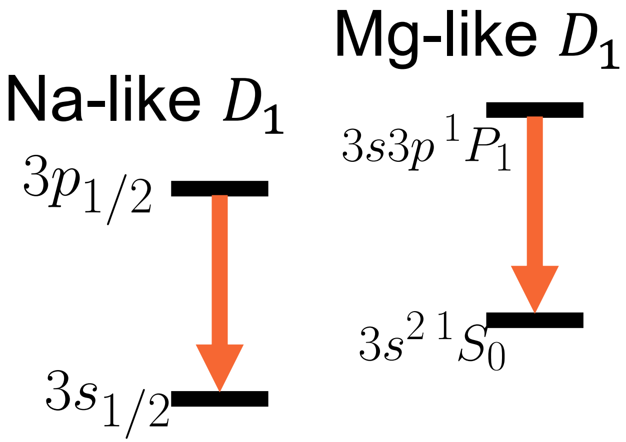
HCI radius measurements: experiment

- Na-like transition energies are far into the extreme ultraviolet (EUV) for Z>50, (80-200) eV or (5-15) nm. Measurements made using EUV Spectrometer.

- When looking at transition energy differences, calibration uncertainty largely cancels.







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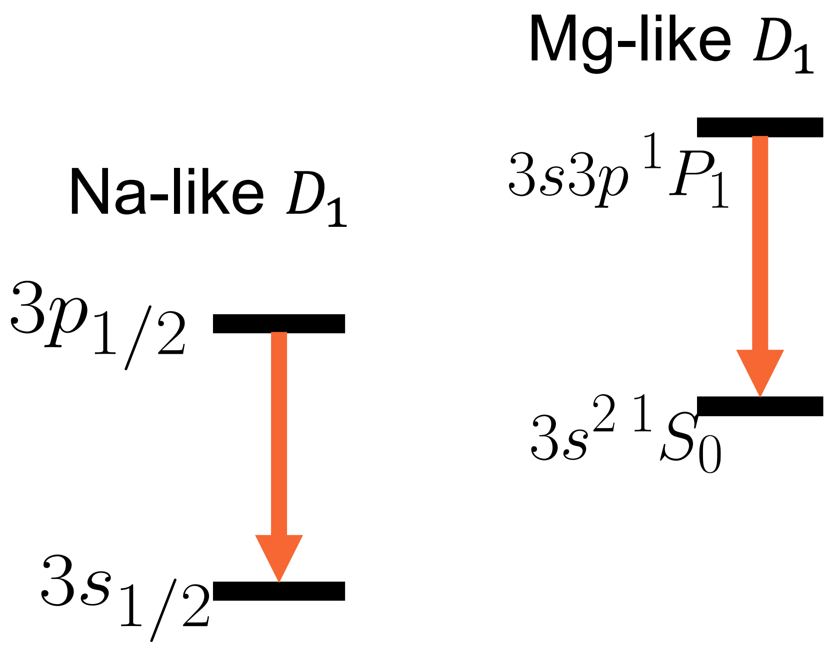
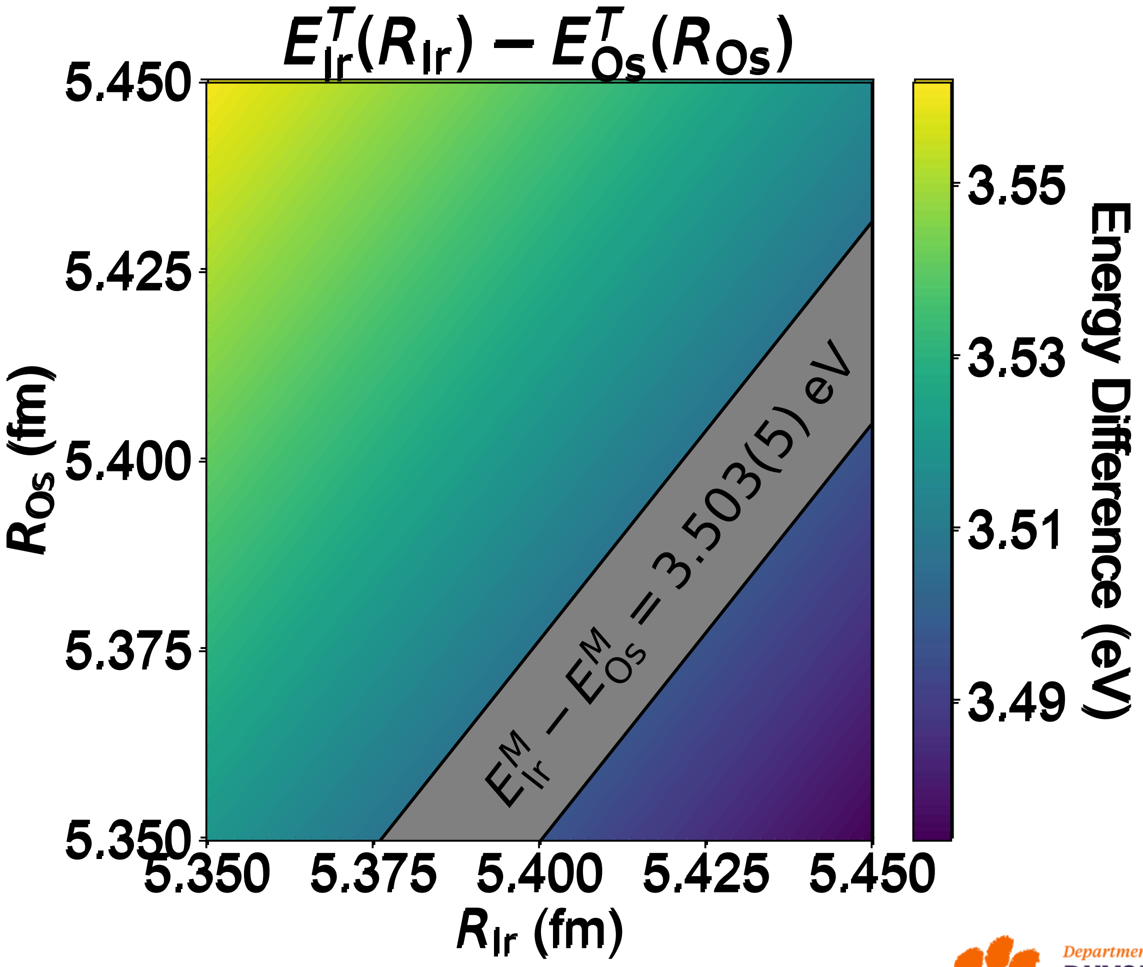
DOI: 10.1103/PhysRevResearch.7.L012024

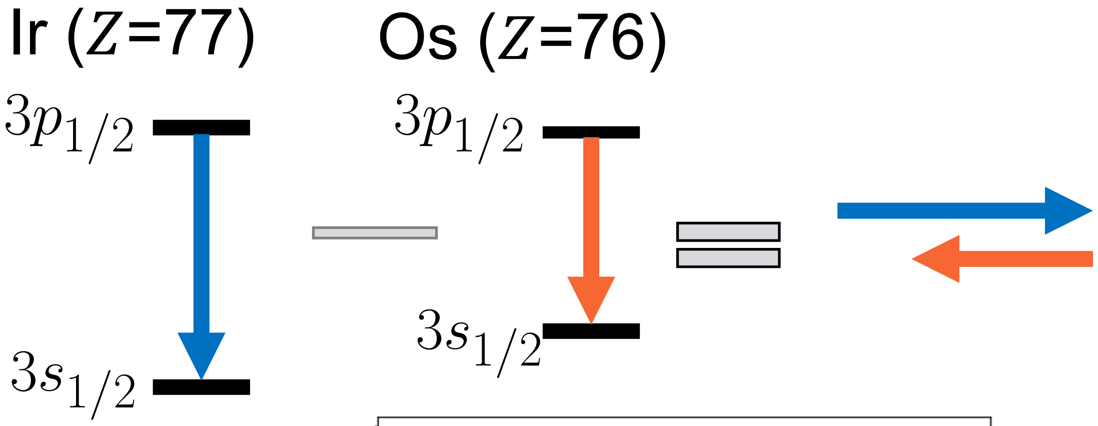
HCI Radius Measurements: Theory

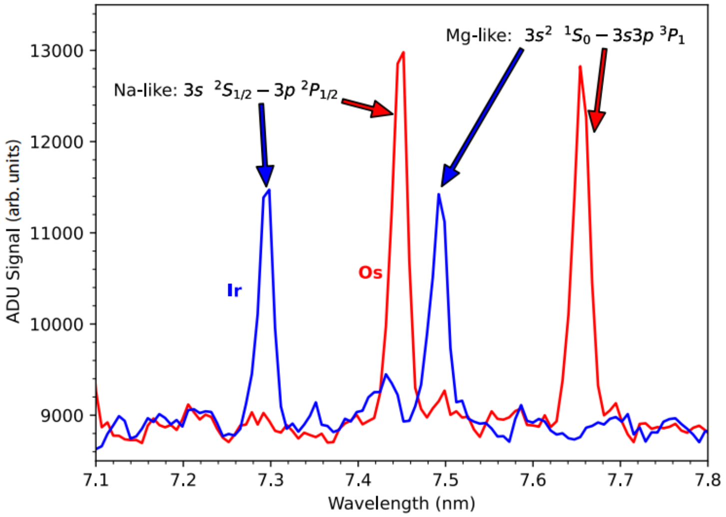
- Transition energies calculated using relativistic many body perturbation theory (RMBPT).
- Recent result: Mg-like calculation using RMBPT + configuration interaction to handle strong n=3 state mixing.



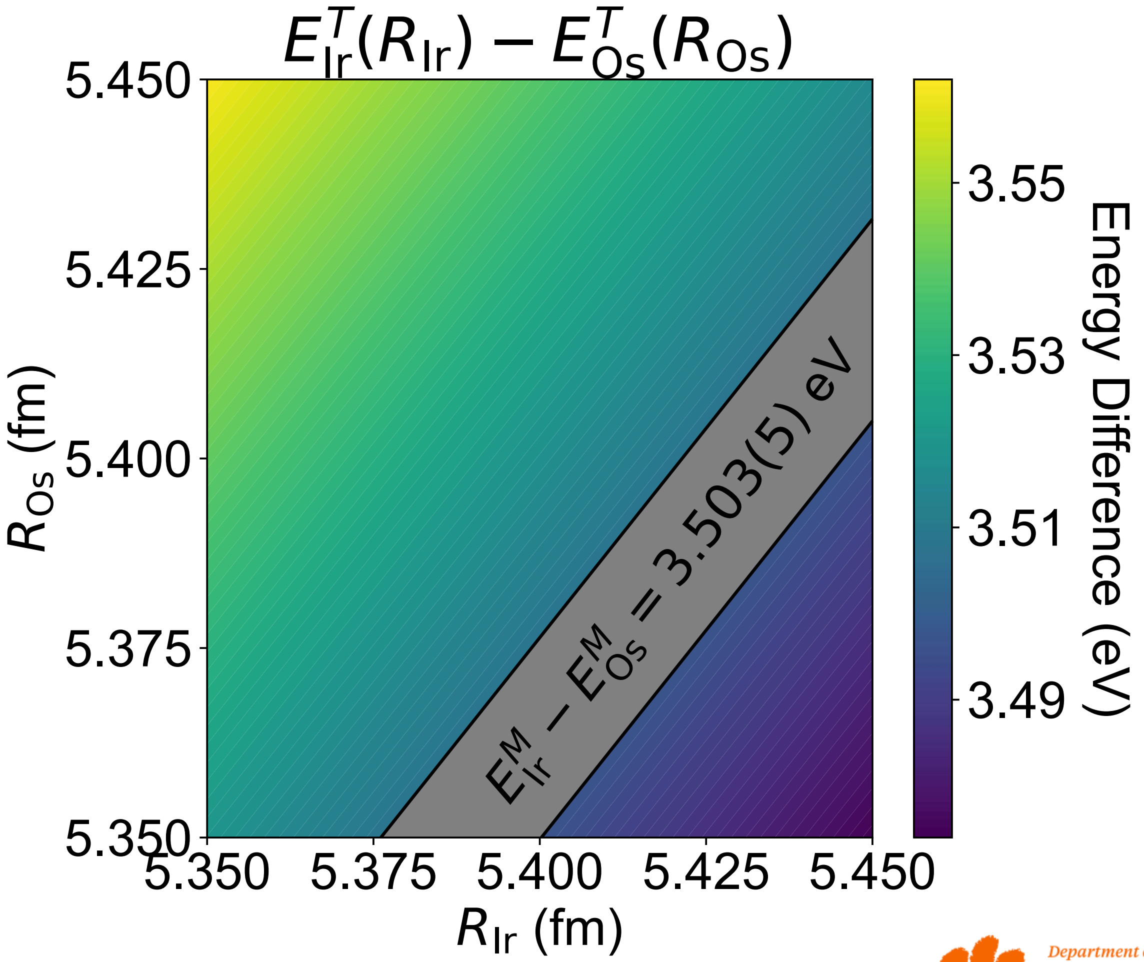
HCI Radius Measurements







HCI radius measurements



$$E^T(R) = E^T(R_0) + S(R - R_0)$$

$$R_B - \frac{S_A}{S_B}R_A = \left(E^M_B - E^M_A\right) - \left[E^T_B(R_{B_0}) - E^T_A(R_{A_0})\right] + \left(R_{A_0} - \frac{S_B}{S_A}R_{B_0}\right)$$

$$E[\rho(r, \theta, \phi)]$$

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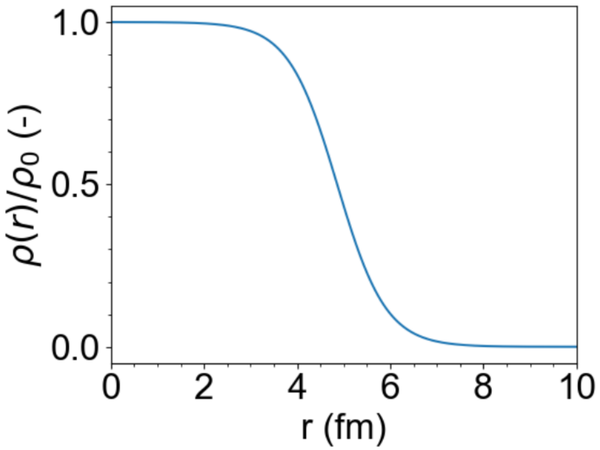
Nuclear model dependence

2-parameter Fermi distribution (spherical nuclei)

$$\rho(r) = \frac{\rho_0}{1 + \exp\frac{4\ln(3)(r-c)}{t}}$$



3-parameter Fermi distribution (deformed nuclei)

$$\rho(r,\theta) = \frac{\rho_0}{1 + \exp\frac{4\ln(3)[r-c(1+\beta_2 Y_{20}(\theta))]}{t}}$$

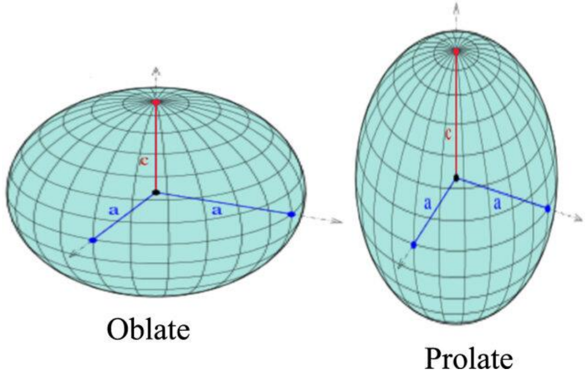


Image Credit: E. Yazdankish Phys. Scr. 98, 115309 (2023)

Nuclear model dependence

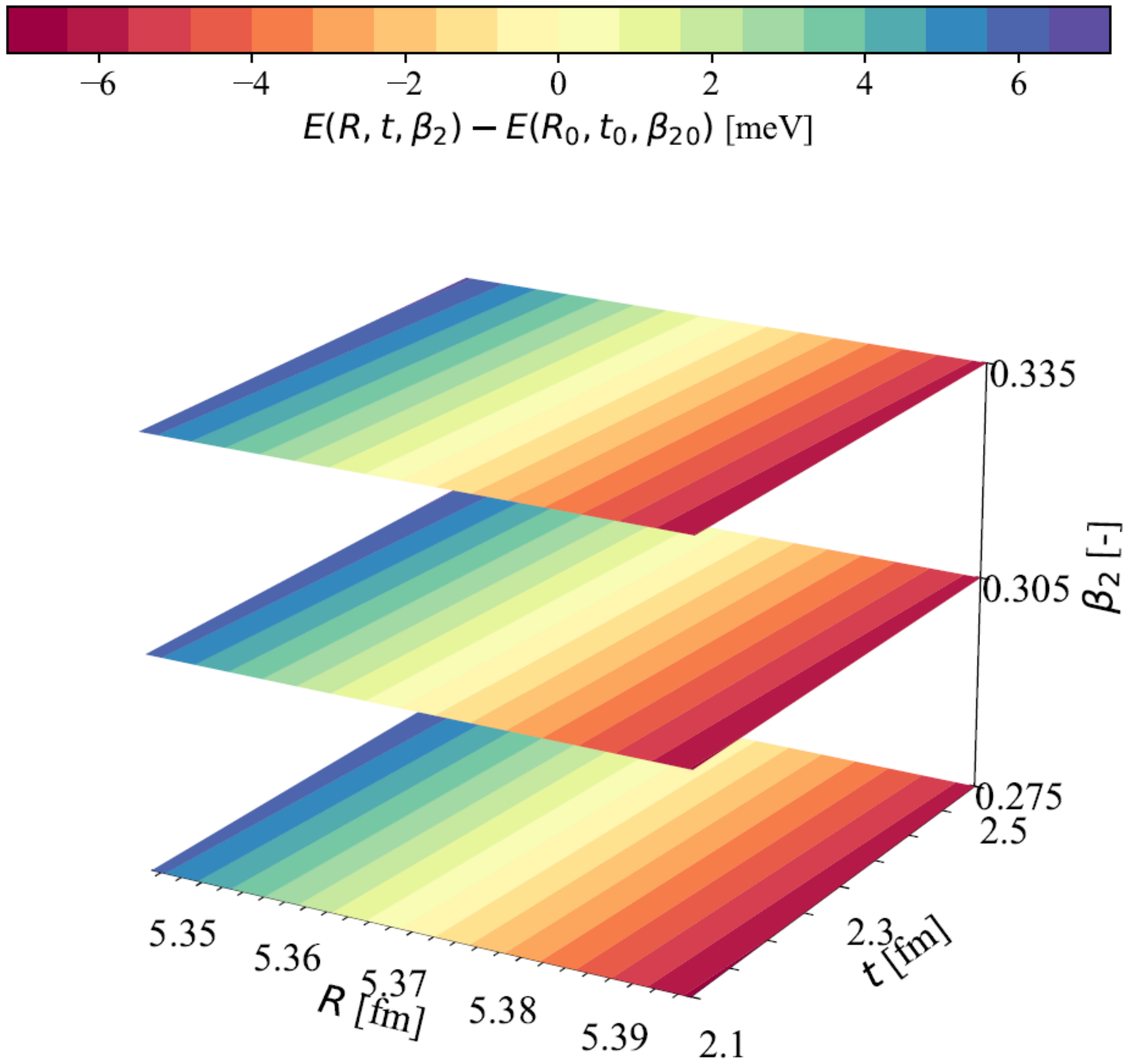
$$S \equiv \frac{\partial E(R,t,\beta_2)}{R}$$

$$S_t \equiv \frac{\partial E(R,t,\beta_2)}{\partial \beta_2}$$

$$S_{\beta_2} \equiv \frac{\partial E(R,t,\beta_2)}{\partial t}$$

$$S \gg S_t \approx S_{\beta_2}$$

Mostly Model Independent



HCI radius measurements

$$\left[\Delta(R_B - \frac{S_A}{S_B}R_A)\right]^2 =$$

$$\left[\frac{\Delta\left(E_B^M - E_A^M\right)}{S_B}\right]^2 + \left\{\frac{\Delta[E_B^T(R_{0B}) - E_A^T(R_{0A})]}{S_B}\right\}^2$$

→ Uncertainty in experimental and theoretical energy differences.

$$+ \left\{\frac{\Delta S_B}{S_B}[(R_B - R_{0B}) - (S_A/S_B)(R_A - R_{0A})]\right\}^2$$

→ Uncertainty in radial sensitivity coefficient

$$+ \left(\frac{S_{\beta_2 B}\Delta\beta_{2B}}{S_B}\right)^2 + \left(\frac{S_{tB}\Delta t_B}{S_B}\right)^2$$

$$+ \left(\frac{S_{\beta_2 A}\Delta\beta_{2A}}{S_B}\right)^2 + \left(\frac{S_{tA}\Delta t_A}{S_B}\right)^2 .$$

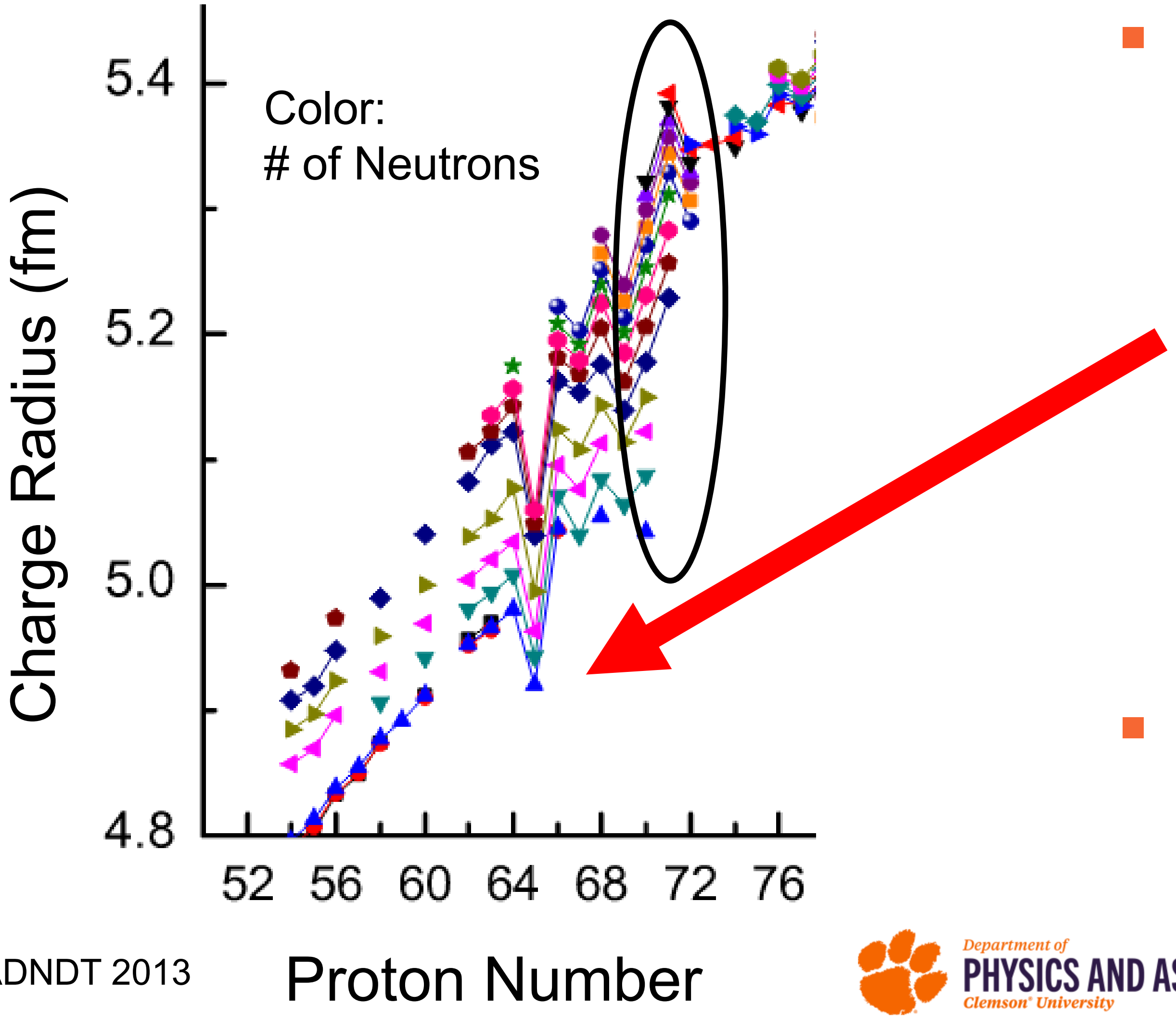
→ Uncertainty in other nuclear parameters.

Previous HCI Radius Determinations

- Xe136 – Xe124 radius difference from Na-like
  - R. Silwal PRA 98, 052502 (2018)
- Xe136 – Xe124 radius difference from Na-like, Mg-like, Al-like
  - R. Silwal PRA 101, 062512 (2020)
- Natural abundance Ir – Os radius difference from Na-like
  - A. Hosier PRR 7, L012024 (2025)
- Reanalysis of Li-like Pb, Bi
  - V. Yerokhin PRA 113, 012804 (2026)

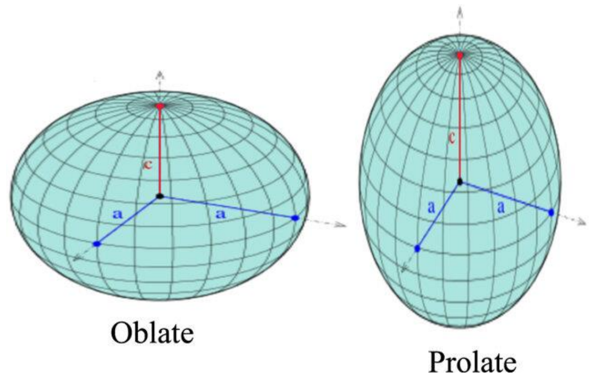
Puzzling Odd-Even Staggering (OES) of Rare Earth Nuclei

The Puzzle



Angeli ADNDT 2013

- Rare earth nuclei are **highly deformed**, causing an odd/even stagger in charge radii



- Exception: Lu (Z=71)?

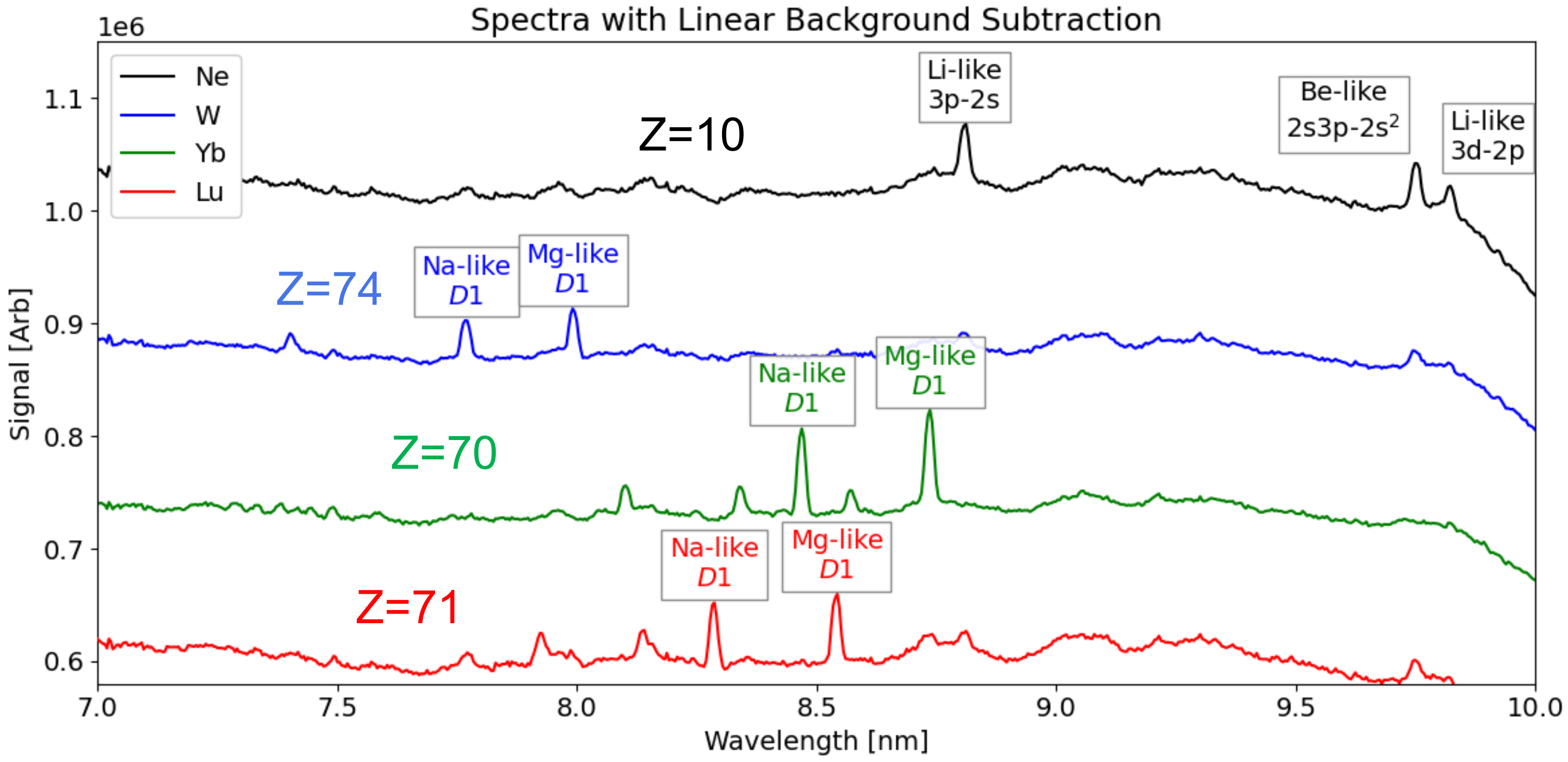
Image Credit: E. Yazdankish Phys. Scr. 98, 115309 (2023)

Lu radius measurement

- Ytterbium (Z=70) radii are known to 0.006 fm
- Lutetium (Z=71) radii are known to 0.030 fm
- Determine Lu-Yb scaled radius difference – lower radius uncertainty of Lu radii.

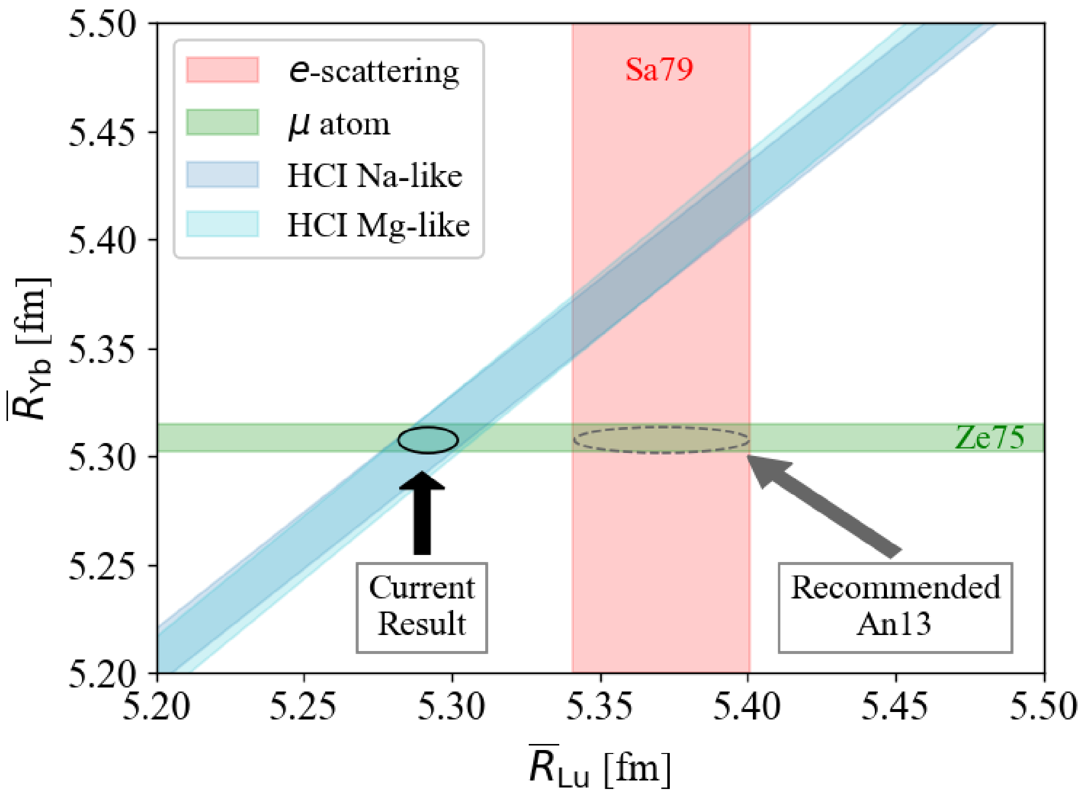
Lu-Yb: The Experiment

- 5 day run, injected Lu, Yb, W, and Ne into Tokyo EBIT with electron beam energy of 10 keV.
- Use W and Ne spectra for calibration
- Note: Inject Ne with W, Lu and Yb to get better signal!

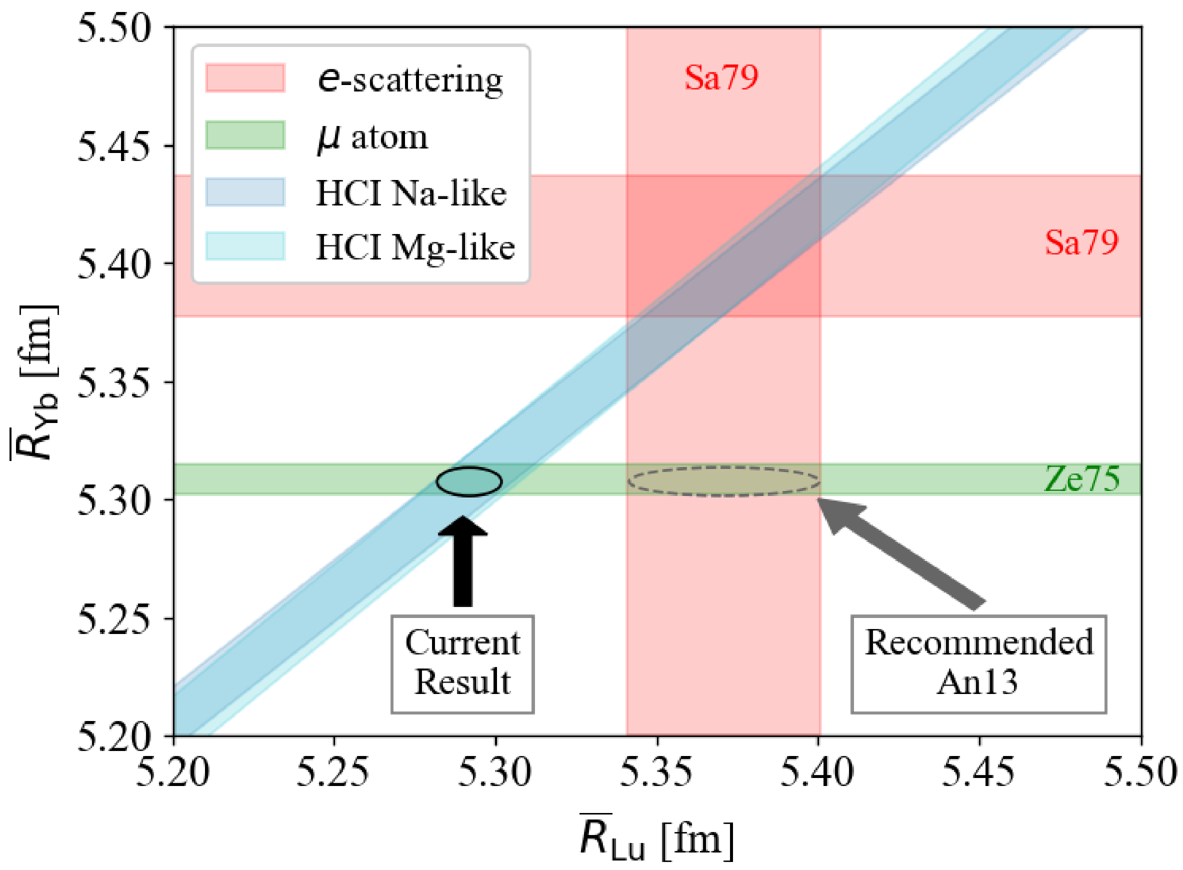


Lu-Yb: Radius Result

- Our Lu-Yb difference (blue region) + previous muonic atom measurement (green region) gave a Lu radius of 5.291(11) fm, **factor of 3 reduction in uncertainty**!
- Problem: Previous measurement of Lu radius was 5.37(3) fm?



Lu-Yb: Radius Result



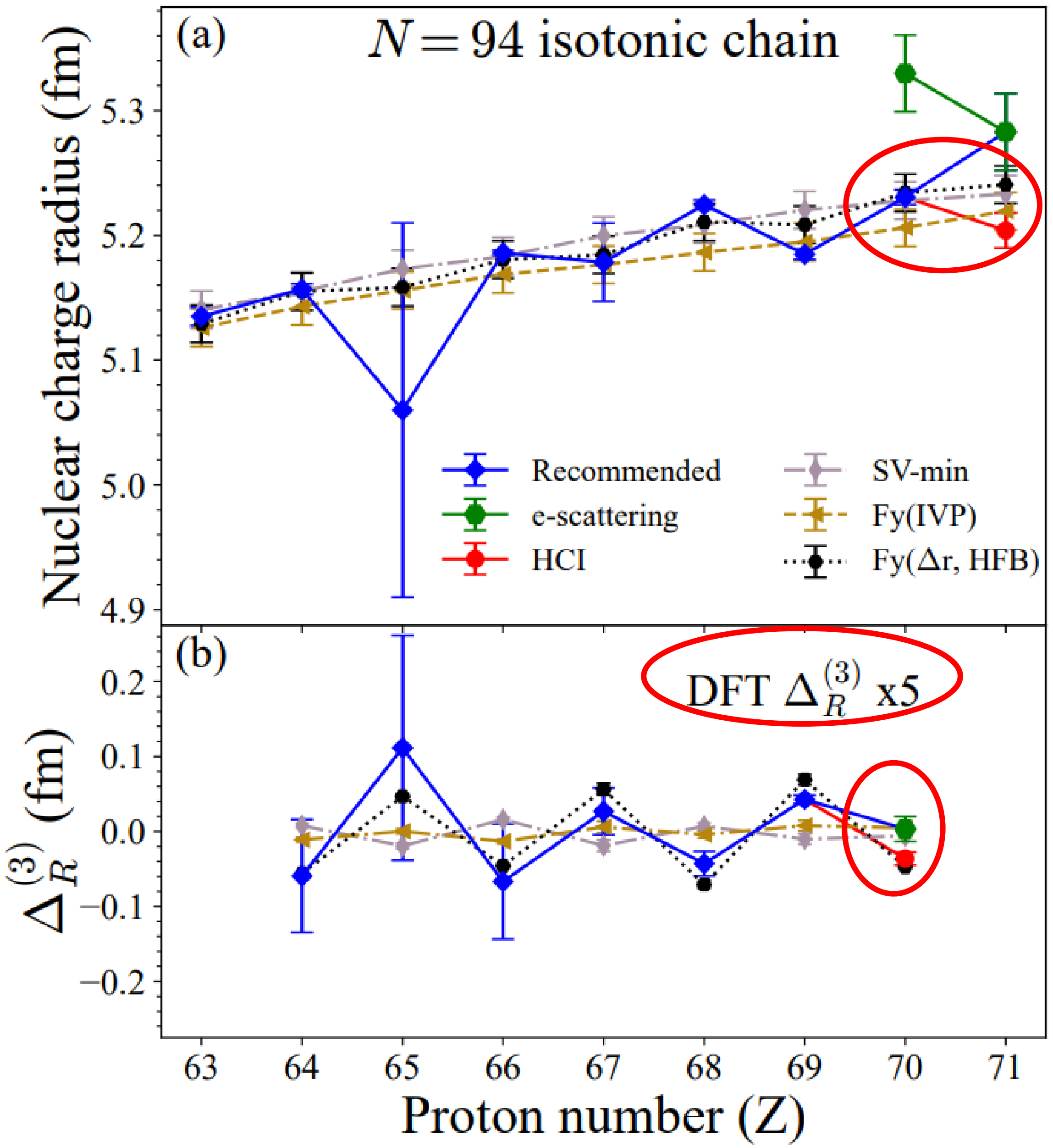
- Interesting problem: that same e- scattering paper (Sa79) also measured Yb and got a different result than the muonic measurement!
- Maybe the radii from Sa79 are systematically too high?

E. Takacs arXiv:2511.19395
H. Staiger arXiv:2511.20537

- Theory predicts absolute charge radii quite well, all theoretical error bars overlap with experimental ones

$$\Delta_R^{(3)} = \frac{1}{2}\left(R_{N-1} - 2R_N + R_{N+1}\right)$$

- However, theory predicts dramatically less stagger than seen in experiment, at least a factor of two or more.



E. Takacs arXiv:2511.19395
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SCIFORWARD
To Guide Our Strategic Priorities and Hold Us Accountable to Our Aspirations

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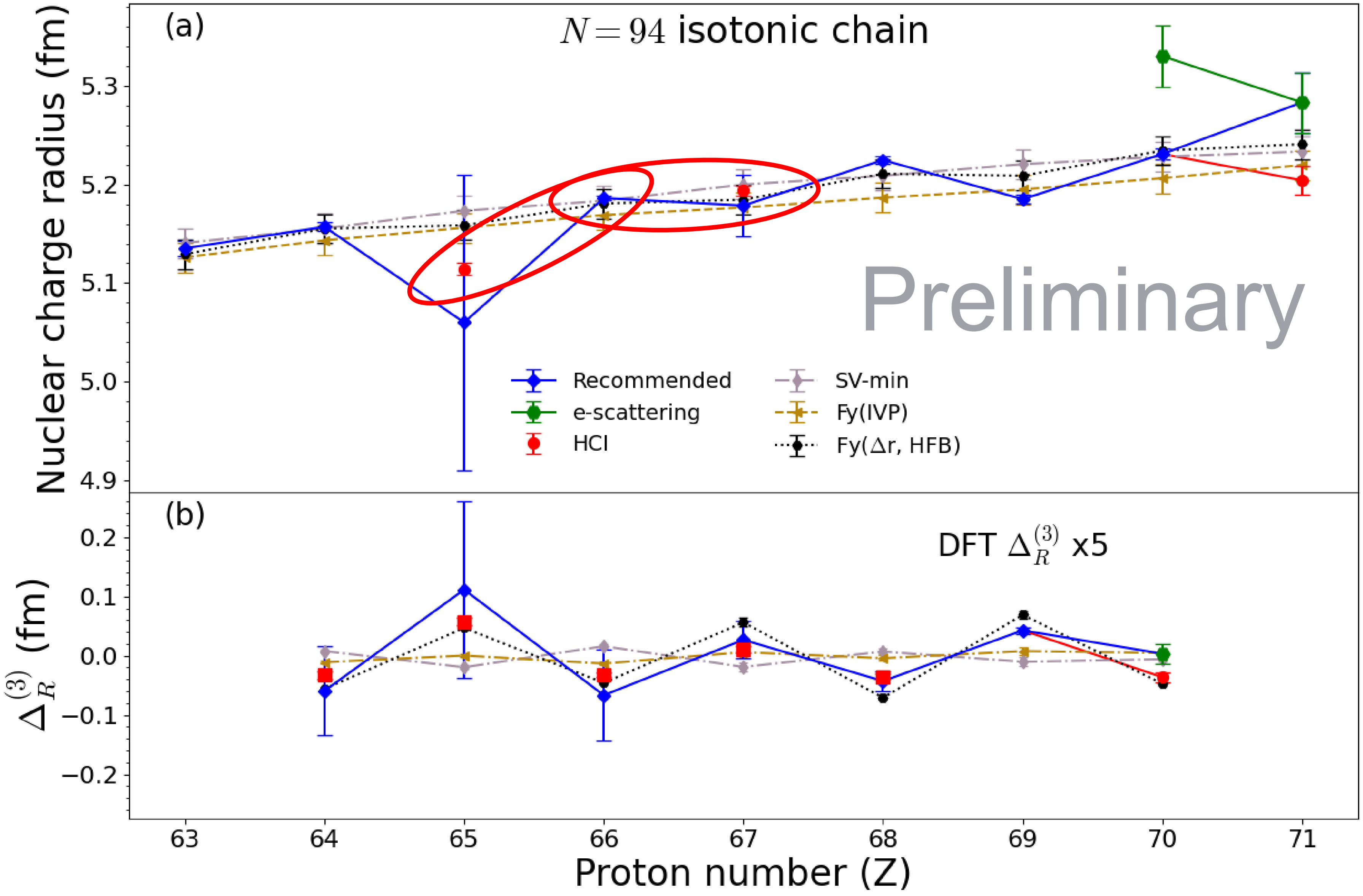

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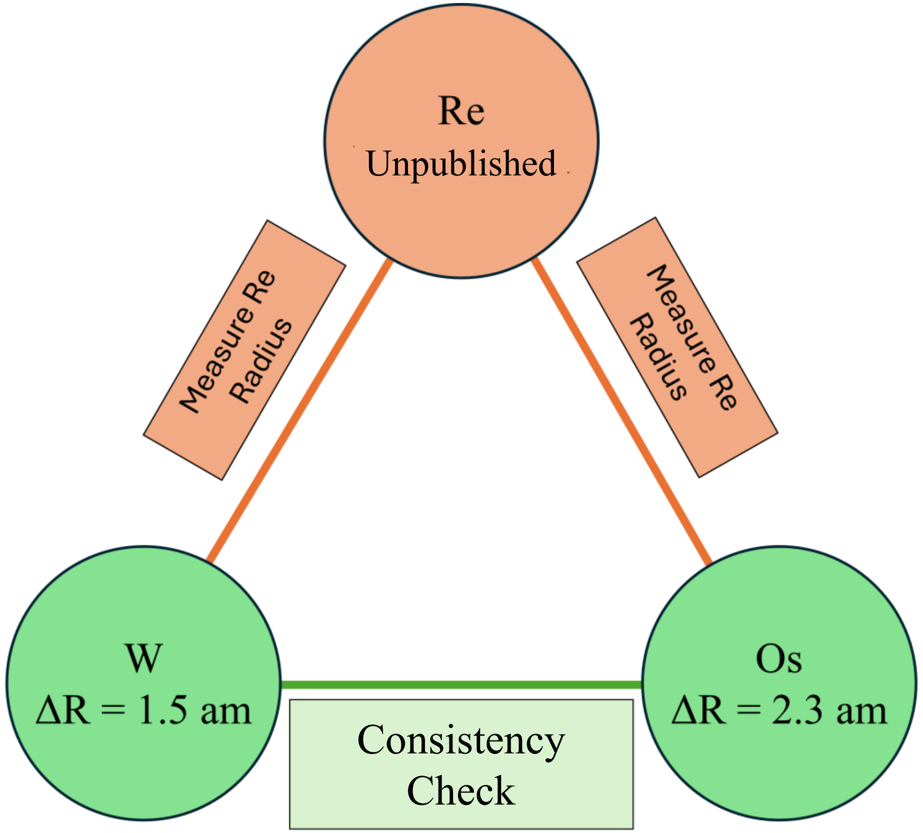
Outlook and Conclusions

Planned Experiments

- Isotopically pure W, Re, Os



- Th229 – Th232 radius difference measurement

- Injection using laser desorption near trapping region, similar to:
  - Ch. Schweiger Rev. Sci. Instrum. 90, 123201 (2019)

- Developing injection scheme.

Conclusions

- Highly charged ions allow for unique inter-element studies of charge radius trends.
- Further measurements of charge radii in the rare earth region are required to explain the tension with nuclear theory
- Combining information from different techniques provides unique pathways to charge radius information.

Acknowledgements

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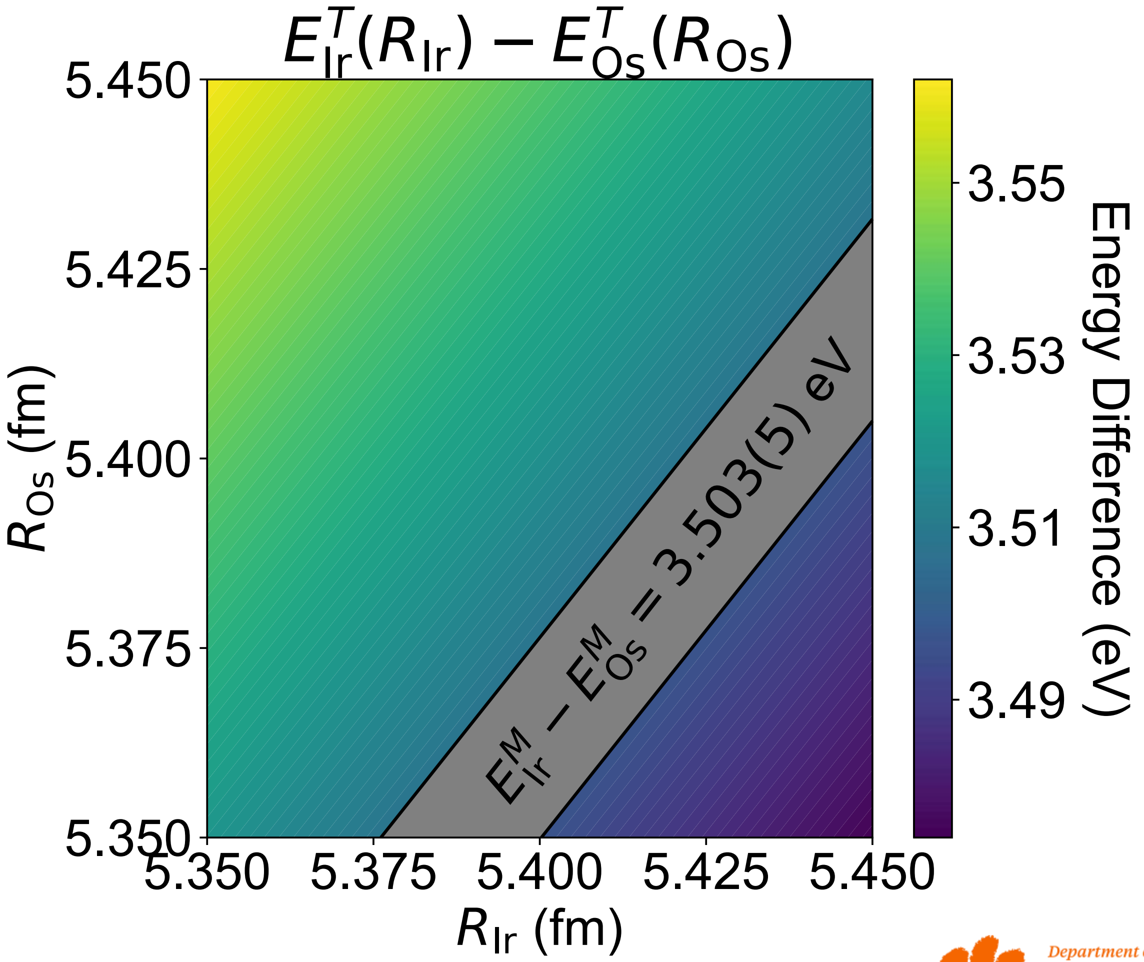
Backup Slides

HCI Radius Measurements



$$E^T(R) = E^T(R_0) + S(R - R_0)$$

$E^T(\mathrm{R})$ linear → scaled radius difference still holds with natural abundance averaged energies and radii.

$$\bar{R}_A \equiv \sum w_{iA} R_{iA}$$

$$\bar{E}_A \equiv \sum w_{iA} E_{iA}$$

w_{iA} = Abundance of isotope i of element A.

Nuclear Model Dependence

$$E(R)$$

$$E[\rho(r,\theta,\phi)]$$

$$P(R|E^M) = P(E^M|R)P(R)$$

$$P(\rho(r,\theta,\phi)|E^M) \propto P(E^M|\rho(r,\theta,\phi))P(\rho(r,\theta,\phi))$$

P(R|E^M): Posterior
P(E^M | R): Likelihood
P(R): Prior

Uniform prior -> Maximum likelihood

Likelihood is easy, but how do we place a prior over a function?

Nuclear Model Dependence

We know that the nuclear charge distribution:

- Has total charge Z|e|
- Is continuous and differentiable
- Has finite extent (is zero beyond some cutoff radius)
- Is mostly flat near the center (incompressibility)

Nuclear Model Dependence: Basis Expansion

- Elastic electron scattering measures scattering cross sections for various levels of momentum transfer, provides the most constraints on the shape of the charge distribution.

$$\rho(r) = \begin{cases} \sum_{\nu=1}^{N} a_\nu j_0(q_\nu r), & r < R_c \\ 0, & r > R_c \end{cases}$$

$$\frac{\Delta\langle r^2\rangle}{\langle r^2\rangle} \gg \frac{\Delta\left(\langle r^2\rangle/\langle r^4\rangle\right)}{\langle r^2\rangle/\langle r^4\rangle}$$

j0 = spherical Bessel function of zeroth order,
q_v = v pi / R_c,
R_c = cutoff radius

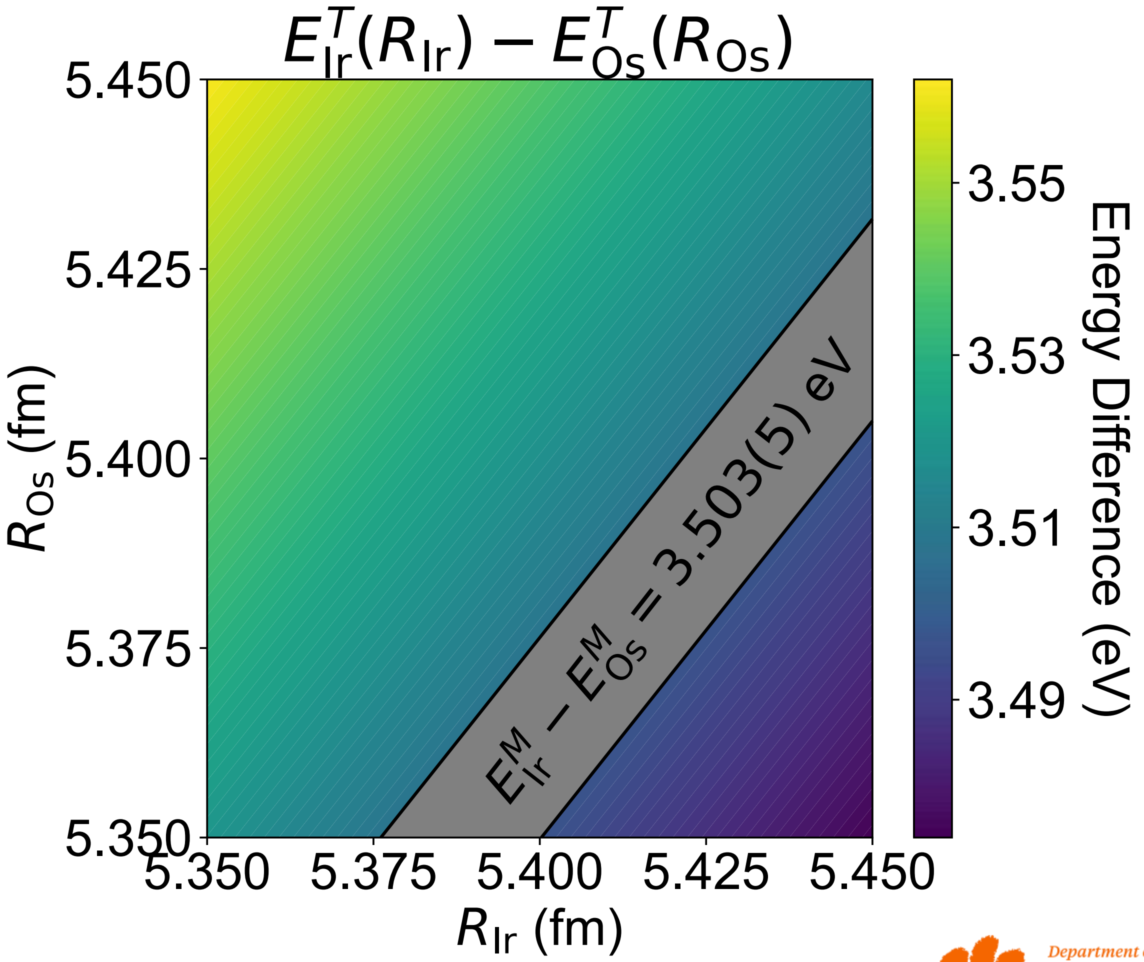
Nuclear Model Dependence: Method of Moments

- Define energy as a strict function of some non-radius moment of charge distribution. Take ratio of that moment to the <r^2> moment from e- scattering/muonic data.

"Barrett Moment" $\langle r^k e^{-r\alpha} \rangle$ + $V^{e-} \equiv \frac{\langle r^k e^{-r\alpha} \rangle}{\langle r^2 \rangle}$

"Seltzer Moment" $\Lambda^{AA'} = \delta\langle r^2 \rangle^{AA'} + \frac{C_2}{C_1}\delta\langle r^4 \rangle^{AA'} + \frac{C_3}{C_1}\delta\langle r^4 \rangle^{AA'} + \cdots$

HCI radius measurements



$$E^T(R) = E^T(R_0) + S(R - R_0)$$

$$R_B - \frac{S_A}{S_B} R_A = \left(E^M_B - E^M_A\right) - \left[E^T_B(R_{B_0}) - E^T_A(R_{A_0})\right] + \left(R_{A_0} - \frac{S_B}{S_A} R_{B_0}\right)$$

HCI Radius Measurements: Theory

- Error in transition energies comes from omitted corrections
- Omitted corrections scale smoothly with Z, errors in nearby elements are highly correlated.
- ≈factor of 20 reduction in uncertainty using energy differences compared to individual energies.