Second-order effective Hamiltonian of QCD

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Motivation for the Front Form (FF)

- Relativistic scattering (EIC)
- Rise of quantum computing

Motivation for my work

- Develop FF framework for quantum computers
- FF QCD Hamiltonian needs to be renormalized

Outline

- Wick's diagrams
- Canonical Hamiltonian
- Zero modes, $p^+ > \epsilon^+$ cutoff
- UV renormalization (t_r cutoff)
- IR structure (gluon mass m_g)
- Self-adjoint extensions

Momentum representation of fields

$$\psi(x) = \sum_\sigma \int \frac{dp^+ d^2p^\perp}{16\pi^3 p^+} \theta(p^+) \left[u_\sigma(p) e^{-ipx} b_{p\sigma} + v_\sigma(p) e^{ipx} d^\dagger_{p\sigma} \right]$$

$$\phi(x) = \int \frac{dp^+ d^2p^\perp}{16\pi^3 p^+} \theta(p^+) \left[e^{-ipx} a_p + e^{ipx} a^\dagger_p \right]$$

$$\begin{aligned}
\Psi(q) &= \int dx^- d^2x^\perp \; e^{\frac{i}{2}q^+x^- - iq^\perp x^\perp} \; \psi(x) \\
&= \sum_\sigma \frac{\theta(q^+) u_\sigma(q) b_{q\sigma} + \theta(-q^+) v_\sigma(-q) d^\dagger_{-q\sigma}}{|q^+|} \\
\Phi(q) &= \frac{\theta(q^+) a_q + \theta(-q^+) a^\dagger_{-q}}{|q^+|}
\end{aligned}$$

PHYSICAL REVIEW D **112,** 096003 (2025)

Second-order renormalized Hamiltonian of Yukawa theory

Kamil Serafin,[*] Carter M. Gustin, and Peter J. Love[†]
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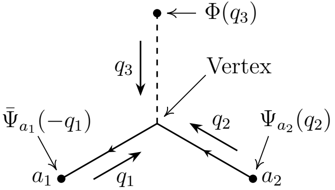
(Received 6 August 2025; accepted 23 September 2025; published 4 November 2025)



Wick's diagram

$$g \int [q_1 q_2 q_3] \, \tilde\delta_{123} \, \mathcal{N} \left[\bar\Psi(-q_1) \Psi(q_2) \Phi(q_3) \right]$$

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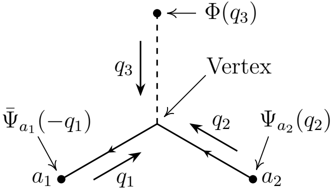
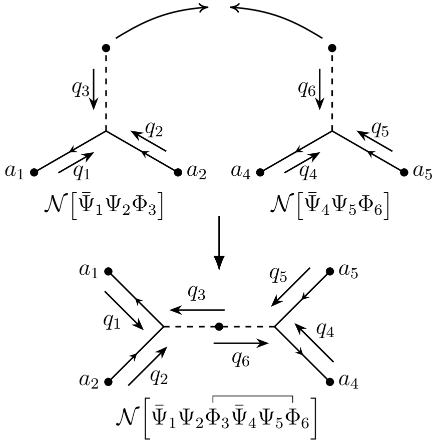


$$g\int[q_1q_2q_3]\,\tilde{\delta}_{123}\,\mathcal{N}\left[\bar{\Psi}(-q_1)\Psi(q_2)\Phi(q_3)\right]$$

$$\overbrace{\Phi_i\Phi_j} = \Phi_i\Phi_j - \mathcal{N}(\Phi_i\Phi_j)$$



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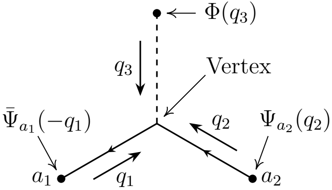
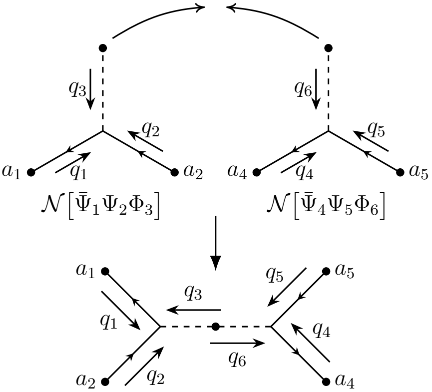




$$g\int[q_1q_2q_3]\,\tilde\delta_{123}\,\mathcal{N}\left[\bar\Psi(-q_1)\Psi(q_2)\Phi(q_3)\right]$$

$$\overline{\Phi_i\Phi_j} = \Phi_i\Phi_j - \mathcal{N}(\Phi_i\Phi_j)$$

$$g^2\int[q_1q_2q_3q_4q_5]\,\tilde\delta_{1245}\,\tilde\delta_{45.3}\,B_{t,123.45(-3)}\,\frac{\theta(q_3^+)}{q_3^+}\mathcal{N}\left[\bar\Psi(-q_1)\Psi(q_2)\bar\Psi(-q_4)\Psi(q_5)\right]$$

Canonical Hamiltonian of QCD

Lagrangian density

$$\mathcal{L} = \bar{\psi}\left(\gamma_\mu i\partial^\mu - m\right)\psi - g\bar{\psi}\gamma_\mu\psi A^\mu - \frac{1}{4}F^{\mu\nu\, c}F^c_{\mu\nu}$$

$$F^{\mu\nu\, c} = \partial^\mu A^{\nu\, c} - \partial^\nu A^{\mu\, c} - gf^{cab}A^{\mu\, a}A^{\nu\, b}$$

$$\downarrow$$

Hamiltonian density

$$\mathcal{H} = \mathcal{H}_{\psi^2+A^2} + \mathcal{H}_{jA} + \mathcal{H}_{A^4} + \mathcal{H}_{\psi AA\psi} + \mathcal{H}_{jj}$$

$$\downarrow$$

Canonical Hamiltonian

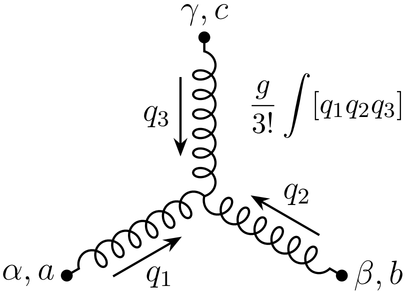
$$H_{\text{canonical}} = \int d^3x\mathcal{H}$$

Canonical Hamiltonian of QCD

$$
\begin{aligned}
\mathcal{H}_{\psi^2+A^2} &= \mathcal{N}\left(\bar{\psi}\frac{\gamma^+}{2}\frac{(i\partial^\perp)^2+m^2}{i\partial^+}\psi\right)+\mathcal{N}\left\{\frac{1}{2}A^{i\,a}\left[m_g^2+(i\partial^\perp)^2\right]A^{i\,a}\right\}\\
\mathcal{H}_{jA} &= j^a_{q\,\mu}A^{\mu a}+j^a_{g\,\mu}A^{\mu a}\\
\mathcal{H}_{A^4} &= \frac{1}{4}g^2 f^{abc}f^{ade}\mathcal{N}\left(A^b_\alpha A^c_\beta A^{\alpha\,d}A^{\beta\,e}\right)\\
\mathcal{H}_{\psi AA\psi} &= \frac{1}{2}g^2\mathcal{N}\left(\bar{\psi}\gamma^i A^{ia}T^a\frac{\gamma^+}{i\partial^+}\gamma^j A^{jb}T^b\psi\right)\\
\mathcal{H}_{jj} &= \frac{1}{2}\mathcal{N}\left[\left(j_q^{+a}+3j_g^{+a}\right)\frac{1}{(i\partial^+)^2}\left(j_q^{+a}+3j_g^{+a}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
j_q^{\mu a} &= g\mathcal{N}\left(\bar{\psi}\gamma^\mu T^a\psi\right)\\
j_g^{\mu c} &= \frac{g}{3}if^{abc}\left[A^a_\alpha(x)i\partial^\mu A^{\alpha b}(x)-2A^{\alpha a}(x)i\partial_\alpha A^{\mu b}(x)\right]
\end{aligned}
$$

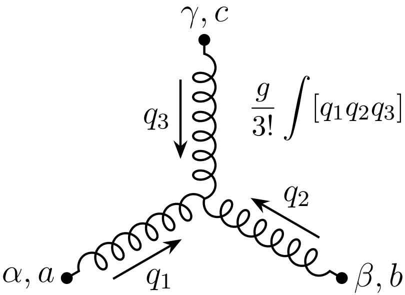
Canonical Hamiltonian of QCD



$$\frac{g}{3!}\int [q_1 q_2 q_3]\, \tilde{\delta}_{123} F^{abc}_{\alpha\beta\gamma}(q_1, q_2, q_3)\, G^{\alpha a}(q_1) G^{\beta b}(q_2) G^{\gamma c}(q_3)$$

$$F^{abc}_{\alpha\beta\gamma}(q_1, q_2, q_3) = i f^{abc}[(q_{2\gamma} - q_{1\gamma}) g_{\alpha\beta} + (q_{3\alpha} - q_{2\alpha}) g_{\beta\gamma} + (q_{1\beta} - q_{3\beta}) g_{\gamma\alpha}]$$

Canonical Hamiltonian of QCD

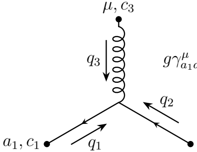


$$\frac{g}{3!}\int [q_1 q_2 q_3]\,\tilde{\delta}_{123} F^{abc}_{\alpha\beta\gamma}(q_1, q_2, q_3)\, G^{\alpha a}(q_1) G^{\beta b}(q_2) G^{\gamma c}(q_3)$$

$$F^{abc}_{\alpha\beta\gamma}(q_1, q_2, q_3) = i f^{abc}[(q_{2\gamma} - q_{1\gamma}) g_{\alpha\beta} + (q_{3\alpha} - q_{2\alpha}) g_{\beta\gamma} + (q_{1\beta} - q_{3\beta}) g_{\gamma\alpha}]$$

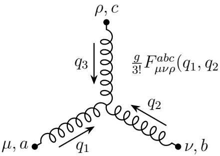
$$J^{c}_{g\,\mu}(q) = \frac{g}{3!}\int [q_1 q_2]\,\tilde{\delta}_{12.q}\, F^{abc}_{\alpha\beta\mu}(q_1, q_2, -q)\, G^{\alpha\, a}(q_1)\, G^{\beta\, b}(q_2)$$

Canonical Hamiltonian of QCD



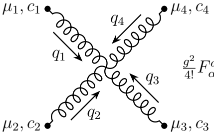
$g\gamma^{\mu}_{a_1 a_2} T^{c_3}_{c_1 c_2}$

$\mathcal{N}\left[\bar{\Psi}_{a_1,c_1}(-q_1)\Psi_{a_2,c_2}(q_2)G^{c_3}_{\mu}(q_3)\right]$



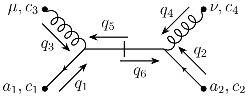
$\frac{g}{3!}F^{abc}_{\mu\nu\rho}(q_1, q_2, q_3)$

$\mathcal{N}\left[G^{\mu\, a}(q_1)G^{\nu\, b}(q_2)G^{\rho\, c}(q_3)\right]$



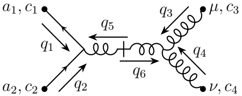
$\frac{g^2}{4!}F^{abcd}_{\alpha\beta\gamma\delta}$

$\mathcal{N}\left[G^{\alpha\, a}(q_1)G^{\beta\, b}(q_2)G^{\gamma\, c}(q_3)G^{\delta\, d}(q_4)\right]$



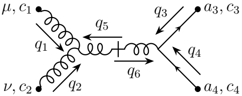
$\frac{g^2}{2q_5^+}\left[\gamma_\mu\gamma^+\gamma_\nu\right]_{a_1 a_2} T^{c_3}_{c_1,c_5}T^{c_4}_{c_5,c_2}$

$\mathcal{N}\left[\bar{\Psi}_{a_1,c_1}(-q_1)\Psi_{a_2,c_2}(q_2)G^{\mu\, c_3}(q_3)G^{\nu\, c_4}(q_4)\right]$



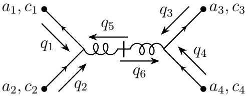
$\frac{g^2}{2q_5^+}\left[\gamma^+\right]_{a_1 a_2}\frac{q_3^+ - q_4^+}{2}g_{\mu\nu}T^{c_5}_{c_1,c_2}if^{c_5 c_3 c_4}$

$\mathcal{N}\left[\bar{\Psi}_{a_1,c_1}(-q_1)\Psi_{a_2,c_2}(q_2)G^{\mu\, c_3}(q_3)G^{\nu\, c_4}(q_4)\right]$



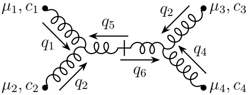
$\frac{g^2}{2q_5^+}\frac{q_1^+ - q_2^+}{2}g_{\mu\nu}\left[\gamma^+\right]_{a_3 a_4} if^{c_5 c_1 c_2}T^{c_5}_{c_3,c_4}$

$\mathcal{N}\left[G^{\mu\, c_1}(q_1)G^{\nu\, c_2}(q_2)\bar{\Psi}_{a_3,c_3}(-q_3)\Psi_{a_4,c_4}(q_4)\right]$



$\frac{g^2}{2q_5^+}\left[\gamma^+\right]_{a_1 a_2}\left[\gamma^+\right]_{a_3 a_4} T^{c_5}_{c_1,c_2}T^{c_5}_{c_3,c_4}$

$\mathcal{N}\left[\bar{\Psi}_{a_1,c_1}(-q_1)\Psi_{a_2,c_2}(q_2)\bar{\Psi}_{a_3,c_3}(-q_3)\Psi_{a_4,c_4}(q_4)\right]$



$\frac{g^2}{2q_5^+}\frac{q_1^+ - q_2^+}{2}g_{\mu_1\mu_2}\frac{q_3^+ - q_4^+}{2}g_{\mu_3\mu_4}if^{c_5 c_1 c_2}if^{c_5 c_3 c_4}$

$\mathcal{N}[G^{\mu_1\, c_1}(q_1)G^{\mu_2\, c_2}(q_2)G^{\mu_3\, c_3}(q_3)G^{\mu_4\, c_4}(q_4)]$

Regulators

- Momentum cutoff ϵ^+: $|q^+| > \epsilon^+$

- UV cutoff t_r: $e^{-t_r(q_1^- + \cdots + q_n^-)^2}$

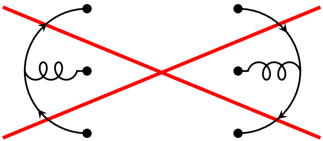
- IR regulator m_g: $q^- = \dfrac{m_g^2 + (q^\perp)^2}{q^+}$

Regulators

- Momentum cutoff ϵ^+: $|q^+| > \epsilon^+$
- UV cutoff t_r: $e^{-t_r(q_1^- + \cdots + q_n^-)^2}$ Regulates changes of q^-
- IR regulator m_g: $$q^- = \frac{m_g^2 + (q^\perp)^2}{q^+}$$

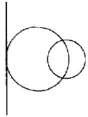
Small momentum cutoff $p^+ > \epsilon^+$

- Cutoff removes zero-mode interactions



$\epsilon^+ \to 0$

- Zero-mode counterterms (in the future)



What to expect?

$\to$ Shifts of bare masses and coupling constants [Burkardt, Nucl.Phys. (2000)]

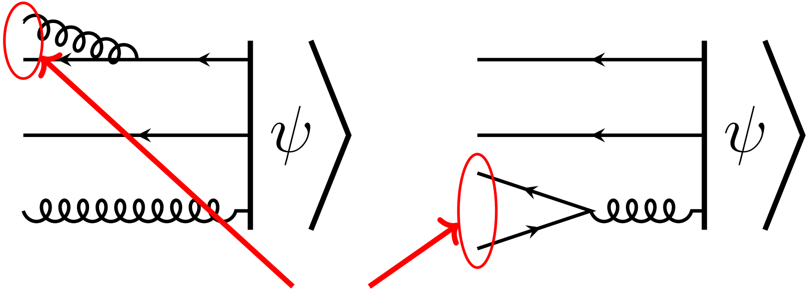
$\to$ Large effective quark masses (χSB)

$\to$ Spin-flip terms, etc.

Canonical Hamiltonian is ill defined

$$\langle \psi | \psi \rangle < \infty \quad \longrightarrow \quad \big\| H | \psi \rangle \big\| = \infty$$

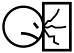
Domain of H is trivial!



Unlimited energy!

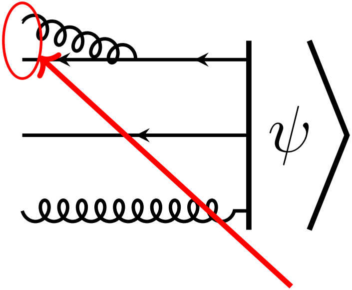
Canonical Hamiltonian is ill defined

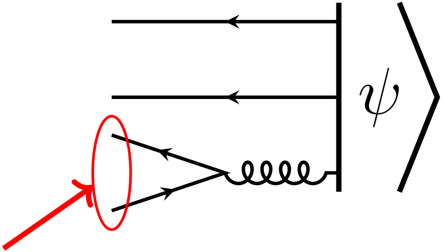
$$\langle\psi|\psi\rangle < \infty \quad \longrightarrow \quad \big\|H|\psi\rangle\big\| = \infty$$



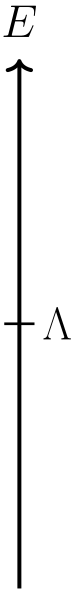
Domain of H is trivial!

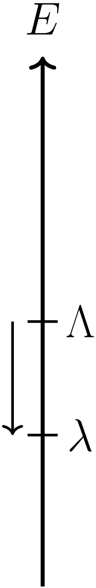
Introduce UV cutoff:
$H \to H(\Lambda)$



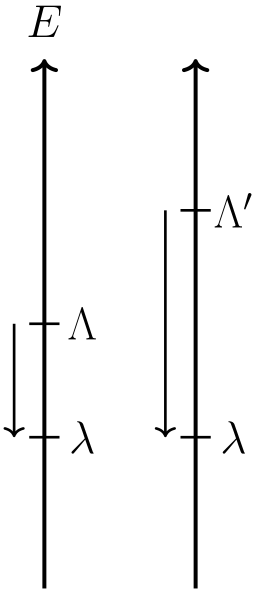


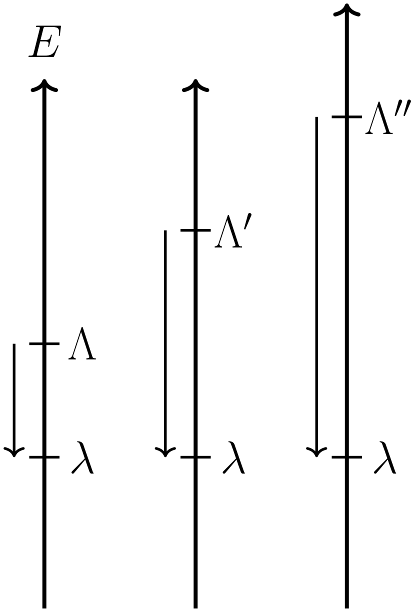
Unlimited energy!


E
Λ

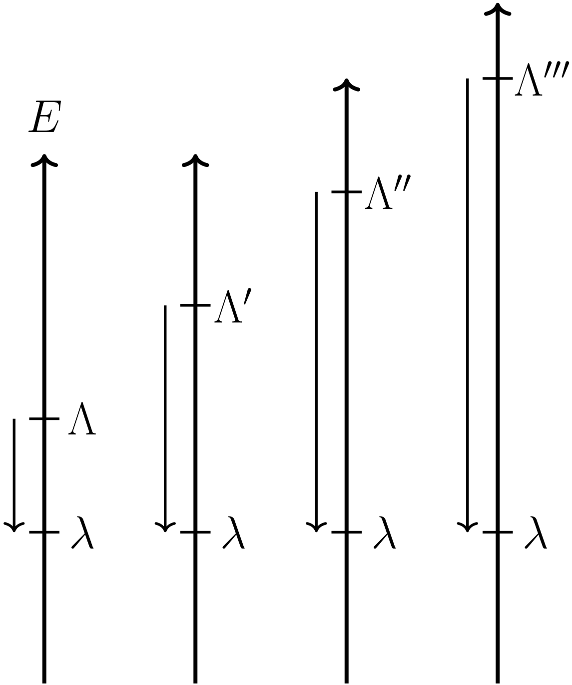


$$H(\lambda) = U^\dagger H(\Lambda) U$$


E
Λ
λ
Λ'
λ



E
Λ
λ
Λ'
λ
Λ''
λ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Λ
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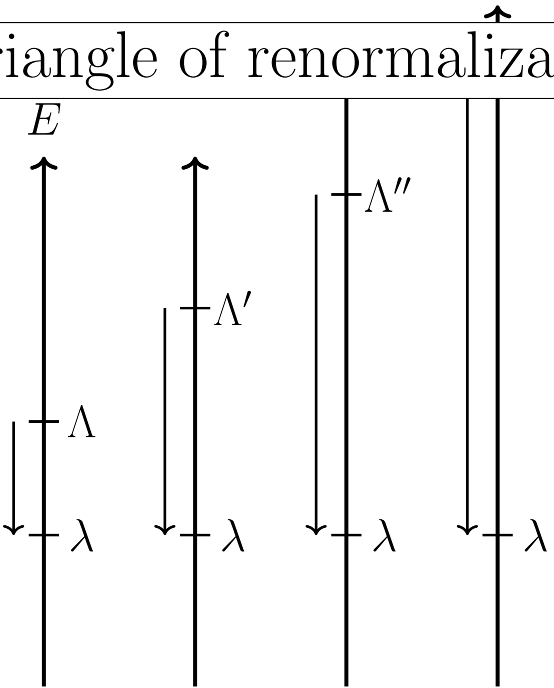
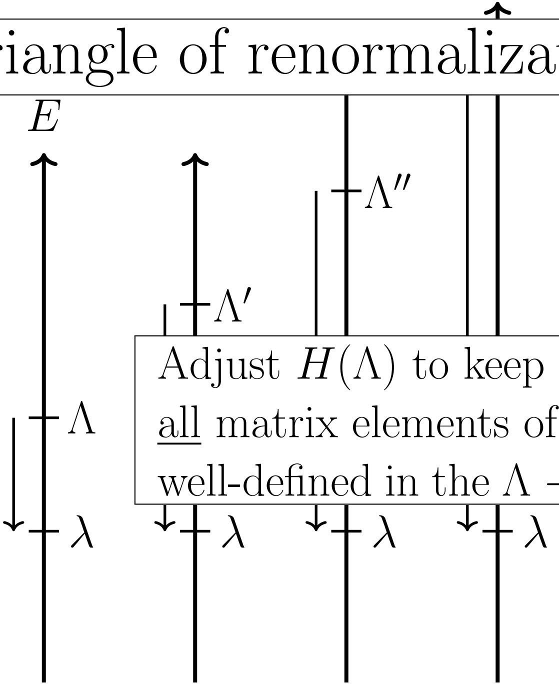
λ
Λ'''
λ

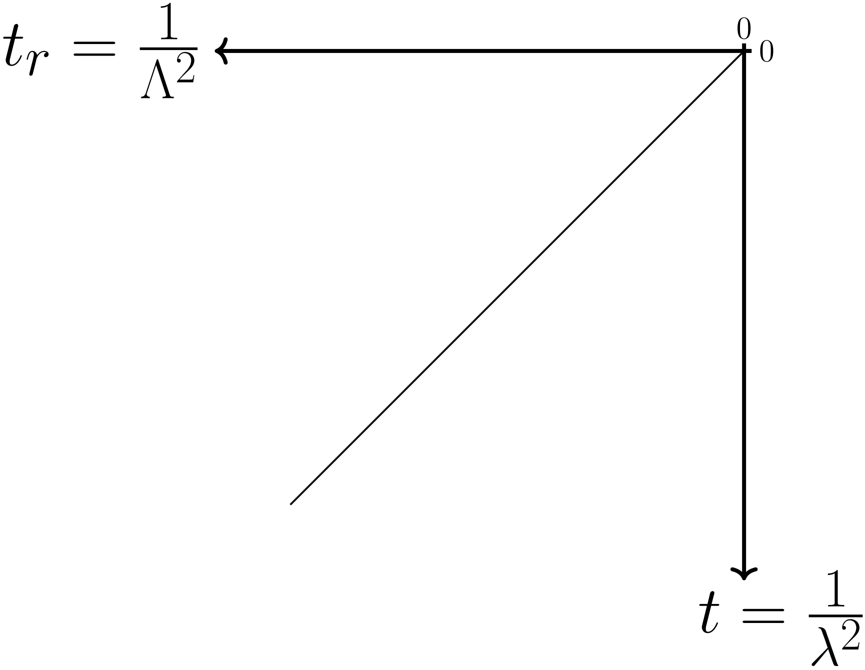
Triangle of renormalization



Triangle of renormalization

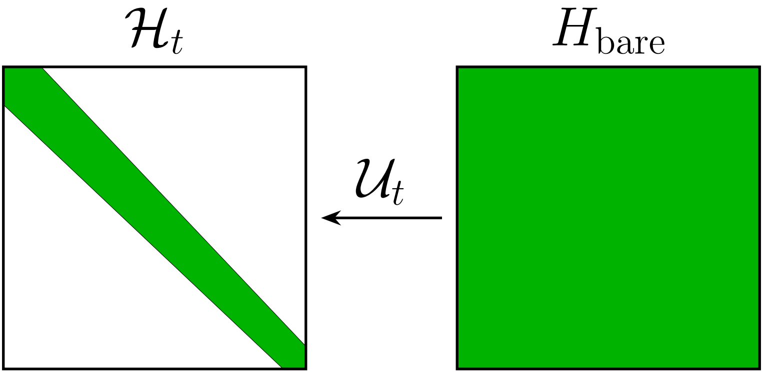


Adjust $H(\Lambda)$ to keep
<u>all</u> matrix elements of $H(\lambda)$
well-defined in the $\Lambda \to \infty$ limit


$t_r = \frac{1}{\Lambda^2}$
0
0
$t = \frac{1}{\lambda^2}$

RGPEP

Renormalization group procedure for effective particles



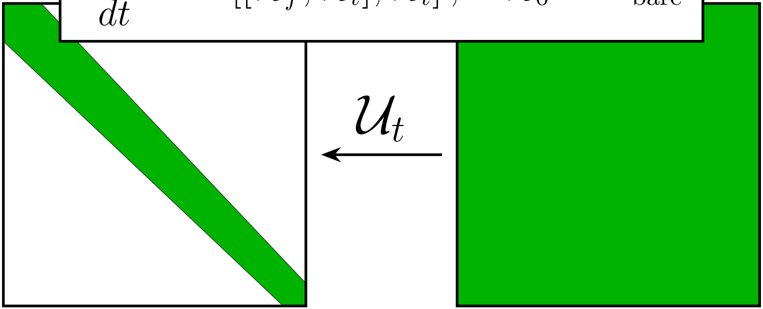
$$a_t = \mathcal{U}_t \, a \, \mathcal{U}_t^\dagger \, , \quad a_t^\dagger = \mathcal{U}_t \, a^\dagger \, \mathcal{U}_t^\dagger$$

$$\mathcal{H}_t = \mathcal{U}_t^\dagger H_{\rm bare} \mathcal{U}_t \, , \quad \mathcal{U}_0 = 1$$

RGPEP

Renormalization group procedure for effective particles

$$\frac{d\mathcal{H}_t}{dt} = [[\mathcal{H}_f, \mathcal{H}_t], \mathcal{H}_t] \,, \quad \mathcal{H}_0 = H_{\rm bare}$$



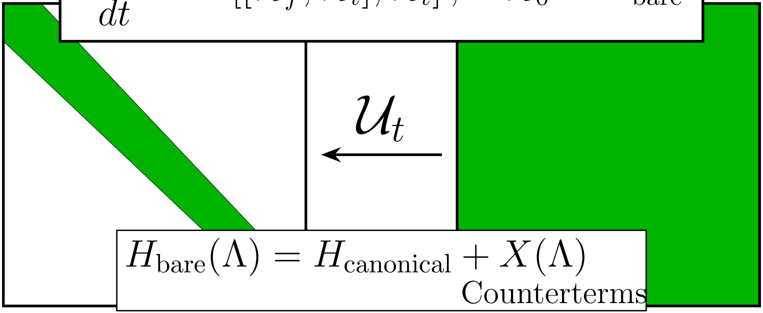
$$a_t = \mathcal{U}_t \, a \, \mathcal{U}_t^\dagger \,, \quad a_t^\dagger = \mathcal{U}_t \, a^\dagger \, \mathcal{U}_t^\dagger$$

$$\mathcal{H}_t = \mathcal{U}_t^\dagger H_{\rm bare} \mathcal{U}_t \,, \quad \mathcal{U}_0 = 1$$

RGPEP

Renormalization group procedure for effective particles

$$\frac{d\mathcal{H}_t}{dt} = [[\mathcal{H}_f, \mathcal{H}_t], \mathcal{H}_t], \quad \mathcal{H}_0 = H_{\text{bare}}$$



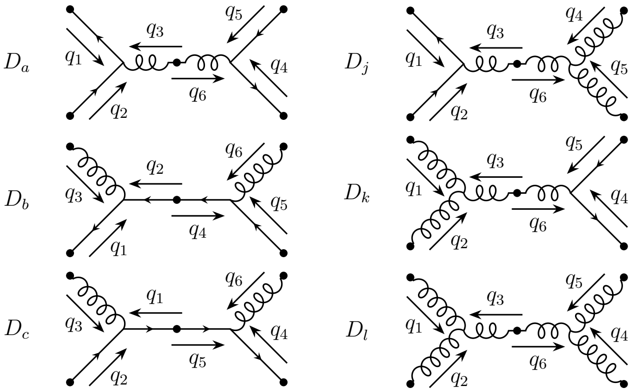
$$H_{\text{bare}}(\Lambda) = H_{\text{canonical}} + \underbrace{X(\Lambda)}_{\text{Counterterms}}$$

Matrix elements (Observables)	$t = 0$	$t > 0$
Unrenormalized	Finite (Infinite)	Infinite (Infinite)
Renormalized	Infinite (Finite)	Finite (Finite)

Effective Hamiltonians perturbatively

Step 1: $\mathcal{H}_t = \mathcal{H}_{t,0} + \mathcal{H}_{t,1} + \mathcal{H}_{t,2} + \dots$

Step 2: Use Wick's diagrams.



Counterterms

- Need to check all matrix elements, but...
- the Hamiltonian is unbounded

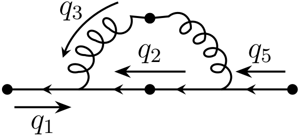
$\rightarrow$ define its domain:

$$F_0 = \Big\{ |\psi\rangle : \text{supp}(|\psi\rangle) \text{ in finitely many Fock sectors and wave functions compactly supported} \Big\}$$

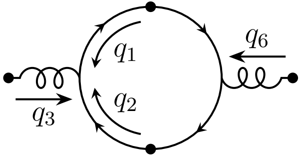
F_0 is dense in the Fock space

Counterterms

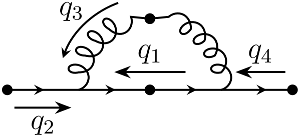
D_d



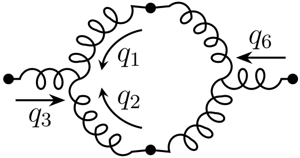
D_f



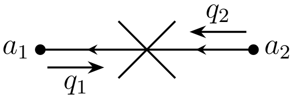
D_e



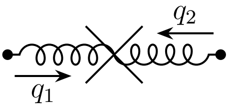
D_m



D_h



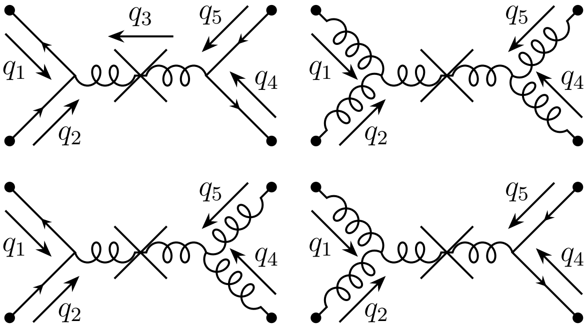
D_i



Counterterms

$$- \frac{S_I}{2} \int [q_1 q_2 q_3 q_4 q_5] \tilde{\delta}_{123} \tilde{\delta}_{45.3} \frac{m_g^2}{[m_g^2 + (q_3^\perp)^2]^2} \sqrt{\frac{\pi}{2t_r}} \delta(q_3^+)$$

$$\times \mathcal{N} \left[\tilde{J}_K^{+\,c}(q_1, q_2) \tilde{J}_L^{+\,c}(q_4, q_5) \right]$$



Infrared structure of front-form QCD

- Gluon mass m_g to regulate IR singularities
- m_g transforms IR into UV (front form!)
- **Result:** as $(m_g \to 0^+)$

$$\begin{array}{rcll} \text{Color singlets:} & \langle\psi| H |\phi\rangle & = & \text{finite} \\ \text{Color nonsinglets:} & \langle\psi| H |\phi\rangle & \sim & \log(m_g) \end{array}$$

- Serafin, Gustin, Love, arXiv:2606.24699.
- Serafin, Gómez-Rocha, More, Głazek, Phys. Rev. D 109 (2024), 016017.

Casimir operators in QCD

$$C_2 = \sum_{a=1}^{8} \hat{T}^a \hat{T}^a \ , \quad C_3 = \sum_{a,b,c=1}^{8} d^{abc} \hat{T}^a \hat{T}^b \hat{T}^c$$

$$\begin{aligned}
\hat{T}^a &= \sum_{\sigma,c_1,c_2} \int \frac{dp^+ d^2p^\perp}{16\pi^3 p^+} \theta(p^+) [T^a_F]_{c_1 c_2} b^\dagger_{p\sigma c_1} b_{p\sigma c_2} \\
&+ \sum_{\sigma,c_1,c_2} \int \frac{dp^+ d^2p^\perp}{16\pi^3 p^+} \theta(p^+) [T^a_{\bar{F}}]_{c_1 c_2} d^\dagger_{p\sigma c_1} d_{p\sigma c_2} \\
&+ \sum_{\sigma,c_1,c_2} \int \frac{dp^+ d^2p^\perp}{16\pi^3 p^+} \theta(p^+) [T^a_A]_{c_1 c_2} a^\dagger_{p\sigma c_1} a_{p\sigma c_2} \ ,
\end{aligned}$$

Casimir operators in QCD

$$C_2 = \sum_{a=1}^{8} \hat{T}^a \hat{T}^a \ , \quad C_3 = \sum_{a,b,c}^{8} d^{abc} \hat{T}^a \hat{T}^b \hat{T}^c$$

$$\begin{aligned} [T_F^a]_{c_1 c_2} &= T^a_{c_1 c_2} \\ [T_{\bar{F}}^a]_{c_1 c_2} &= -T^a_{c_2 c_1} \\ [T_A^a]_{c_1 c_2} &= -i f^{a c_1 c_2} \end{aligned}$$

$$\begin{aligned} \hat{T}^a &= \sum_{\sigma, c_1, c_2} \int \frac{dp^+ d^2 p^\perp}{16\pi^3 p^+} \theta(p^+) [T_F^a]_{c_1 c_2} b^\dagger_{p\sigma c_1} b_{p\sigma c_2} \\ &+ \sum_{\sigma, c_1, c_2} \int \frac{dp^+ d^2 p^\perp}{16\pi^3 p^+} \theta(p^+) [T_{\bar{F}}^a]_{c_1 c_2} d^\dagger_{p\sigma c_1} d_{p\sigma c_2} \\ &+ \sum_{\sigma, c_1, c_2} \int \frac{dp^+ d^2 p^\perp}{16\pi^3 p^+} \theta(p^+) [T_A^a]_{c_1 c_2} a^\dagger_{p\sigma c_1} a_{p\sigma c_2} \ , \end{aligned}$$

Casimir operators in QCD

$$\left[\hat{T}^a, b^\dagger_{p\sigma c}\right] = \sum_{c'} [T^a_F]_{c'c}\, b^\dagger_{p\sigma c'}$$

$$\left[\hat{T}^a, d^\dagger_{p\sigma c}\right] = \sum_{c'} [T^a_{\bar{F}}]_{c'c}\, d^\dagger_{p\sigma c'}$$

$$\left[\hat{T}^a, a^\dagger_{p\sigma c}\right] = \sum_{c'} [T^a_A]_{c'c}\, a^\dagger_{p\sigma c'}$$

$$\hat{T}^a\,|0\rangle = 0$$

$$C_2 b^\dagger_{p\sigma c}\,|0\rangle = C_F b^\dagger_{p\sigma c}\,|0\rangle$$

$$C_2 d^\dagger_{p\sigma c}\,|0\rangle = C_F d^\dagger_{p\sigma c}\,|0\rangle$$

$$C_2 a^\dagger_{p\sigma c}\,|0\rangle = C_A a^\dagger_{p\sigma c}\,|0\rangle$$

$$C_F = \textstyle\sum_{a,c'} [T^a_F]_{cc'}\,[T^a_F]_{c'c} = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$$

$$C_A = \textstyle\sum_{a,c'} [T^a_A]_{cc'}\,[T^a_A]_{c'c} = N_c = 3$$

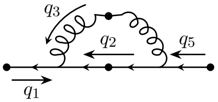
Casimir operators in QCD

$$
\begin{aligned}
\hat{T}^a\hat{T}^a &= \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) \\
&+ \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right] \\
&+ \int [q]\frac{C_A|q^+|}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{ja}(-q)G^{ja}(q)\right]
\end{aligned}
$$

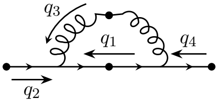
$$
\begin{aligned}
H_{\text{kinetic}} &= \int [q]\,\frac{m^2+(q^\perp)^2}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right] \\
&+ \int [q]\,\frac{m_g^2+(q^\perp)^2}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{i\,a}(-q)G^{i\,a}(q)\right]
\end{aligned}
$$

Casimir operators in QCD

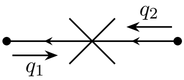
$$
\begin{aligned}
\hat{T}^a\hat{T}^a &= \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) \\
&+ \int [q]\frac{C_F|q^+|}{q^+}\,\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right] \\
&+ \int [q]\frac{C_A|q^+|}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{ja}(-q)G^{ja}(q)\right]
\end{aligned}
$$



(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2 G_3\bar{\Psi}_4\Psi_5 G_6\right]$



(e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2 G_3\bar{\Psi}_4\Psi_5 G_6\right]$

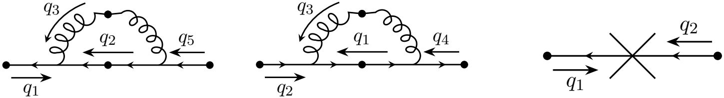


(h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

Casimir operators in QCD

$$\hat{T}^a\hat{T}^a = \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) + \int[q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right]$$

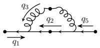
$$\boxed{\delta m_{t,2}^2 = \frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right) - \gamma - \frac{7}{2}\right] + o(1) \quad (m_g \to 0, m \to 0)}$$



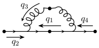
(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2 G_3\bar{\Psi}_4\Psi_5 G_6\right]$ (e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2 G_3\bar{\Psi}_4\Psi_5 G_6\right]$ (h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

Casimir operators in QCD

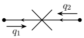
$$\hat{T}^a\hat{T}^a = \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) + \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right] + \int [q]\frac{C_A|q^+|}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{ja}(-q)G^{ja}(q)\right]$$



(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$

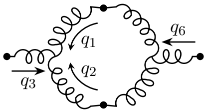


(e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$

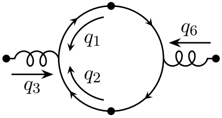


(h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

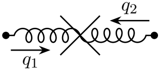
$$\frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right)-\gamma\right]$$



(m) $\mathcal{N}\left[G_1G_2G_3G_4G_5G_6\right]$



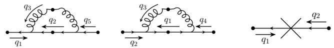
(f) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



(i) $\frac{1}{2}\mathcal{N}\left[G_1G_2\right]$

Casimir operators in QCD

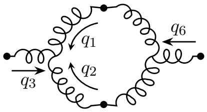
$$\hat{T}^a\hat{T}^a = \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) + \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right]$$



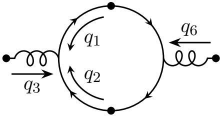
(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$ (e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$ (h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

$$\frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right) - \gamma\right]$$

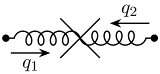
$$\delta\mu^2_{t,g,2}(q) = \frac{C_A g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right) - \gamma - \frac{23}{6}\right] + o(1) \quad (m_g \to 0)$$



(m) $\mathcal{N}\left[G_1G_2G_3G_4G_5G_6\right]$



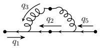
(f) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



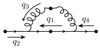
(i) $\frac{1}{2}\mathcal{N}\left[G_1G_2\right]$

Casimir operators in QCD

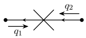
$$\hat{T}^a\hat{T}^a = \mathcal{N}\left(\hat{T}^a\hat{T}^a\right) + \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right]$$



(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



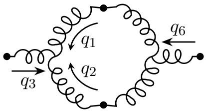
(e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



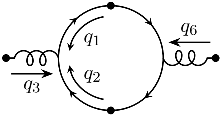
(h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

$$\frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right) - \gamma\right]$$

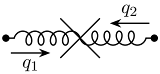
$$\lim_{m_g\to 0}\delta\mu^2_{t,q,2}(q) = \frac{g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\cdot\frac{1}{3} + o(1) \quad (m\to 0)$$



(m) $\mathcal{N}\left[G_1G_2G_3G_4G_5G_6\right]$



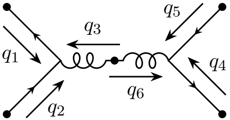
(f) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



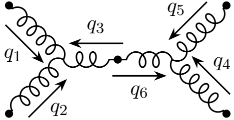
(i) $\frac{1}{2}\mathcal{N}\left[G_1G_2\right]$

Casimir operators in QCD

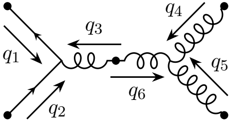
$$
\begin{aligned}
\hat{T}^a\hat{T}^a &= \mathcal{N}\left(\hat{T}^a\hat{T}^a\right)\\
&+ \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right]\\
&+ \int [q]\frac{C_A|q^+|}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{ja}(-q)G^{ja}(q)\right]
\end{aligned}
$$



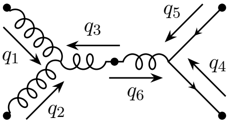
(a) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2\overline{G_3\bar{\Psi}_4\Psi_5G_6}\right]$



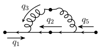
(l) $\mathcal{N}\left[G_1G_2\overline{G_3G_4G_5G_6}\right]$



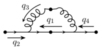
(j) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2\overline{G_3G_4G_5G_6}\right]$



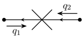
(k) $\mathcal{N}\left[G_1G_2\overline{G_3\bar{\Psi}_4\Psi_5G_6}\right]$



(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



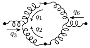
(e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$



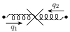
(h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

$$\frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4t}\right)-\gamma\right]$$

$$\frac{C_A g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4t}\right)-\gamma\right]$$



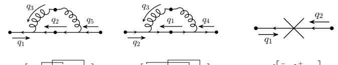
(m) $\mathcal{N}\left[G_1G_2G_3G_4G_5G_6\right]$



(i) $\frac{1}{2}\mathcal{N}\left[G_1G_2\right]$

Casimir operators in QCD

$$\hat{T}^a\hat{T}^a = \mathcal{N}\left(\hat{T}^a\hat{T}^a\right)$$
$$+ \int [q]\frac{C_F|q^+|}{q^+}\mathcal{N}\left[\bar{\Psi}(q)\frac{\gamma^+}{2}\Psi(q)\right]$$
$$+ \int [q]\frac{C_A|q^+|}{q^+}q^+\mathcal{N}\left[\frac{1}{2}G^{ja}(-q)G^{ja}(q)\right]$$



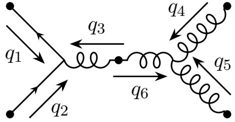
(d) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$ (e) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$ (h) $\mathcal{N}\left[\bar{\Psi}_1\frac{\gamma^+}{2q_2^+}\Psi_2\right]$

$$\frac{C_F g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right)-\gamma\right]$$

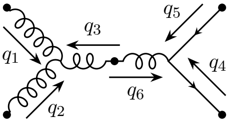
$$\frac{C_A g^2}{16\pi^2}|q^+|\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(q^+)^2}{8m_g^4 t}\right)-\gamma\right]$$

$$\frac{g^2}{16\pi^2}\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(\mathcal{P}^+)^2}{8m_g^4 t}\right)-\gamma\right]\mathcal{N}\left(\hat{T}^c\hat{T}^c\right)+O(1) \quad (m_g \to 0)$$

(a) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3\bar{\Psi}_4\Psi_5G_6\right]$ (l) $\mathcal{N}\left[G_1G_2G_3G_4G_5G_6\right]$



(j) $\mathcal{N}\left[\bar{\Psi}_1\Psi_2G_3G_4G_5G_6\right]$

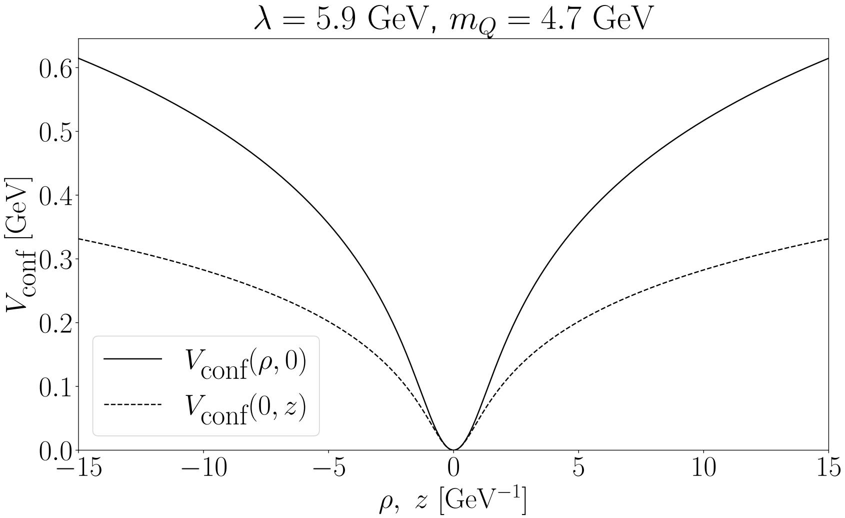


(k) $\mathcal{N}\left[G_1G_2G_3\bar{\Psi}_4\Psi_5G_6\right]$

Casimir operators in QCD

$$\mathcal{H}_{t,2} = \frac{g^2}{16\pi^2}\sqrt{\frac{\pi}{2t}}\left[\log\left(\frac{(\mathcal{P}^+)^2}{8m_g^4 t}\right) - \gamma\right]\hat{T}^c\hat{T}^c + O(1) \quad (m_g \to 0)$$

Logarithmic confining potential



K.Serafin et al. Phys.Rev.D (2024)

R.Perry (1994)

Self-adjoint extensions

- Effective Hamiltonian is a form on F_0.
- The form likely corresponds to a symmetric operator.
- Need to extend to a self-adjoint operator.

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- Effective Hamiltonian is a form on F_0.
- The form likely corresponds to a symmetric operator.
- Need to extend to a self-adjoint operator.
- **Hypothesis:** different self-adjoint extensions correspond to different vacuum physics.

Conclusion

Result: effective renormalized Hamiltonian of QCD

$\rightarrow$ Well-defined quadratic form (symmetric operator) in the color-singlet subspace.

$\rightarrow$ We found a Casimir operator times $\log \frac{1}{m_g}$.

$\rightarrow$ No divergences due to the gluon mass and omission of zero-mode counterterms at the level of matrix elements (color singlet).

$\rightarrow$ Suitable for nonperturbative diagonalization.

Thank you for attention!

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