

Spin-orbit correlation within heavy quarkonium

Xianghui Cao

University of Science and Technology of China, Hefei, China

In collaboration with

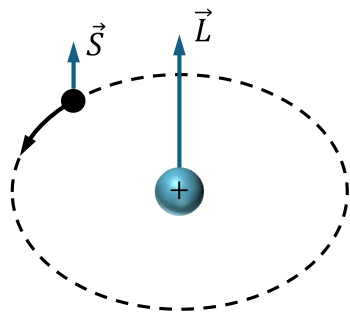
Tian-Yang Hu (IMP), Si-Qi Xu (IMP), Qin-Tao Song (ZZU), Wei-Jie Du (IMP), Yang Li (USTC)



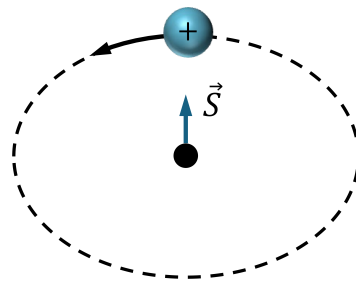
Light-Cone 2026: Applications at EIC era
Stony Brook University, Stony Brook, July 22



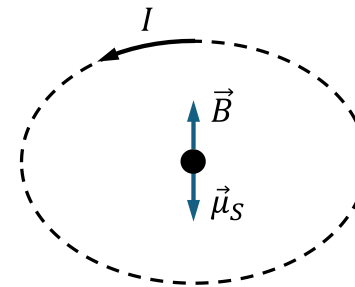
Spin-orbit correlation in atoms



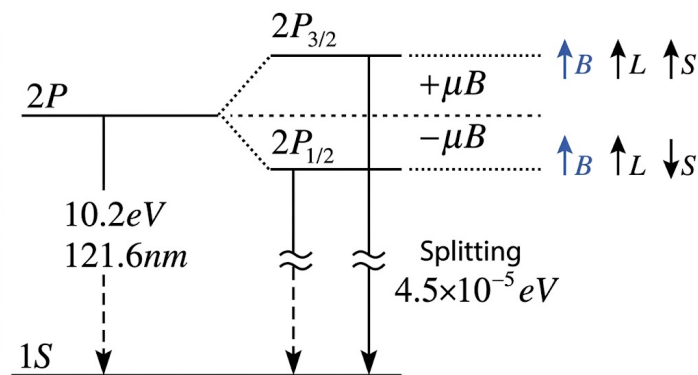
nuclei frame



electron frame



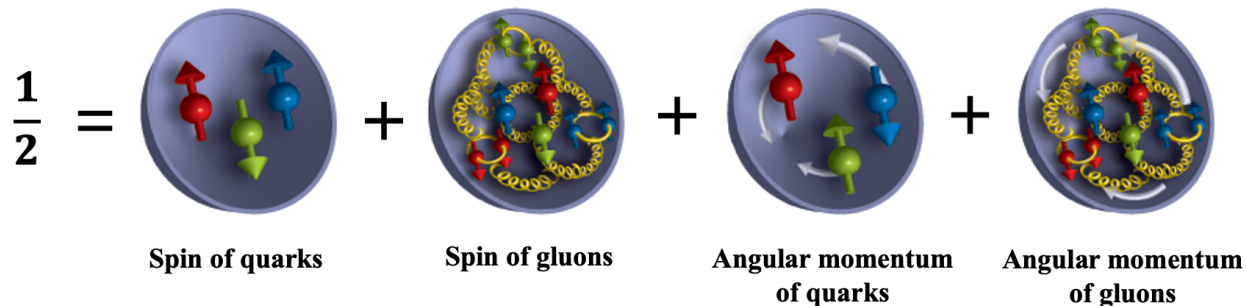
magnetic field from nuclei motion



$$H_{ls} = \frac{1}{2m_e^2 c^2} \frac{Ze^2}{4\pi\epsilon_0 r^3} \vec{L} \cdot \vec{S}$$

Fine structure of hydrogen atom

Spin decomposition



- Ji's spin sum rule: [Ji:1996ek]
(gauge invariant)

$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$$

- Jaffe-Manohar spin sum rule: [Jaffe:1989jz]
(infinite momentum frame, light-cone gauge)

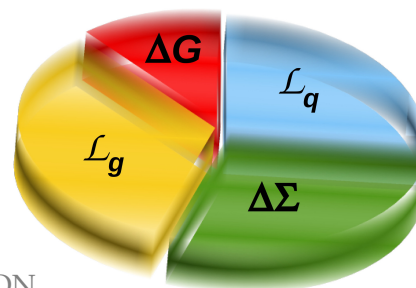
$$\frac{1}{2}\Delta\Sigma + S_g + L_q + L_g = \frac{\hbar}{2}$$

Can quark spin and OAM couple in hadrons?

- quark spin $\Delta\Sigma \sim 30\%$
- quark orbital angular momentum (OAM) L_q
(measurable in EIC and EicC)

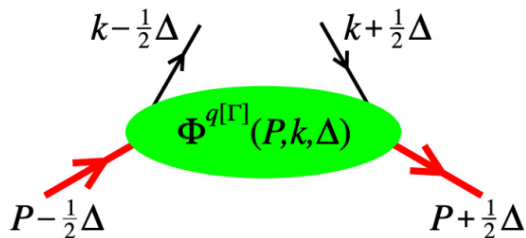
[Accardi:2012qut]

■ Gluon Spin ■ Gluon angular momentum
■ Quark Spin ■ Quark Angular Momentum



Spin-orbit correlation in QCD

[Meissner:2009ww]



$$\Phi_{\text{GTMD}}^{q[\Gamma]} = \int \frac{dz^- d^2 z_\perp}{2(2\pi)^3} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle \Big|_{z^+=0}$$

- GTMD contains information from GPD, TMD and beyond

[Lorce:2011kd]

$$\Phi_{\text{GTMD}}^{q[\gamma^+]} = \frac{1}{2M} \bar{u}(p') \left[F_{1,1}^q + \frac{i\sigma^{i+} k_\perp^i}{P^+} F_{1,2}^q + \frac{i\sigma^{i+} \Delta_\perp^i}{P^+} F_{1,3}^q + \frac{i\sigma^{ij} k_\perp^i \Delta_\perp^j}{M^2} F_{1,4}^q \right] u(p)$$

$$\Phi_{\text{GTMD}}^{q[\gamma^+ \gamma_5]} = \frac{1}{2M} \bar{u}(p') \left[-\frac{i\varepsilon_\perp^{ij} k_\perp^i \Delta_\perp^j}{M^2} G_{1,1}^q + \frac{i\sigma^{i+} \gamma_5 k_\perp^i}{P^+} G_{1,2}^q + \frac{i\sigma^{i+} \gamma_5 \Delta_\perp^i}{P^+} G_{1,3}^q + i\sigma^{+-} G_{1,4}^q \right] u(p)$$

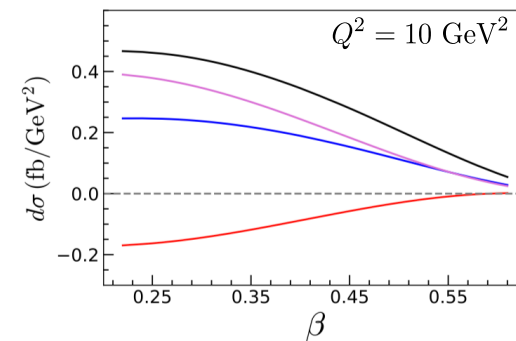
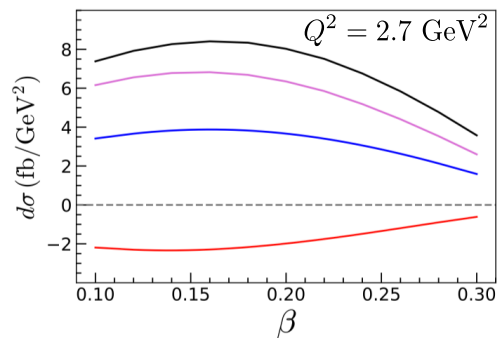
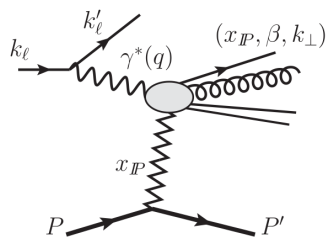
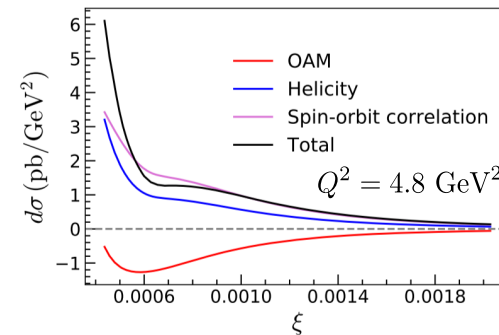
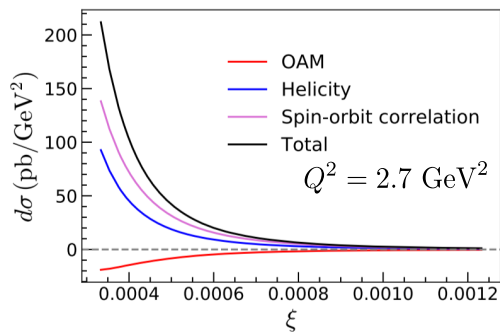
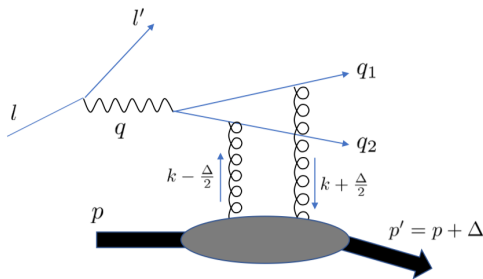
- Quark orbit angular momentum and **spin-orbit correlation**

$$L_z^q = - \int dx d^2 k_\perp \frac{k_\perp^2}{M^2} F_{1,4}^q(x, k_\perp) \Big|_{\Delta=0}, \quad C_z^q = \int dx d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^q(x, k_\perp) \Big|_{\Delta=0}$$

Spin-orbit correlation in experiments

[Bhattacharya:2024sck]

- Gluon spin-orbit correlations are non-negligible in exclusive di-jet production and SIDDIS



$$\hat{S}_z^q = \int d^3x \frac{1}{2} \bar{\psi} \gamma^+ \gamma_5 \psi = \frac{1}{2} (\hat{N}^{qR} - \hat{N}^{qL}), \quad \text{quark spin}$$

$$\hat{L}_z^q = \int d^3x \frac{1}{2} \bar{\psi} \gamma^+ (\vec{x} \times i \overleftrightarrow{D})_z \psi = \hat{L}_z^{qR} + \hat{L}_z^{qL}, \quad \text{quark OAM}$$

$$\hat{C}_z^q = \int d^3x \frac{1}{2} \bar{\psi} \gamma^+ \gamma_5 (\vec{x} \times \overleftrightarrow{D})_z \psi = \hat{L}_z^{qR} - \hat{L}_z^{qL} \sim \hat{L}_z^q \hat{S}_z^q \quad \text{quark spin-orbit correlation}$$

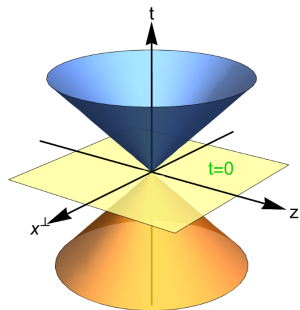
where q_R/q_L denotes the contribution of right/left-hand fermion and \hat{N} is the quark number operator

- Spin-orbit correlation (SOC) defined from parity-odd energy-momentum tensor (EMT):

$$\begin{aligned} \hat{C}_z^q &= \int d^3x (x^1 \hat{T}_{q5}^{+2} - x^2 \hat{T}_{q5}^{+1}), & \hat{T}_{q5}^{\mu\nu} &= \frac{1}{2} \bar{\psi} \gamma^\mu \gamma_5 i \overleftrightarrow{D}^\nu \psi \\ \hat{L}_z^q &= \int d^3x (x^1 \hat{T}_q^{+2} - x^2 \hat{T}_q^{+1}), & \hat{T}_q^{\mu\nu} &= \frac{1}{2} \bar{\psi} \gamma^\mu i \overleftrightarrow{D}^\nu \psi \end{aligned}$$

Light-front quantization

equal time quantization



✓ Time:

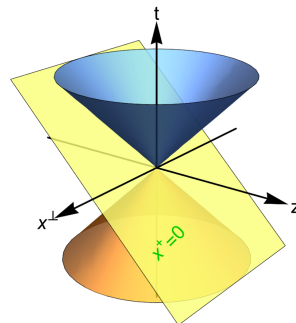
$$t \equiv x^0$$

✓ Hamiltonian:

$$H \equiv P^0$$

✓ Dispersion relation: $p^0 = \sqrt{\vec{p}^2 + m^2}$

light-front quantization



$$t \equiv x^+ = x^0 + x^3$$

$$H \equiv P^- = P^0 - P^3$$

$$p^- = (\vec{p}_\perp^2 + m^2)/p^+$$

light-front coordinates

$$x^\pm = x^0 \pm x^3$$

$$\vec{x}^\perp = (x^1, x^2)$$



[Dirac:1949cp]

Light-front quantization is a Hamiltonian method of the quantum field theory

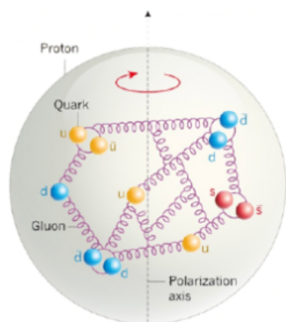
[Review: Brodsky:1997de]

$$(P^+ \hat{P}^- - \vec{P}_\perp^2) |\psi_h\rangle = M_h^2 |\psi_h\rangle$$

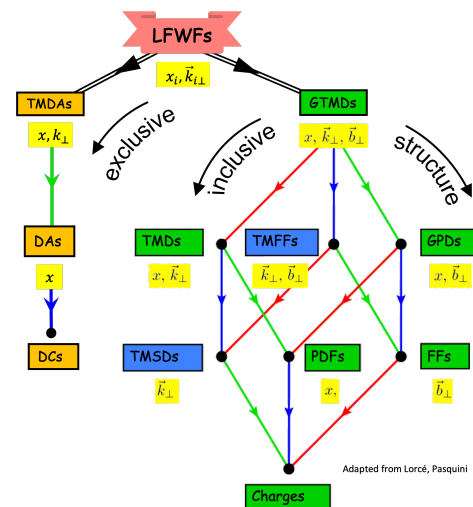
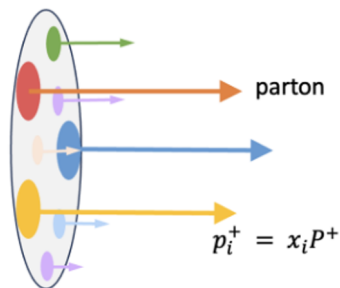
Light-front wave functions

$$|\psi_h(P, j, \lambda)\rangle = \sum_{n=1}^{\infty} \int [dx_i d^2 k_{i\perp}]_n \psi_{h/n}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

- Light-front physics measures hadron structures in high-energy scattering experiments
- Light-front wave functions (LFWFs) provide full information about the hadron structure
- LFWF representation offers simple microscopic interpretations



$P_z \rightarrow \infty$



[Lorce:2011kd]

Adapted from Lorce, Pasquini

Microscopic interpolation of form factors

- Drell-Yan-West formula for charge form factor:

[Drell:1969km, West:1970av, Brodsky:1980zm]

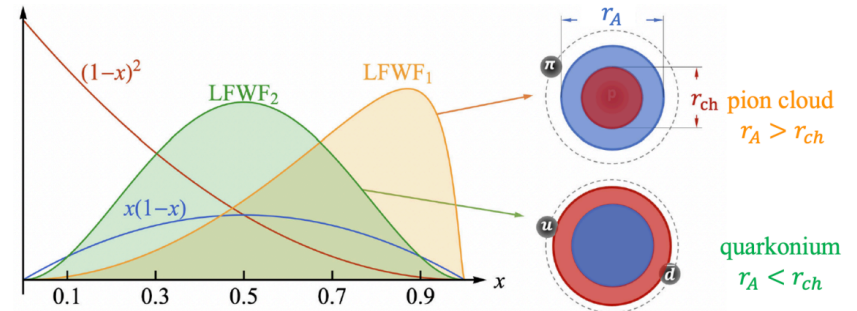
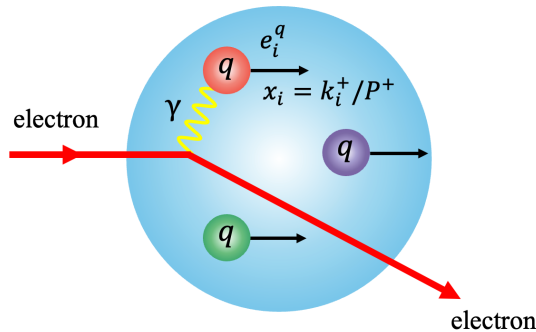
$$\rho_{\text{ch}}(r_{\perp}) = \int [dx_i dr_{i\perp}]_n |\tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})|^2 \sum_j e_j \delta^{(2)}(r_{\perp} - r_{j\perp}) \equiv \langle \sum_j e_j \delta^{(2)}(r_{\perp} - r_{j\perp}) \rangle$$

- Brodsky-Hwang-Ma-Schmidt formula for gravitational form factor A :

[Brodsky:2000ii]

$$\mathcal{A}(r_{\perp}) = \int [dx_i dr_{i\perp}]_n |\tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})|^2 \sum_j x_j \delta^{(2)}(r_{\perp} - r_{j\perp}) \equiv \langle \sum_j x_j \delta^{(2)}(r_{\perp} - r_{j\perp}) \rangle$$

Matter density $\mathcal{A}(r_{\perp})$ mainly samples the valence partons $x_j \sim O(1)$; wee parton $x_j \ll 1$ contributions suppressed



[Hackett:2023nkr]

$$r_{\text{ch}}^{\pi} \sim 1.6 r_A^{\pi}$$

Microscopic interpolation of spin-orbit correlation

$$C_{LS}^z(r_{\perp}) = \left\langle \sum_j \delta^{(2)}(r_{\perp} - r_{j\perp}) L_j^z S_j^z \right\rangle$$

- Spin-orbit correlation defined from quantum many-body formula
- The orbit angular momentum operator $L_j^z = -(\vec{r}_{j\perp} \times i \overleftrightarrow{\nabla}_{j\perp})_z / 2$
- The quantum average is defined as

$$\langle \hat{O} \rangle = \int [dx_i d^2 r_{i\perp}] \psi_n^* (\{x_i, \vec{r}_{i\perp}\}) \hat{O} \psi_n (\{x_i, \vec{r}_{i\perp}\})$$

Heavy meson on the light front

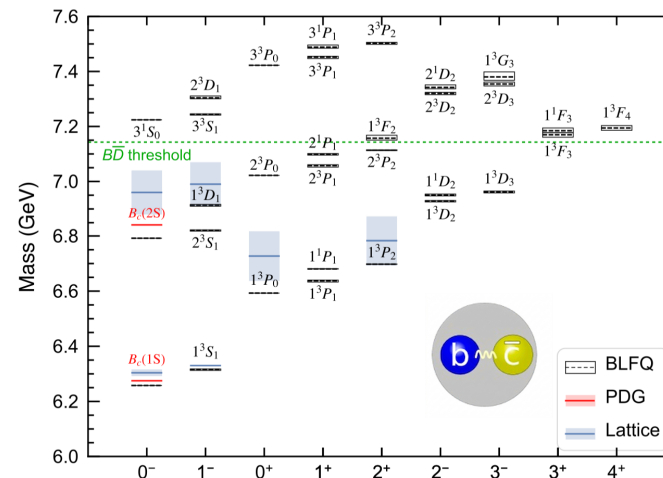
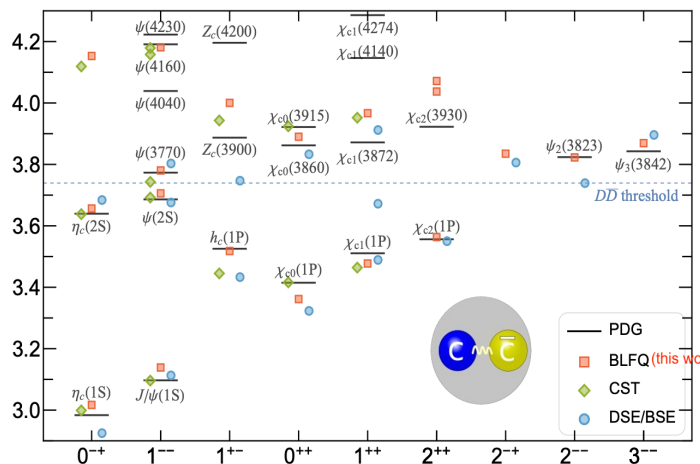
- Effective Hamiltonian in the $q\bar{q}$ Fock sector

[Li:2017mlw, Tang:2018myz]

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{s'}(k') \gamma_{\mu} u_s(k) \bar{v}(\bar{k}) \gamma^{\mu} v_{\bar{s}'}(\bar{k}')$$

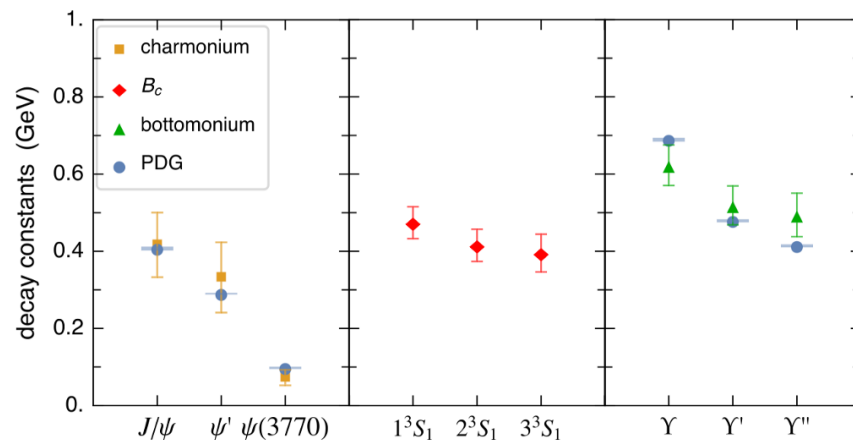
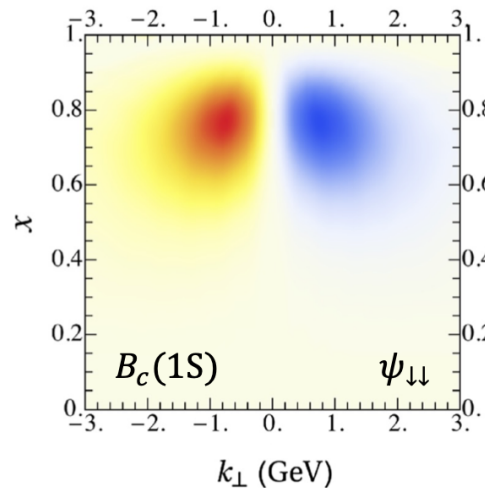
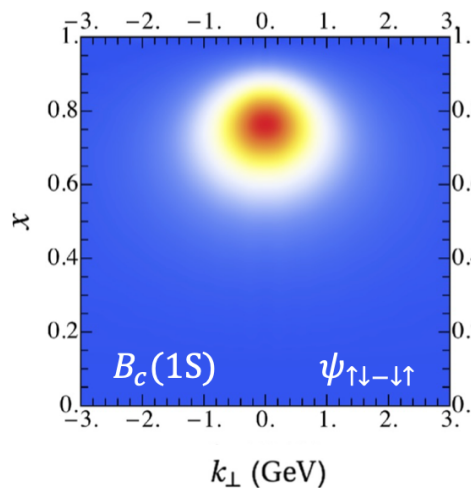
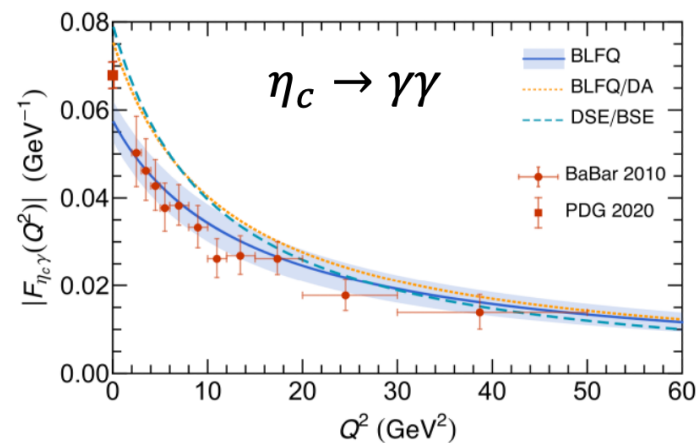
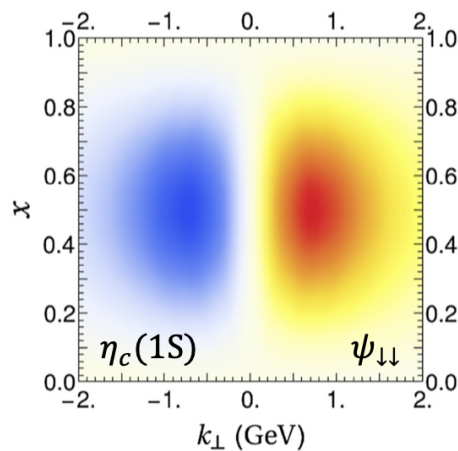
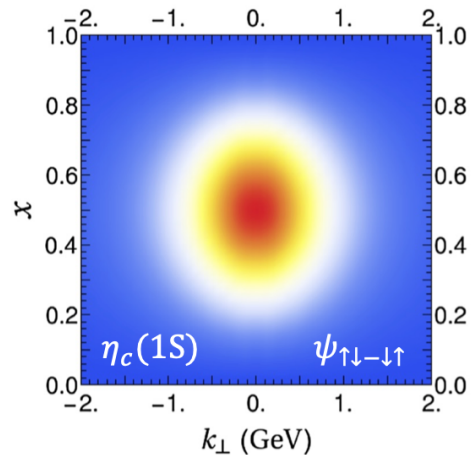
one gluon exchange

- Two/three parameters ($m_q, m_{\bar{q}}, \kappa$) are fixed by fitting the mass spectrum



Light-front wave functions of heavy mesons

[Li:2021ejv, Tang:2018myz]



Covariant light-front decomposition

- Hadron matrix elements for spin-0 hadrons:

[Carbonell:1998rj, Cao:2024rul]

$$\langle P + \frac{1}{2}\Delta | \hat{T}_{q5}^{[\mu\nu]}(0) | P - \frac{1}{2}\Delta \rangle = i\varepsilon^{\mu\nu\Delta P} \tilde{F}_q(\Delta^2) + M^2 \frac{\varepsilon^{\mu\nu P\omega}}{\omega \cdot P} \tilde{A}_1(\Delta^2) + M^2 \frac{i\varepsilon^{\mu\nu\Delta\omega}}{\omega \cdot P} \tilde{A}_2(\Delta^2)$$

$$\langle P + \frac{1}{2}\Delta | \hat{T}_{q5}^{\{\mu\nu\}}(0) | P - \frac{1}{2}\Delta \rangle = \frac{iP^{\{\mu}\varepsilon^{\nu\}P\Delta\omega}}{\omega \cdot P} \tilde{S}_1(\Delta^2) + \frac{\Delta^{\{\mu}\varepsilon^{\nu\}P\Delta\omega}}{\omega \cdot P} \tilde{S}_2(\Delta^2) + \frac{i\omega^{\{\mu}\varepsilon^{\nu\}P\Delta\omega}}{(\omega \cdot P)^2} \tilde{S}_3(\Delta^2)$$

where $P = (p + p')/2$, $\Delta = p' - p$, $\varepsilon^{\mu\nu\Delta P} = \varepsilon^{\mu\nu\rho\sigma} \Delta_\rho P_\sigma$. M is the hadron mass. $\omega^\mu = (1, 0, 0, -1)$ is a null vector indicating the light-front direction

- $\tilde{A}_i(\Delta^2)$ and $\tilde{S}_i(\Delta^2)$ are spurious form factors appears due to the violation of the full Lorentz symmetry
- SOC receives contributions from both physical + spurious form factors

[Lorce:2025ayr]

$$\hat{C}_z^q = \int d^3x (x^1 \hat{T}_{q5}^{+2} - x^2 \hat{T}_{q5}^{+1})$$

$$\langle P + \frac{1}{2}\Delta | \hat{T}_{q5}^{+i} | P - \frac{1}{2}\Delta \rangle = iP^+ \epsilon^{ij} \Delta_\perp^j (\tilde{F}_q(\Delta_\perp^2) + \tilde{S}_1(\Delta_\perp^2))$$

Light-front wave function representation

- SOC density:

$$C_z^q(r_\perp) = r_\perp^1 \hat{\mathcal{T}}_{q5}^{+2}(\vec{r}_\perp) - r_\perp^2 \hat{\mathcal{T}}_{q5}^{+1}(\vec{r}_\perp)$$

- Parity-odd EMT density:

$$\hat{\mathcal{T}}_{q5}^{\mu\nu} = \int \frac{d^2\Delta_\perp}{(2\pi)^2 2p^+} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} \langle p' | \hat{T}_{q5}^{\mu\nu}(0) | p \rangle$$

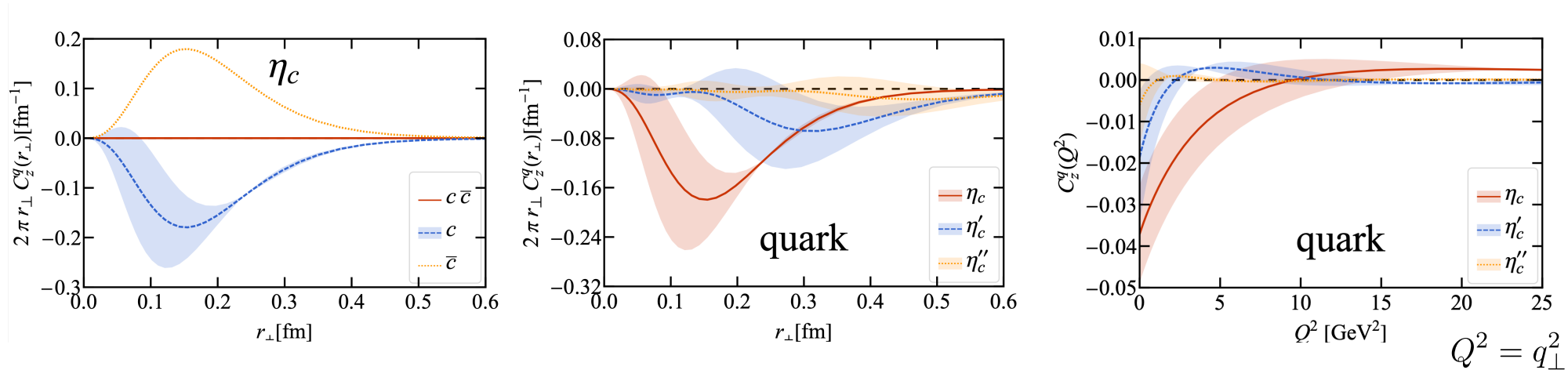
- Light-front wave function representation for quark sectors:

$$C_z^q(r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} [\tilde{\mathcal{F}}^q(\Delta_\perp^2) + \Delta_\perp^2 \frac{d\tilde{\mathcal{F}}^q(\Delta_\perp^2)}{d\Delta_\perp^2}] \quad \tilde{\mathcal{F}}^q(\Delta_\perp^2) = \tilde{F}^q(\Delta_\perp^2) + \tilde{S}_1(\Delta_\perp^2)$$

$$= \sum_{s, \bar{s}} \int [dx d^2r_\perp] \tilde{\psi}_{s\bar{s}}^*(x, \vec{r}_\perp) \sum_{j=q, \bar{q}} \delta^{(2)}(r_\perp - r_{j\perp}) \eta_j \underbrace{s_j (\vec{r}_{j\perp} \times (-i \vec{\nabla}_{j\perp}))_z}_{L_j^z S_j^z} \tilde{\psi}_{s\bar{s}}(x, \vec{r}_\perp)$$

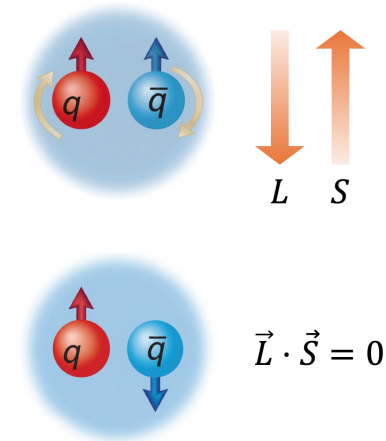
$\eta_j = \pm 1$ for quarks/antiquarks

Spin-orbit correlation: η_c

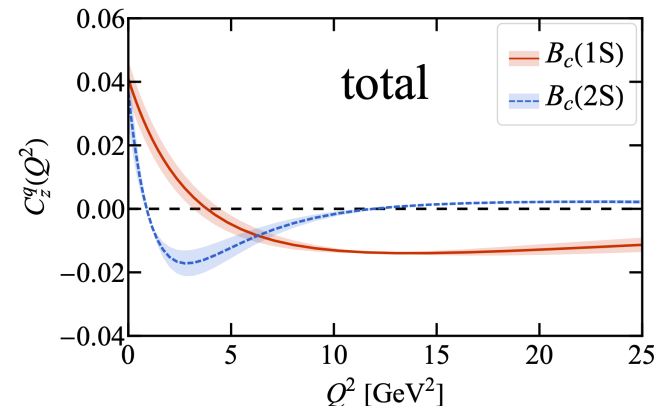
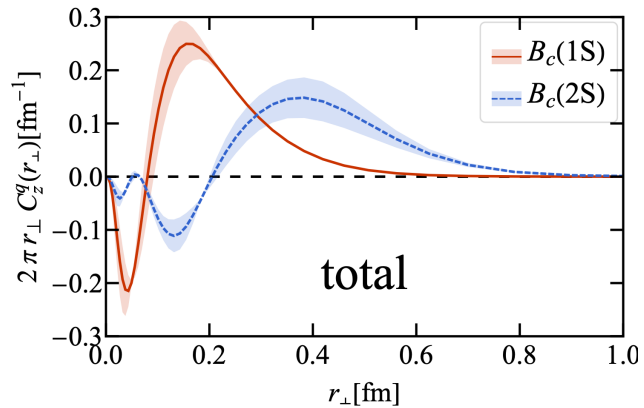
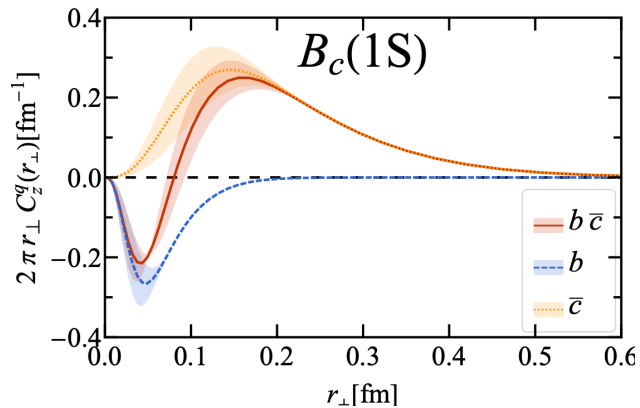


- Quark spin and OAM are anti-aligned in spin-0 hadrons
- $C_z^q(0) = 0$ is enforced by charge conjugation (C) symmetry
- Magnitude of SOC decreases as the radial excitation increases
- The SOC form factor is defined as:

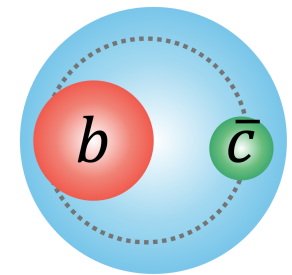
$$C_z^q(Q^2) = \int d^2 r_{\perp} e^{-i\vec{q}_{\perp} \cdot \vec{r}_{\perp}} C_z^q(r_{\perp})$$



Spin-orbit correlation: B_c

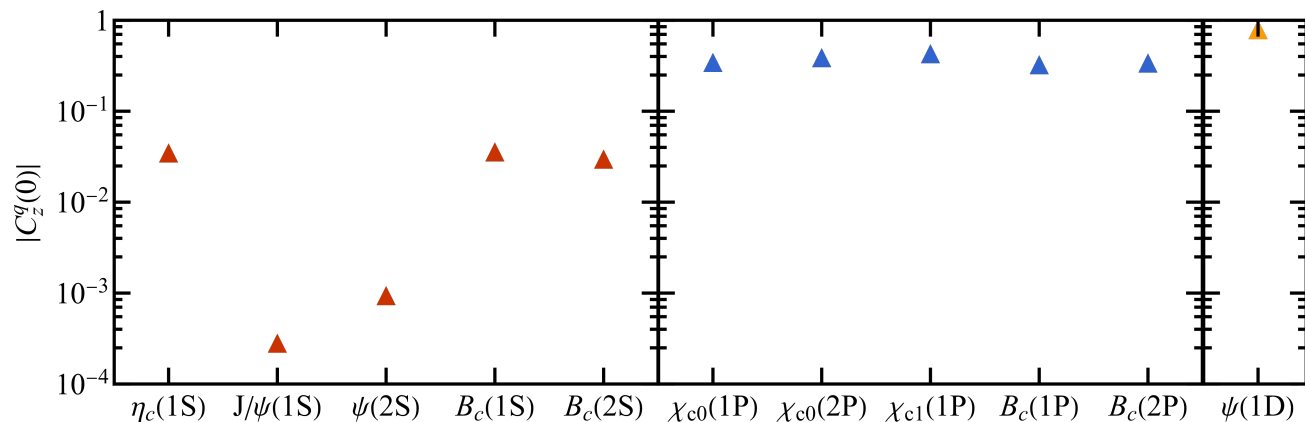


- b quark dominates at small r_{\perp} , while \bar{c} quark dominates at large r_{\perp}
- The global spin and OAM are parallel
- Spin-0 hadrons have non-zero spin-orbit correlations

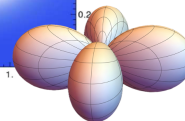
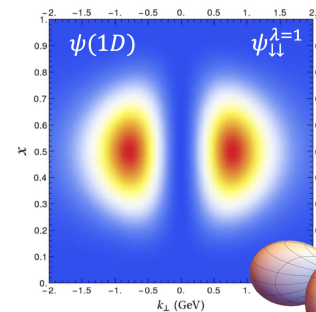
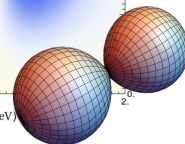
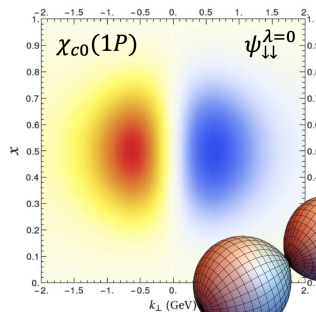
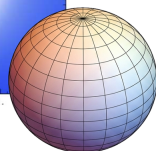
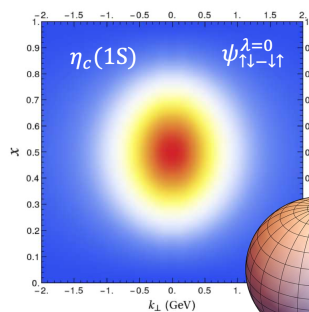


$$m_b \sim 3m_{\bar{c}}$$

Summary of SOC in heavy mesons



angular
excitations



Summary

- Spin-orbit correlations provide a novel probe of hadron spin structures
- We obtain a non-perturbative quantum many-body formula for spin-orbit correlations under the light-front Hamiltonian method
- We apply this formula to heavy mesons solved from an effective Hamiltonian
- This formula can be further extended to proton and other hadrons

Based on: arXiv: 2606.06263

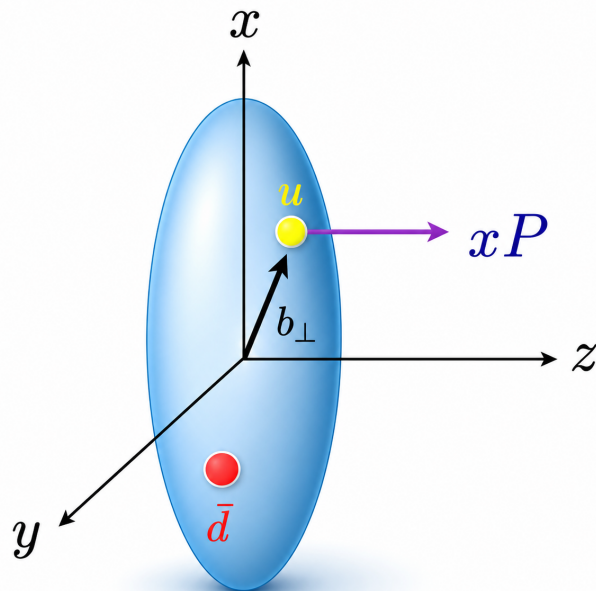
Thank you!

Backup Slides

Spin structure of spin-0 mesons

[QCDSF:2007ifr, Lorce:2025ayr]

Pion Spin: $S = 0$



Longitudinal spin structure is trivial

$$\langle \pi(p') | \bar{\psi} \gamma^3 \gamma^5 \psi | \pi(p) \rangle = 0$$

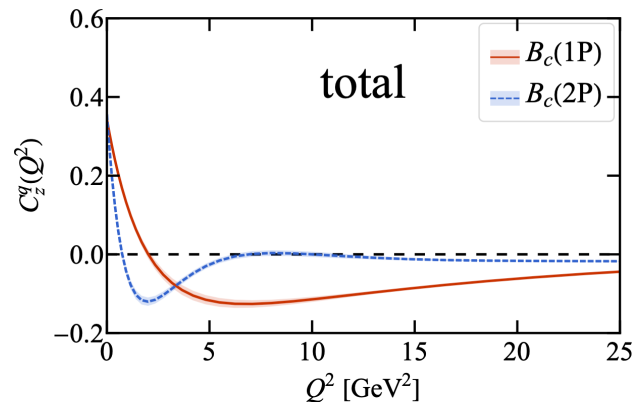
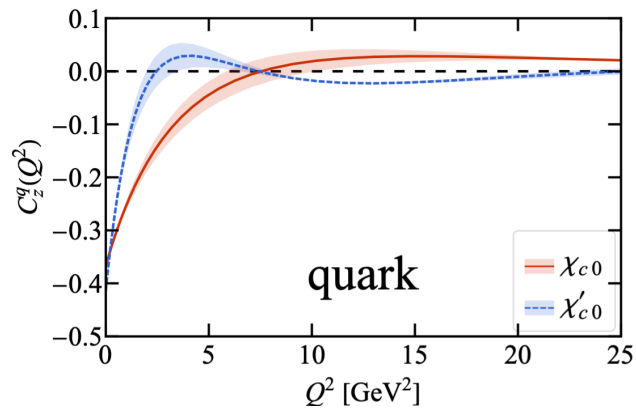
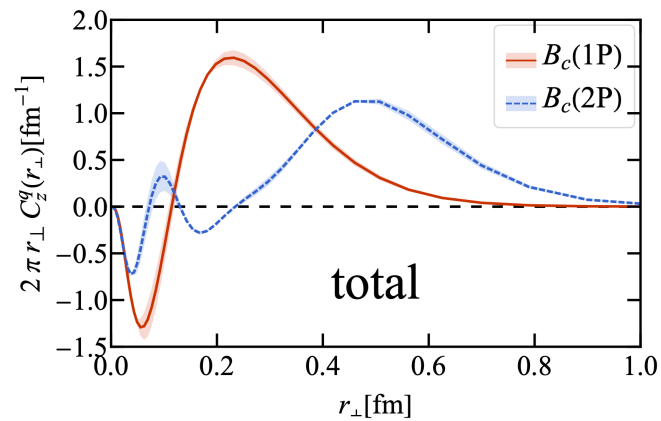
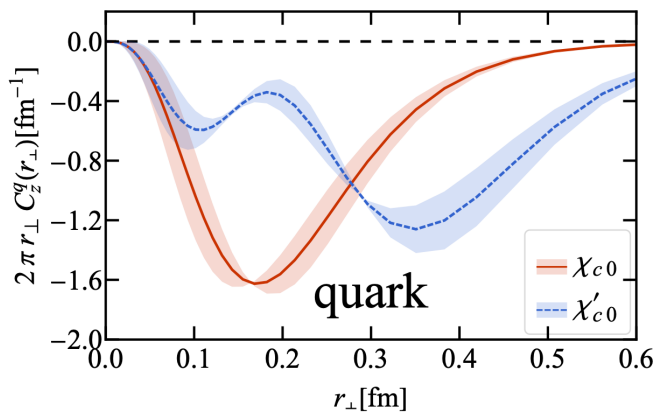
Nonzero spin-orbit correlation (twist-3 GPD)

$$C_z^q = - \int dx x G_2^q(x, \xi, 0)$$

$$\frac{i \epsilon_T^{\mu\nu} \Delta_\nu}{2P^+} G_2^q = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-\frac{z}{2}) \gamma_T^\mu \gamma_5 \psi(\frac{z}{2}) | p \rangle$$

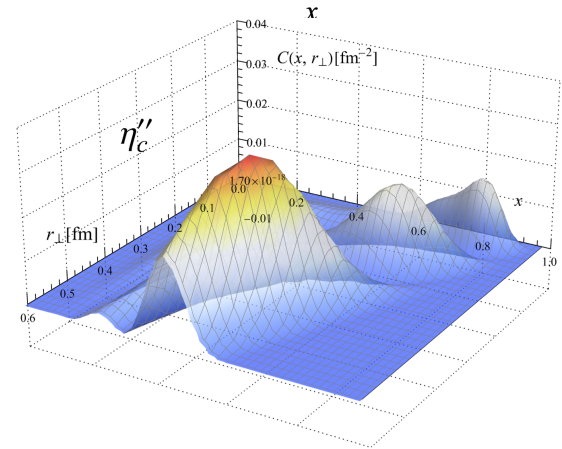
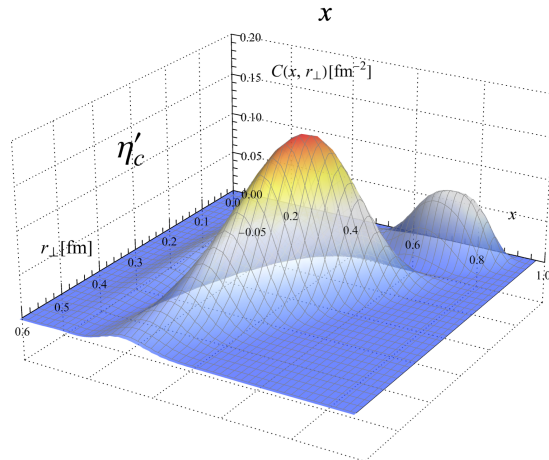
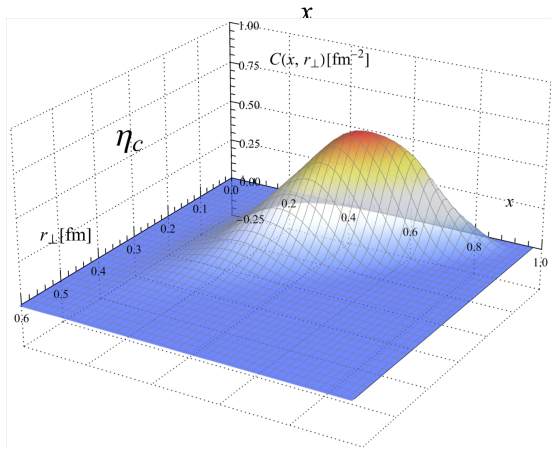
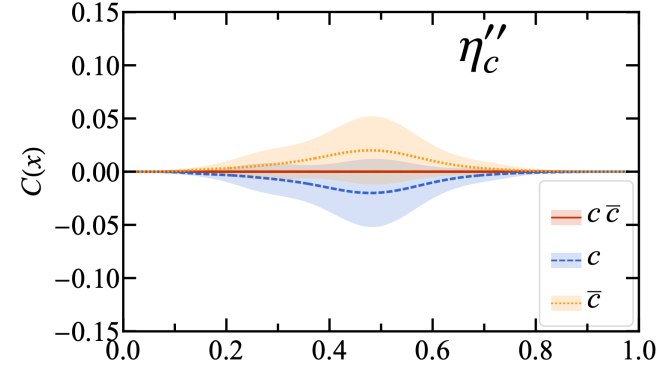
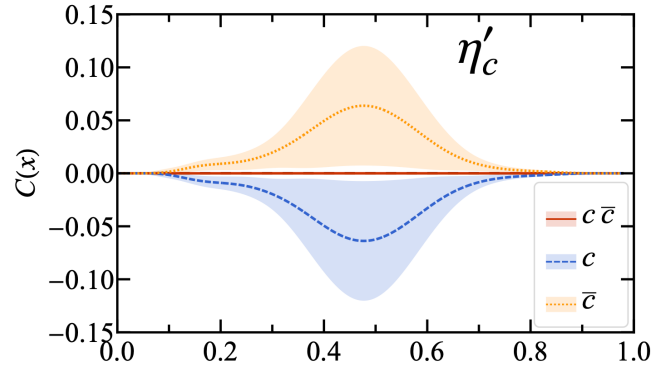
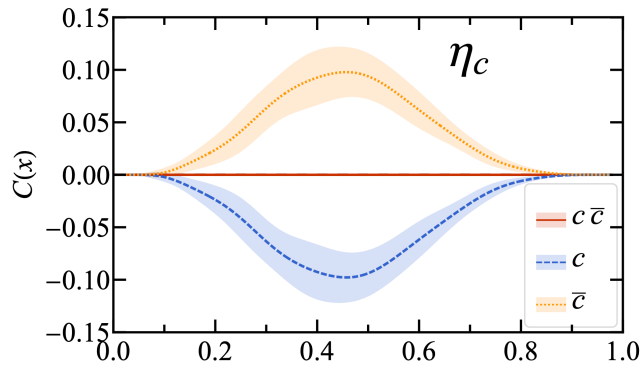
A new probe of the spin structure in spin-0 hadrons

Spin-orbit correlation: P-wave



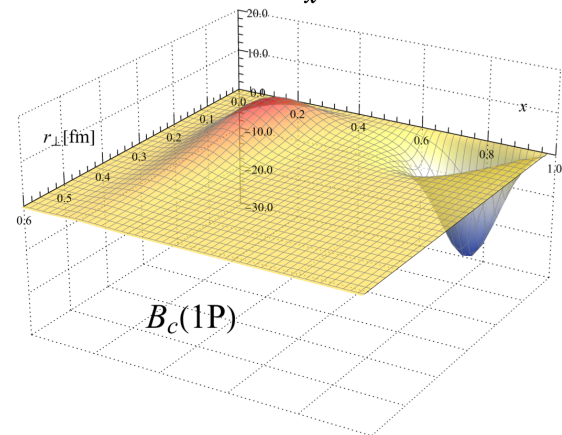
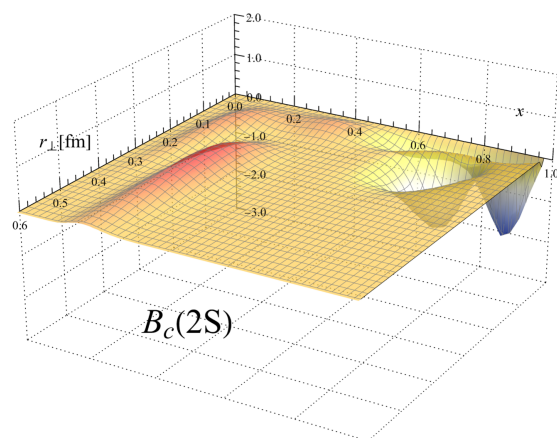
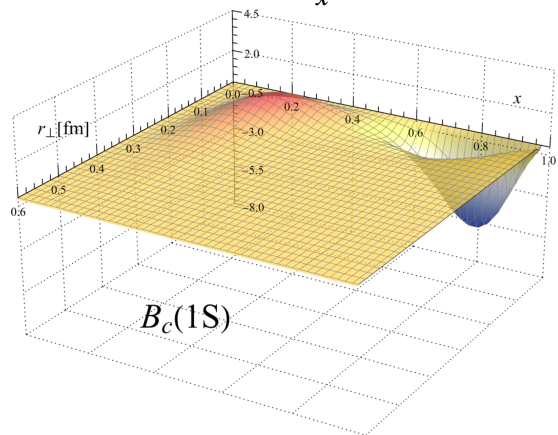
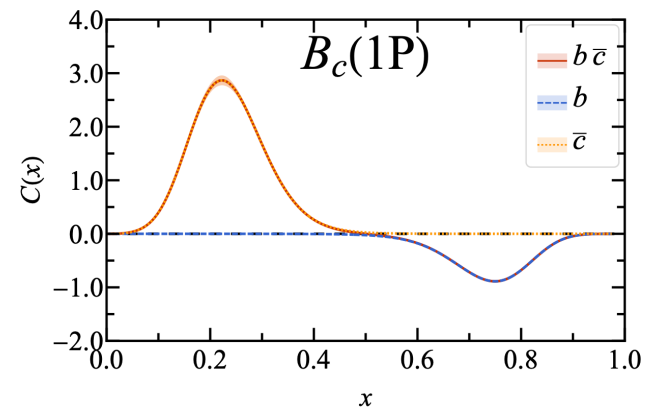
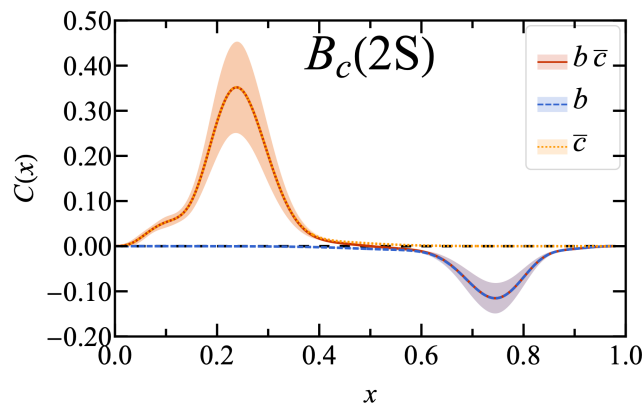
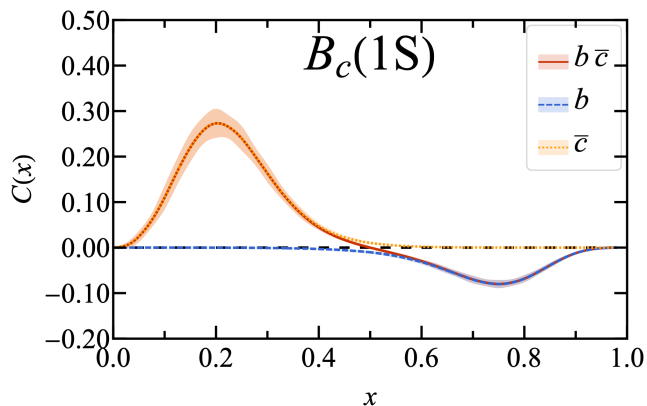
Parton distribution

$$C_z^q(x) = \frac{m_q}{M} \mathcal{H}_1^q(x, 0, 0) - \frac{1}{2} \mathcal{H}^q(x, 0, 0) - x \mathcal{G}_2^q(x, 0, 0)$$



Parton distribution

$$C_z^q(x) = \frac{m_q}{M} \mathcal{H}_1^q(x, 0, 0) - \frac{1}{2} \mathcal{H}^q(x, 0, 0) - x \mathcal{G}_2^q(x, 0, 0)$$



How to access SOC

- Relation with other form factors:

[Lorce:2025ayr]

$$\tilde{F}^q(q^2) = \frac{1}{2} \left[-F^q(q^2) + \frac{m_q}{M} H^q(q^2) \right] = -\frac{1}{2} F^q(q^2) + \mathcal{O}\left(\frac{m_q}{M}\right)$$

$$\langle p' | \bar{\psi} \gamma^\mu \psi(0) | p \rangle = 2P^\mu F^q(q^2)$$

$$\langle p' | \bar{\psi} i \sigma^{\mu\nu} \gamma_5 \psi(0) | p \rangle = \frac{i \epsilon^{\mu\nu\Delta P}}{M} H^q(q^2)$$

- Relation with leading-twist GPDs:

$$C_z^q(x) = \frac{m_q}{M} \mathcal{H}_q^1(x, 0, 0) - \frac{1}{2} \mathcal{H}^q(x, 0, 0) - x \mathcal{G}_2(x, 0, 0)$$

Model parameters

TABLE I. Summary of the model parameters.

	N_f	κ (GeV)	m_c (GeV)	m_b (GeV)	r.m.s (MeV)	$\overline{\delta_j M}$ (MeV)	$N_{\max} = L_{\max}$	Ref.
$c\bar{c}$	4	0.966	1.603	...	31	17	32	[12]
$b\bar{b}$	5	1.389	...	4.902	38	8	32	[12]
$b\bar{c}$	4	1.196	1.603	4.902	37	6	32	This work