Back-to-back dijet production in DIS: TMD framework up to twist-3 for all Bjorken-x

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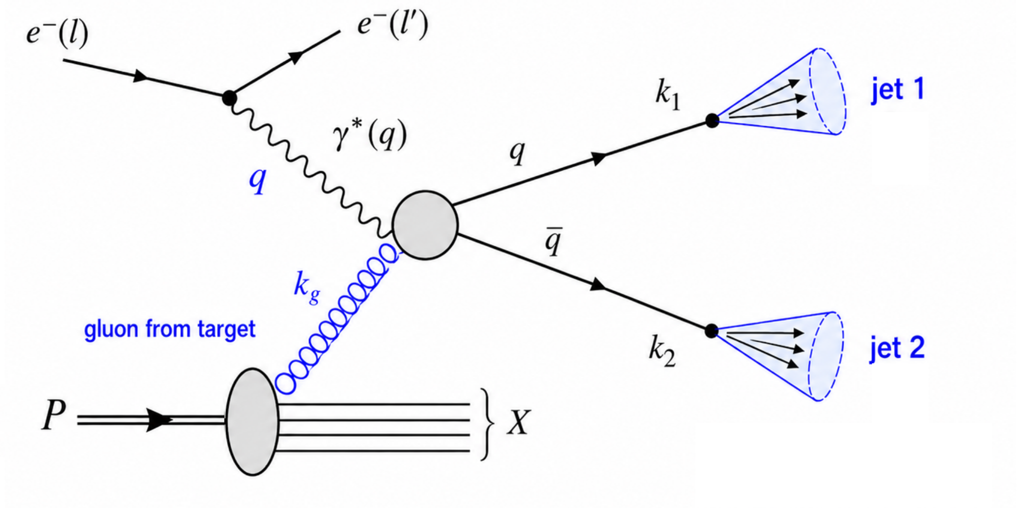
Based on arXiv:2602.15137 and a forthcoming paper

Dijet in DIS: what are we studying?

- Back-to-back dijets in DIS probe transverse partonic structure.

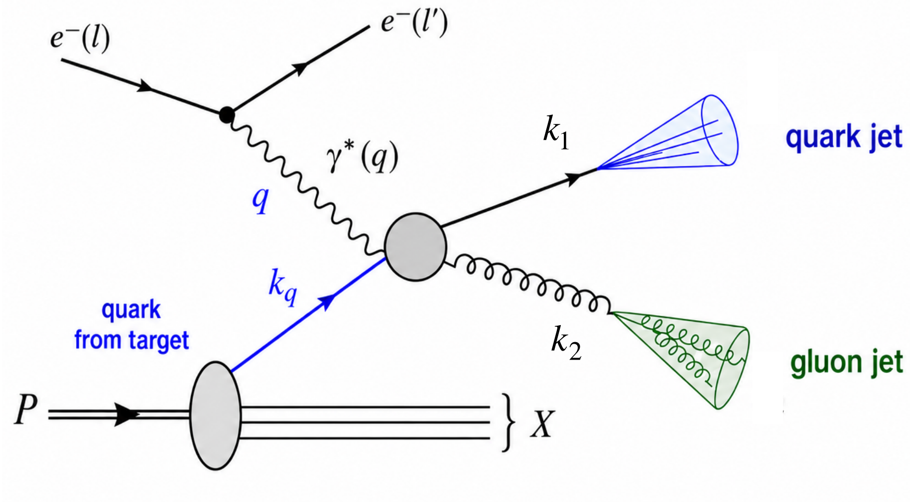
Gluon TMD sector



$\gamma^*(q) + p(P) \rightarrow q(k_1) + \bar{q}(k_2) + X$

- Background gluon field
- Sensitive to gluon TMDs

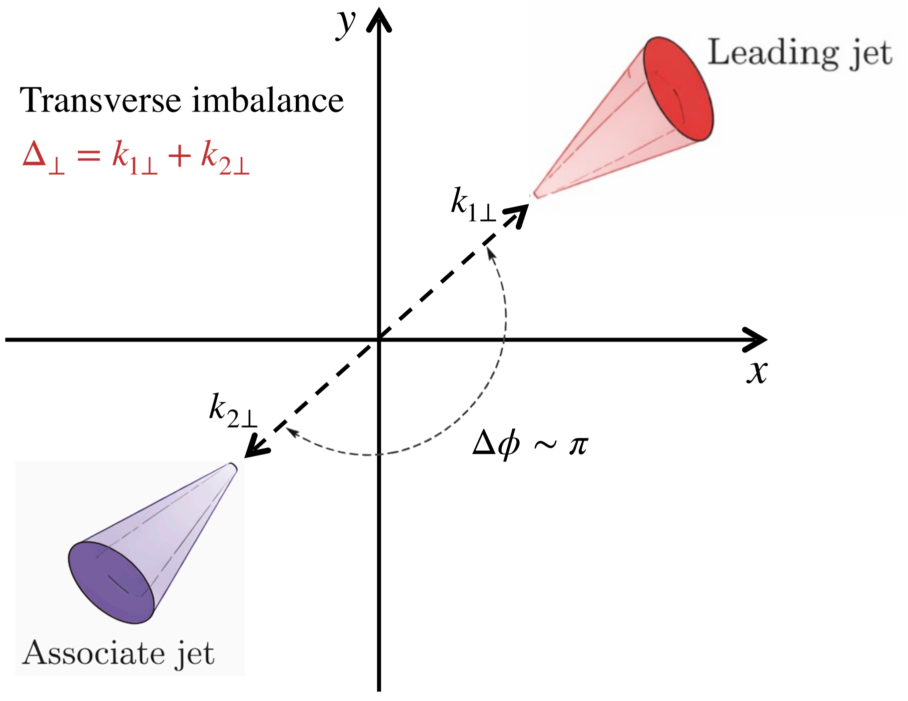
Quark TMD sector



$\gamma^*(q) + p(P) \rightarrow q(k_1) + g(k_2) + X$

- Background quark field
- Sensitive to quark TMDs

Back-to-back kinematics

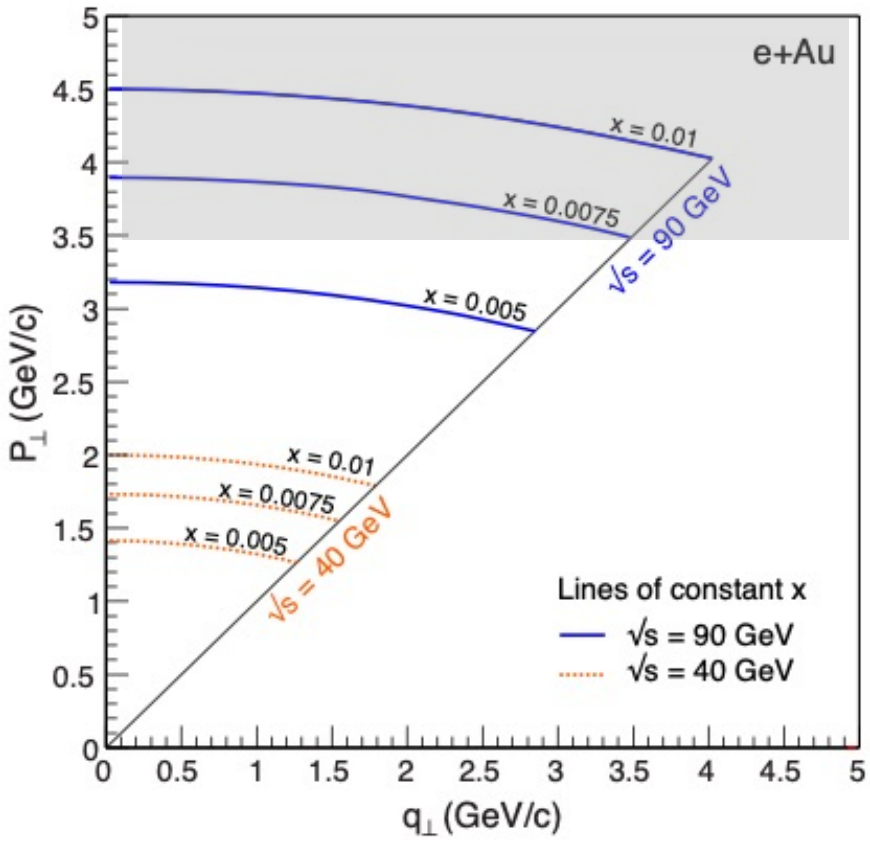


Transverse plane

- Hard splitting scale: $P_\perp \sim |k_{1\perp}| \sim |k_{2\perp}|$
- Transverse momentum imbalance:

  $\Delta_\perp = k_{1\perp} + k_{2\perp}$
- Back-to-back limit: $\Delta_\perp \ll P_\perp$
- This limit allows a TMD expansion in $\Delta_\perp / P_\perp$.

Kinematic reach of EIC dijet production



Dumitru, Skokov & Ullrich, PRC 99, (2019)

- Perturbative dijets require a hard transverse scale:

$$P_\perp \gtrsim 3.5\text{GeV}$$

- In this region, accessible x is typically

$$x \sim 10^{-2}$$

—not parametrically in the strict $x \ll 1$ limit

→ **need general-x**

Existing Results for DIS Dijet Production

	finite-/large-x TMD (moderate to large x)	small-x CGC ($x \ll 1$)	our results (General-x)
Gluon TMDs	Leading-twist (NLO) [1-4, …]	• Up to twist-3 (LO): Eikonal + subeikonal [5,6] • Leading twist (NLO) Eikonal [7-9, …]	• Up to twist-3 (LO): $q\bar{q}$ channels arXiv:2602.15137
Quark TMDs	Standard leading-twist TMD framework [1, …]	• Leading-twist (LO): Subeikonal [10]	• Up to twist-3 (LO): $q\bar{q}$ and qg channels paper to appear

[1] Collins (2011)
[4] del Castillo et al., JHEP 03 (2022)
[7] Caucal et al. JHEP 11 (2022)
[10] Altinoluk et al. PRD 108 (2023)
[2] Mulders & Rodrigues (2001)
[5] Dominguez et al. (2011)
[8] Caucal et al. JHEP 08 (2023)
[3] del Castillo et al, JHEP01(2021)
[6] Altinoluk et al. PRD 111 (2025)
[9] Caucal et al. PRL 132 (2024)

Background field method

$$C_\mu = A_\mu + B_\mu$$

Full gauge field = quantum field + background field

Tree level (this work)

- Work in a fixed background B_μ
- Compute propagator in B_μ
- Wilson lines and TMD operators emerge

Operator structure emerges

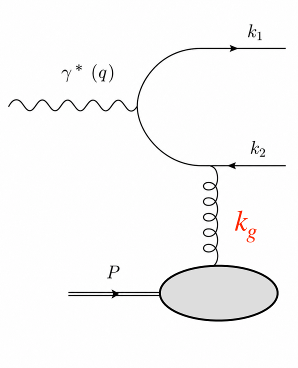
Loop level (future extension)

- Integrate out quantum modes A_μ
- Loop corrections introduce divergences
- Scale separation and evolution emerge

Evolution and factorization

A unified general-x TMD framework

Following the general-x TMD factorization framework (MSTT) proposed in Refs. [Mukherjee, Skokov, Tarasov & Tiwari, PRD 109(2024), Mukherjee, Skokov, Tarasov & Tiwari, PRD 111 (2025)], we impose **no hierarchy** between k_g^+ and $k_{g\perp}$.



small-x ← → moderate-x

Small-x TMD (Wilson-line regime)	Standard TMD factorization (Sudakov regime)

This framework: general-x TMD

BFKL evolution ($k_g^+ \ll k_{g\perp}$)

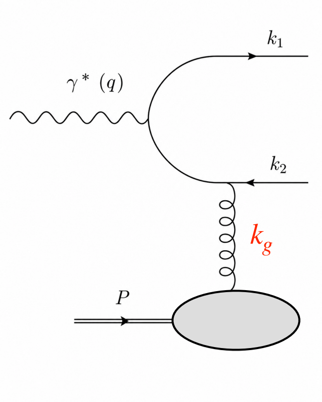
CSS evolution ($k_{g\perp} \ll k_g^+$)

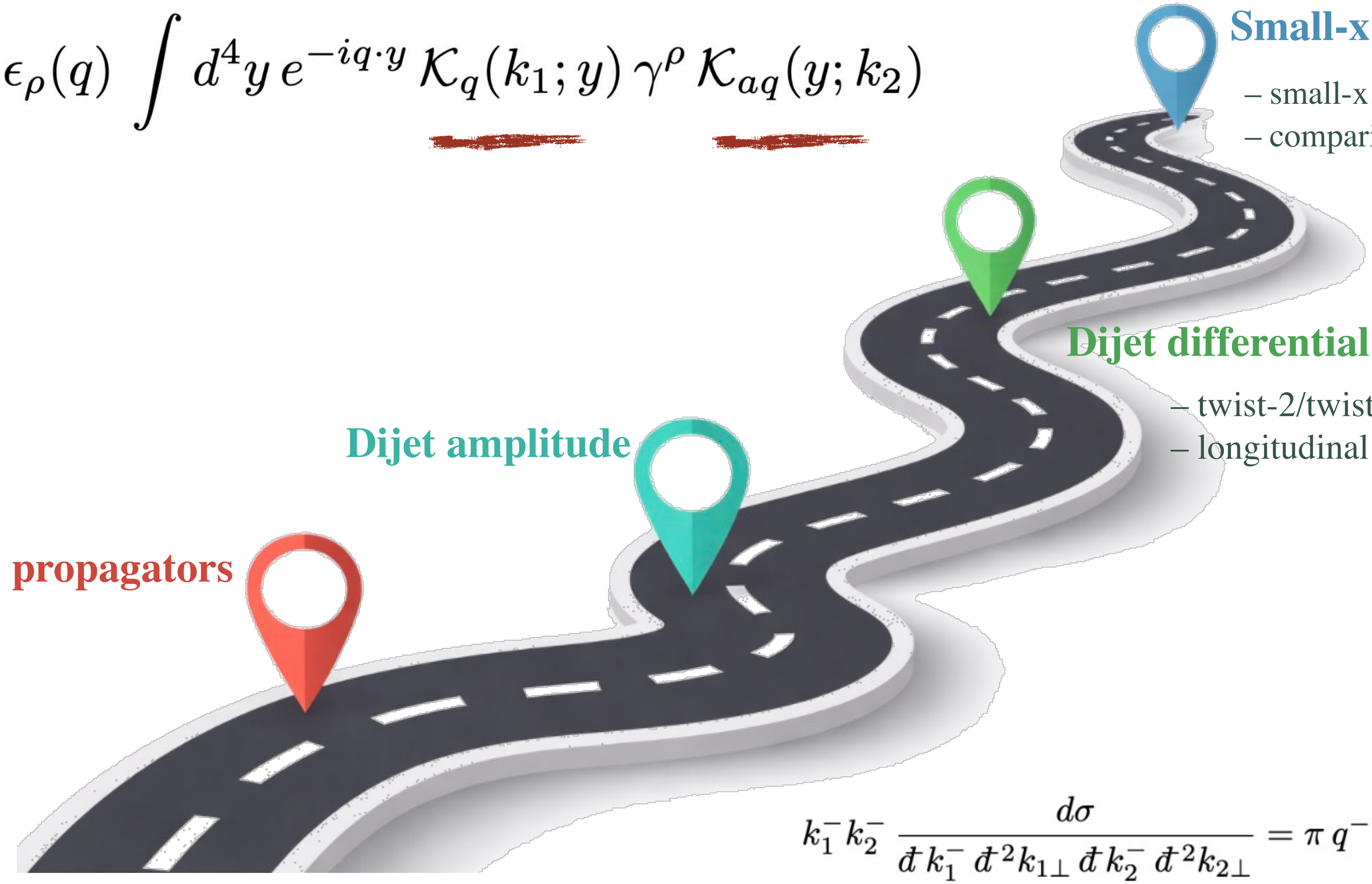
$\Delta_\perp / P_\perp \ll 1$ (Back-to-back)

Roadmap of this work

Amputated quark and antiquark

$$i\mathcal{M} = -ie\,\epsilon_\rho(q)\int d^4y\, e^{-iq\cdot y}\,\mathcal{K}_q(k_1;y)\,\gamma^\rho\,\mathcal{K}_{aq}(y;k_2)$$





Small-x consistency

– small-x limit
– comparison with CGC

Dijet differential cross-section

– twist-2/twist-3 operators
– longitudinal and transverse

Dijet amplitude

Background field propagators

squared amplitude

$$k_1^- k_2^- \frac{d\sigma}{đk_1^-\, đ^2k_{1\perp}\, đk_2^-\, đ^2k_{2\perp}} = \pi\, q^-\, \delta(k_1^- + k_2^- - q^-)\,|i\mathcal{M}|^2$$

Background-field quark propagator: gradient expansion

$$\mathcal{K}_q(k_1;y) = \bar{u}(k_1) \lim_{k_1^2\to 0} (k_1|\not{p}\frac{i}{\not{\mathrm{P}}}|y) \qquad \mathrm{P}^\mu = p^\mu + A^\mu$$

$$= \bar{u}(k_1) \lim_{k_1^2\to 0} k_1^2 (k_1|\frac{1}{\mathrm{P}^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}|y) = \bar{u}(k_1) \lim_{k_1^2\to 0} k_1^2 (k_1|\frac{1}{2p^-p^+ + 2p^-A_- - i\partial^-A_- - \mathrm{P}_\perp^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}|y)$$

Gradient expansion =

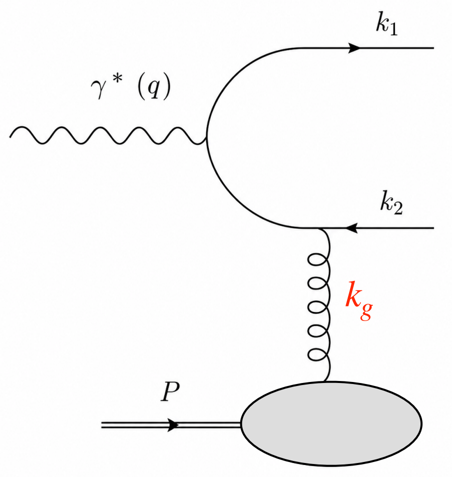
Wilson line + dynamics in target F_{+-} + field-strength insertions F_{-i} + Spin-field coupling σF

$$\lim_{k_1^2\to 0} k_1^2 (k_1|\frac{1}{\mathrm{P}^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}|y) = i e^{ik_1\cdot y}\Big[-i[\infty, y^-]_{y_\perp} + \textit{background field insertions}\Big]$$

Retains full-phase

- Details are given in eq.(21) of arXiv:2602.15137

Dijet amplitude



$$i\mathcal{M} = -ie\,\epsilon_\rho(q)\int d^4y\, e^{-iq\cdot y}\,\mathcal{K}_q(k_1;y)\,\gamma^\rho\,\mathcal{K}_{aq}(y;k_2)$$

quark propagator: $\mathcal{K}_q(k_1;y) = \bar{u}(k_1)\lim_{k_1^2\to 0}(k_1|\not{p}\,\frac{i}{\not{P}}|y) \propto e^{ik_1\cdot y}$

antiquark propagator: $\mathcal{K}_{aq}(y;k_2) = \lim_{k_2^2\to 0}(y|\frac{i}{\not{P}}\not{p}|-k_2)v(k_2) \propto e^{ik_2\cdot y}$

$$e^{ik_1\cdot y}\times e^{ik_2\cdot y}\times e^{-iq\cdot y} = e^{i(k_1+k_2-q)\cdot y} = e^{ik_g\cdot y}$$

Reconstructs full-phase, with no $k_g^+/k_{g\perp}$ hierarchy

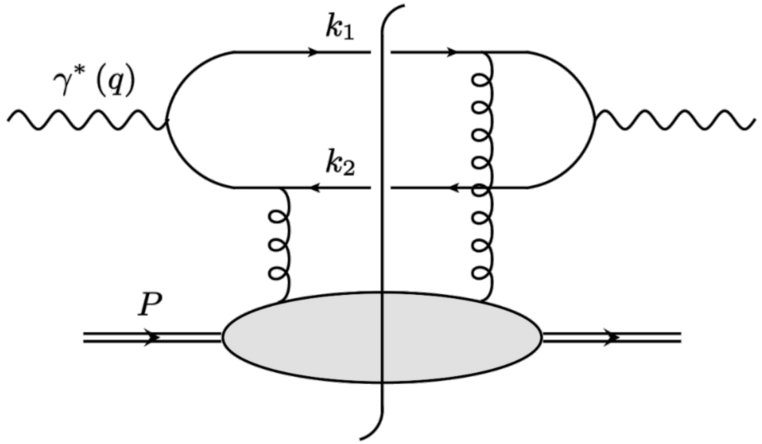
- Details are given in eq.(27) of arXiv:2602.15137

Dijet differential cross-section

Differential cross-section

$$k_1^- k_2^- \frac{d\sigma}{đ k_1^- \, đ^2 k_{1\perp} \, đ k_2^- \, đ^2 k_{2\perp}} = \pi\, q^-\, \delta(k_1^- + k_2^- - q^-)\, |i\mathcal{M}|^2$$

Longtudinally polarized	Transversely polarized
$\epsilon_-(q)\,\epsilon_-^*(q) = \frac{Q^2}{q^{-2}}$	$\frac{1}{2}\sum_{\lambda=\pm1} \epsilon_k^\lambda(q)\,\epsilon_l^{\lambda *}(q) = -\frac{1}{2} g_{kl}$



Leading-twist contribution to the cross section.

Operator structure up to twist-3

Kinematic twist

expansion in $\Delta_\perp / P_\perp$ →

dynamic twist

$\tau = d - s$ ↓

	twist-2 (leading)	twist-3 (linear)
twist-2	$C_1 \otimes F_{-i}F_{-j}$ ✔	$C_2 \otimes F_{-i}F_{-j}$ $C_2 \propto \mathcal{O}\left(\Delta_\perp / P_\perp\right)$ ✔
twist-3	$C_3 \otimes F_{ij}F_{-j}$ $C_4 \otimes F_{+-}F_{-j}$ $C_5 \otimes F_{-i}F_{-k}F_{-l}$ ✔	$C_6 \otimes F_{ij}F_{-j}, \ \ldots$ ✘

Differential cross-section: the longitudinally polarized photon

complete phase

$$|i\mathcal{M}|_L^2 \approx e^2 \int dx^- d^2x_\perp dy^- d^2y_\perp\, e^{ik_g^+(x^- - y^-) - i\Delta_\perp(x_\perp - y_\perp)} \frac{16\, z^2\bar{z}^2\, \epsilon_f^2}{(P_\perp^2 + \epsilon_f^2)^4} \mathrm{Tr}\left[O_1 + O_2 + O_3 + O_4\right]$$

$$\epsilon_f^2 = z\bar{z}Q^2$$

Kinematic twist-3

$$O_1 = \left[2P^iP^j - (\bar{z} - z)(P^i\Delta^j + P^j\Delta^i) + 8(\bar{z} - z)\frac{(\Delta\cdot P)P^iP^j}{P_\perp^2 + \epsilon_f^2}\right]\overline{F}_{-i}(x^-, x_\perp)\,\overline{F}_{-j}(y^-, y_\perp),$$ dynamic leading-twist

$$O_2 = \frac{(z - \bar{z})}{z\bar{z}\, q^-} P^iP_\perp^2\left[\overline{F}_{-i}(x^-, x_\perp)\,\overline{F}_{+-}(y^-, y_\perp) + \overline{F}_{+-}(x^-, x_\perp)\,\overline{F}_{-i}(y^-, y_\perp)\right],$$ dynamic twist-3

$$O_3 = P^i\left[\mathcal{H}_{kl}(z)\,\overline{F}_{-i}(x^-, x_\perp)\,\mathbb{D}_{kl}{}^\dagger(l^+, y^-, y_\perp) + \mathcal{H}_{kl}(z)\,\mathbb{D}_{kl}(l^+, x^-, x_\perp)\,\overline{F}_{-i}(y^-, y_\perp)\right]_{l^+=0},$$

$$O_4 = P^i\left[\mathcal{H}_{kl}(\bar{z})\,\overline{F}_{-i}(x^-, x_\perp)\,\mathbb{D}_{kl}(-l^+, y^-, y_\perp) + \mathcal{H}_{kl}(\bar{z})\,\mathbb{D}_{kl}^\dagger(-l^+, x^-, x_\perp)\,\overline{F}_{-i}(y^-, y_\perp)\right]_{l^+=0},$$

dynamic twist-3

where we defined

$$\mathcal{H}_{kl}(z) \equiv \delta_{kl} - \frac{4P^kP^l}{P_\perp^2 + \epsilon_f^2} + \frac{2P^kP^l}{zq^-}\frac{\partial}{\partial l^+}.$$

Small-x limit: Matching to CGC

From general-x TMD factorization to high-energy eikonal physics

1 Full longitudinal phase (general-x TMD framework):

$$e^{ixk_g^+(x^- - y^-)}$$

2 Small-x limit (x → 0), taking phase expansion:

$$e^{ixP^+(x^- - y^-)} \approx 1 + ix\,P^+(x^- - y^-)$$

3 Operator reduction:

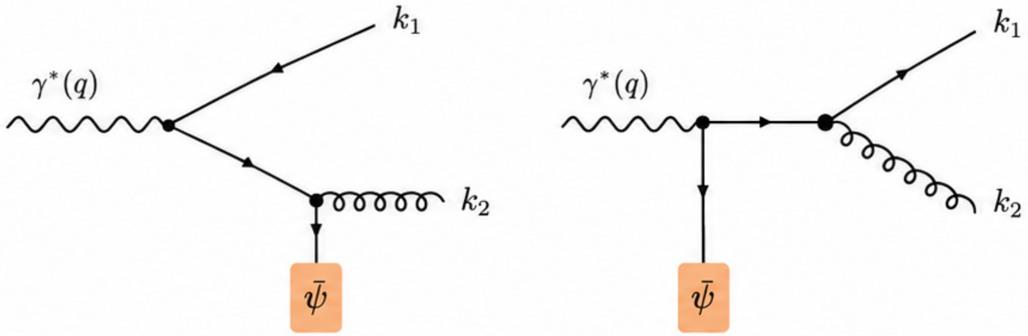
- $F_{-i}F_{-j}$ → eikonal, subeikonal
- $F_{-i}F_{-j}F_{-k}$ → eikonal, subeikonal
- $F_{+-}F_{-j}$, $F_{ij}F_{-j}$ → subeikonal

✓ **Reproduces eikonal and subeikonal CGC expressions in Ref.** [Altinoluk, Beuf, Czajka & Marquet, PRD 111 (2025)]

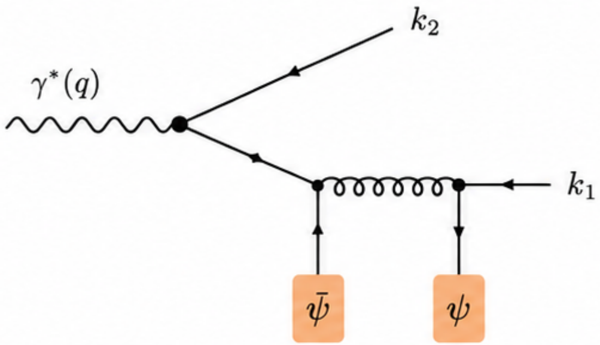
Companion quark sector

The same background-field method also probes the **quark** content of the target.

- **Quark-gluon dijet channel:**
  starts already at leading twist
  probes quark TMD operators



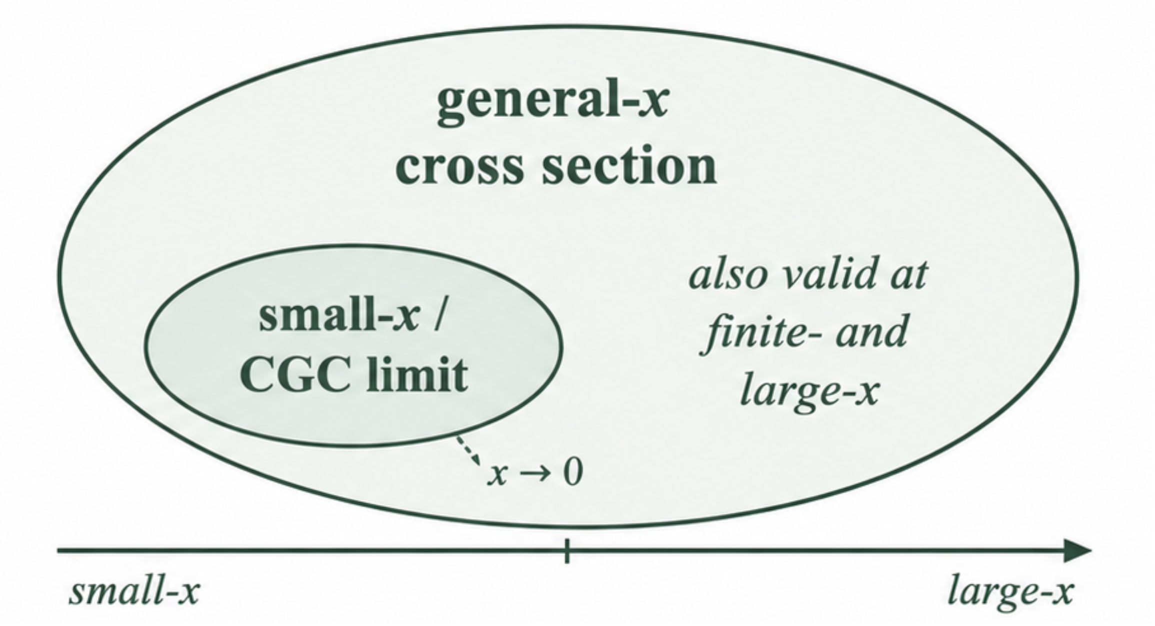
- **Quark-antiquark dijet channel:**
  appears first at twist three
  requires two background-quark insertions



Together with the **gluon sector**, these channels complete the quark/gluon TMD description at **twist-three accuracy**.

Summary & Outlook

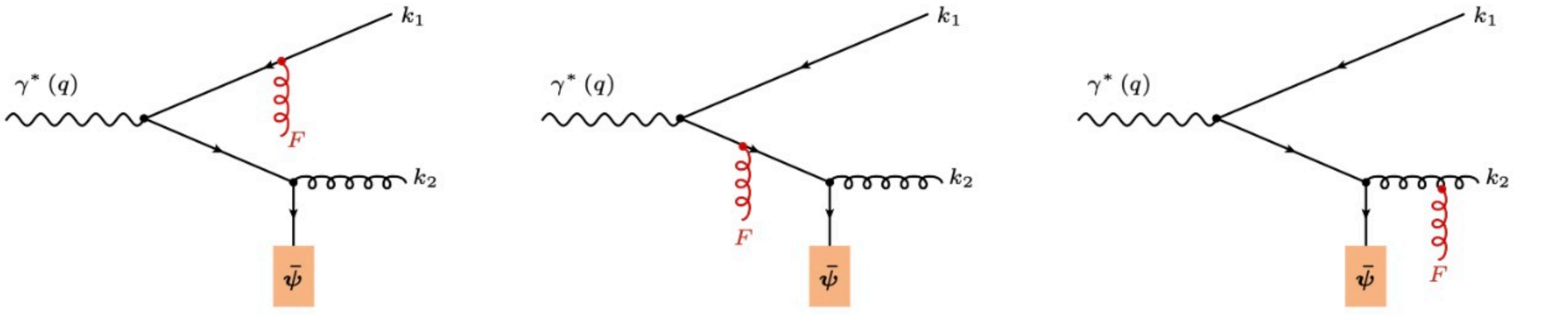
Theoretical input for extracting quark and gluon TMDs across the full-x region at the EIC.



Outlook: full NLO calculation and factorization test.

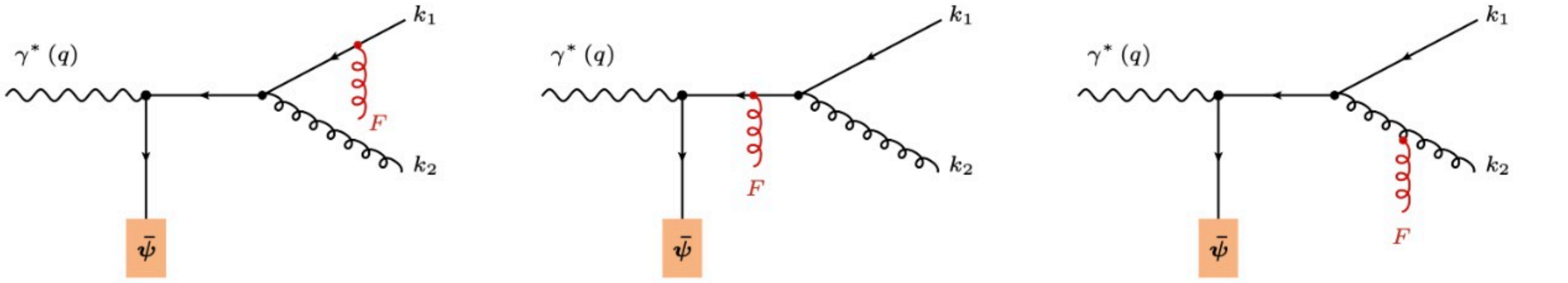
Thank you!

Backup: Companion quark sector



(a) $(A_1,\ H_0,\ \mathcal{K}_0)$ (b) $(A_0,\ H_1,\ \mathcal{K}_0)$ (c) $(A_0,\ H_0, \mathcal{K}_1)$

FIG. 3. Three-body contributions from the first topology. The three panels correspond to the cases where the field– strength insertion originates from A_1, H_1, and $\mathcal{K}_1$ respectively. The shaded box denotes the background quark field $\bar{\psi}$. The red F denotes the explicit field-strength insertion.



(a) $(A_1,\ L_0,\ \mathcal{K}_0)$ (b) $(A_0,\ L_1,\ \mathcal{K}_0)$ (c) $(A_0,\ L_0, \mathcal{K}_1)$

FIG. 4. Three-body contributions from the second topology. The three subfigures correspond to the cases where the field–strength insertion originates from A_1, L_1, and $\mathcal{K}_1$, respectively. The shaded box denotes the background quark field ψ. The produced quark and gluon carry momenta k_1 and k_2, respectively.