Title: Electron and neutrino scattering off of light nuclei

W. Polyzou
University of Iowa

NSF Eager collaboration

Project Participants: O. Shebeko, A. Arslanaliev, Y. Kostylenko, V. Chahar, J. Golak, H. Kamada, D. Ramírez, R. Skibiński, K. Topolnicki, H. Witała, B. Martin, W. P.

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References: https://arxiv.org/abs/2604.24457, https://arxiv.org/abs/2601.00534

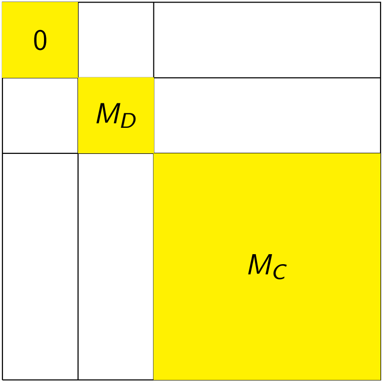
Background:

- 1958 - Clothed-particle representation (CPR) proposed by O.W. Greenberg and S.S. Schweber.
- 2000 - present - Development as a method for realistic computations by A. Shebeko, M. Shirokov and collaborators.
- 2016 - Kharkiv NN potential from CPR, H. Kamada, A. Shebeko, A. Arslanaliev.
- 2024 - Many-body currents from CPR, Y. Kostylenko (Thesis), A. Shebeko.
- 2024 - Applications to electroweak interactions, EAGER collaboration.

Goal of CPR

- Separate a particle's self interaction from the inter-particle interactions.
- Representation where the vacuum and single-particle states are eigenstates of the Hamiltonian.
- Preserve Poincaré invariance.
- Consistent currents (from gauge covariant derivatives; transform like 4 vector densities in the one γ, W of Z exchange approximation)
- Formulation of scattering asymptotic conditions.

Structure of the mass operator, (M), in CPR



General comments

- A well-defined canonical transformation changes the bare mass to the physical one-body mass, this cannot be realized as a unitary transformation.
- This requires a new vacuum functional.
- The transformed mass operator has the physical mass spectrum.
- Since the mass is a Casimir operator for the Poincaré group, the dynamical unitary representation of the Poincaré group is also block diagonal.

Details

- The physics problem.
- Model.
- Clothed particle representation of strong interaction.
- Scattering in the clothed particle representation.
- Gell-Mann Goldberger two-potential approximation.
- Currents and dynamics
- Selected results.
- Future directions.

Physics goal

- The physics goal is to understand how leptons interact with nuclear targets at the few-GeV energy scale.
- Required elements:
  - A realistic non-perturbative treatment of the strong interaction.
  - A dynamical unitary representation of the Poincaré group.
  - A consistent treatment of hadronic currents.
  - Cluster properties - the interactions and currents in few-body systems appear in larger nuclear targets.

Starting point:

- Model: Nucleons exchanging $\pi, \rho, \sigma, \eta, \delta, \omega$ mesons.
- Hamiltonian formulation - non-perturbative calculations reduced to linear algebra.
- Noether's theorem $\Rightarrow$ Poincaré generators.
- Hamiltonian and boost generators have interactions (current work in instant form, front-form is being investigated).
- Consistent currents - replace derivatives in Lagrangian by gauge covariant derivatives.

Scattering equivalent representations; justification for CPR

- Ekstein's theorem:

$$S = S' \Leftrightarrow$$

$$H' = AHA^\dagger \qquad AA^\dagger = I \qquad \lim_{t\to\pm\infty} \|(A-I)e^{iH_0 t}|\psi\rangle\| = 0$$

- H, H' are experimentally indistinguishible

- Cannot distinguish models based on H and H'. Result can be restricted to a given energy range since $S(E) = S'(E)$.

- A is independent of H.

- It is not necessary to know A.

- Currents are representation dependent.

Clothed particle representation

- A $*$ automorphism, W, on the algebra of creation and annihilation operators is constructed to transform the creation and annihilation operators to new "clothed" creation and annihilation operators, $a = W(a_c, a_c^\dagger) a_c W^*(a_c, a_c^\dagger)$.

- W is constructed so when the Poincaré generators are expressed in terms of the clothed creation and annihilation operators **the interactions** have no terms with less that 2 transformed (clothed) creation or annihilation operators.

- A vacuum functional is **defined** on this algebra by the requirement that it is annihilated by the transformed (clothed) annihilation operators. The clothed creation operators applied to this vacuum are assumed to generate a representation of the physical Hilbert space.

- The new **vacuum** and physical **one-particle** states, created by applying the clothed particle creation operator to the new vacuum, are **exact eigenstates** of the Hamiltonian.

- The Hamiltonian, expressed in terms of the clothed particle creation and annihilation operators, is **non-local**.

- The simplest interaction is a short-range **2-2** (Nucleon-Nucleon) interaction.

- There are **no-vertices**; meson production is due a short-range 2 - 3 interaction (i.e. $NN \rightarrow NN\pi$).

- While W and the choice of vacuum takes care of the mass and vacuum renormalization, UV singularities remain.

Interaction does not change the new vacuum or one-particle states

$$\int a_c^\dagger(\mathbf{p}_{1f})a_c^\dagger(\mathbf{p}_{2f})V_c(\mathbf{p}_{1f},\mathbf{p}_{2f};\mathbf{p}_{1i},\mathbf{p}_{2i})a_c(\mathbf{p}_{1i})a_c(\mathbf{p}_{2i})d\mathbf{p}_{1f}d\mathbf{p}_{2f}d\mathbf{p}_{2i}d\mathbf{p}_{1i}\times$$

$$|0\rangle = 0$$

$$\int a_c^\dagger(\mathbf{p}_{1f})a_c^\dagger(\mathbf{p}_{2f})V_c(\mathbf{p}_{1f},\mathbf{p}_{2f};\mathbf{p}_{1i},\mathbf{p}_{2i})a_c(\mathbf{p}_{1i})a_c(\mathbf{p}_{2i})d\mathbf{p}_{1f}d\mathbf{p}_{2f}d\mathbf{p}_{2i}d\mathbf{p}_{1i}\times$$

$$\int a_c^\dagger(\mathbf{p})|0\rangle f(\mathbf{p})d\mathbf{p} = 0$$

- The gauge covariant currents, when expressed in terms of the clothed particle creation and annihilation operators, have **many-body parts** (i.e. consistent currents).

- The Hamiltonian and current operators are **unchanged**; they are just expressed in terms of different operators.

- The clothing operator W is used to express the Poincaré generators in terms of the clothed particle creation and annihilation operators.

- The **vacuum and representation** of the Hilbert space are changed.

Construction of W

General decomposition of the Hamiltonian

$$H = H_m + g(V_g + V_b)$$

where $\mathbf{H}_m$ is the free Hamiltonian with **physical masses**, V_g is the part of the remainder that has at least two creation and 2 annihilation operators (**good terms**), and V_b involves the remaining terms, which are called **bad terms**.

Choose $W(a_c, a_c^\dagger)$, $W(a_c, a_c^\dagger)W^*(a_c, a_c^\dagger) = I$, of the form

$$W = e^{iR} = e^{i\sum g^n r_n}.$$

to eliminate the bad terms from H. This done order by order in g. The leading term satisfies:

$$[r_1, H_m] = -V_b; \qquad r_1 = -i \lim_{\epsilon \to 0} \int_0^\infty dt e^{iH_m t} V_b e^{-iH_m t} e^{-\epsilon t}$$

- r_1 removes the bad terms of order g but generates good and bad terms of order g^2. r_2 is constructed to eliminate bad terms of order g^2. The process can be continued to all orders.

- The construction is **not unique**, but **consistent**, and if the regularized W^*W''s satisfy the short-range (Ekstein) condition then W and W' should give the **same scattering operator**.

- Current work is based on a recursive construction. The calculation at order g^2 gives a realistic non-local **nucleon-nucleon interaction, consistent currents, and dynamical boost generators**.

- Meson production first appears at order g^3. This is not treated in this work.

Scattering, interpretation of $a(\mathbf{p})$, relation to Haag-Ruelle scattering asymptotic condition

$$\Omega_\pm|f_1 f_2\rangle = \lim_{t\to\pm\infty} B_{1hr}(t)B_{2hr}(t)|0\rangle =$$

$$B_{hr}(t) := -i\left(f(\mathbf{x},-t)\partial_t\phi_{hr}(\mathbf{x},t) - \phi_{hr}(\mathbf{x},t)\partial_t f(\mathbf{x},-t)\right)$$

$$\phi_{hr}(x) = \frac{1}{(2\pi)^4}\int \chi_m(-p^2)e^{-ip\cdot(x-y)}\phi(y)d^4y d^4p$$

- $\chi(m^2)=1$; $\chi(\sigma(M^2)/m)=0$.
- $B_{hr}(t)$ **asymptotically** approaches a single-physical-particle creation operator.
- Gives strong limit - can use standard time-independent methods.

$$\Omega_\pm|f_1 f_2\rangle = \lim_{t\to\pm\infty} e^{-iHt}\int a_c^\dagger(\mathbf{p}_1)a_c^\dagger(\mathbf{p}_2)|0\rangle f(\mathbf{p}_1,-t)f(\mathbf{p}_2,-t)d\mathbf{p}_1 d\mathbf{p}_2$$

Dynamics:

- The order g^2 Hamiltonian has a short-range two-body interaction, called the **Kharkiv potential** that has a Deuteron bound state.
- Because the clothed creation operators create physical one-particle eigenstates out of the physical vacuum, they can be used to formulate scattering asymptotic conditions based on strong limits:

$$S = \Omega_+^\dagger \Omega_-$$

$$\Omega_- = s \lim_{t \to \pm\infty} e^{iHt} \int \prod a_c^\dagger(\mathbf{p}_i) f(\mathbf{p}_i) e^{-iE_i(\mathbf{p}_i)t} |0\rangle d\mathbf{p}_i$$

A **sufficient condition** for convergence is

$$\int_0^{\pm\infty} \| V \int \prod a_c^\dagger(\mathbf{p}_i) f(\mathbf{p}_i) e^{-iE_i(\mathbf{p}_i)t} |0\rangle \| \, dt < \infty$$

Scattering:

- The product $\prod_{i=1}^{n} a_c^\dagger(\mathbf{p}_i)|0\rangle$ does not create n-particle states, but because the interactions are short ranged, these states asymptotically look like n-particle states when the states are asymptotically separated.
- For a large class of short-range interactions with massive particles in the final state the integrand falls off like $t^{-3(n-1)/2}$.
- There is a similar short-range condition on the boost interaction which ensures the invariance of the scattering operator, S.

Electroweak probes of light nuclei - use **Gell-Mann-Goldberger two-potential formula**:

$$H = H_0 + V_s + V_w$$

The **chain rule** for wave operators gives

$$S = \lim_{t\to\infty} e^{iH_0 t} e^{-iHt} e^{-iHt} e^{iH_0 t} =$$

$$\lim_{t\to\infty} e^{iH_0 t} e^{-iH_s t} e^{iH_s t} e^{-iHt} e^{-iHt} e^{iH_s t} e^{-iH_s t} e^{iH_0 t} =$$

$$\lim_{t\to\infty} \Omega^\dagger_{+s} e^{iH_s t} e^{-iHt} e^{-iHt} e^{iH_s t} \Omega_{-s} =$$

$$\Omega^\dagger_{+s} \left(I - i \int V_I(t) dt + \frac{(-i)^2}{2!} \int dt_1 dt_2 T(V_I(t_1) V_I(t_2)) \cdots \right) \Omega_{-s}$$

Interaction term

$$V_I(t) = We \int d\mathbf{x}(\bar{\Psi}_{nc}(\mathbf{x},t)\gamma_n^{\mu}\Psi_{nc}(\mathbf{x},t)+$$

$$\bar{\Psi}_{pc}(\mathbf{x},t)\gamma_p^{\mu}\Psi_{pc}(\mathbf{x},t)+\bar{\Psi}_{e}(\mathbf{x},t)\gamma_e^{\mu}\Psi_{e}(\mathbf{x},t))A_{\mu}(\mathbf{x},t)W^*$$

- Creation and annihilation operators are expressed in terms of clothed creation and annihilation operators.
- Uses the same clothing transformation, W, used to construct the interaction and Poincaré generators
- In the clothed particle representation the hadronic current has both one- and many-body parts

Hadronic many-body currents

$$J_h^\mu(0) = e^{iR} J_c^\mu(0) e^{-iR} =$$

$$J_c^\mu(0) + i[R, J_c^\mu(0)] - \frac{1}{2}[R,[R, J_c^\mu(0)]] + \cdots$$

- This expansion includes both dynamical and one and two-body currents.
- The one-body parts generate nucleon form factors.

Structure of a typical two-body current matrix element in the CPR

$$\langle \mathbf{p}_1', \nu_1', \mathbf{p}_2', \nu_2' | J^{\mu}_{\pi NN}(0) | \mathbf{p}_1, \nu_1, \mathbf{p}_2, \nu_2 \rangle =$$

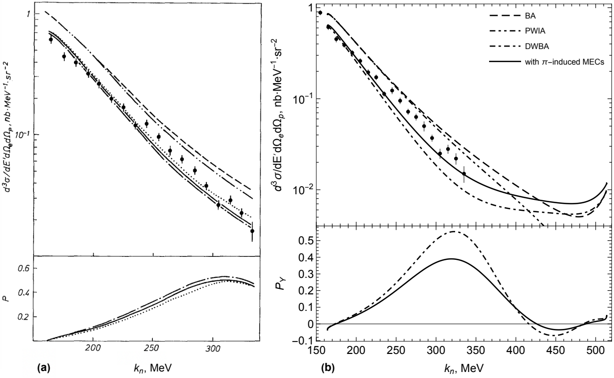
$$\frac{e g_\pi^2 m^2}{2(2\pi)^6} T_3 \frac{h_{c\pi}(\mathbf{p}_1', \mathbf{p}_1) h_{c\pi}(\mathbf{p}_2', \mathbf{p}_2)}{\sqrt{E_1' E_2' E_1 E_2}} (p_1' - p_1 + p_2 - p_2')^{\mu} \times$$

$$\frac{\bar{u}(\mathbf{p}_1', \nu_1') \gamma_5 u(\mathbf{p}_1, \nu_1)}{(p_1' - p_1)^2 - m_\pi^2} \frac{\bar{u}(\mathbf{p}_2', \nu_2') \gamma_5 u(\mathbf{p}_2, \nu_2)}{(p_2' - p_2)^2 - m_\pi^2} +$$

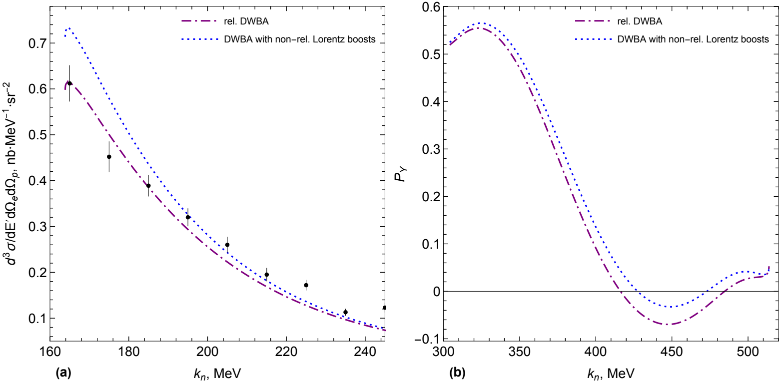
seagull (pair) terms.

Where h_π are cutoff functions and T^3 is an isospin matrix.

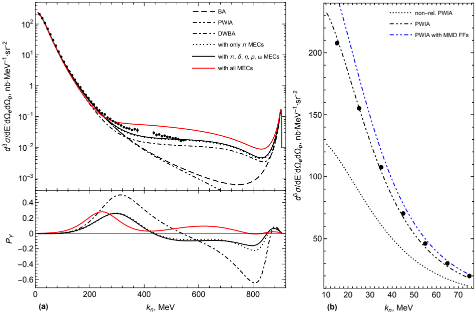
Selected results



Panel a: Differential cross section (upper part) and induced polarization (lower part) of outgoing protons versus the neutron momentum for the Saclay kinematics. Left panel - Paris potential and MEC, right panel CPR calculations.



Role of the Lorentz boosts in the Deuteron electrodisintegration observables for the Saclay kinematics. The dash-dotted curves corresponds to the relativistic FSI calculation, while the dotted curves – with a non-relativistic approximation to the Lorentz boosts. (a): the differential cross section, (b): the induced polarization of knocked-out protons.



Differential cross section (upper part) and induced polarization (lower part) of knocked-out protons versus the neutron momentum for the kinematics of Ulmer et. al.. Calculations without MECs: PWIA (dash-double dotted curve), BA (dashed curve), FSI (dash-dotted curve). Calculations with FSI and MECs: pion-induced MECs only (dotted), with MECs induced by the $\pi, \delta, \eta, \rho, \omega$ mesons (solid), and with full set of MECs (red solid).

Future directions

- Extend to weak currents.
- Light-front formulation. The clothing transformation works the same way with light-front generators. The clothing procedure removes the ill-defined terms that act on the vacuum. Initially examining the two-body interaction in $\phi^3(x)$ field theory. Still a lot to learn.
- Next order in g - pion production.

A term from the light-front 2-body CPR interaction in a $\lambda\phi^3(x)$ field theory.

$$\cdots + \frac{\delta(p_1'^+ + p_2'^+ - p_1^+ - p_2^+)\delta(\mathbf{p}_{1\perp}' + \mathbf{p}_{2\perp}' - \mathbf{p}_{1\perp} - \mathbf{p}_{2\perp})}{\sqrt{p_1^+ p_2^+ p_2'^+ (p_1^+ + p_2^+ - p_2'^+)}} \times$$

$$\lim_{\epsilon \to 0} \frac{\lambda}{3!(2\pi)^{5/2}} \frac{\theta(p_1^+)\theta(p_2^+)\theta(-p_1^+ - p_2^+)\theta(p_1'^+)\theta(p_1^+ + p_2^+ - p_2'^+)}{8(p_2^- - p_1^- - i\epsilon)(p_1^+ + p_2^+)} + \cdots$$

Summary:

- Physics from local meson-exchange field theory.
- Clothed particle representation solves the problem of separating the interaction into a part that dresses a particle from the part the allows two particles to interact.
- Hamiltonian, Poincaré generators, and currents are unchanged, but expressed in terms different operators.
- Strong interaction dynamics satisfies cluster properties. Two-body interaction in two-body problem is same as two-body interaction in the N-body problem
- Vacuum and representation of the Hilbert space change.
- Leading approximation gives realistic non-local Hamiltonian with Deuteron bound state and consistent many-body currents.
- Non-relativistic limit consistent with other realistic non-relativistic calculations

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