# JIMWLK on a quantum computer

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Based on: A. A. Agrawal, E. Budd, A. F. Kemper, A. Tarasov, V. V. Skokov, S.S.T : 2603.02516v1

**JIMWLK**: Jalillian-Marian,Iancu,Mclerran,Weigert,Leonidov and Kovner

- JIMWLK evolution equation: small-x observables
- Important to understand high energy observables, gluon saturation, proton spin puzzle etc
- Current method of solving: Map to Langevin equation

*J.P.Blaizot,E.Iancu,H.Weigert*
*K.Rummukainen and H.Weigert*

# Motivation

**JIMWLK**: Jalillian-Marian,Iancu,Mclerran,Weigert,Leonidov and Kovner

- JIMWLK evolution equation: small-x observables
- Important to understand gluon saturation, high energy collisons
- Current method: Map to Langevin equation

Problems

- Computationally expensive for better statistics
- Higher order JIMWLK lacks a Langevin formulation
- Not all observables can be evolved

**Need a new method to solve JIMWLK!**

*J.P.Blaizot,E.Iancu,H.Weigert*
*K.Rummukainen and H.Weigert*

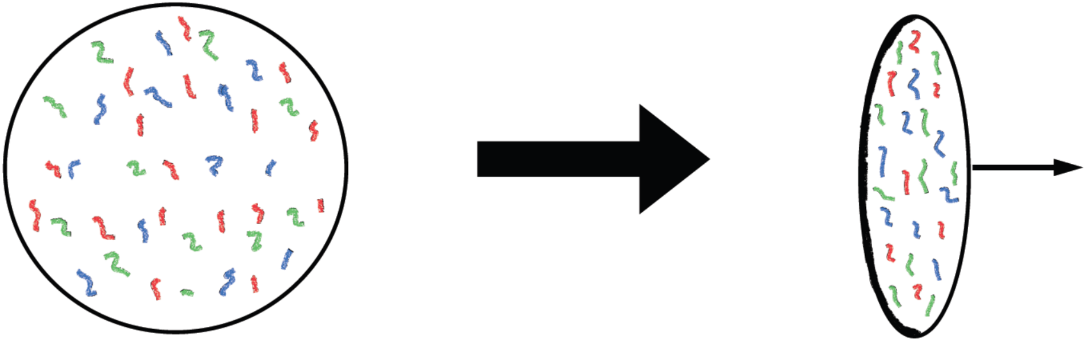
# Lindblad-JIMWLK evolution

We propose a new algorithm to solve JIMWLK on quantum computers

- Evolve lindblad equation in a lattice gauge theory motivated basis
- No statistical noise
- Compute the evolution of a more general class of “off-diagonal” observables

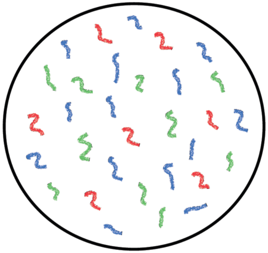
A first study: Toy model of JIMWLK for SU(2)

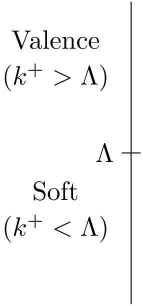
*Lin, Lin (2024)*
*N.Kleo, J.Stryker, M.Savage (2019)*



Nucleus flattens due to lorentz contraction

QCD degrees of freedom separated into valence and soft based on rapidity



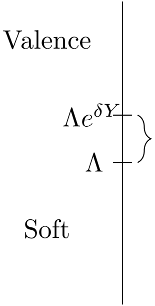



W[j]: Distribution of valence color charges

# JIMWLK

Functional fokker-planck JIMWLK: Evolution of the weight functional $W_Y[j]$

$$\partial_Y W_Y = H_{JIMWLK} W_Y$$




Scattering cross-sections averaged over different configurations of $W[j]$

# JIMWLK

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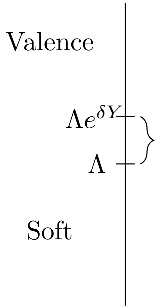
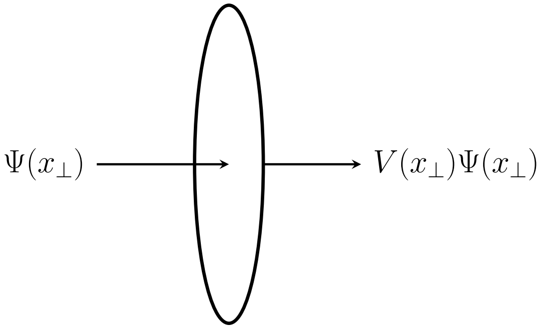



Too complex: rewrite as a langevin equation

# JIMWLK-Langevin

Work with infinite wilson lines

$$V_j(x_\perp) = \mathcal{P}\exp\left[i\int_{-\infty}^{\infty} dx^-\, \tau_j^a\, \alpha^a(x^-, x_\perp)\right]$$




Eikonal scattering of a color charge on a nucleus at high energy: Wilson lines

# JIMWLK-Langevin

Work with infinite Wilson lines

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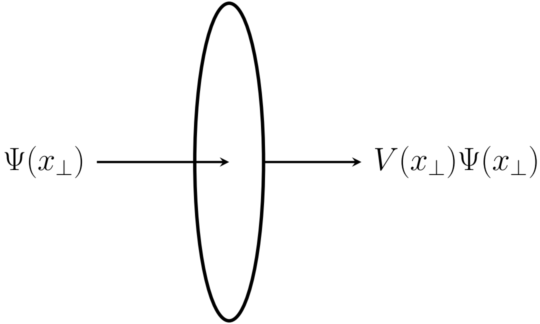


Write JIMWLK as a Langevin equation on Wilson lines

The procedure summarized: Evolving the wilson lines

- Generate an initial set of wilson lines from $W_Y[j]$
- Evolve the Wilson lines $V^Y_{\alpha_n}(x_\perp)$ for multiple realizations of gaussian noise $\xi$
- Compute expectation values by using Wilson lines at rapidity Y and average over noise and initial conditions

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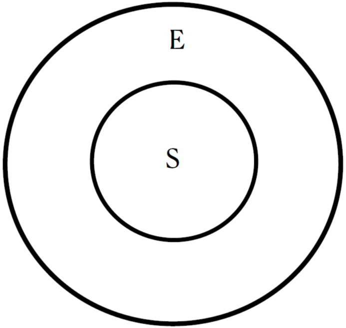
Problems with Langevin-JIMWLK

- Computationally expensive: requires statistical averaging over noise and initial conditions
- NLL JIMWLK and helicity JIMWLK do not admit langevin formulation

Need a new method: Lindblad formulation of JIMWLK

# An alternative

Write JIMWLK as a Lindblad evolution for open quantum systems




**Lindblad evolution:** Evolution of system density matrix $\rho_S$ in the presence of environment

Interaction parameterized by **jump operators** $Q_\alpha$

*N.Armesto, F. Dominguez, A.Kovner, M.Lublinsky, V.V.Skokov (2019)*

# An alternative

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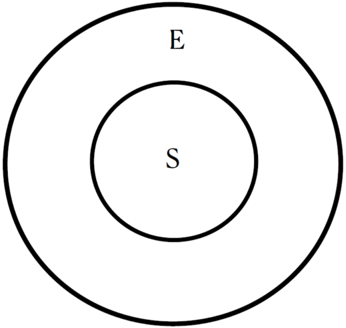


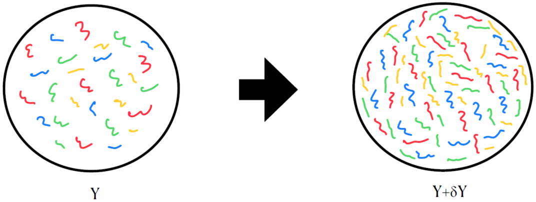

**Lindblad evolution:** Evolution of system density matrix $\rho_S$ in the presence of environment

Interactions parameterized by **jump operators** $Q_\alpha$

$$\partial_t \rho_S(t) = -i[H, \rho_S(t)] + \boxed{\sum_\alpha Q_\alpha \rho_S(t) Q_\alpha^\dagger - \frac{1}{2}\{Q_\alpha^\dagger Q_\alpha, \rho_S(t)\}}$$

# JIMWLK as Lindblad equation

Write JIMWLK as a Lindblad evolution for open quantum systems




time → rapidity

system → target(valence)

environment → soft degrees of freedom

$$\frac{d}{dY}\rho_T(Y) = \int \frac{d^2 z_\perp}{2\pi}\Big[Q_i^a[z_\perp],\Big[Q_i^a[z_\perp],\rho_T(Y)\Big]\Big]$$

*N.Armesto, F. Dominguez, A.Kovner, M.Lublinsky, V.V.Skokov (2019)*
*M.Li, A.Kovner (2020)*

# What do we want?

**Ultimate goal**: Simulate JIMWLK with an initial density matrix

- Difficult on a quantum computer
- Reduce 2 dimensions to one dimension
- Work with SU(2) instead of SU(3)
- Need a discretization/truncation of field space

**Reduced goal**: Simulate reduced JIMWLK in 0 dimensions (2 points) with SU(2)

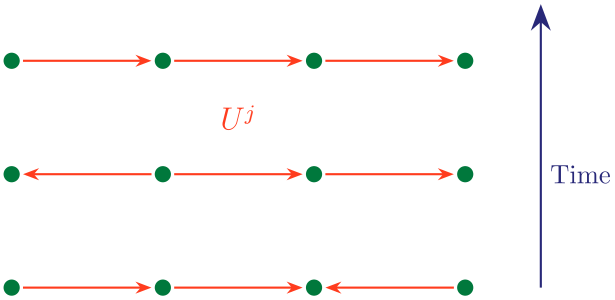
# Choice of basis

**Reduced goal**: Simulate reduced JIMWLK in 0 dimensions (2 points) with SU(2), starting with an initial density matrix.

$$\frac{d}{dY}\rho_T(Y) = a_\perp^2 \sum_{z_\perp} z_\perp \left[Q_i^a[z_\perp], \left[Q_i^a[z_\perp], \rho_T(Y)\right]\right]$$

Main problem: Choose a basis for truncated hilbert space of valence dof

# Hamiltonian lattice gauge theory

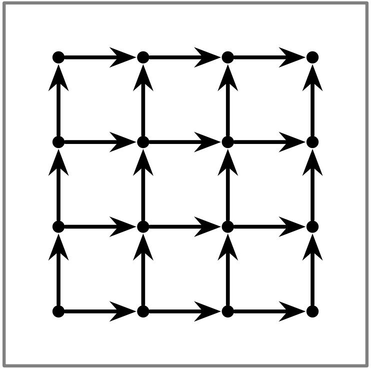



Space: Discretized, Time: Continuous

# The setup

Wilson links are the degrees of freedom

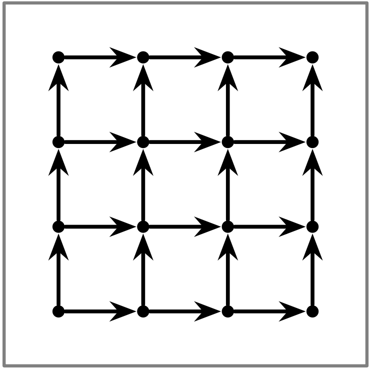
$$\mathbf{U}^j(\vec{r},\mu) = \exp\left\{ig\,a_s\,\tau_j^a\,A_\mu^a(\vec{r})\right\}$$



Spatial lattice at a fixed time slice

Conjugate to Wilson links: Electric fields

$$
\begin{aligned}
[\mathbf{E}^a, \mathbf{U}] &= \tau^a \mathbf{E}^a \,, \\
[\mathbf{E}^a, \mathbf{E}^b] &= i f^{abc}\, \mathbf{E}^c \,, \\
[\mathbf{F}^a, \mathbf{U}] &= -\mathbf{F}^a \tau^a \,, \\
[\mathbf{F}^a, \mathbf{F}^b] &= i f^{abc}\, \mathbf{F}^c \,.
\end{aligned}
$$

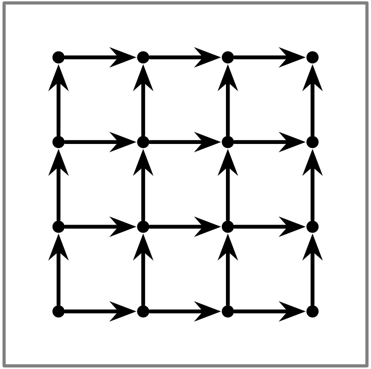


Spatial lattice at a fixed time slice

# The setup

Two types of bases possible

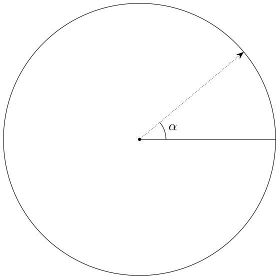
- **Group element basis**: Eigenstates of the U operator
- **Electric field basis**: Eigenstates of the E operator



Spatial lattice at a fixed time slice

# Group element basis

$$\mathbf{U}^j(\vec{r},\mu)|\alpha\rangle = U^j(\vec{r},\mu,\alpha)|\alpha\rangle$$

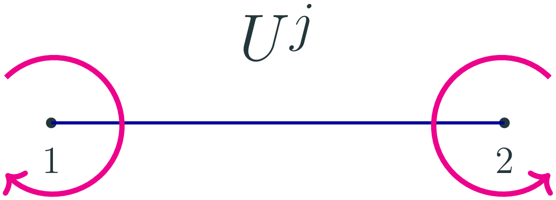



- $\alpha$ runs from 0 to $2\pi$: is compact
- useful for formulating initial conditions

# Hilbert space: Electric field basis

$$\mathbf{E}^a\mathbf{E}^a|j,m,n\rangle = j(j+1)|j,m,n\rangle$$

$$\mathbf{F}^a\mathbf{F}^a|j,m,n\rangle = j(j+1)|j,m,n\rangle$$

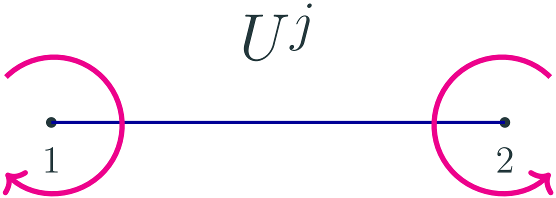



$$|j,m,n\rangle = \mathbf{U}^j_{mn}\,|0\rangle$$

*T. Brynes, Y. Yamamoto (2006)*

# Hilbert space: Electric field basis

- j: Casimir
- m: Left rotation
- n: Right rotation



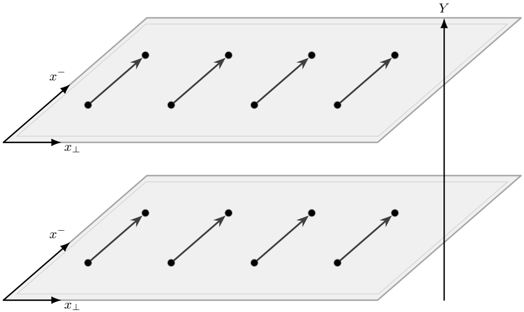

$$|j, m, n\rangle = \mathbf{U}^j_{mn} |0\rangle$$

$$\mathrm{U}^j \to V(1)\, U^j\, V^{-1}(2)$$

Hilbert space of a rigid rotor

*T. Brynes, Y. Yamamoto (2006)*

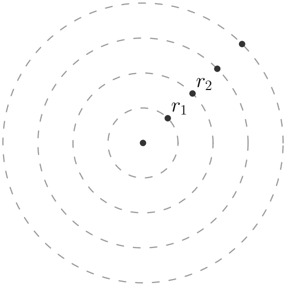
# Mapping to JIMWLK setup




Discretize the infinite wilson link along $x^-$ direction

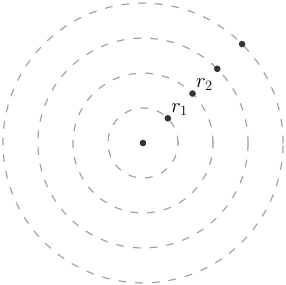
# Space discretization

We approximate the JIMWLK jump operator with an azimuthally symmetric one



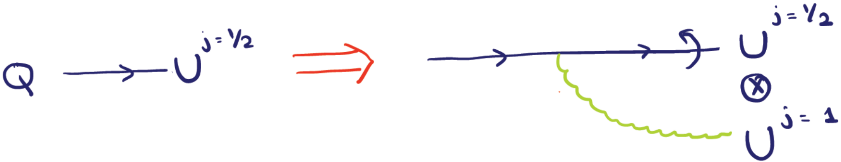

Consider two radial lattice points

# Hilbert space




- $j^{r_1}, j^{r_2} < j_{max}$
- $-j^r < n^r < j^r$
- For $j_{max} = 1/2$, 2 radial points: 25 states

# Jump operators




$$U^{1/2}_{m_L m_R}(\alpha, x)\, U^{1}_{n_L\, n_R}(\alpha, x) = \sum_{J=1/2}^{3/2} C^{J,M_L}_{j,m_L\, j'n_L}\, C^{J,M_R}_{jm_R\, j,m_R}\, U^{J}_{M_L\, M_R}(\alpha, x)$$

Using this we can compute jump operators in the representation basis

# What do we want?

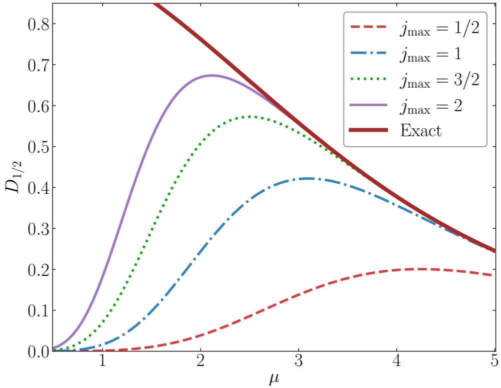
- Work with a maximum j: $j_{max}$
- Write initial conditions in group element basis
- Check convergence in representation basis
- Evolve using Lindblad-JIMWLK and check convergence of evolution

Work with a gaussian density matrix in group element basis and do a basis transformation

$$\langle \alpha | \rho | \alpha' \rangle \approx e^{-\frac{2\,(\alpha^2 + \alpha'^2)}{\mu^2}}$$

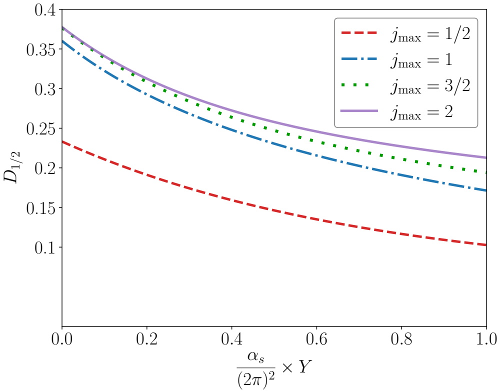
*N.Armesto, F. Dominguez, A.Kovner, M.Lublinsky, V.V.Skokov (2019)*

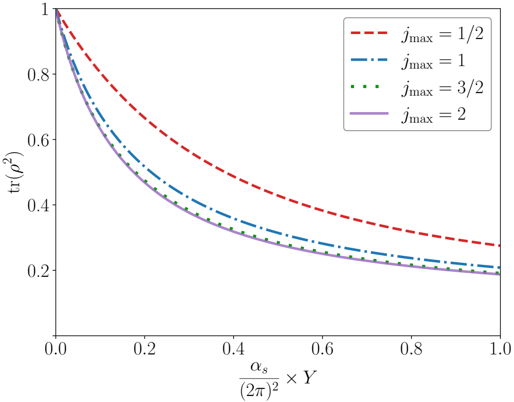
# Convergence of the initial condition



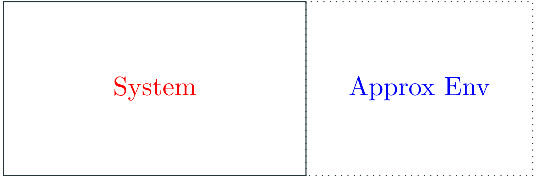

$$\hat{D}_{1/2} = \mathrm{tr}_c(\mathbf{U}^{\dagger\, j}(r_1)\, \mathbf{U}^{1/2}(2))$$

# Convergence of evolution



$\mathcal{L}^E =$

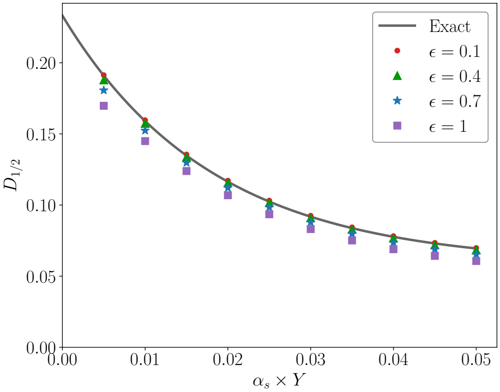


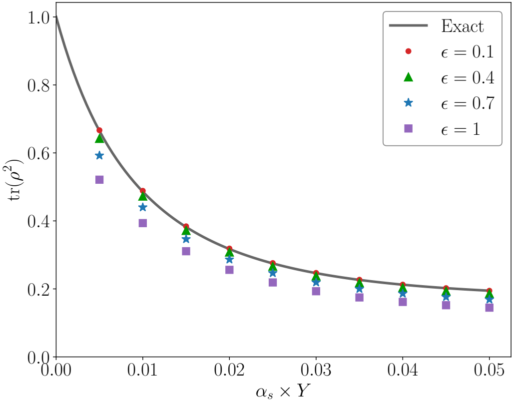

- Enlarge the hilbert space to make the lindbladian operator unitary

$$\frac{d|\rho\rangle^E}{dY} = \mathcal{L}^E|\rho\rangle^E$$

*Schlimgen et al (2021, 2022)*

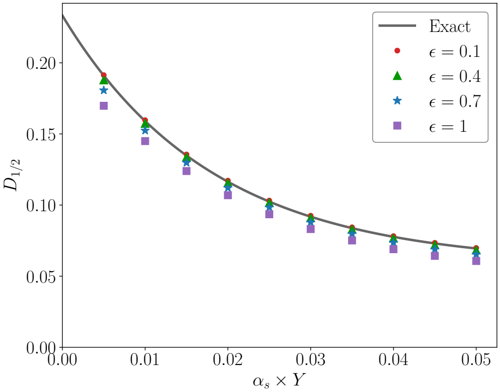
# Implementation on quantum computers

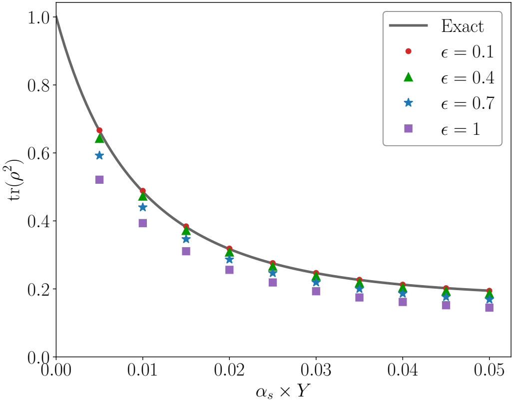



$\epsilon$: Error parameter of the enlargement scheme

# Implementation in quantum computers




$\epsilon \to 0$: Exact but noisy on quantum computers

# Summary

**JIMWLK in 2-dimensions (SU(3))**

Reduce dimensions ↓ 2 points

**JIMWLK in 0-dimensions (SU(3))**

Reduce generators ↓ 3 colors

**JIMWLK in 0-dimensions (SU(2))**

Truncate field space ↓ $j_{max}$

**QM JIMWLK in 0-dimensions (SU(2))**

# Future directions

Relax the approximations

- Generalize to infinite wilson lines
- Work with the SU(3) group
- Intelligent truncations to include more transverse points
- Compute more interesting (off-diagonal) observables

*Ciaverella et al (2021)*