Higher-Order Structure of Hamiltonian Truncation Effective Theory

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Hamiltonian Truncation

V. P. Yurov and A. B. Zamolodchikov, Int. J. Mod. Phys. A 5, 3221 (1990).

Physical system $\longrightarrow$ $H = H_0 + V$

Free Hamiltonian:

$$H_0 \left|\mathcal{E}_i\right\rangle = \mathcal{E}_i \left|\mathcal{E}_i\right\rangle$$

exactly solvable

Interaction Term:

$$V_{ij} = \left\langle\mathcal{E}_i\right| V \left|\mathcal{E}_j\right\rangle$$

matrix elements

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matrix elements

We first solve H_0 and then diagonalize the hamiltonian H in the free eigenbasis

The $\lambda\phi^4$ theory in (1+1)d

A minimal interacting quantum field theory

What does it describe?

- A real scalar field with local quartic self-interaction

- Landau–Ginzburg description of systems in the Ising universality class:

$$\mathbb{Z}_2 : \phi \to -\phi$$

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Hamiltonian split for HT

$$H = H_0 + V$$

$$H_0 = \frac{1}{2} \int dx \left[(\partial_t \phi)^2 + (\partial_x \phi)^2 + m^2 \phi^2 \right]$$

$$V = \frac{\lambda}{4!} \int dx \, \phi^4$$

Why $\lambda\phi^4$ theory in (1+1)d?

Simple UV, strongly coupled IR, non-perturbative spectrum

$$[\lambda] = 2 \longrightarrow \boxed{g_{\text{eff}} \sim \frac{\lambda}{E^2}}$$

The ϕ^4 perturbation is relevant in 1+1 dimensions

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UV / short distances

Perturbation is small and we can use PT

IR / long distances

PT breaks down, so we need *nonperturbative* tools

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Qualitative analogy to QCD

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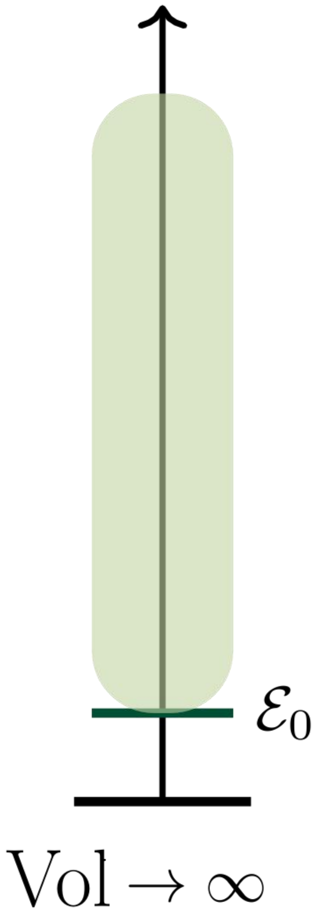
Hamiltonian Truncation targets the low-energy spectrum

How to obtain a finite free basis



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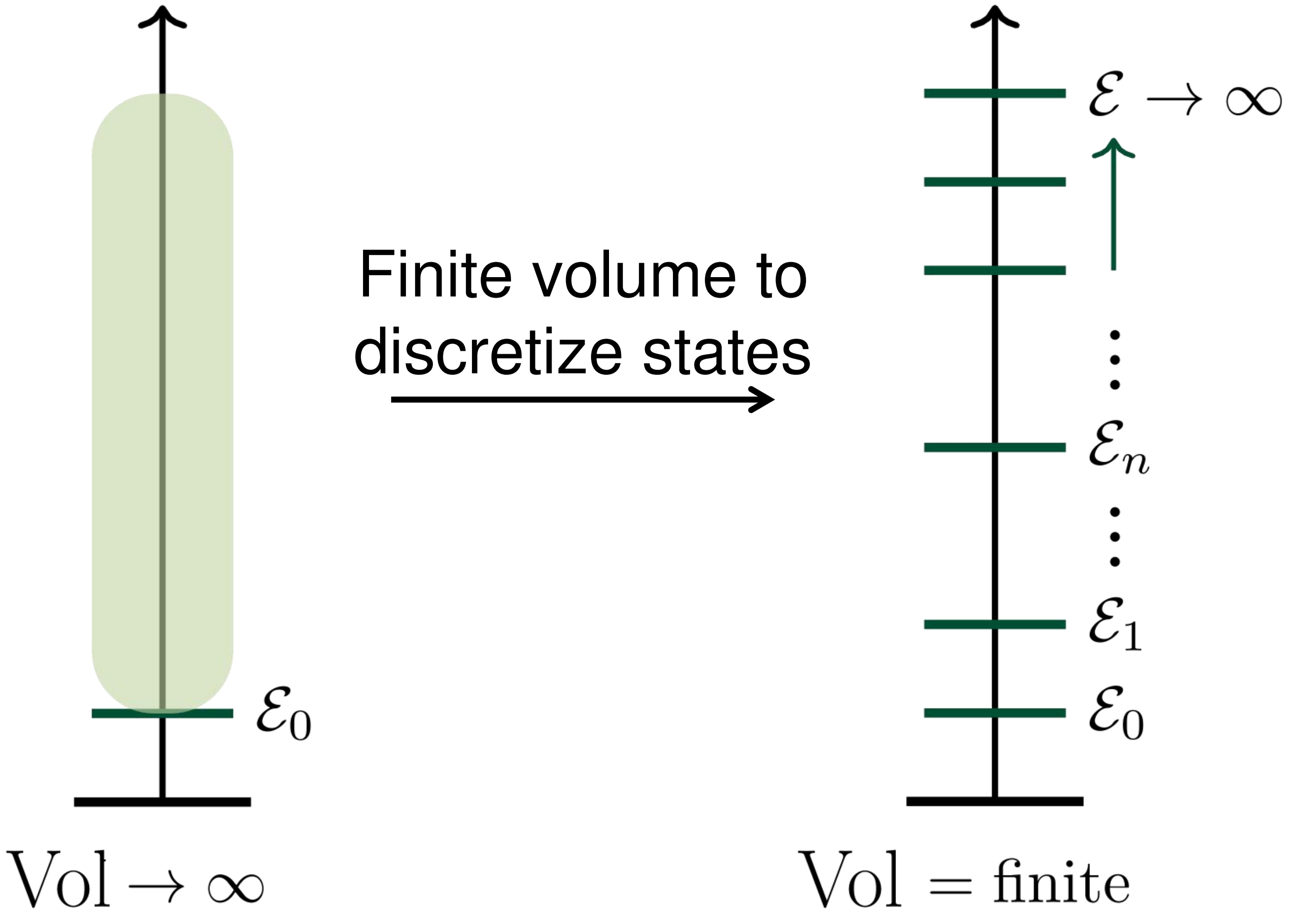

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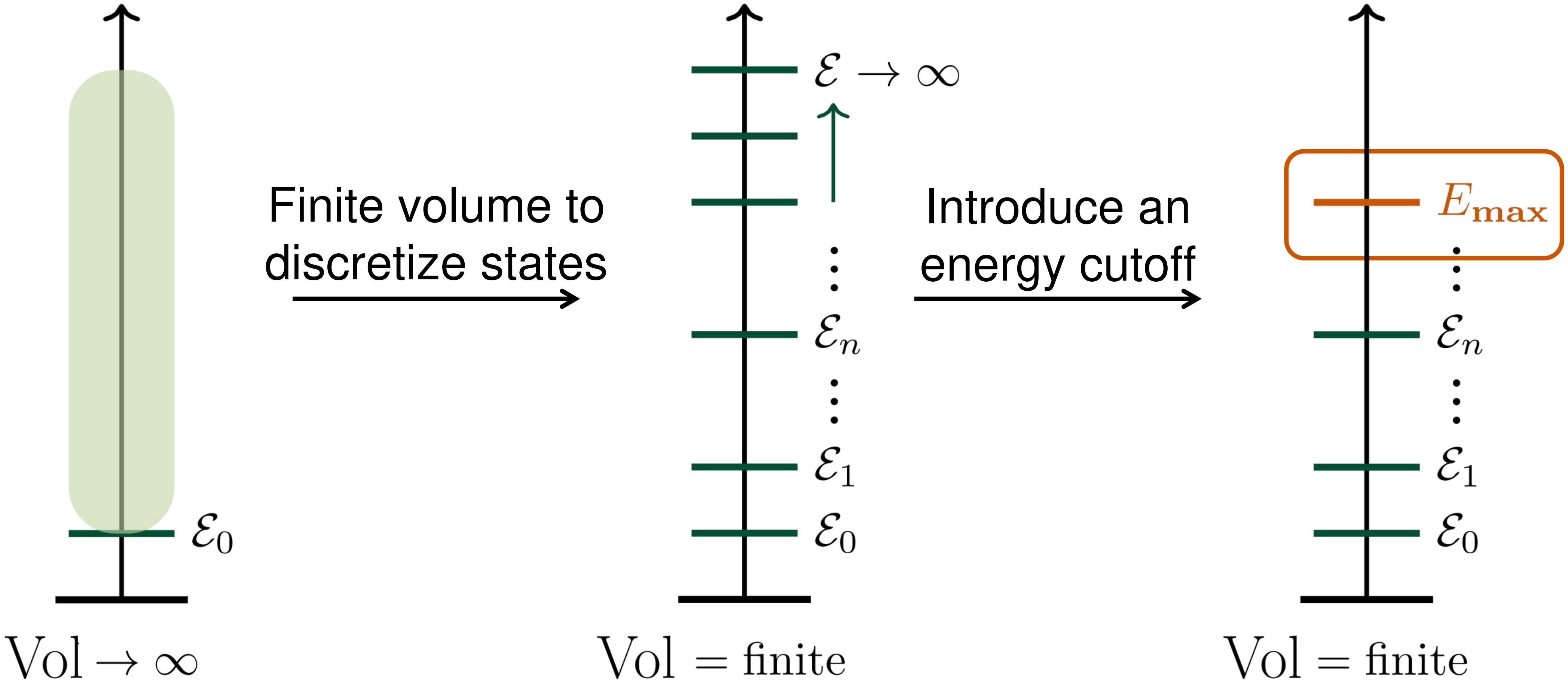
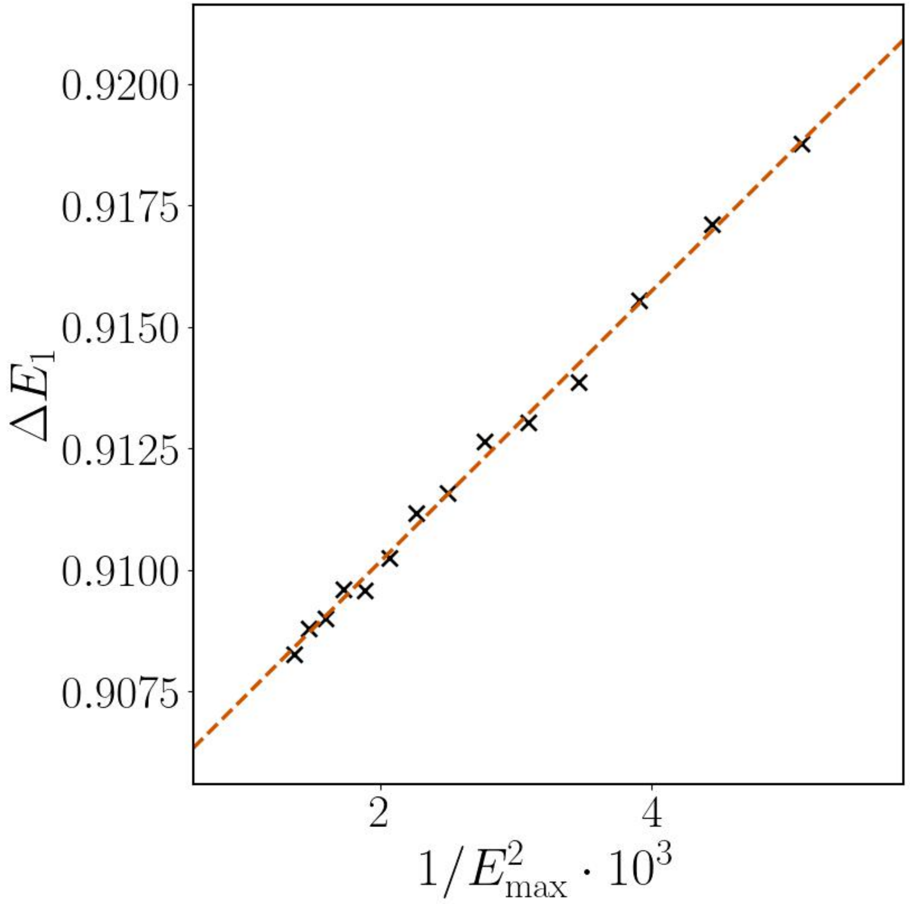


Numerical Results

Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$

Accuracy $\sim E_{\rm max}^{-2}$



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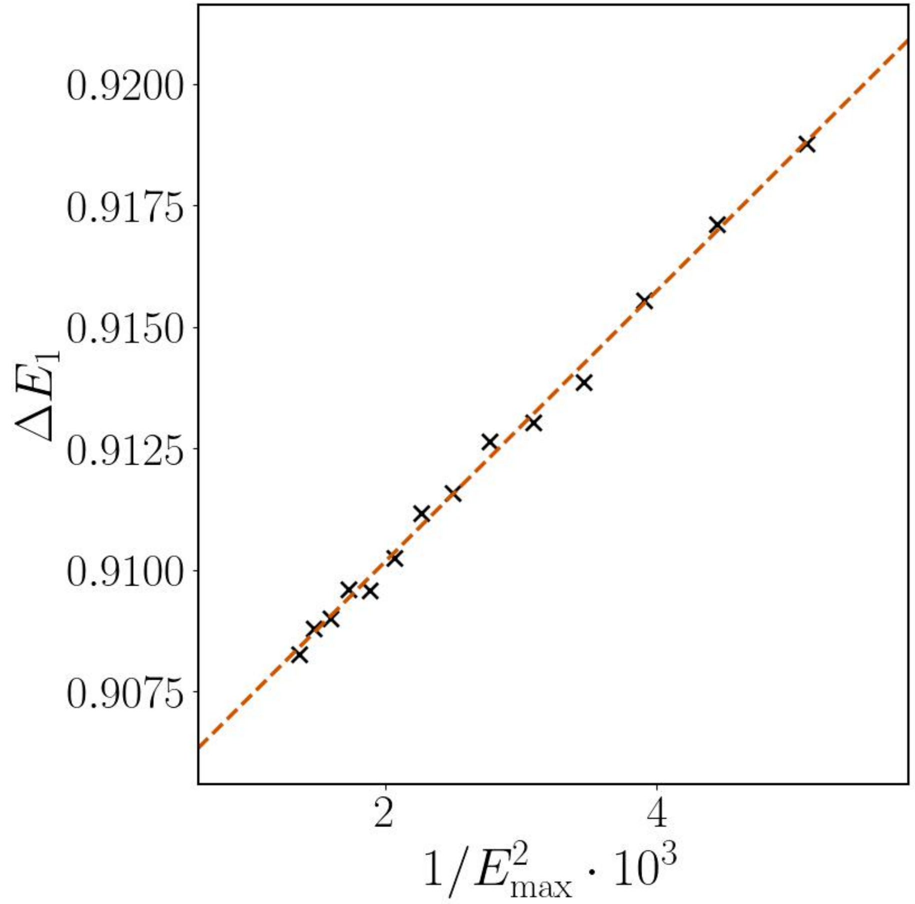
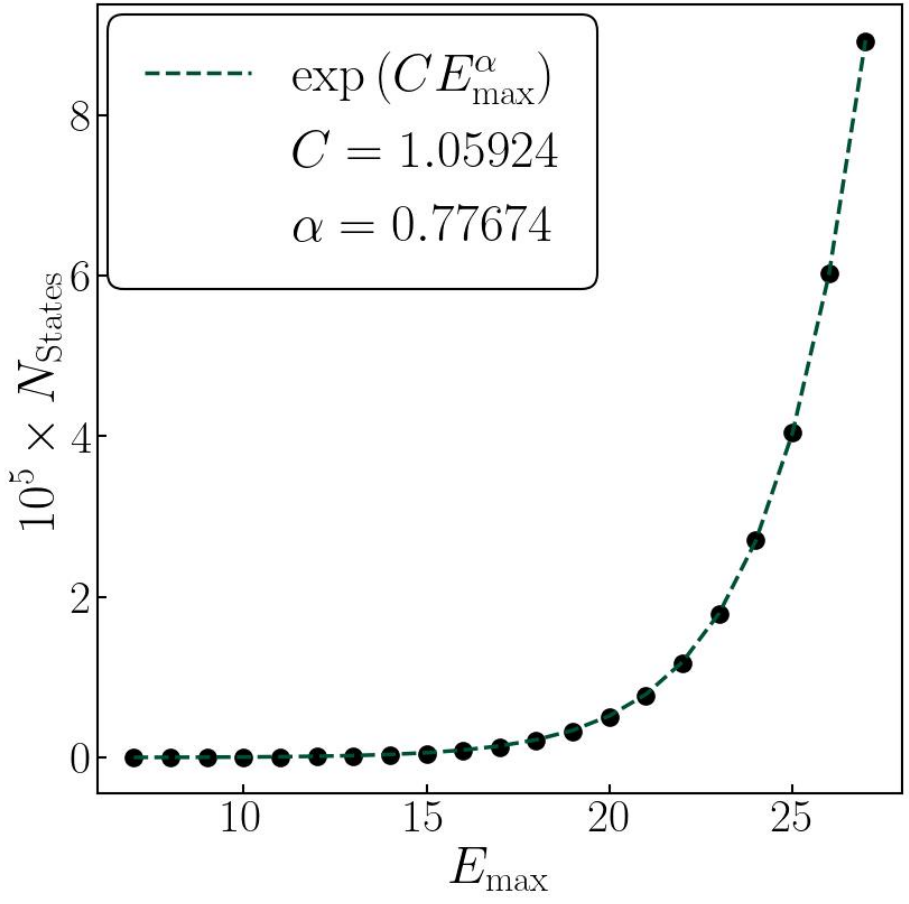


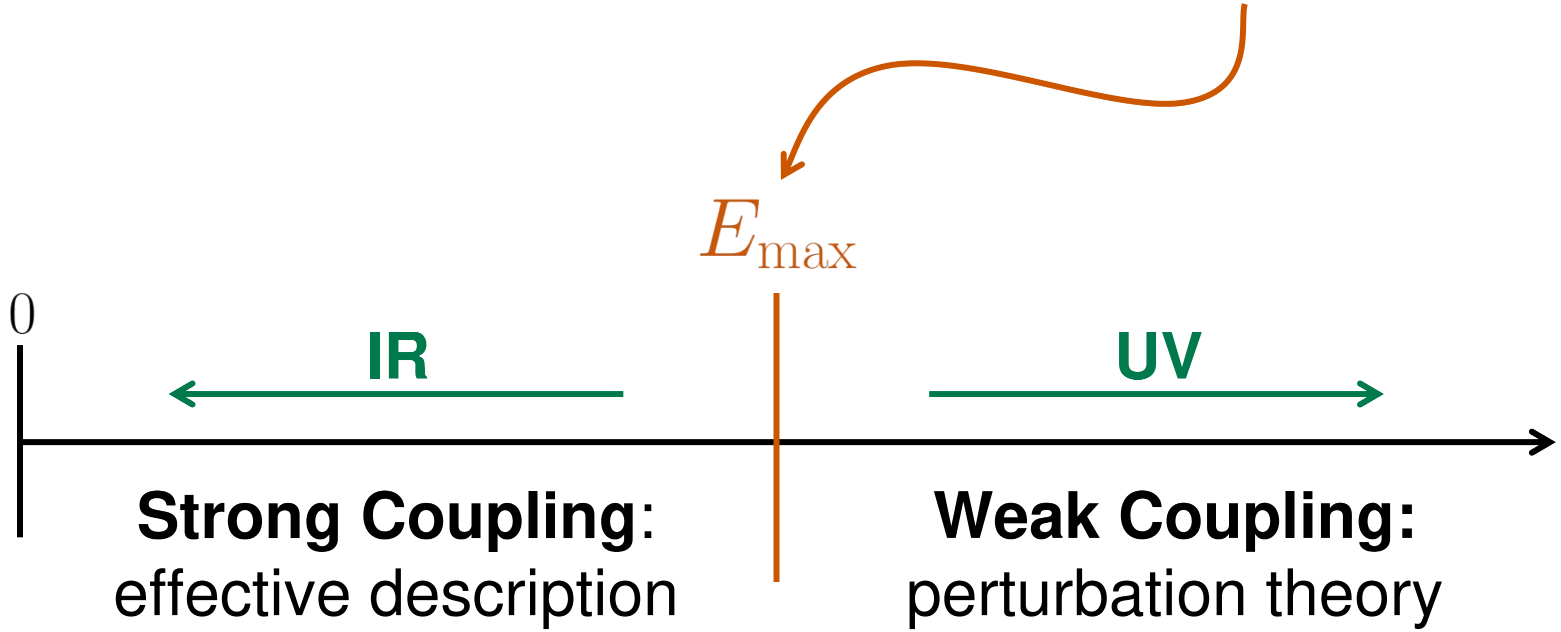
Computational cost $\sim \exp(\sqrt{E_{\max}})$

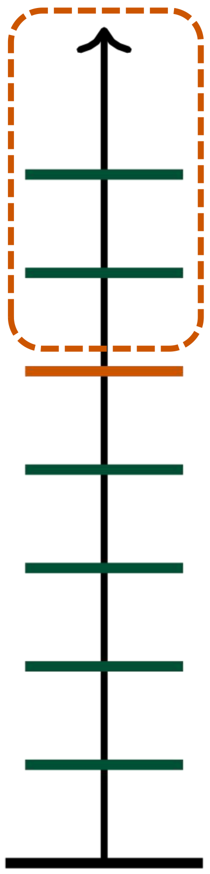


Effective field theory to improve HT

T. Cohen, K. Farnsworth, R. Houtz, and M. A. Luty, SciPost Phys. 13, 011 (2022)

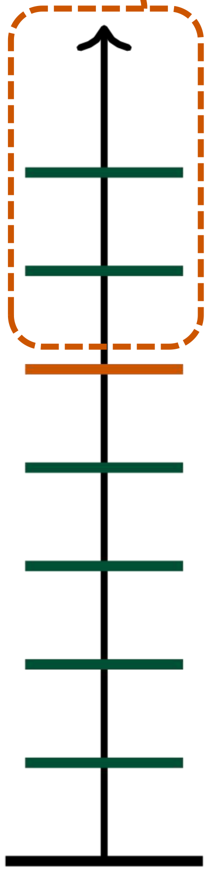
- EFT applies with a clear scale hierarchy: $E_{\rm IR} \ll E_{\rm max}$

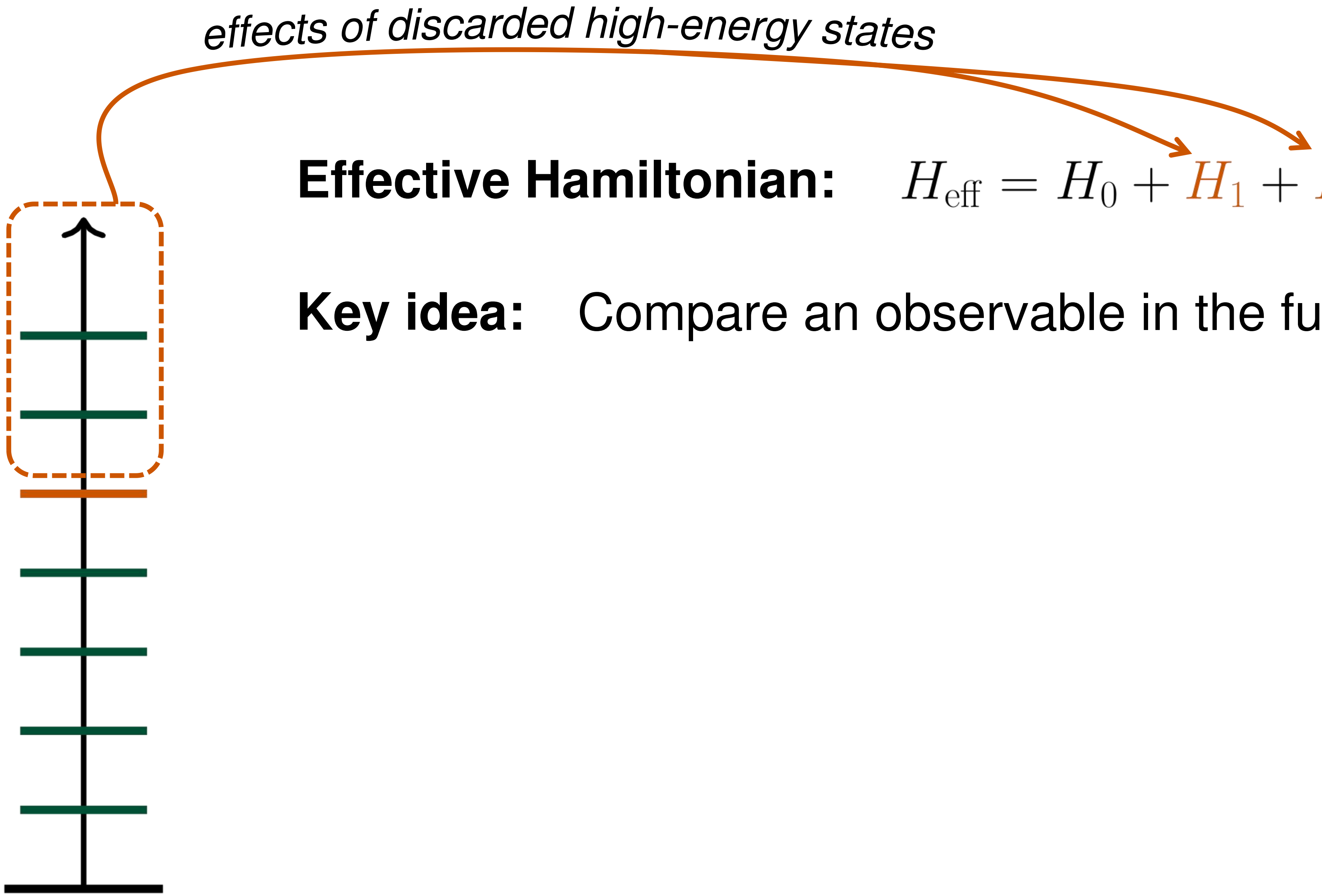




effects of discarded high-energy states

Effective Hamiltonian: $H_{\text{eff}} = H_0 + H_1 + H_2 + \cdots \ , \quad H_n \sim \mathcal{O}(V^n)$





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Key idea: Compare an observable in the full and effective theories

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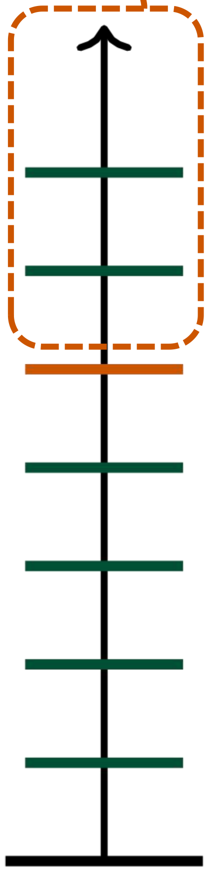

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Requirements:

1. Determine H_{eff} uniquely
2. Allow systematic expansion in $E_{\text{IR}}/E_{\text{max}}$
3. Ensure separation of scales at each order

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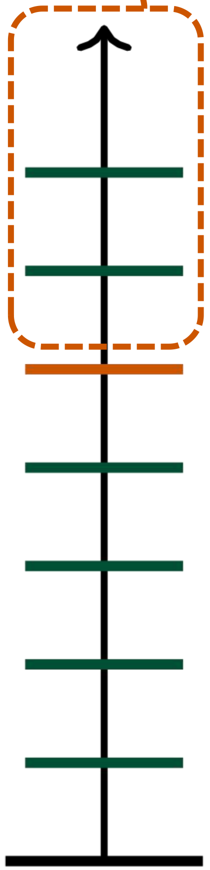

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Approach: Match the transition matrix

$$\lim_{t_f \to \infty} \langle f | \, U_{IP}(t_f, 0) \, | i \rangle = \delta_{ij} + \frac{\langle \boldsymbol{f} | \boldsymbol{T} | \boldsymbol{i} \rangle}{E_{fi} + i\epsilon}$$

Matching the Transition Matrix

$$\langle f | T | i \rangle_{\text{full}} = \langle f | T | i \rangle_{\text{eff}}$$

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Order $\mathcal{O}(V) \longrightarrow \langle f | H_1 | i \rangle_{\text{eff}} = \langle f | V | i \rangle$

Trivial condition ensuring that:

$$H_{\text{eff}} = H + \mathcal{O}(V^2)$$

Matching the Transition Matrix

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Trivial condition ensuring that:

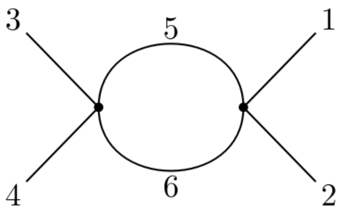
$$H_{\text{eff}} = H + \mathcal{O}(V^2)$$

Order $\mathcal{O}(V^2)$ → $\langle f|\, H_2\, |i\rangle_{\text{eff}} = \sum_{\alpha}^{>} \frac{\langle f|\, V\, |\alpha\rangle \langle \alpha|\, V\, |i\rangle}{E_{f\alpha}}$

→ Only states excluded by the truncation contribute!

We perform the matching using a diagrammatic approach

Example of a diagrammatic computation



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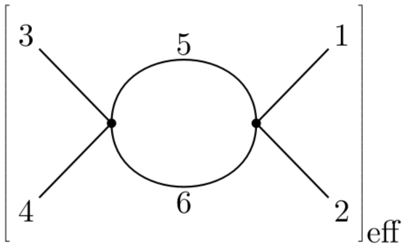

$$= \frac{1}{8}\left(\frac{\lambda}{2\pi R}\right)^2 \sum_{1,\dots,6} \frac{\delta_{12,56}\delta_{34,56}}{2\omega_5\, 2\omega_6} \frac{\langle f|\, \phi_4^{(-)}\phi_3^{(-)}\phi_2^{(+)}\phi_1^{(+)}\, |i\rangle}{\omega_3+\omega_4-\omega_5-\omega_6}$$

$$\times\, \Theta(\mathcal{E}_f - E_{\max} - \omega_3 - \omega_4 + \omega_5 + \omega_6)$$

Allows systematic expansion in $E_{\mathrm{IR}}/E_{\max}$

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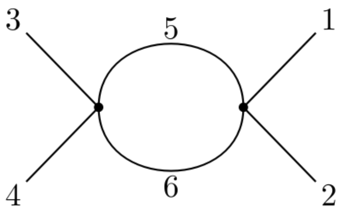

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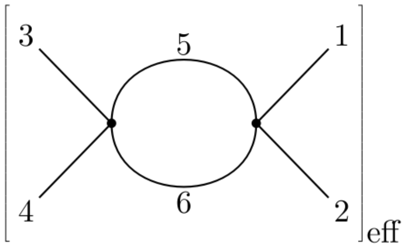

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Allows systematic expansion in $E_{\rm IR}/E_{\rm max}$

First Order: **Local approximation**

$$\simeq -\frac{\lambda^2}{128\pi R}\int dx\, \langle f|[\phi^{(-)}]^2[\phi^{(+)}]^2|i\rangle \sum_k \frac{\Theta(-E_{\rm max}+2\omega_k)}{\omega_k^3} + \mathcal{O}\left(\frac{\mathcal{E}_{i,f}}{E_{\rm max}}\right)$$

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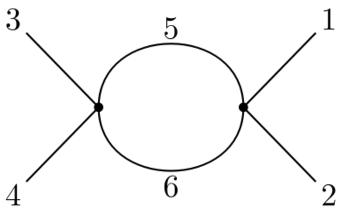

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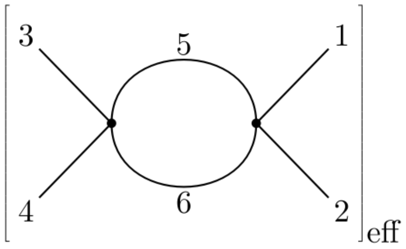

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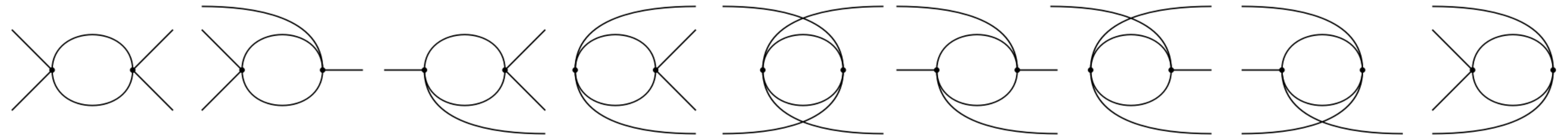
The initial and final states may contain spectator particles, which makes the effective interaction nonlocal.

Combining all diagrams

Local approximation



$$H_2 \simeq \frac{\delta\lambda}{4!}\int \mathrm{d}x : \phi^4 : + \frac{\delta m^2}{2}\int \mathrm{d}x : \phi^2 :$$

$$\delta\lambda = \frac{3\lambda^2}{16\pi R}\sum_k \frac{\Theta(2\omega_k - E_{\max})}{\omega_k^3}$$

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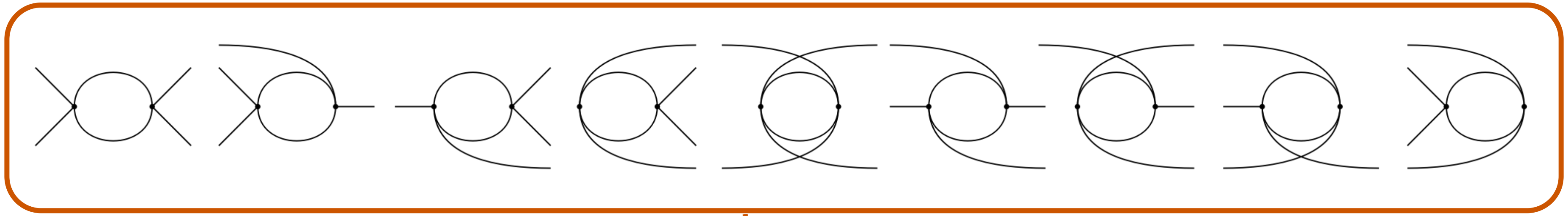

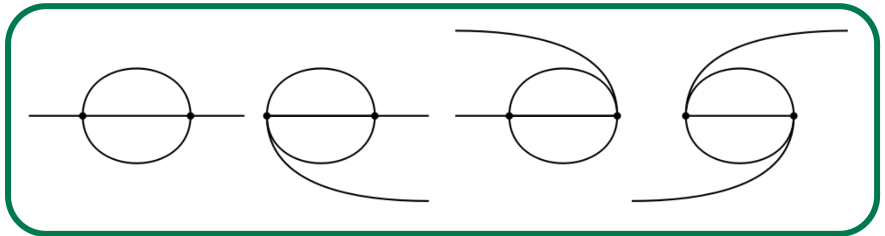

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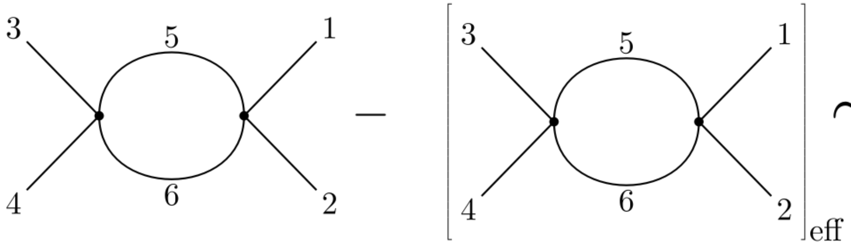
Second-order expansion

E. Demiray, K. Farnsworth, and R. Houtz, arXiv:2507.15941 (2025)



$$\sim \Theta(\mathcal{E}_f - E_{\max} - \omega_3 - \omega_4 + \omega_5 + \omega_6)$$

Expand the cutoff constraint:

$$\Theta(X + \mathcal{E}_f) = \Theta(X) + \mathcal{E}_f\, \delta(X) + \cdots$$

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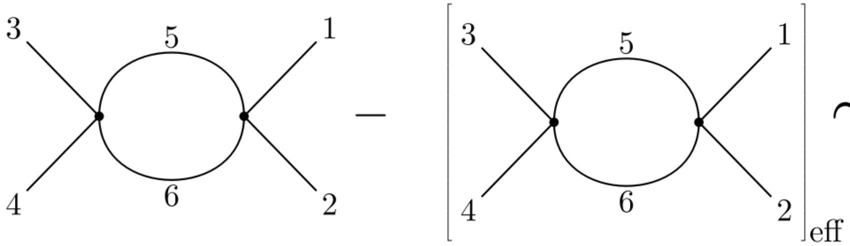

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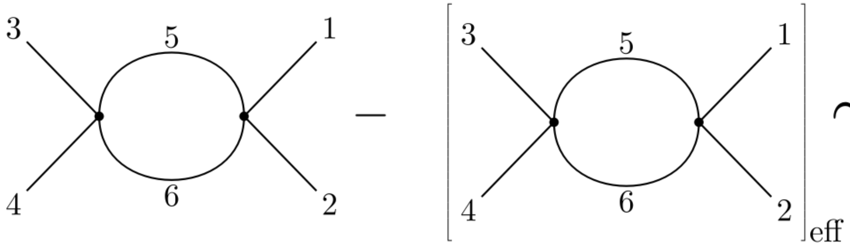

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First term gives the previous local corrections

Subleading terms encode *nonlocal* effects:

$$H_2 \sim [H_0, :\phi^2:],\ [H_0, :\phi^4:],\ \{H_0, :\phi^2:\},\ \{H_0, :\phi^4:\}$$

HTET expansion: mixed power counting

$1/E_{\max}$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(V^4)$
$\mathcal{O}(E_{\max}^{-2})$	local		
$\mathcal{O}(E_{\max}^{-3})$	1st nonlocal		
$\mathcal{O}(E_{\max}^{-4})$	NNLoc: 2nd non local	local	
$\mathcal{O}(E_{\max}^{-5})$	3rd nonlocal	1st nonlocal	
$\mathcal{O}(E_{\max}^{-6})$	4th nonlocal	2nd nonlocal	local
	$\vdots$	$\vdots$	$\vdots$

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	⋮	⋮	⋮

Key Point

$\mathcal{O}(E_{\max}^{-4})$ gets contributions from both $\mathcal{O}(V^2)$ and $\mathcal{O}(V^3)$

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A. Maestri, S. Rodini, B. Pasquini, arXiv:2602.13019 (2026)

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Why continuum-first matching?

makes the distributional nature of the cutoff expansion unambiguous

$$\Theta(X + \mathcal{E}_f) = \Theta(X) + \mathcal{E}_f\,\delta(X) + \boxed{\frac{\mathcal{E}_f^2}{2}\,\delta'(X)} + \cdots$$

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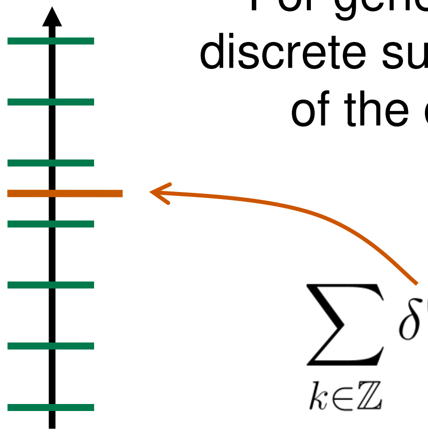
Finite-volume first

discretize before expanding

For generic parameters, the discrete sum misses the support of the delta distributions



$k \in \mathbb{Z}$

$$\sum_{k \in \mathbb{Z}} \delta^{(n)}(2\omega_k - E_{\max})$$

ambiguous prescription

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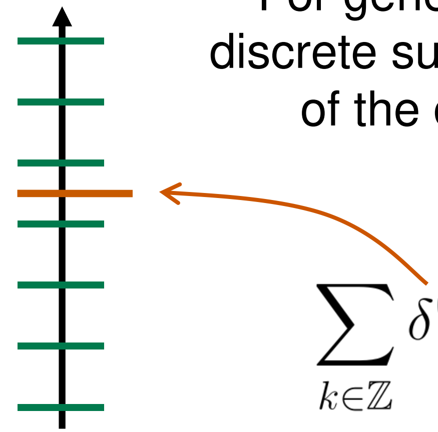

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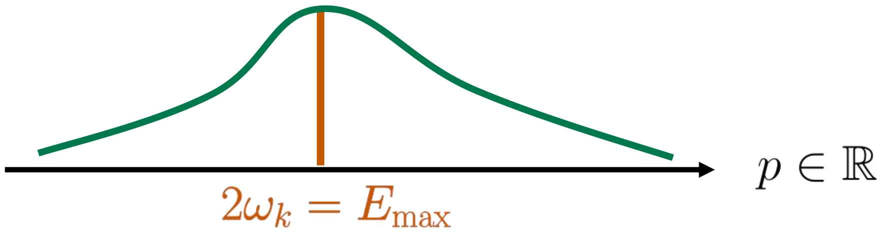


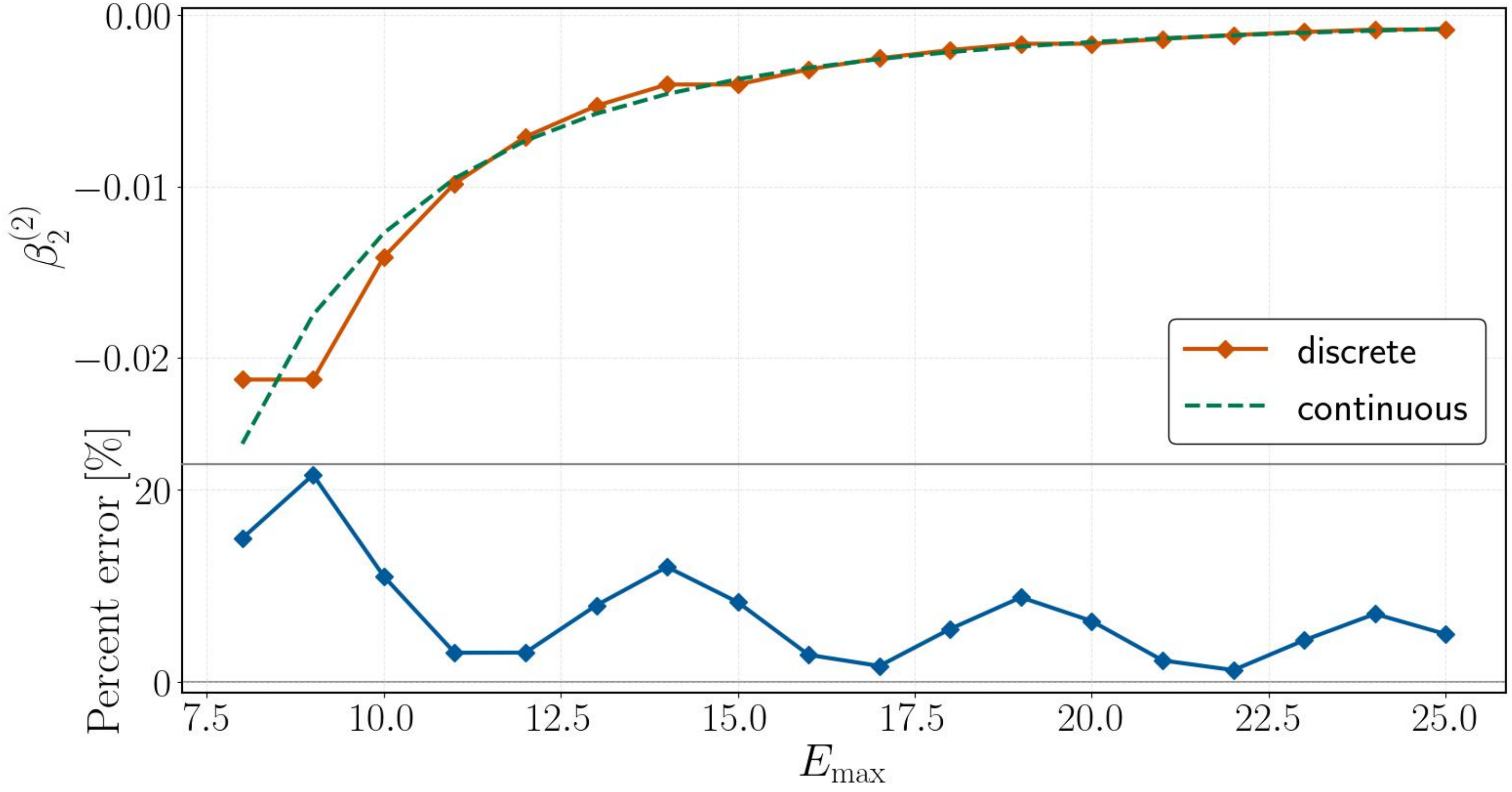
$$\int \mathrm{d}p\; f(p)\delta^{(n)}(2\omega(p) - E_{\max})$$

distributions are well-defined

Why continuum-first matching?

$$H_2 \sim \frac{\beta_2^{(2)}}{4!} \int \mathrm{d}x \; [H_0, :\phi^4:]$$

Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$



Finite-volume first

$$\beta_2^{(2)} = -\frac{3\lambda^2}{64\pi R} \sum_k \frac{\Theta(2\omega_k - E_{\max})}{\omega_k^4}$$

Continuum-first

$$\beta_2^{(2)} = -\frac{3\lambda^2}{64\pi} \int_{-\infty}^{+\infty} dp \, \frac{\Theta(2\omega(p) - E_{\max})}{\omega(p)^4}$$

Third-order expansion: NNLoc Corrections

Structure and contributions of NNLoc corrections in the four-external-leg sector

1. Energy Monomials

$$E_f^m\, E_i^l \longrightarrow H_0^m : \phi^4 : H_0^l$$

$$m + l \leq 2$$

Nonlocality: dependence on spectator energies through H_0

$$H_0 : \phi^4 : H_0,\ \ H_0^2 : \phi^4 :,\ \ : \phi^4 : H_0^2$$

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2. External Frequencies

New!

$$\omega_p^2\ \phi_p^\pm \rightarrow -\partial_t^2 \phi_p^\pm \rightarrow [H_0, [H_0, \phi_p^\pm]]$$

$$\Omega_4 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2$$

NNLoc corrections feature irreducible dependence on individual external legs energy

$$\Omega_4 : \phi^4 :$$

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2. External Frequencies

New!

$$\omega_p^2 \, \phi_p^\pm \longrightarrow -\partial_t^2 \phi_p^\pm \longrightarrow [H_0, [H_0, \phi_p^\pm]]$$

$$\Omega_4 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2$$

NNLoc corrections feature irreducible dependence on individual external legs energy

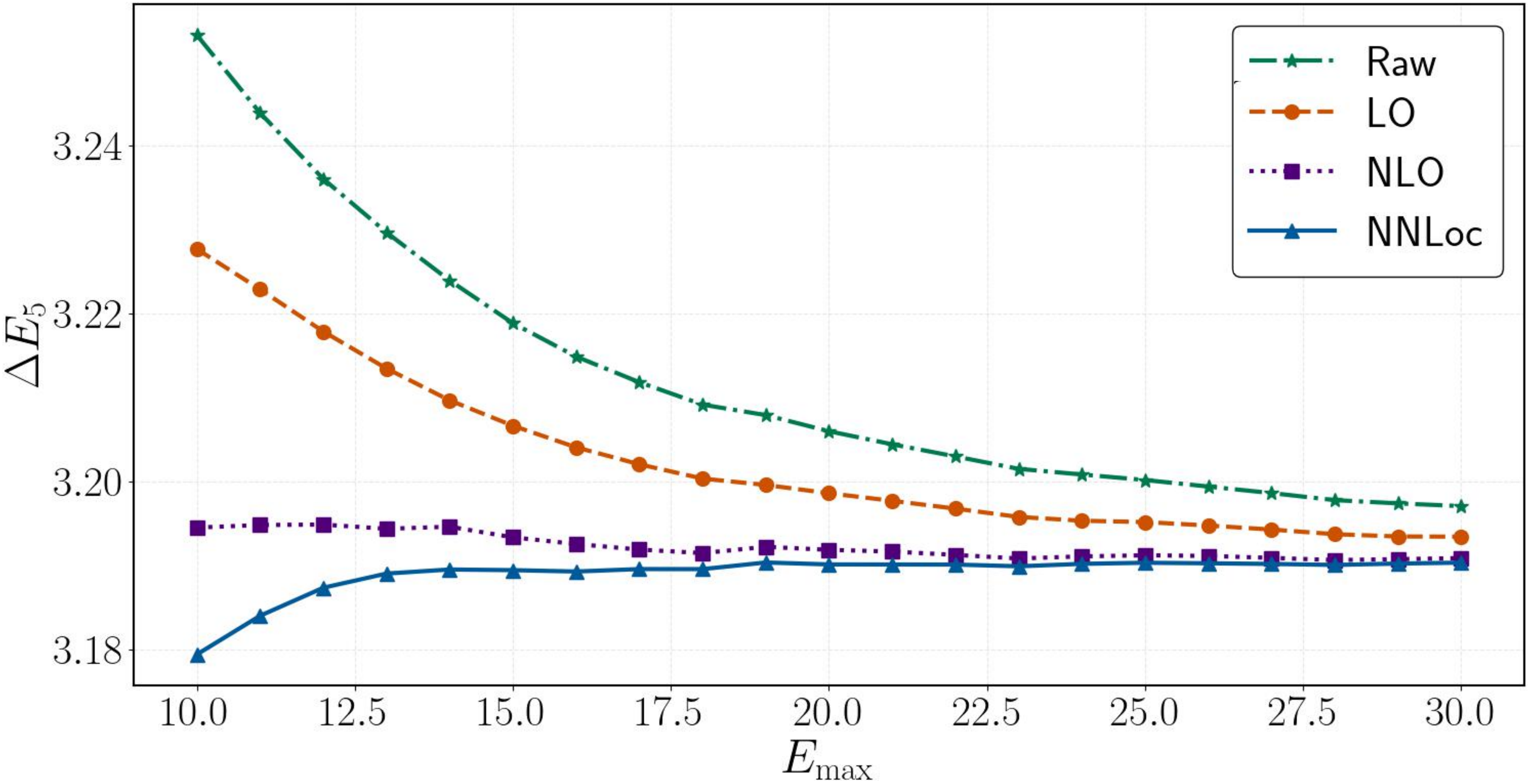
$$\Omega_4 : \phi^4 :$$

Take-home message:

A systematic HTET expansion is possible, but each higher order in the cutoff expansion requires an increasingly rich operator basis

Numerical Results

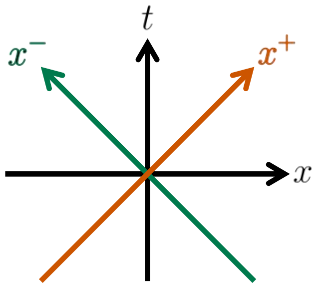
Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$



- Effective corrections flatten the $E_{\max}$ dependence.
- **NNLoc** captures the next cutoff trend, but $\mathcal{O}(E_{\max}^{-4})$ accuracy requires $\mathcal{O}(V^3)$ terms.

HTET on the Light-Front

A. Maestri, S. Rodini, B. Pasquini, *Work in progress...*



Equal-time HTET

$$H = H_0 + V$$

Cutoff on the free energy

$$\mathcal{E} < E_{\text{max}}$$

Light-front HTET

$$M^2 = 2P^+(P_0^- + P_{\text{int}}^-)$$

Cutoff on the free invariant mass

$$M_0^2 < \Lambda^2$$

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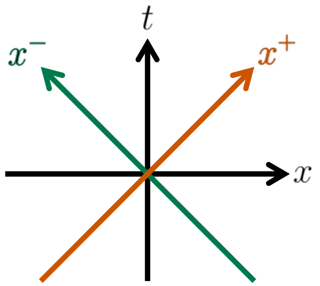

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Same EFT Logic

Truncate the basis and restore the effect of discarded states through transition-matrix matching, improving convergence with the cutoff.

Light-front setup

1. We work with a compactified light-front direction: $\phi(x^+, x^- + 2L) = \phi(x^+, x^-)$
2. At fixed harmonic resolution K, DLCQ gives a finite basis: $P^+ = \frac{\pi}{L} K$
3. We work in the zero-mode-truncated theory: $\phi(x) = \phi_0(x^+) + \varphi(x^+, x^-)$, with $\phi_0(x^+) \to 0$

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Invariant-mass split

$$M^2 = M_0^2 + M_{\rm int}^2$$

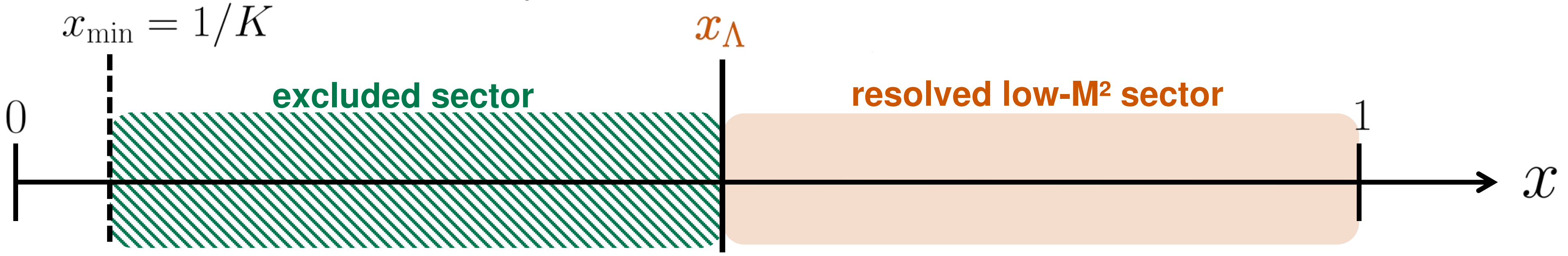
$$M_0^2 = m^2 K \sum_{j \geq 1} \frac{n_j}{j}$$

$$M_{\rm int}^2 = 2P^+ \frac{\lambda}{4!} \int_{-L}^{L} \mathrm{d}x^- \; :\varphi^4(x^-):$$

Clear EFT separation

The clean limit separates resolved low-M^2 states from endpoint/high-multiplicity states

$$M_0^2 = m^2 \sum_i \frac{1}{x_i} \longrightarrow x > x_\Lambda = m^2/\Lambda^2$$



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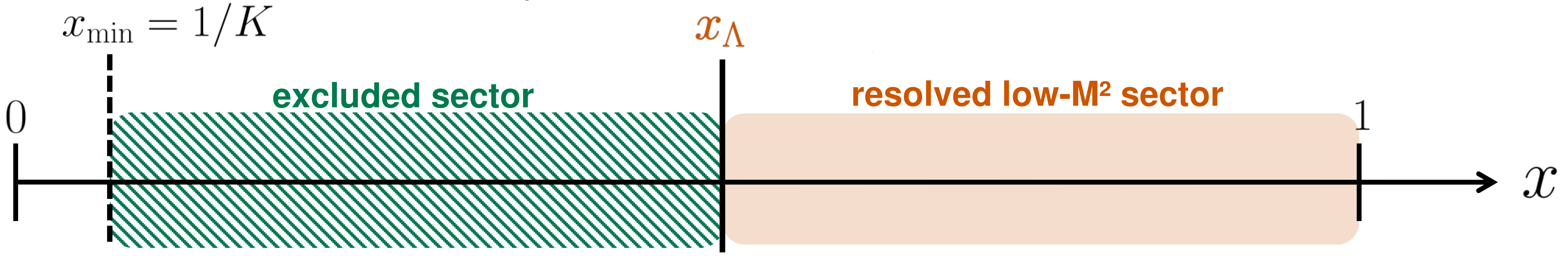

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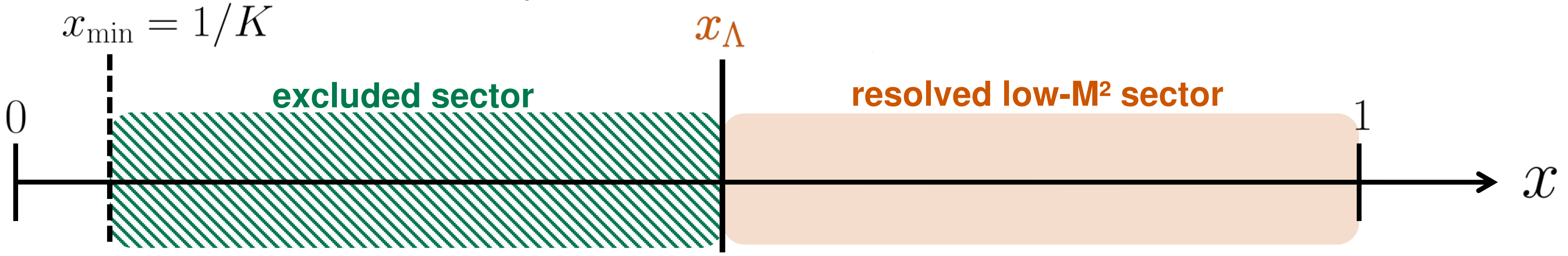


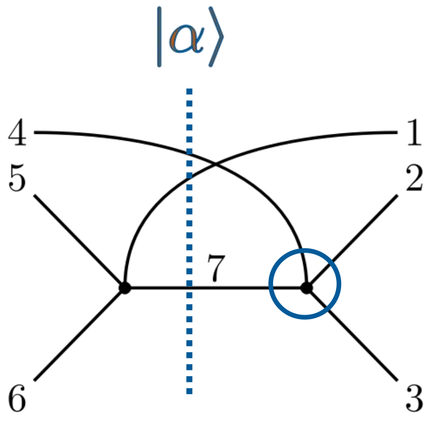
- Require x_Λ to be resolved on the DLCQ grid $\longrightarrow \Lambda^2 \ll m^2 K$
- Scale hierarchy $\longrightarrow m^2 \ll \Lambda^2$

Clear EFT regime

$$m^2 \ll \Lambda^2 \ll m^2 K$$

Light-front HTET: New Diagrams!

Transition-matrix matching at second order → Only intermediate states $|\alpha\rangle$ above the cutoff contribute



Light-front HTET: New Diagrams!

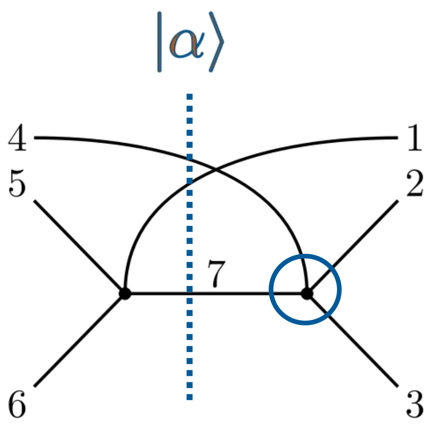
Transition-matrix matching at second order → Only intermediate states $|\alpha\rangle$ above the cutoff contribute

Equal-time case

- $E_\alpha = \omega_{p_1} + \omega_{p_4} + \omega_{p_7}$
- $\omega_{p_1}, \omega_{p_4} \ll E_{\max}$

$$E_\alpha \ll E_{\max}$$

No matching contribution



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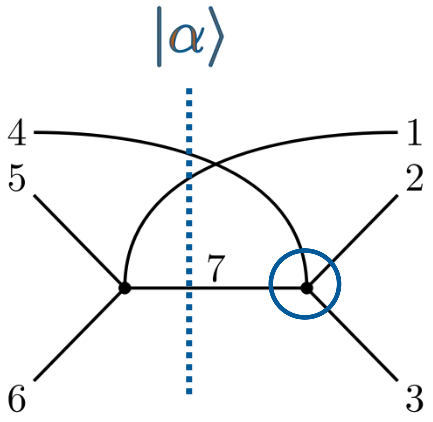


Light-front case

- $M^2_{0,\alpha} = M^2_{0,p_1} + M^2_{0,p_4} + M^2_{0,p_7}$
- $M^2_{0,p_1},\ M^2_{0,p_4} \ll \Lambda^2$

$$M^2_{0,p_7} = \frac{m^2 K}{p_7} = \frac{m^2 K}{p_2 + p_3 - p_4}$$

Possible denominator cancellations

$$M^2_{0,\alpha} \lesseqgtr \Lambda^2$$

New!

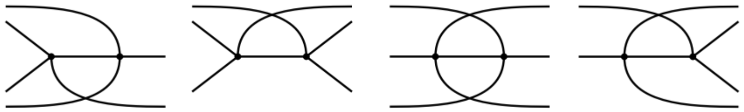
General picture of Light-front HTET

Current work: operator basis, zero-mode effects, and numerical tests in DLCQ

Clear EFT regime

$$m^2 \ll \Lambda^2 \ll m^2 K$$

New diagrams



Zero-mode effects

S.S. Chabysheva, J.R. Hiller, arXiv:2502.01775 (2025).

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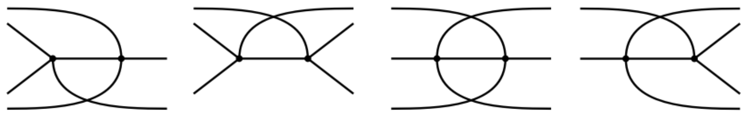

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Goal

Reach the low-energy spectrum with fewer DLCQ states by integrating out the high-invariant-mass sector

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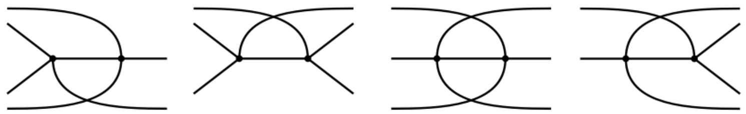

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Future directions...

1. **Extend to more complex theories (Schwinger model, QCD)**
2. **Generalize to higher dimensions (2+1d)**

Thanks for your attention!

SCAN ME

A. Maestri, S. Rodini and B. Pasquini,
Higher-Order Structure of Hamiltonian Truncation Effective Theory