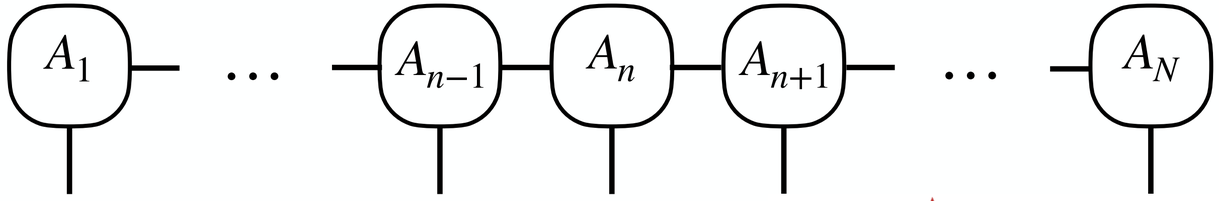
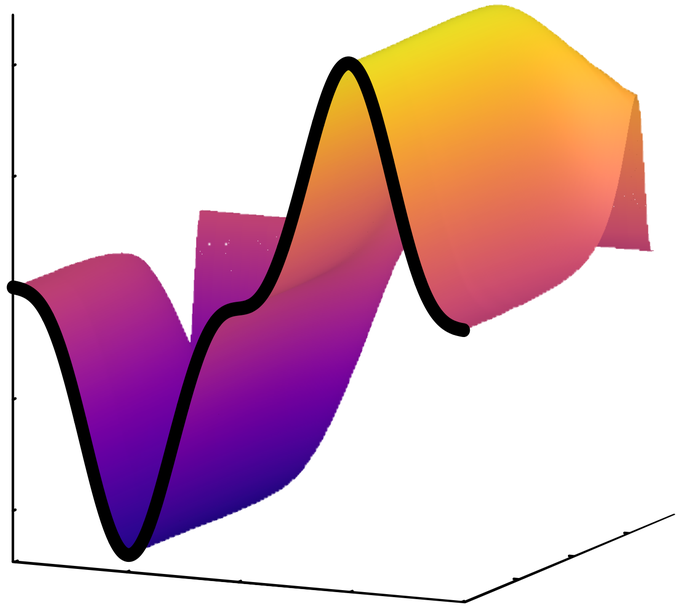
Quasi-GPDs for the Schwinger model using tensor networks

Jake Montgomery

Stony Brook University





Center for Frontiers in Nuclear Science

Stony Brook University

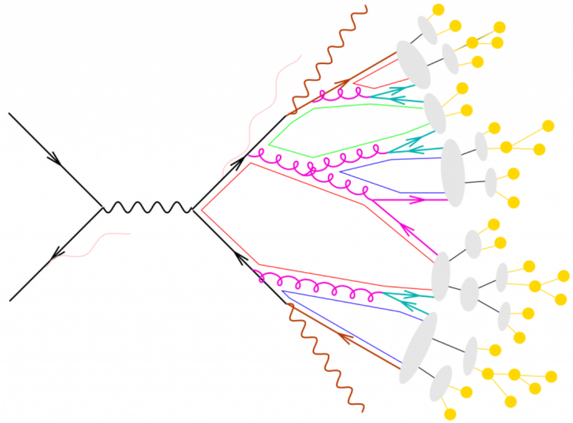
Light Cone 2026, June 26, 2026

CFNS, Stony Brook University

Real-time dynamics of QFTs

- Want to study real time scattering of QCD in 3+1D

$$\sim \int d^4x\, e^{ip\cdot x} \langle P_f | \, T\{\hat{O}(x)\hat{O}(0)\} \, | P_i \rangle$$



- Need understanding of the partonic content of the theory

$$\frac{d\sigma}{dxdQ^2} = \sum_i f_{i/A}(x) \otimes F(x, Q^2)$$

- Nonperturbative at low energies:

  Simulations on lattices in Euclidean time

  Use Hamiltonian formulation?

- Very difficult: use the Schwinger model as test bed

- Want to do these simulations on a quantum hardware

  Jordan, Lee, Preskill

  → PDFs are near-term goal

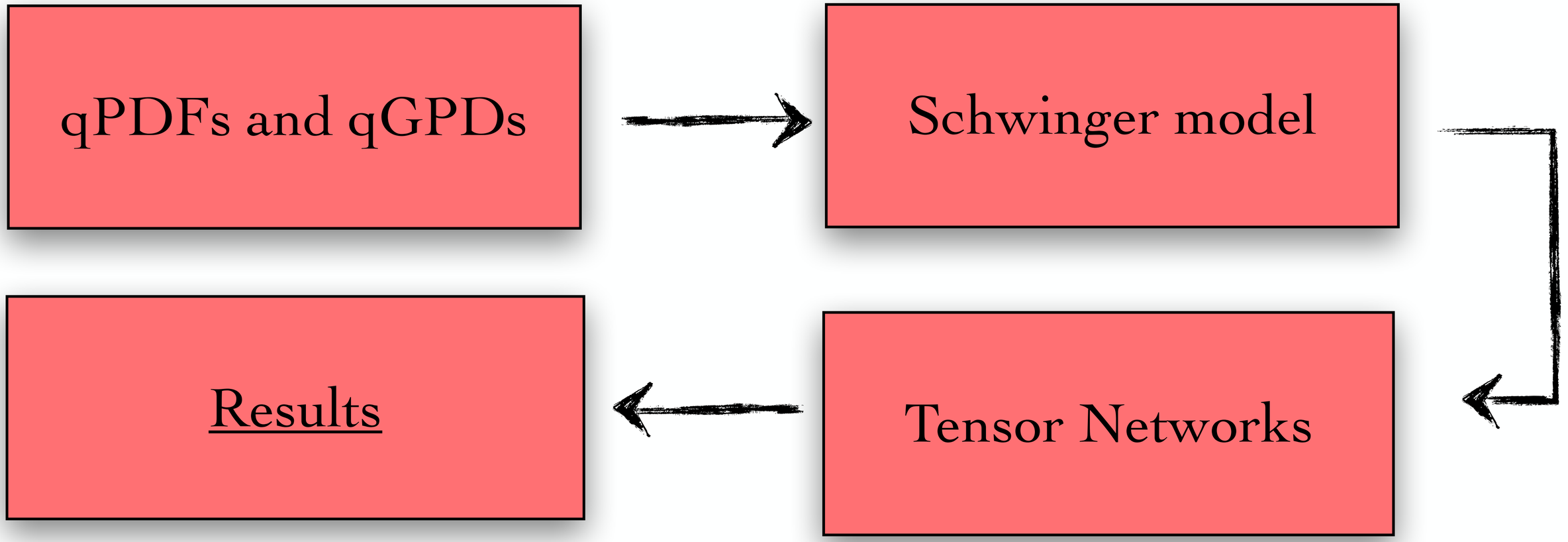
- In collaboration with:

Sebastian Grieninger

Felix Ringer

Ismail Zahed

Outline



Quasi-PDFs

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$$x = \frac{k^+}{P^+}$$

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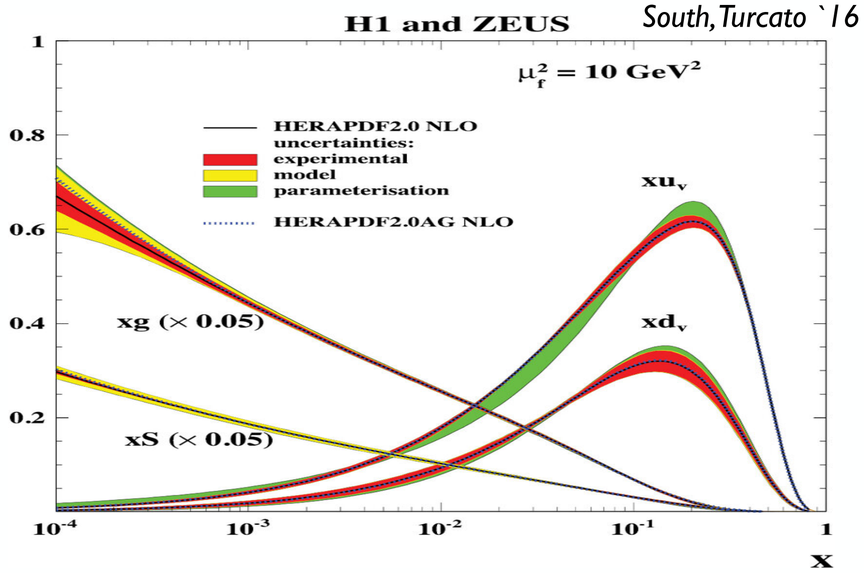
Collins, '11

$$f_i(x) = \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{-ixz^-P^+} \langle \eta \,|\, \overline{\psi}_i(0,z^-) W(z^-,0)\, \gamma^+ \psi_i(0,0) \,|\, \eta \rangle$$



South, Turcato `16

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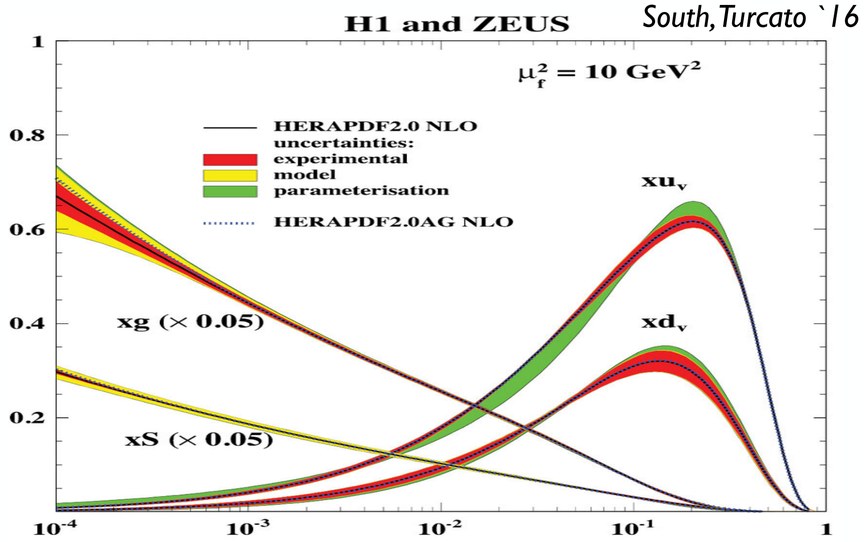

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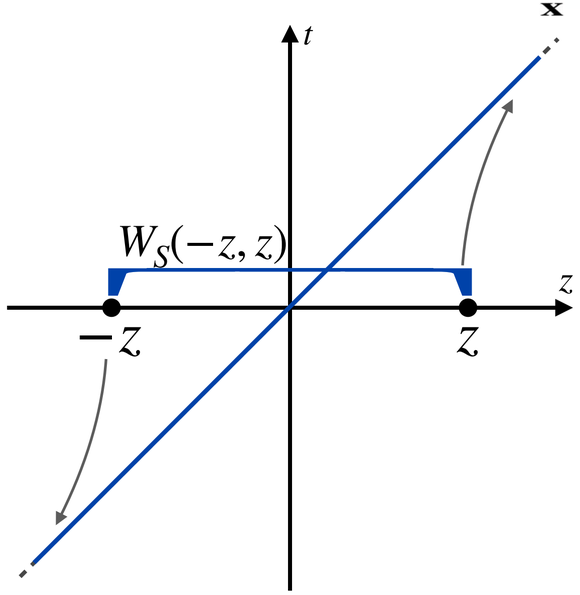
- qPDFs (accessible through lattice MC) *Ji. '13*

** For velocities $|v| \leq 1$ $\chi = \dfrac{1}{2} \ln\left(\dfrac{1+v}{1-v}\right)$

$$q_\eta(x,\chi) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-izxP^1(\chi)} \langle \eta \,|\, e^{-i\chi\hat{K}} \overline{\psi}(-z) W_S(-z,z)\ \gamma^0 \psi(z) e^{i\chi\hat{K}} \,|\, \eta \rangle$$

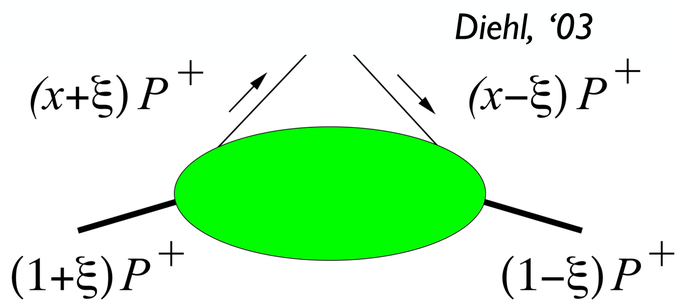


Quasi-GPDs

- GPDs:

$$F_p(x,\xi,t) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dz^-}{2\pi}e^{ixz^-P^+}\langle p_f(\xi)\,|\,\overline{\psi}_i(0,z^-)W(z^-,0)\;\gamma^+\,\psi_i(0,0)\,|\,p_i(\xi)\rangle$$

$$\xi = \frac{(P-P')^+}{(P+P')^+}$$



extension to include transverse information for exclusive processes

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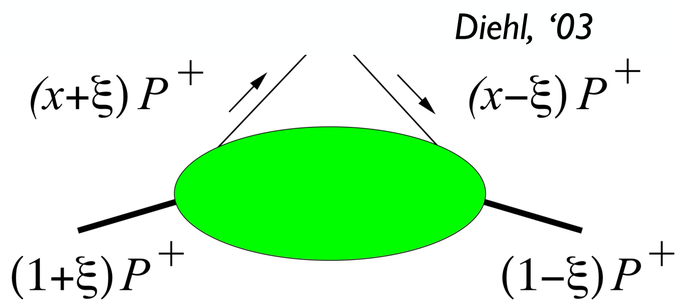
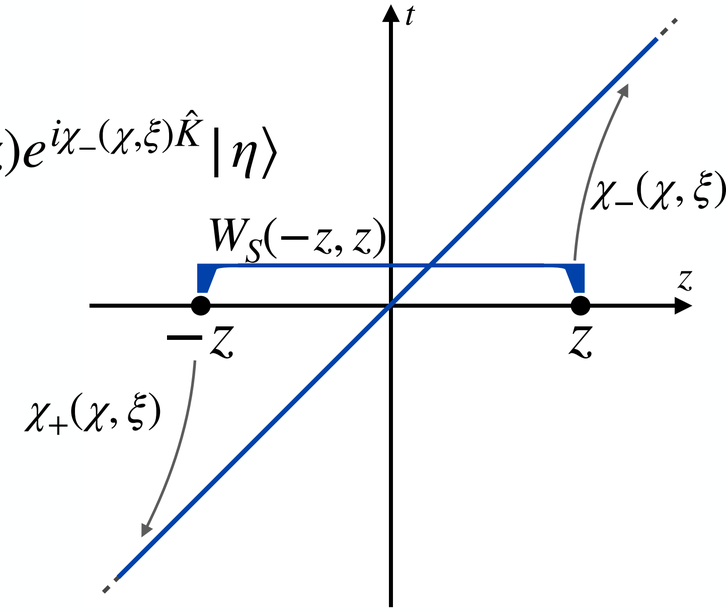


extension to include transverse information for exclusive processes

- qGPDs *Ji. '13* ** In 1+1D, $t = -\dfrac{4\xi^2 m_\eta^2}{1-\xi^2}$

$$H_\eta(x,\xi,\chi) = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-izxP^1(\chi)} \langle \eta\,|\,e^{-i\chi_+(\chi,\xi)\hat{K}}\overline{\psi}(-z)W_S(-z,z)\,\gamma^0\,\psi(z)e^{i\chi_-(\chi,\xi)\hat{K}}\,|\,\eta\rangle$$

$$\chi_\pm = \sinh^{-1}\left(\frac{P^1(\chi)}{m_\eta}(1\mp\xi)\right)$$



Schwinger Model

- QED in 1+1D:

$$\mathscr{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Schwinger `50s

  - Confining → toy model for QCD

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$$\hat{\psi}_n = \begin{pmatrix} \hat{\varphi}_n \\ \hat{\varphi}_{n+1} \end{pmatrix}$$

$$\hat{\varphi}_n = \prod_{m<n} (iZ_m)(X - iY)_n$$

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Kogut, Susskind `70s

$$\mathbb{H} = \frac{ag^2}{2} \sum_{n=0}^{N-2} \left(\frac{1}{2} \sum_{m=0}^{n} (-1)^m + Z_m \right)^2 + \frac{1}{4a} \sum_{n=0}^{N-2} \left(X_{n+1}X_n + Y_{n+1}Y_n \right) + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n = \sum_n h_n$$

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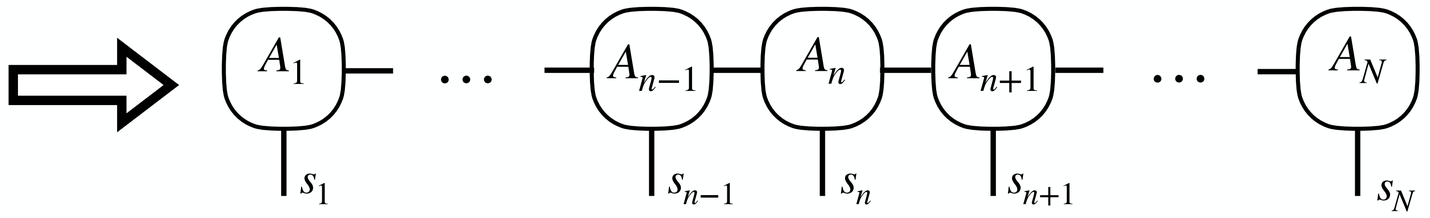
$$\mathbb{K} = \int dx\, x\mathscr{H}(x) \to \sum_n anh_n$$

$$\mathbb{P} = \frac{-1}{4a}\sum_{n=0}^{N-2}\left(X_{n+2}Z_{n+1}Y_n - Y_{n+2}Z_{n+1}X_n\right)$$

Tensor Networks

- Matrix product states (MPS)

$$|\Psi(A)\rangle = \sum_{\{s\}} \left(\prod_{n=1}^{N} A_n^{s_n} \right) |s_1, s_2, \ldots, s_N\rangle$$



Penrose graphical notation

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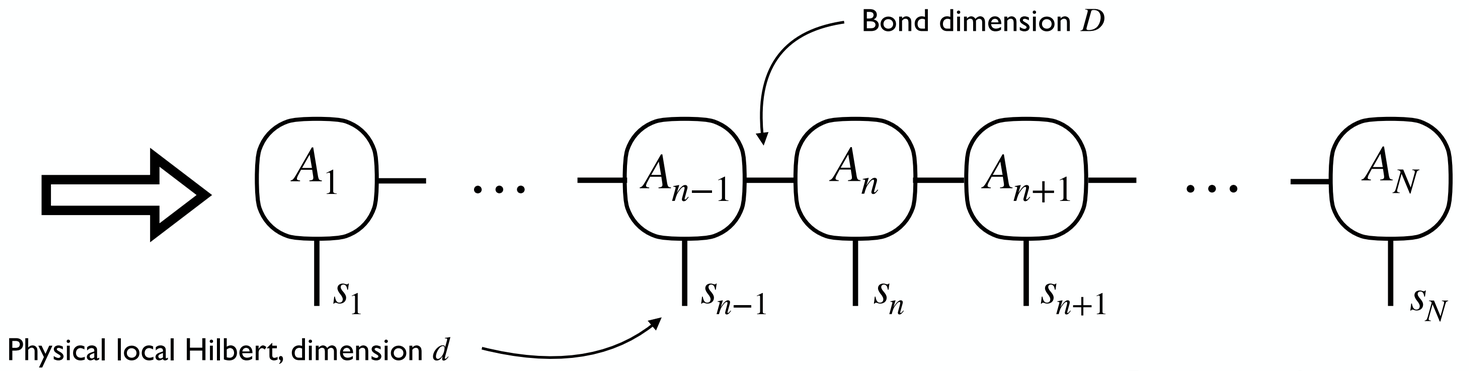

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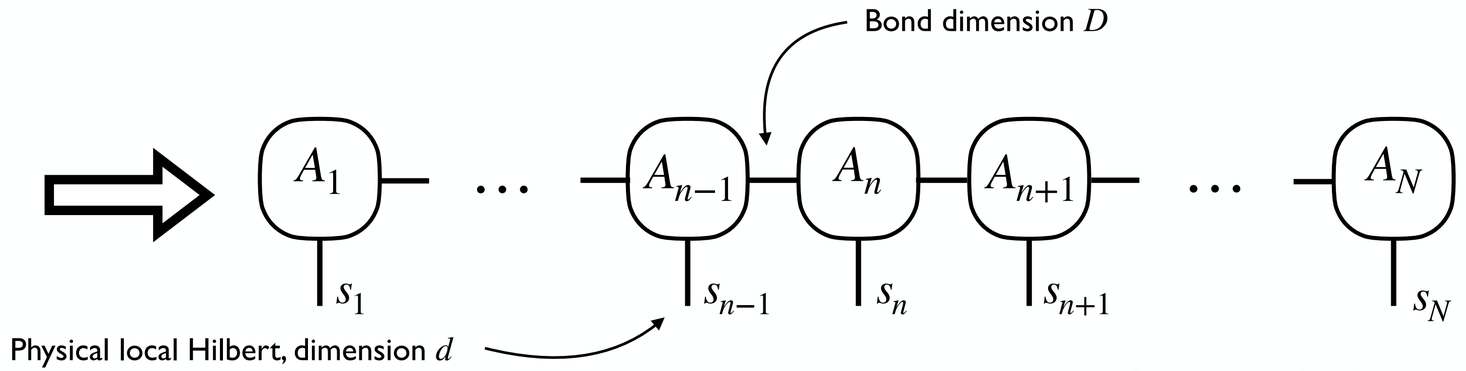

Penrose graphical notation

- Vacuum and hadron state preparation

$$|\Omega\rangle \approx |\Psi(A)\rangle = \underset{|\Psi(A)\rangle}{\text{argmin}} \left[\frac{\langle \Psi(A) | \mathbb{H} | \Psi(A) \rangle}{\langle \Psi(A) | \Psi(A) \rangle} \right]$$

$\langle \Psi(A) | \hat{O} | \Psi(A) \rangle$ costs $\mathcal{O}(ND^3d)$

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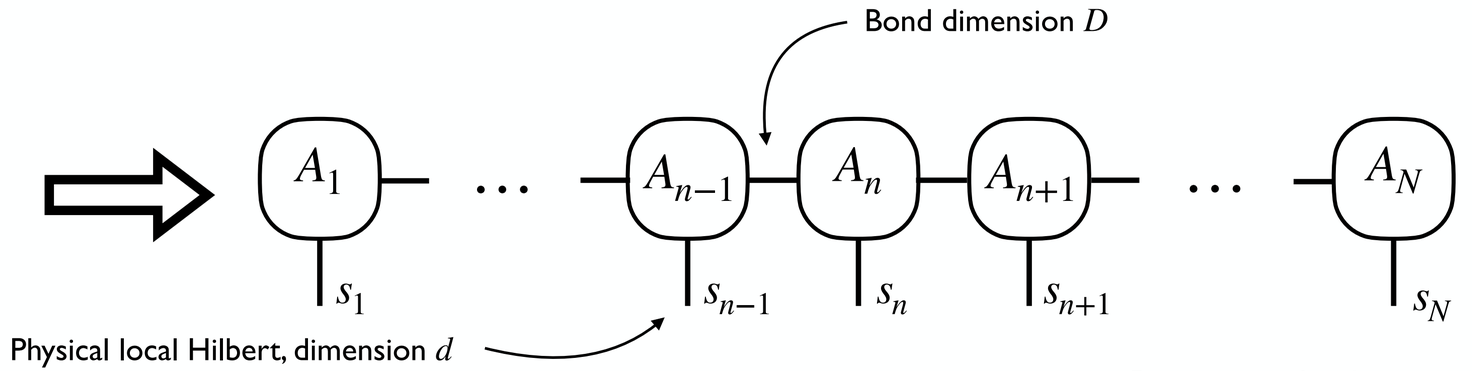

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- Unitary evolution

$$\frac{d}{d\chi}|\eta(\chi)\rangle = -i\mathcal{P}_A\mathbb{K}|\eta(\chi)\rangle \longrightarrow |\eta(\chi)\rangle = e^{-i\chi(\mathcal{P}_A\mathbb{K}\mathcal{P}_A)}|\eta\rangle$$

Approaching the light cone

- State preparation: small variances achievable $\delta E_\Omega / m_\eta \sim \delta E_\eta / m_\eta \sim 10^{-4}$
- In continuum:

$$\langle \eta | e^{i\hat{K}\chi} : \hat{H} : e^{-i\hat{K}\chi} | \eta \rangle = m_\eta \cosh(\chi) \qquad \langle \eta | e^{i\hat{K}\chi} : \hat{P} : e^{-i\hat{K}\chi} | \eta \rangle = m_\eta \sinh(\chi)$$

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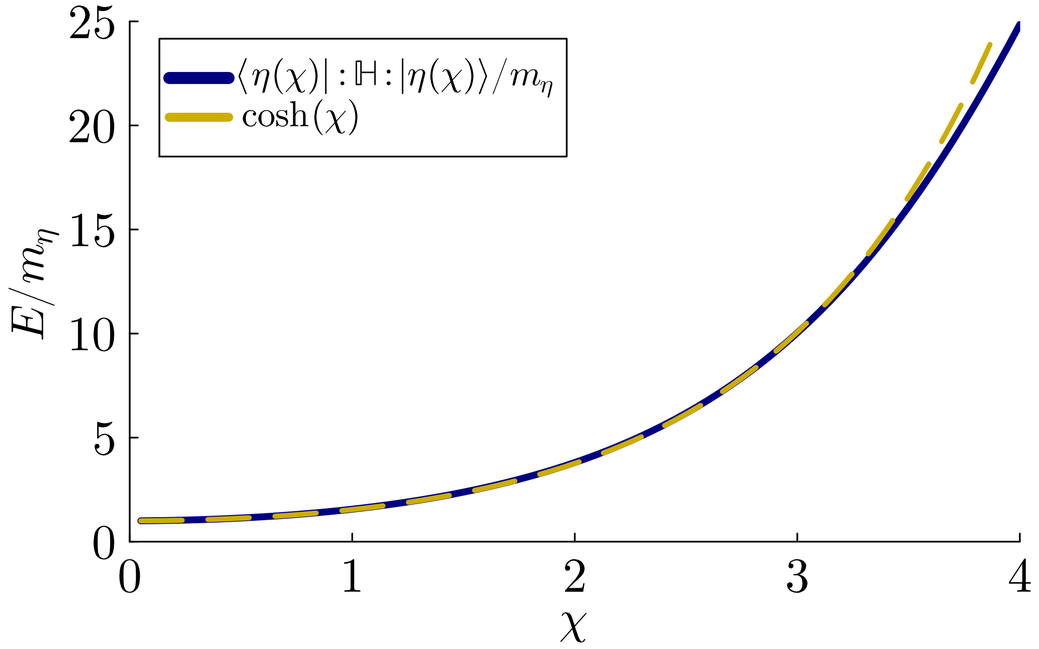

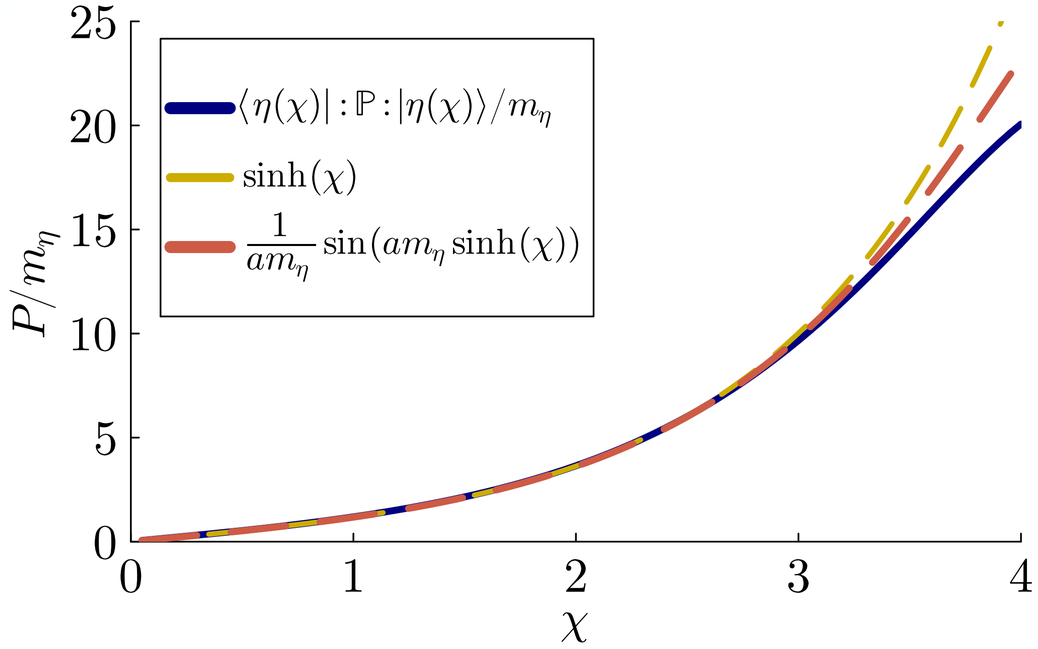

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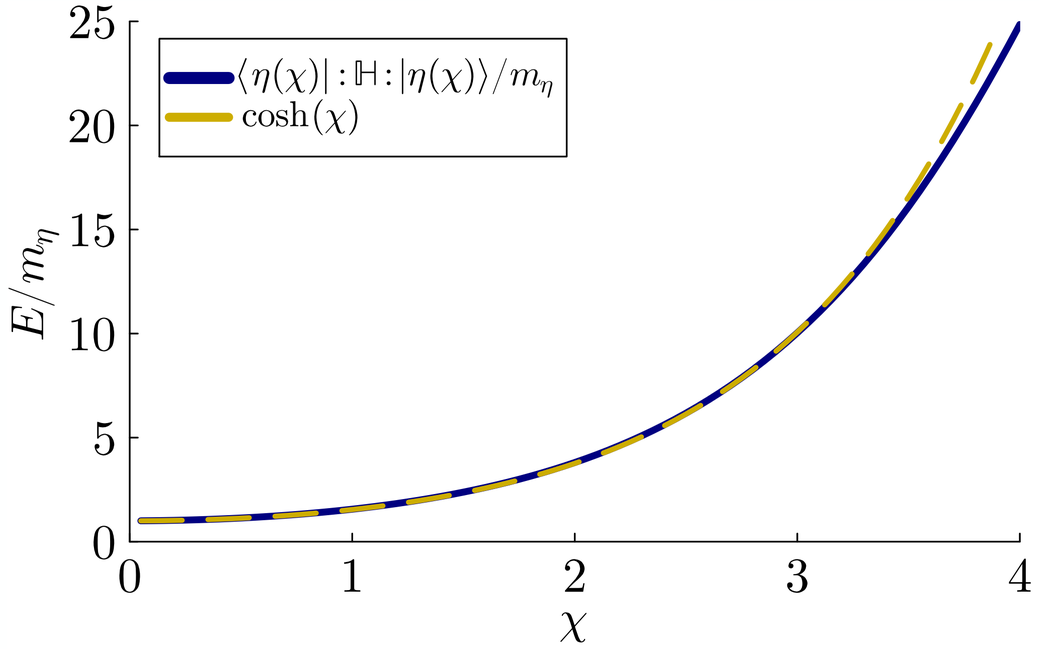
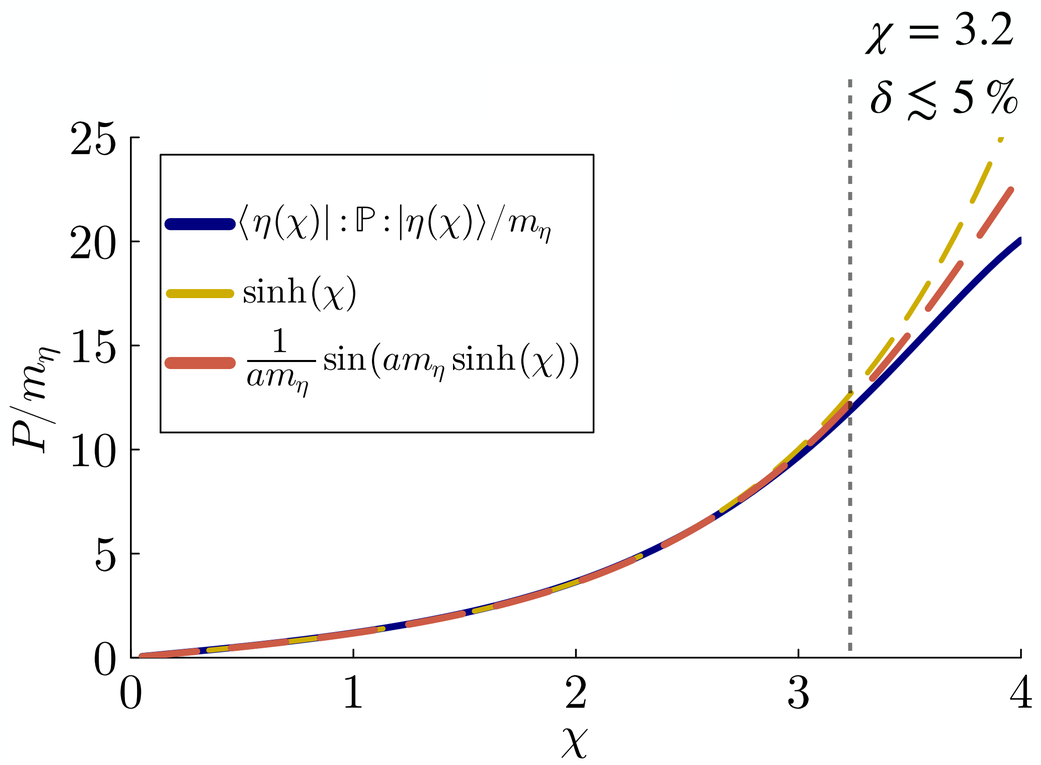
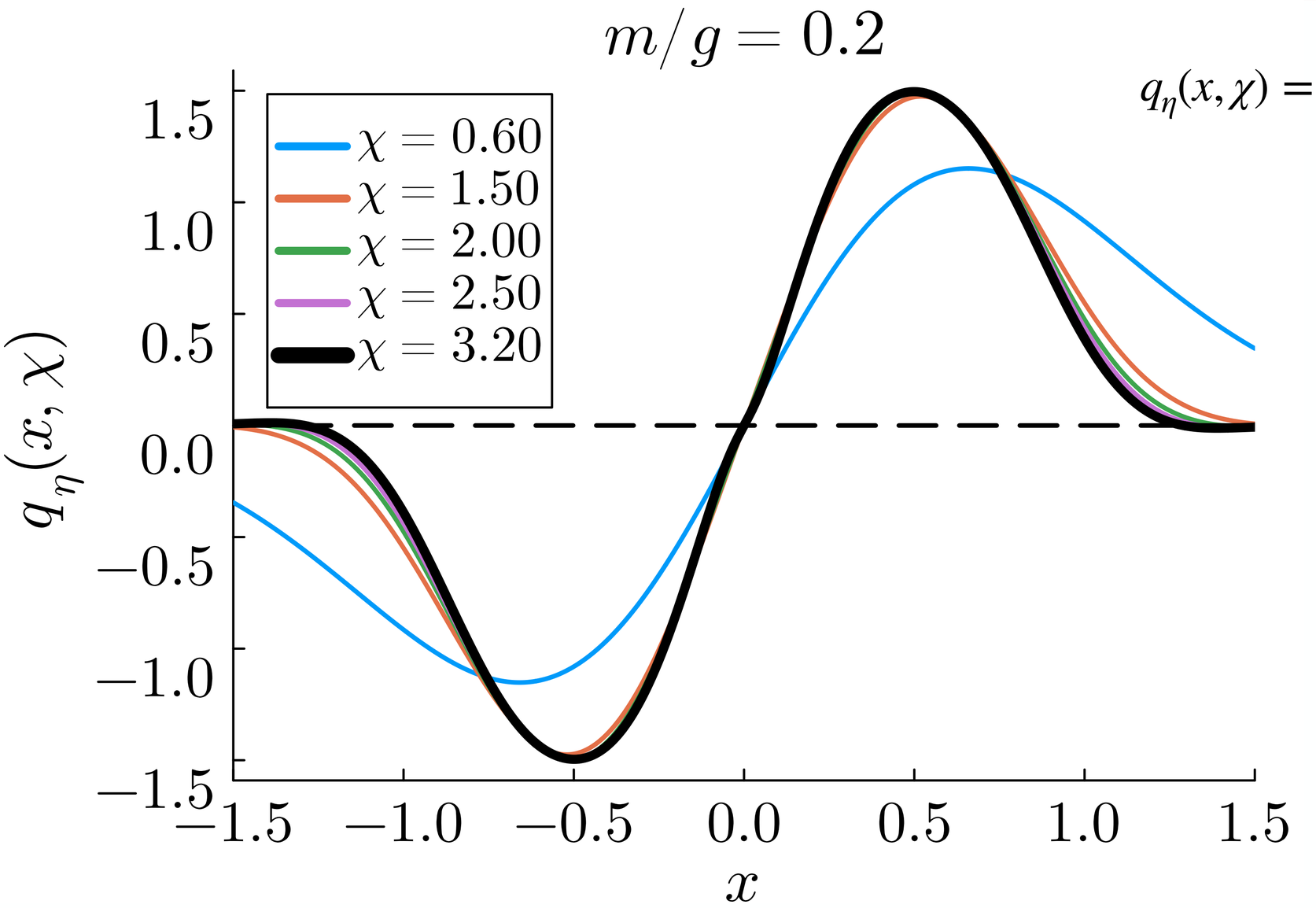




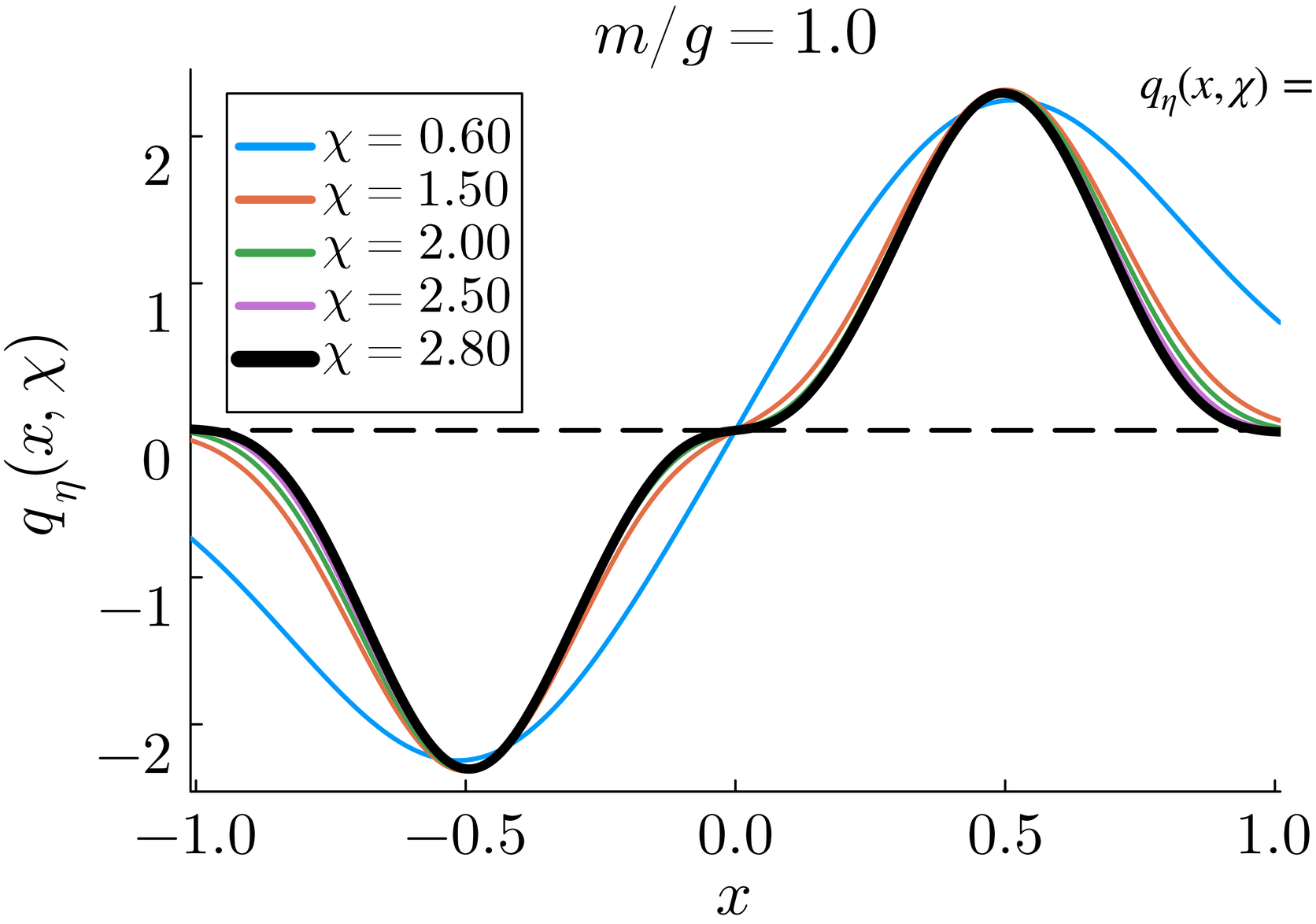
qPDFs



$$q_\eta(x,\chi) = \frac{2a}{2\pi}\sum_z e^{ixzP^1(\chi)} \times$$

$$\langle \eta(\chi) \,|\, \overline{\psi}(-z) W_S(-z,z)\gamma^0 \psi(z) \,|\, \eta(\chi)\rangle$$

qPDFs



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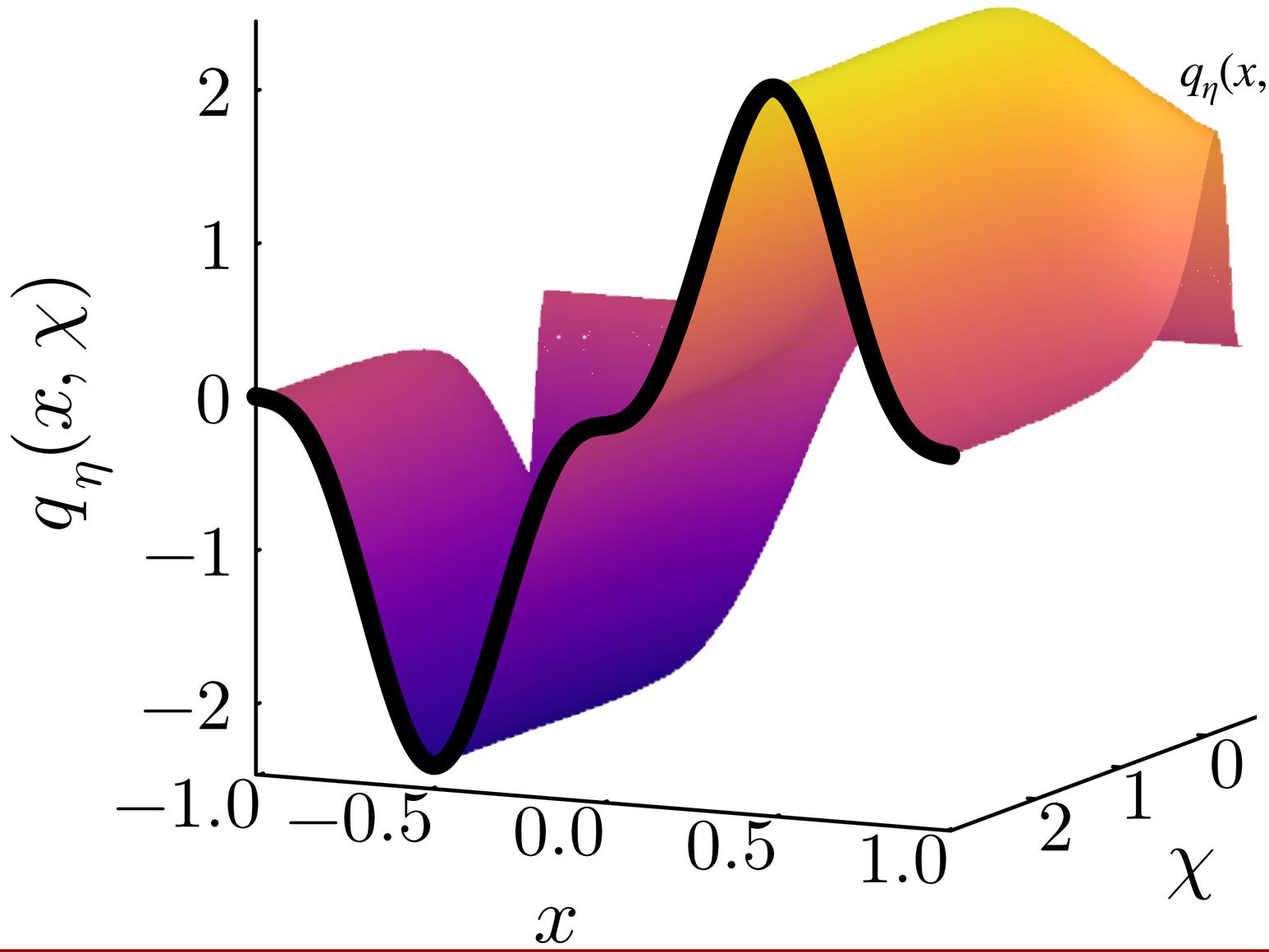

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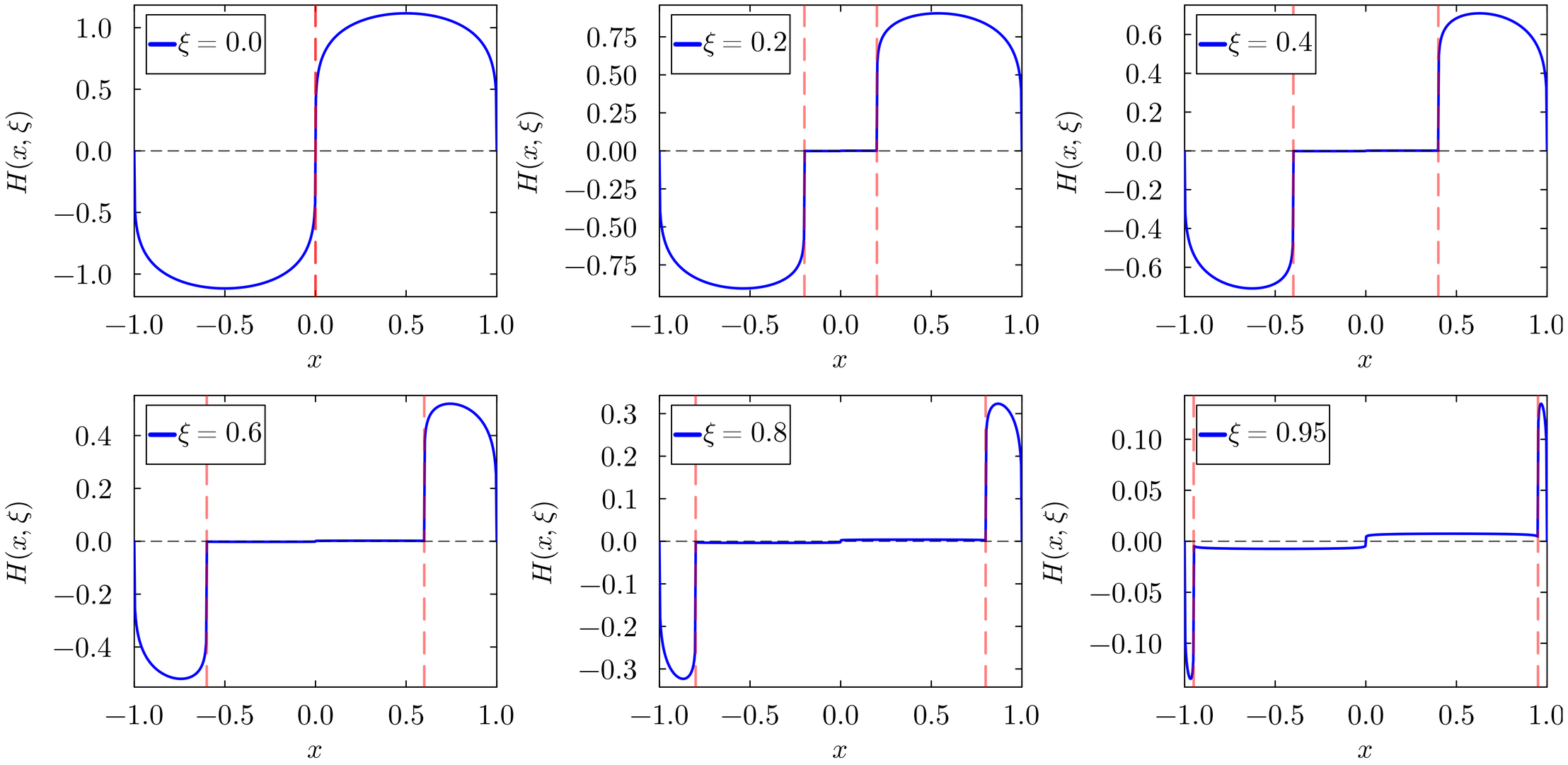
qGPDs

- Analytic calculations: 2 particle approximation for DGLAP, WKB for ERBL



qGPDs

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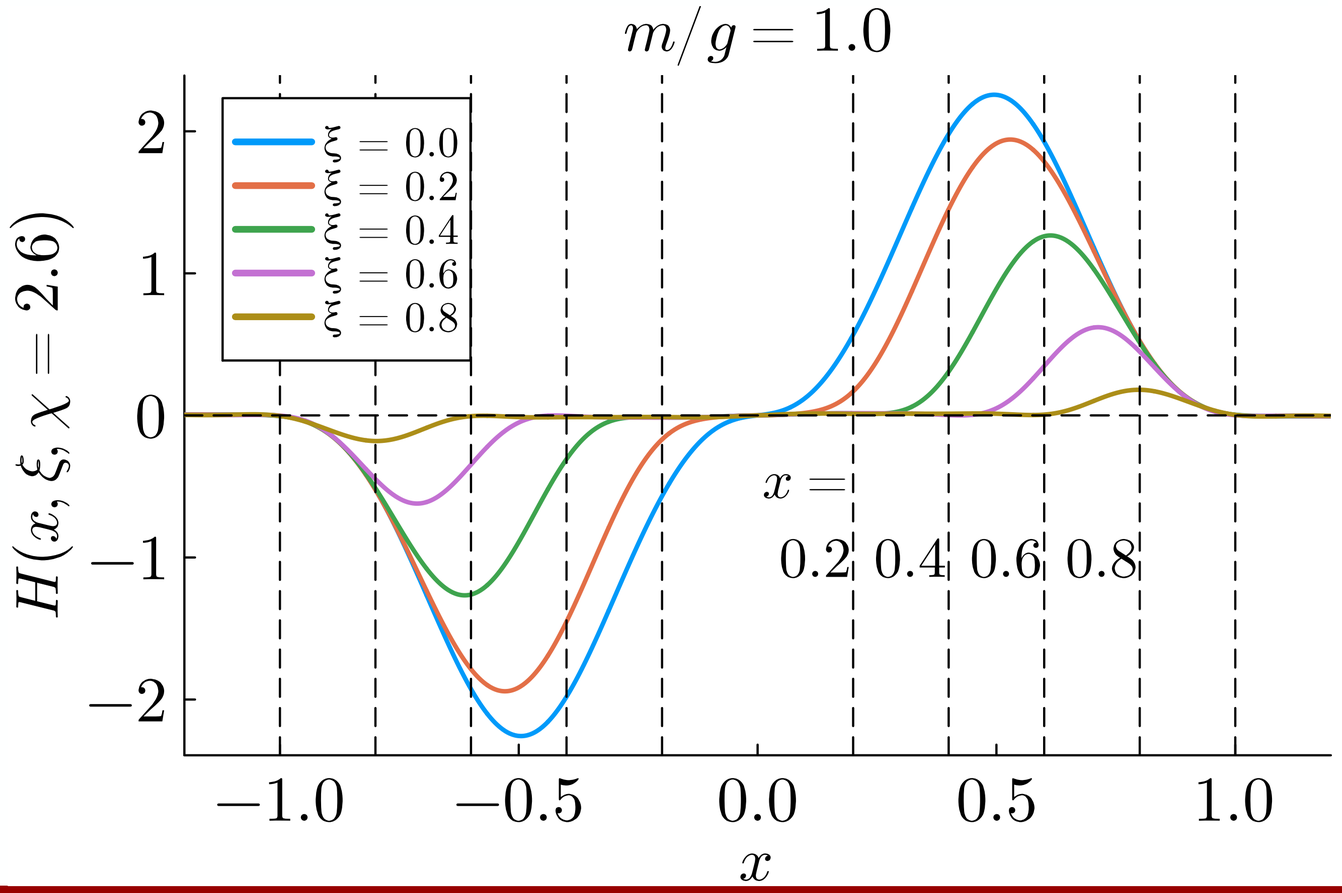

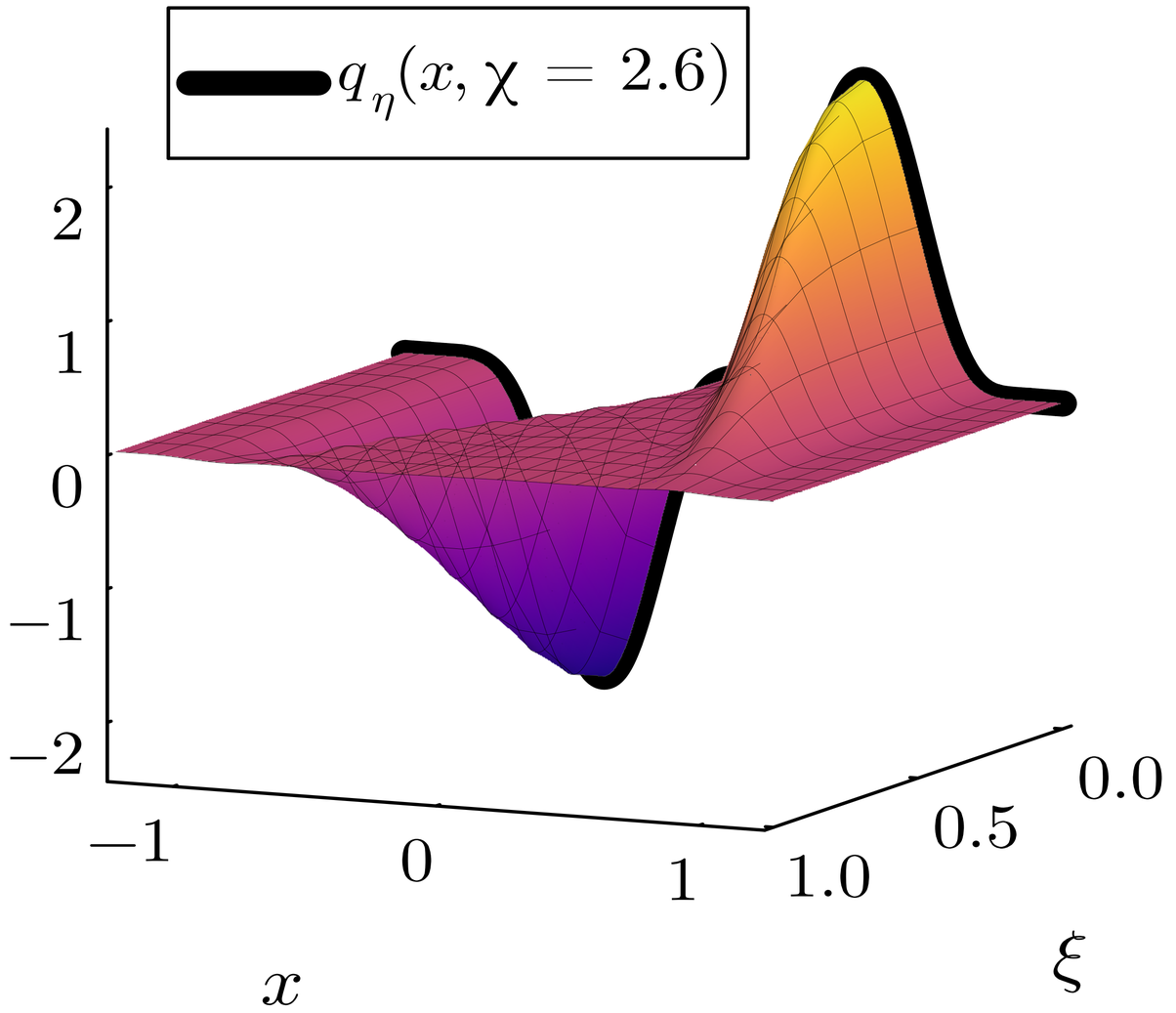

Conclusion

- Quasi PDFs/GPDs for Schwinger model accessible using tensor networks
  - PDFs exhibit convergence at modest χ
  - Provide benchmark for future quantum simulations (in progress…)
  - Can be crosschecked with Euclidean time MC methods
  - qGPDs and exhibit correct properties

Stony Brook University