Updates on the Gluon PDF from Large Momentum Effective Theory

William Good[1]
Light Cone 2026
06.26.2026
Works in Collaboration with:
Alex NieMiera[1], Huey-Wen Lin[1], Fei Yao[2], Joshua Lin[3], and Yong Zhao[3]

MICHIGAN STATE UNIVERSITY 1

Brookhaven National Laboratory 2

Argonne NATIONAL LABORATORY 3

An Introduction to PDFs and Lattice QCD Methods

Introduction and Background

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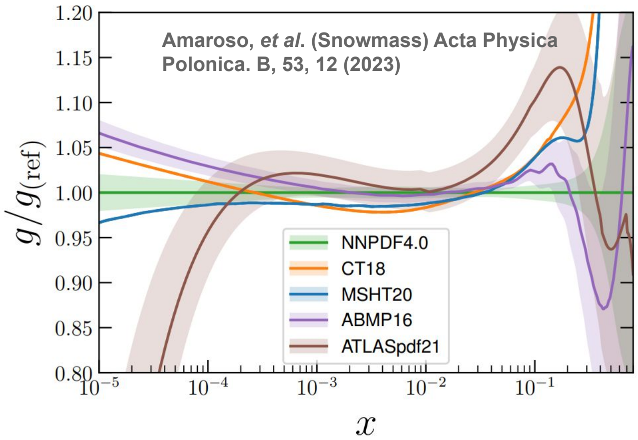

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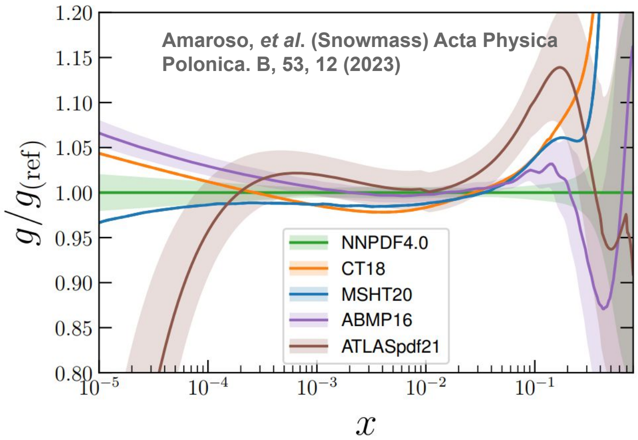

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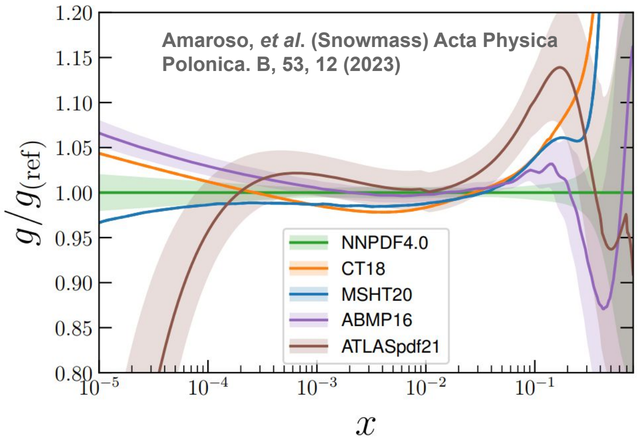

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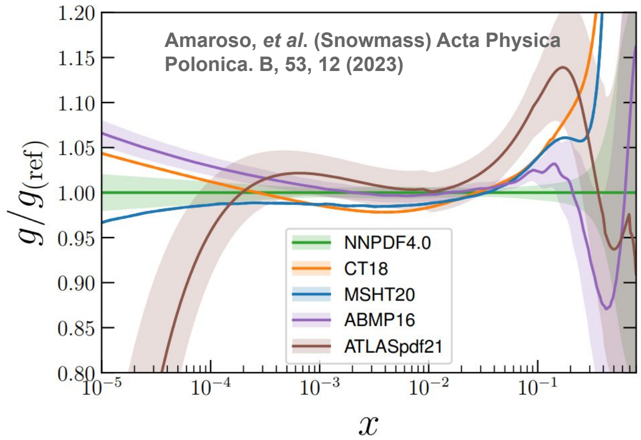
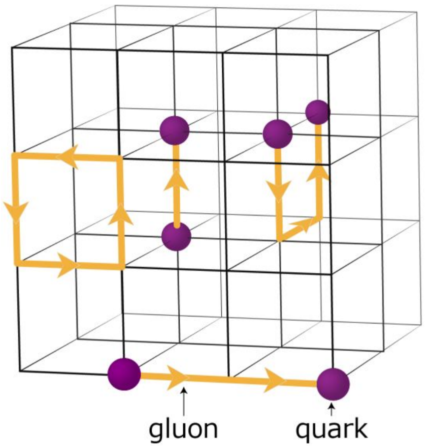
- Lattice QCD provides *ab initio* methods to calculate PDFs, though the gluon is notoriously challenging compared to quark PDFs and there are systematics that need to be controlled if gluon PDFs are to be seriously compared with and contribute to phenomenological studies
- I will present an introduction to these methods, discuss our recent systematic study of the gluon PDF from LaMET, before sharing some progress on Coulomb Gauge gluon PDFs



Lattice QCD

Image credit: JICFuS (2013) (?)



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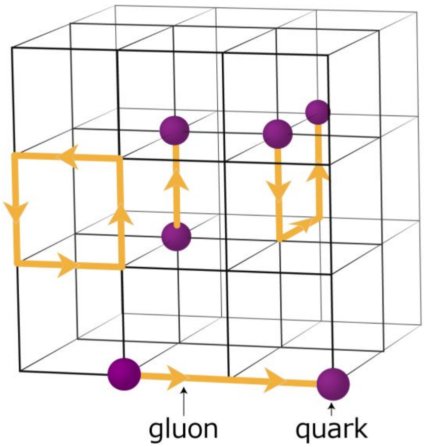

- In the simplest terms, LQCD is a method by which we estimate QCD observables via Monte Carlo sampling of Euclidean path integrals

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O[A, \psi, \bar{\psi}] e^{-S_E[A,\psi,\bar{\psi}]} \approx \frac{1}{N} \sum_{\substack{U_n \text{ with} \\ \text{probability} \\ \propto \mathrm{e}^{-S[U_n]}}} O[U_n]$$

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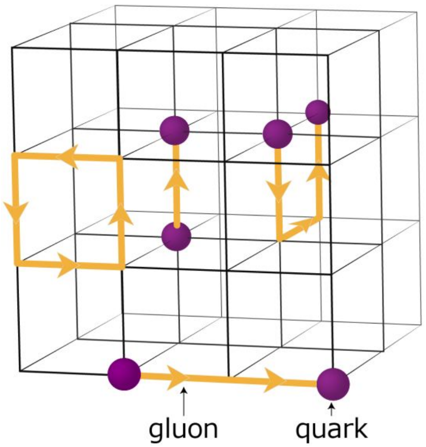

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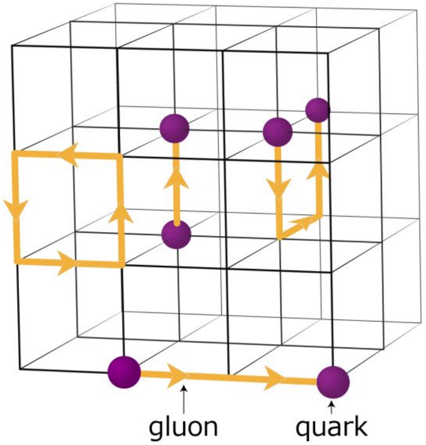

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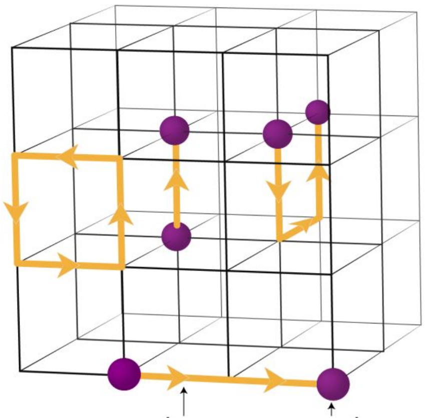

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  - “Classically” these are the main sources of systematic uncertainty in LQCD, and continuum-infinite volume-physical limit extrapolations can be made to remove them
- The fact that LQCD is a Euclidean theory provides some challenges for computing PDFs, and methods to solve this introduce new systematics, which must be handled just as carefully as if not more than the “classical” systematics

Ji, PRL 110:262002 (2013)
Ji, Sci. China Phys. Mech. Astron. 57:1407 (2014)

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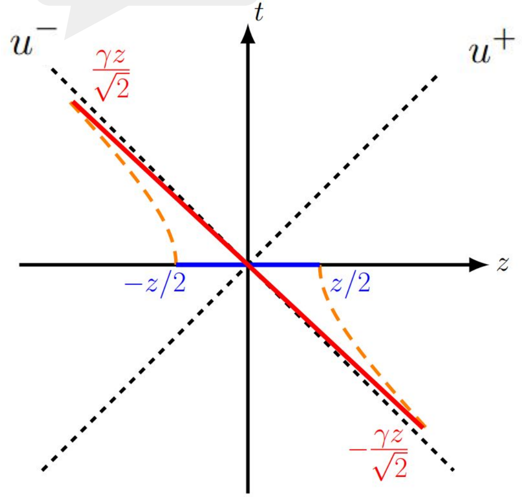
PDFs are formally defined by light-cone correlations of quarks (or gluons):

$$f(x) = \int dz^- e^{ixP^+z^-} \langle P|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^+ W\left(-\frac{z}{2},+\frac{z}{2}\right)\psi\left(+\frac{z}{2}\right)|P\rangle$$

$$u^\pm = \frac{u^0 \pm u^3}{\sqrt{2}}$$



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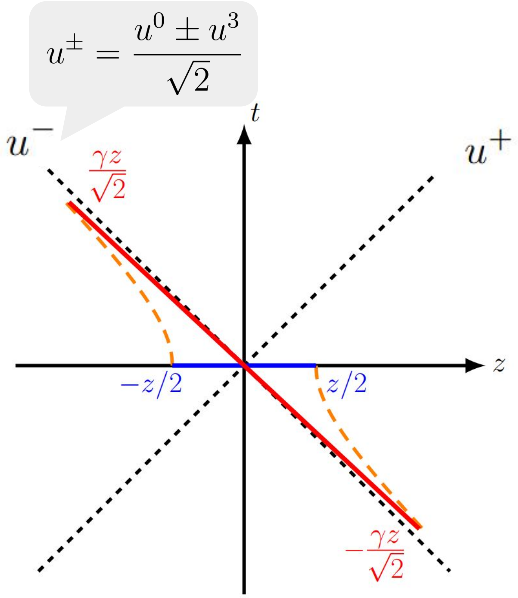

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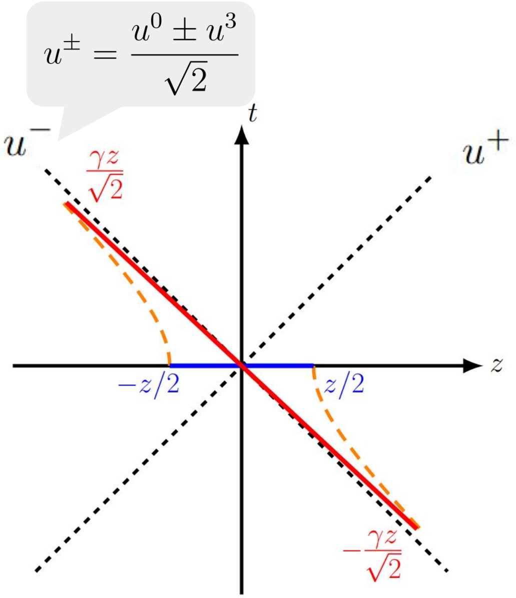

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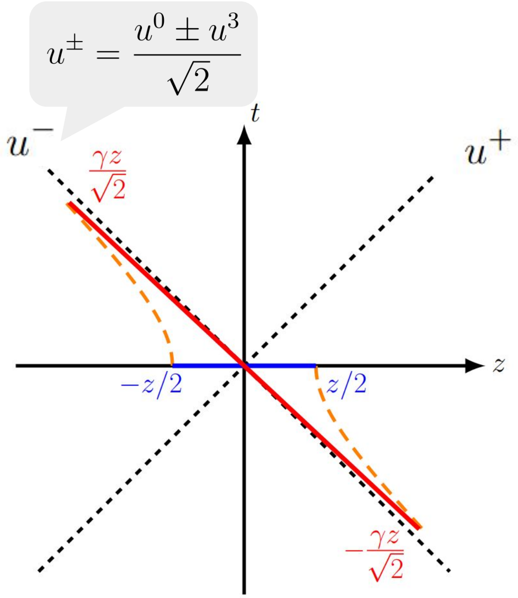

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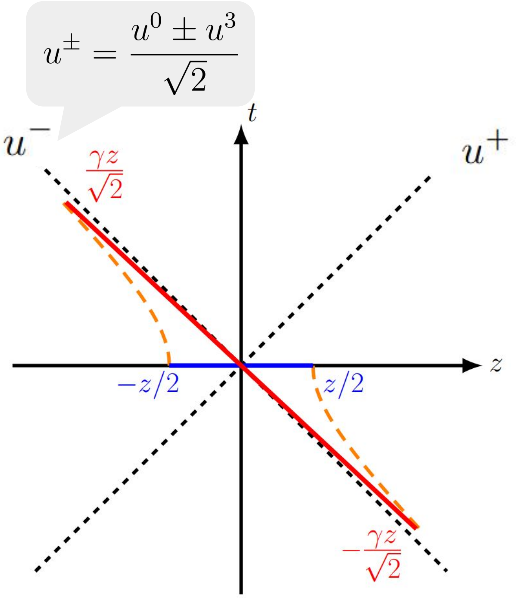

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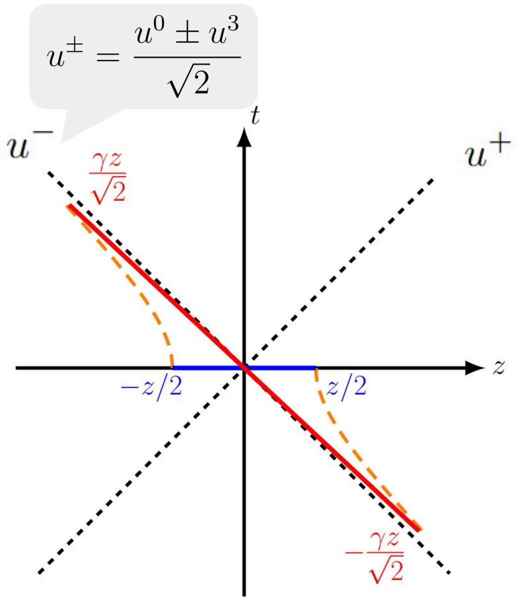

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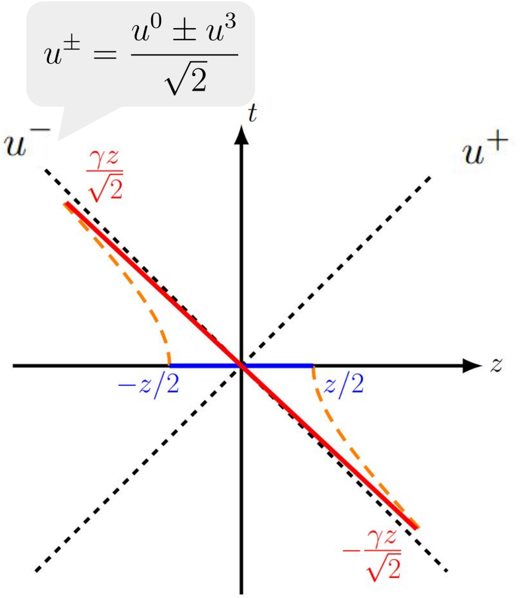

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Only reliable in the mid-x region!

A Systematic Study of the Nucleon Gluon PDF from LaMET

Gluon Quasi-PDF Operator and Renormalization

Ji, *et al*. NPB. 964:115311 (2021)

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- Short distances are easy, while at long distances, we need to be more clever…

$$h^R(z,P_z) = \frac{h^B(z,P_z,1/a)}{h^B(z,P_z=0,1/a)}\theta(z_s-|z|) + T_s\frac{h^B(z,P_z,1/a)}{Z_R(z,1/a)}\theta(|z|-z_s)$$

This slide is courtesy of Alex NieMiera!!!

Self Renormalization

LPC, J. Nuc. Phys B 115443 (2021)
Balitsky, *et. al.*, Phys. Rev. D.105.014008 (2022)

The self renormalization factor is defined as

$$Z_R(z,1/a) = \exp\left[\frac{kz}{a\ln(a\Lambda_{\mathrm{QCD}})} + m_0 z + f(z)a^2 + \frac{5C_A}{3b_0}\ln\left(\frac{\ln(1/(a\Lambda_{\mathrm{QCD}}))}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right) + \ln\left(1+\frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right)\right]$$

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The final two terms come from resummation of leading and sub-leading logarithmic divergences

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$$+\begin{cases}\ln(C_{0,\mathrm{NLO}}(z,\mu)) + m_0 z, & z_0 \le z \le z_1\\ g(z), & z_1 < z\end{cases}$$

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For the gluon, the Wilson coefficient is defined as

$$C_{0,\mathrm{NLO}}(z,\mu)=1+\frac{\alpha_sC_A}{4\pi}\left(\frac{5}{3}\ln\left[\frac{z^2\mu^2}{4e^{-2\gamma_E}}\right]+3\right)$$

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In addition to the parameters above, we also fit g(z), the residual contribution containing intrinsic non-perturbative physics.

The MSULat Setup

Follana *et al*. PRD 75:054502, 2007.
A. Bazavov, *et al*. [MILC], PRD 82:074501, 2010.
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a (fm)	0.0888(8)	0.1207(11)	0.1510(20)
$L^3 \times T$	$32^3 \times 96$	$24^3 \times 64$	$16^3 \times 48$
M_π^{val} (GeV)	0.313(1)	0.309(1)	0.319(3)
$M_{\eta_s}^{\text{val}}$ (GeV)	0.698(7)	0.6841(6)	0.687(1)
P_z (GeV)	{0, 1.31, 1.75, 2.18}	{0, 1.28, 1.71, 2.14}	{0, 1.54, 2.05, 2.57}
N_{cfg}	1026	1013	900
$N_{\text{meas}}^{\text{2pt}}$ $(P_z = 0)$	196,992	64,832	129,600
$N_{\text{meas}}^{\text{2pt}}$ $(P_z \neq 0)$	1,378,944	1,555,968	950,400

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Important Note: Gauge Link Smearing

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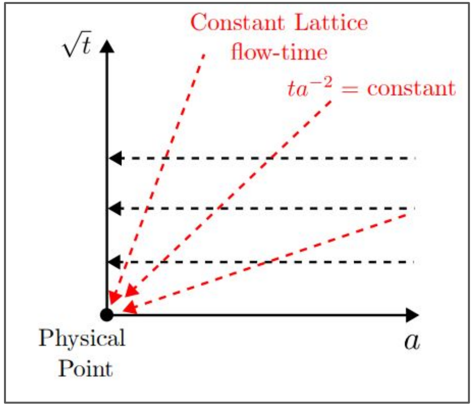

Figure from Joshua Lin

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- We find little dependence on the path we take to approach this limit

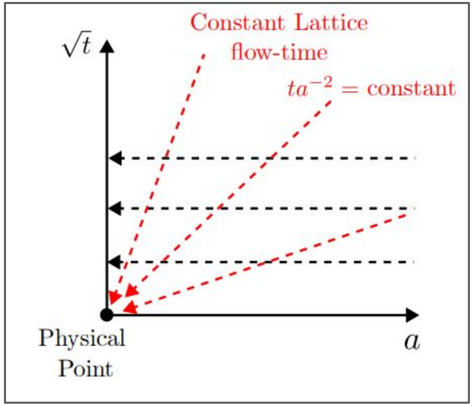

Figure from Joshua Lin

NieMiera, *et al.* arXiv 2511.14708 (2025)

Renormalization Factors

NieMiera, *et al*. arXiv 2510.17758 (2025)

Renormalization Factors

We find a very reasonable fit and parameters,

Parameter	W333
k	0.633(26)
d	−0.0008(78)
Λ_{QCD} (GeV)	0.264(77)
m_0 (GeV)	0.13(25)
χ^2/dof	0.81(62)

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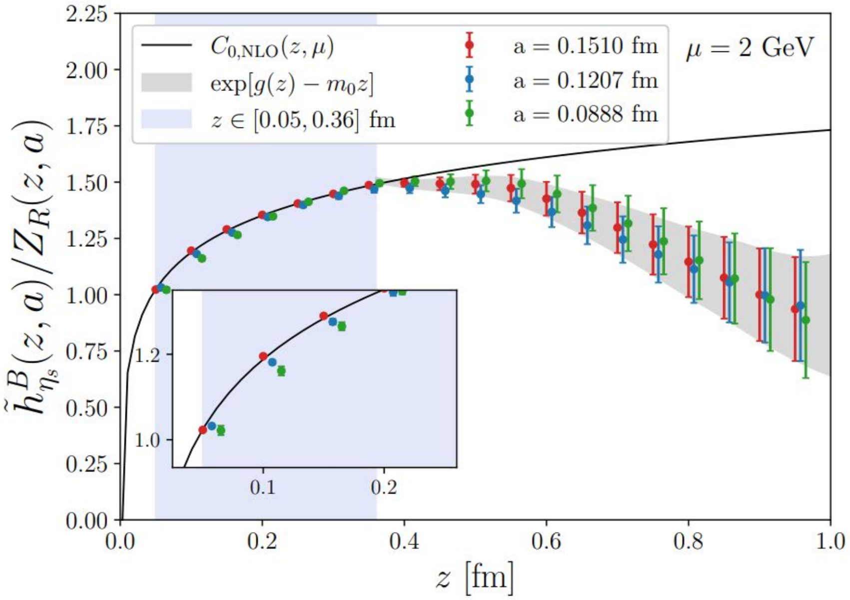

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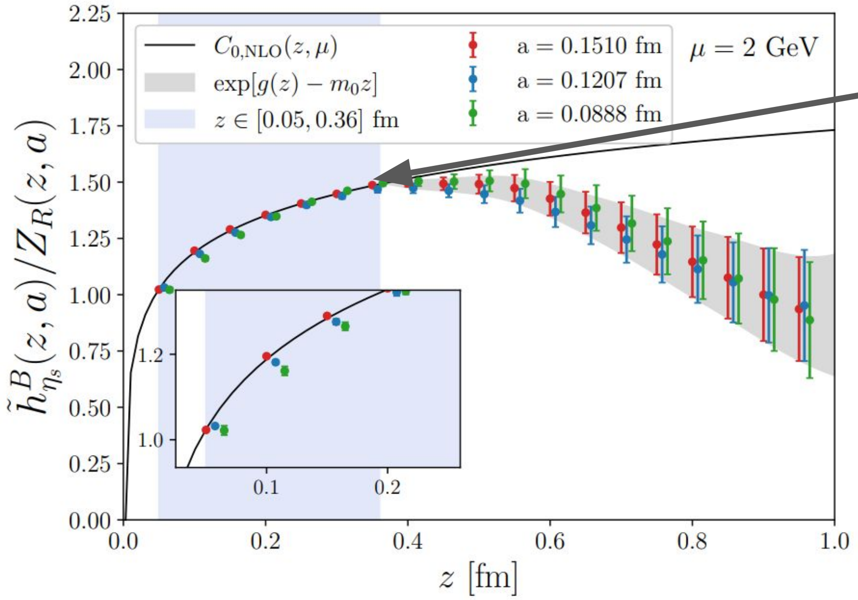

Agreement with the perturbative Wilson coefficient at short distances

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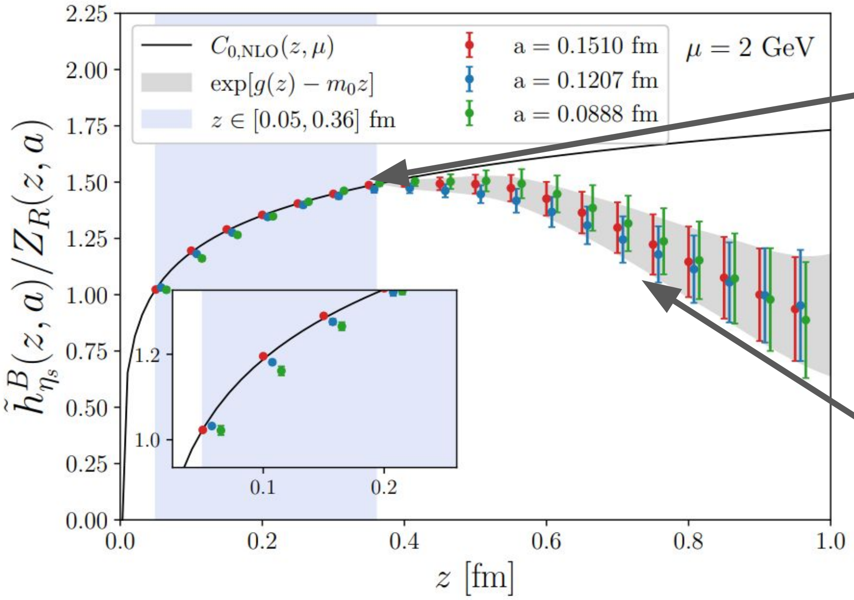

Agreement with the perturbative Wilson coefficient at short distances

Lattice spacing dependence mostly removed

Long Distance Physics and Continuum Extrapolation

Long Distance Physics and Continuum Extrapolation

- We apply a common extrapolation form, which has recently been given stronger theoretical motivations

$$\tilde{h}^R(z, P_z) \approx A \frac{e^{-m\nu}}{|\nu|^d}$$

Gao, *et al*. PRL 128:142003 (2022)
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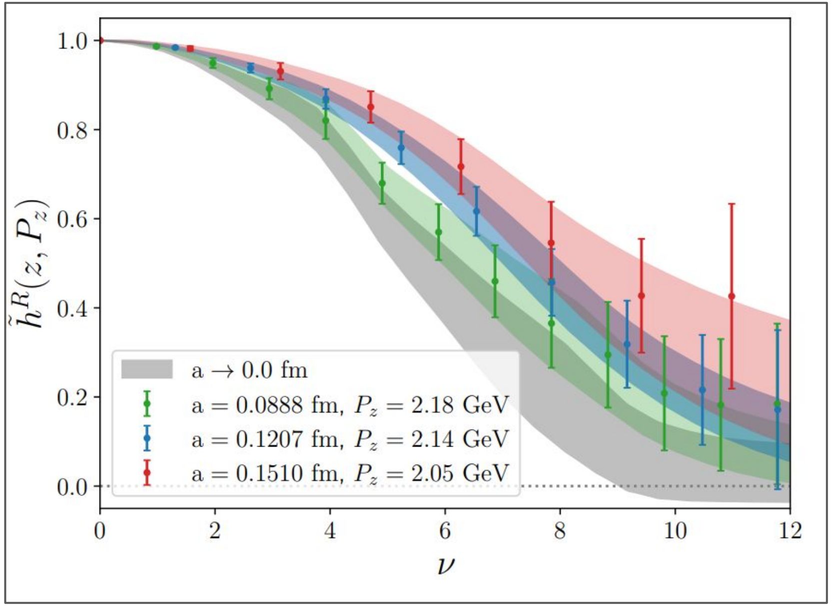

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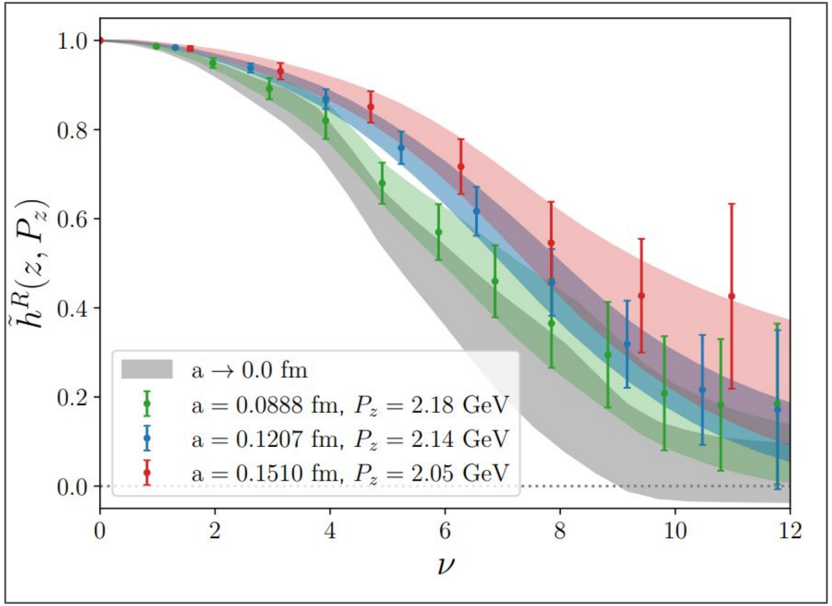
- We see some residual lattice spacing dependence (less than 2 standard deviations in most regions), which we wash out via a continuum extrapolation:

$$\tilde{h}^{\text{latt}}(z, P_z) = \tilde{h}^{\text{cont}}(z, P_z) + c(z)a^2$$



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Final PDFs

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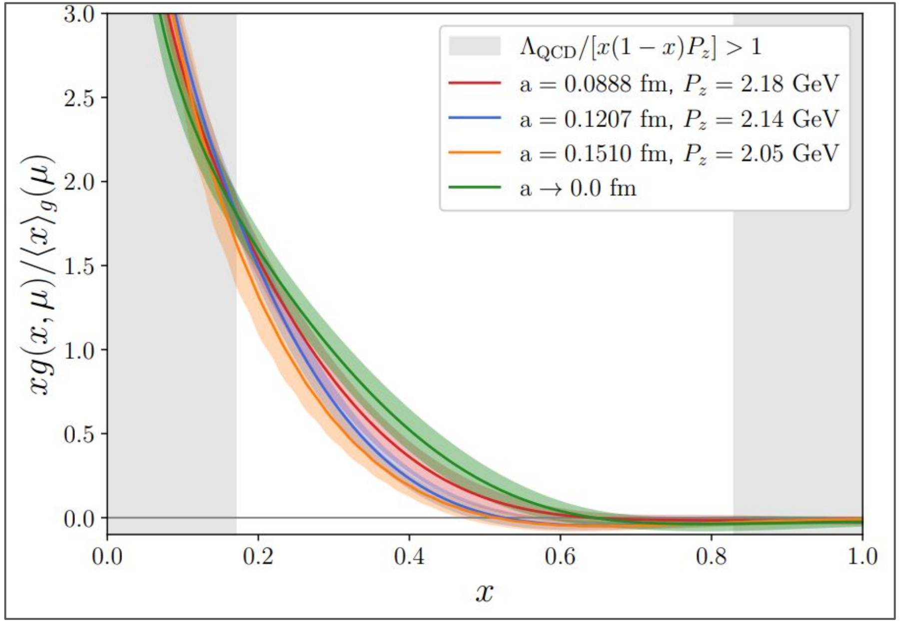
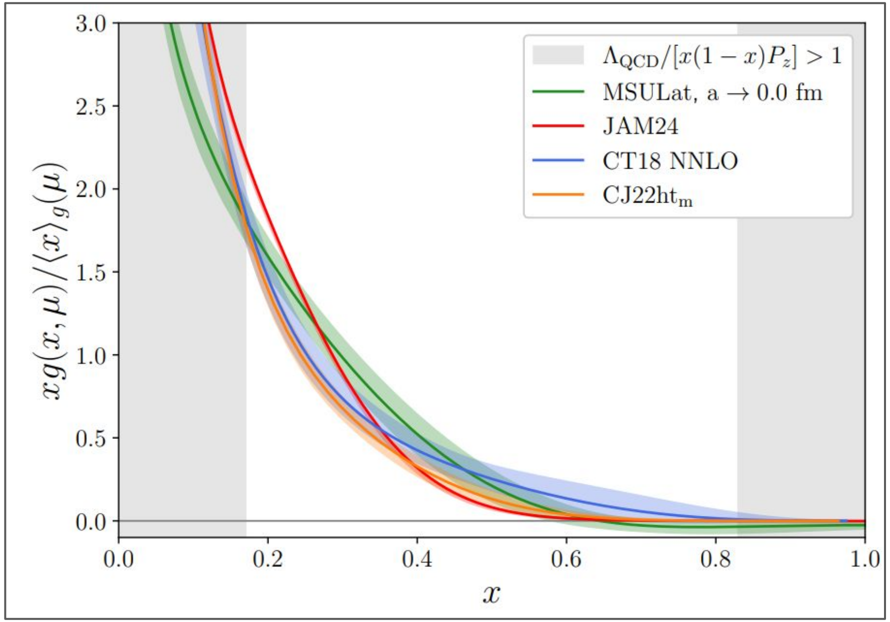
Anderson, et al. arXiv 2501.00665 (2024)
Hou, et al. (CTEQ) PRD 103(1):014013 (2021)
Cerutti, et al. PRD 111:094013 (2025)

We compare across the different lattice spacings and against phenomenological PDFs





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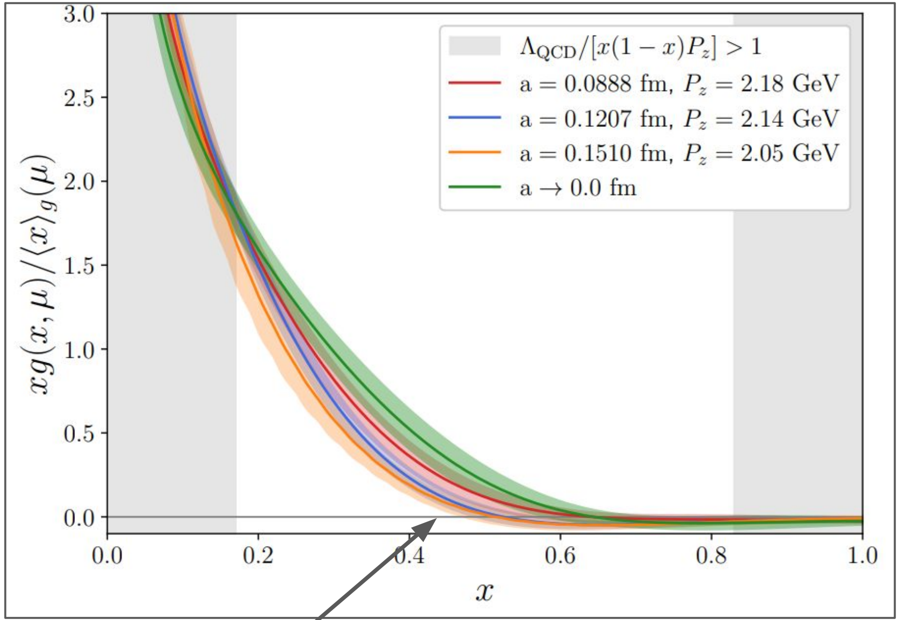
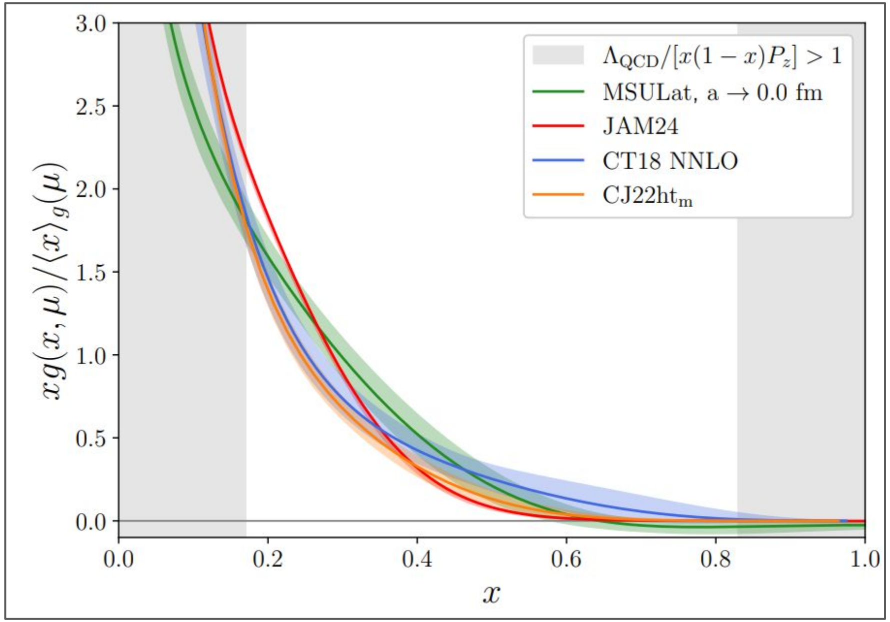
We compare across the different lattice spacings and against phenomenological PDFs





Lattice spacing seems to have a strong impact on whether the PDF goes below zero or not

Final PDFs

NieMiera, *et al.* arXiv 2510.17758 (2025)

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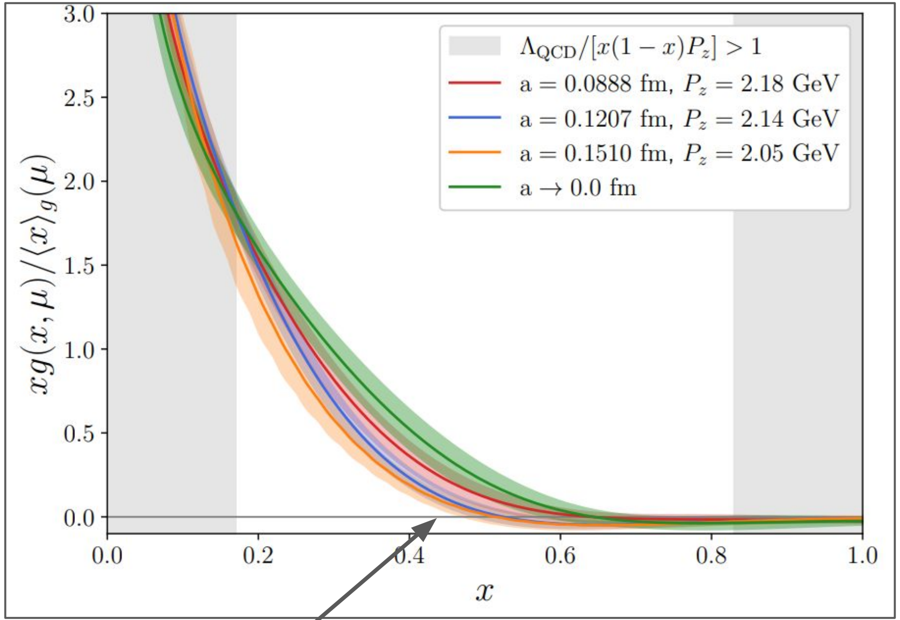

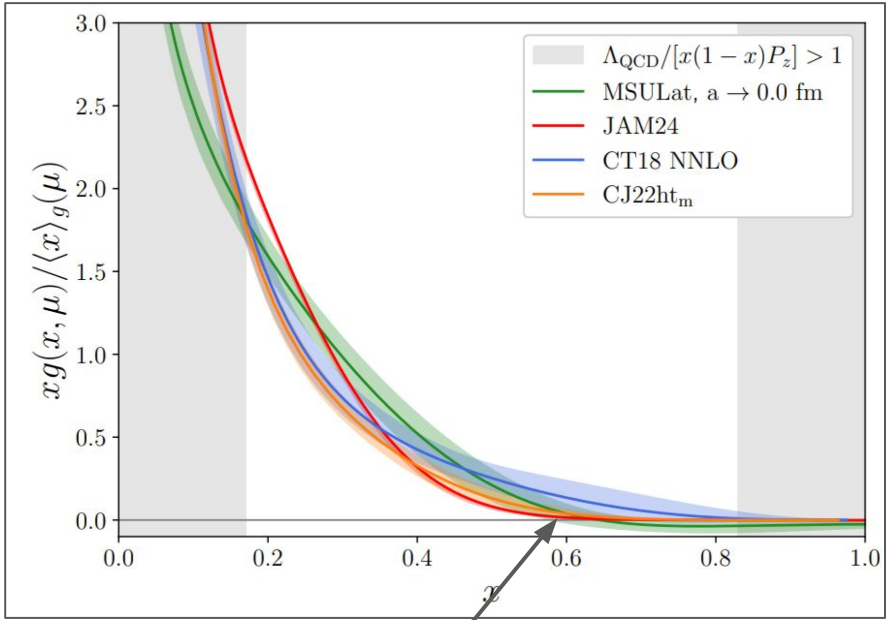

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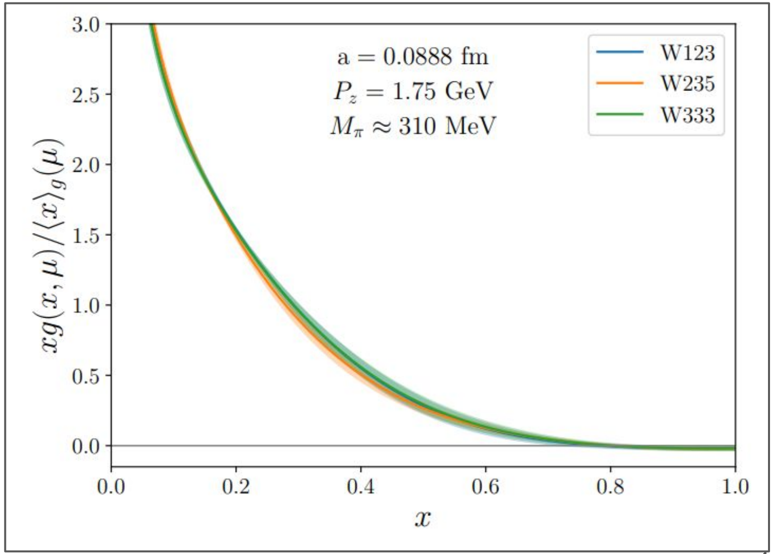

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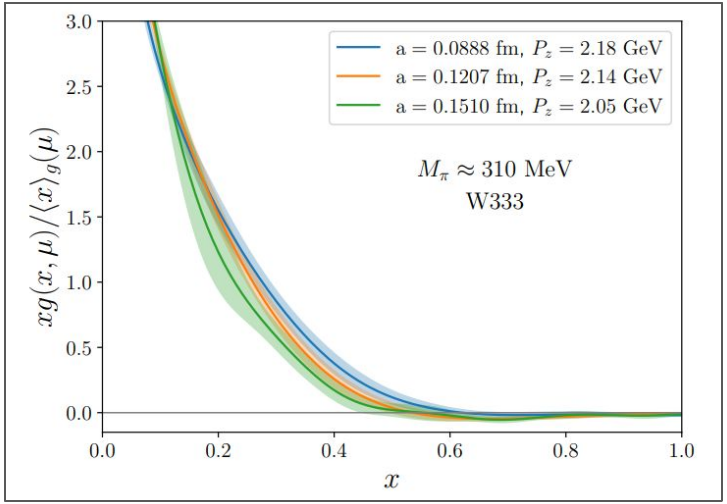

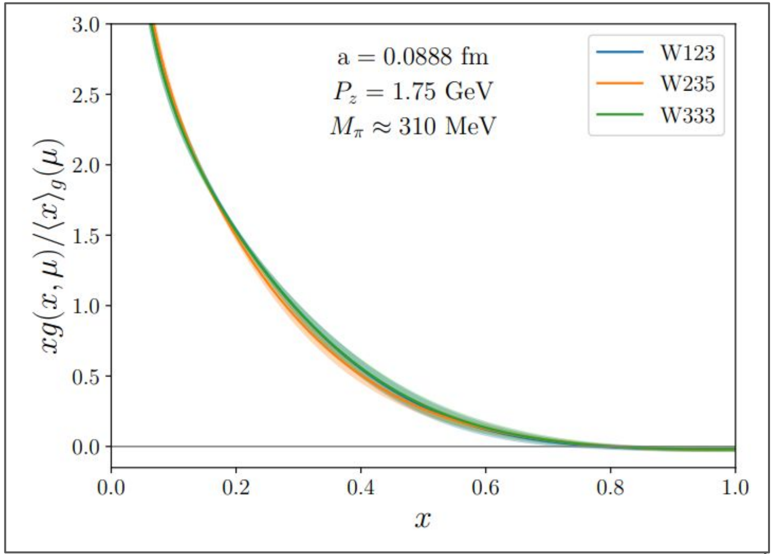

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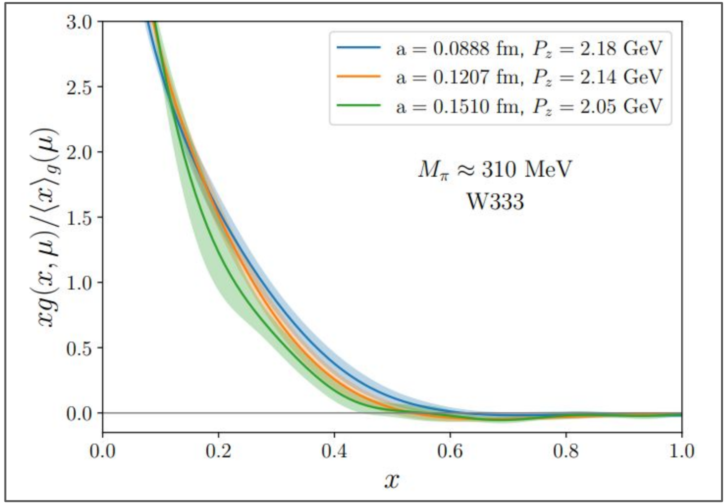

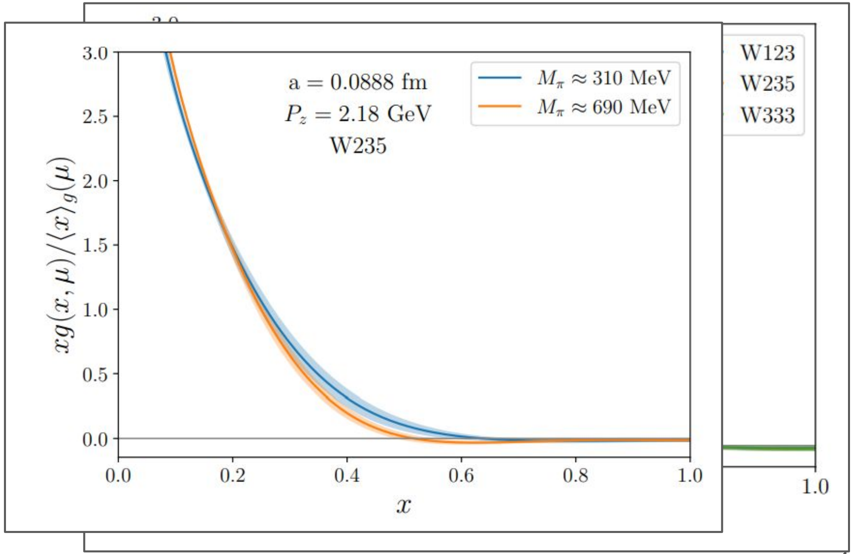

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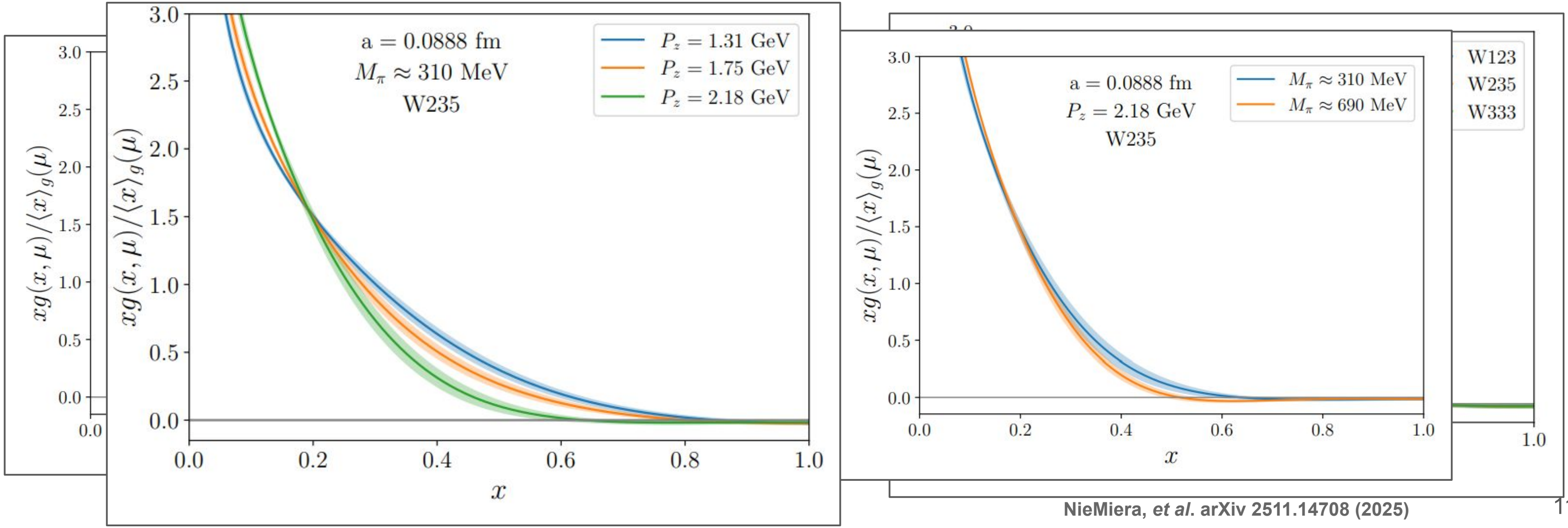

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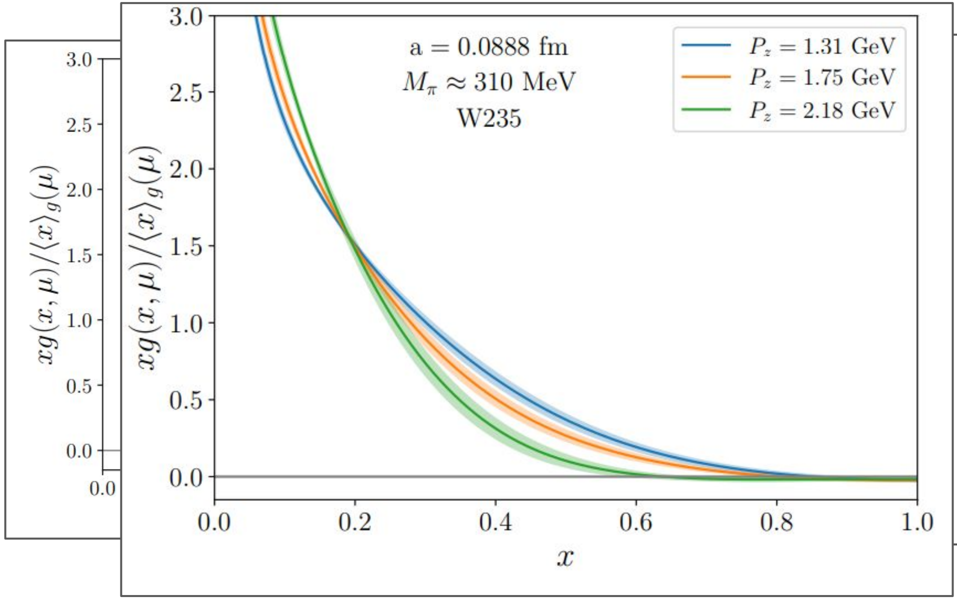

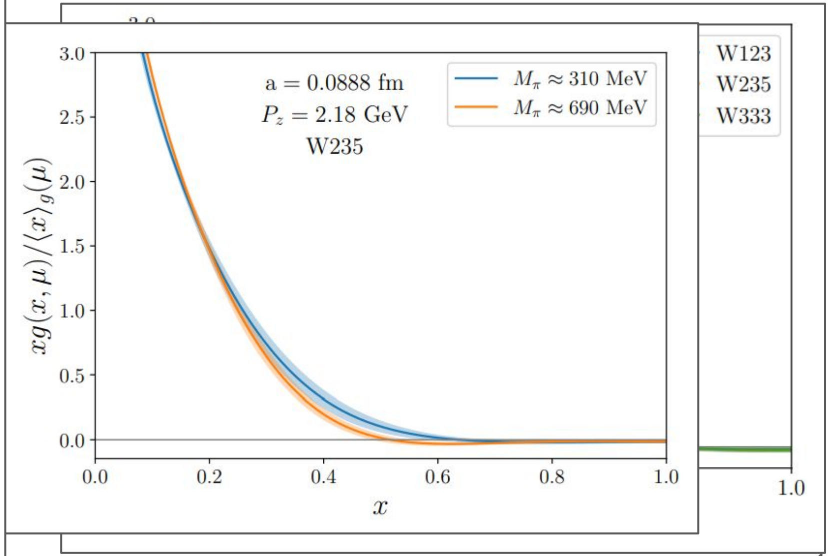

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Progress Towards Gluon PDFs in the Coulomb Gauge

Coulomb Gauge PDFs

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- A main issue with PDFs from LaMET it constraining the large-*z* signal, which decays exponentially because of the linear divergence in the Wilson line, which ensures gauge invariance (GI) of the matrix elements

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Bonus: the renormalization is no longer z-dependent, making renormalization much easier, and allowing x-dependent "quasi-PDF matrix elements" to be extracted directly from lattice 3pt correlators. **Grebe, *et al*. arXiv:2606.16877**

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- Matching to relevant lattice renormalization schemes is straightforward

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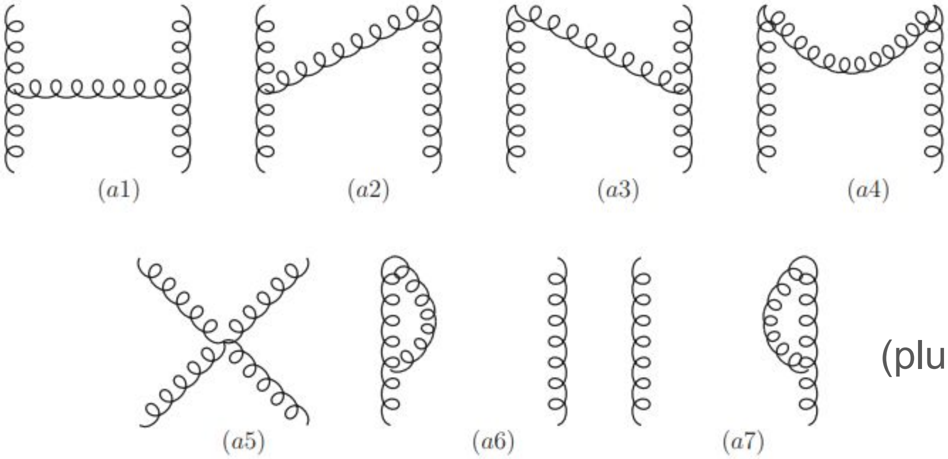

(plus self energy terms)

Wang, *et al*. PRD 100:074509 (2019)

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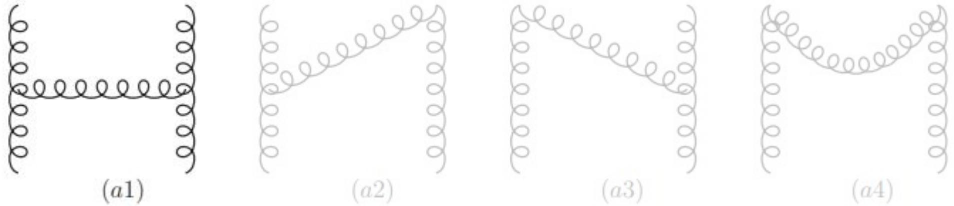

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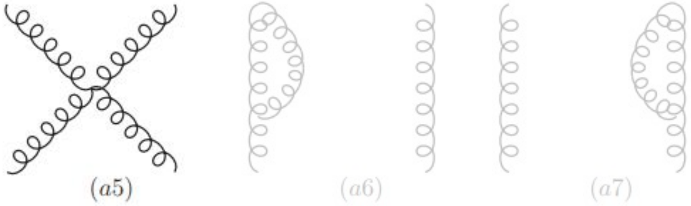

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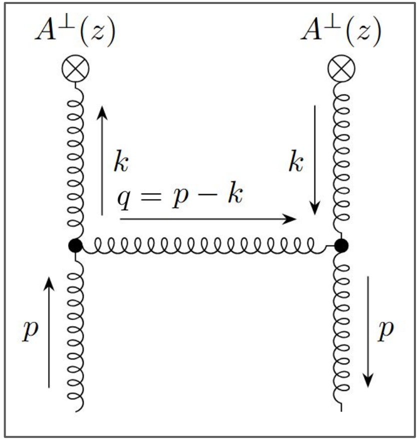

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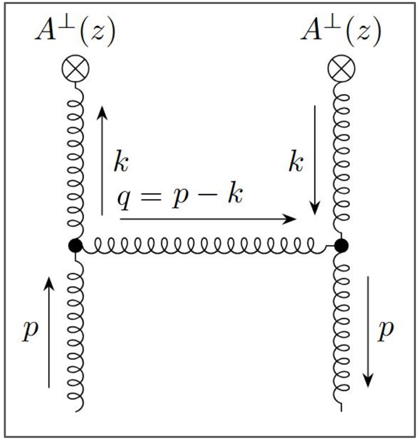

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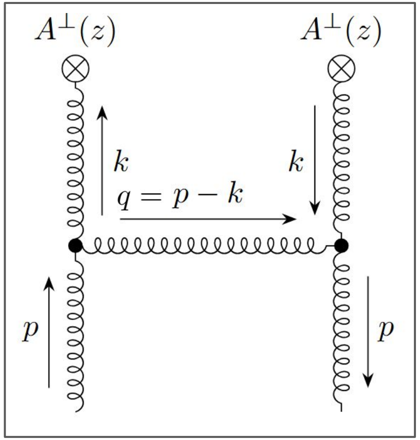

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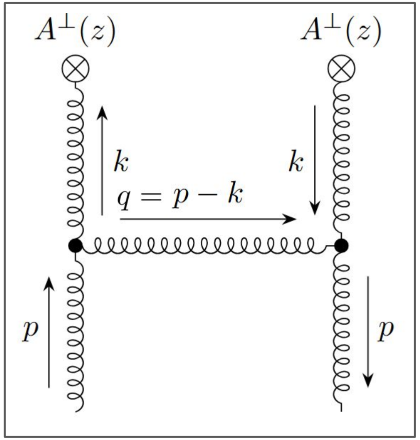

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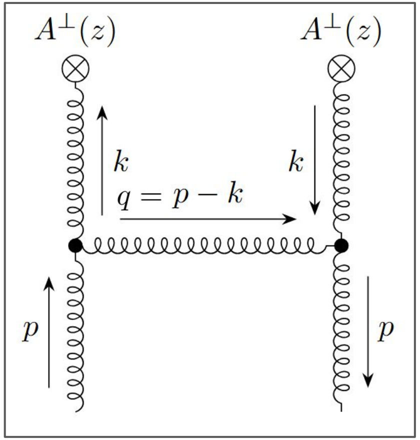

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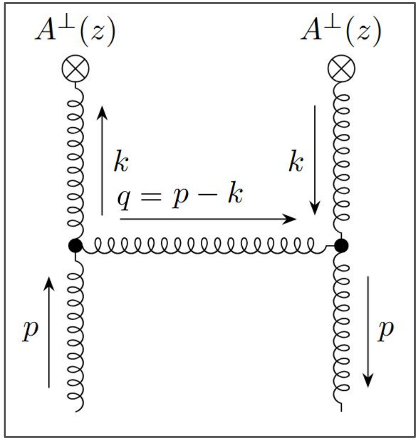

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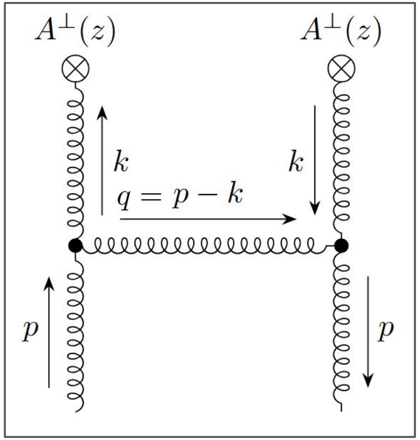

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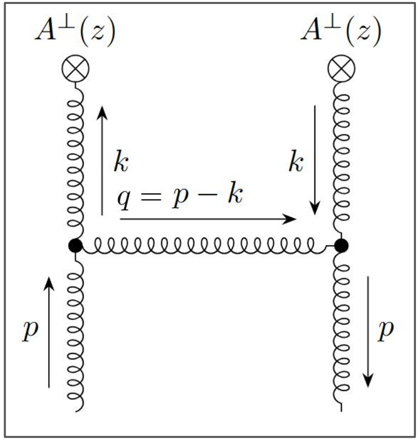

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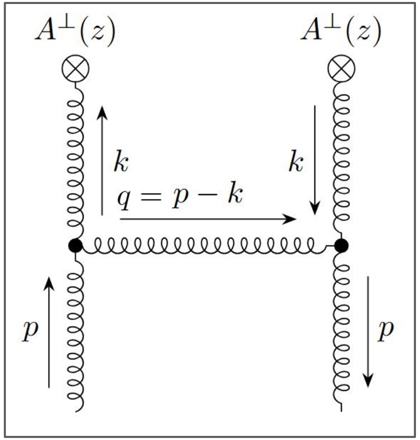

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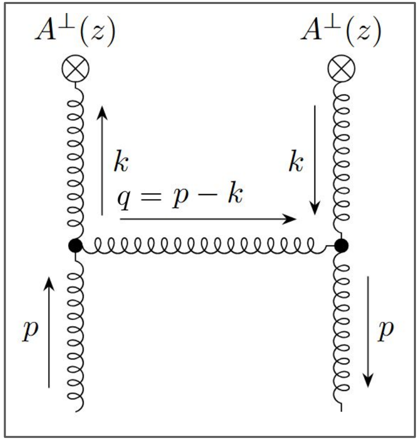

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$$\tilde{g}^{(1)}(x,p_z) \propto \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \sum_i \frac{A_i(x)\vec{k}_\perp^{2n_i}}{(\vec{q}^2)^{a_i}(\vec{k}^2)^{b_i}(q^2)^{c_i}(k^2)^{d_i}}$$

D = 4-2ϵ

- In general, we recover hypergeometric functions with parameters depending on ϵ, which hide singularities at x=1:

$${}_2F_1(a, b-\epsilon; c; \frac{2x-1}{x^2})$$ with $$a, b, c \in \mathbb{Z}/2$$

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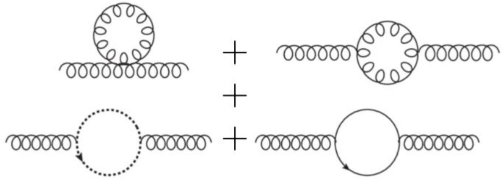

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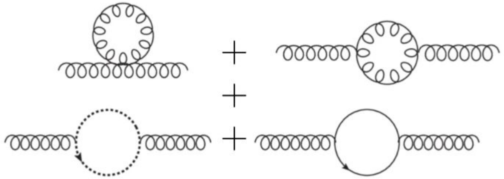

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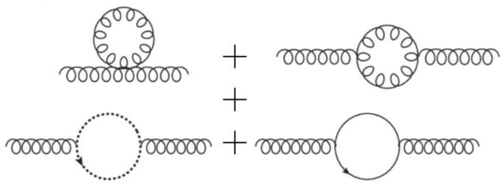

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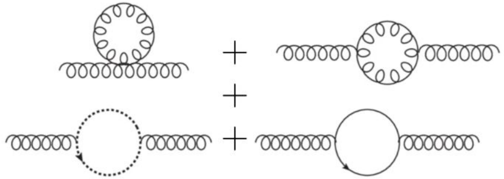

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- Using an old calculation with some ambiguity we recover the IR divergence up to a log(2) term…

$$-\frac{P_{gg}(x)}{\epsilon_{\text{IR}}} = -\frac{2C_A}{\epsilon_{\text{IR}}}\left(\frac{x}{(1-x)_+} + x(1-x) + \frac{1-x}{x}\right) + \frac{\beta_0}{2}\delta(1-x)$$

$$+\,\delta(1-x)\log(2)$$

Discussion and Outlook

LaMET Higher-Twist Effects

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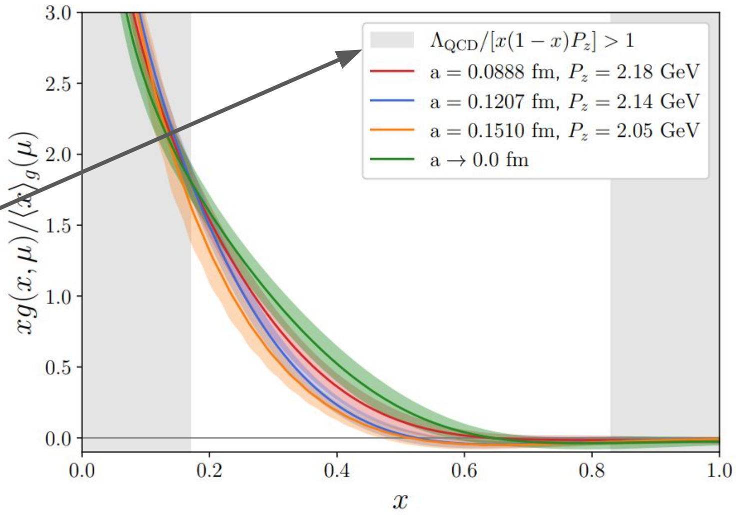
NieMiera, *et al.* arXiv 2510.17758 (2025)



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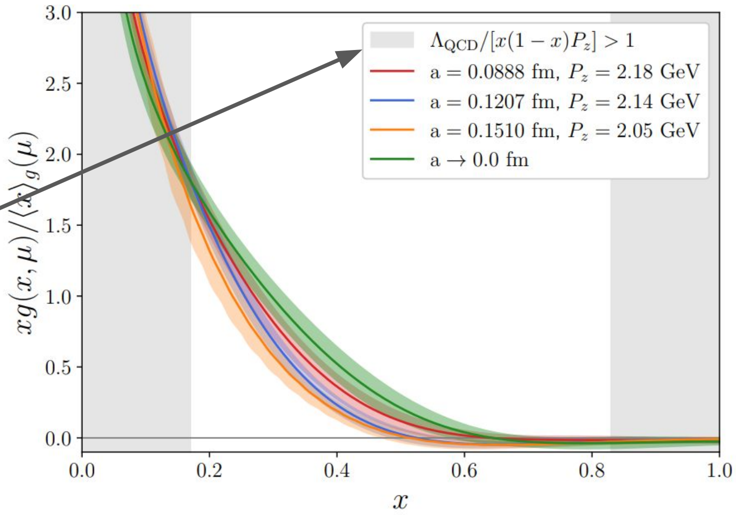

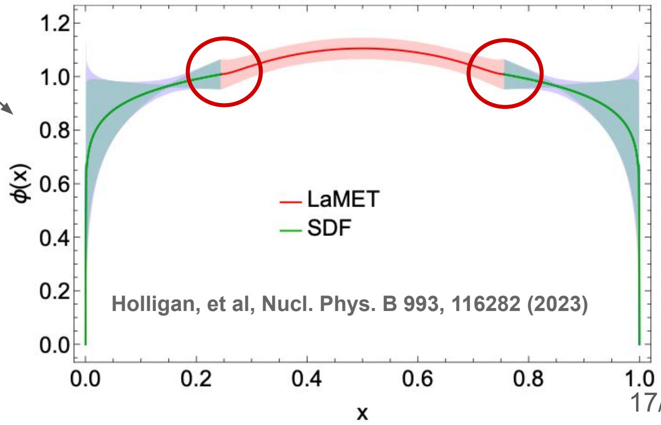

Holligan, et al, Nucl. Phys. B 993, 116282 (2023)

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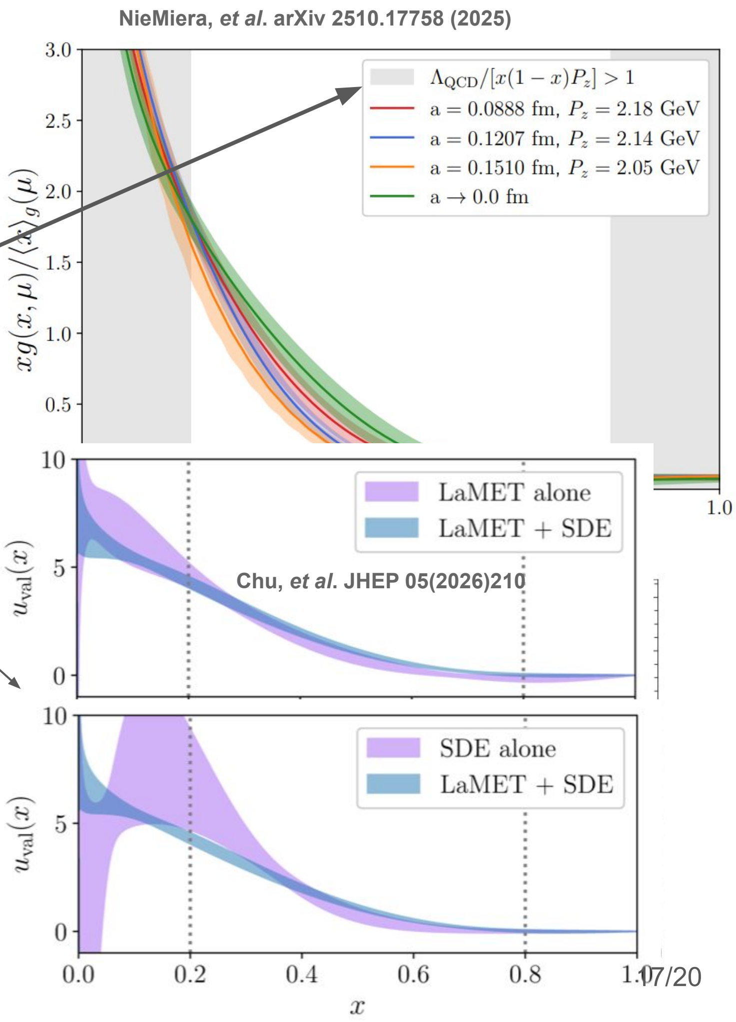

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- Other early attempts at using additional lattice data to constrain the LaMET PDF are black and white or ignore systematics
- These higher-twist systematics are not well understood, but there has to be a better way to probe them; otherwise, it's hard to say what we should really take away from LaMET PDFs

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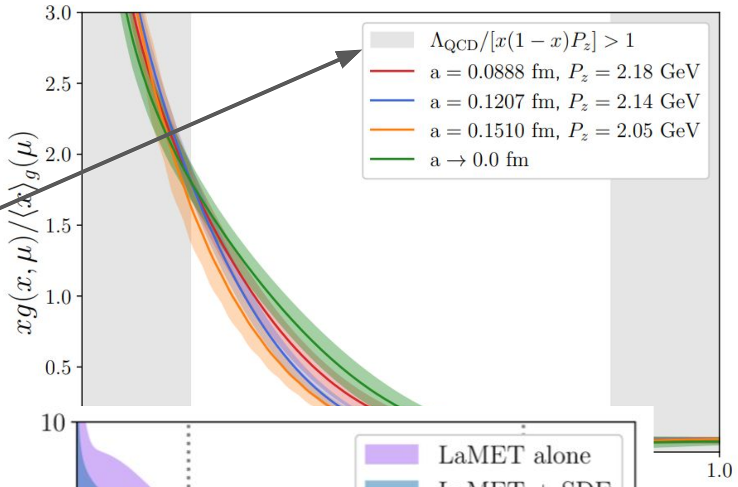

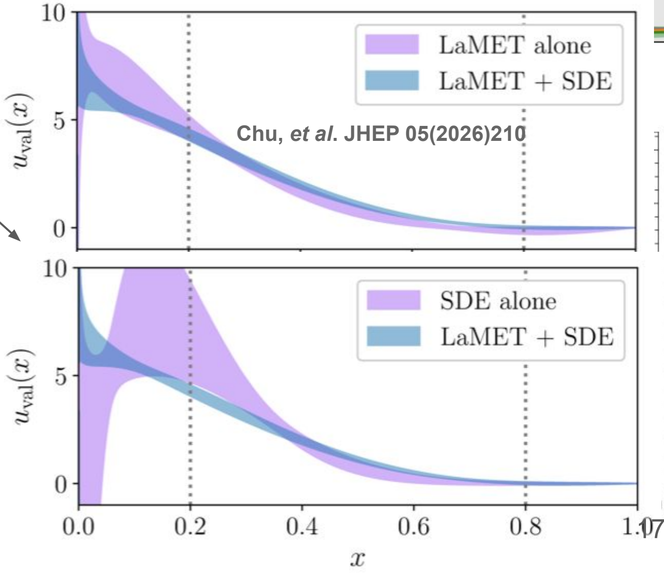

Chu, *et al.* JHEP 05(2026)210

Towards Ground Truth: LaMET + Pseudo-PDF (+ Experiment)

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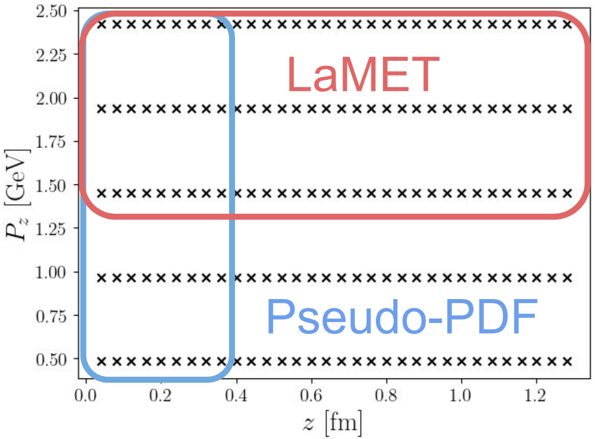

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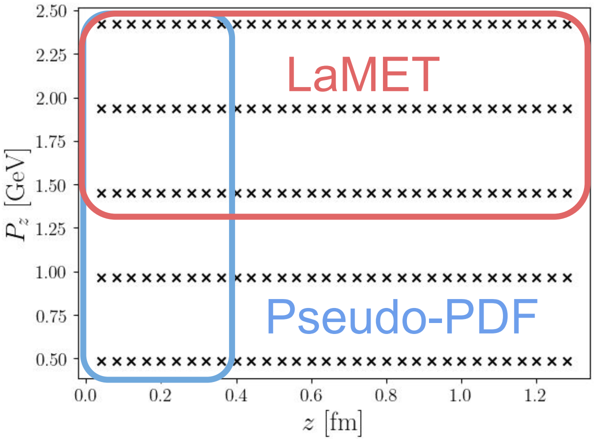
- A joint analysis using state of the art statistical and machine learning techniques, with proper handling of systematics should be able to extract the ground truth PDF



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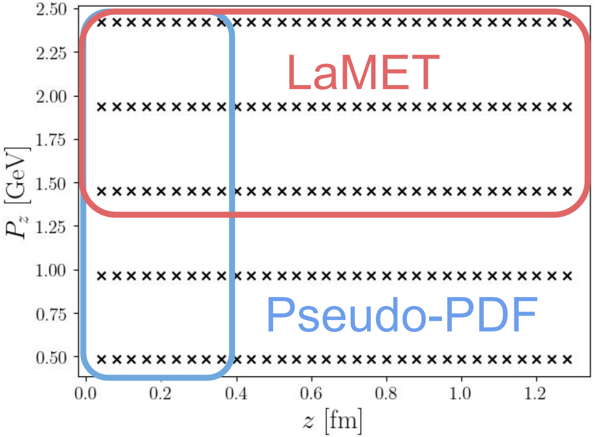
- A joint analysis using state of the art statistical and machine learning techniques, with proper handling of systematics should be able to extract the ground truth PDF
- Bonus: Since pseudo-PDF data can be handled similarly to experimental data, this framework should easily extend to include experimental data and other lattice systematics



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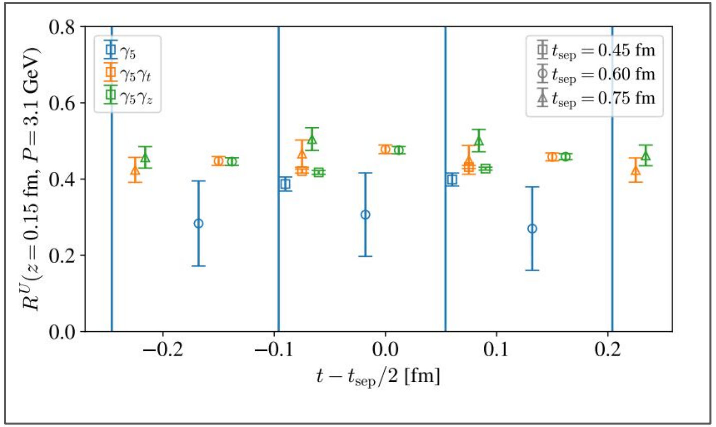

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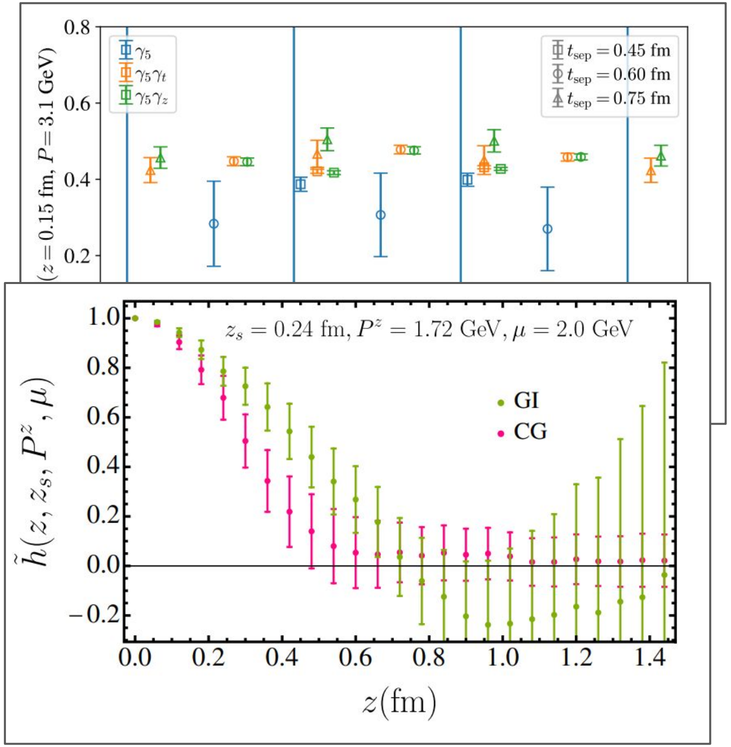

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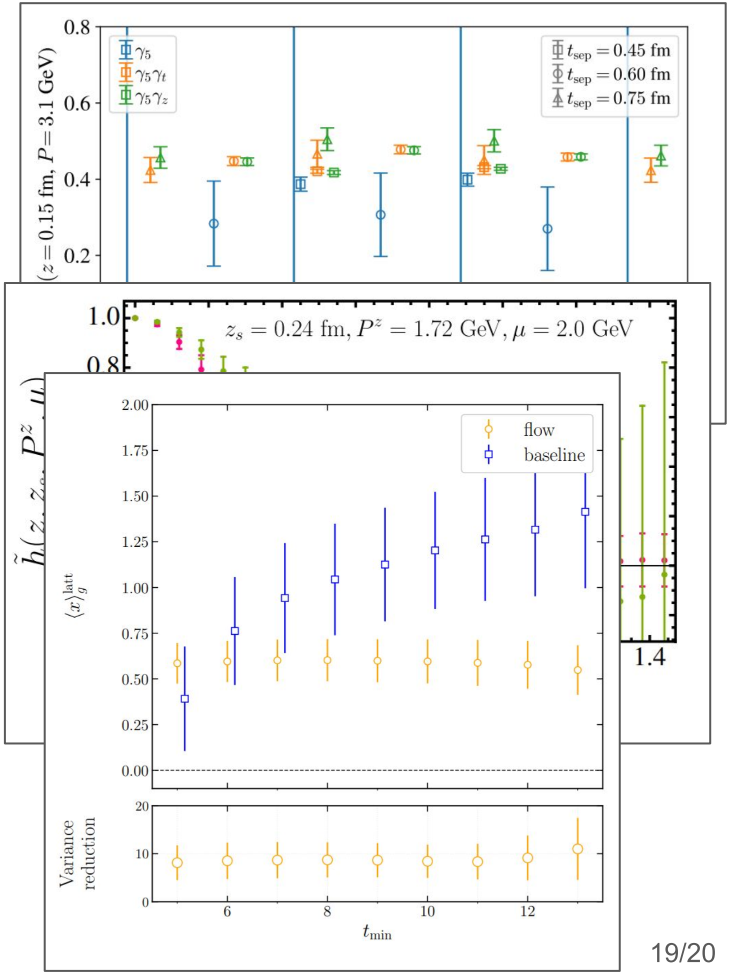
  Abbott, *et al*. arXiv:2603.02984



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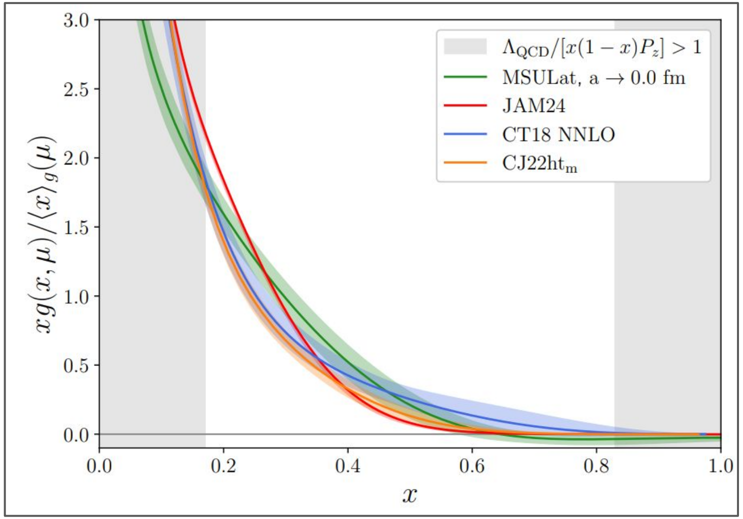
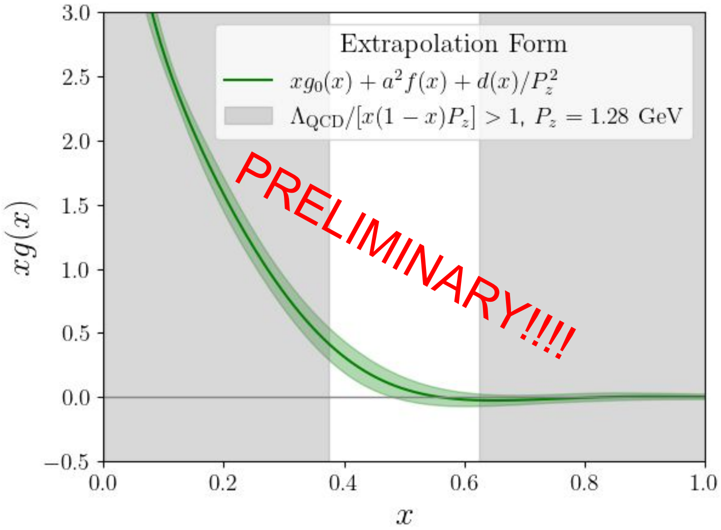
- Severals methods to improve the signal at higher momenta, larger distances, and smaller pion masses have been proposed recently that should really push our ability to improve both statistics and systematics
- The gluon PDF will likely remain behind the non-singlet quark PDFs in terms of signal, but we don’t need the same level of precision as quarks to contribute to global fits, meaning that the gluon provides a unique opportunity

Backup

Infinite Momentum Extrapolation

- Infinite momentum extrapolations have been suggested to alleviate the bad momentum convergence on our gluon PDFs. We tried this and the results look good, but mind that the infinite momentum extrapolation is only valid within the region where the smallest momentum PDF is valid (extrapolating non-convergent series is not safe!)





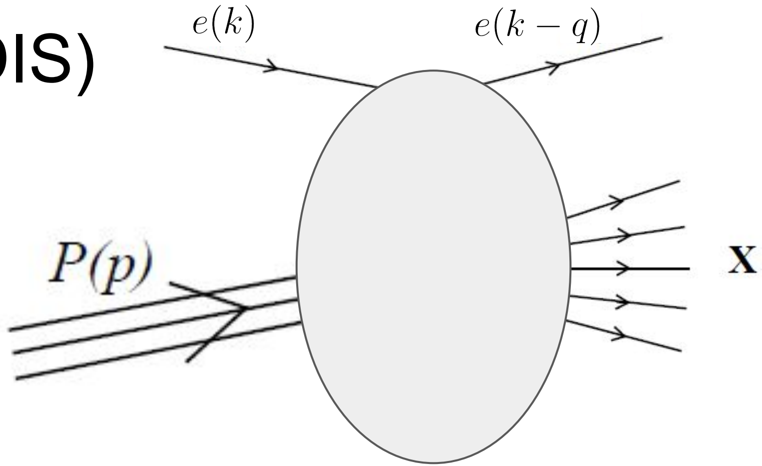
Systematics to Resolve in the Gluon Case

Systematic	LaMET Status	Pseudo-PDF Status
Lattice Spacing	Early continuum extrapolations	Early continuum extrapolations
Pion Mass	Effect smaller than statistics	Early extrapolations
Gauge Link smearing	Small effect	Small effect
Renormalization	Full control via self renormalization	Good control at short distances via ratio renormalization
Power corrections	Strong momentum power corrections	Parameterized corrections, stick to small distances
Model dependence	Asymptotic form with strong motivations	Fixed functional form, some ML methods
Perturbative Scale dependence	Not addressed	Not addressed

Example: Deep Inelastic Scattering (DIS)

- Want to calculate the cross section for

$$e(k) + P(p) \rightarrow e(k-q) + X(p+q)$$



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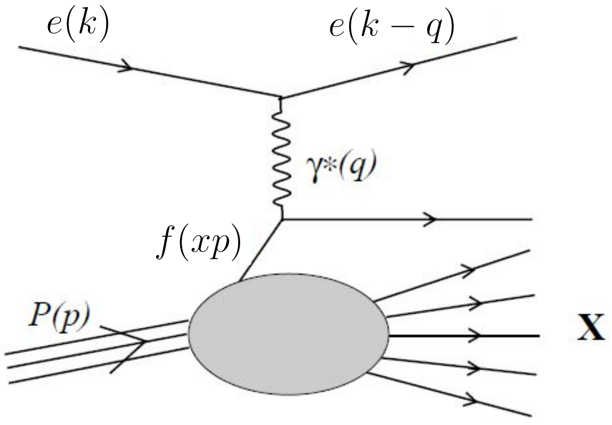

$$\hat{\sigma}_{ef}(xp, q)$$

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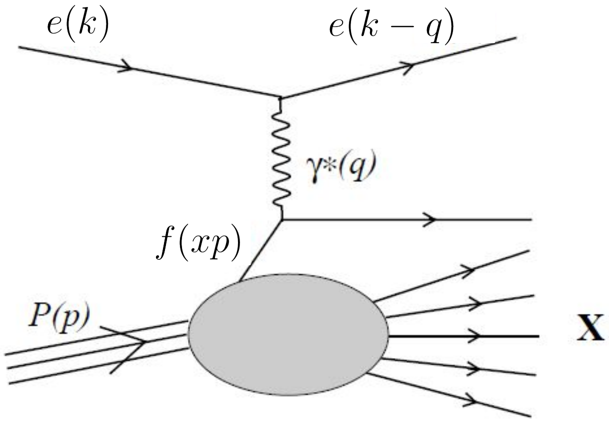

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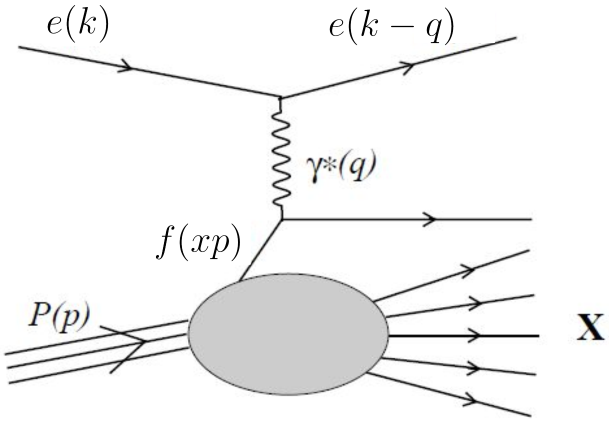

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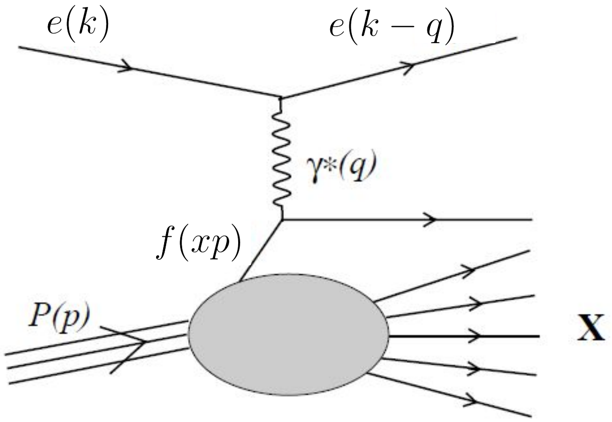

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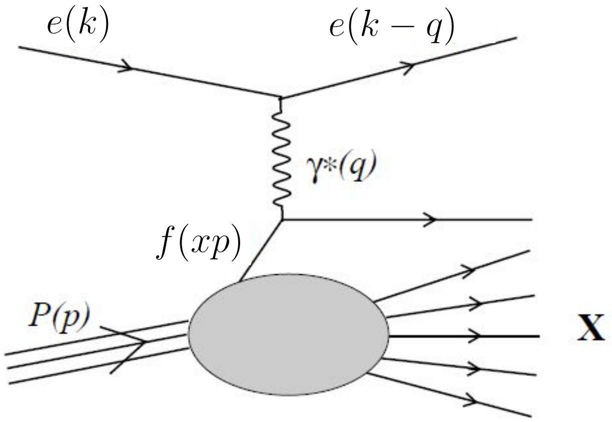

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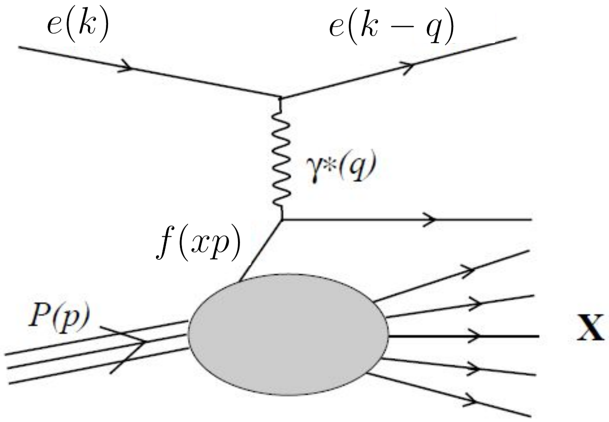

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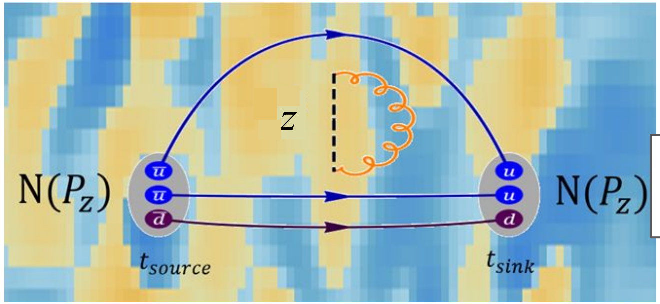
- One can use the same PDF for different cross sections
  - Structure functions inherit the same factorization forms as well
- The PDF is defined on the light cone, but the cross section is completely physical

MSULat 3pt Methodology

- We can then insert operators into the 2pt correlators to construct three-point (3pt) correlators, which allow us to extract matrix elements from the data

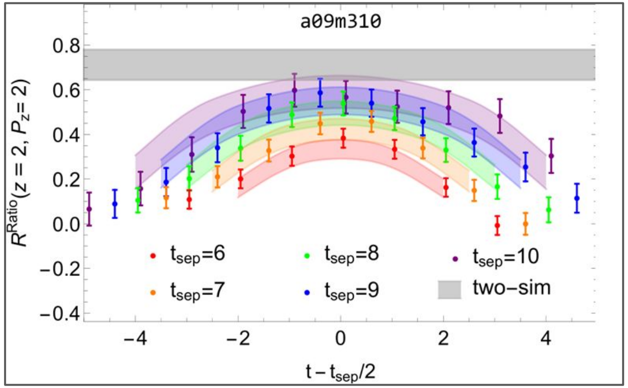


$$C_N^{2\text{pt}}(P_z,t) = |A_{N,0}|^2 e^{-E_{N,0}t} + |A_{N,1}|^2 e^{-E_{N,1}t} + \ldots$$

$$C_N^{3\text{pt}}(z,P_z,t,t_{\text{sep}}) = |A_{N,0}|^2 \langle 0|O_g|0\rangle e^{-E_{N,0}t_{\text{sep}}} + |A_{N,0}||A_{N,1}|\langle 0|O_g|1\rangle e^{-E_{N,1}(t_{\text{sep}}-t)} e^{-E_{N,0}t}$$
$$+ |A_{N,0}||A_{N,1}|\langle 1|O_g|0\rangle e^{-E_{N,0}(t_{\text{sep}}-t)} e^{-E_{N,1}t} + |A_{N,1}|^2\langle 1|O_g|1\rangle e^{-E_{N,1}t_{\text{sep}}} + \ldots$$

- MSULat mostly uses two state fits, which we often demonstrate with ratio plots

$$R(t,t_{\text{sep}},P_z,z) = \frac{C_{3\text{pt}}(t,t_{\text{sep}},P_z,z)}{C_{2\text{pt}}(t_{\text{sep}},P_z)} \xrightarrow[t_{\text{sep}}\to\infty]{} \langle P_z|O(z)|P_z\rangle$$



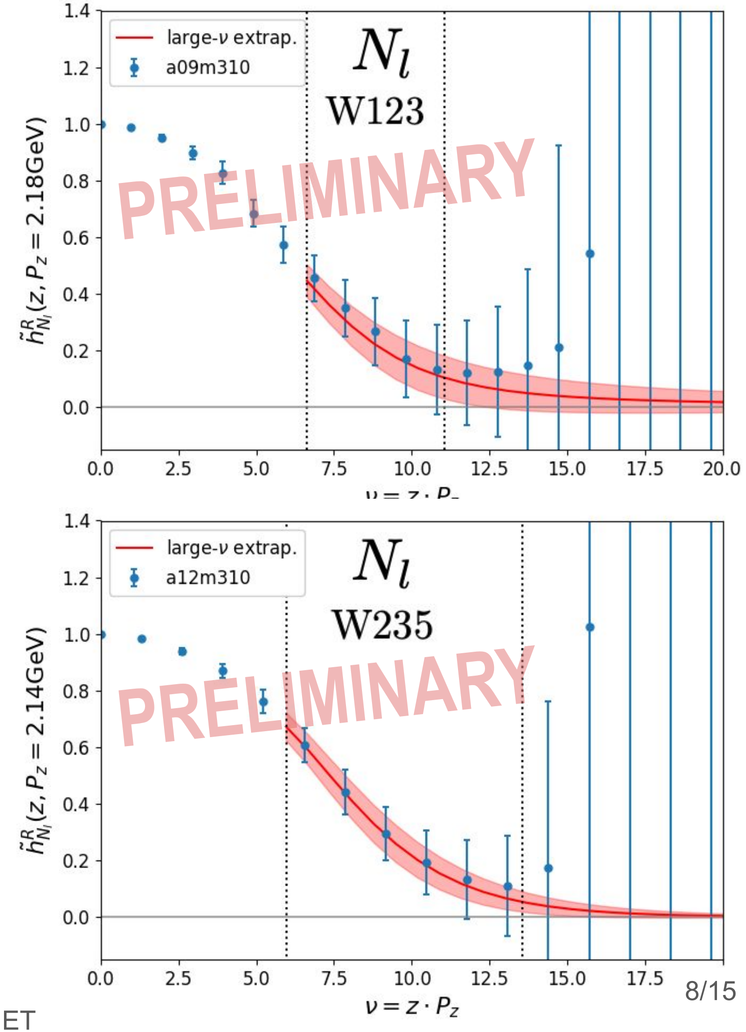
Large-ν Extrapolation

We apply a physically motivated extrapolation form to fit the renormalized matrix elements at intermediate - large distances with $z \gtrsim 0.6$ fm ,

$$\tilde{h}^R(z, P_z) \approx A \frac{e^{-m\nu}}{|\nu|^d}$$

Gao, et al. Phys. Rev. Lett. 128, 142003 (2022)

We're able to obtain reasonable extrapolations across all ensembles and pion masses for each smearing scheme (that's ~50 extrapolations!); however, future work should explore possible systematics more carefully.



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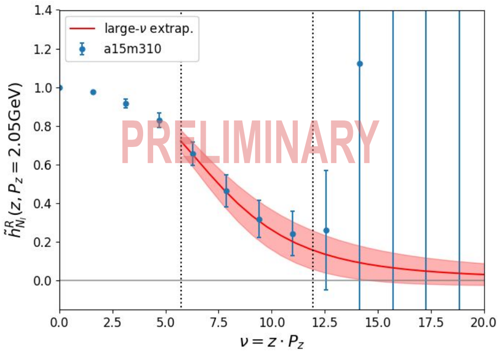

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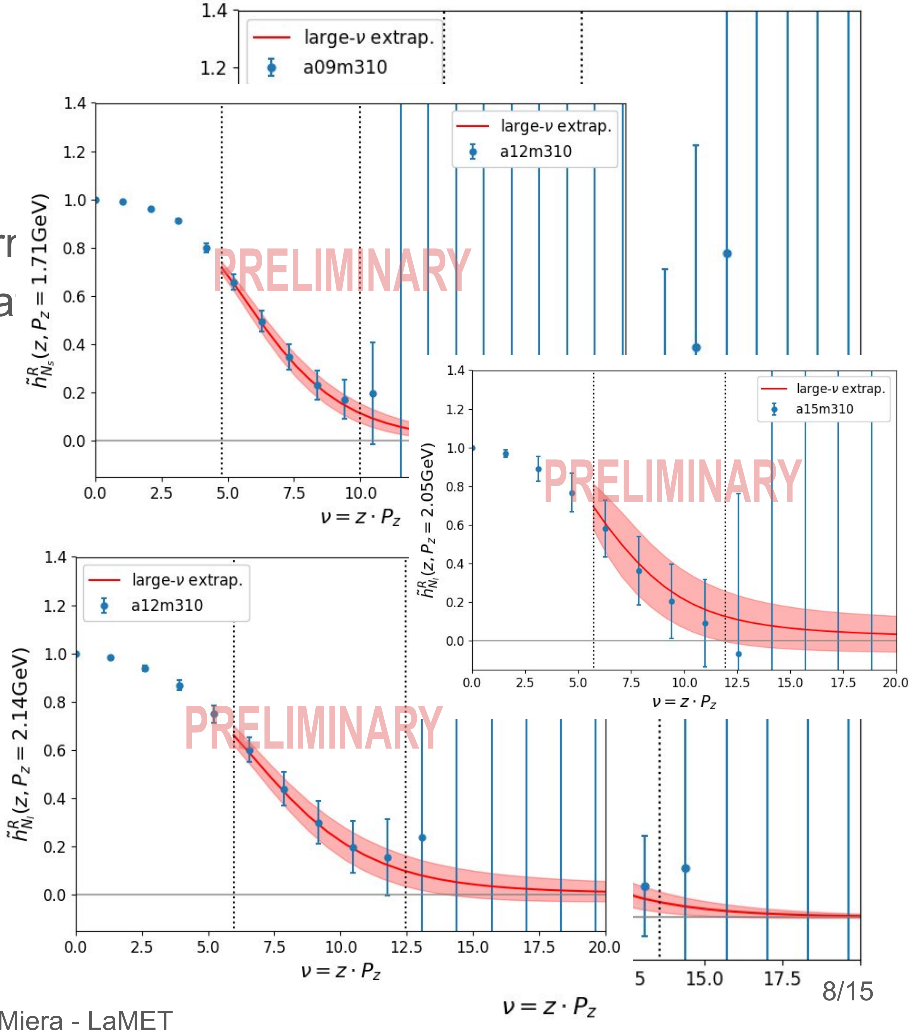

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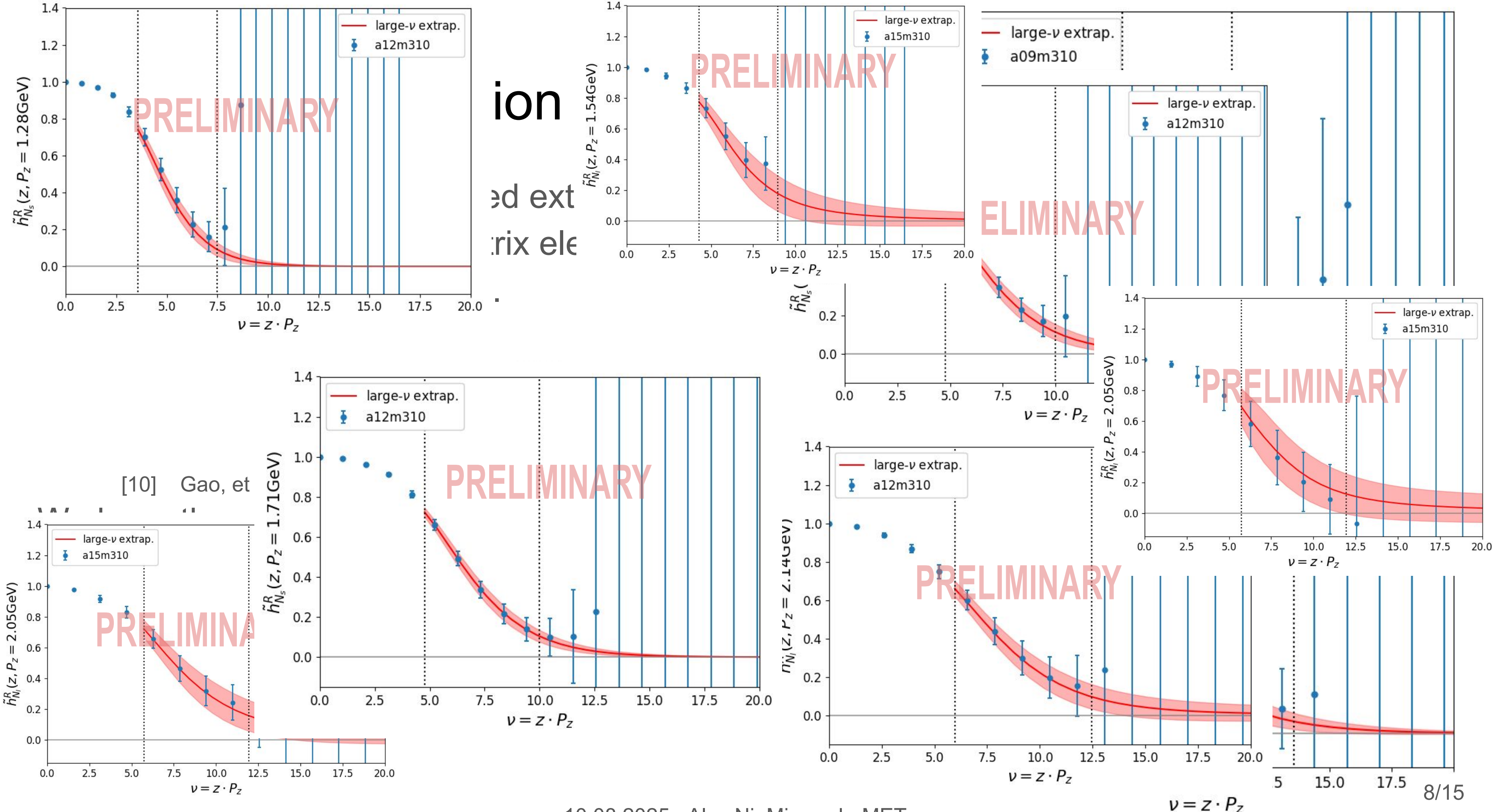
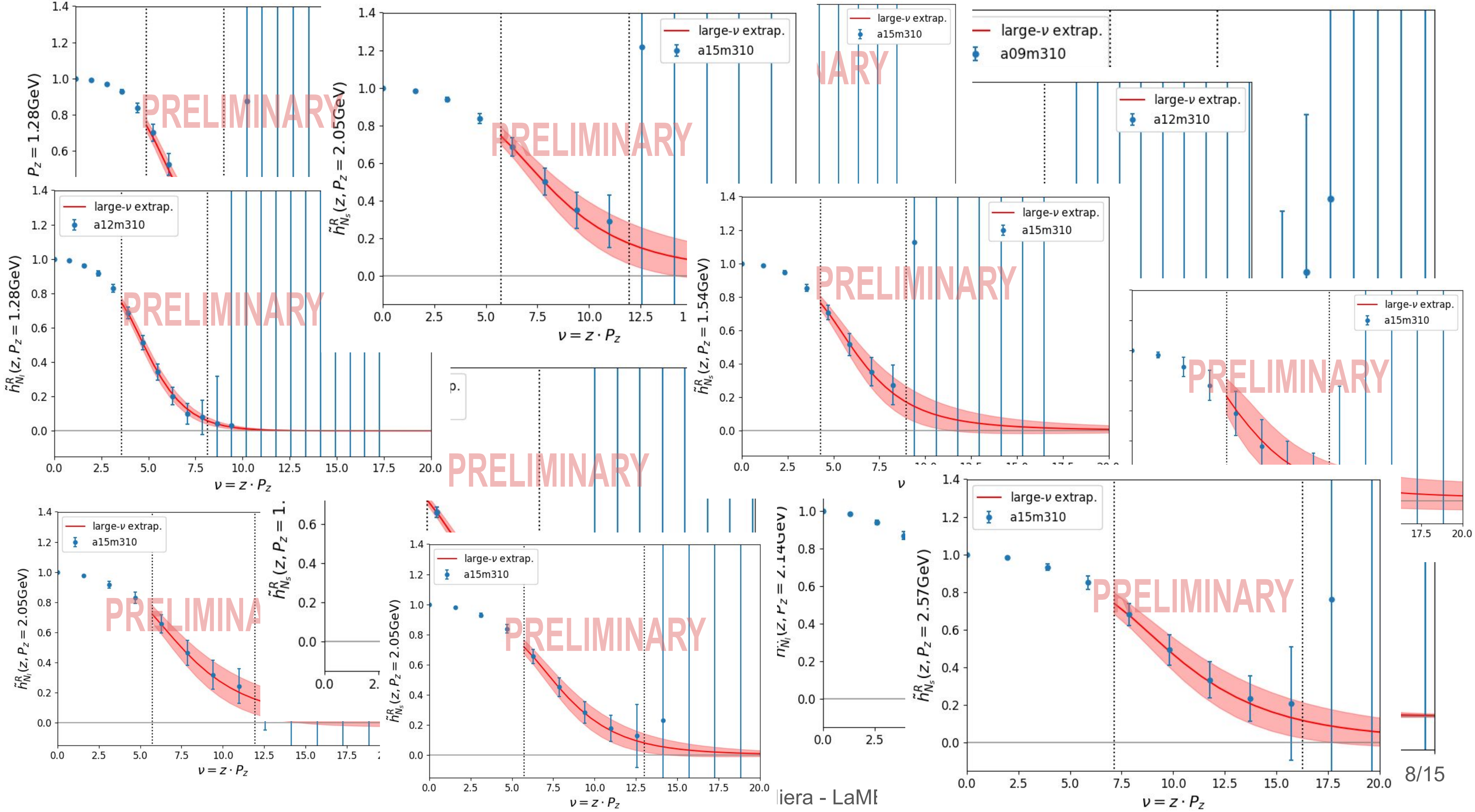




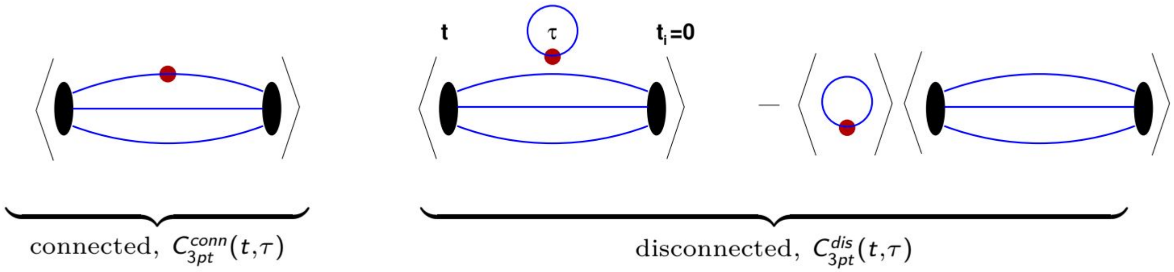
PRELIMINARY
large-ν extrap.
a15m310
a12m310
a09m310
$\nu = z \cdot P_z$
$\tilde{h}^R_{N_l}(z, P_z = 1.28\text{GeV})$
$\tilde{h}^R_{N_s}(z, P_z = 2.05\text{GeV})$
$\tilde{h}^R_{N_s}(z, P_z = 1.54\text{GeV})$
$\tilde{h}^R_{N_l}(z, P_z = 2.05\text{GeV})$
$\tilde{h}^R_{N_s}(z, P_z = 2.57\text{GeV})$

Disconnected Diagrams

- To calculate the Euclidean correlators on the lattice, we compute three-point (3pt) correlation functions:

$$C_N^{3\mathrm{pt}}(z, P_z; t, \tau) = \langle 0|\Gamma \int d^3y\, e^{-iyP_z} \chi(\vec{y}, t) \mathcal{O}_q(z, \tau) \chi(\vec{0}, 0)|0\rangle$$

- The wick contraction gives different diagrams:



- The disconnected pieces are much noisier and require large 2pt statistics to improve signal

Gambhir (2017). Dissertations, Theses, and Masters Projects. William & Mary. Paper 1516639772.