Lattice QCD Study of Nonzero Skewness GPDs

M. Chu, Phys.Rev.D 112 (2025) 9, 094510

M. Chu, arxiv: 2607.xxxx (in preparation)

Min-Huan Chu

Adam Mickiewicz University

23/06/2026

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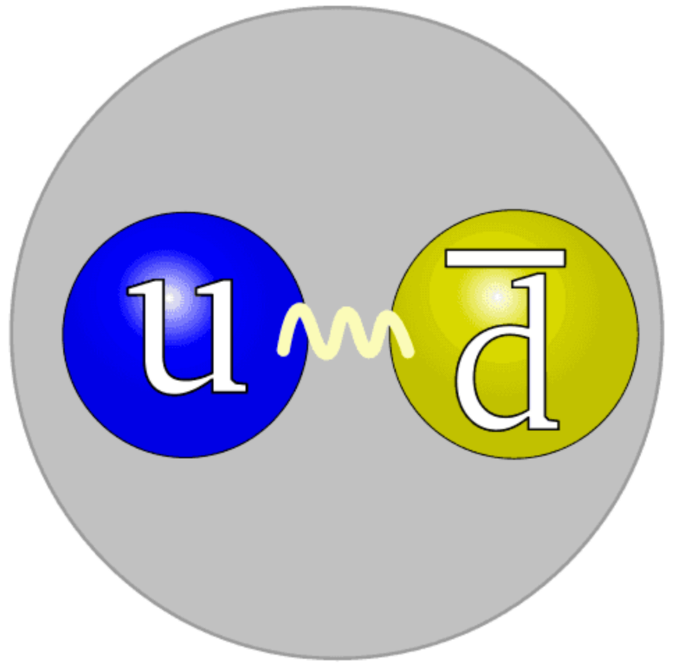
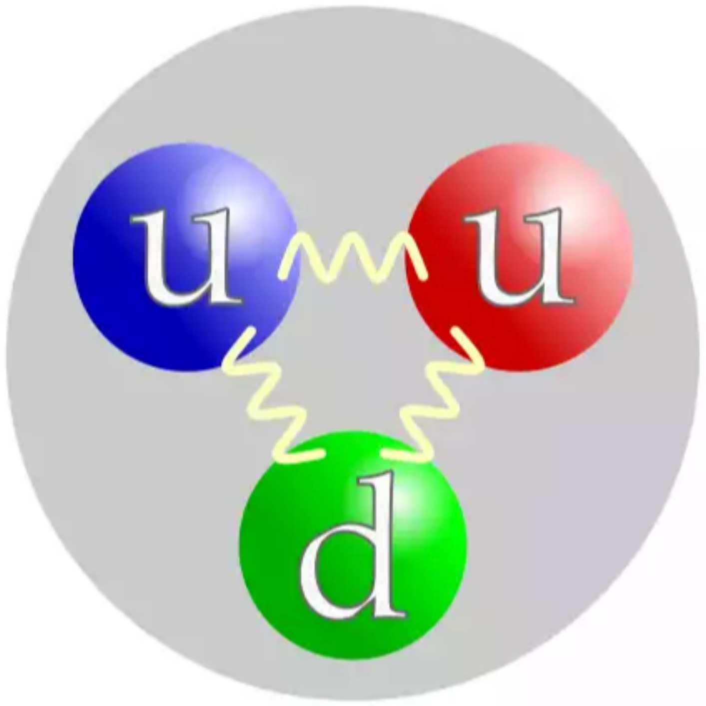
Outline

- GPDs
- LaMET approach
- SDE approach
- Summary and outlook





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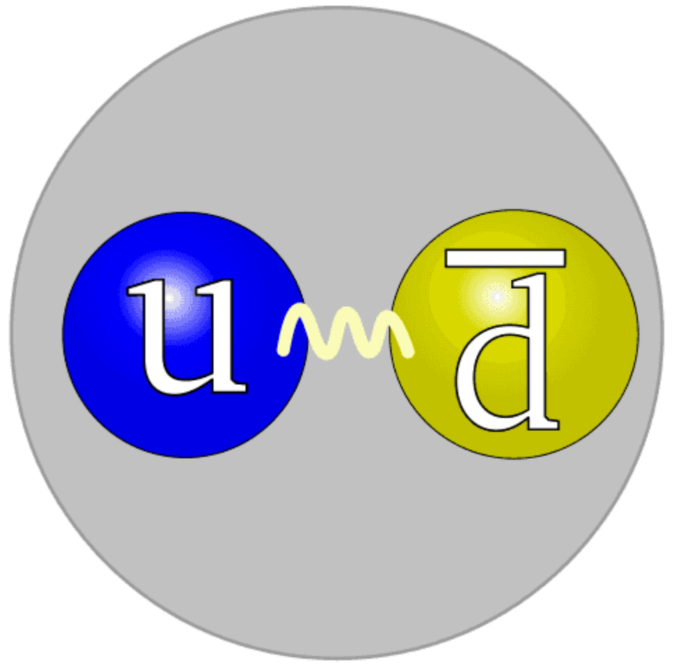
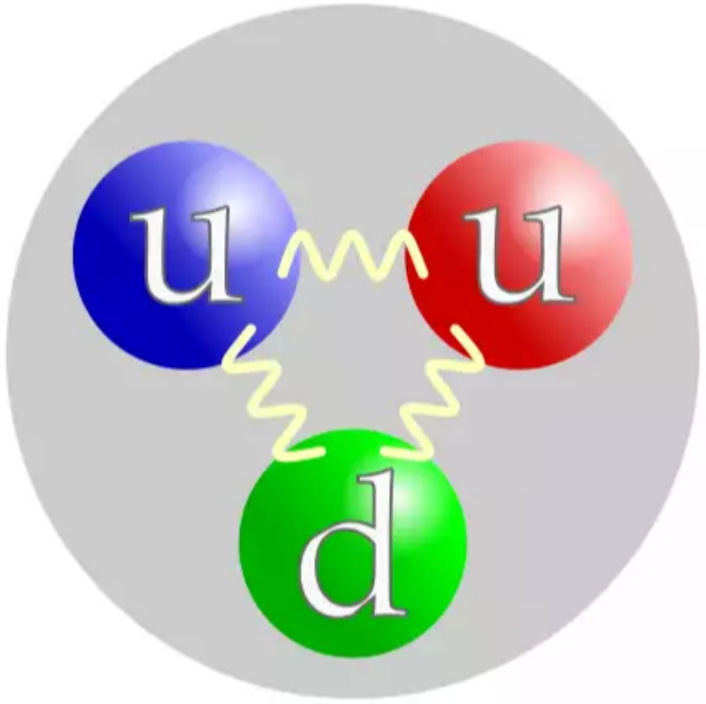
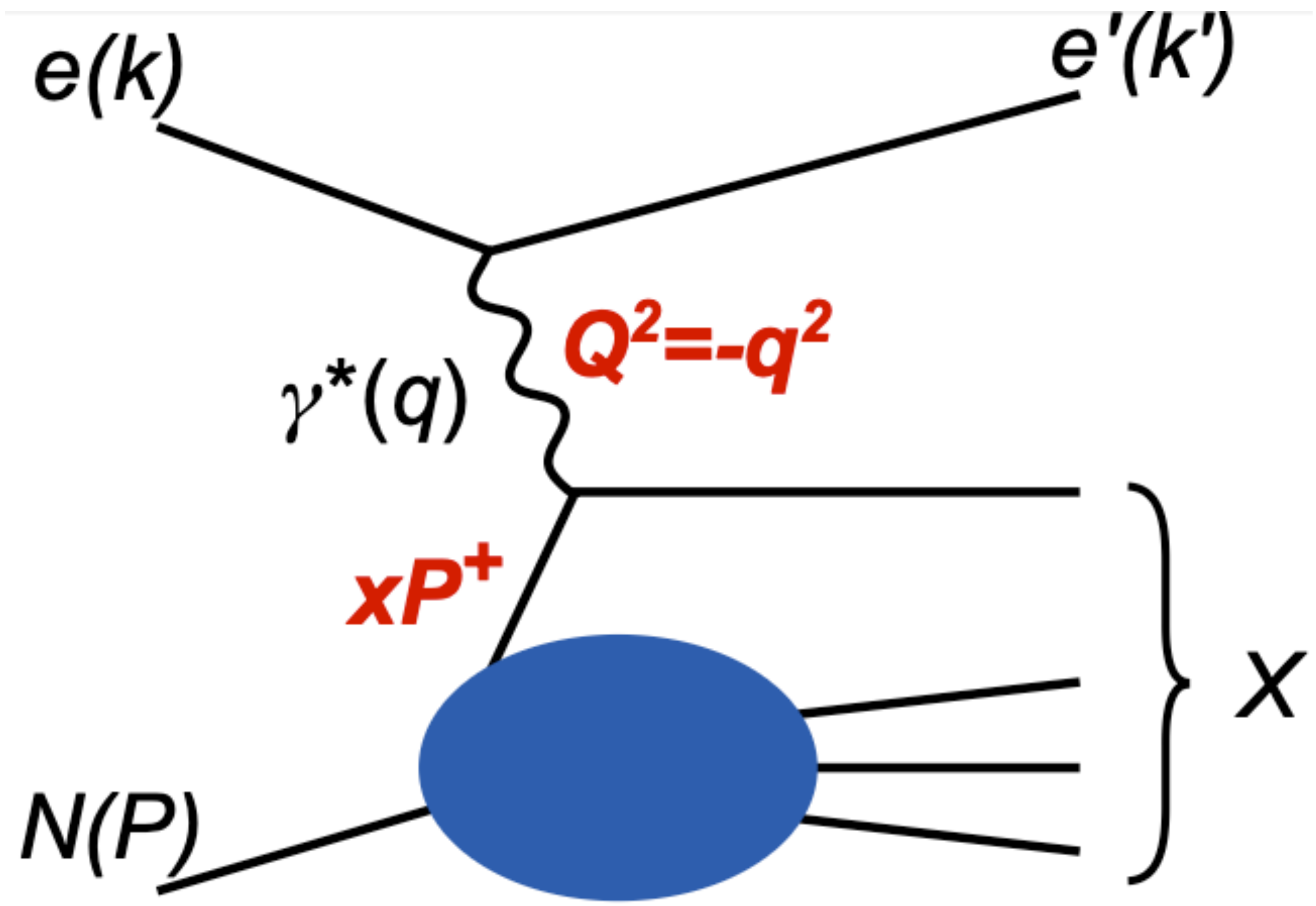




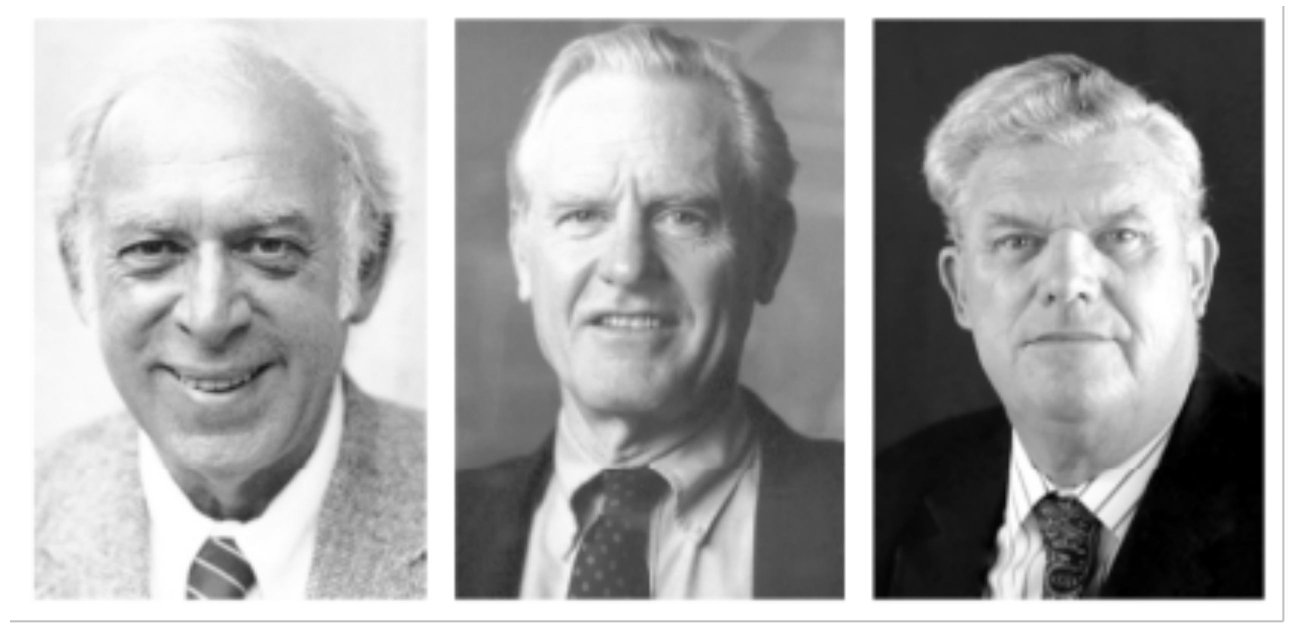
Deep Inelastic Scattering Process



hadronic part of cross section

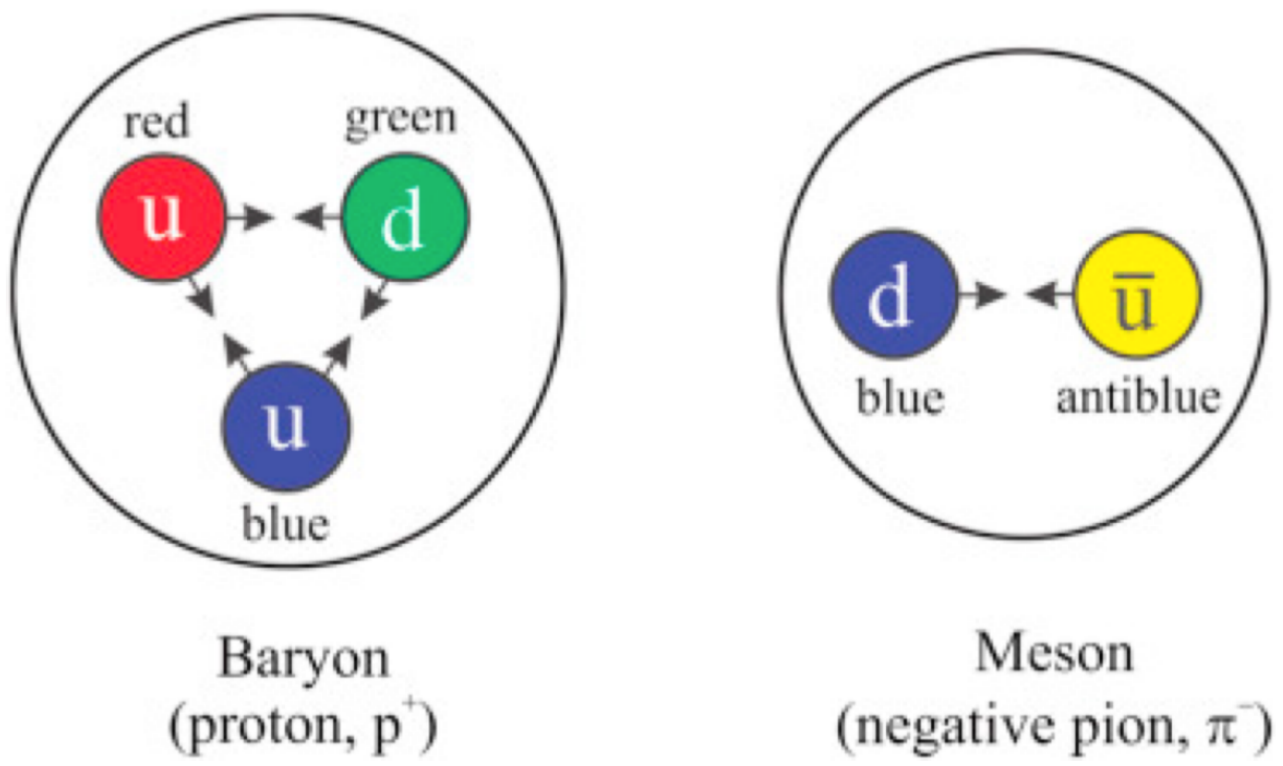
$$\frac{d\sigma}{d\Omega} \propto q(x)$$

Quarks in hadrons

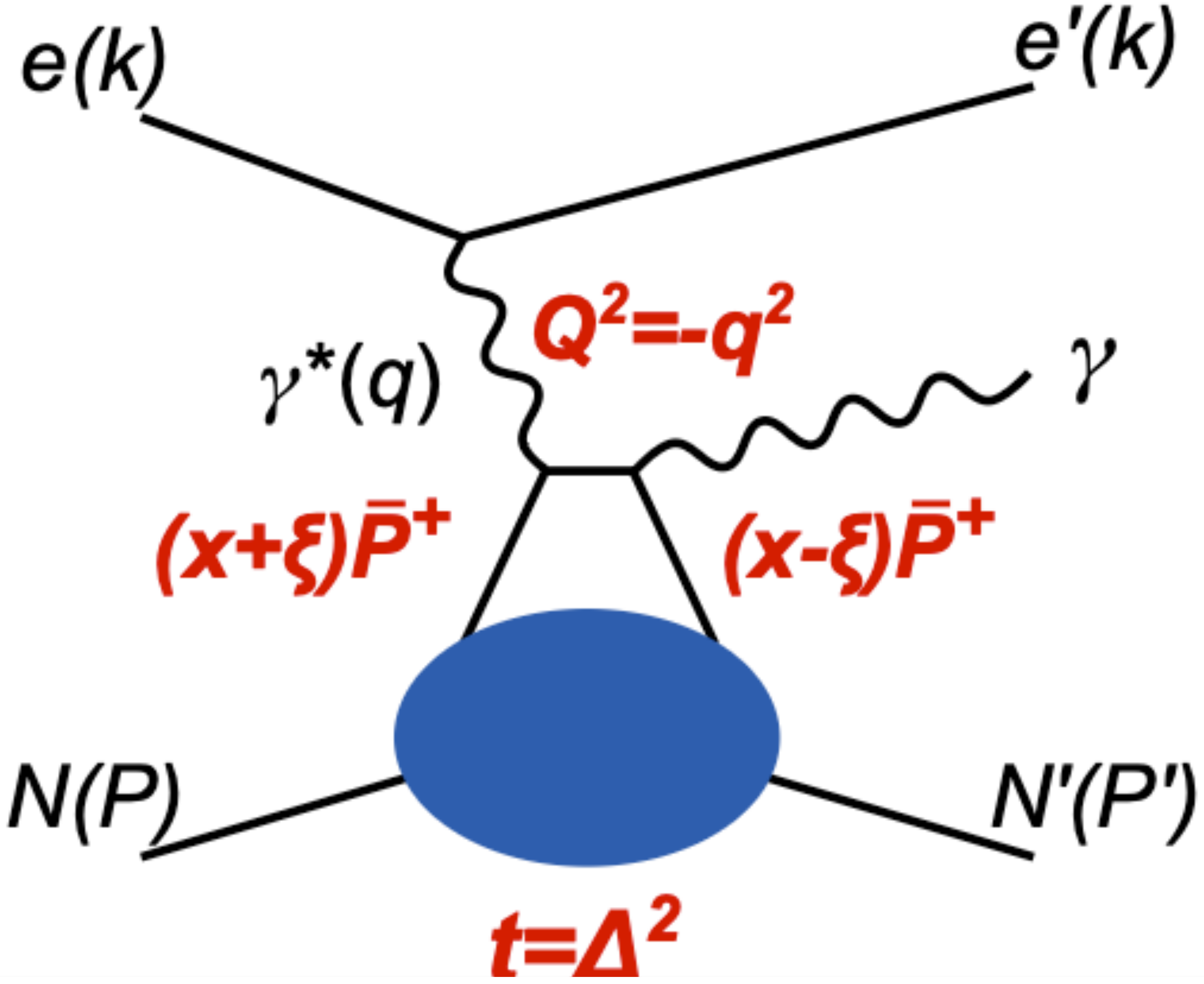


J. Friedman
H. Kendall
R. Taylor
Nobel prize in 1990

pictures



Deep Virtual Compton Scattering



angular momentum
Ji's sum rule

nucleon tomographic figures

hadronic part of cross section related to $q(x, \xi, t)$

$$q(x,\xi,t) = \bar{u}(P')\left[\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E^q(x,\xi,t)\right]u(P)$$

$H(x, \xi, t)$ $E(x, \xi, t)$

forward limit $\xi \to 0,\ t \to 0$

$q(x)$

GPDs are important inputs!

GPDs

- Experimental analysis

global fitting

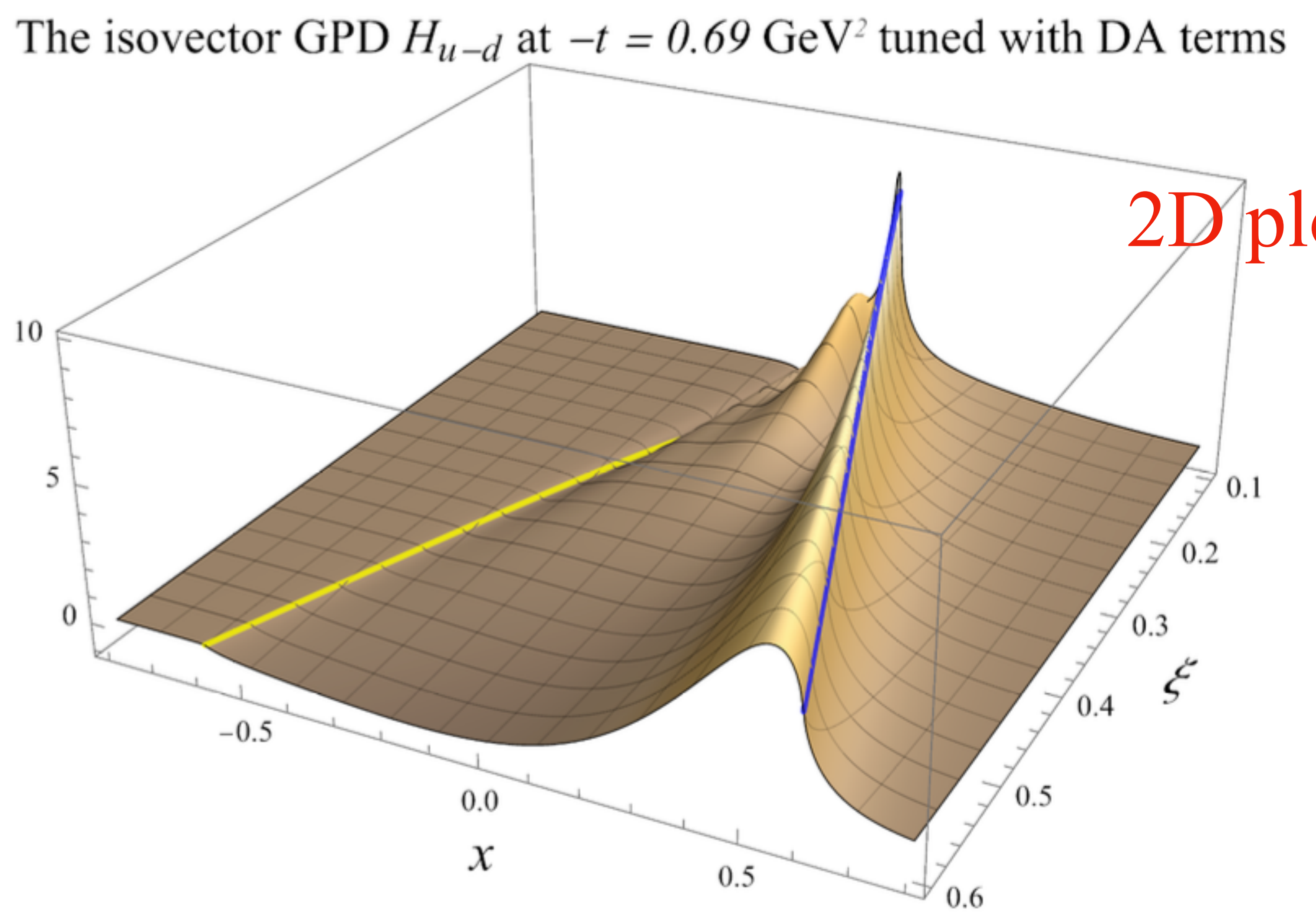
1. Kumerički et al., EPJ Web Conf. 112 (2016) 01012
2. Guo et al., JHEP 05 (2023) 150
3. Burkert et al., Nature 557 (2018) 7705, 396-399

…

- Phenomenological results

1. Moutarde et al., Eur.Phys.J.C 78 (2018) 11, 890
2. Hannaford-Gunn et al., Phys.Rev.D 110 (2024) 1, 014509
3. Dupre et al., Phys.Rev.D 95 (2017) 1, 011501

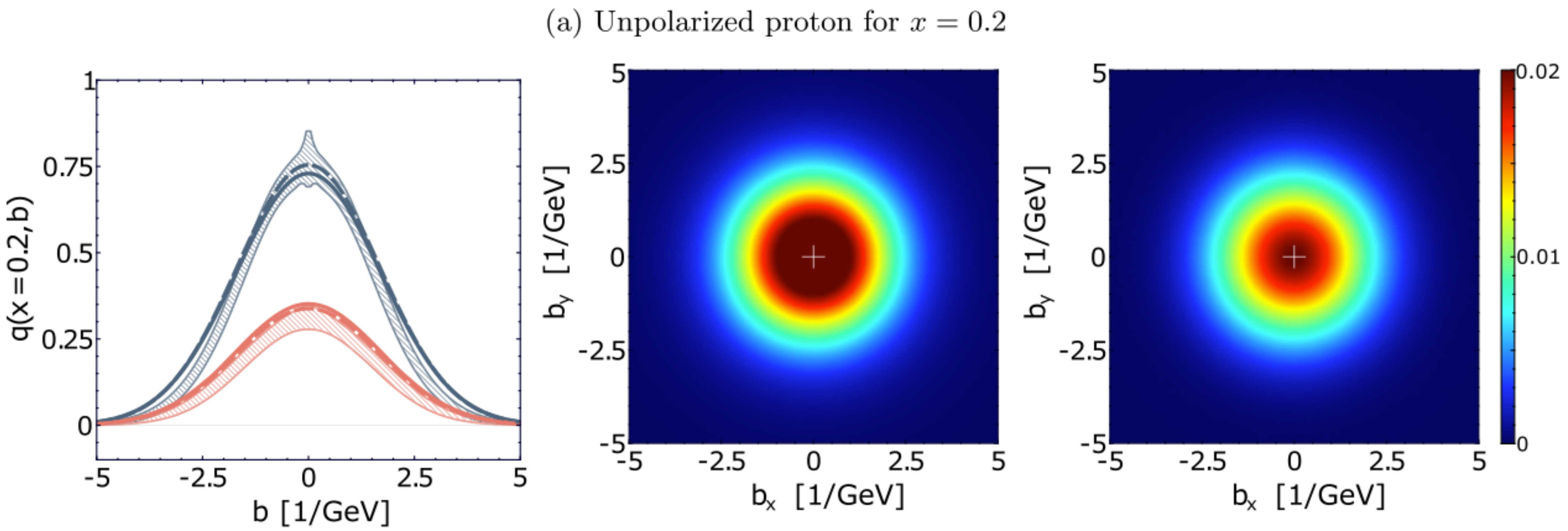
…



2D plots for $H_{u-d}(x, \xi)$

Y. Guo et al., JHEP 05 (2023) 150

tomographic figures



K. Cichy et al., Phys.Rev.D 110 (2024) 11, 114025

GPDs

two approaches in lattice determination of GPDs

matrix element

LaMET

renormalization: RI/MOM, hybrid….

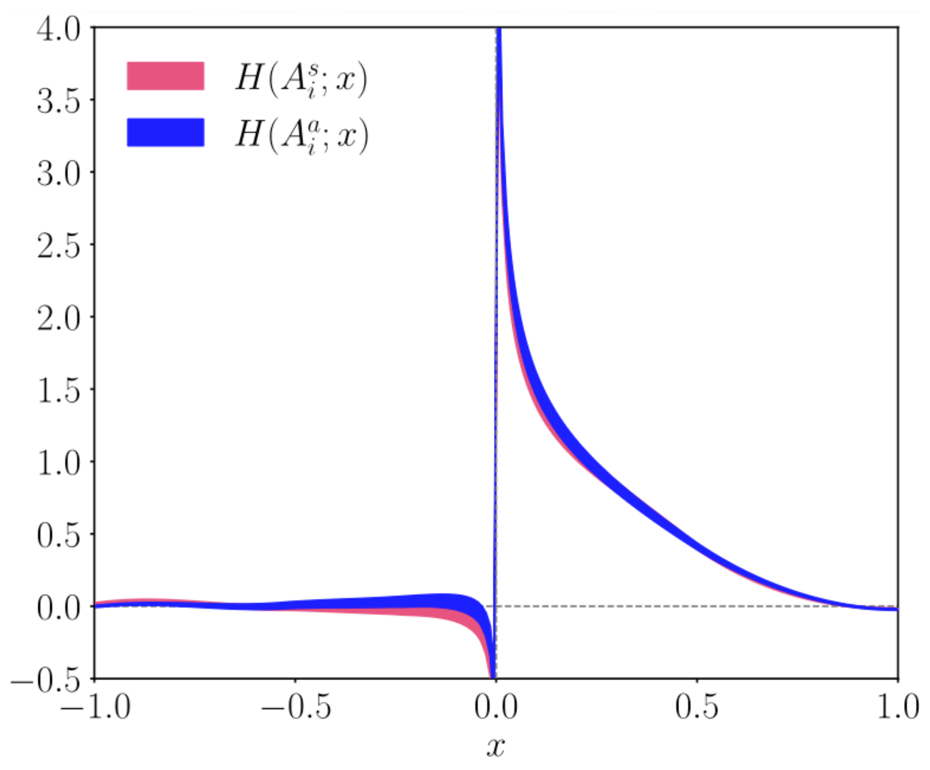
quasi-correlation

FT

quasi-distribution

matching

x dependent GPDs



SDE

renormalization: double ratio

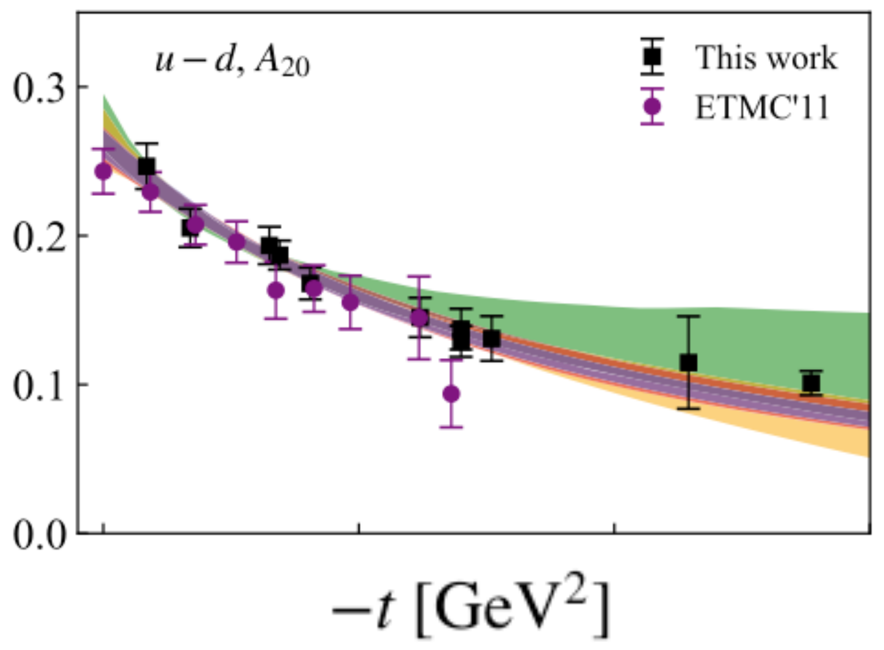
pseudo-correlation

matching

matched-correlation

fit

Melin moments for GPDs



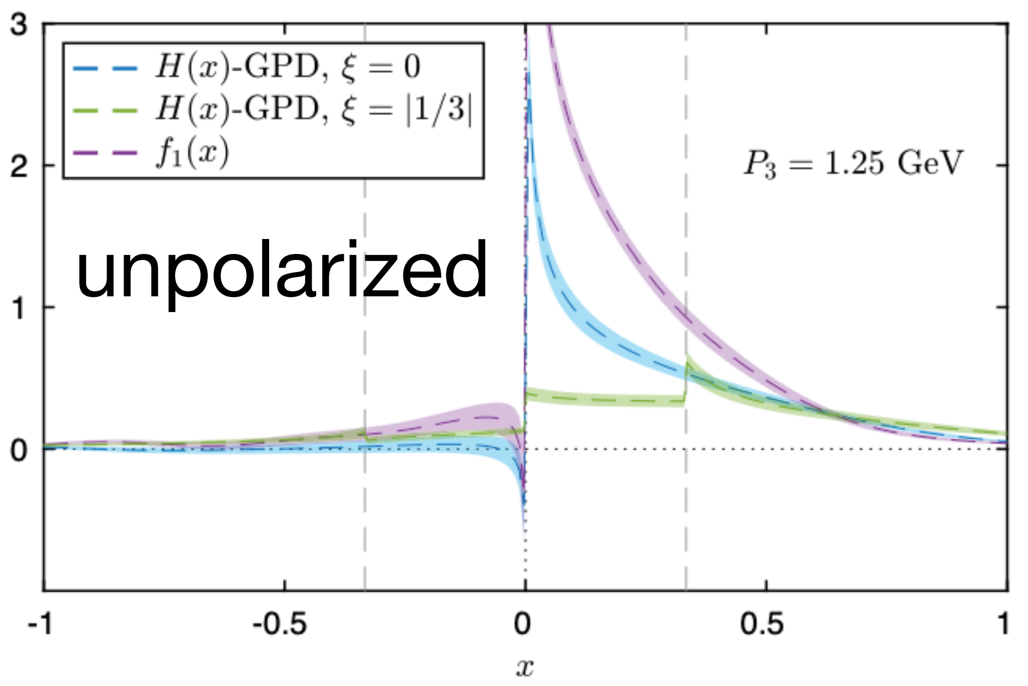
GPDs

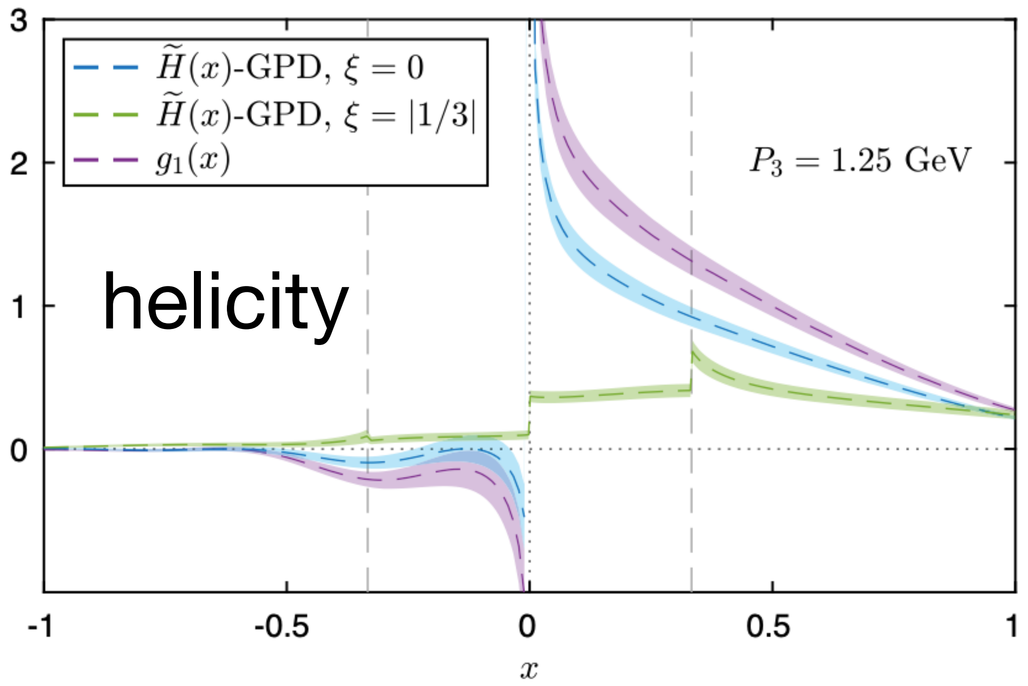
Lattice results of GPDs LaMET

- **symmetric frame**

ETMC, Phys.Rev.Lett. 125 (2020) 26, 262001 (unpolarized+helicity+one ξ)

ETMC, Phys.Rev.D 105 (2022) 3, 034501(transversity)



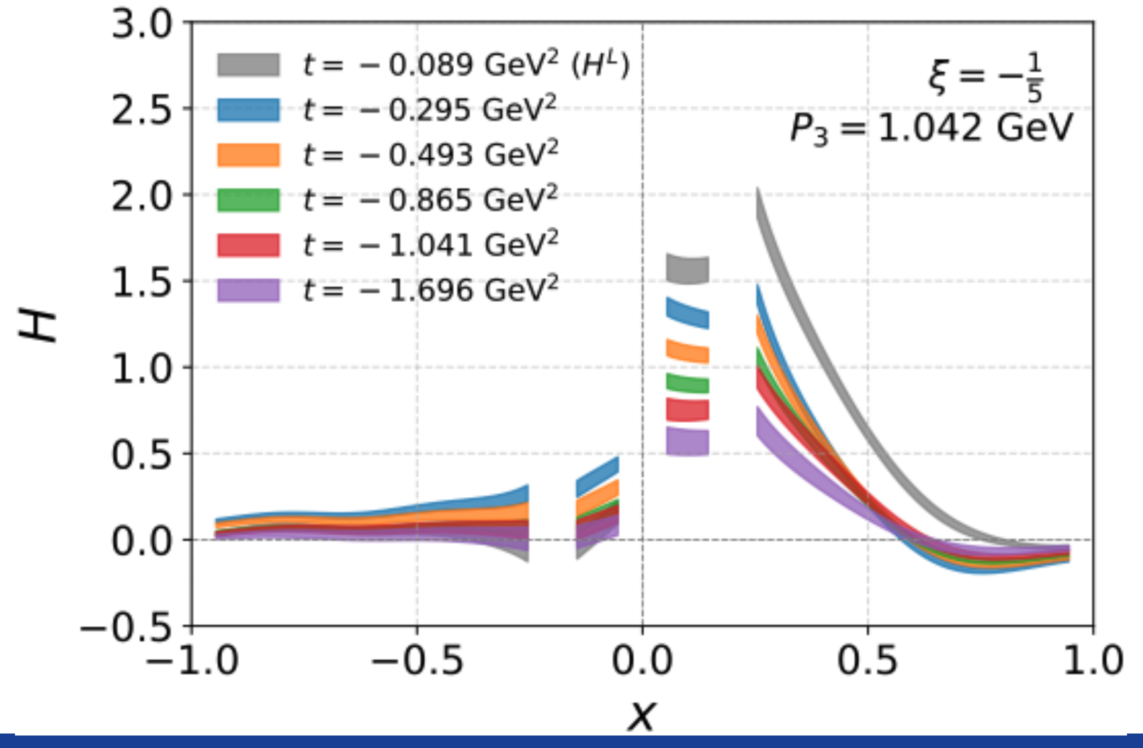


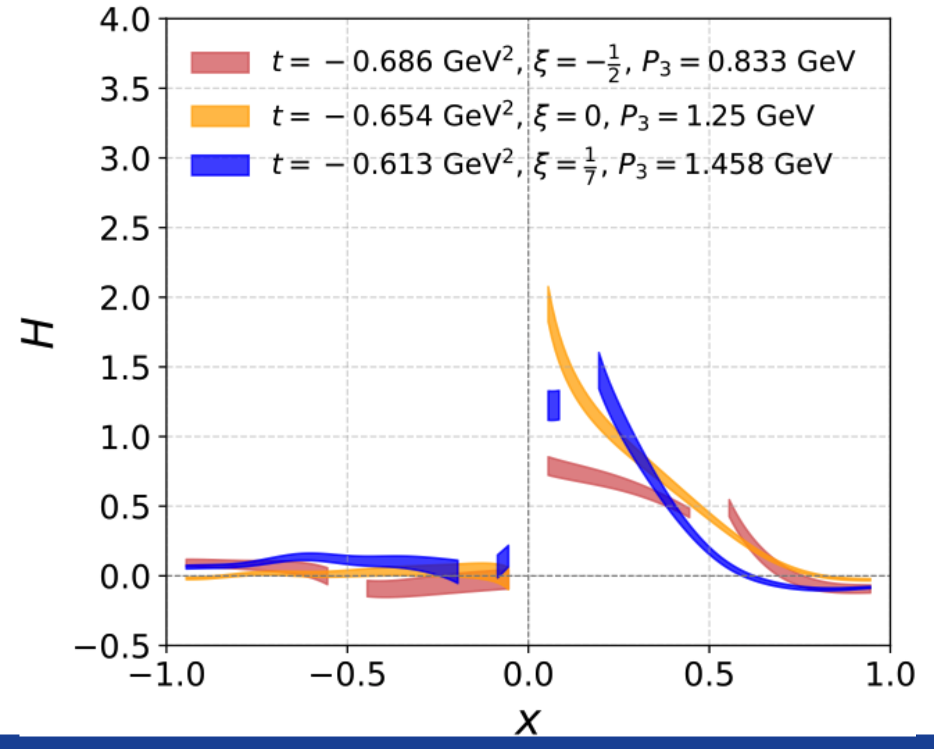
- **asymmetric frame**

ETMC, Phys.Rev.D 106 (2022) 11, 114512 (unpolarized)

ETMC, Phys.Rev.D 109 (2024) 3, 034508 (axial-vector)

ETMC, Phys.Rev.D 112 (2025) 9, 094510 (unpolarized + ξ dependent)





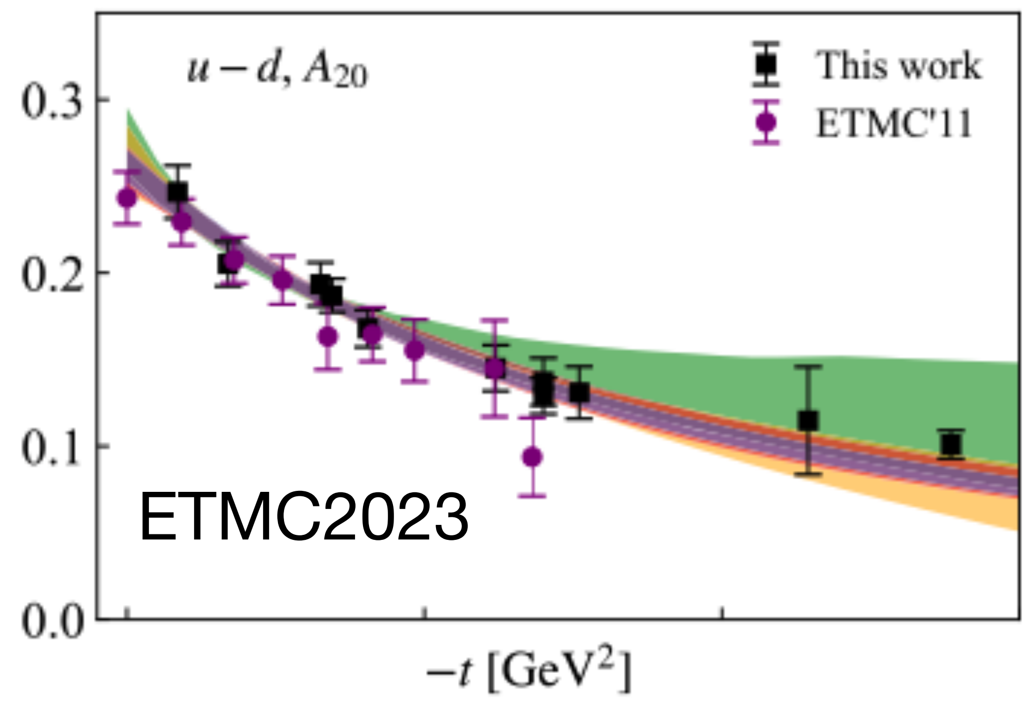
Lattice results of GPDs SDE

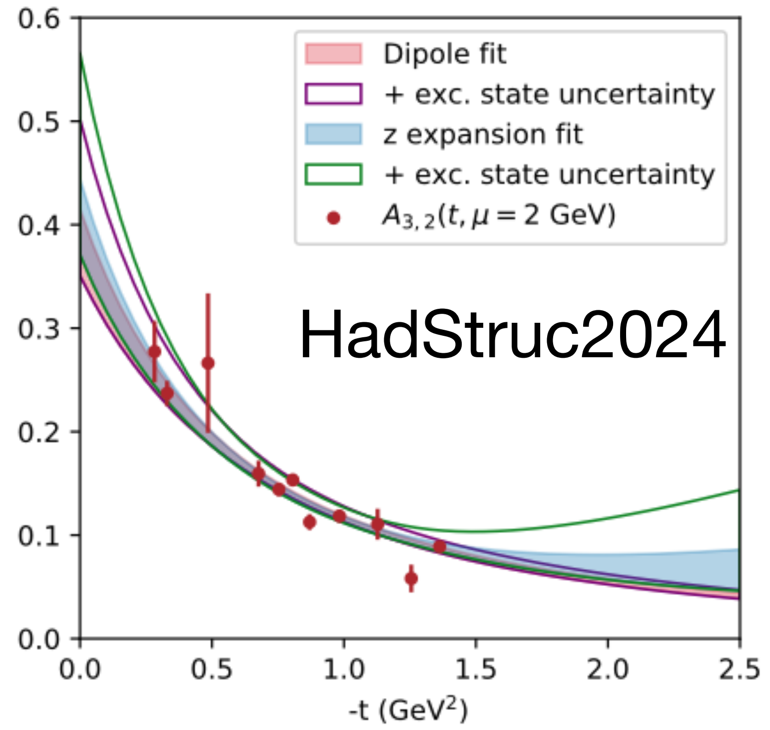
- **short-distance expansion (moments)**

ETMC, Phys.Rev.D 108 (2023) 1, 014507 (unpolarized + moments)

ETMC, Phys.Rev.D 110 (2024) 5, 054502 (unpolarized + reconstruction)

HadStruc, JHEP 08 (2024) 162 (unpolarized + moments + ξ dependent)





- **GPD features**

unpolarized → polarized (helicity, transversity…)

$\xi = 0 \rightarrow \xi \neq 0$

ANN reconstruction could be applied for all cases !

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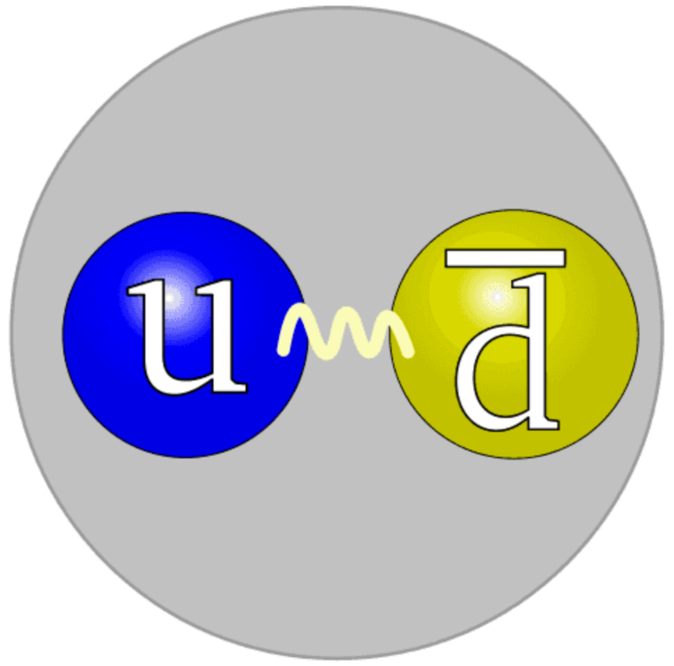
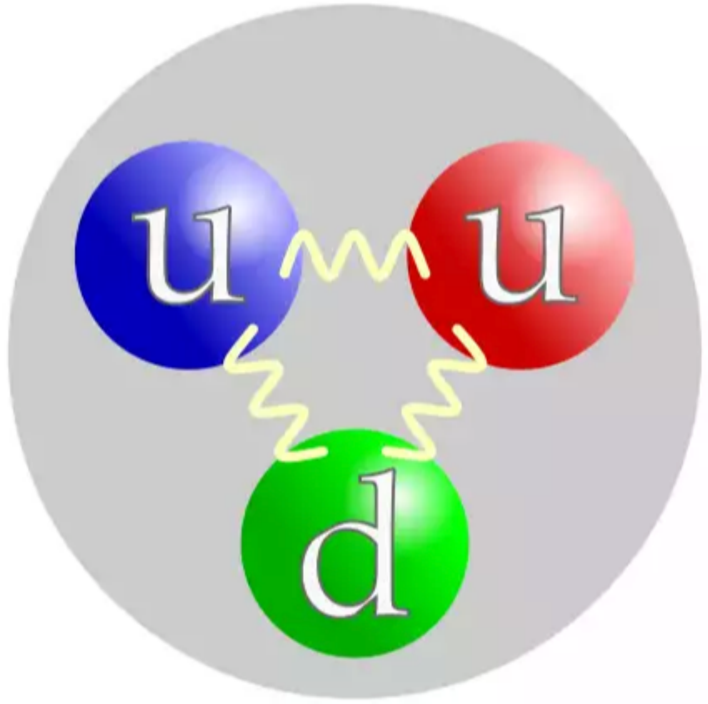




LaMET approach

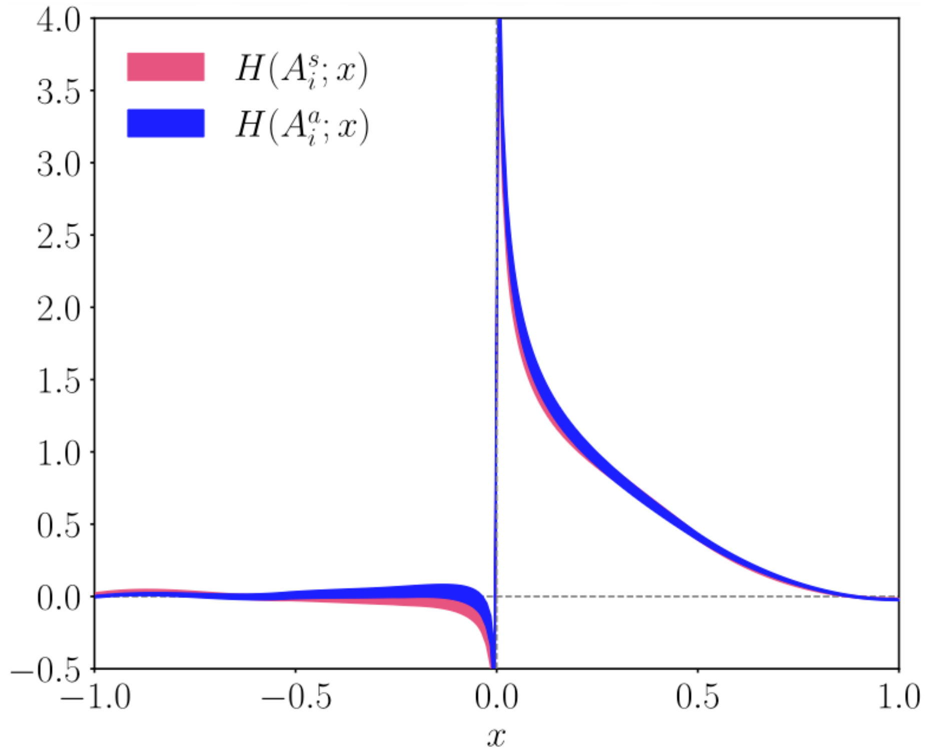
two approaches in lattice determination of GPDs

matrix element

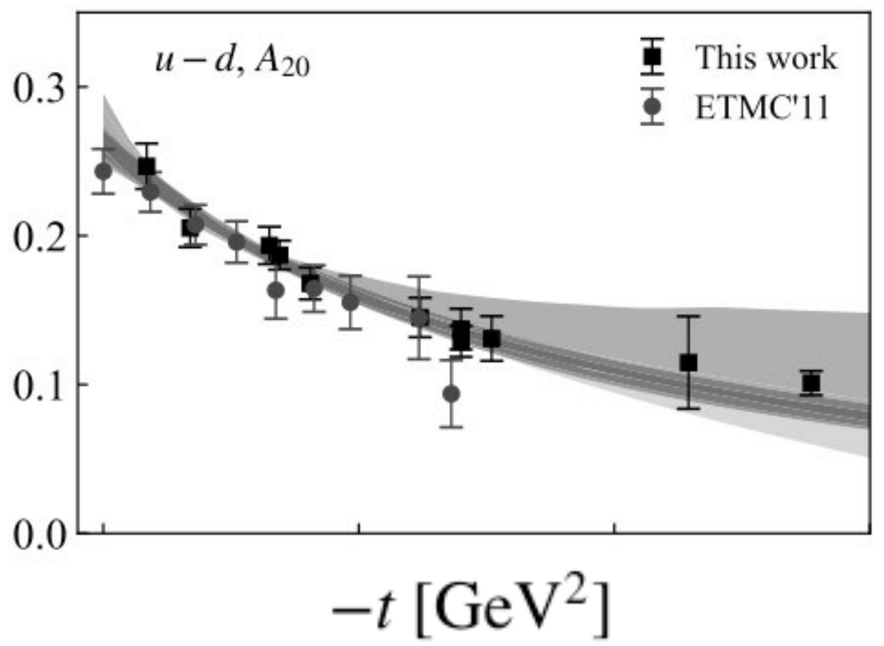
RI/MOM → quasi-correlation → FT → quasi-distribution → matching → x dependent GPDs

renormalization: double ratio → pseudo-correlation → matching → matched-correlation → fit → Melin moments for GPDs

LaMET



SDE



LaMET approach

LaMET

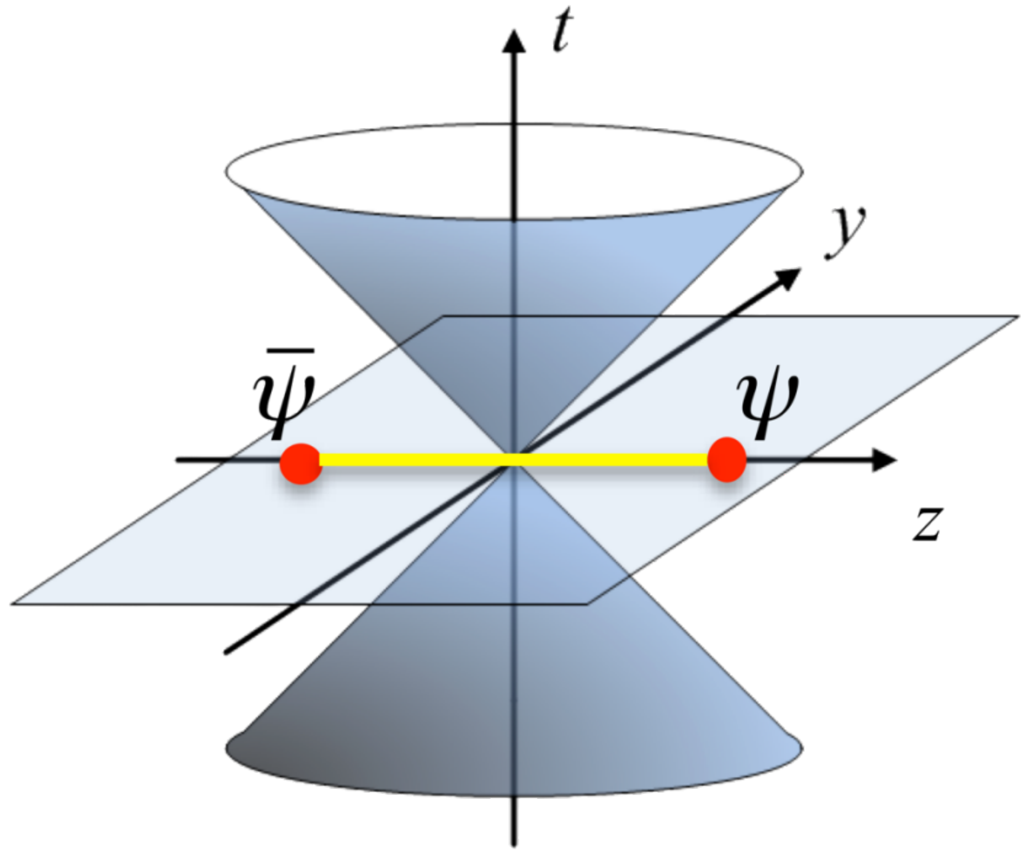
Equal time correlation

$$\tilde{\phi}(z) \sim \langle P^z | \bar{\psi}(\frac{z}{2})\Gamma U(\frac{z}{2}, -\frac{z}{2})\psi(-\frac{z}{2}) | P^z \rangle$$

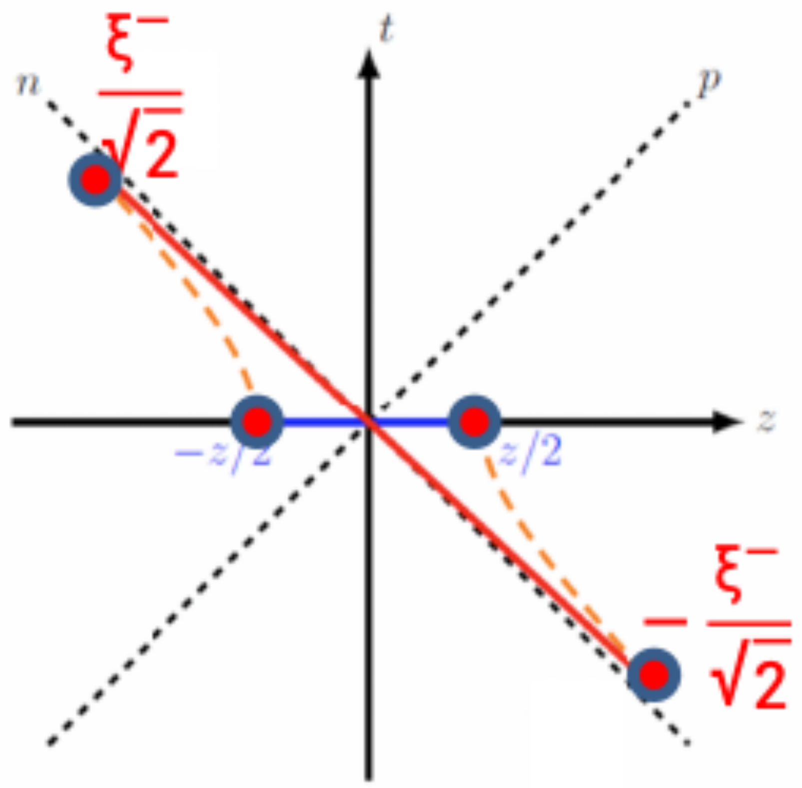
LaMET

Light-cone correlation:

$$\phi(x) \sim \langle P^+ | \bar{\psi}(\frac{\xi^+}{\sqrt{2}})\Gamma U(\frac{\xi^+}{\sqrt{2}}, -\frac{\xi^+}{\sqrt{2}})\psi(-\frac{\xi^+}{\sqrt{2}}) | P^+ \rangle$$

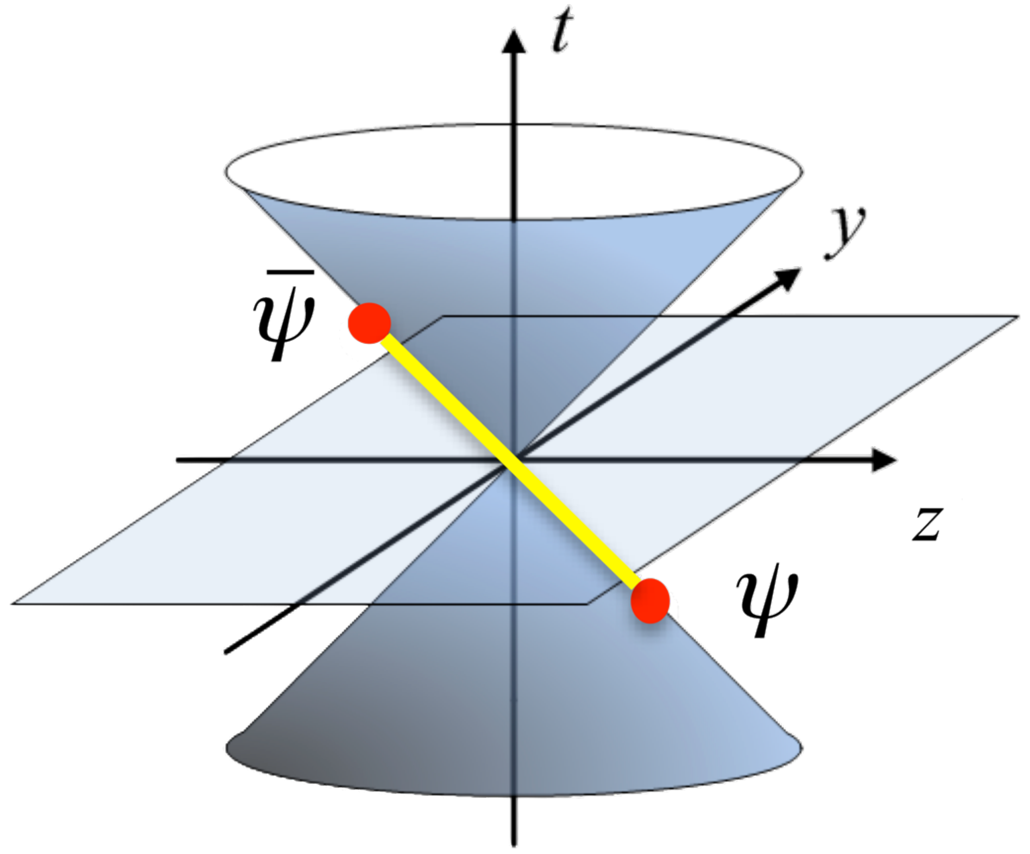


Lorentz transformation



X. Ji, Phys.Rev.Lett. 110 (2013) 262002

matching



Due to the IR structure are only based on states, then the difference between $\phi(x)$ and $\tilde{\phi}(x)$ is only UV structure, which can be perturbatively determined.

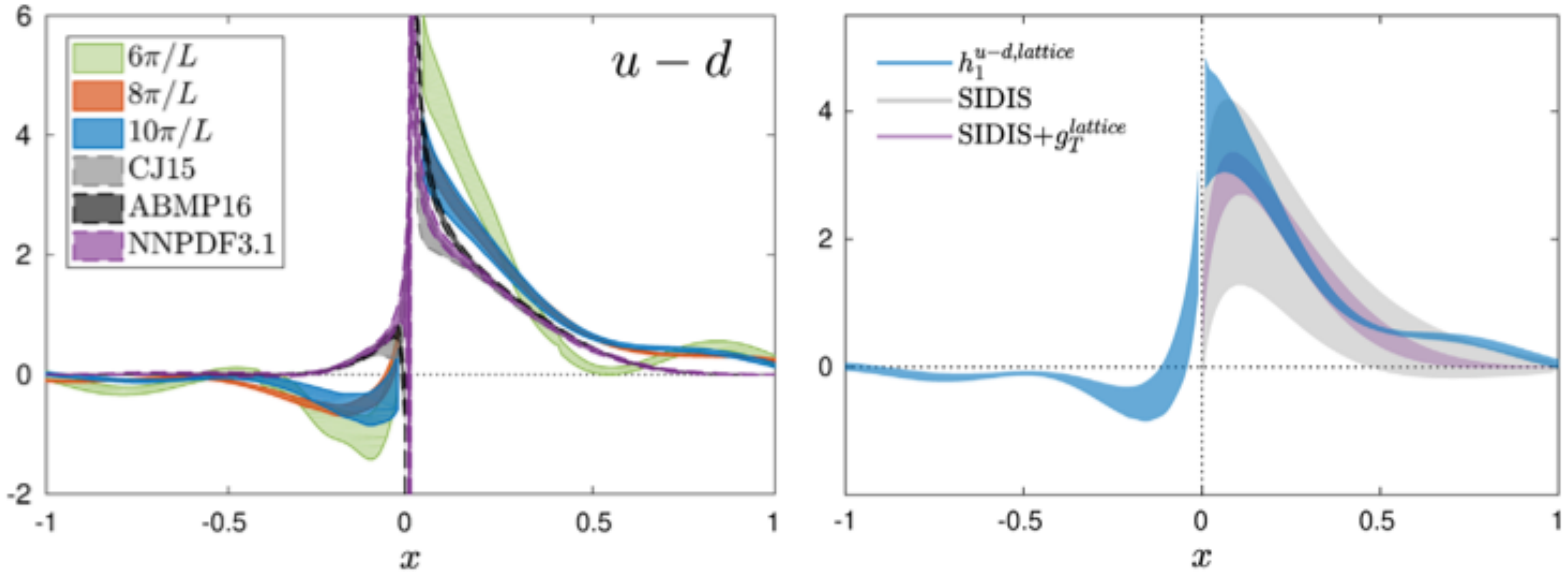
$$\tilde{\phi}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|} C(\frac{y}{x}, \frac{\mu}{xP^z})\phi(x, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(yP^z)^2}, \frac{\Lambda^2_{QCD}}{((1-y)P^z)^2}\right)$$

LaMET approach

achievements

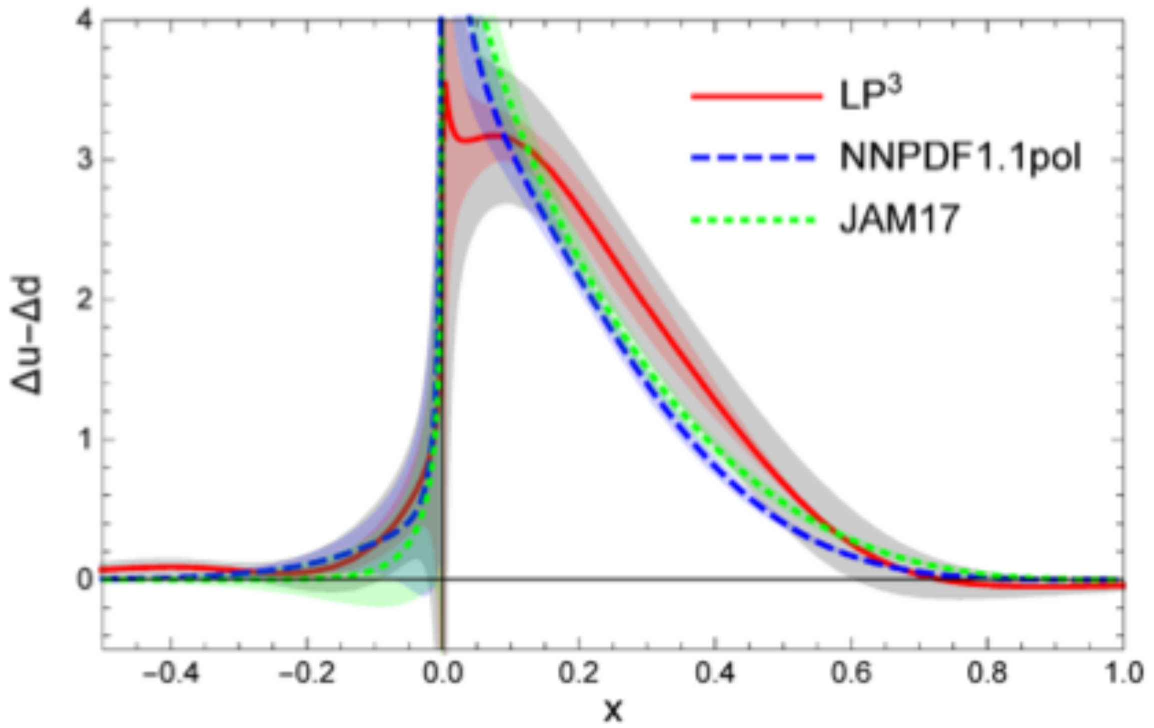
Proton unpolarized quark PDF

C. Alexandrou, et,al., Phys.Rev.D. 98 (2018) 9, 091503



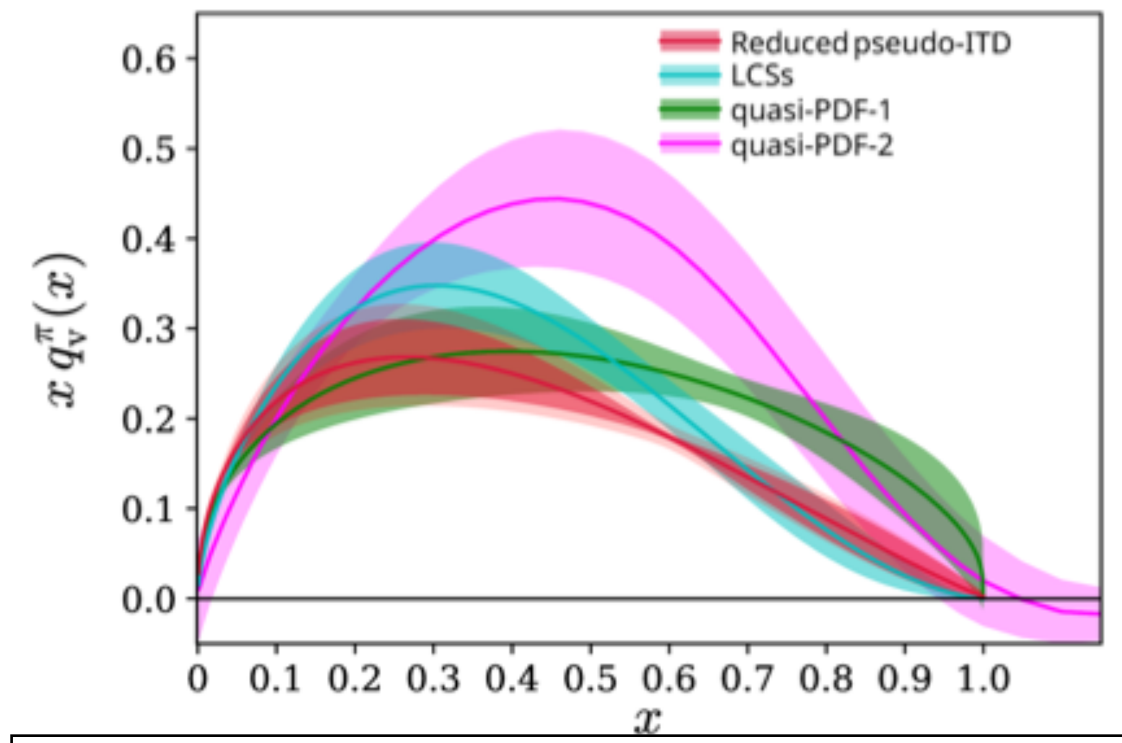
C. Alexandrou, et,al., Phys.Rev.Lett. 121 (2018) 11, 112001

Proton helicity quark PDF



H. Lin, et,al., Phys.Rev.Lett. 121 (2018) 24, 242003

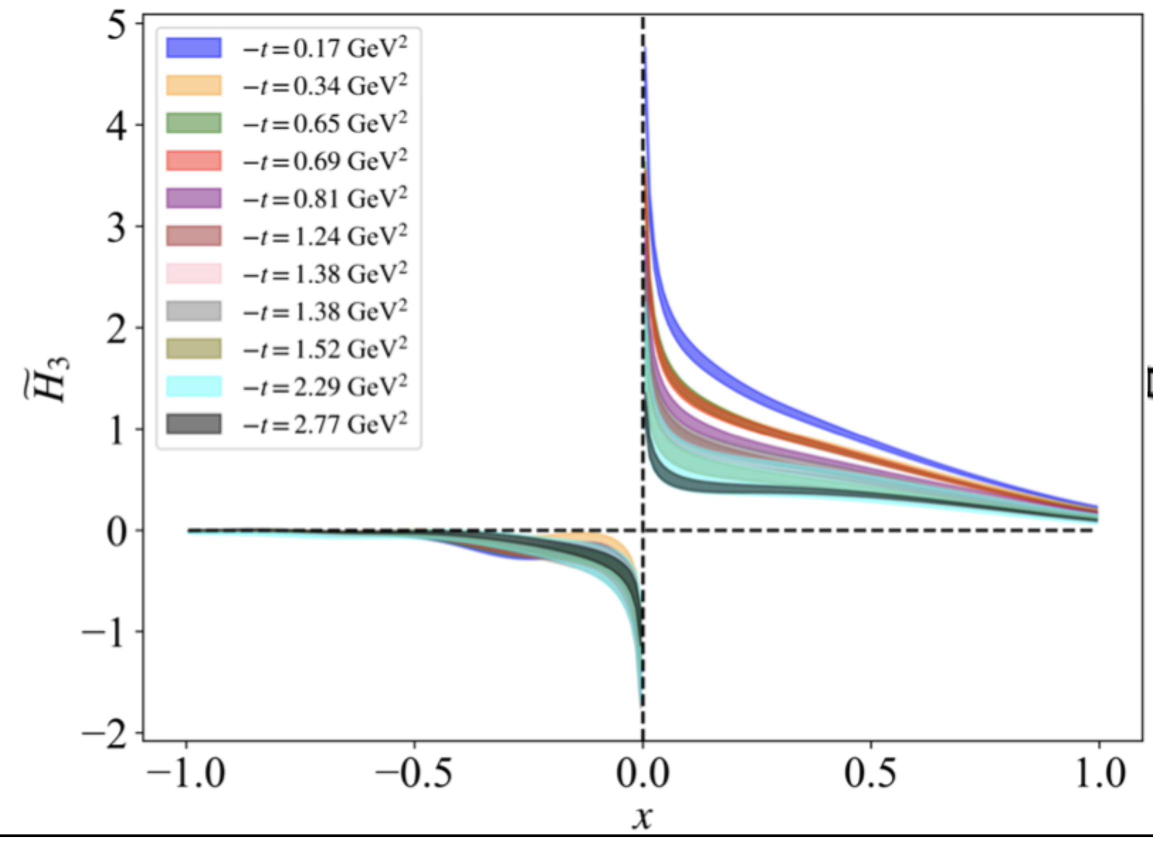
Pion valence quark PDF



B. Joo, et,al., Phys.Rev.D 100 (2019) 11, 114512

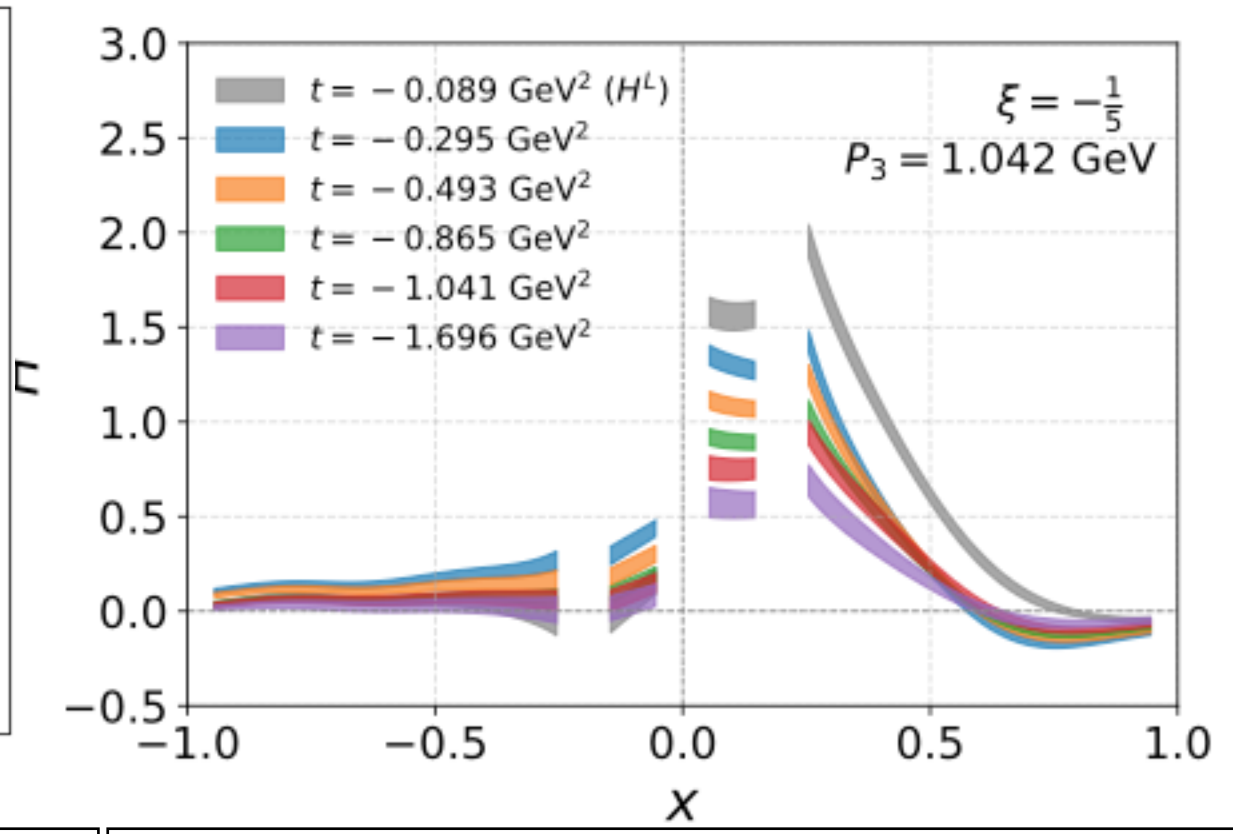
achievements

Proton axial vector GPD



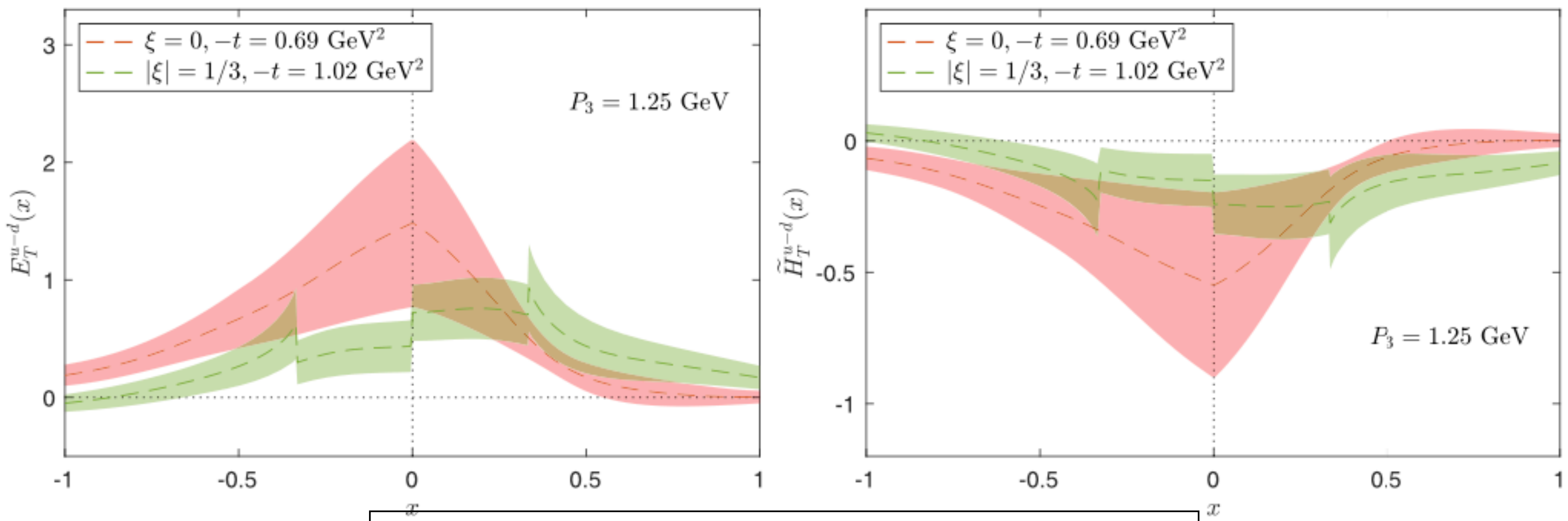
S. Bhattacharya et al., Phys.Rev.D 109 (2024) 3, 034508

Proton unpolarized GPD related to this talk



M. Chu, et,al., Phys.Rev.D 112 (2025) 9, 094510

Proton transversity quark GPD



C. Alexandrou, et,al., Phys.Rev.D 105 (2022) 3, 034501

LaMET approach

definition of GPDs

$$F^\mu(z,P,\Delta) = \langle P_f | \bar{q}(-\frac{z}{2})\gamma^\mu W(-\frac{z}{2},\frac{z}{2})q(\frac{z}{2}) | P_i \rangle$$
$$= \bar{u}(P_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(P_i,\lambda)$$

In $\Delta_\perp = 0$ case:

$$F^{\mu,L}(z,P,\Delta_L) = \bar{u}(P_f,\lambda')\left[\frac{P^\mu}{m}A_1^L + mz^\mu A_2^L + im\sigma^{\mu z}A_4^L\right]u(P_i,\lambda)$$

definition of GPDs

Projecting to 16 components:

amplitudes

$$\Pi_\mu(\Gamma_\nu) = K\,\mathrm{Tr}\left[\Gamma_\nu\left(\frac{-i\not{P}_f + m}{2m}\right)\tilde{F}^\mu\left(\frac{-i\not{P}_i + m}{2m}\right)\right]$$

$$\Pi_\mu(\Gamma_\nu) = \sum_{i=0}^{8} C_i A_i$$

decomposition

H and E GPDs

$$H = A_1 - 2\xi A_3$$

$$E = -A_1 + 2\xi A_3 + 2A_5 + 2P_3 z A_6 - 4\xi P_3 z A_8$$

RI/MOM renormalization

FT and matching

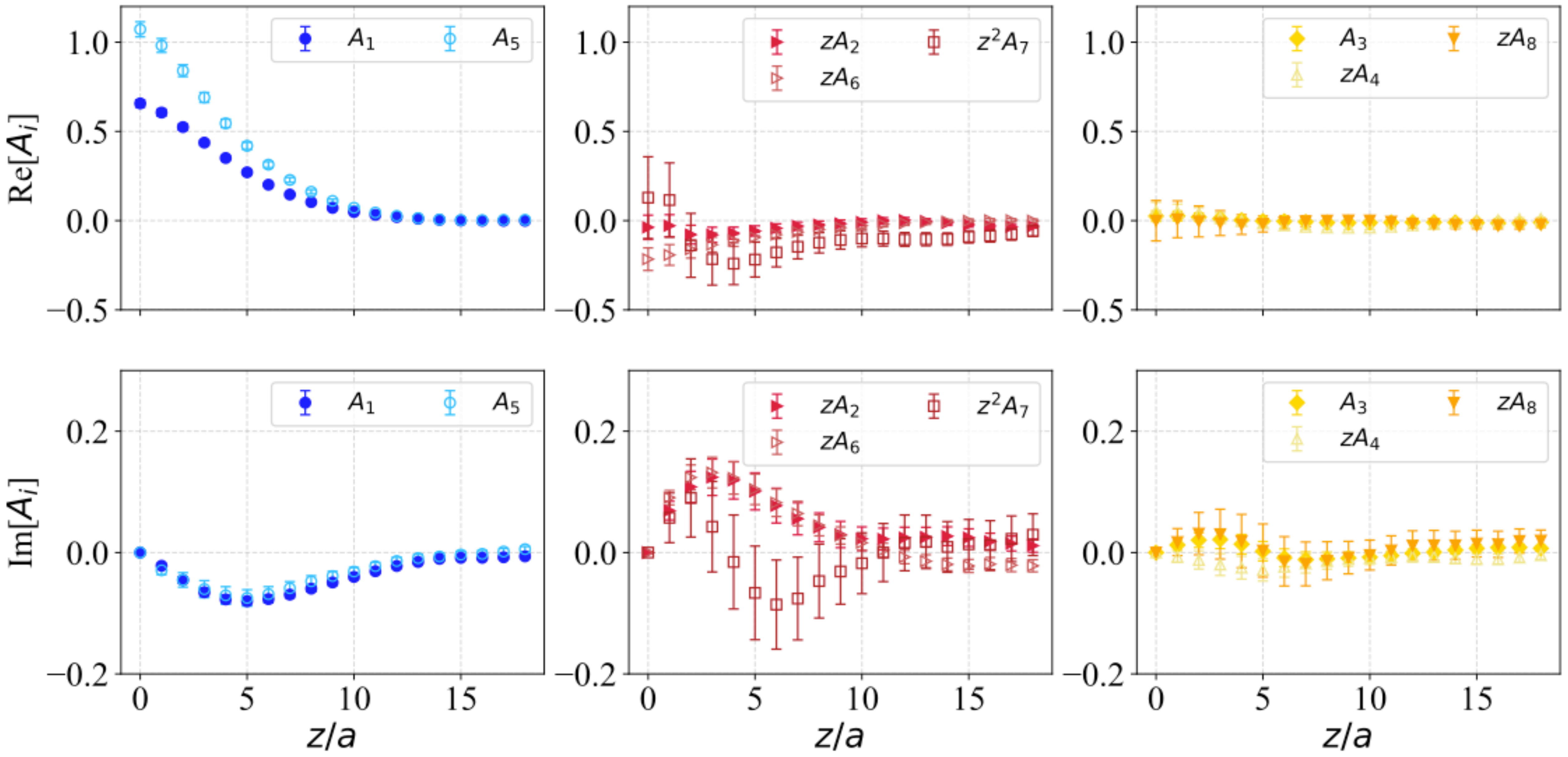
$$H(x,\xi,t),\ \ E(x,\xi,t)$$

light-cone H and E GPDs

LaMET approach

numerical results: amplitudes

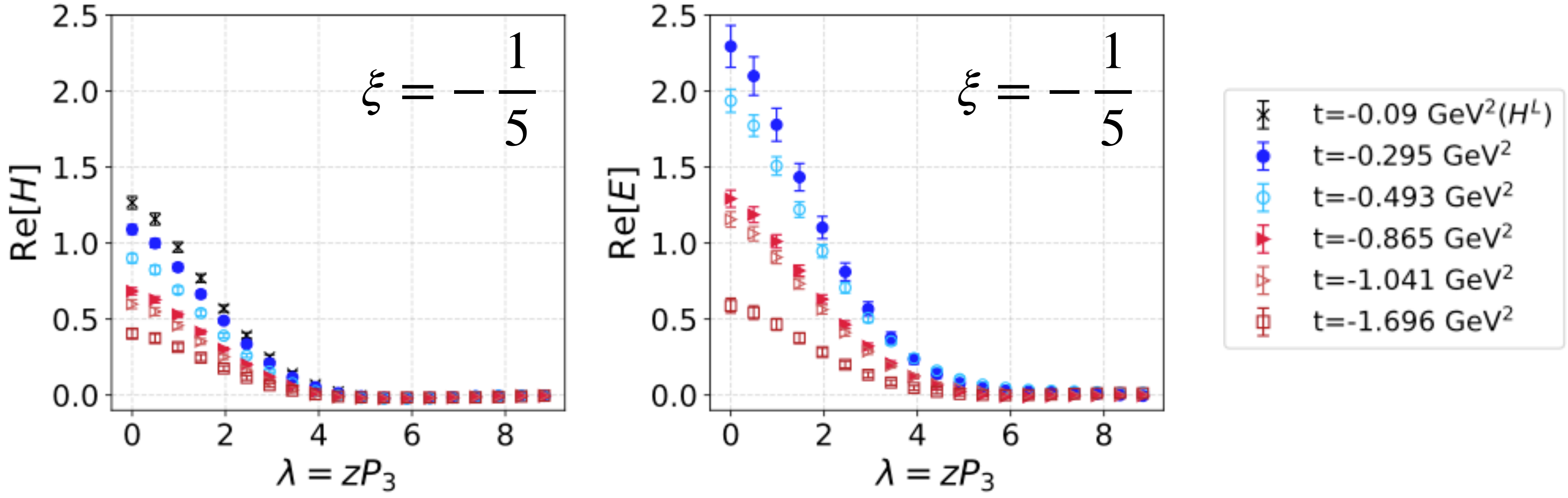
8 amplitudes



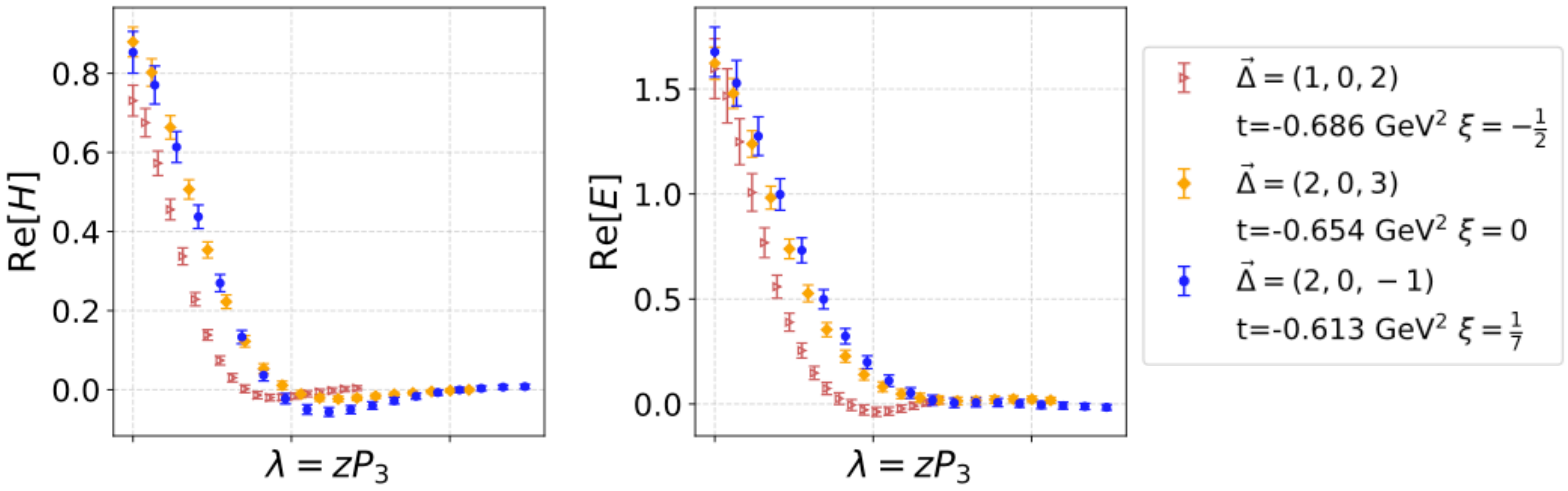
same as $\xi = 0$ case: A_1 and A_5 dominate

different from $\xi = 0$ case: A_2, A_6 and A_7 may have signal

numerical results: H and E GPDs



same as $\xi = 0$: H/E primarily decay with $-t$

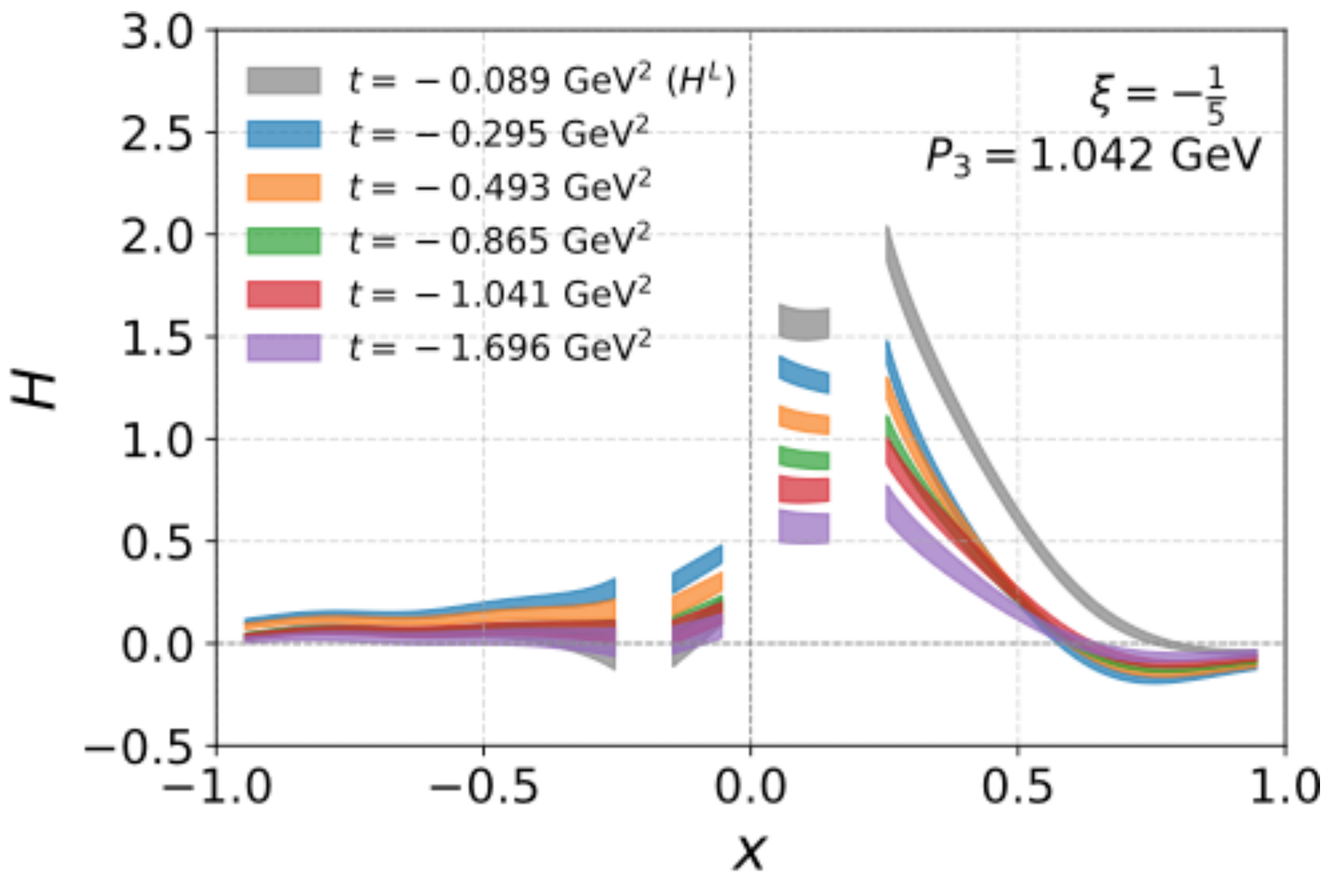


different from $\xi = 0$: ξ accelerates the decay

LaMET approach

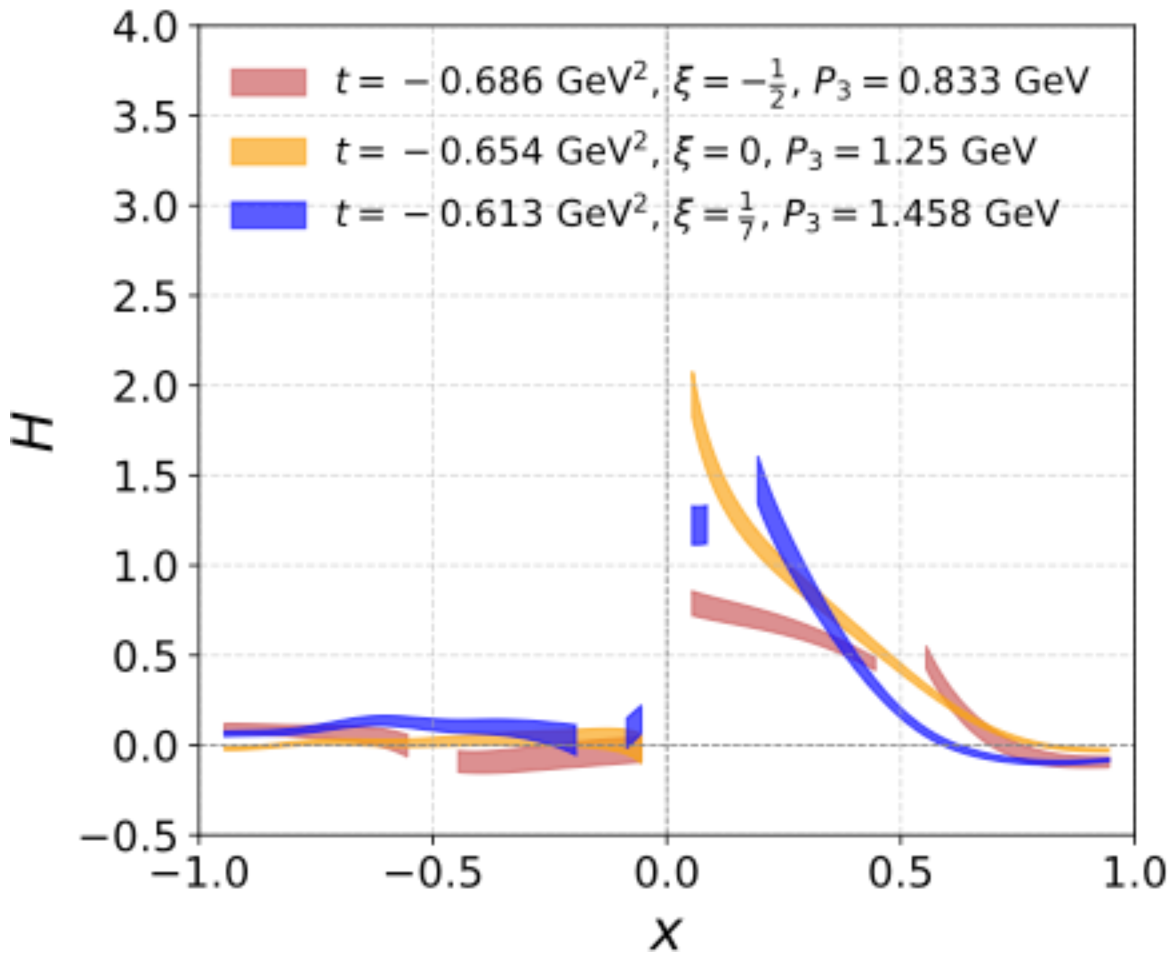
numerical results: light-cone GPDs

unreliable region: power correction at $x \to \pm|\xi|$ and $x \to \pm 1$, BG at $x \to 0$ (limitation of light-cone results)



H/E decay with $-t$

difficult to obtain information in the ERBL region



H/E suppress in ERBL but grows with ξ increase

discontinuity in unreliable region causes underwhelming results

solutions for limitations

1. renormalization: from RI/MOM to hybrid
2. power correction: larger hadron momentum
3. FT: neuron network reconstruction (in progress)
4. lattice: continuum limit + physical pion mass (in progress)

$H(x, \xi, t), \quad E(x, \xi, t)$

angular momentum

elastic form factor

nucleon tomographic

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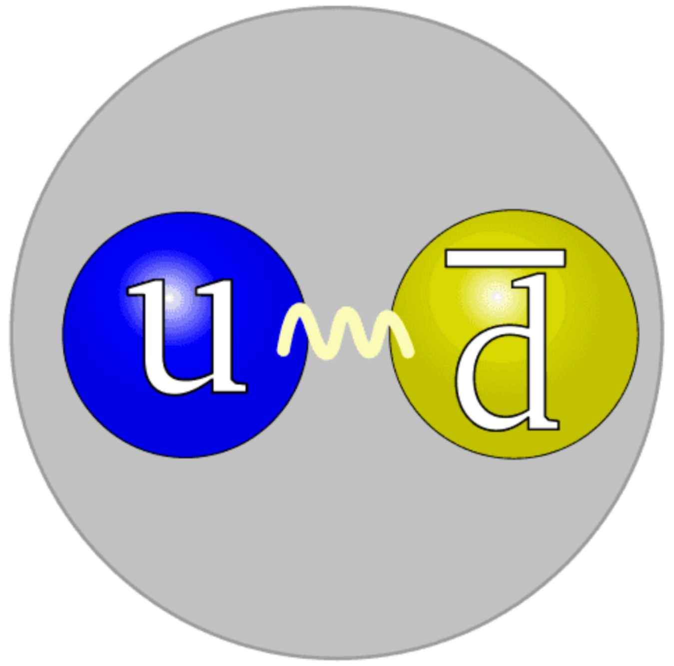
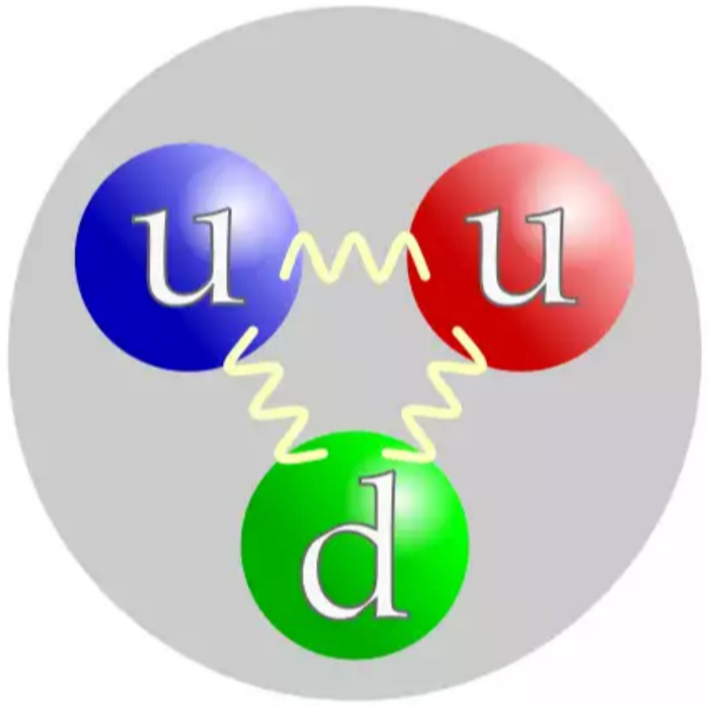




two approaches in lattice determination of GPDs

matrix element

LaMET

renormalization: RI/MOM, hybrid….

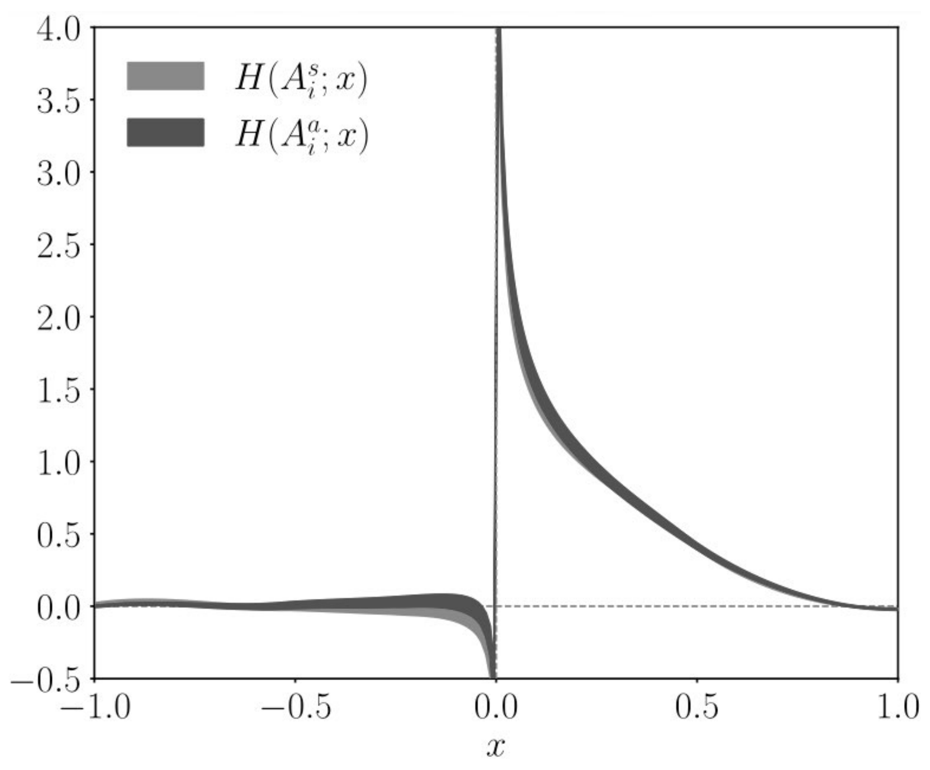
quasi-correlation

FT

quasi-distribution

matching

x dependent GPDs



SDE

renormalization: double ratio

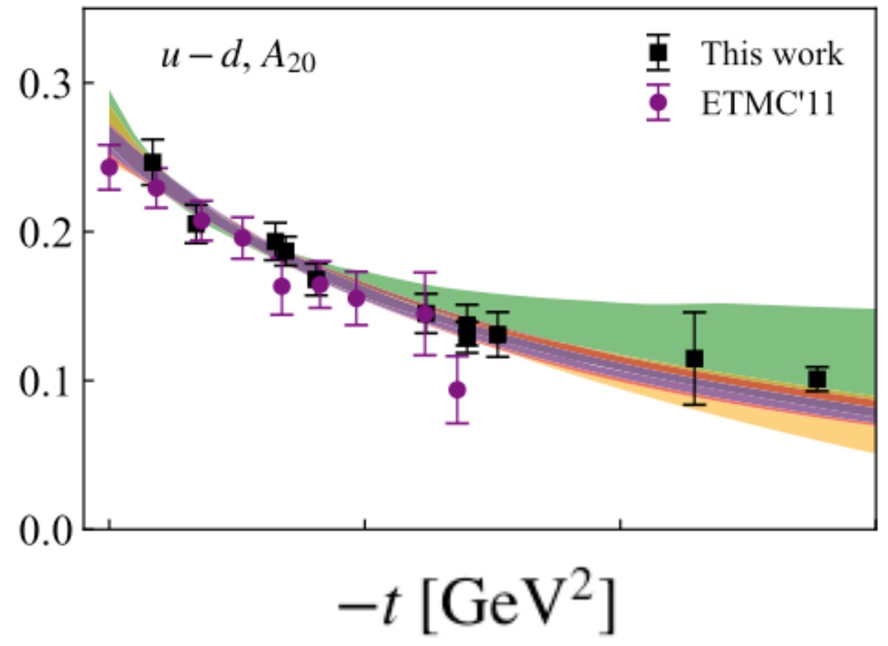
pseudo-correlation

matching

matched-correlation

fit

Melin moments for GPDs



SDE approach

main idea

Melin moments for equal time correlation

parametrization of $\bar{F}_{n+1}(\xi, t, z^2)$ and fit

$$F(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} C_n(z^2, \mu^2) \otimes \bar{F}_{n+1}(\xi, t, z^2) + \mathcal{O}(z^2 \Lambda^2_{QCD})$$

lattice ME (double ratio)

valid when z is small

main idea

Melin moments for light-cone correlation

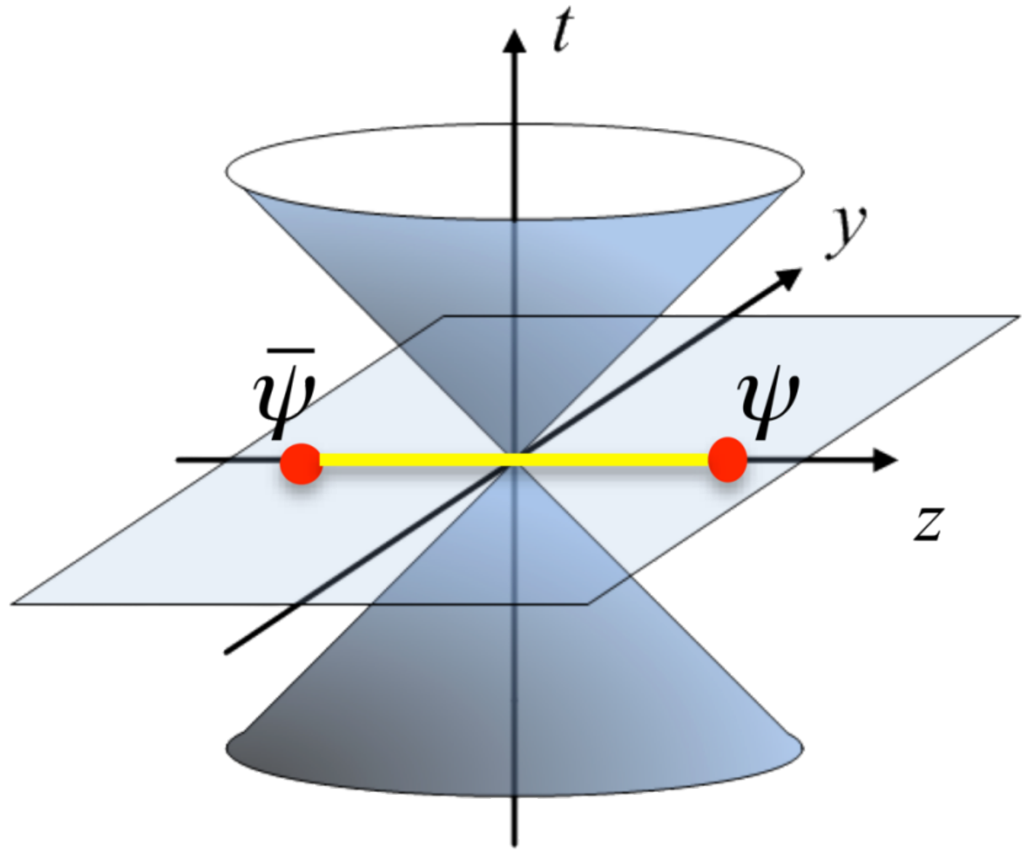
$$\bar{F}_n(\xi, t, z^2) = \int_{-1}^{1} dx\, x^{n-1} \bar{F}(x, \xi, t, z^2)$$

FT

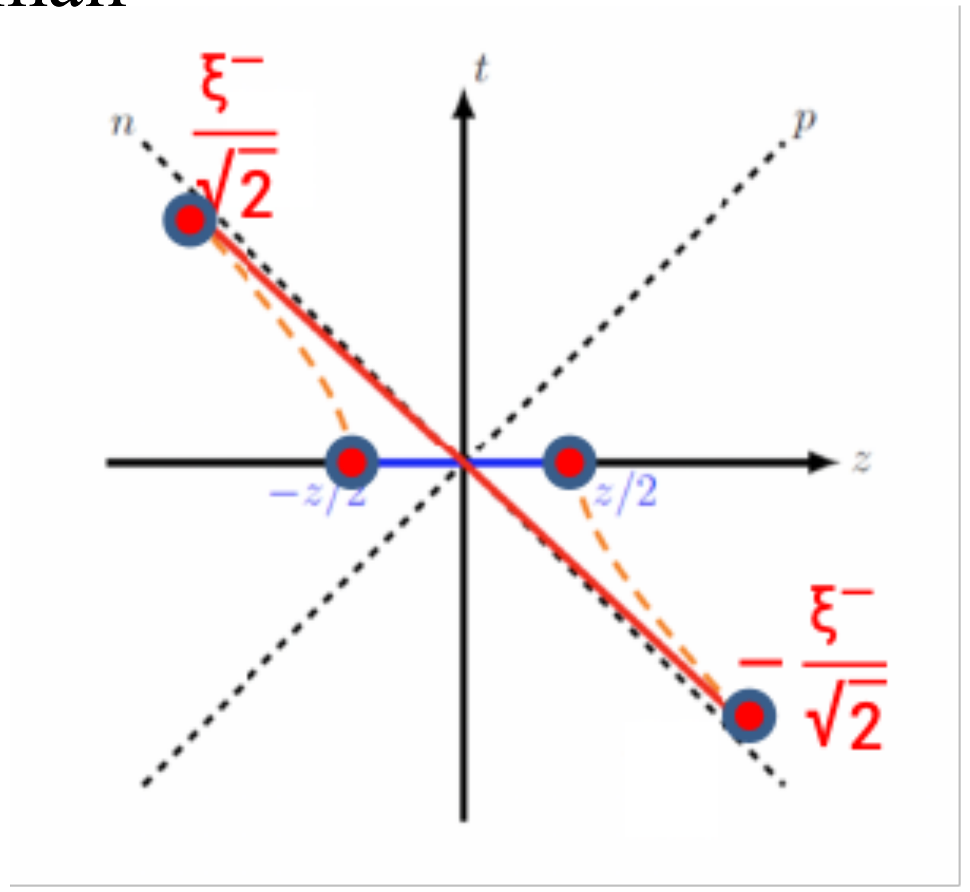
$$\bar{F}(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} \bar{F}_{n+1}(\xi, t, z^2)$$

coordinate matching

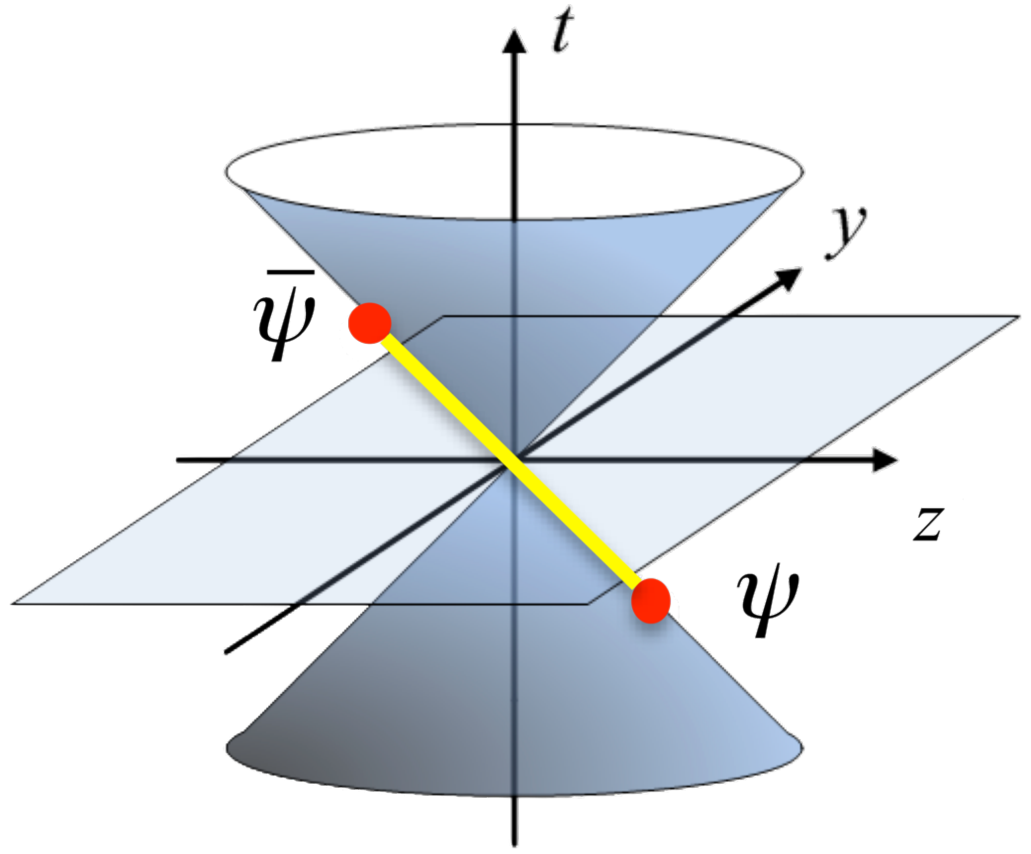
A.V. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025



Lorentz transformation



matching

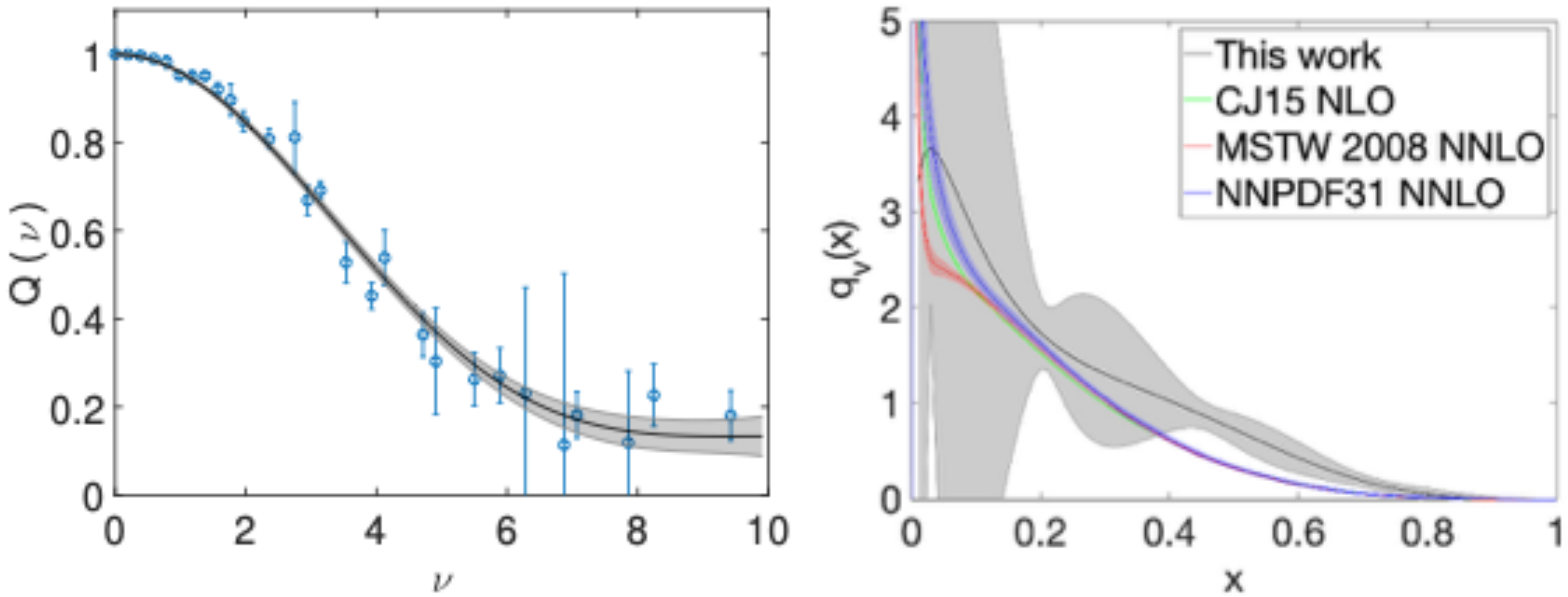


SDE approach

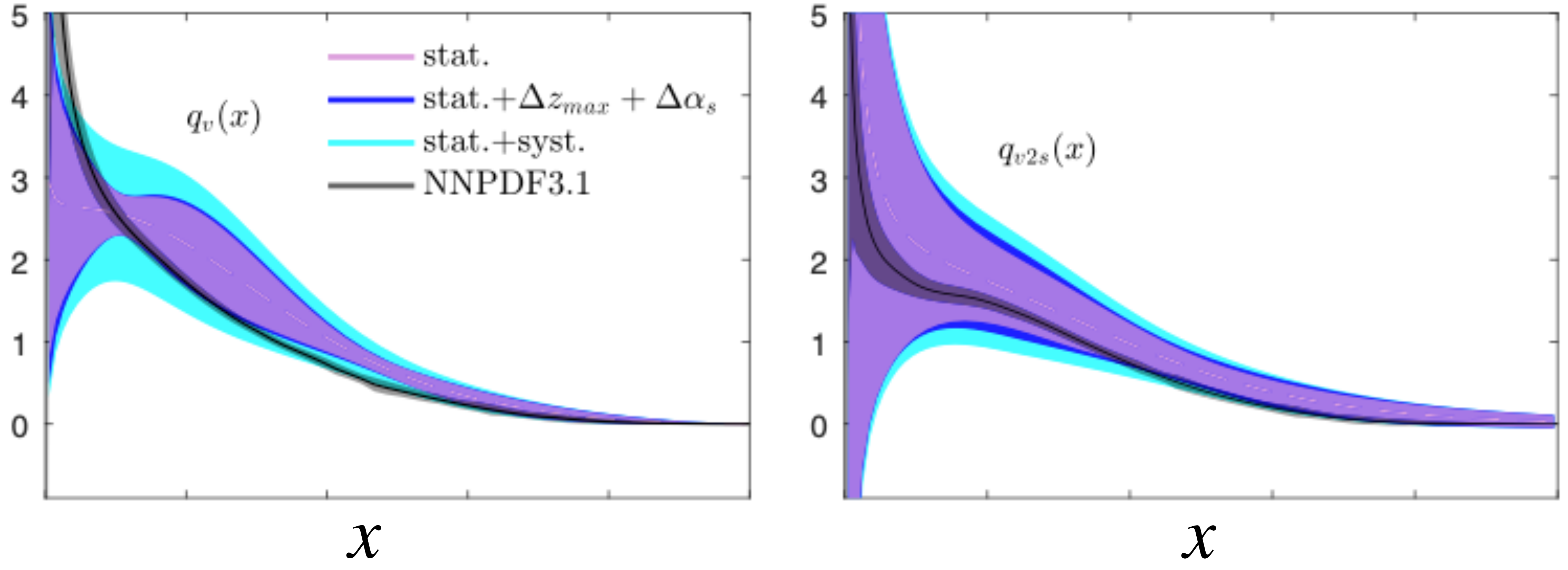
achievements

Proton unpolarized quark PDF

B. Joó, et,al., JHEP 12 (2019) 081



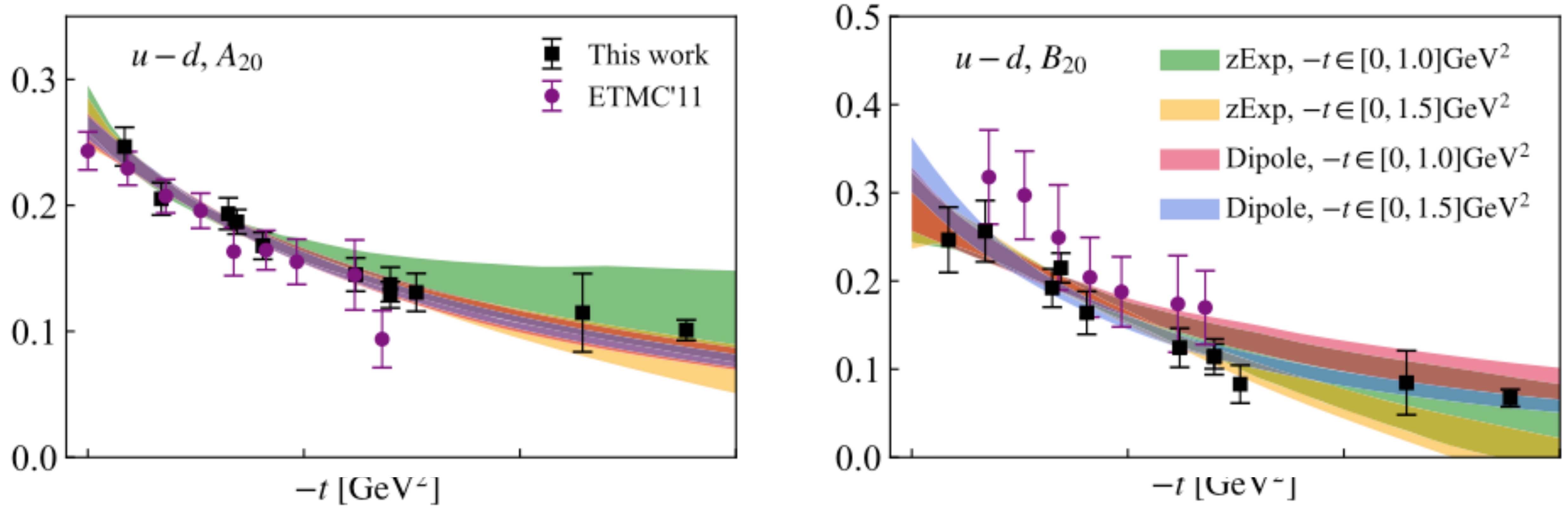
M Bhat, et,al., Phys.Rev.D 103 (2021) 3, 034510



achievements

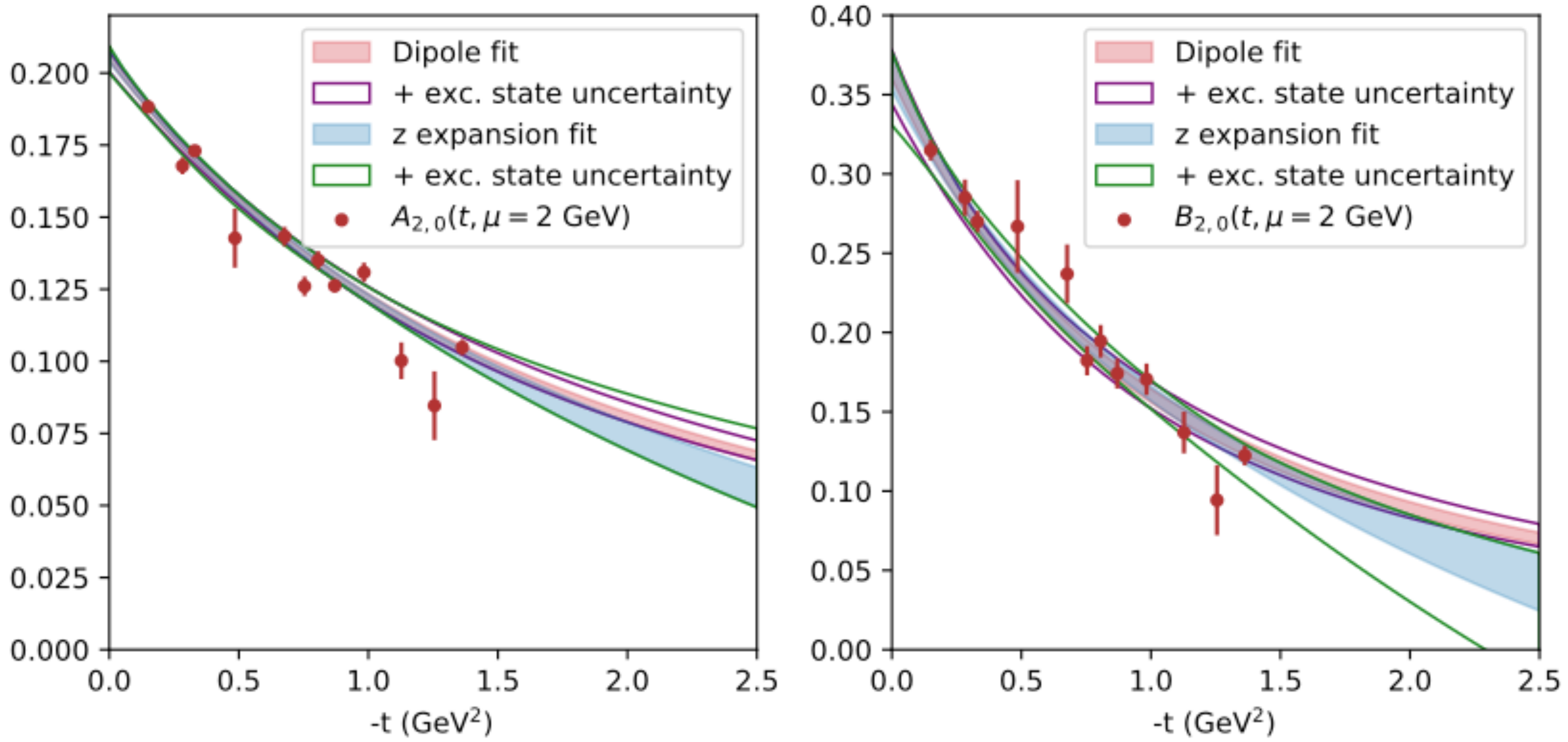
Proton unpolarized quark GPD at zero skewness

S. Bhattacharya, et,al., Phys.Rev.D 108 (2023) 1, 014507



Proton unpolarized quark GPD at nonzero skewness

H. Dutrieux, et,al., JHEP 08 (2024) 162



SDE approach

from GPDs to moments

$$\bar{F}_n(\xi, t, z^2) = \int_{-1}^{1} dx\, x^{n-1} \bar{F}(x, \xi, t, z^2)$$

↓ zero skewness

$$\int_{-1}^{1} dx\, x^n H(x, \xi = 0, t) = \sum_{i=0,\text{even}}^{n} A_{n+1,i}(t)$$

$$\int_{-1}^{1} dx\, x^n E(x, \xi = 0, t) = \sum_{i=0,\text{even}}^{n} B_{n+1,i}(t)$$

from GPDs to moments

double ratio $$F(\nu, \xi, t, z^2) = \frac{F_0(\nu, \xi, t, z^2)/\mathrm{PDF}_0(\nu,0)}{\mathrm{PDF}_0(0,z^2)/\mathrm{PDF}_0(0,0)}$$

$$F(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} C_n(z^2, \mu^2) \otimes \bar{F}_{n+1}(\xi, t, z^2)$$

H. Dutrieux, et,al., JHEP 08 (2024) 162

first four moments at $\mu = 2$ GeV

lattice data

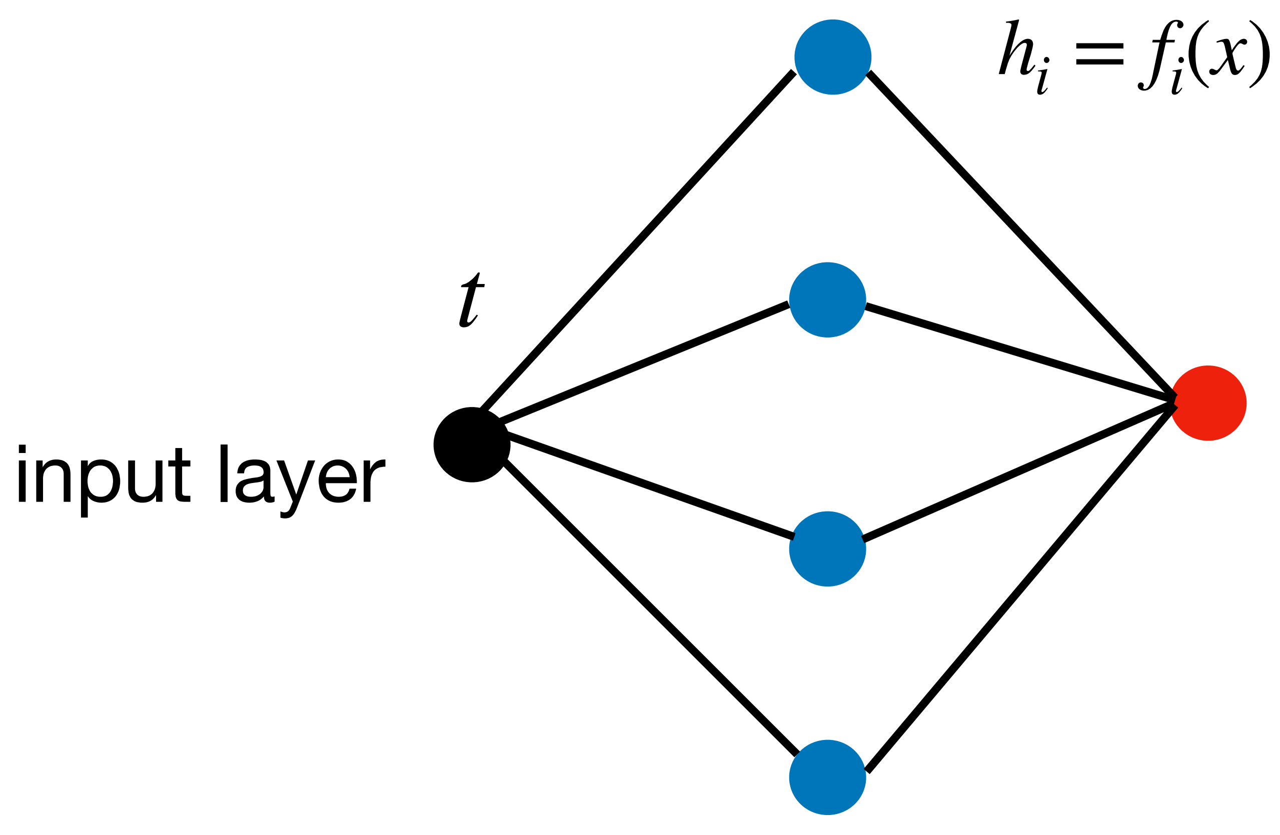
$$\mathrm{Re}[H(\nu, \xi, t, z^2)] = \bar{C}_{1,0}(z^2\mu^2) A_{1,0}(t) - \frac{\nu^2}{2} \left[\xi^2 \bar{C}_{3,2}(z^2\mu^2) A_{1,0}(t) + \bar{C}_{3,0}(z^2\mu^2) \left(A_{3,0}(t) + \xi^2 A_{3,2}(t) \right) \right]$$

$$\mathrm{Im}[H(\nu, \xi, t, z^2)] = -\nu \bar{C}_{2,0}(z^2\mu^2) \left(A_{2,0}(t) + \xi^2 C_2(t) \right) + \frac{\nu^3}{6} \left[\xi^2 \bar{C}_{4,2}(z^2\mu^2) \left[A_{2,0}(t) + \xi^2 C_2(t) \right] + \bar{C}_{4,0}(z^2\mu^2) \left(A_{4,0}(t) + \xi^2 A_{4,2}(t) + \xi^4 C_4(t) \right) \right]$$

parametrization

SDE approach

ANN reconstruction



$h_i = f_i(x) = \tanh(w_i x + b_i)$

output layer: $A_{n,i}(t)$, $B_{n,i}(t)$

$$y_{\rm ANN} = \sum_i W_i h_i + B$$

$$\chi^2(p_k) = \sum_i^N (y_{\rm ANN}(x_i, p_k) - y_{\rm data}(x_i))^2 \xrightarrow{\text{gradient decent}} p_k^{\rm new} = p_k^{\rm old} - r\frac{\partial \chi^2}{p_k}$$

learning process

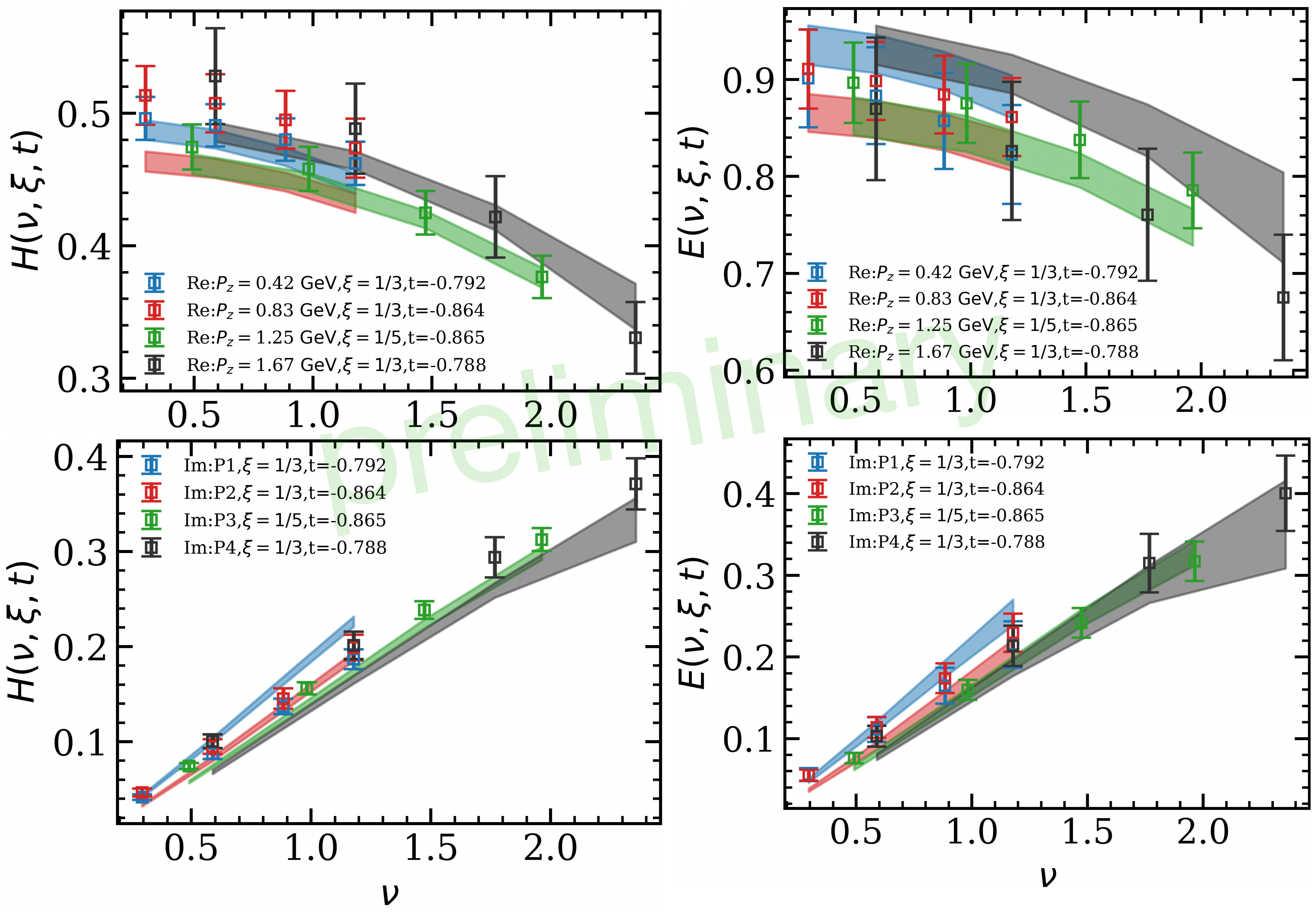
$$A_{n,i}(t) = {\rm ANN}_{{\rm a}_{n,i}}(t),\ B_{n,i}(t) = {\rm ANN}_{{\rm b}_{n,i}}(t)$$

previous: dipole fits and z expansion fit

this work: ANN fit, model independent

fit quality

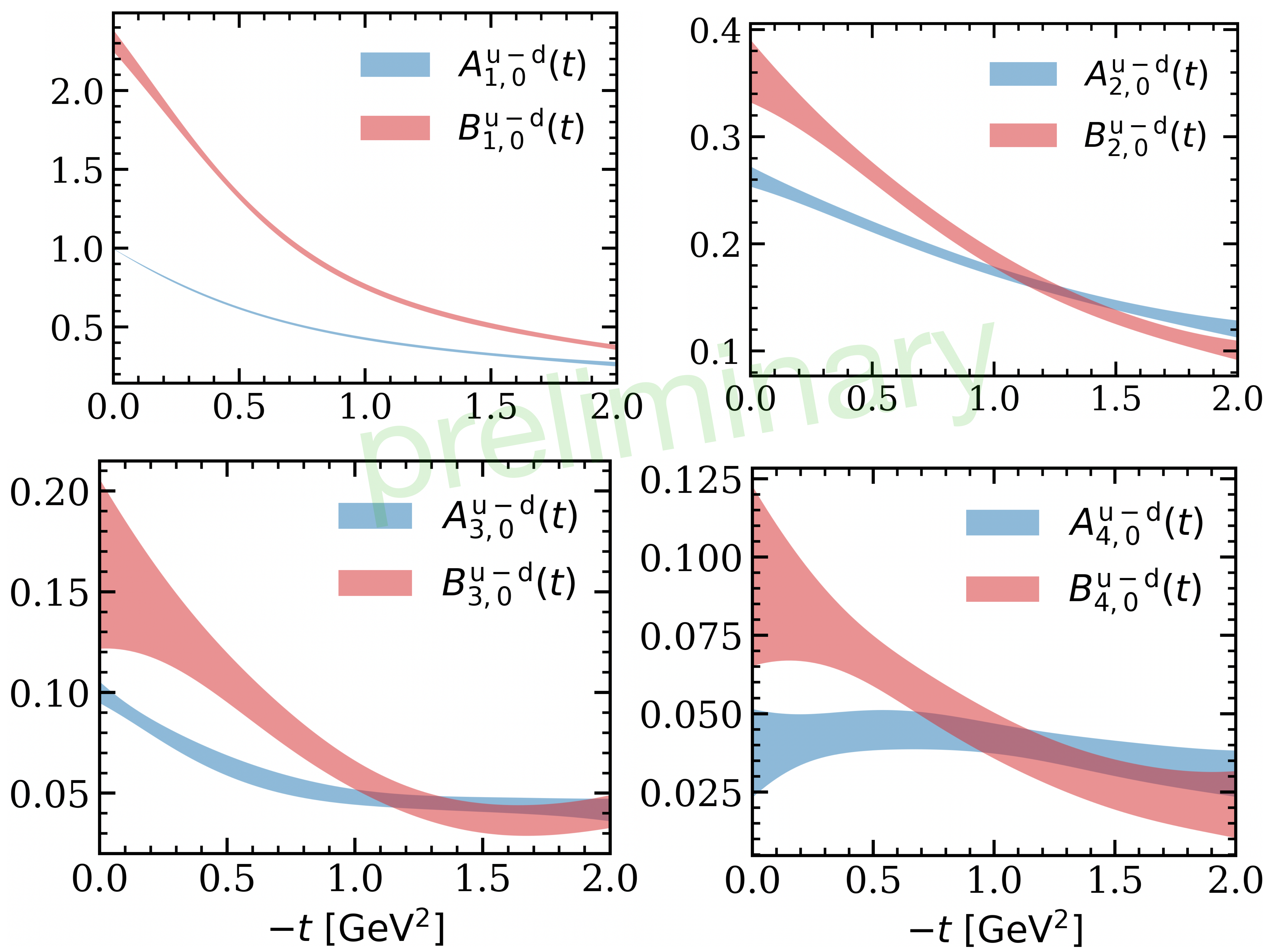
lattice data with $z = [1,2,3,4]\ a$



$\chi^2_{\rm total} \sim 1.5$

SDE approach

moments results



Other higher moments are almost comparable with zero.

Ji's sum rule

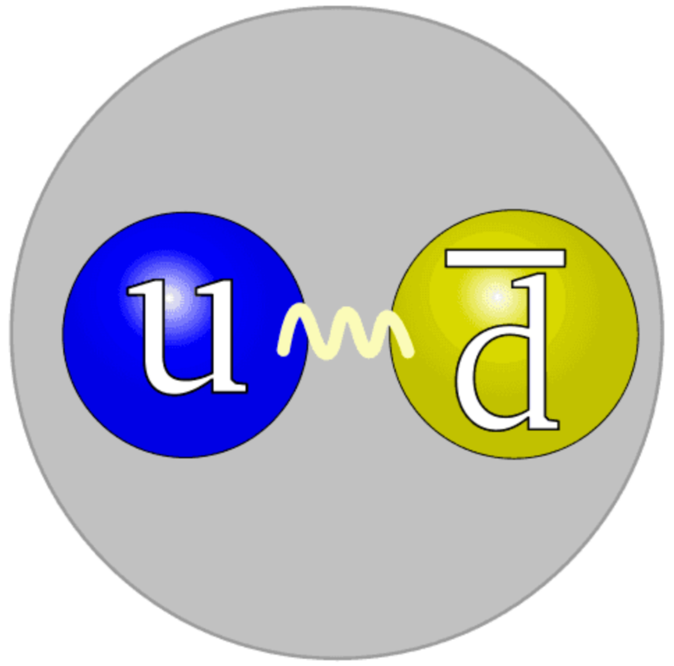
	HadStruc (2024)	ETMC (2023)	ETMC (2026)	This work (preliminary)
J (u-d)	0.288(21)	0.281(32)	0.295(30)	0.312(16)

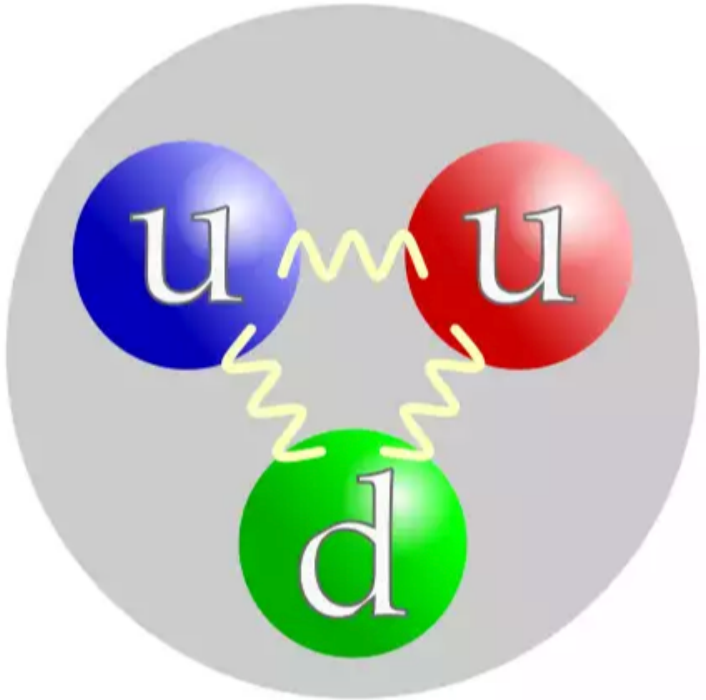
Future plan

1. include RGR (done);
2. two loop matching;
3. cooperation with LaMET;
3. lattice: continuum limit + physical pion mass (in progress).

Outline

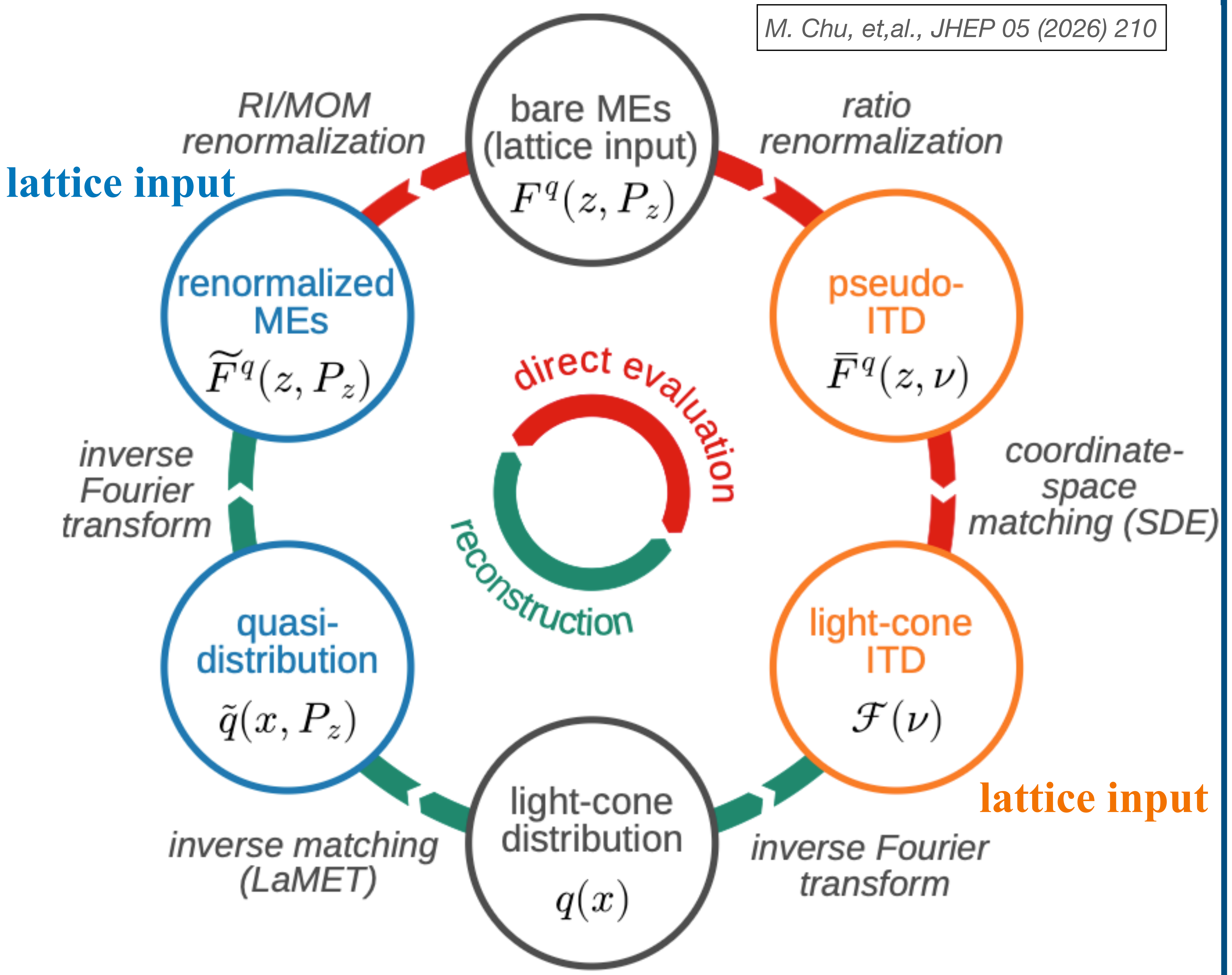
- GPDs
- LaMET approach
- SDE approach
- Summary and outlook



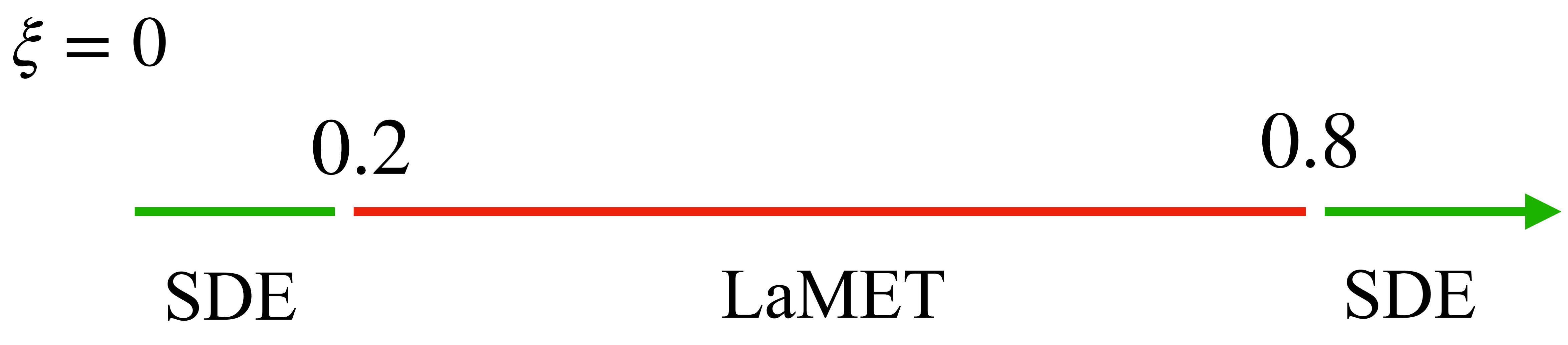


Summary and outlook

unified ANN reconstruction



unified ANN reconstruction



Three major issues

1. renormalization and matching of LaMET
2. When $\xi \neq 0$, reliable region of LaMET depends on ξ.
3. How could SDE cooperate with LaMET?

Summary and outlook

- Lattice QCD calculations of GPDs with non-zero skewness are essential for understanding the internal 3D structure of hadrons.

- The project presents results from LaMET and SDE frameworks, extracting light-cone GPDs and its Mellin moments.

- In the future, applying a unified ANN reconstructions will help improving the x dependent results of GPDs.

Thank you!