Brookhaven National Laboratory
RBRC RIKEN BNL Research Center

GPD and gravitational form factors at large momentum transfer

Yoshitaka Hatta
BNL/RIKEN BNL

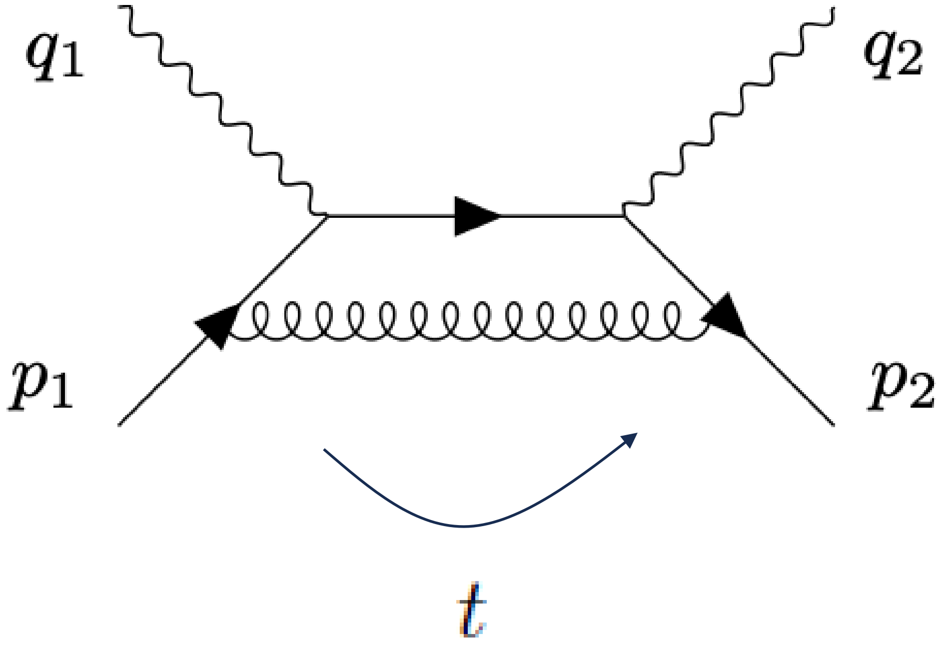
Based on work with Jakob Schoenleber, 2508.01529 (JHEP)

Light-cone 2026, Stony Brook U.

Preamble: Sudakov double logs in DVCS?

Bhattacharya, YH, Vogelsang 2305.09431

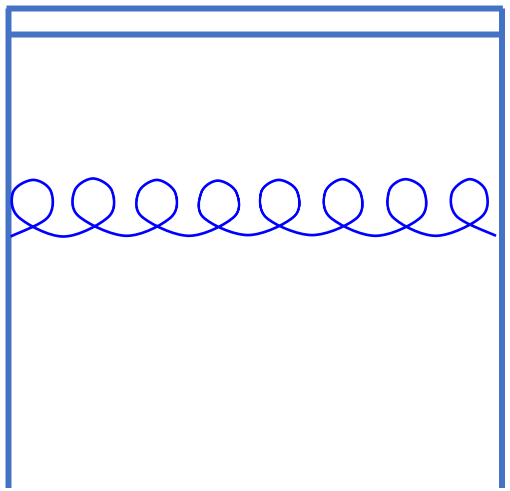
NLO DVCS, quark channel, use momentum transfer t as the infrared regulator



$$\sim P_{qq}(x,\xi)\ln\frac{Q^2}{-t}$$

$$-\frac{1}{1-x}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\frac{Q^2}{-t}+\frac{1}{2}\ln^2\frac{Q^2}{-t}+\cdots\right)$$

Quark GPD $f_q(x,\xi,t)$ contains exactly the same divergence



$$\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}}{\epsilon_{\rm UV}}\left[\frac{1+x^2-2\xi^2}{(1-\xi^2)(1-x)_+}+\left(\frac{3}{2}-\ln(1-\xi^2)\right)\delta(1-x)\right]$$

$$-\,\delta(1-x)\left(\frac{1}{\epsilon_{\rm IR}^2}+\frac{3}{2}\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}$$

→ Can be absorbed into GPD via infrared matching   Bhattacharya, YH, Vogelsang (2023)

But this means GPDs develop Sudakov logs at large-t   cf. Huang, Kroll, Morii (2001); Radyushkin, Zhao (2021)

Resummation to all orders? No discussion in the GPD literature

GPD and GFF at large-t important in threshold meson production and wide-angle Compton scattering

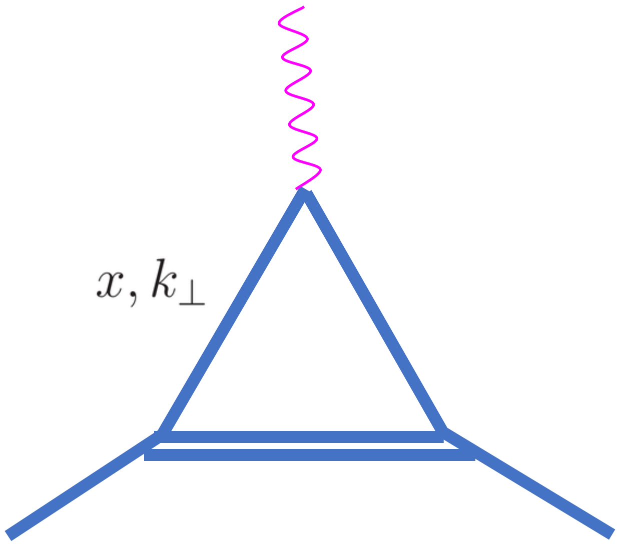
Form factor at large-t: pre-QCD

Drell, Yan (1970)
West (1970)

Feynman (or `soft’ or `overlap’) contribution to the elemag form factor

$$F(t) \sim \int dx \int d^2k_\perp \Psi(x,k_\perp)\Psi(x,k_\perp + (1-x)q_\perp)$$

Light-cone wavefunctions



At large $t = -q_\perp^2$, x-integral dominated by $x \to 1$

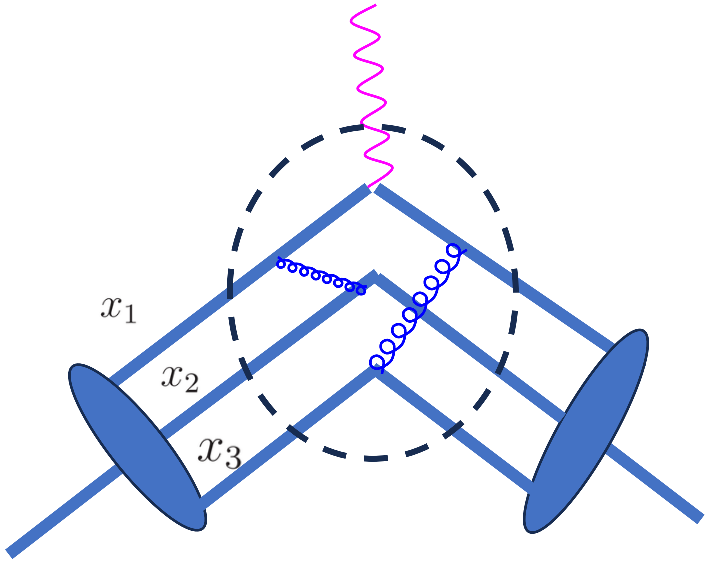
`Active’ parton carries the total momentum of the proton.
`Spectators' are soft

Emergent `hard-collinear' scale $\Lambda^2 \ll \quad k^2 \sim \dfrac{-k_\perp^2}{1-x} \sim \sqrt{|t|}\Lambda \ll |t|$

Form factor at large-t: QCD factorization

Efremov, Radyushkin (1980)
Brodsky, Lepage (1980)

Large momentum transfer democratically shared by 3 quarks
→ hard gluon exchange



$$F(t) \sim H \otimes DA \otimes DA$$ + power corrections

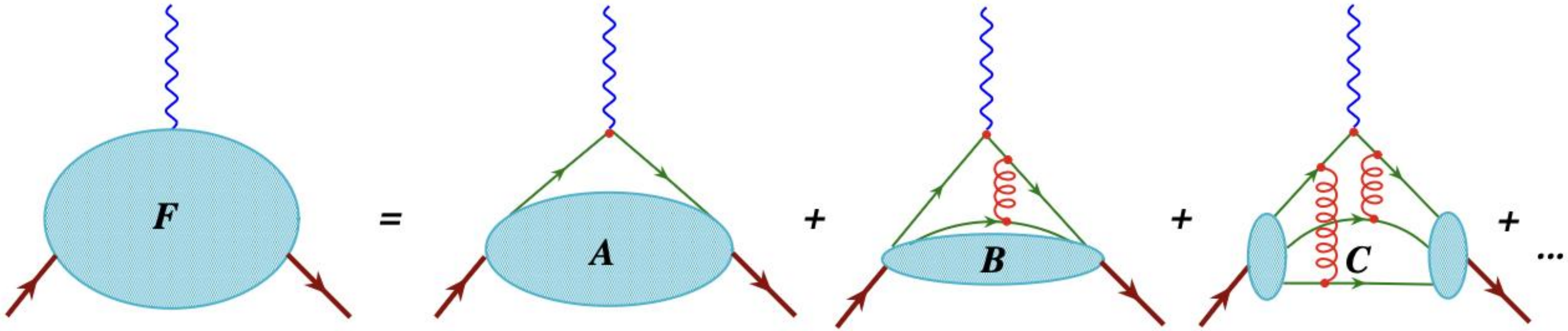
$$H \sim \frac{\alpha_s}{\pi}\frac{1}{t}$$ for pion

$$H = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{t^2}$$ for proton

<-- Factorization breaking soft divergence at two-loops α_s^4/t^2 Duncan, Mueller (1980)

Form factors at large-t: Who wins?

figure from Braun, Lenz, Wittmann (2006)



Feynman contribution

$$\sim \frac{\Lambda^6}{t^3}$$

← Higher twist?

Hard scattering contribution

$$\sim \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\Lambda^4}{t^2}$$

Feynman contribution dominant until

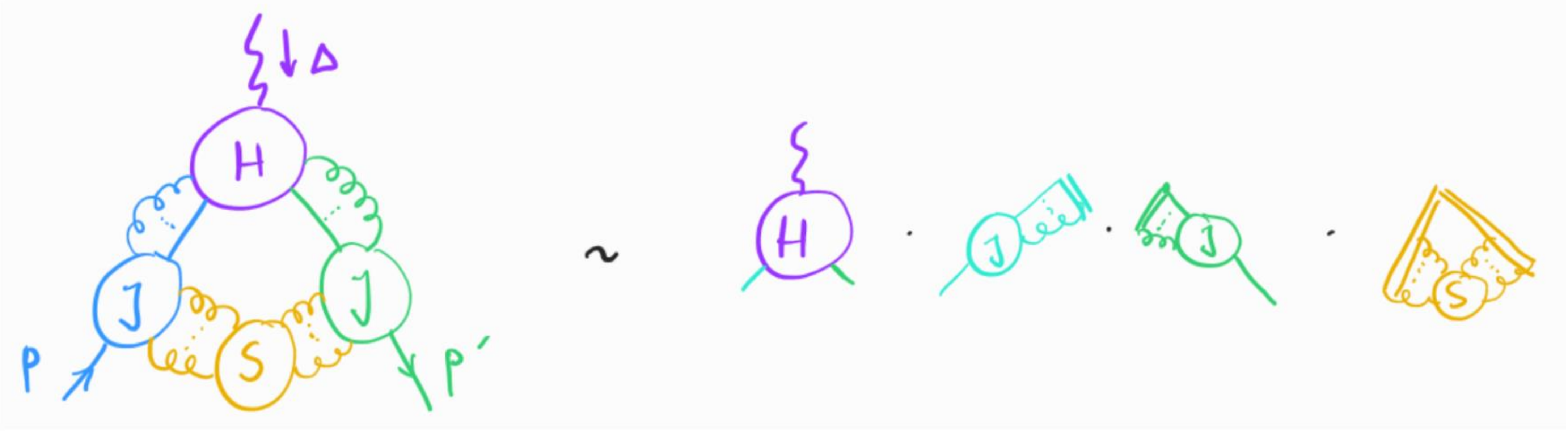
$$|t| \sim \frac{\Lambda^2}{(\alpha_s/\pi)^2} = 10 \sim 100 \mathrm{GeV}^2$$

Sudakov form factor in QED/QCD

$$F(t) = \langle p'|\bar{\psi}(0)\gamma^\mu\psi(0)|p\rangle$$

$$= 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{\mu_F^2}{-t} - 3\ln\frac{\mu_F^2}{-t} - 8 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2) \quad \sim \exp\left(-\frac{\alpha_s C_F}{4\pi}\ln^2\frac{-t}{\mu^2}\right)$$

All order factorization Collins, Soper, Sterman, Korchemsky,...



Factorization of nonlocal Sudakov form factor

YH, Schoenleber, 2508.01529

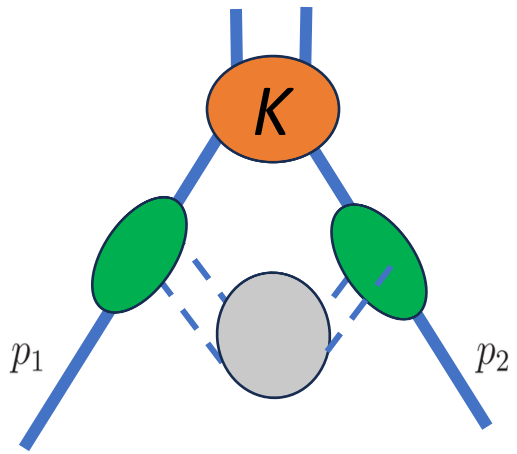
Nonlocal generalization of Sudakov = quark GPD of a quark

$$\langle p_2|\bar{\psi}(0)\gamma^\mu\psi(0)|p_1\rangle \to \langle p_2|\bar{\psi}(-z/2)\gamma^\mu[-z/2, z/2]\psi(z/2)|p_1\rangle$$

Power-counting parameter

$$\lambda \equiv \frac{\sqrt{-p^2}}{\sqrt{-t}} \ll 1$$

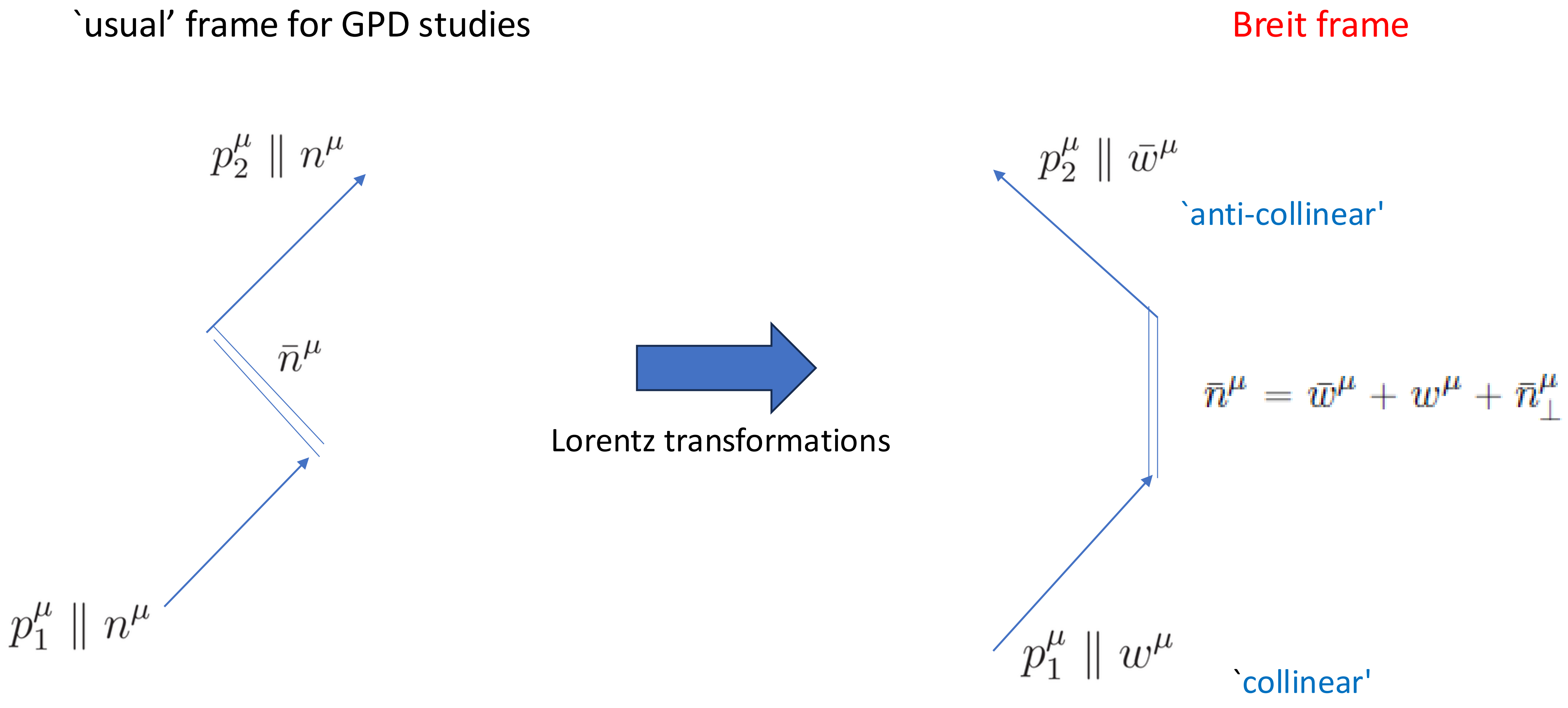
Parton virtuality $-p^2$ to be specified later



factorization

$$F(x, y, \xi, t, p_1^2, p_2^2, \mu_{\mathrm{UV}}) = K(x, y, \xi, t, \mu_{\mathrm{UV}}, \mu_F) J(p_1^2, \mu_F) J(p_2^2, \mu_F) S\left(\frac{p_1^2 p_2^2}{-t}, \mu_F\right) + \mathcal{O}(\lambda)$$

Breit frame for GPD at large-t



Soft collinear effective theory (SCET)

$$k^\mu = \frac{\bar{w}\cdot k}{2} w^\mu + \frac{w\cdot k}{2}\bar{w}^\mu + k_\perp^\mu$$

$$k_c^\mu \sim (1, \lambda^2, \lambda)\sqrt{-t}, \qquad k_{\bar{c}}^\mu \sim (\lambda^2, 1, \lambda)\sqrt{-t}, \qquad k_{us}^\mu \sim (\lambda^2, \lambda^2, \lambda^2)\sqrt{-t},$$

collinear anticollinear ultrasoft

$$\lambda \equiv \frac{\sqrt{-p^2}}{\sqrt{-t}} \ll 1$$

Field operator decomposition

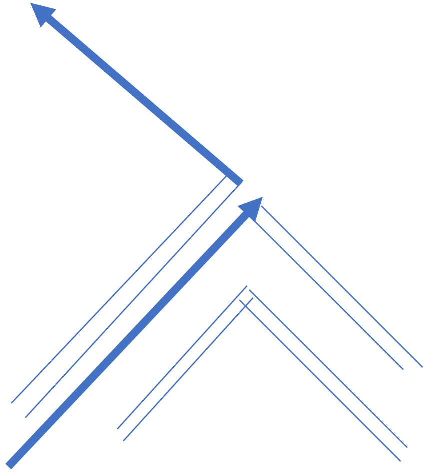
$$\psi = \psi_c + \psi_{\bar{c}} + \psi_{us}, \qquad A^\mu = A_c^\mu + A_{\bar{c}}^\mu + A_{us}^\mu.$$

To leading power, soft/collinear sectors decouple in the effective Lagrangian

$$L \approx L_s + L_c + L_{\bar{c}}$$

Factorization in SCET

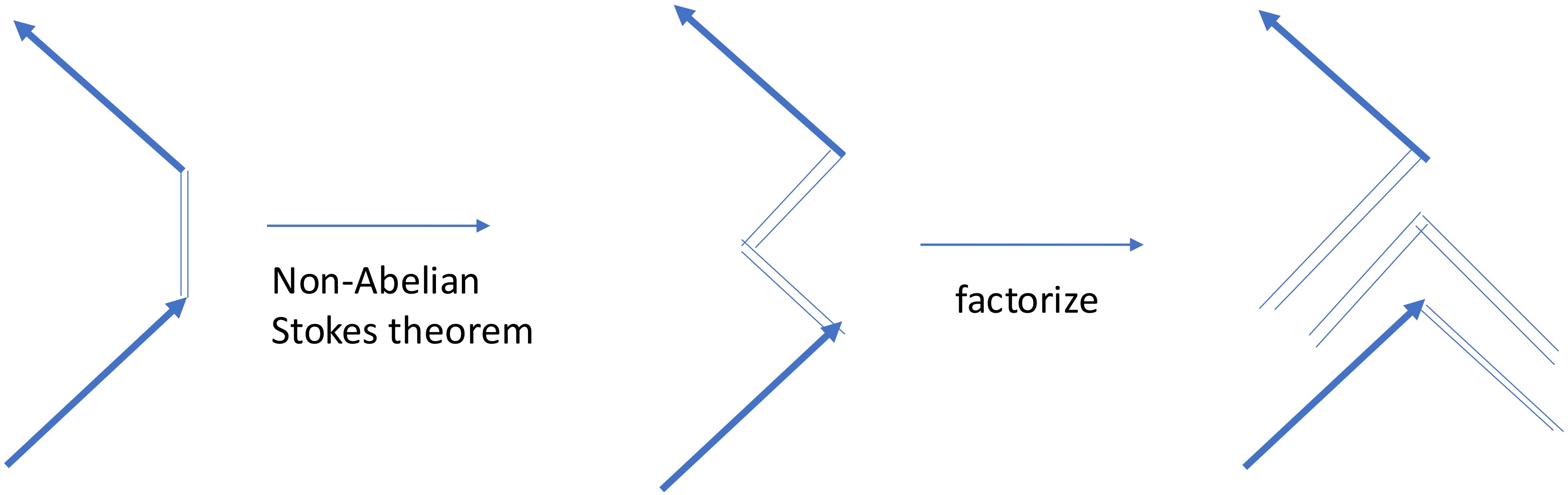
Local Sudakov



$$\langle p_2|\,\bar{\psi}(x)\gamma^{\mu}\psi(x)\,|p_1\rangle$$

$$= C(t,\mu_F)\,\langle p_2|\,\bar{\chi}_{\bar{c}}(x)\,|0\rangle\,\gamma_{\perp}^{\mu}\,\langle 0|\,Y_{\bar{w}}^{\dagger}(0)\,Y_w(0)\,|0\rangle\langle 0|\,\chi_c(x)|p_1\rangle + \mathcal{O}(\lambda)$$

nonlocal Sudakov (GPD)



One-loop test

Hard part

$$K_{\rm DGLAP}(x,y,\xi,t,\mu_{\rm UV},\mu_F) = \delta(x-y) + \frac{\alpha_s C_F}{2\pi}\Bigg[\frac{\delta(x-y)}{2}\left\{-\ln^2\frac{\mu_F^2}{-t} - 3\ln\frac{\mu_F^2}{-t} + \frac{\pi^2}{6} - \ln^2\frac{y^2-\xi^2}{(1-x)^2}\right\}$$
$$+\left(\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu_{\rm UV}^2}{-t}\right)\left\{\frac{x^2+y^2-2\xi^2}{y^2-\xi^2}\left[\frac{\theta(y-x)}{y-x}\right]_+ + \delta(x-y)\left(\frac{3}{2} + \ln\frac{(1-x)^2}{y^2-\xi^2}\right)\right\}$$
$$+\frac{(x^2+y^2-2\xi^2)\ln(y^2-\xi^2)-(x-y)^2}{y^2-\xi^2}\left[\frac{\theta(y-x)}{y-x}\right]_+ - 2\frac{x^2+y^2-2\xi^2}{y^2-\xi^2}\left[\frac{\theta(y-x)\ln(y-x)}{y-x}\right]_+\Bigg].$$

$$K_{\rm ERBL}(x,y,\xi,t,\mu_{\rm UV}) = \frac{\alpha_s C_F}{2\pi}\Bigg[\left(\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu_{\rm UV}^2}{-t}\right)\frac{x+\xi}{y+\xi}\left(\frac{1}{2\xi} + \frac{1}{y-x}\right) - \frac{x+\xi}{2\xi(y+\xi)}$$
$$+\frac{1}{2\xi(y-x)(y^2-\xi^2)}\Bigg\{(x-y)(x+\xi)(y+\xi)\ln\frac{\xi^2-x^2}{4\xi^2} + 2\xi x^2\ln\frac{2\xi(y+\xi)}{(x+\xi)(y-x)}$$
$$+2\xi^3\ln\frac{\xi+x}{\xi-x} + 2\xi y^2\ln\frac{(y+\xi)(\xi-x)}{2\xi(y-x)} + 4\xi^3\ln\frac{y-x}{y+\xi}\Bigg\}\Bigg].$$

integrate over x

$$1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{\mu_F^2}{-t} - 3\ln\frac{\mu_F^2}{-t} - 8 + \frac{\pi^2}{6}\right)$$

Independent of ξ, as it should.
But only after a highly-nontrivial cancellation between the DGLAP and ERBL regions

Jet function

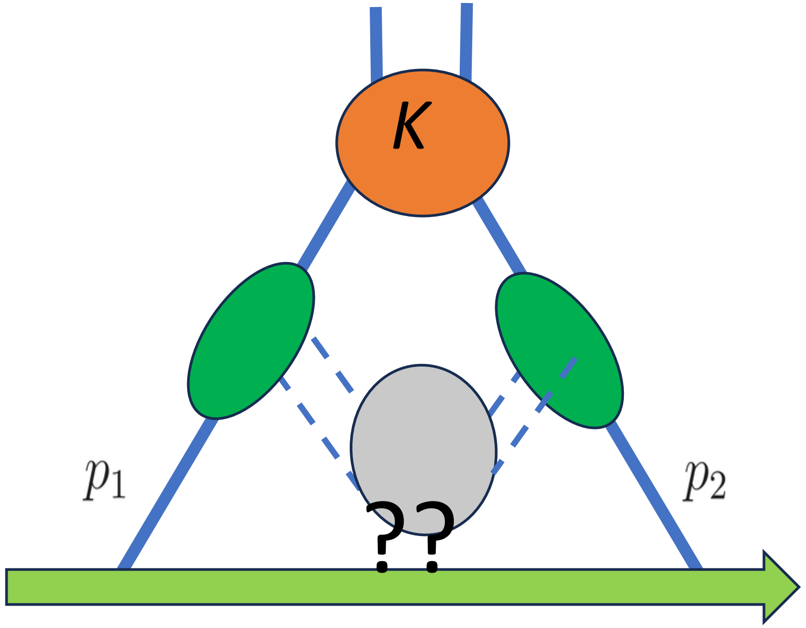
$$J(p^2,\mu_F) = 1 + \frac{\alpha_s C_F}{8\pi}\left(2\ln^2\frac{\mu_F^2}{-p^2} + 3\ln\frac{\mu_F^2}{-p^2} + 7 - \frac{\pi^2}{3}\right)$$

Soft function

$$S(t,p_1^2,p_2^2,\mu_F) = 1 - \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{-t\mu_F^2}{p_1^2p_2^2} + \frac{\pi^2}{2}\right)$$

RG-improvement standard, Sudakov suppression in K

From quark GPD to proton GPD



Embed the quark-in-quark GPD to quark-in-hadron GPD

parton virtuality should be chosen to be the hard-collinear scale

$$p_1^2, p_2^2 \sim \sqrt{-t}\Lambda$$

Drell-Yan; West;
Duncan-Mueller

Power-counting parameter is now

$$\lambda \equiv \frac{\sqrt{-p^2}}{\sqrt{-t}} = \frac{\sqrt{\Lambda}}{(-t)^{1/4}}$$

Two kinematical regimes need to be considered separately:
Hard-collinear scale perturbative or nonperturbative

1. Asymptotic regime

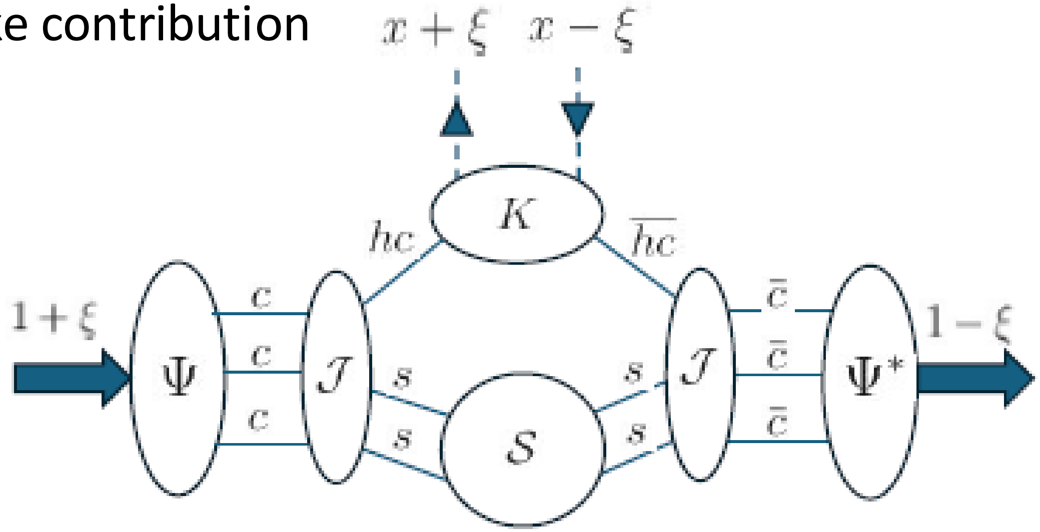
$$10 \sim 25\ \mathrm{GeV}^2 \ll |t| \to \infty,$$

Hard-collinear scale perturbative $p_{1,2}^2 \sim \sqrt{-t\Lambda} \gg 1\mathrm{GeV}^2$

Integrate the scale $\sqrt{-t\Lambda}$ - -> SCET factorization of Feynman-like contribution

Kivel, Vanderhaeghen (2010,2014); Kivel (2012)

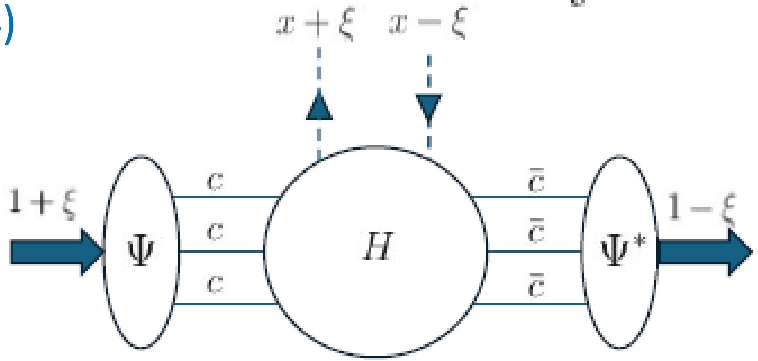
$$\frac{\alpha_s^4(\sqrt{t\Lambda})}{t^2}$$



Same topology as in Duncan-Mueller

Small correction to the standard contribution $\frac{\alpha_s^2(t)}{t^2}$

Hoodbhoy, Ji, Yuan (2004)



2. Pre-asymptotic regime (region that matters in practice)

Hard collinear scale nonperturbative

$$p_{1,2}^2 \sim \sqrt{-t}\Lambda < 1\mathrm{GeV}^2 \qquad \Lambda^2 \ll |t| \lesssim 10 \sim 25\ \mathrm{GeV}^2,$$

Factorization

YH, Schoenleber (2025)

$$H_q^{\mathrm{Feyn}}(x,\xi,t,\mu_{\mathrm{UV}}) = K_{qq}(x,\xi,t,\mu_{\mathrm{UV}},\mu_F)U(\mu_F,\mu_{hc})f_1^q(t,\mu_{hc}) + \mathcal{O}(\lambda^{10})$$

Sudakov

$$\langle P_2|\bar{\chi}_c(0)\bar{\not{n}}Y_{\bar{w}}^{\dagger}(0)Y_w(0)\chi_c(0)|P_1\rangle = f_1^q(t,\mu_{hc})\bar{N}(P_2)\bar{\not{n}}_{\perp}N(P_1)$$

Same matrix element as in the factorization of elemag form factor

Kivel, Vanderhaeghen (2010,2012,2014)

Remarks:

1 $H_q^{\mathrm{Feyn}}(x,t) \approx \delta(1-x) f_1^q(t)$ to leading order, under strict power-counting

2 Power-counting argument

$$H_q^{\mathrm{Feyn}}(x,t),\, F_{q1}^{\mathrm{Feyn}}(t),\, A_q^{\mathrm{Feyn}}(t) \sim \frac{1}{t^2}$$

not $1/t^3$
as is often assumed.

3 Gluon GPD

$$H_g^{\mathrm{Feyn}} \sim K_{gg} \frac{1}{t^{5/2}} + K_{gq} \frac{\alpha_s}{t^2}$$

4 Meson GPD

$$H_q^{\mathrm{Feyn}} \sim \frac{1}{t^{3/2}}$$ Power-suppressed

$$H_q^{\mathrm{hard}} \sim \frac{\alpha_s}{t}$$ α_s -suppressed

→ Roughly of the same size

Ratio of form factors perturbatively calculable!

YH, Schoenleber, 2508.01529

The sole nonperturbative input $f_1^q(t)$ cancels when taking ratios

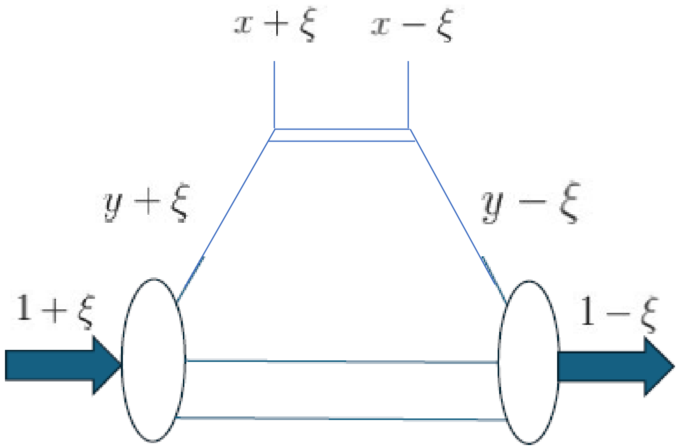
$$\frac{A_q^{\rm Feyn}(t,\mu_{\rm UV})}{F_{1q}^{\rm Feyn}(t)} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{8}{3}\ln\frac{-t}{\mu_{\rm UV}^2} - \frac{52}{9}\right) + \mathcal{O}\left(\alpha_s^2, \frac{1}{\sqrt{-t}}\right) \approx 0.85$$

Recently tested in lattice QCD Dutrieux et al. 2604.21476

Using numerical values from our previous study [55] gives 0.96, while values from our GPR reconstruction give 0.54.

Overlap representation of GPD/GFF

Diehl, Feldmann, Jakob, Kroll (2001)



$y = 1$ in strict power counting.
One parton carries 100% of proton momentum

More realistically, $1 - y \sim \frac{\Lambda}{\sqrt{-t}} \ll 1$ Drell, Yan, West

quark-in-quark GPD at tree level

$$H_q(x,\xi,t) = \sum_{N=3}^{\infty} \int dy d^2 p_\perp \int \prod_{i=1}^{N-1} dx_i d^2 k_{\perp i} \delta(x-y)$$
$$\times \Phi_N^* \left(\frac{y-\xi}{1-\xi}, p_\perp + \frac{1-y}{1-\xi}\frac{\Delta_\perp}{2}; \frac{x_i}{1-\xi}, k_{\perp i} - \frac{x_i}{1-\xi}\frac{\Delta_\perp}{2} \right)$$
$$\times \Phi_N \left(\frac{y+\xi}{1+\xi}, p_\perp - \frac{1-y}{1+\xi}\frac{\Delta_\perp}{2}; \frac{x_i}{1+\xi}, k_{\perp i} + \frac{x_i}{1+\xi}\frac{\Delta_\perp}{2} \right)$$

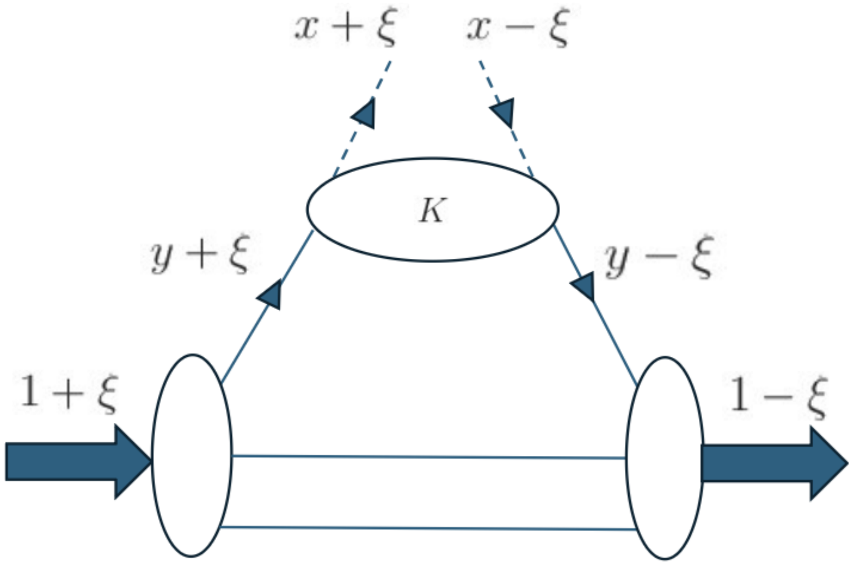
Light-cone wavefunctions

QCD-improved overlap representation

$$H_q^{\text{Feyn}}(x,\xi,t,\mu_{\text{UV}}) = \sum_N \int_x^1 dy \int d^2p_\perp \prod_i dx_i d^2p_{\perp i} \underline{K_{\text{DGLAP}}(x,y,\xi,t,\mu_{\text{UV}},\mu_F)} U(\mu_F,\mu_{hc}) \Phi_N^* \Phi_N \,.$$

$$\delta(x-y) + \mathcal{O}(\alpha_s)$$

Sudakov



All-order Sudakov factor

$$\exp\left(-\frac{\alpha_s C_F}{2\pi}\ln^2\frac{-t}{\mu_{hc}^2}\right) \sim \exp\left(-\frac{\alpha_s C_F}{8\pi}\ln^2\frac{-t}{\Lambda^2}\right)$$

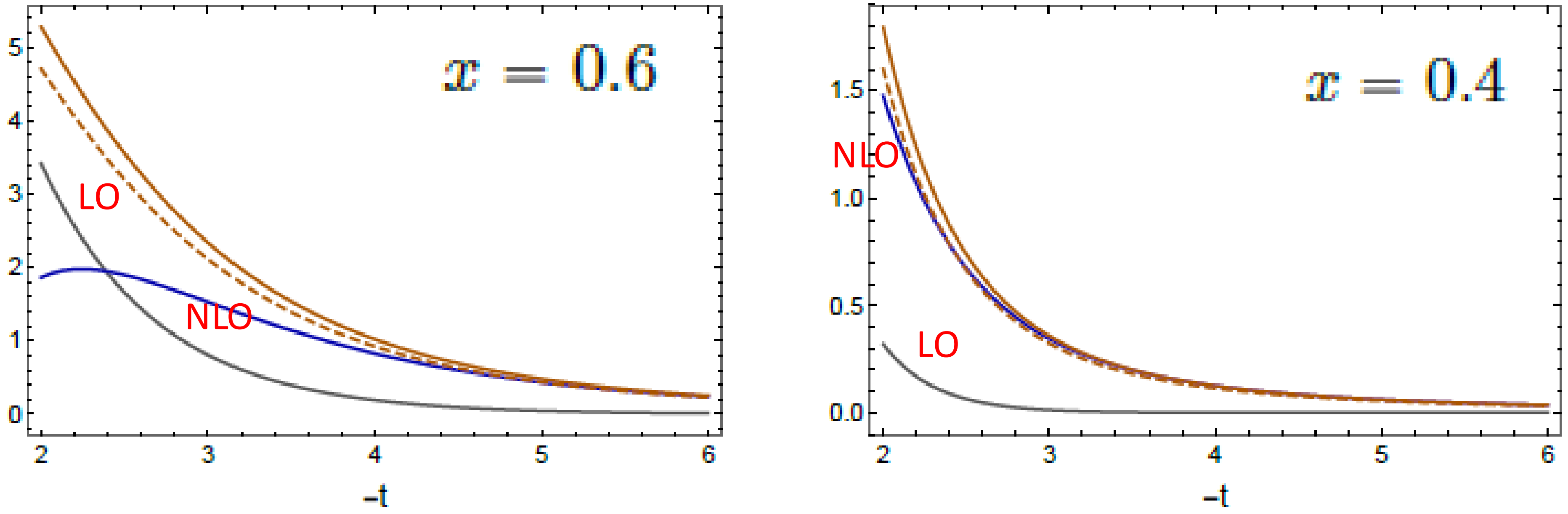
relatively mild suppression

Use the model $\Phi_3 \propto (1-y)^{\delta} \exp\left(-\frac{p_\perp^2 + k_{\perp 1}^2 + k_{\perp 2}^2}{\Lambda^2}\right)$

$$H_q^{\text{Feyn}}(x) \sim (1-x)^{2\delta+1} \exp\left(\frac{5t}{6\Lambda^2}(1-x)^2 + \cdots\right) + \mathcal{O}\left(\frac{\alpha_s}{\pi}\left(\frac{\Lambda^2}{-t}\right)^{\delta+1}\right)$$

Leading term exponentially suppressed
Becomes a power-law after x-integral

NLO decays only by power-law



Gravitational form factor at large, but not too large-t

$$A_q(t) \sim U\left(\frac{\Lambda^2}{-t}\right)^{\delta+1} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\Lambda^2}{-t}\right)^2,$$

$$D_q(t) \sim U\frac{\alpha_s}{\pi}\left(\frac{\Lambda^2}{-t}\right)^{\delta+2} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\Lambda^2}{-t}\right)^3$$

Feynman contribution
YH, Schoenleber (2025)

Hard scattering contribution
Tanaka (2018); Tong, Ma, Yuan (2022)

Preferred value from SCET power-counting $\delta = 1$

Dipole parametrization $A_q(t) \sim \frac{1}{(1-t/m^2)^2}$

This is just the Feynman contribution dominating form factors for realistic values of t.
Large-t limit of this is NOT connected to the hard scattering contribution.

Summary

Resummation of nonlocal Sudakov form factor achieved for the first time

QCD factorization for the Feynman (soft) contribution to proton GPD.

Resummation of all the large logs in exclusive processes with two hard scales

$$\Lambda_{\rm QCD}^2 \ll |t| \ll Q^2$$

Large-t behavior of GFF revised.