EFFECT OF TMD EVOLUTION ON SIVERS ASYMMETRY AT EIC

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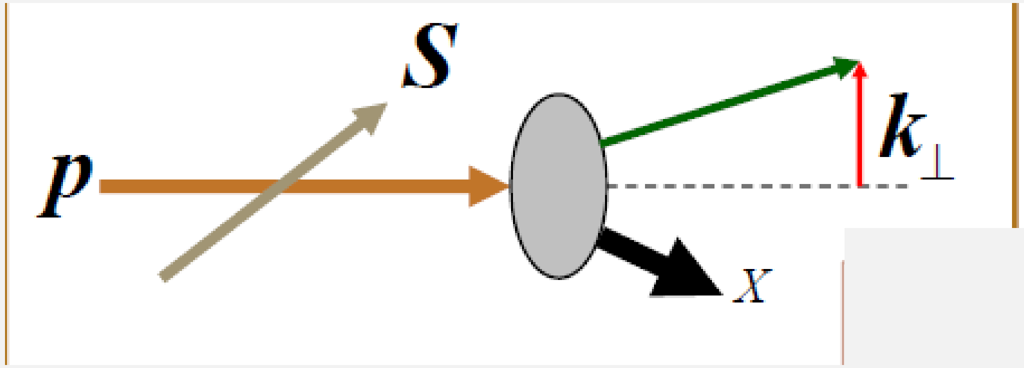
Sivers Function TMD



$$f_{q/p,S}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\, S\cdot(\hat{p}\times\hat{k}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\, S\cdot(\hat{p}\times\hat{k}_\perp)$$

Sivers function TMD (D. Sivers. (1990)) is related to the probability to find an unpolarized quark inside a transversely polarized nucleon

It includes the correlation between the quark intrinsic transverse momentum and the transverse spin of the proton

Gives the distribution of unpolarized quarks/ gluons in a transversely polarized nucleon, which is not left-right symmetric with respect to the plane formed by the transverse momentum and spin of the nucleon.

In some models it is related to the orbital angular momentum of the quarks

M. Burkardt, Nucl. Phys. A735, 185 (2004)

T-odd function ; combines with another T-odd effect to give an observable asymmetry

NON-ZERO SIVERS ASYMMETRY

The Sivers function introduces an asymmetry, for example, in the azimuthal angle of the observed final state hadron in semi-inclusive deep inelastic scattering (SIDIS) and in the azimuthal angle correlations of the lepton pair in Drell-Yan process or back-to-back jets in pp collision, called the Sivers asymmetry.

The first transverse moment of the Sivers function is related to the twist-three Qiu-Sterman function.

Qiu and Sterman, PRL (1991)

Experimental information on a nonzero Sivers function for quarks was first obtained from HERMES and COMPASS

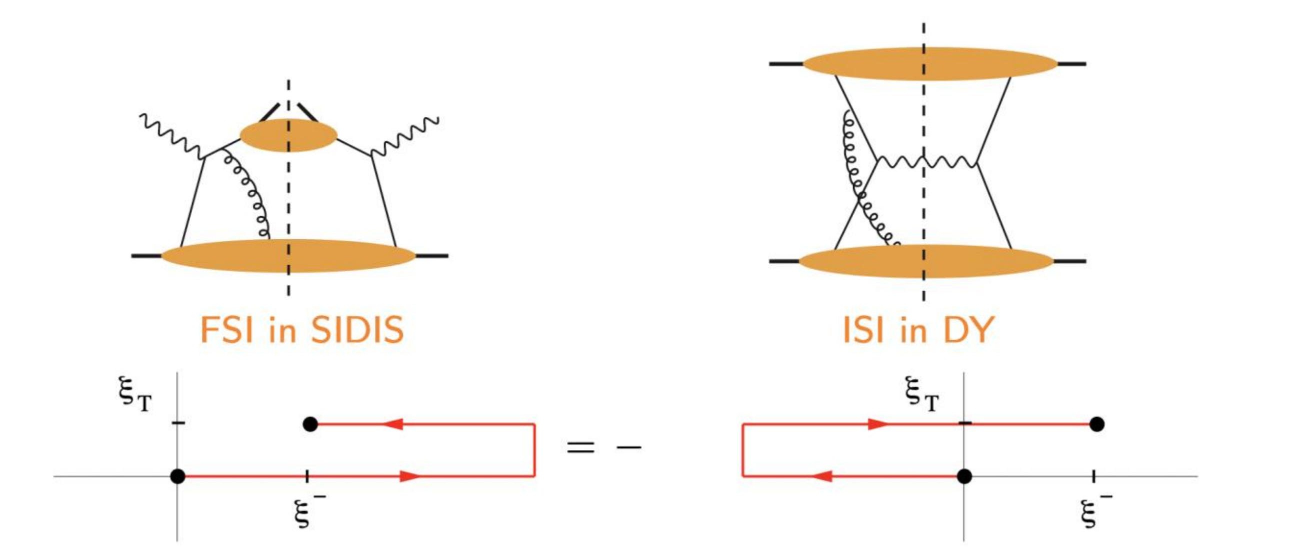
HERMES Collaboration, PRL94, 012002 (2005).

COMPASS Collaboration, PLBB 717, 383 (2012)

Parametrization of the Sivers function incorporating the TMD evolution has been obtained

M. G. Echevarria, A. Idilbi, Z.-B. Kang, and I. Vitev, PRD 89, 074013 (2014).

SIVERS FUNCTION: GAUGE LINK



$$f_{1T}^{\perp\,[DY]}(x, \boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x, \boldsymbol{k}_\perp^2)$$

However, more complex processes have complex gauge link structure, and factorization is not always guaranteed

Sivers function is a T-odd object and initial and final state interactions play an important role in Sivers asymmetry. They are resummed into the gauge link or Wilson line in the operator definition of the Sivers function that is needed for color gauge invariance.

Sivers function in Drell-Yan process is same in magnitude but opposite in sign compared to the Sivers function probed in semi-inclusive DIS

Collins, PLB (2002); Boer, Mulders, Pijlman, Nucl. Phy. B (2003)

GLUON SIVERS FUNCTION

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Very little is known about GSF apart from a positivity bound

P. J. Mulders and J. Rodrigues, Phys. Rev. D 63, 094021 (2001).

Gluon TMDs need two gauge links for color gauge invariance unlike quark TMDs where there is one gauge link in the operator

Depending on the gauge link in the operator structure there can be two different gluon Sivers function, f-type and d-type

Bomhof and Mulders, JHEP 02, 029 (2007), Buffing, AM, Mulders, PRD 88, 054027 (2013)

Burkardt's sum rule, which states that the total transverse momentum of all quarks and gluon in a transversely polarized proton is zero, still leaves some room for GSF (30 %), moreover d type GSF is not constrained by it.

M. Burkardt, Phys. Rev. D 69, 091501 (2004)

GLUON SIVERS FUNCTION IN J/ψ PRODUCTION PROCESSES

Semi-inclusive J/ψ production in eP collision is a good cannel to probe gluon TMDs including gluon Sivers function, only f type gluon TMD contributes

For low transverse momentum region, TMD factorization is expected to hold and for large transverse momentum collinear factorization is applicable. In the intermediate region, results from these two formalisms should match

TMD factorized description of the process needs smearing effects to be taken into account in the form of TMD shape functions. The perturbative tail of the shape function can be obtained through a matching procedure.

M. G. Echevarria, JHEP (2019), Boer et al, JHEP (2023)

Also gluon TMDs can be probed in back-to-back production of J/ψ and photon/jet/pion, TMD factorization is expected to be valid. The small scale is provided by the transverse momentum of the pair. By varying the invariant mass of the pair scale evolution of the TMDs can be studied

Gluon Sivers function in pp collision at RHIC

U. D'Alesio, F. Murgia, and C. Pisano, Journal of High Energy Physics 2015, 1 (2015); U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Taels, Physical Review D 99, 036013 (2019)

Gluon Sivers function in di-jet production at EIC

Echevarria, Garcia, Scimemi; e-Print: 2603.00375 [hep-ph]

PRODUCTION OF J/Ψ IN NRQCD

In NRQCD the heavy quark pair is produced in the hard process either in color octet or in color singlet configuration

Then they hadronize to form a color singlet quarkonium state of given quantum numbers through soft gluon emission

Hard process is calculated perturbatively and soft process is given in terms of long distance matrix elements (LDMEs) that are determined from data

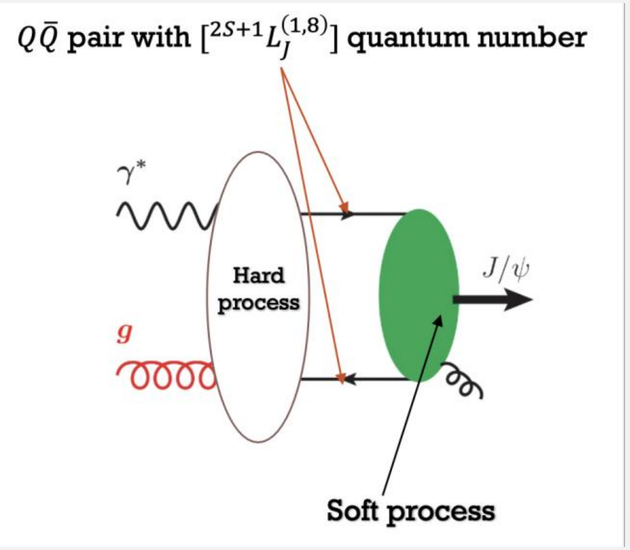
The LDMEs are categorized by performing an expansion in terms of the relative velocity of the heavy quark v in the limit $v \ll 1$

The theoretical predictions are arranged as double expansions in terms of v as well as α_s.

C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976).
E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
R. Baier and R. Ruckl, Phys. Lett. B 102B, 364 (1981).
R. Baier and R. Ruckl, Nucl. Phys. B201, 1 (1982).
E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).
P. L. Cho and A. K. Leibovich, Phys. Rev. D 53, 150 (1996).

PRODUCTION OF J/Ψ IN NRQCD

J/ψ is a bound state of charm quark and anti-quark ($Q\bar{Q}$)



Long distance matrix elements (LDMEs) : Describes hadronization of of $Q\bar{Q}[n]$ states into final quarkonium state

NRQCD factorization

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

Perturbative short distance coefficient

Subprocess cross section for formation of heavy quark pair in particular color, angular momentum and spin state "n" : $^{2S+1}L_J$, calculated by perturbative QCD

G. T. Bodwin et al, PRD51 (1995), Lepage 95

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/Ψ AND JET

$$e(l) + p^{\uparrow}(P) \to J/\psi(P_\psi) + \text{jet}(P_j) + X,$$

$$q^2 \approx -2E_e E'_e(1-\cos\theta)$$

Consider the limit when 4 momentum of virtual photon $q^2 = -Q^2 \to 0$

$$E_\Psi E_j \frac{d\sigma}{d^3\boldsymbol{P}_\Psi d^3\boldsymbol{P}_j} = \frac{d\sigma}{d^2\boldsymbol{P}_{\Psi\perp} dz d^2\boldsymbol{P}_{j\perp} dz_1}$$
$$= \frac{1}{4(2\pi)^2}\frac{1}{zz_1}\sum_a \int dx_\gamma dx_a d^2\boldsymbol{p}_{a\perp} f_{\gamma/e}(x_\gamma)$$
$$\times f_{a/p}(x_a, p_{a\perp})$$
$$\times \delta^4(q+p-P_\Psi-P_j)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2,$$

When the exchanged photon is quasi-real, the interaction takes place through the Weizsäker-Williams distribution function of the electron, this describes the density of photons inside the electron and is given by

$$f_{\gamma/e}(x_\gamma) = \frac{\alpha}{2\pi}\left[2m_e^2 x_\gamma\left(\frac{1}{Q^2_{\text{min}}} - \frac{1}{Q^2_{\text{max}}}\right) + \frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2_{\text{max}}}{Q^2_{\text{min}}}\right],$$

$$Q^2_{\text{min}} = m_e^2\frac{x_\gamma^2}{1-x_\gamma},$$

$$z = \frac{P\cdot P_h}{P\cdot q} \text{ and } z_1 = \frac{P\cdot P_j}{P\cdot q}$$

Both CS and CO contributions are Considered $^3S_1^{(1,8)}$, $^1S_0^{(8)}$ and $^3P_J^{(8)}$

Dominant subprocess

$$\gamma(q) + g(p) \to J/\psi(P_\psi) + g(P_j)$$

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/Ψ AND JET

Outgoing J/ψ and jet are almost back $\boldsymbol{q}_\perp = \boldsymbol{P}_{\Psi\perp} + \boldsymbol{P}_{j\perp}$ $\boldsymbol{K}_\perp = (\boldsymbol{P}_{\Psi\perp} - \boldsymbol{P}_{j\perp})/2.$ $|\boldsymbol{q}_\perp| \ll |\boldsymbol{K}_\perp|$

For transversely polarized proton

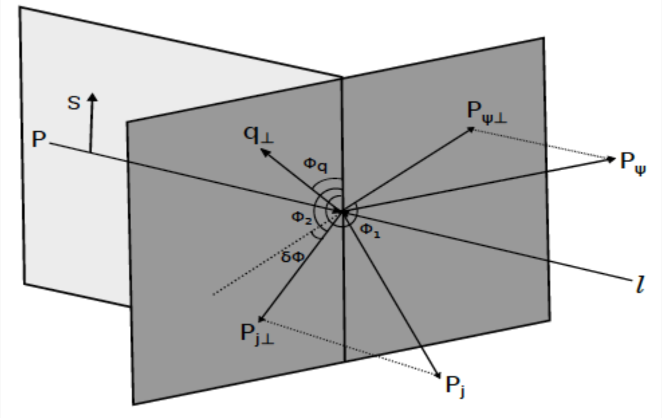
$$\frac{d\sigma^{\uparrow(\downarrow)}}{d^2\boldsymbol{q}_\perp dz d^2\boldsymbol{K}_\perp} = \frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma) f_{a/p^{\uparrow(\downarrow)}}(x_a, q_\perp)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2.$$

Weighted Sivers asymmetry

$$A_N^{W(\phi_q)} \equiv \frac{\int d\phi_q W(\phi_q)(\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow)}{\int d\phi_q (\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow)} \equiv \frac{\int d\phi_q W(\phi_q)\mathrm{d}\Delta\sigma(\phi_q)}{\int d\phi_q 2\mathrm{d}\sigma},$$

Weight factor for Sivers asymmetry

$$-\sin(\phi_q) = \frac{(\boldsymbol{S}\times\hat{\boldsymbol{P}})\cdot\boldsymbol{q}_\perp}{|\boldsymbol{S}\times\hat{\boldsymbol{P}}||\hat{\boldsymbol{P}}\times\boldsymbol{q}_\perp|}.$$



SIVERS ASYMMETRY

Cross section for transversely polarized proton

$$\begin{aligned}
\mathrm{d}\Delta\sigma &\equiv \frac{d\sigma^{\uparrow}}{d^2\boldsymbol{q}_{\perp}dz d^2\boldsymbol{K}_{\perp}} - \frac{d\sigma^{\downarrow}}{d^2\boldsymbol{q}_{\perp}dz d^2\boldsymbol{K}_{\perp}} \\
&= \frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma)\Delta\hat{f}_{a/p}(x_a,\boldsymbol{q}_{\perp}) \\
&\times \frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2,
\end{aligned}$$

$\gamma + g \to J/\psi + g$

$$\begin{aligned}
|\mathcal{M}[{}^1S_0^{(8)}]|^2 = {} & \frac{3\pi^3 e_c^2\alpha_s^2\alpha}{4M}\langle 0|\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle \frac{128}{s_1^2t_1^2u_1^2(M^2+u_1)^2}\{8M^{14}+4M^{12} \\
& \times(4(s_1+t_1)+7u_1)+2M^{10}(8s_1^2+17u_1(s_1+t_1)+12s_1t_1+8t_1^2+19u_1^2) \\
& +2M^8(7s_1^3+4u_1(s_1^2+5s_1t_1+t_1^2)+5s_1^2t_1+5s_1t_1^2+6u_1^2(s_1+t_1)+7t_1^3 \\
& +13u_1^3)+2M^6(2s_1^4+4u_1(s_1^3+t_1^3)+s_1^3t_1+u_1^2(-17s_1^2+7s_1t_1-17t_1^2) \\
& +2s_1^2t_1^2+s_1t_1^3-8u_1^3(s_1+t_1)+2t_1^4+5u_1^4)+2M^4(6u_1^3(-3s_1^2+s_1t_1-3t_1^2) \\
& -6u_1^2(s_1+t_1)(s_1^2+t_1^2)+u_1(s_1-t_1)^2(s_1^2+4s_1t_1+t_1^2)-6u_1^4(s_1+t_1) \\
& -3s_1t_1(s_1-t_1)^2(s_1+t_1)+u_1^5)+M^2(-2u_1^4(5s_1^2-11s_1t_1+5t_1^2)+u_1^2 \\
& \times(-2s_1^4+s_1^3t_1-5s_1^2t_1^2+s_1t_1^3-2t_1^4)-2u_1^5(s_1+t_1)-6u_1^3(s_1-t_1)^2(s_1+t_1) \\
& +s_1t_1u_1(s_1-t_1)^2(s_1+t_1)-s_1t_1(s_1-t_1)^2(2s_1+t_1)(s_1+2t_1)) \\
& +s_1t_1u_1(3u_1^2(s_1^2+s_1t_1+t_1^2)+(s_1-t_1)^2(s_1^2+s_1t_1+t_1^2)+8u_1^3(s_1+t_1)+8u_1^4)\}
\end{aligned}$$

Some of the subprocess contributions are given below :

$\gamma + q(\bar{q}) \to J/\psi + q(\bar{q})$

$$|\mathcal{M}({}^3S_1^{(1)})|^2 = 0,$$

$$|\mathcal{M}({}^3S_1^{(8)})|^2 = \frac{-2(4\pi)^3e_c^2\alpha_s^2\alpha}{9M^3\hat{s}\,\hat{t}}\langle 0|\mathcal{O}_8^{J/\psi}({}^3S_1)|0\rangle[\hat{s}^2+\hat{t}^2+2\hat{u}M^2],$$

$$|\mathcal{M}({}^1S_0^{(8)})|^2 = \frac{-4(4\pi)^3e_c^2\alpha_s^2\alpha}{3M}\langle 0|\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle\frac{\hat{s}^2+\hat{t}^2}{\hat{u}(\hat{s}+\hat{t})^2},$$

$$|\mathcal{M}({}^3P_0^{(8)})|^2 = \frac{-16(4\pi)^3e_c^2\alpha_s^2\alpha}{9M^3}\langle 0|\mathcal{O}_8^{J/\psi}({}^3P_0)|0\rangle\frac{(\hat{s}^2+\hat{t}^2)(\hat{u}-3M^2)^2}{\hat{u}(\hat{s}+\hat{t})^4},$$

$$|\mathcal{M}({}^3P_1^{(8)})|^2 = \frac{-32(4\pi)^3e_c^2\alpha_s^2\alpha}{9M^3}\langle 0|\mathcal{O}_8^{J/\psi}({}^3P_1)|0\rangle\frac{(\hat{s}^2+\hat{t}^2)\hat{u}+4M^2\hat{s}\hat{t}}{(\hat{s}+\hat{t})^4},$$

Rajesh, Kishore, AM, PRD 98,014007(2018);
Kishore, AM, Rajesh, PRD 101, 054003 (2020)

SIVERS FUNCTION PARAMETRIZATION

Gaussian parametrization

$$\Delta^N f_{a/p^\uparrow}(x_a, q_\perp) = 2\frac{\sqrt{2e}}{\pi}\mathcal{N}_a(x_a) f_{a/p}(x_a)\sqrt{\frac{1-\rho}{\rho}} q_\perp \frac{e^{-q_\perp^2/\rho\langle q_\perp^2\rangle}}{\langle q_\perp^2\rangle^{3/2}}.$$

$$f_{a/p}(x_a, q_\perp) = \frac{1}{\pi\langle q_\perp^2\rangle} f(x_a) e^{-q_\perp^2/\langle q_\perp^2\rangle}.$$

$$\mathcal{N}_a(x_a) = N_a x_a^\alpha (1-x_a)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta},$$

Satisfies positivity bound

Best fit parameters for Sivers function are obtained from SIDIS and RHIC data

Evolution	a	N_a	α	β	ρ	$\langle q_\perp^2\rangle$ GeV^2	Notation
DGLAP	g [7]	0.65	2.8	2.8	0.687	0.25	SIDIS1
	g [7]	0.05	0.8	1.4	0.576	0.25	SIDIS2
	g [8]	0.25	0.6	0.6	0.1	1.0	SIDIS3
TMD	u [13]	0.106	1.051	4.857		0.38	TMD-a
	d [13]	−0.163	1.552	4.857		0.38	TMD-b

U. D’Alesio, F. Murgia, and C. Pisano, J. High Energy Phys. (2015) 119 ; U. D’Alesio, C. Flore, F. Murgia, C. Pisano, and P. Taels, Phys. Rev. D 99, 036013 (2019).

INCORPORATING TMD EVOLUTION

Renormalization group and Collins-Soper equations are obtained by taking scale evolution with respect to μ and ζ

light-cone (rapidity) divergences in TMD factorization is regulated by the the rapidity scale ζ

$$f(x_a, b_\perp, \mu) = \int d^2 p_{a\perp} e^{-i b_\perp \cdot p_{a\perp}} f(x_a, p_{a\perp}, \mu),$$

J. Collins, Foundations of Perturbative QCD (Cambridge University Press, Cambridge, England, 2013); S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, Phys. Rev. D 85, 034043 (2012).

$$f(x_a, b_\perp, Q_f, \zeta) = f(x_a, b_\perp, Q_i) R_{\text{pert}}(Q_f, Q_i, b_*) R_{NP}(Q_f, b_\perp)$$ — A

Perturbative and non-perturbative parts are given by

$$R_{\text{pert}}(Q_f, b_*) = \exp\left\{-\int_{c/b_*}^{Q_f} \frac{d\mu}{\mu}\left(A \log\left(\frac{Q_f^2}{\mu^2}\right) + B\right)\right\},$$

$$R_{NP}(Q_f, b_\perp) = \exp\left\{-\left[g_1^{\text{TMD}} + \frac{g_2}{2}\log\frac{Q_f}{Q_0}\right] b_\perp^2\right\}$$

Depends on specific TMD

Anomalous dimensions are obtained order by order in perturbative series

$A_1 = C_A,\ A_2 = \frac{1}{2} C_F\left(C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} C_A N_f\right)$ $B_1 = -\frac{1}{2}\left(\frac{11}{3} C_A - \frac{2}{3} N_f\right),$ Gluons

$A_1 = C_F,\ A_2 = \frac{1}{2} C_F\left(C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} N_f\right)$ $B_1 = -\frac{3}{2} C_F$ Quarks

Echevarria, Idilbi, Kang, Vitev PRD 89 (2014)

TMD EVOLUTION

Initial scale of TMDs $Q_i = c/b_*(b_\perp)$ $c = 2e^{-\gamma_\epsilon}$

Large $b_\perp$ region perturbation theory breaks down

b* prescription allows us to use perturbation theory in the region where it is valid, while including nonperturbative effects separately.

$$b_*(b_\perp) = \frac{b_\perp}{\sqrt{1+(\frac{b_\perp}{b_{\max}})^2}}$$

$$b^*(b_\perp) \approx b_{max} \quad as \quad b_\perp \to \infty$$
$$b^*(b_\perp) \approx b_\perp \quad as \quad b_\perp \to 0$$

Unpolarized TMD

$$f(x_a,b_\perp,Q_i)=\sum_{i=g,q}\int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/a}(x_a/\hat{x},b_\perp,\alpha_s,Q_i) f_{i/p}(\hat{x},c/b_*) + \mathcal{O}(b_\perp \Lambda_{\rm QCD}),$$

perturbatively calculated process independent coefficient function, different for each type of TMD.

Sivers function

$$f_{1T}^{\perp}(x_a,p_{a\perp},Q_f) = -\frac{1}{2\pi p_{a\perp}}\int_0^\infty db_\perp b_\perp J_1(p_{a\perp}b_\perp) f_{1T}^{\prime\perp}(x_a,b_\perp,Q_f),$$

Derivative of Sivers function related to Qiu-Sterman function

$$f_{1T}^{\prime\perp}(x_a,b_\perp,Q_i) \simeq \frac{M_p b_\perp}{2} T_{a,F}(x_a,x_a,Q_i),$$

Derivative of Sivers function follows the same evolution equation as (A)

Assumed to be given in terms of collinear pdfs

$$T_{a,F}(x_a,x_a,Q_i) = \mathcal{N}_a(x_a) f_{a/p}(x_a,Q_i).$$

Echevarria, Idilbi, Kang, and Vitev, Phys. Rev. D 89, 074013 (2014); Kouvaris, Qiu, Vogelsang, Yuan, Phys. Rev. D 74, 114013 (2006).

SIVERS FUNCTION

$$(a)\ \mathcal{N}_g(x_g) = (\mathcal{N}_u(x_g) + \mathcal{N}_d(x_g))/2,$$
$$(b)\ \mathcal{N}_g(x_g) = \mathcal{N}_d(x_g).$$

Boer and Vogelsang, Phys. Rev. D 69, 094025 (2004).

$$\mathrm{d}\Delta\sigma = -\frac{1}{\pi M_p}\frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a\int\frac{dx_\gamma}{x_\gamma}db_\perp b_\perp J_1(q_\perp b_\perp)f'^{\perp}_{1T}(x_a,b_\perp,Q_f)f_{\gamma/e}(x_\gamma)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2\sin(\phi_q),$$

$$2d\sigma = \frac{1}{2(2\pi)^2\pi}\frac{1}{z(1-z)s}\sum_a\int\frac{dx_\gamma}{x_\gamma}db_\perp b_\perp J_0(q_\perp b_\perp)f_{a/p}(x_a,b_\perp,Q_f)f_{\gamma/e}(x_\gamma)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2.$$

Assuming contributions only from u and d quarks and gluons, we have checked that TMD-(a) parametrization satisfies the Burkardt sum rule, and violation is about 1%; TMD-(b) violates the sum rule by about 19 %

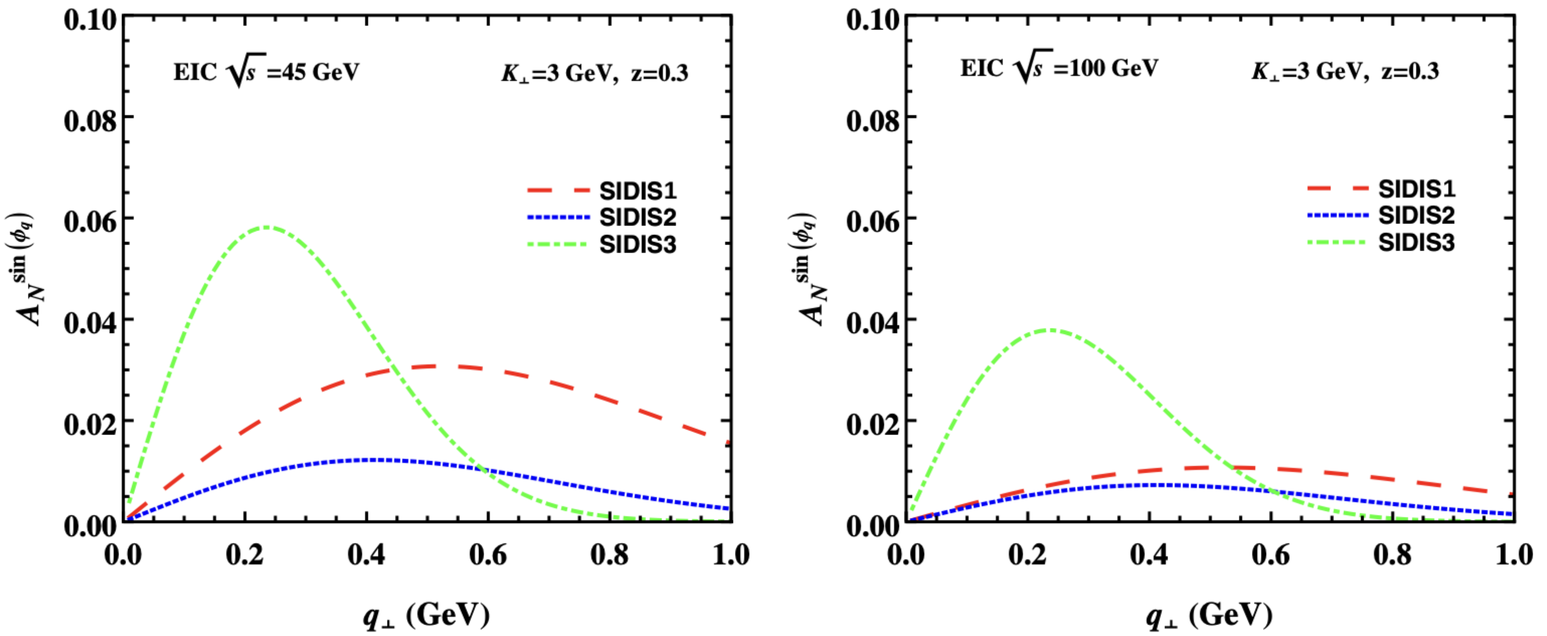
Upper bound of Sivers asymmetry is obtained by saturating the positivity constraint

$$|\Delta^N f_{a/p^\uparrow}(x_a,q_\perp)| \le 2f_{a/p}(x_a,q_\perp),$$
$$\frac{q_\perp}{M_p}|f^{\perp}_{1T}(x_a,q_\perp)| \le f_{a/p}(x_a,q_\perp).$$

$$\begin{aligned}\Delta\hat{f}_{a/p^\uparrow}(x_a,\boldsymbol{q}_\perp) &\equiv \hat{f}_{a/p^\uparrow}(x_a,\boldsymbol{q}_\perp) - \hat{f}_{a/p^\downarrow}(x_a,\boldsymbol{q}_\perp)\\ &= \Delta^N f_{a/p^\uparrow}(x_a,q_\perp)\hat{\boldsymbol{S}}\cdot(\hat{P}\times\hat{\boldsymbol{q}}_\perp)\\ &= -\Delta^N f_{a/p^\uparrow}(x_a,q_\perp)\sin(\phi_q)\\ &= -\frac{2}{M_p}f^{\perp}_{1T}(x_a,q_\perp)\hat{\boldsymbol{S}}\cdot(\hat{P}\times\boldsymbol{q}_\perp).\end{aligned}$$

Sivers function in Trento convention

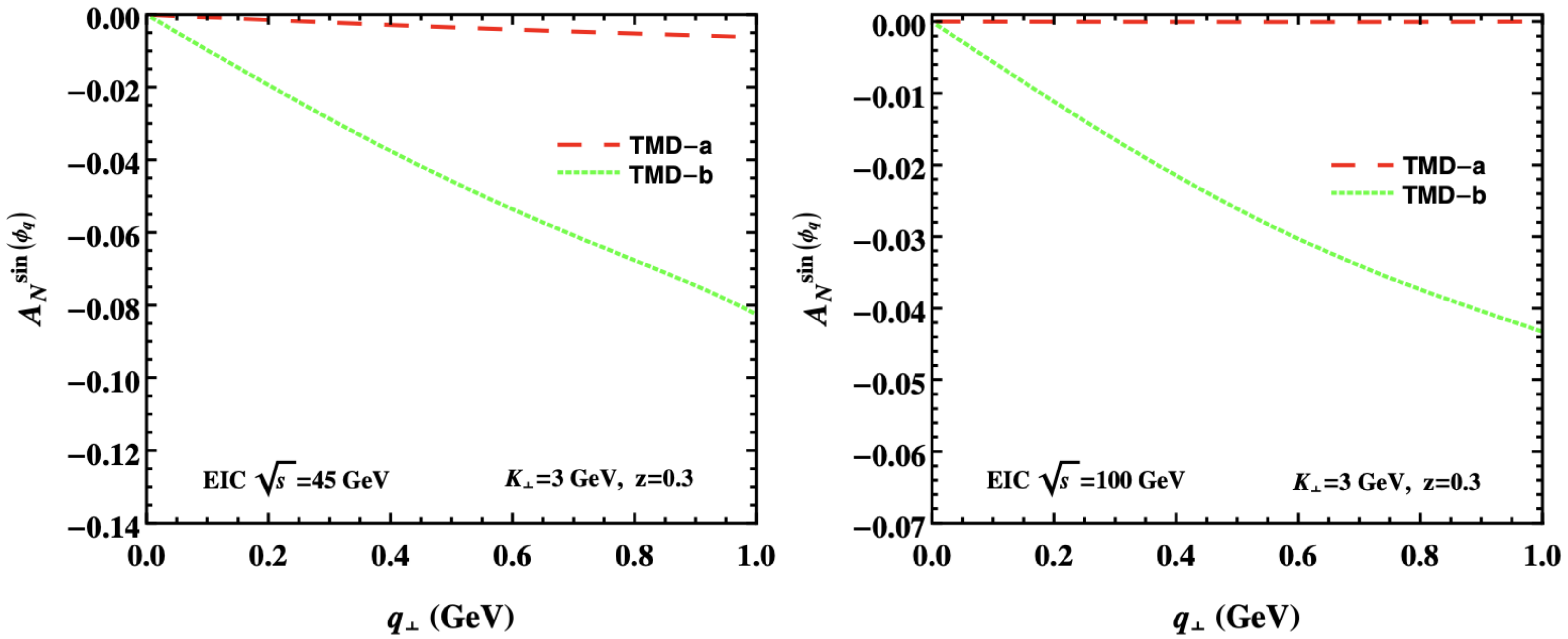
SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/Ψ AND JET



The weighted Sivers asymmetry in the $e+p^\uparrow \to J/\psi+\text{jet}+X$ process as a function of $q_\perp$ at EIC (a) $\sqrt{s}=45$ GeV and (b) $\sqrt{s}=100$ GeV using the DGLAP evolution approach for SIDIS1, SIDIS2, and SIDIS3 GSF parametrization sets which are given in Table I.

Kishore, AM, Rajesh, PRD 101, 054003 (2020)

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/Ψ AND JET



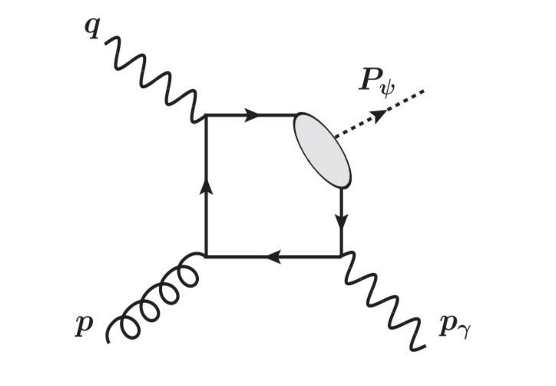
The weighted Sivers asymmetry in the $e + p^{\uparrow} \rightarrow J/\psi + \text{jet} + X$ process as a function of $q_\perp$ at EIC (a) $\sqrt{s} = 45$ GeV and (b) $\sqrt{s} = 100$ GeV using the TMD evolution approach for TMD-a and TMD-b GSF parametrization sets which are given in Table I.

Kishore, AM, Rajesh, PRD 101, 054003 (2020)

SIVERS ASYMMETRY IN J/Ψ AND PHOTON ELECTRO PRODUCTION

We consider the following process where the proton can be unpolarized or transversely polarized

$$e(l) + p^{\uparrow}(P) \to e(l') + J/\psi(P_\psi) + \gamma(p_\gamma) + X,$$



$$P^\mu = n_-^\mu + \frac{M_p^2}{2} n_+^\mu \approx n_-^\mu,$$
$$q^\mu = -x_B n_-^\mu + \frac{Q^2}{2x_B} n_+^\mu \approx -x_B P^\mu + (P \cdot q) n_+^\mu,$$

$$l^\mu = \frac{1-y}{y} x_B n_-^\mu + \frac{1}{y}\frac{Q^2}{2x_B} n_+^\mu + \frac{\sqrt{1-y}}{y} Q \hat{l}_\perp^\mu,$$

$$Q^2 = x_b y S \qquad S = (P+l)^2$$

$$y = \frac{P \cdot q}{P \cdot l} \qquad x_B = \frac{Q^2}{2P \cdot q}$$

Partonic subprocess : virtual photon-gluon fusion

$$d\sigma = \frac{1}{2S} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_\psi}{(2\pi)^3 2E_\psi} \frac{d^3 p_\gamma}{(2\pi)^3 2E_\gamma} \times \int dx d^2 p_T (2\pi)^4 \delta^4(q + p - P_\psi - p_\gamma) \times \frac{1}{Q^4} L^{\mu\nu}(l,q) \Phi_g^{\rho\sigma}(x, p_T) H_{\mu\rho} H^*_{\nu\sigma}.$$

J/ψ production in color singlet channel in virtual photon-photon fusion is suppressed due to a larger gluon density

Use TMD factorization

D. Chakrabarti, R. Kishore, AM, S. Rajesh, *Phys.Rev.D* 107 (2023) 1, 014008

SIVERS ASYMMETRY IN BACK-TO-BACK J/ψ AND JET ELECTRO PRODUCTION

$$e^-(l) + p(\mathrm{P}) \to e^-(l') + J/\psi(\mathrm{P}_\psi) + \mathrm{jet}(\mathrm{P}_j) + X,$$

$$Q^2 = -q^2, \qquad s = (P+l)^2, \qquad W^2 = (P+q)^2,$$

$$x_B = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot \mathrm{P}_\psi}{P\cdot q}.$$

Use TMD factorization in the kinematics where the outgoing J/ψ and (gluon) jet are almost back-to back

Use NRQCD to calculate the J/ψ production

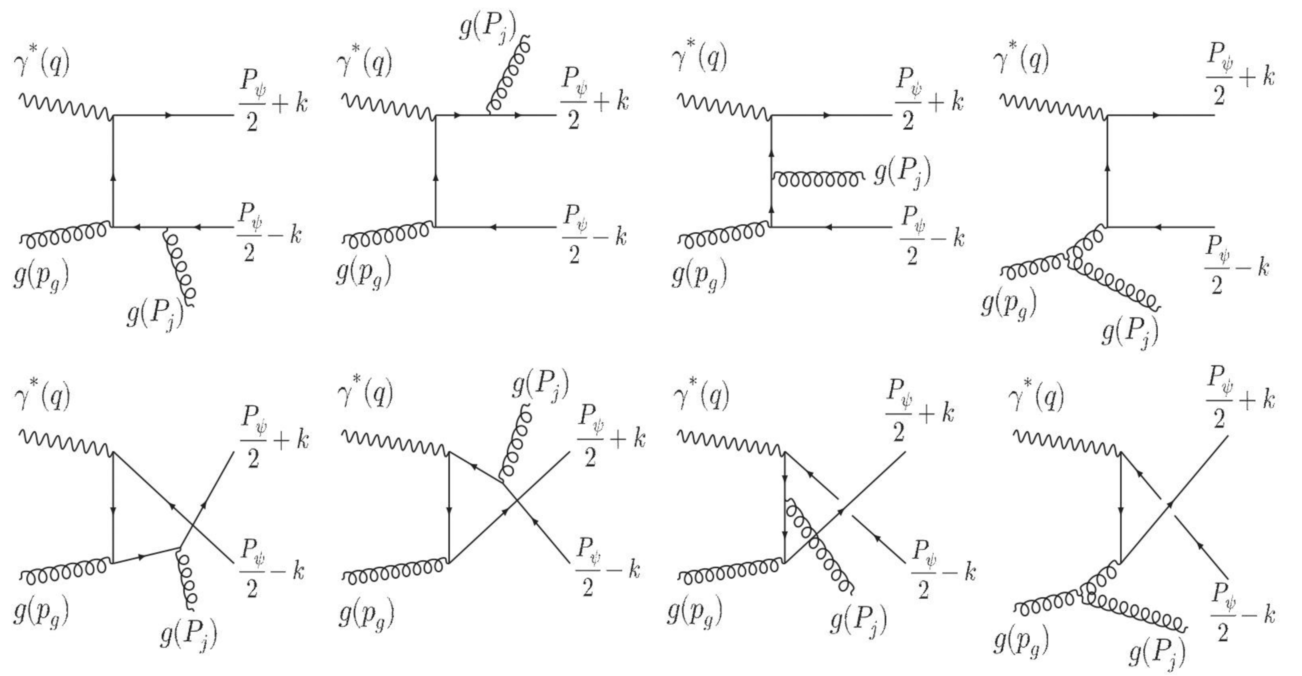


FIG. 1. Feynman diagrams for the partonic process $\gamma^*(q) + g(p_g) \to J/\psi(\mathrm{P}_\psi) + g(\mathrm{P}_j)$.

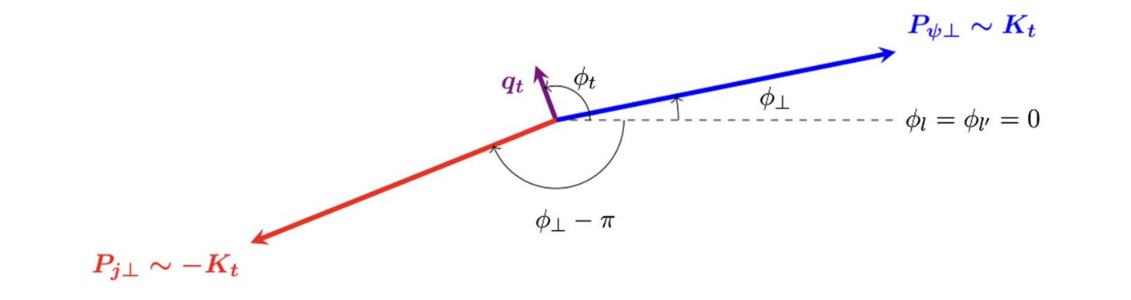
BACK-TO-BACK PRODUCTION OF J/Ψ AND JET

$$d\sigma=\frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_\psi}{2E_\psi(2\pi)^3}\frac{d^3P_j}{2E_j(2\pi)^3}\times\int dx d^2\mathbf{p}_T(2\pi)^4\delta^4(q+p_g-P_j-P_\psi)\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\nu\nu'}(x,\mathbf{p}_T^2)\mathcal{M}_{\mu\nu}^{g\gamma^*\to J/\psi g}\mathcal{M}_{\mu'\nu'}^{*g\gamma^*\to J/\psi g}.$$

$$\mathcal{M}(\gamma^* g\to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g)=\sum_{L_zS_z}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Psi_{LL_z}(\mathbf{k})\langle LL_z;SS_z|JJ_z\rangle\times\mathrm{Tr}[O(q,p_g,P_\psi,k)\mathcal{P}_{SS_z}(P_\psi,k)],$$

Contribution comes from the color singlet state $({}^3S_1^{(1)})$ And color octet states $({}^3S_1^{(8)},\ {}^1S_0^{(8)},\ {}^3P_{J(0,1,2)}^{(8)})$

In NRQCD, k, the relative momentum of the charm quark is small. Taylor expanded the amplitude about k=0. The first term gives the S wave contribution and second term the p wave contribution. Results depend on LDME set chosen



U. D’Alesio, F. Murgia, C. Pisano, and P. Taels *Phys.Rev.D* 100 (2019) 9, 094016

Raj Kishore, AM, Amol Pawar, M. Siddiqah *Phys.Rev.D* 106 (2022) 3, 034009

TMD EVOLUTION

TMDs are evolved from initial scale $\mu_b = \frac{b_0}{b_T}, \qquad b_0 = 2e^{-\gamma_E},$ To the final scale $\mu = \sqrt{M_\psi^2 + Q^2},$

Perturbative expression for the unpolarized gluon TMD valid in the region $|\boldsymbol{b}_T| << 1/\Lambda_{\mathrm{QCD}},$

$$\tilde{f}_1^g\left(x, \boldsymbol{b}_T; \mu_b, \mu\right) = \sum_{a=q,\bar{q},g} \left(C_{g/a} \otimes f_1^a\right)(x, \mu_b) \times \exp\left(-S_{\mathrm{pert}}(\mu_b, \mu)\right),$$

Echevarria, Idilbi, Kang, and Vitev, Phys. Rev. D 89, 074013 (2014)

Pert coefficient function — Collinear pdf — Pert Sudakov factor

$$C_{g/a}\left(x, \mu_b\right) = \delta_{ga}\delta(1-x) + \sum_{k=1}^{\infty} C_{g/a}^{(k)}(x) \left(\frac{\alpha_s\left(\mu_b\right)}{\pi}\right)^k$$

Boer, D'Alesio, Murgia, Pisano, and Taels, JHEP 09, 040 (2020),

NLL :

$$\begin{aligned}\sum_{a=q,\bar{q},g} \left(C_{g/a} \otimes f_1^a\right)(x, \mu_b) &= \sum_{a=q,\bar{q},g} \int_x^1 \frac{\mathrm{d}x'}{x'} C_{g/a}(x', \mu_b)\, f_1^a\Big(\frac{x}{x'}, \mu_b\Big) \\ &= \sum_{a=q,\bar{q},g} \int_x^1 \frac{\mathrm{d}x'}{x'} \delta_{ga}\, \delta(1-x')\, f_1^a\Big(\frac{x}{x'}, \mu_b\Big) \\ &= f_1^g(x, \mu_b)\,.\end{aligned}$$

Unpolarized gluon TMD

$$\tilde{f}_1^g\left(x, \boldsymbol{b}_T; \mu_b, \mu\right) = f_1^g(x, \mu_b)\, \exp[-S_{\mathrm{pert}}(\mu_b, \mu)]\,.$$

Perturbative Sudakov factor :

$$S_{\mathrm{pert}}(\mu_b, \mu) = \int_{\mu_b^2}^{\mu^2} \frac{\mathrm{d}\eta^2}{\eta^2} \left[A_g(\alpha_s(\eta)) \log\left(\frac{\mu^2}{\eta^2}\right) + B_g(\alpha_s(\eta))\right],$$

TMD EVOLUTION

$$A_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi}\right)^k A_g^{(k)}, \qquad B_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi}\right)^k B_g^{(k)}.$$

$$A_g^{(1)} = \frac{C_A}{2},$$
$$A_g^{(2)} = \frac{C_A}{4}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_f\right],$$
$$B_g^{(1)} = -\frac{C_A}{2}\frac{\beta_0}{6}, \qquad \beta_0 = 11 - \frac{2}{3}n_f.$$

At one loop perturbative Sudakov kernel

$$S_{\text{pert}}^{\text{1-loop}}(\mu^2, \mu_b^2) = -\frac{4A_g^{(1)}}{\beta_0}\left[\ln\left(\frac{\mu^2}{\mu_b^2}\right) - \left(\frac{B_g^{(1)}}{A_g^{(1)}} + L_\mu\right)\ln\left(\frac{L_\mu}{L_{\mu_b}}\right)\right].$$

Maxia, Boer, and Bor, Phys. Rev. D 112, 076034 (2025),

And at NLL it can be written as

$$S_{\text{pert}}^{\text{NLL}}(\mu^2, \mu_b^2) = S_{\text{pert}}^{\text{1-loop}} - \frac{16A_g^{(2)}}{\beta_0^2}\left[\ln\left(\frac{L_\mu}{L_{\mu_b}}\right) - \frac{1}{L_{\mu_b}}\ln\left(\frac{\mu^2}{\mu_b^2}\right)\right],$$

$$L_\eta = \ln\left(\eta^2/\Lambda_{\text{QCD}}^2\right).$$

Perturbative expansion valid for

$$|\boldsymbol{b}_T| \ll 1/\Lambda_{\text{QCD}}.$$

Fourier integral extends from small to large values of b_T. In the large-b_T region, perturbation theory breaks down due to the presence of large logarithmic contributions from higher-order terms : nonperturbative large- b_T region cannot be described using perturbation theory alone.

b_* prescription allows us to use perturbation theory in the region where it is valid, while including nonperturbative effects separately.

TMD EVOLUTION

Introduce modified impact parameter

$$b_*(b_T) = \frac{b_T}{\sqrt{1+\left(\frac{b_T}{b_{\max}}\right)^2}},$$

$$b_{\max} = 1.5 \text{ GeV}^{-1}.$$

$b_T << b_{max}, b_* \approx b_T$ Remains perturbative

$b_T >> b_{\max}, b^* \to b_{\max}$ Prevents scale from entering non-perturbative region

In small b_T region scale may be larger than hard scale, to prevent this introduce a lower cutoff

$$b_{\min} = \frac{b_0}{\mu}.$$

Also use a regulated scale

$$\mu'_{b_*} = \frac{b_0}{\sqrt{b_*^2 + b_{\min}^2}},$$

which ensures the scale associated with b_T always remains a physically meaningful perturbative range.

Maxia, Boer, and Bor, Phys. Rev. D 112, 076034 (2025),

$$\begin{aligned}\tilde{f}_1^g\left(x, \boldsymbol{b}_T; \mu'_{b_*}, \mu\right) &= \tilde{f}_1^g\left(x, \boldsymbol{b}_*; \mu'_{b_*}, \mu\right)\, e^{-S_{\rm NP}} \\ &= f_1^g(x, \mu'_{b_*})\, \exp\left[-S_{\rm pert}(\mu'_{b_*}, \mu)\right]\, e^{-S_{\rm NP}}\end{aligned}$$

Non-perturbative effects are incorporated

$$S_{\rm NP}^{\rm pdf}(b_T, \mu) = b_T^2\left(g_1^{\rm pdf} + \frac{g_2}{2}\ln\frac{\mu}{Q_0}\right),$$

$$S_{\rm NP}^{\rm Sivers}(b_T, \mu) = b_T^2\left(g_1^{\rm Sivers} + \frac{g_2}{2}\ln\frac{\mu}{Q_0}\right),$$

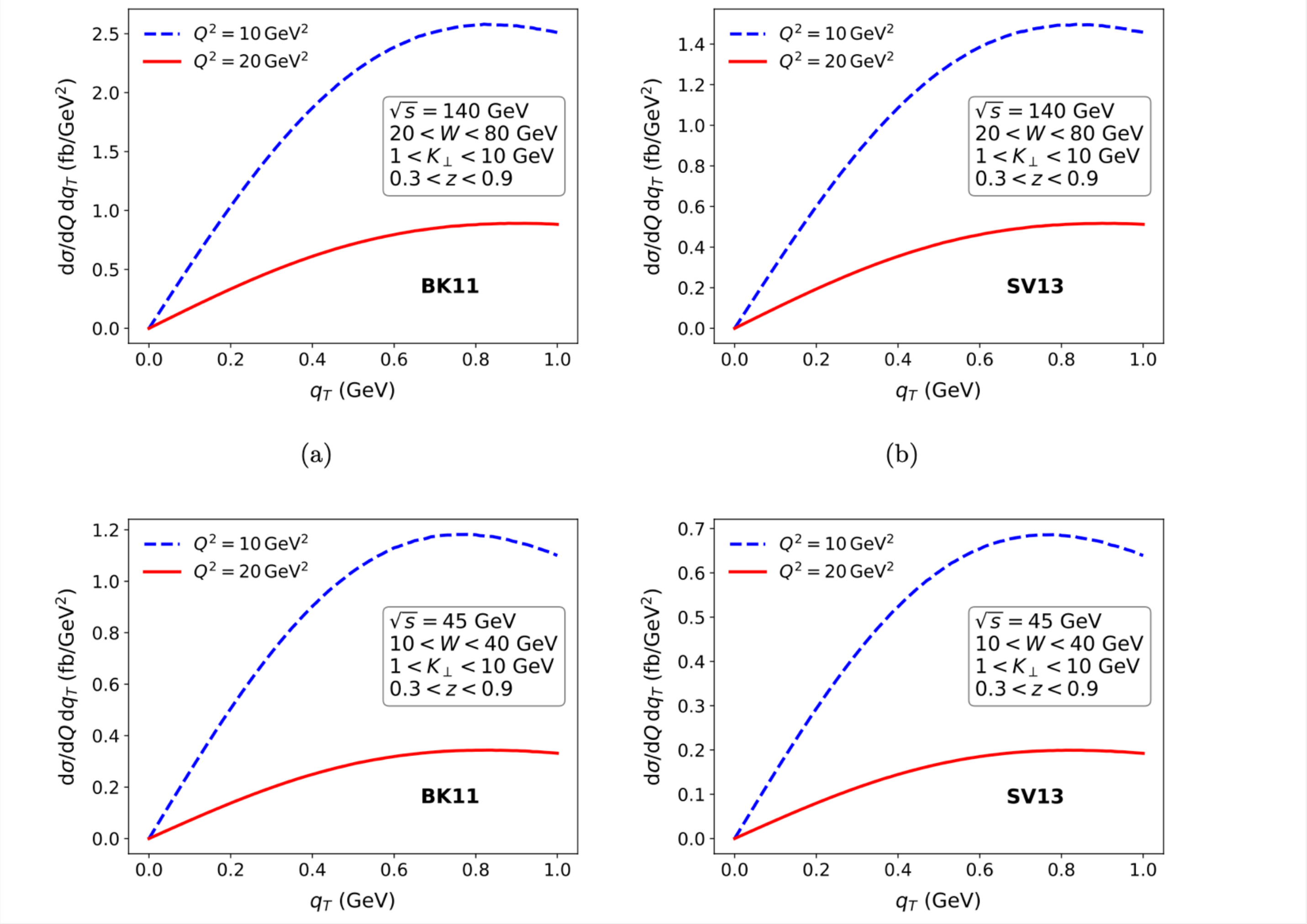
Best fit parameters in non-pert part from

$$\tilde{f}_{1T}^{\prime\perp g}\left(x, \boldsymbol{b}_T; \mu'_{b_*}, \mu\right) = \frac{b_T M_p}{2} T_{g,F}(x, x, \mu'_{b_*})\, \exp\left[-S_{\rm pert}(\mu'_{b_*}, \mu)\right],$$

Use same prescription as before for derivative of Sivers function

- Echevarria, Idilbi, . Kang, Vitev, PRD 89, 074013 (2014)

UNPOLARIZED CROSS SECTION IN BACK-TO-BACK ELECTROPRODUCTION OF J/Ψ AND PHOTON



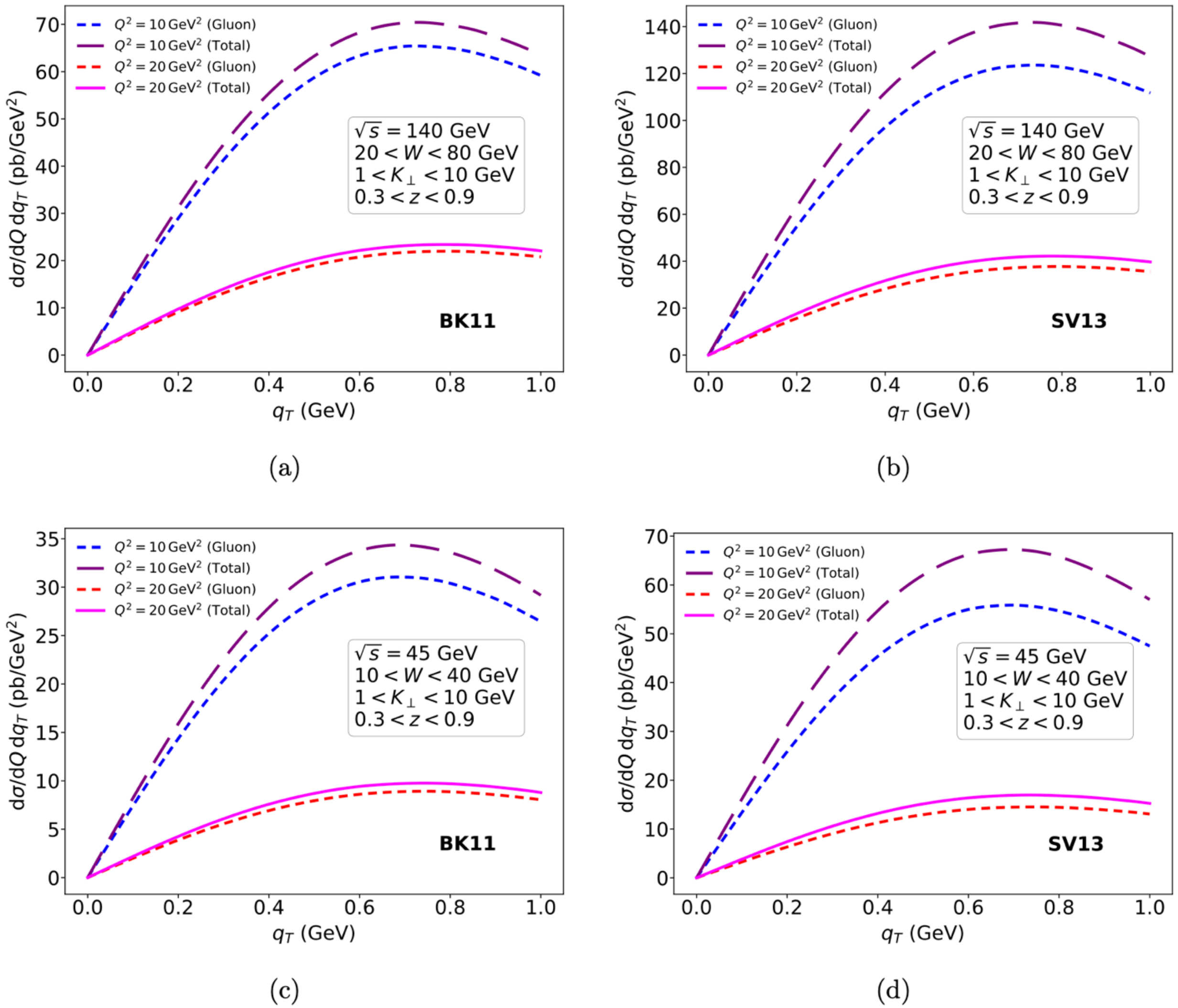
M. Butenschoen and B. A. Kniehl, Physical Review D—Particles, Fields, Gravitation, and Cosmology 84, 051501 (2011)

R. Sharma and I. Vitev, Physical Review C 87, 044905 (2013)

Only the CO $^3S_1$ state contributes. LDME value of this state in the BK11 set is 1.73 times larger than that of SV13.

Arghya Jana, AM, Sangem Rajesh, in preparation

UNPOLARIZED CROSS SECTION IN BACK-TO-BACK ELECTROPRODUCTION OF J/Ψ AND JET



M. Butenschoen and B. A. Kniehl, Physical Review D—Particles, Fields, Gravitation, and Cosmology 84, 051501 (2011)

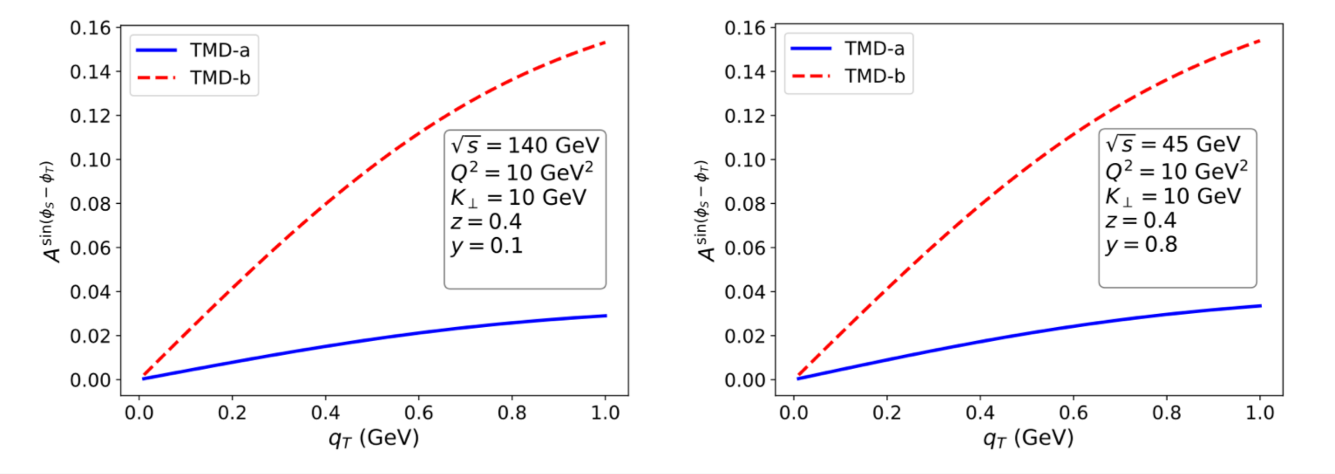
R. Sharma and I. Vitev, Physical Review C 87, 044905 (2013)

Bound on z eliminates fragmentation contribution at low z as well keeping the outgoing gluon hard.

CS and CO contributions are included, also both quark and gluon channels

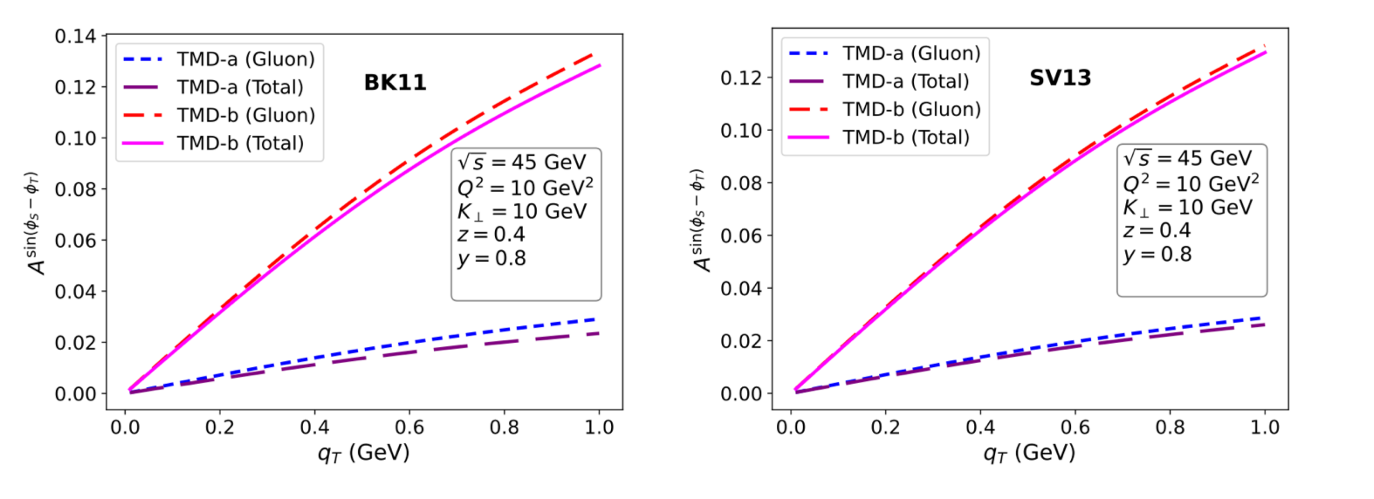
Gluon channel contribution dominates over quark channel : ideal to probe gluon TMD

SIVERS ASYMMETRY



J/ψ and photon

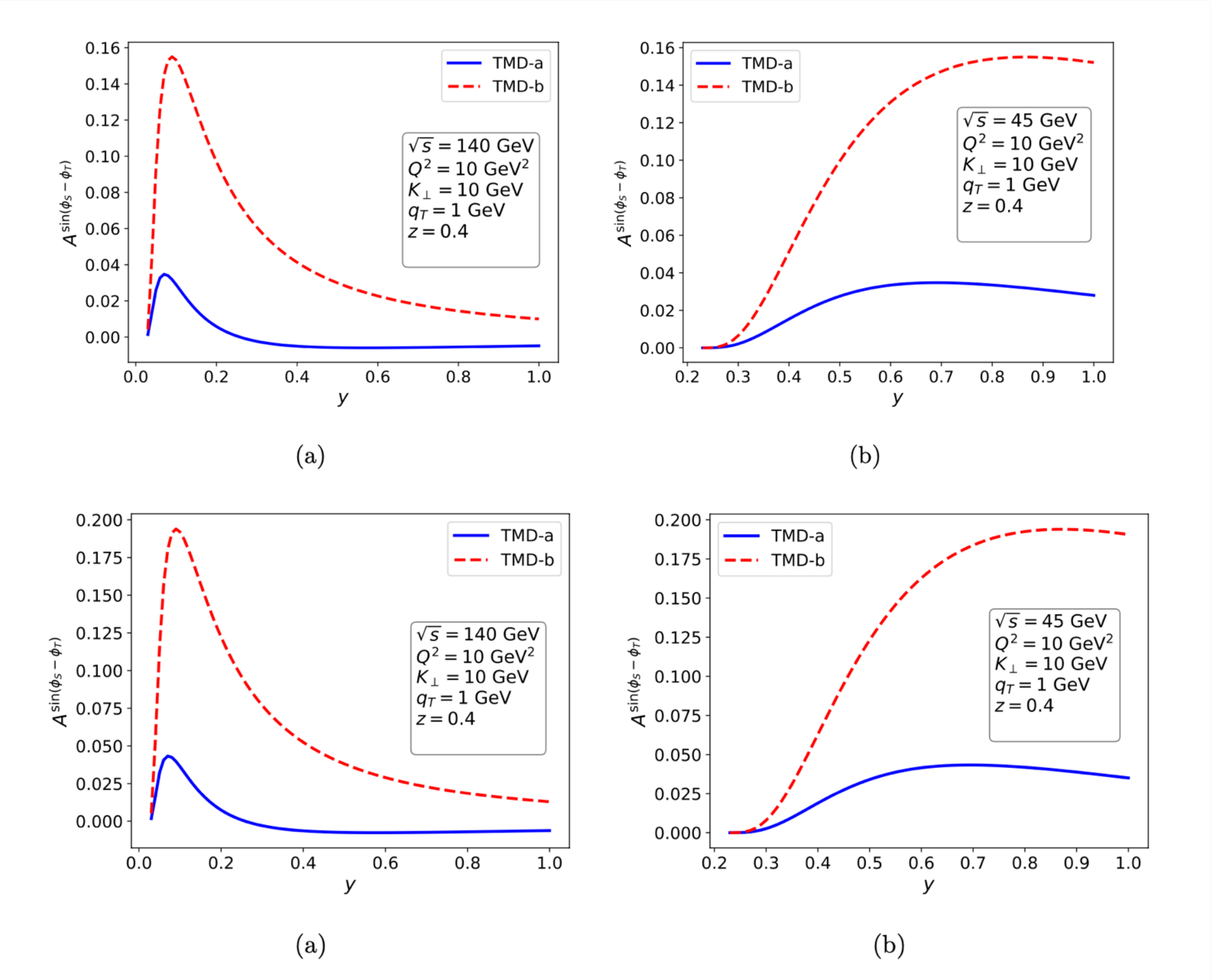
TMD-a : $N_g(x)$ is defined by averaging the $N_u(x)$ and $N_d(x)$ of up-quark and down-quark Sivers functions. Best fit value $N_d(x)$ for the down-quark Sivers function is negative; $N_u(x)$ is positive for the up-quark : smaller value of $N_g(x)$ for GSF in TMD-a parametrization. As a result, the Sivers asymmetry is maximum for the TMD-b parametrization compared to the TMD-a parametrization in all the figures.



J/ψ and jet

Arghya Jana, AM, Sangem Rajesh, in preparation

SIVERS ASYMMETRY



J/ψ and photon

Asymmetry rises at intermediate y, reaches a peak, and then decreases. The cm energy s strongly affects the position of this peak. For larger s peak appears at smaller y and is clearly visible. For smaller s, he peak may not be visible within the kinematic range, resulting in an asymmetry that appears to increase.

J/ψ and jet

Arghya Jana, AM, Sangem Rajesh, in preparation

SUMMARY AND CONCLUSION

J/ψ production processes at the upcoming EIC are useful tools to probe gluon TMDs including gluon Sivers function. In particular, almost back-to-back production of J/ψ and jet(photon) are important as in this kinematics TMD factorization is expected to hold

We presented a study of the effect of TMD evolution in CSS framework on the unpolarized cross section and Sivers asymmetry in almost back-to back production of J/ψ and jet and J/ψ and photon production. J/ψ production is calculated in NRQCD

Cross section is rather small for and J/ψ and photon production; but larger for J/ψ and jets Sivers asymmetry is sizable even after incorporating TMD evolution.

For J/ψ –photon production, cross section depends only on one LDME; asymmetry is independent of LDME and a robust probe of gluon TMD

Further study would involve TMD evolution scheme dependence and choice of specific non-perturbative kernel