Gluon Production in the Saturation Region

José Garrido* (USM & PUCV),
Carlos Contreras (USM),
Eugene Levin (TAU)

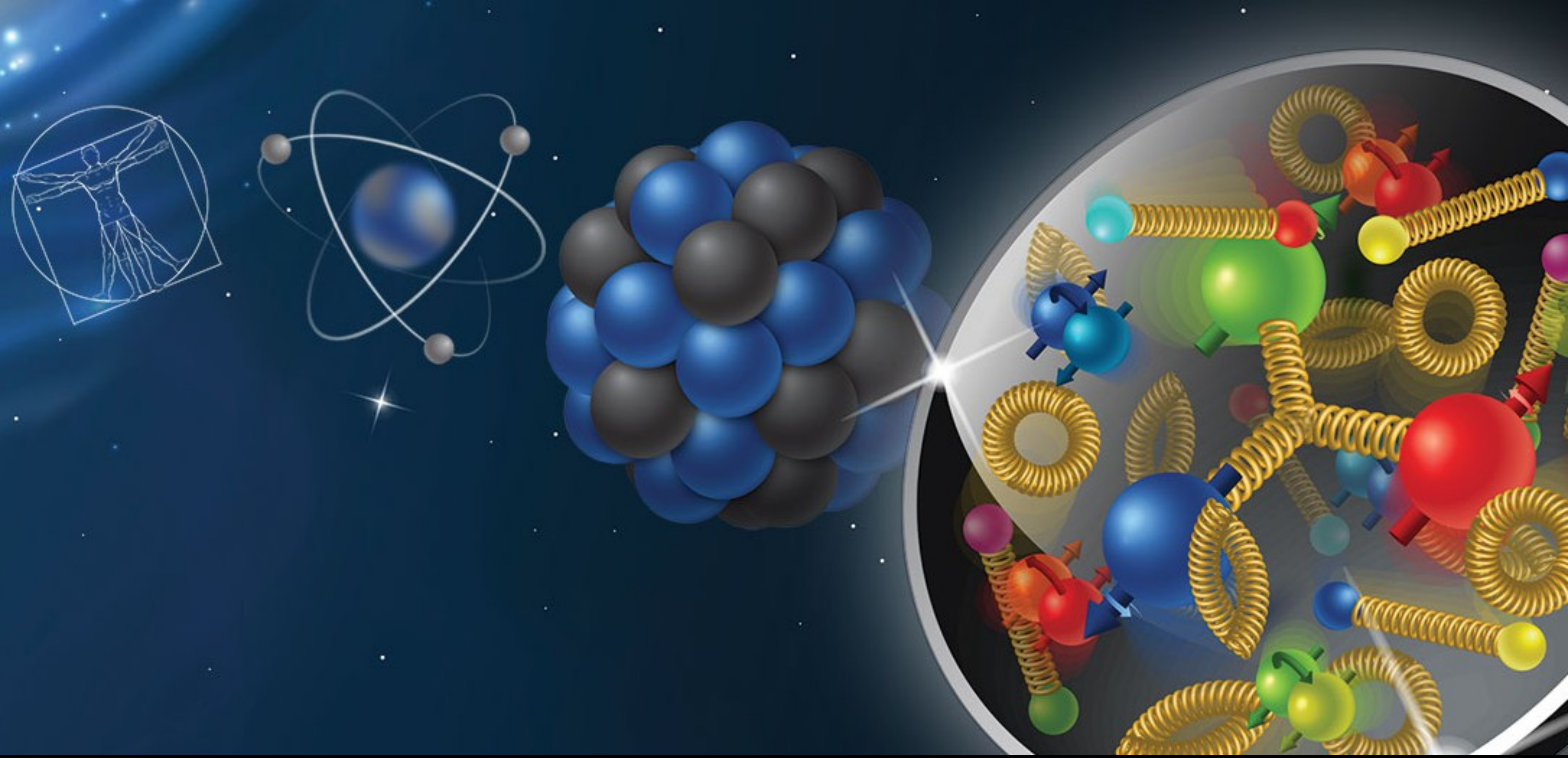
Light Cone 2026, June 24, CFNS-SBU, Stony Brook, NY, USA

Outline

- General concepts of small-x physics:
  - Quarks and gluons in the proton
  - DIS in the dipole picture, GGM/MV formula
  - Small-x evolution, parton saturation, map of high-energy QCD
- Non-linear small-x evolution
  - Non-linear BK and KL evolution equations
  - Solution of BK and KL equations, geometric scaling
- Particle production at small-x
  - AGK cross sections in BFKL pomeron calculus and dipole approach
  - Solution of the linear but with complicated kernel AGK equations
  - Multiplicity distribution and entanglement entropy for produced gluons
- Applications at EIC era

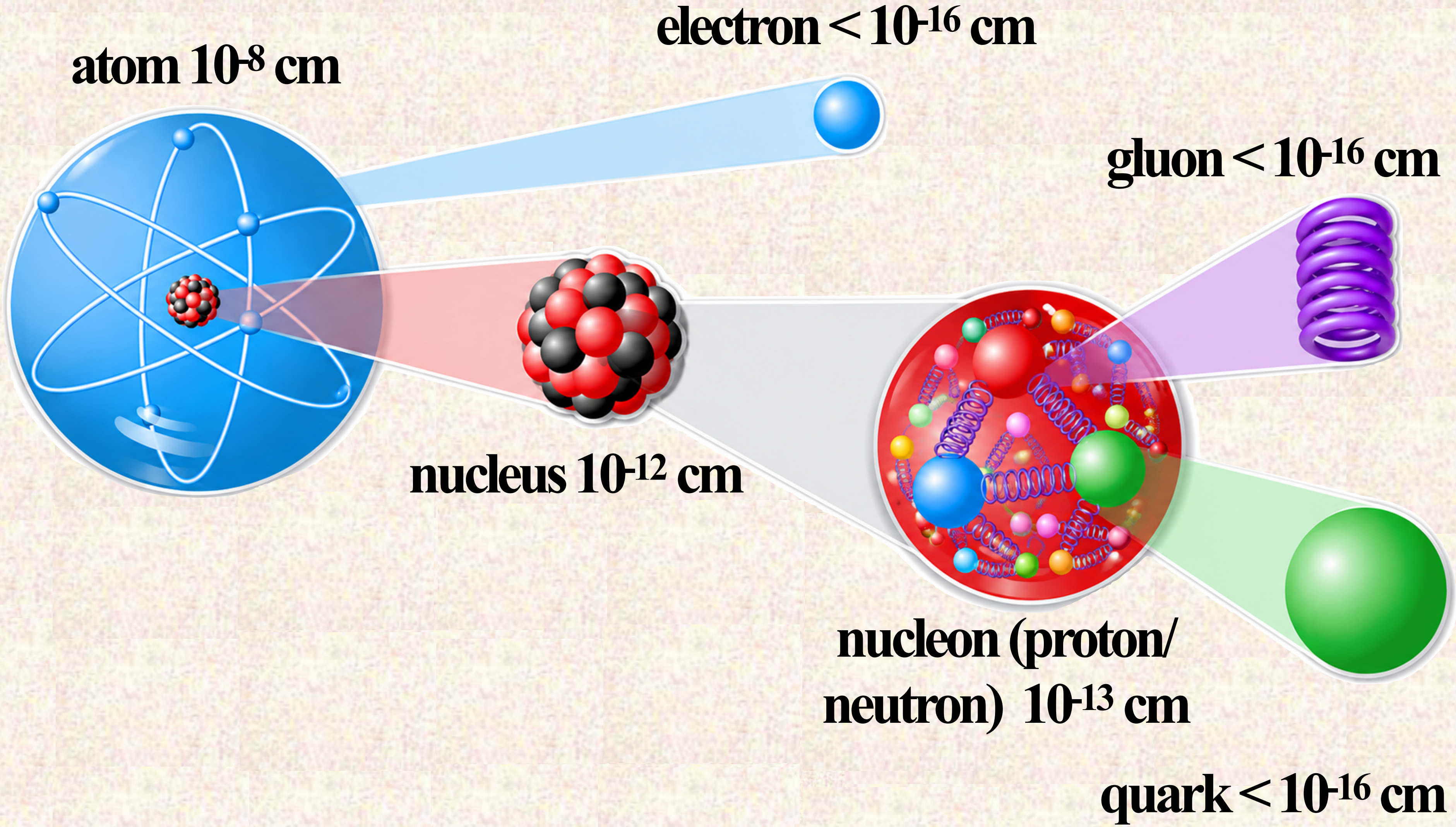
Based on arXiv:2603.21775 [hep-ph],

Phys. Rev. D 113, 114021 (2026)



General concepts of small-x physics

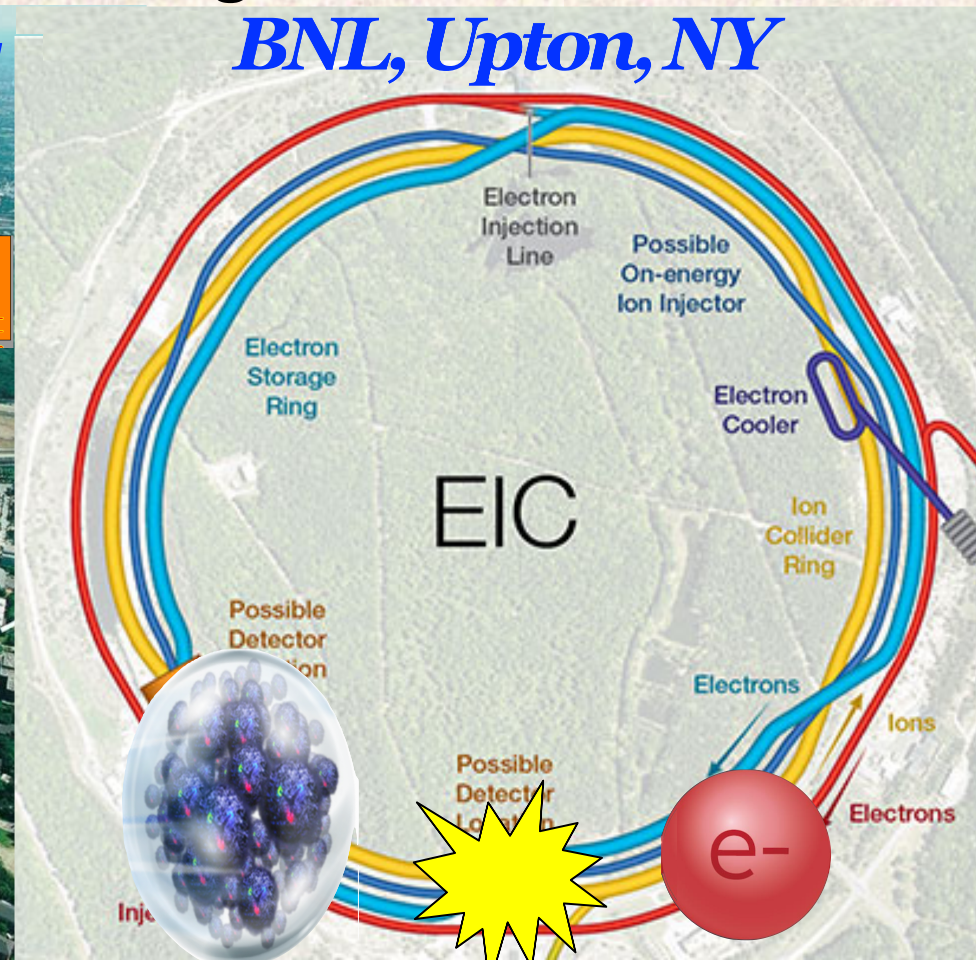
Quarks and Gluons in the Proton



- **We still don't have a clear picture of how quarks and gluons are distributed inside the proton.**

Quarks and Gluons in the Proton

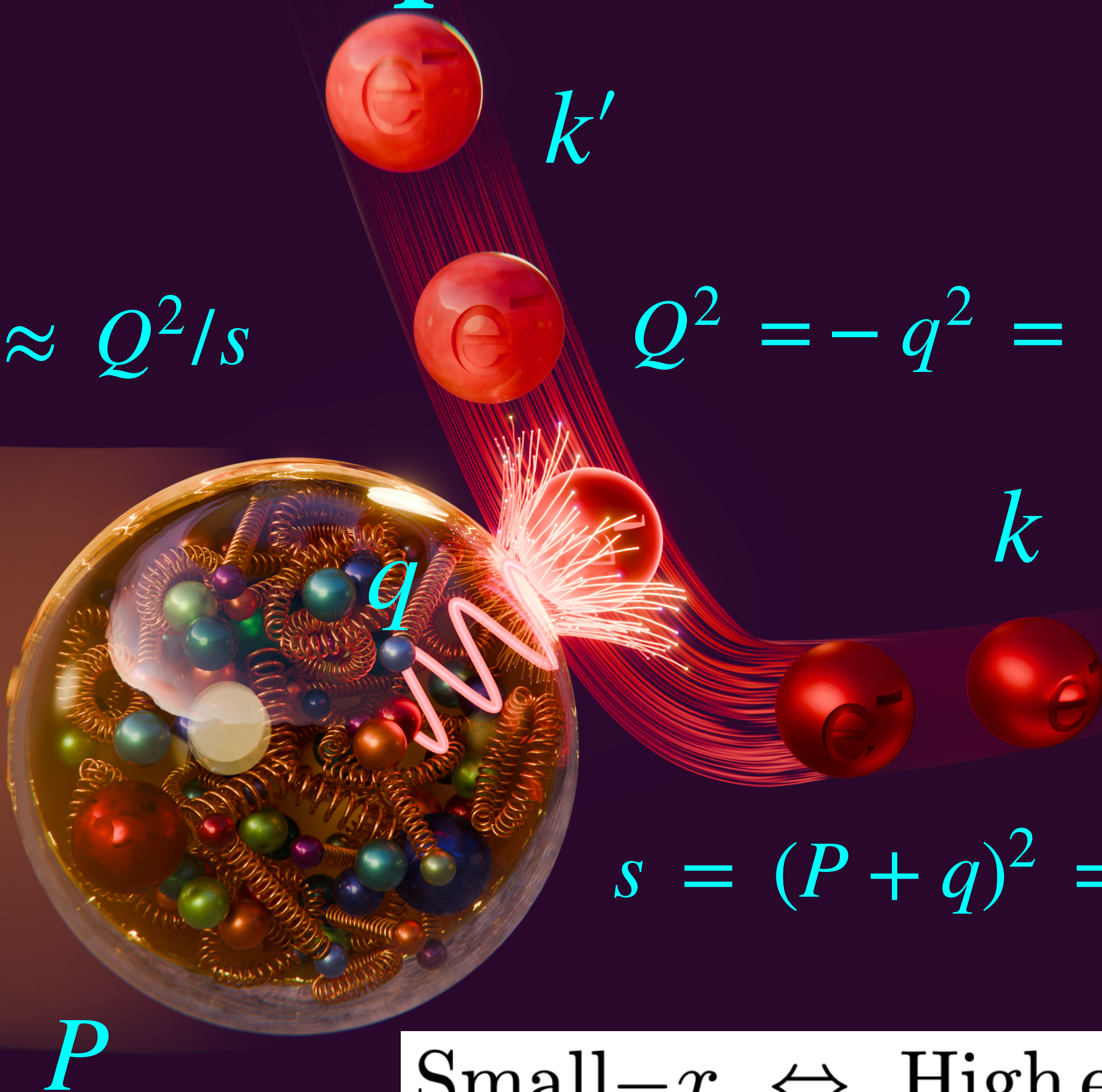
- **By colliding tiny electrons with relatively larger particles such as protons or nuclei in particle accelerators, it help us (nuclear physicists) to unlock the mysteries of the strong force, which binds atomic nuclei together.**



Deep inelastic scattering (DIS) on a proton

$$x = \frac{Q^2}{2P\cdot q} \approx Q^2/s$$

$$Q^2 = -q^2 = (k-k')^2$$



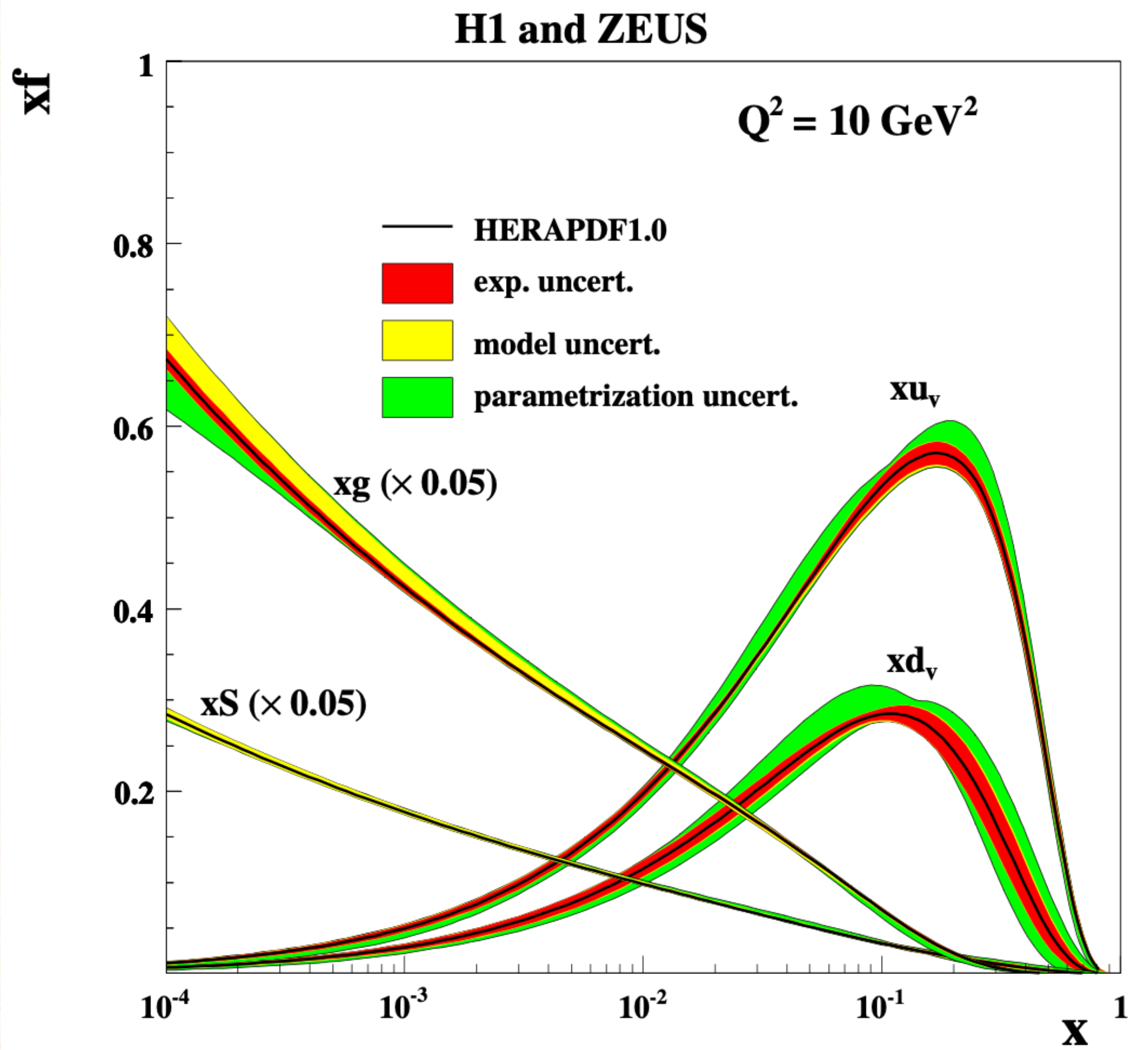
$$y = \frac{Q^2}{sx}$$

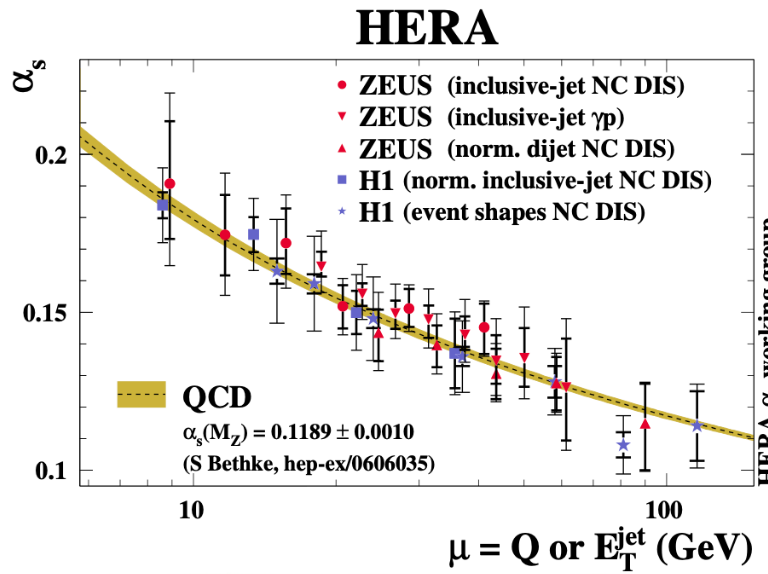
$$s = (P+q)^2 = W^2$$

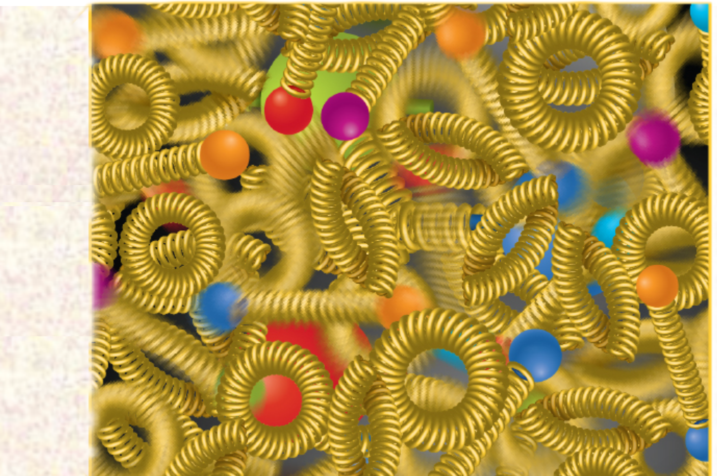
Small$-x$ $\Leftrightarrow$ High energy s

Quarks and Gluons in the Proton

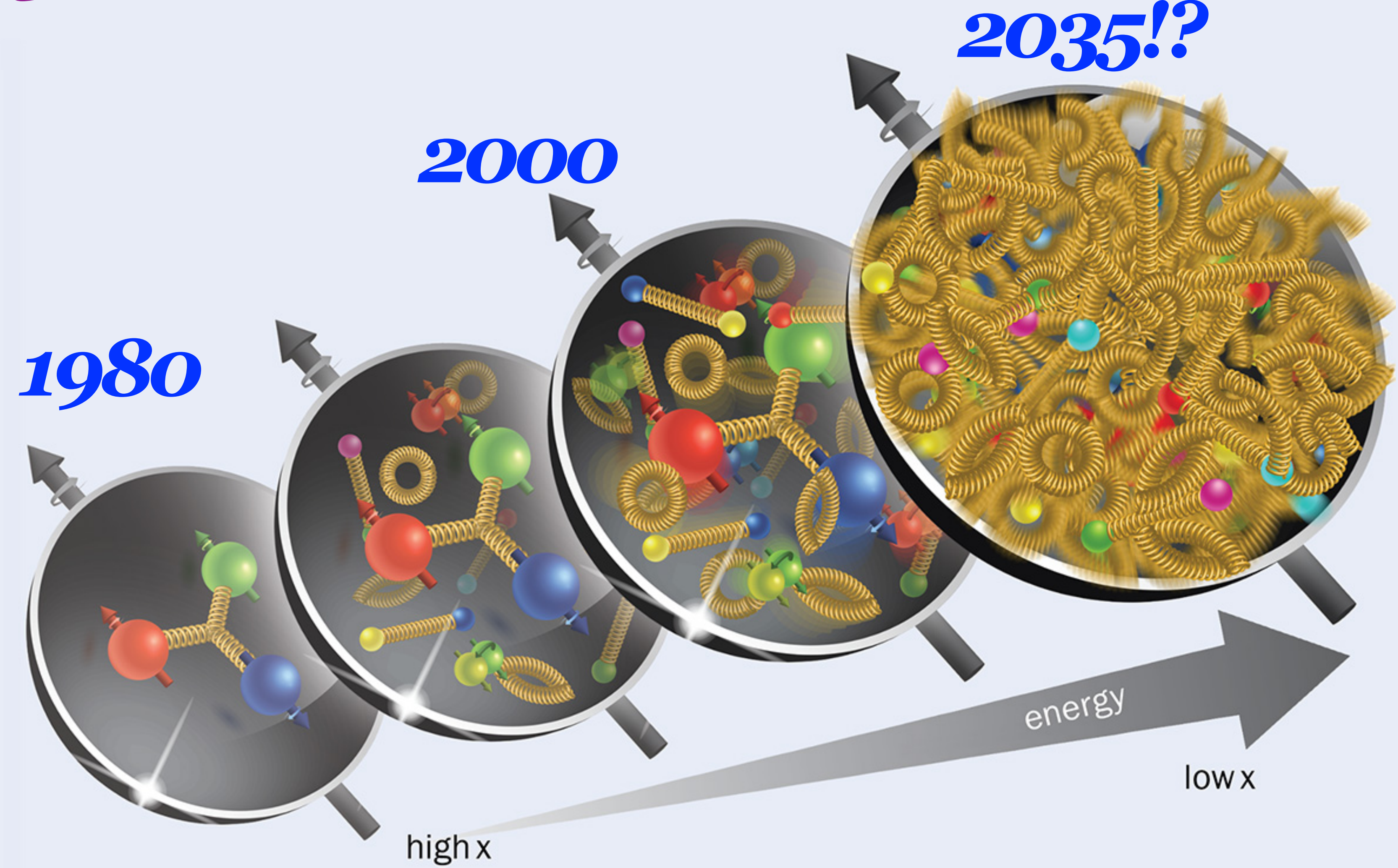
- **There is a huge number of quarks, anti-quarks and gluons at small-x!**







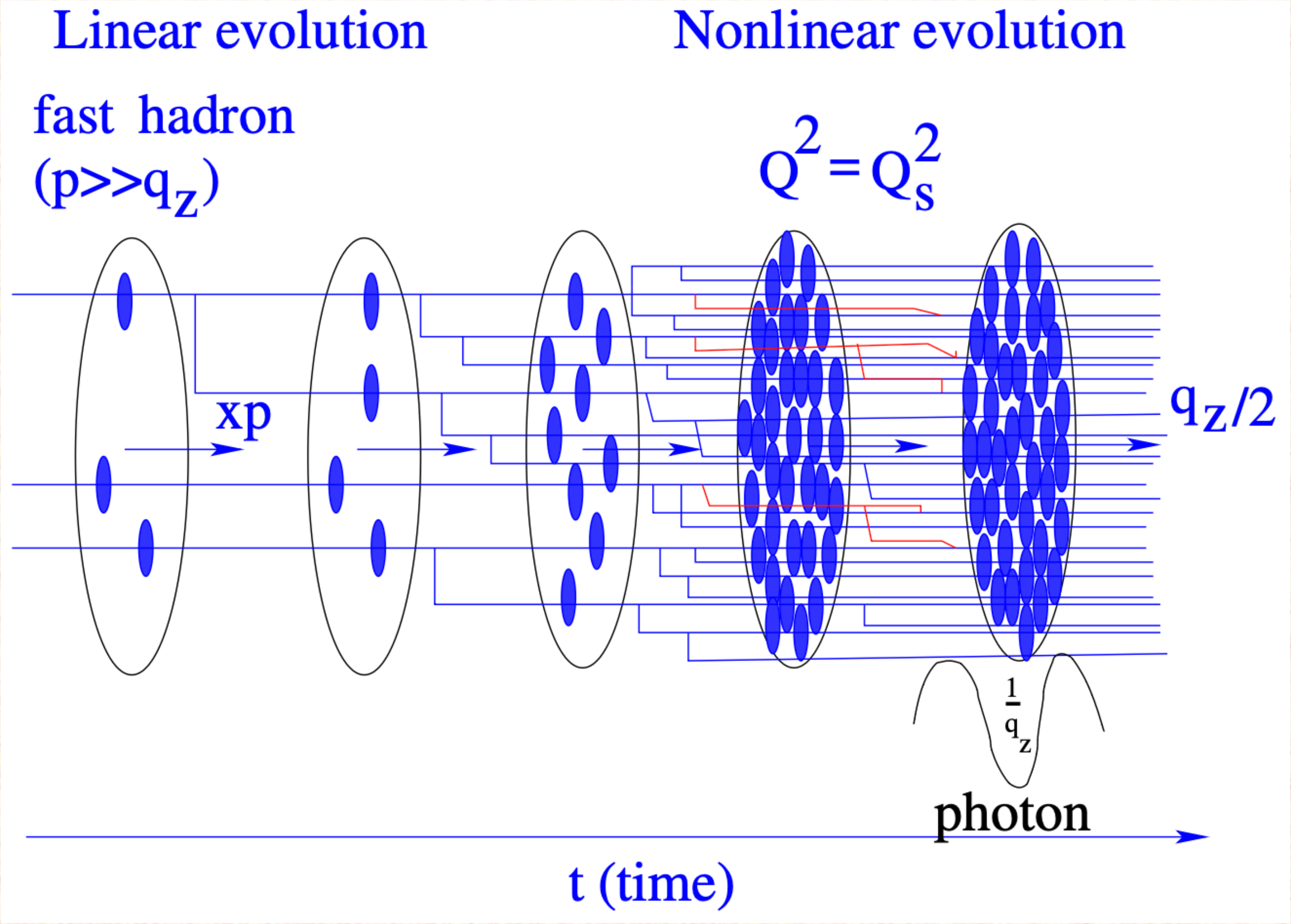
Quarks and Gluons in the Proton



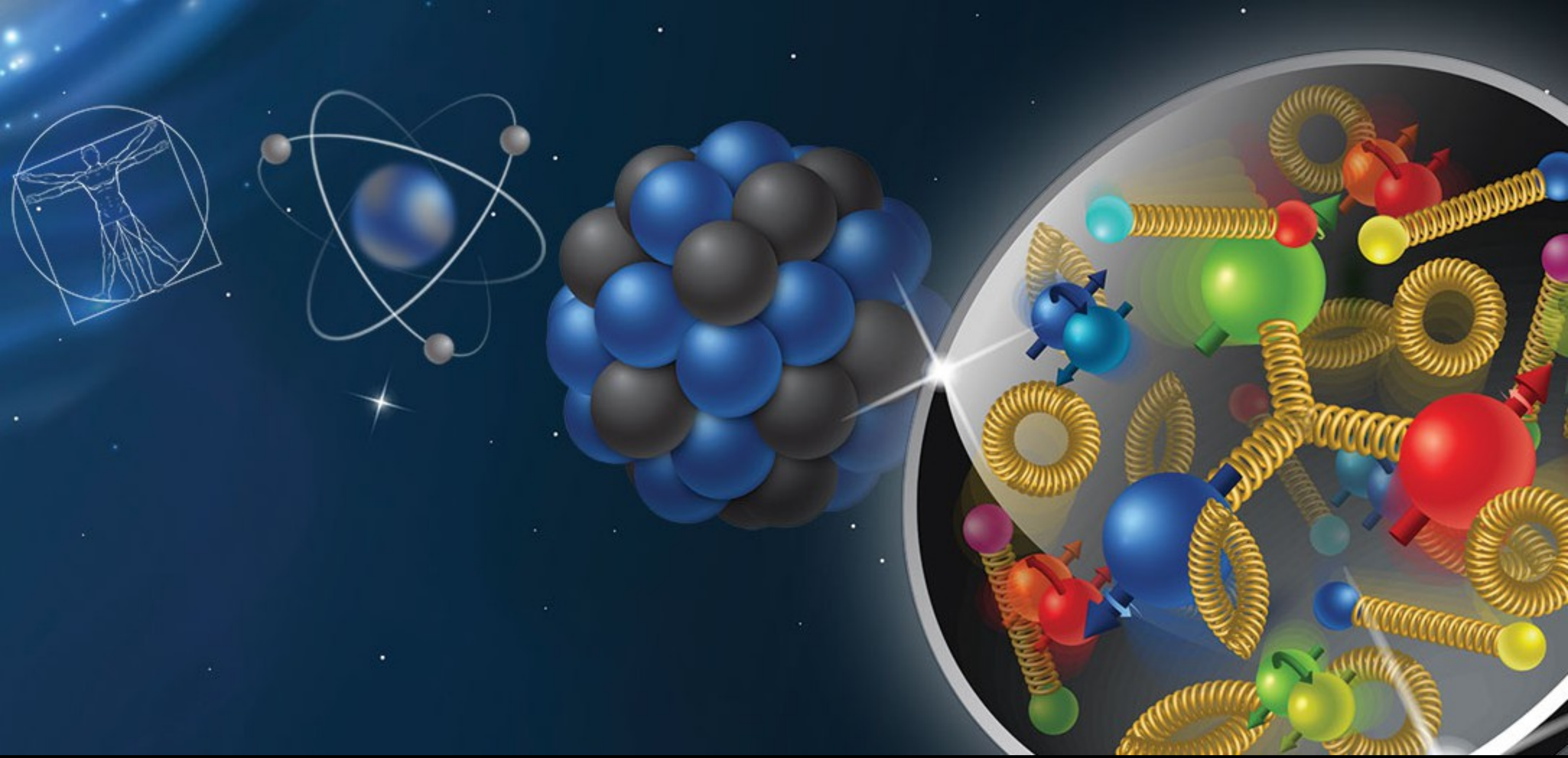
- **Qualitatively we understand that these extra quarks and gluons are emitted by the original three valence quarks in the proton.**

Quarks and Gluons in the Proton

- **The Gribov-Feynman time structure of the parton cascade**



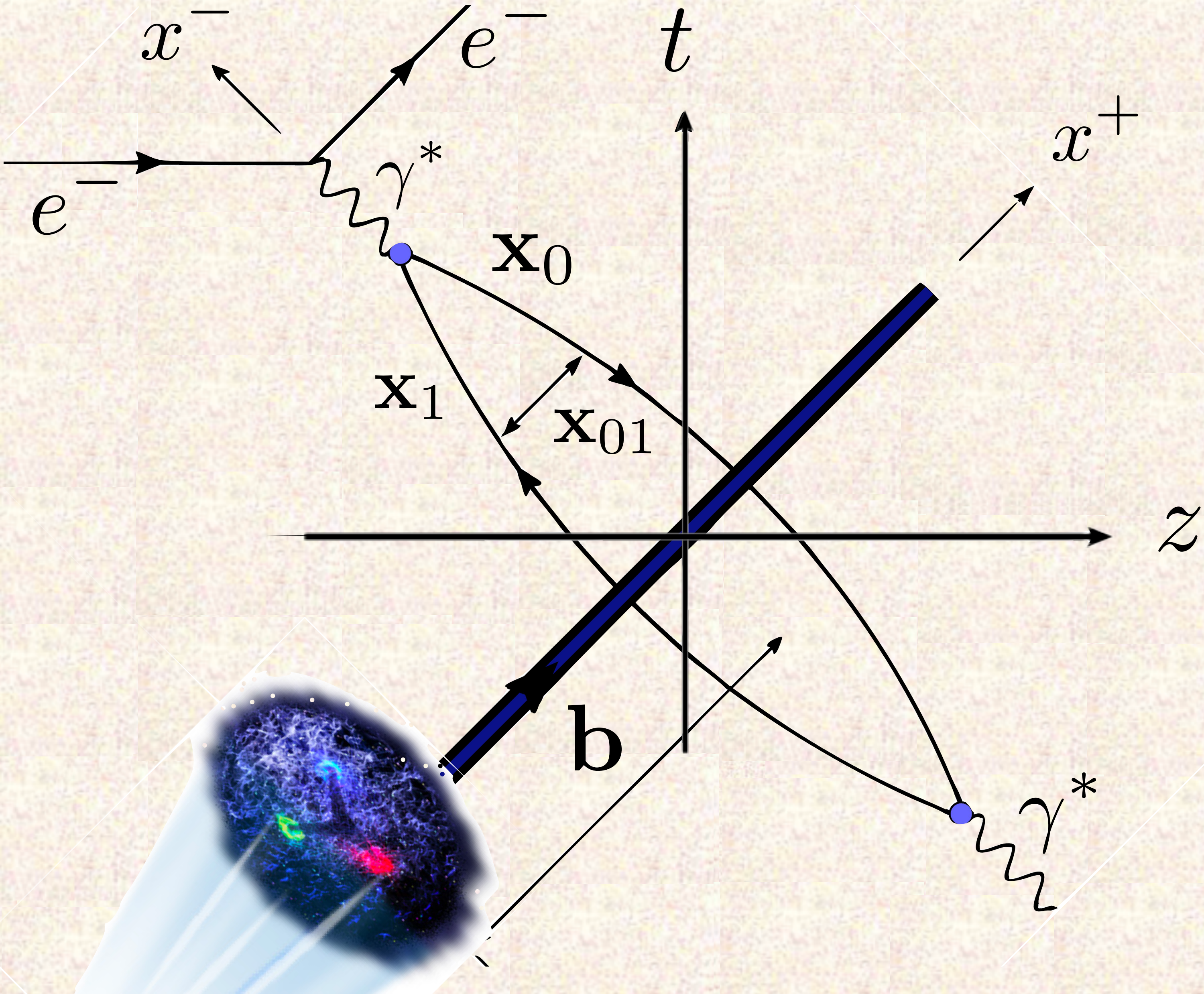
- **The straight solid lines represent gluons.**



The dipole approach to QCD

Dipole picture of DIS

- DIS on a proton at small-x ($x<0.01$ and $Q^2 \geq 1$ GeV^2):

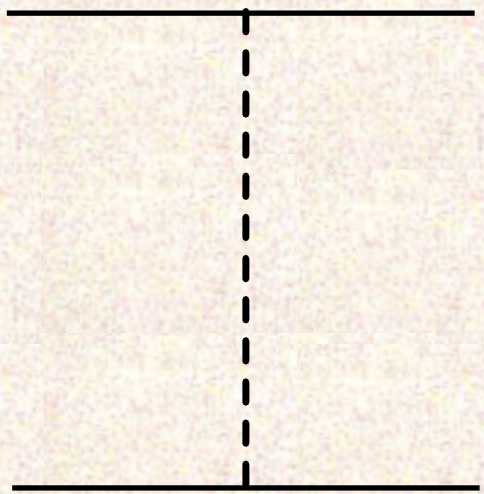


Dipole picture of DIS

- The total DIS cross section is expressed in terms of the quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* p} = \int \frac{d^2 x_{01}}{2\,\pi}\, d^2 b \int_0^1 \frac{dz}{z\,(1\,-\,z)}\, |\Psi^{\gamma^* \to q\bar{q}}(\mathbf{x}_{01}, z, Q^2)|\, N(Y; \mathbf{x}_{01}, \mathbf{b})$$

- How does the dipole interacts with the proton?



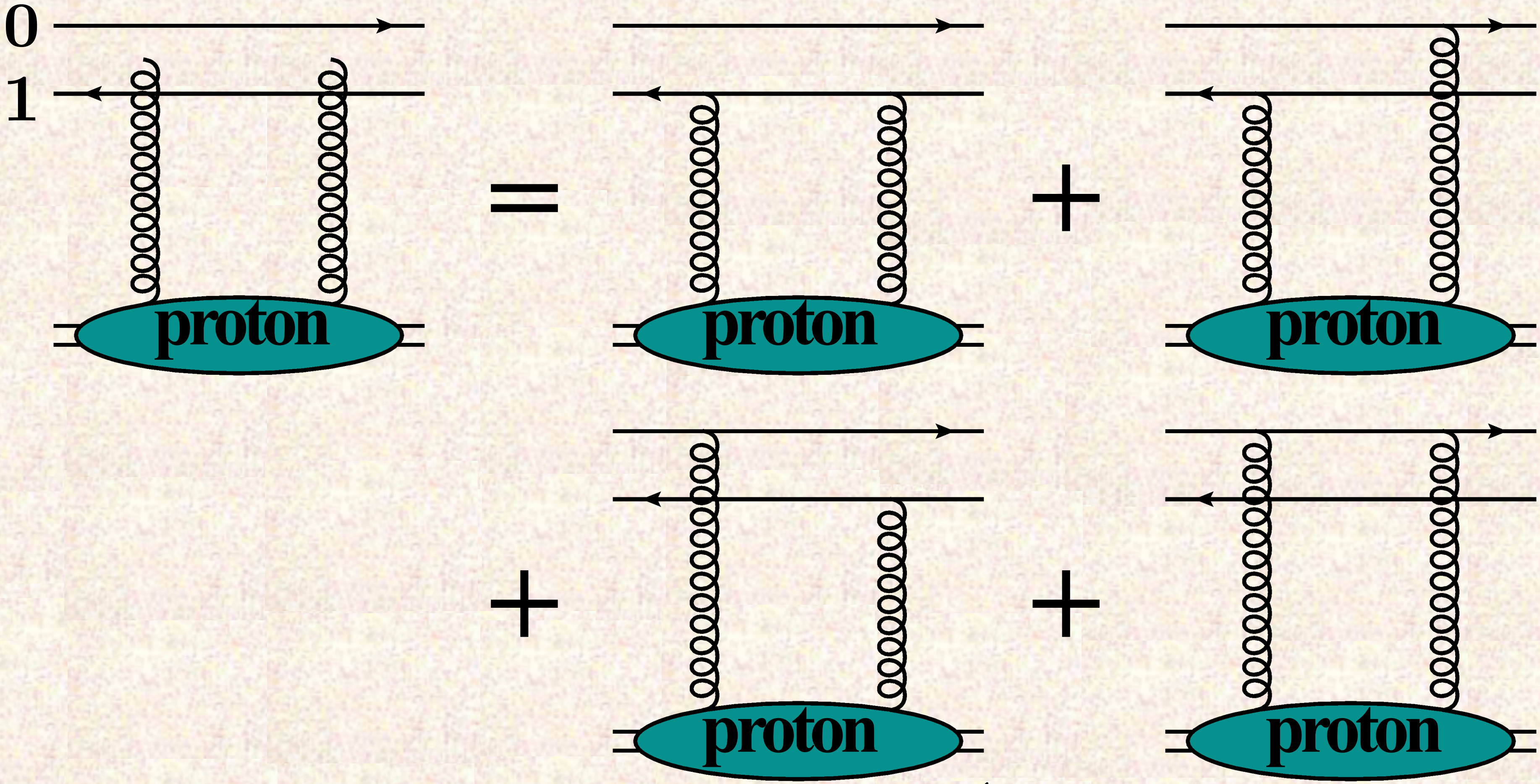
$$\sigma \;\sim\; \ldots\, s^{J-1}$$

J = spin of exchanged particle

- Answer: the gluon exchange dominates (J=1). Spin-2 particles, such as gravitons, luckily are weakly-coupled at the energies of the modern-day accelerators.

Dipole picture of DIS

- **For the 2-gluon exchange in the proton=quark model**

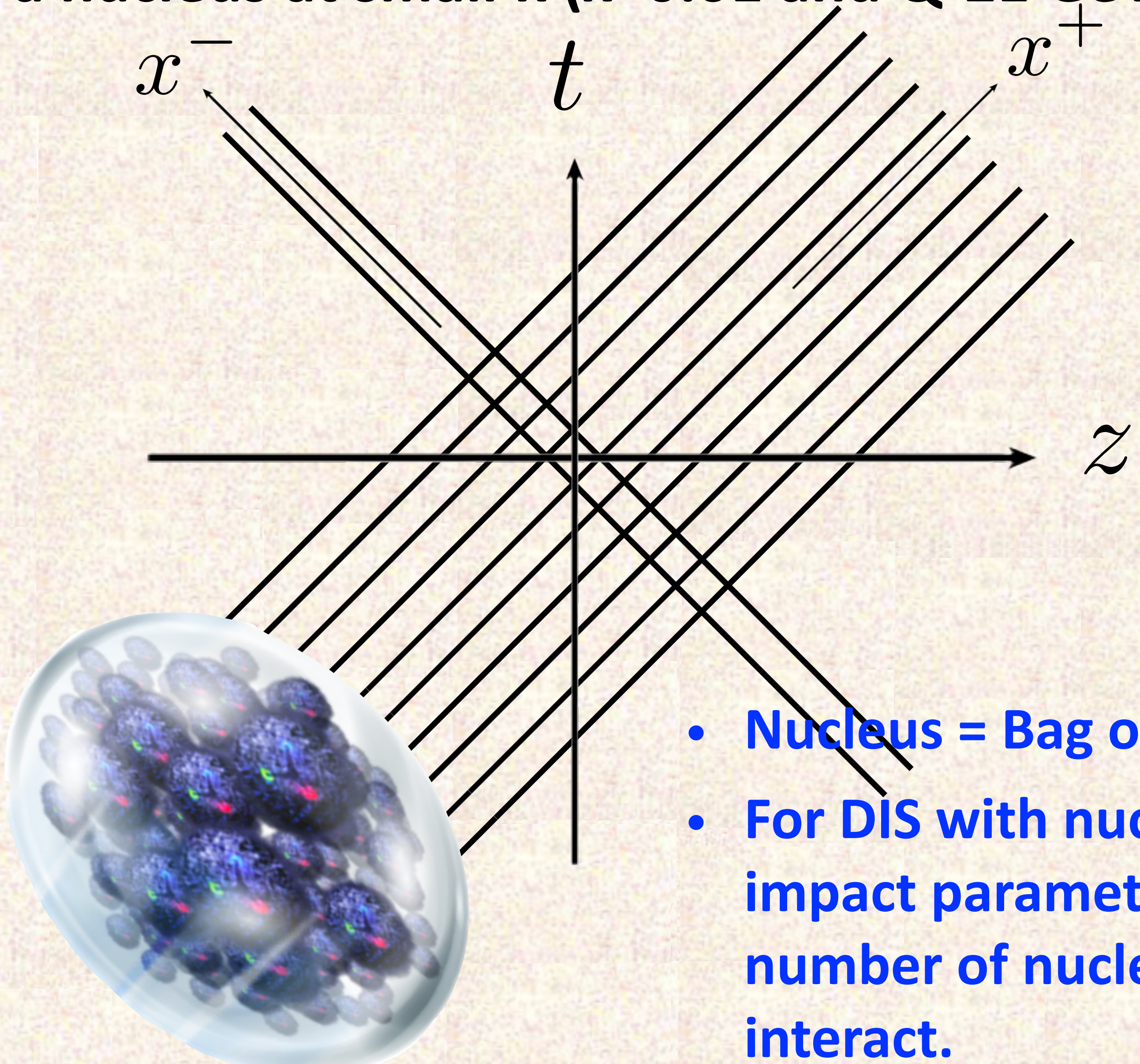


$$N^{2-\text{gluons}}(Y=0,\mathbf{x}_{01},\mathbf{b}) = \frac{1}{4}\,x_{01}^2\,Q_s^2(Y=0,\mathbf{b})$$

where $Q_s(Y=Y_0,b)$ is the initial saturation scale.

Dipole picture of DIS

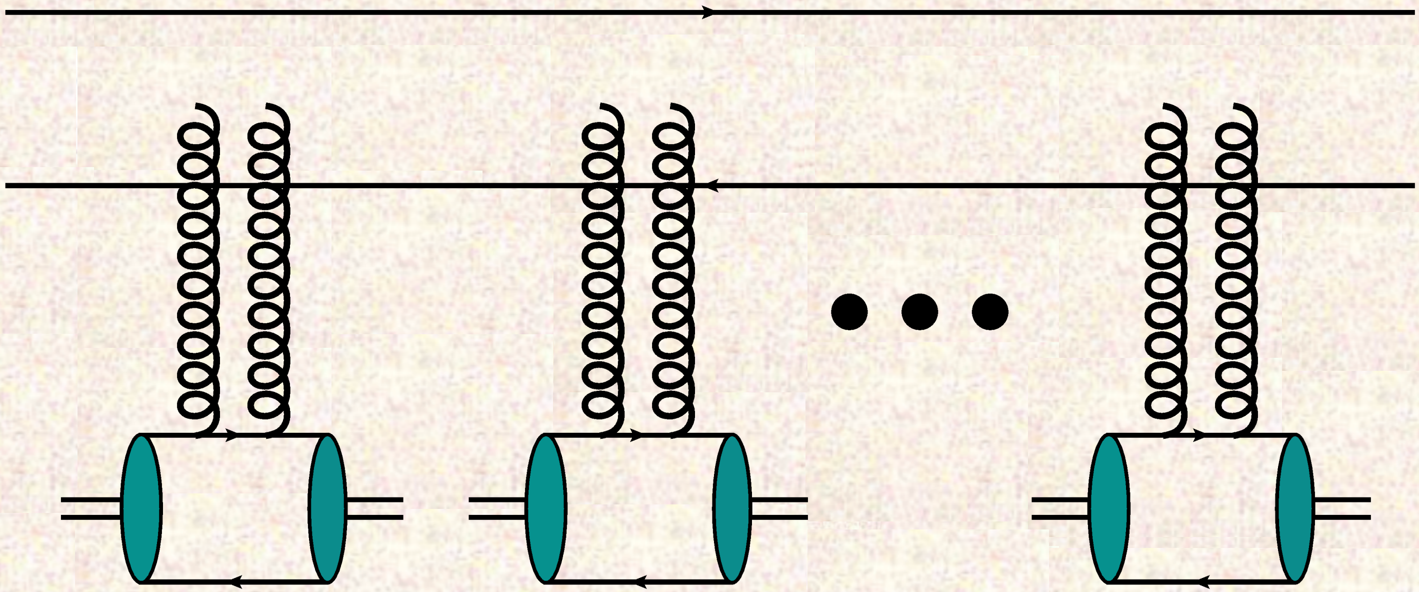
- **DIS on a nucleus at small-x (x<0.01 and $Q^2 \geq 1$ GeV^2):**



- **Nucleus = Bag of dipoles**
- **For DIS with nucleus the impact parameter fixes the number of nucleons that interact.**

Gribov-Glauber-Mueller Picture

- **If the interaction with 1 nucleon becomes strong, we need to account for multiple interactions**



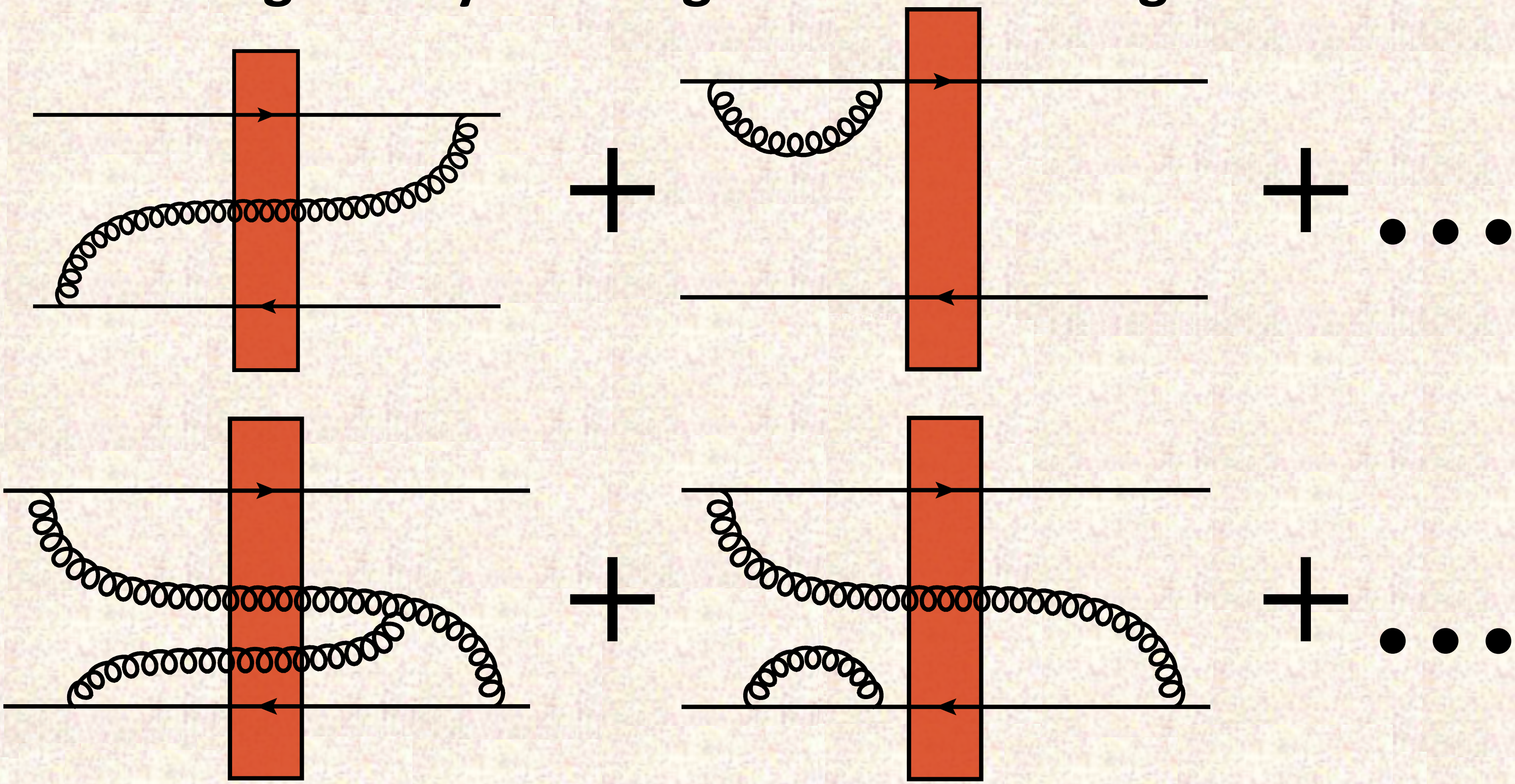
- **For multiple exchanges simply exponentiate (Mueller '90)**

$$N^{(0)}\left(Y = Y_A, \mathbf{x}_{01}, \mathbf{b}\right) \; = \; 1 \; - \; \exp\left(-\frac{1}{4}\, x_{01}^2\, Q_s^2(Y_A, \mathbf{b})\right)$$

- **Since the interaction is local, gluon exchanges with different nucleons do not get entangled and we also do not have any gluon-gluon interactions.**

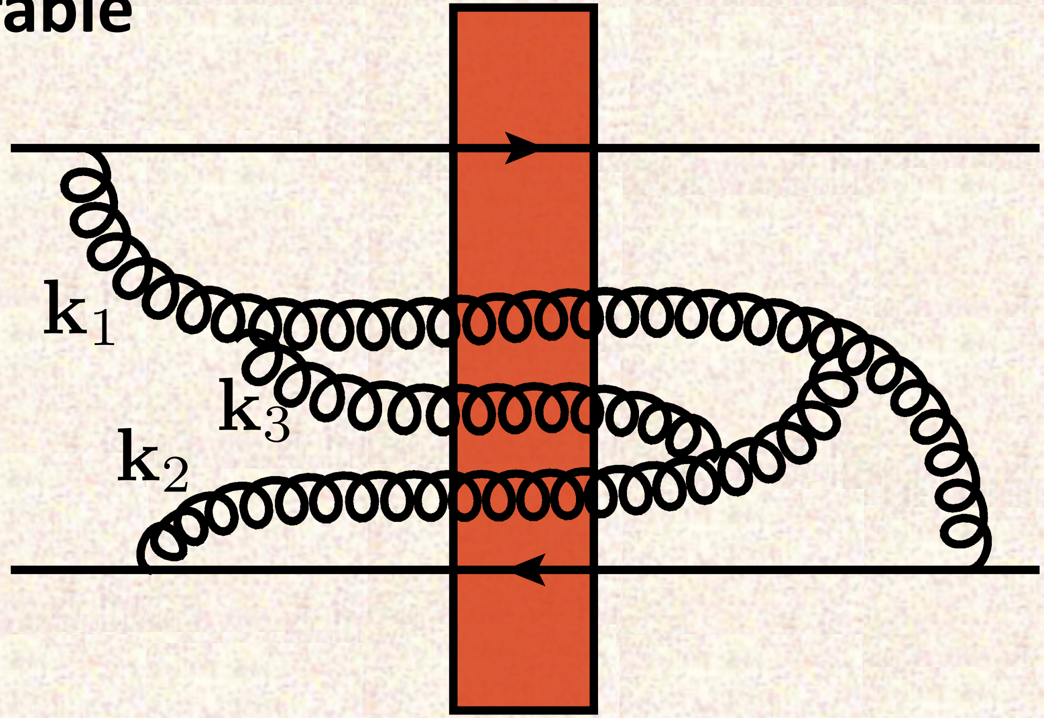
Small-x Evolution

- **The previous result does not include any energy dependence. Energy dependence comes in through quantum corrections, which bring in powers of   . Those are given by the long-lived s-channel gluons:**



Small-x Evolution

- **To give us powers of $\alpha_s \ln(1/x)$ (LLA), the gluons minus momenta have to be ordered and the transverse momenta are comparable**



$$k_1^- \gg k_2^- \gg k_3^- \gg \ldots; \quad k_{1T} \sim k_{2T} \sim k_{3T} \sim \ldots$$

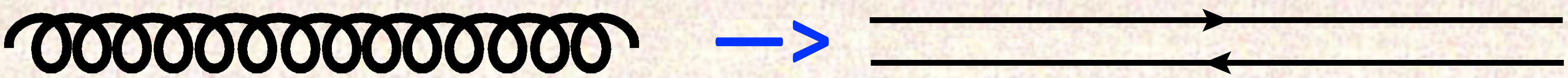
$$\frac{2k_1^-}{k_{1T}^2} \gg \frac{2k_2^-}{k_{2T}^2} \gg \frac{2k_3^-}{k_{3T}^2} \gg \ldots$$

- **Still, hard to resume a gluon cascade due to color factors**

Small-x Evolution

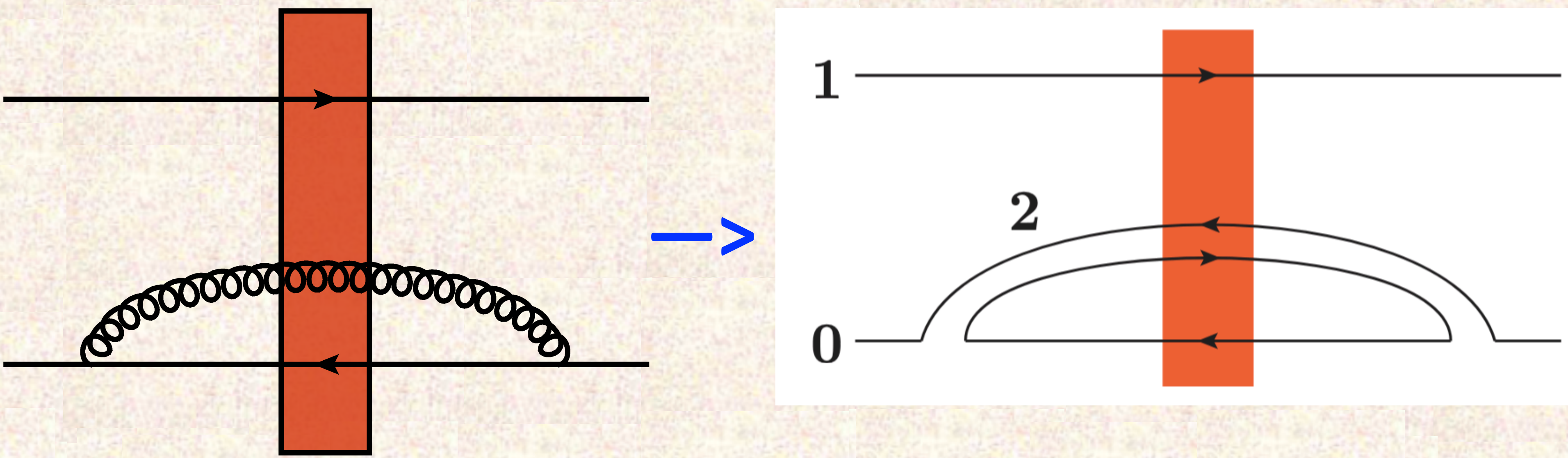
- **We employ the large-N_c limit ('t Hooft '77)**

$$N_c \gg 1$$



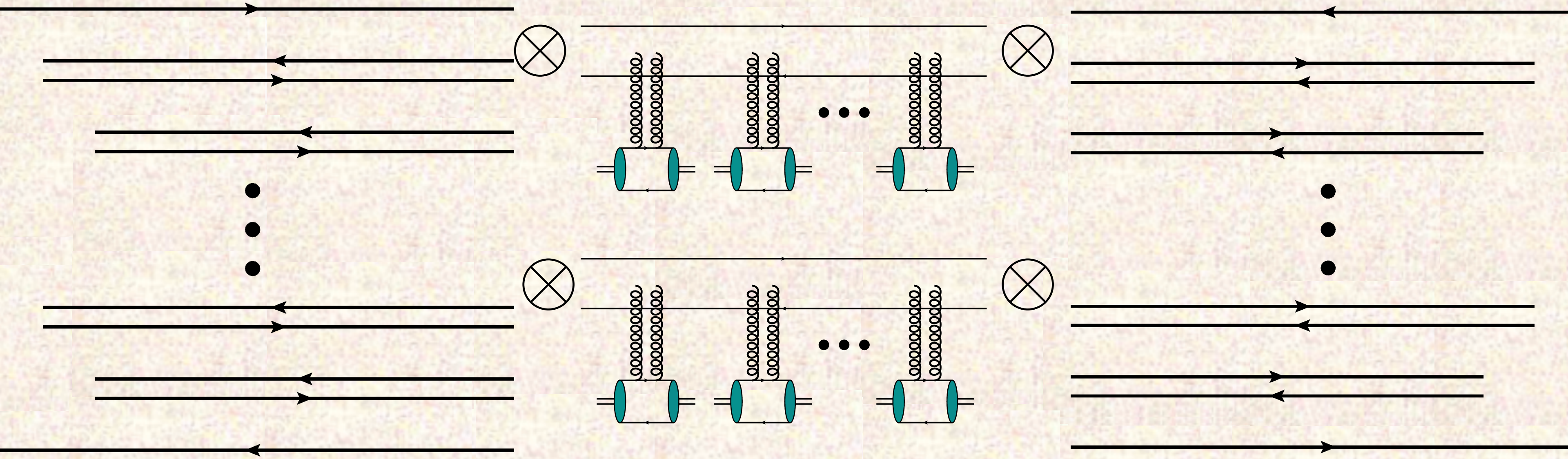
$$N_c \otimes \bar{N}_c = 1 \oplus (N_c^2 - 1) \sim N_c^2 - 1$$

- **We replace the gluon by a qqbar pair. Only planar diagrams contribute**



Small-x Evolution

- **Forward scattering amplitude for dipole–nucleus scattering including small-x evolution:**



- **Instead of calculating the forward dipole–nucleus scattering amplitude N we start with the S-matrix, which is related to N via**

$$S(Y; \mathbf{x}_{01}, \mathbf{b}) = 1 - N(Y; \mathbf{x}_{01}, \mathbf{b})$$

Small-x Evolution

- **The sum of the probability to have one daughter dipole in the parent dipole convoluted with the *S*-matrix for the dipole–nucleus scattering in the GGM approximation, along with the probability to have two dipoles convoluted with their multiple rescattering interactions with the nucleus, etc., is:**

$$S_{dA}\left(Y;\mathbf{x}_{01},\mathbf{b}\right) = \sum_{n=0}^{\infty}(-1)^n \int \prod_{i=1}^{n} d^2x_{01,i}\, d^2b_i$$

$$\times\ \gamma^{BA}(\mathbf{x}_{01,i},\mathbf{x}_{0'1'},\mathbf{b}_i)\,\rho_n(Y;\{\mathbf{x}_{01,i},\mathbf{b}_i\})$$

- **Where gamma is the GGM initial condition and rho the dipole densities**

Non-linear evolution

- **The dipole–nucleus S-matrix obey the following non-linear evolution equation (Balitsky '96; Kovchegov '99):**

$$\frac{\partial S_{dA}(Y, \mathbf{x}_{01}, \mathbf{b})}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{12}^2 x_{02}^2}$$

$$\times \left[S_{dA}(Y; \mathbf{x}_{12}, \mathbf{b}) S_{dA}(Y; \mathbf{x}_{02}, \mathbf{b}) - S_{dA}(Y; \mathbf{x}_{01}, \mathbf{b})\right]$$

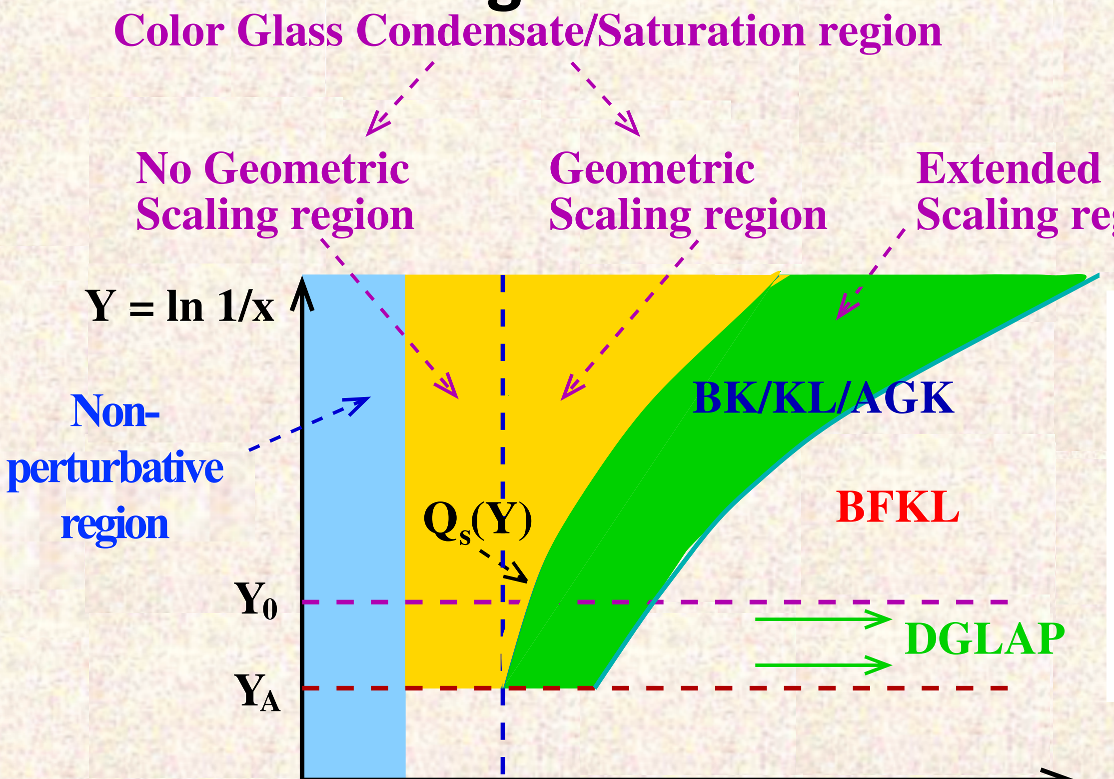
where $Y = \ln 1/x$ **and** $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$, $x_{ij} = |\mathbf{x}_{ij}|$

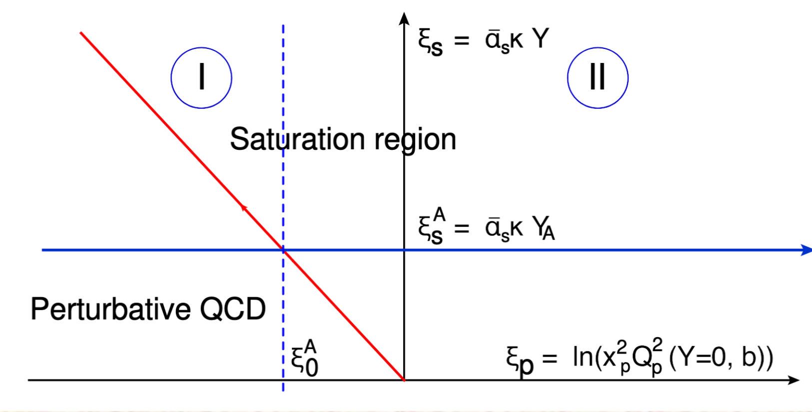
- **The equation resumes power of** $\alpha_S N_c \ln(1/x)$ **(LLA)**
- **The initial condition with energy is given by**

$$S_{dA}(Y = Y_A, \mathbf{x}_{01}, \mathbf{b}) = \exp\left(-\frac{1}{4} x_{01}^2 Q_s^2(Y_A, \mathbf{b})\right)$$

Non-linear evolution

- The nonlinear BK evolution equation describe the transition into the saturation region, along with the physics inside that region.





- Critical line at $x_{01} = 1/Q_s(Y, \mathrm{b})$. It gives the boundary conditions.
- Energy dependence of Qs is $Q_s^2(Y, \mathrm{b}) = Q_{s0}\, e^{\bar{\alpha}_S\, \kappa\, (Y - Y_A)}$

Solution of BK equation

- **For the leading twist BFKL kernel (E. Levin and K. Tuchin '00)**

$$\frac{\bar{\alpha}_S}{2\,\pi}\int d^2 x_2\,\frac{x^2_{01}}{x^2_{12}\,x^2_{02}}\;\rightarrow\;\frac{\bar{\alpha}_S}{2}\int\limits_{Q_s^{-2}(Y,b)}^{x^2_{01}}\frac{d x^2_{02}}{x^2_{02}}\;+\;\frac{\bar{\alpha}_S}{2}\int\limits_{Q_s^{-2}(Y,b)}^{x^2_{01}}\frac{d x^2_{12}}{x^2_{12}}\;\equiv\;\frac{\bar{\alpha}_S}{2}\int\limits_{-\xi_s}^{\xi} d\xi_{02}\;+\;\frac{\bar{\alpha}_S}{2}\int\limits_{-\xi_s}^{\xi} d\xi_{12}$$

where $\xi_{ij}\;=\;\ln(x^2_{ij}\,Q^2_s(Y\;=\;Y_A,\mathbf{b}))$ **and**

- **We have**

$$\frac{\partial\,S_{dA}\,(\xi_s,\xi)}{\partial\,\xi_s}\;=\;\frac{1}{\kappa}\,S_{dA}\,(\xi_s,\xi)\,\int\limits_{-\xi_s}^{\xi}\,d\xi'\,\left(1\;-\;S_{dA}\,(\xi_s,\xi')\right)$$

- **Introducing the auxiliary function**

$$S_{dA}\,(\xi_s,\xi)\;=\;\exp\left(-\,\Omega\,(\xi_s,\xi)\right)$$

Solution of BK equation

- **We can obtain the non-linear equation for** $\Omega\left(\xi,\xi_s\right)$

$$\frac{\partial^2\,\Omega\left(\xi_s,\xi\right)}{\partial\,\xi_s\,\partial\,\xi} = \frac{1}{\kappa}\left(1 - e^{-\,\Omega(\xi_s,\xi)}\right)$$

- **The geometric scaling z=ξs+ξ equation and implicit solution is:**

$$\frac{d^2\,\Omega\left(z\right)}{d\,z^2} = \frac{1}{\kappa}\left(1 - e^{-\,\Omega(z)}\right)$$

$$\Rightarrow \int\limits_{\Omega_0(\xi)}^{\Omega(z)} \frac{d\,\Omega'}{\sqrt{\Omega' + e^{-\,\Omega'} - 1 + \frac{\kappa}{2}\,C_1}} = \sqrt{\frac{2}{\kappa}}\left(z + C_2\right)$$

Solution of BK equation

- **We can assume that ζ ≫ 1 at large z. In this case we have:**
- **1.) Neglecting $e^{-\Omega'}$ in the integrand**

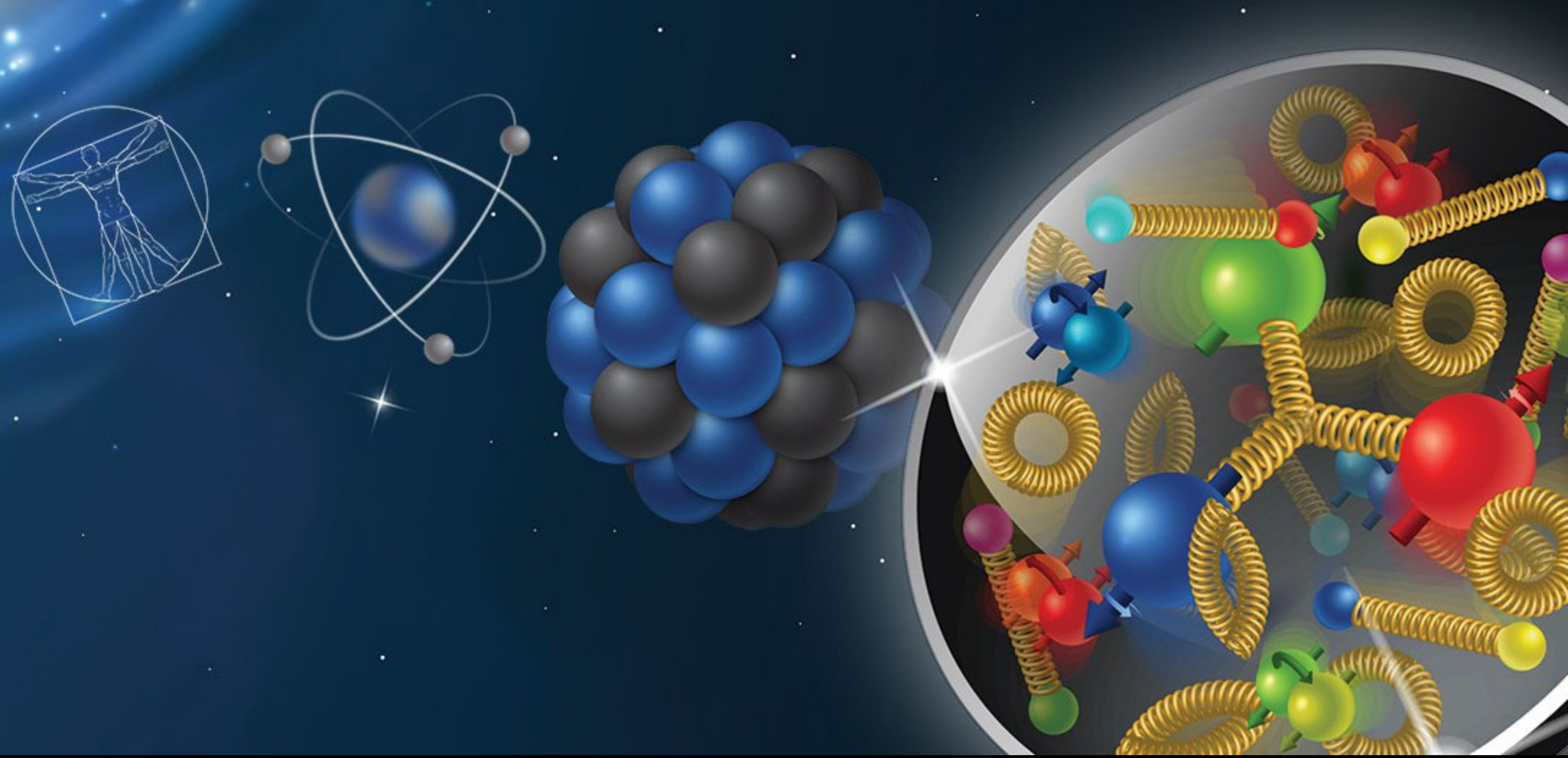
$$\int_{\Omega_0(\xi)}^{\Omega(z)} \frac{d\,\Omega'}{\sqrt{\Omega' - \Omega_0}} = \sqrt{\frac{2}{\kappa}}\,(z + C_2)$$

$$\sqrt{\Omega(z) - \Omega_0(z_A)} = \frac{1}{\sqrt{2\,\kappa}}\,(z + C_2)$$

- **Obtaining** $N(z) = 1 - e^{-\Omega(z)}$

$$= 1 - C\exp\left(-\frac{(z - \tilde{z})^2}{2\,\kappa}\right)$$

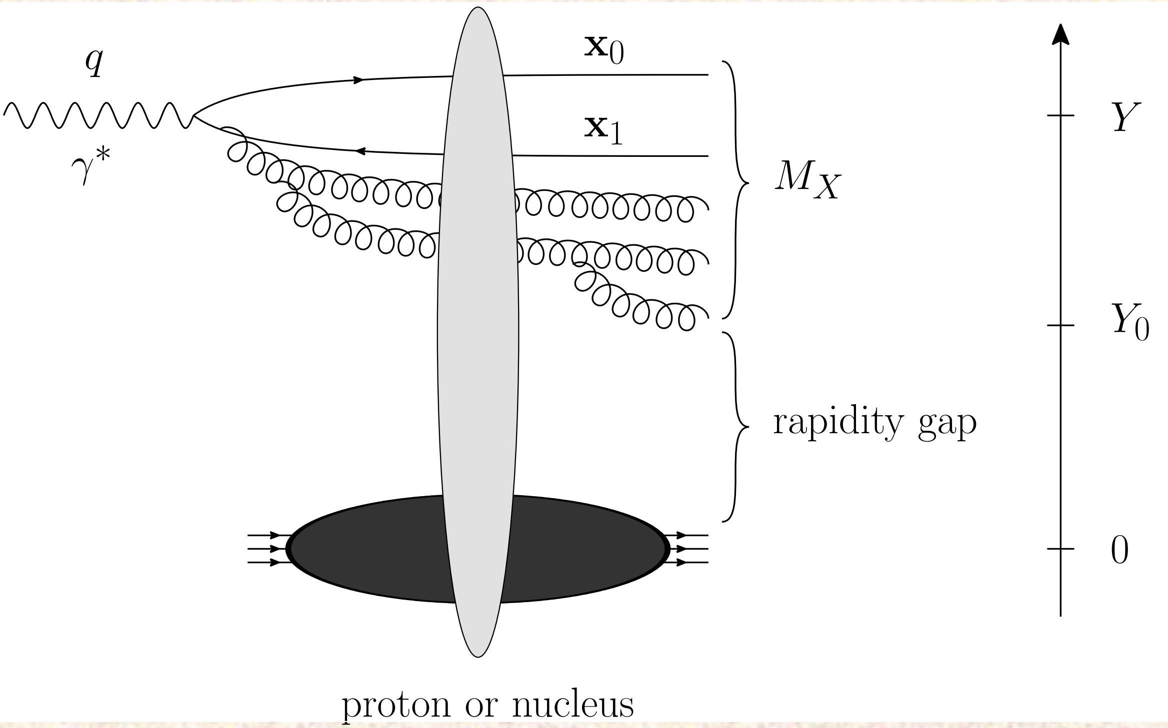
where $\kappa \equiv \frac{\chi(\gamma_{cr})}{1 - \gamma_{cr}}$

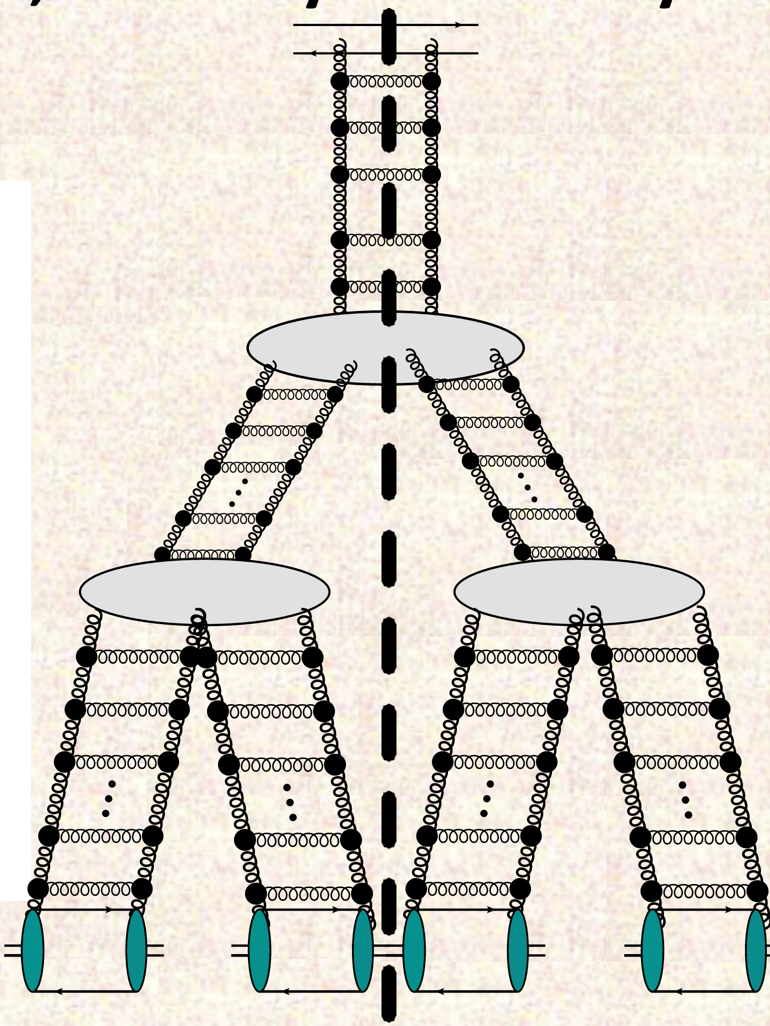


Single diffractive dissociation

Single diffractive dissociation

- At high M_X, in addition to the qqbar, one may have many more gluons produced (y>Y0):





The process

Diffraction evolution as fan diagrams

Diffractive S-matrix

- **We define a new object — the diffractive S-matrix — as:**

$$S^D_{dA}(Y, Y_0; \mathbf{x}_{01}, \mathbf{b}) = \sum_{n=0}^{\infty} (-1)^n \sum_{k=0}^{\infty} (-1)^k$$

$$\times \int \prod_{i=1}^{n} d^2 x_{01,i}\, d^2 b_i \prod_{j=1}^{k} d^2 x_{01,j}\, d^2 b_j$$

$$\times\, \gamma^{BA}(\mathbf{x}_{01,i}, \mathbf{x}_{0'1'}, \mathbf{b}_i)\, \gamma^{BA}(\mathbf{x}_{01,j}, \mathbf{x}_{0'1'}, \mathbf{b}_j)$$

$$\times\, \rho^D_{n,k}(Y_m, Y_0; \{\mathbf{x}_{01,i}, \mathbf{b}_i\})\, \rho_n(Y_0; \{\mathbf{x}_{01,i}, \mathbf{b}_i\})$$

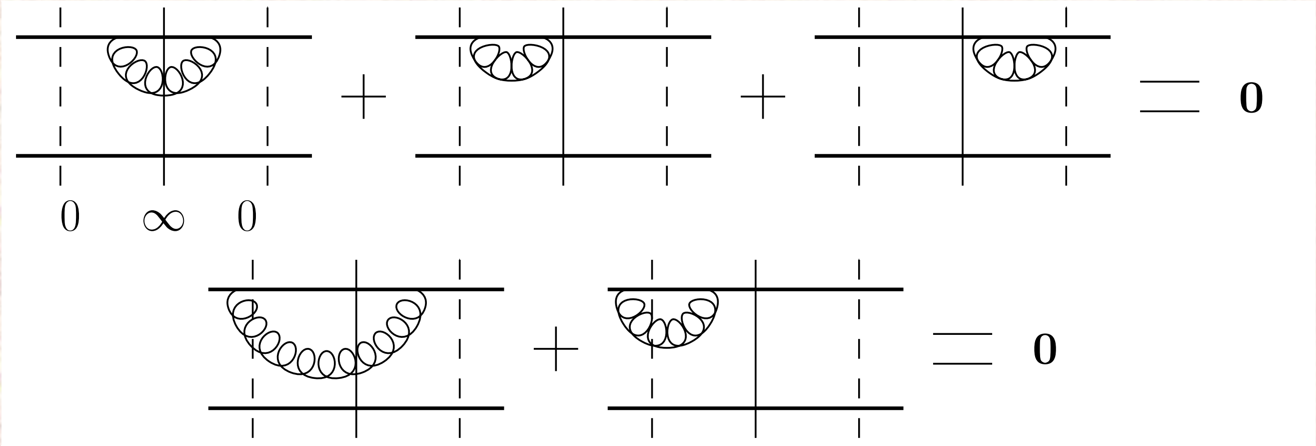
- **where parton densities**

$$\rho^D_{n,k}(r_1, b_1, \ldots, r_n, b_n; \ldots, r_k, b_k; Y, Y_0) =$$

$$\frac{1}{n!} \prod_{i=1}^{n} \frac{\delta}{\delta u_i} \frac{1}{k!} \prod_{i=1}^{k} \frac{\delta}{\delta \bar{u}_i} Z\left(Y, Y_0; [u], [\bar{u}]\right)|_{u=1, \bar{u}=1}$$

Diffractive S-matrix

- **Using the following cancellations of final state interactions for gluons with $y>Y_0$ (Z. Chen, A. Mueller '95)**



- **The equation for the single diffraction has the following form in QCD**

$$\frac{\partial\, S^D_{dA}\left(Y, Y_0; \mathbf{x}_{01}, \mathbf{b}\right)}{\partial\, Y} \;=\; \frac{\alpha_S\, N_c}{2\,\pi^2} \int d^2 x_2\, \frac{x^2_{01}}{x^2_{12}\, x^2_{02}}$$

$$\times\ \left[S^D_{dA}\left(Y, Y_0; \mathbf{x}_{12}, \mathbf{b}\right)\, S^D_{dA}\left(Y, Y_0; \mathbf{x}_{02}, \mathbf{b}\right) \,-\, S^D_{dA}\left(Y, Y_0; \mathbf{x}_{01}, \mathbf{b}\right)\right]$$

I.C. $S^D_{dA}(Y = Y_0, Y_0; \mathbf{x}_{01}, \mathbf{b}) \,=\, S^2_{dA}(Y = Y_0; \mathbf{x}_{01}, \mathbf{b})$

Solution in the saturation region

- **Introducing** $S^{D}_{dA}\left(\xi,\xi_s\right) = \exp\left(-\Omega^{D}\left(\xi,\xi_s\right)\right)$

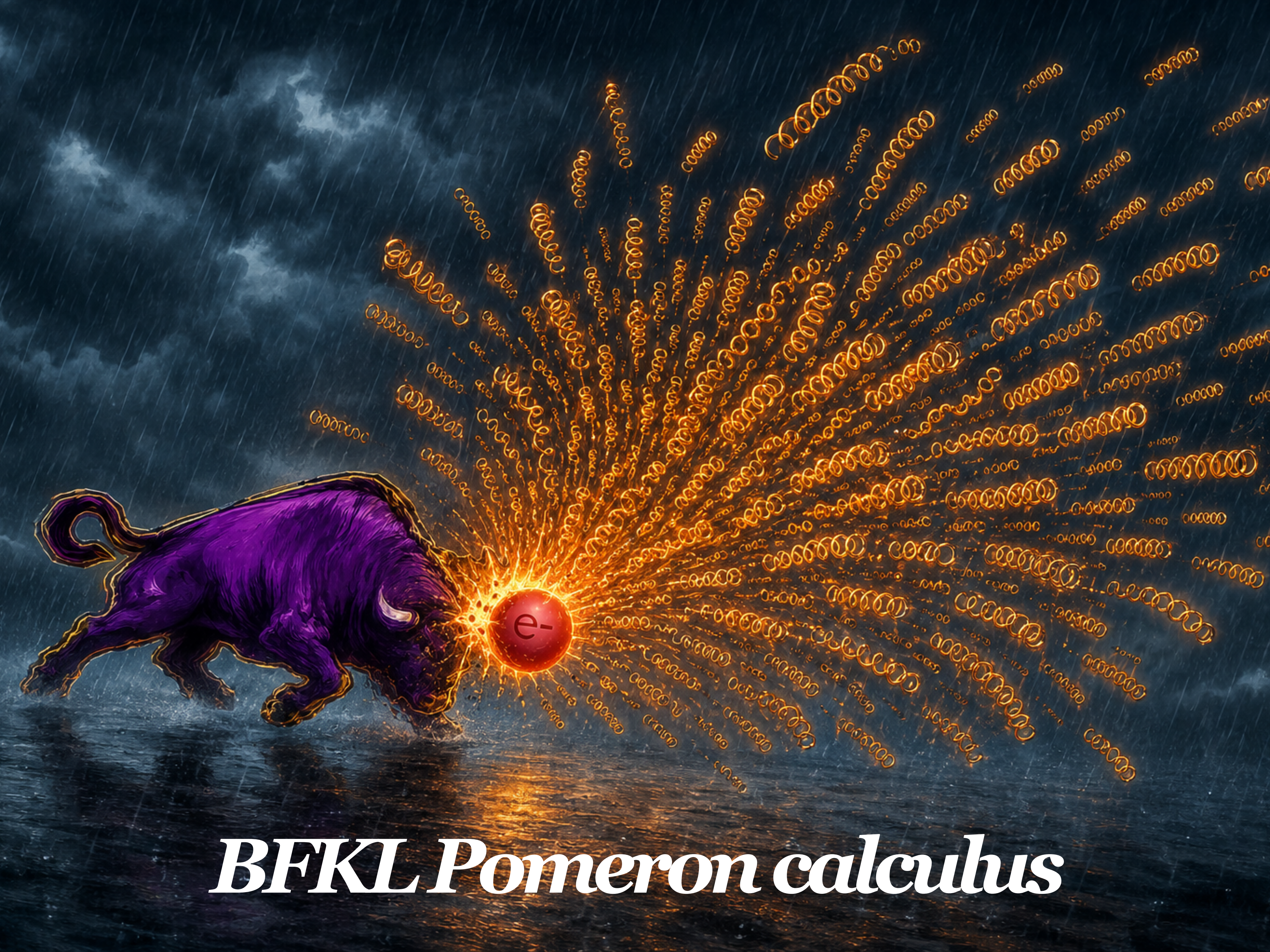
$$\frac{\partial^2\,\Omega^{D}\left(\xi_s,\xi\right)}{\partial\,\xi_s\,\partial\,\xi} = \frac{1}{\kappa}\left(1 - e^{-\,\Omega^{D}\left(\xi_s,\xi\right)}\right)$$

- **The solution to this equation has the form:**

$$S^{D}_{dA}\left(z\right) = S_{I\!P}\left(z_m\right)S^{2}_{dA}\left(z_0\right)$$

- **This notation reflects that the production of gluons with mass M_X occurs in the dipole-Pomeron scattering**


e-
Gluon production
at small-x

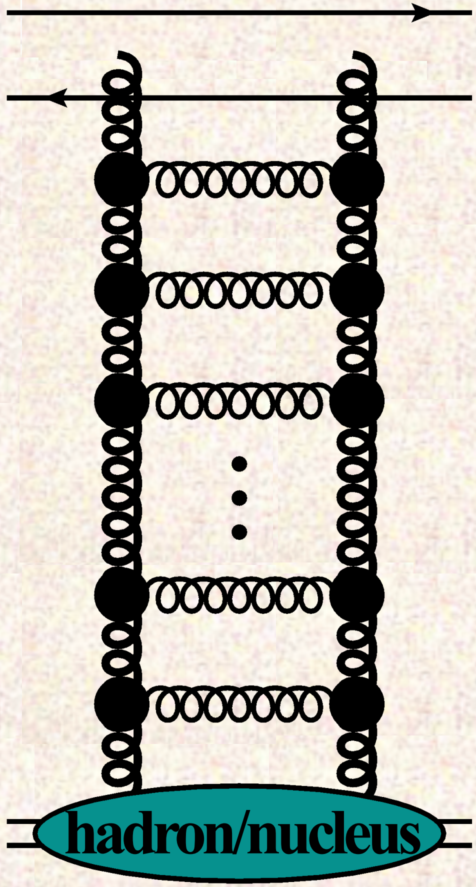

e-
BFKL Pomeron calculus

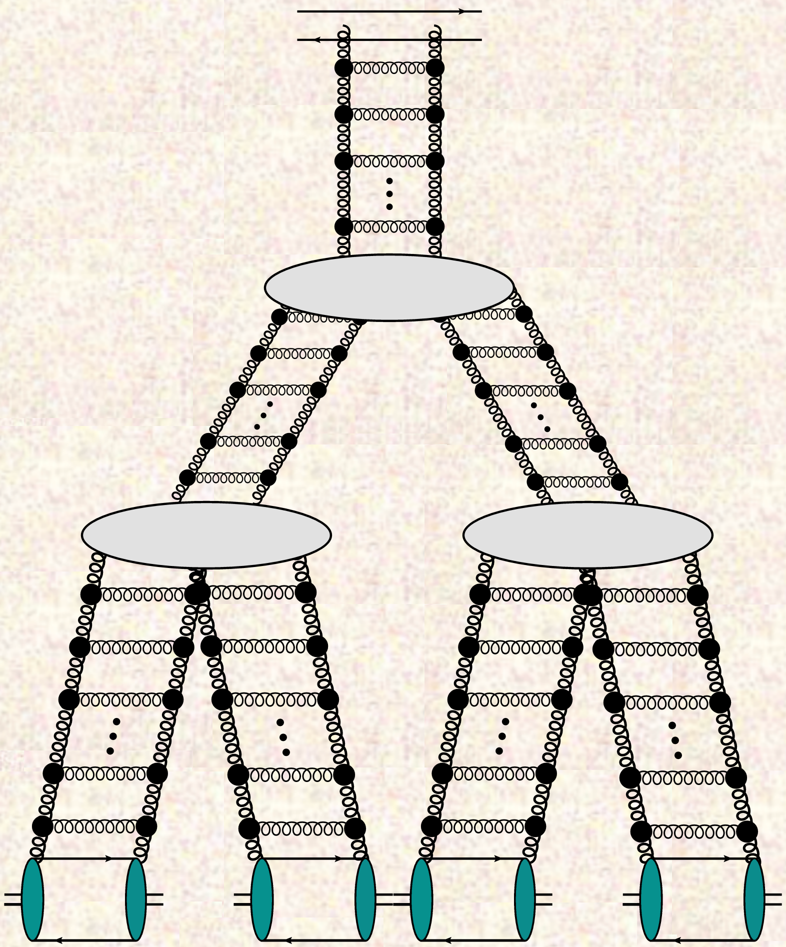
The old times

- The scattering amplitude in the "BFKL Pomeron calculus":

Single Pomeron <-> Linear evolution

Multiple Pomerons <-> Non-linear evolution





- Pomeron=Bound state of two reggeized gluons

The old times

- **The scattering amplitude can also be calculated as the sum of the "fan" diagrams of the BFKL Pomeron calculus.**

$$N(Y, \mathbf{x}_{01}, \mathbf{b}) \,=\, \sum_{k=1}^{\infty} (-1)^{k+1} \underbrace{C_k(\mathbf{x}_{01}, \mathbf{b}) \left(\operatorname{Im} G_{\rm BFKL}(Y, \mathbf{x}_{01}, \mathbf{b})\right)^k}_{= F_k(Y, \mathbf{x}_{01}, \mathbf{b})}$$

- **where F_k is the contribution of the exchange of k-Pomerons to the cross section. This series we do not known except for the initial conditions**

$$N^{(0)}(Y = Y_A, \mathbf{x}_{01}, \mathbf{b}) \,=\, 1 \,-\, \exp\left(-\frac{1}{4}\, x^2_{01}\, Q^2_s(Y_A, \mathbf{b})\right)$$

$$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k!} \left(\frac{x^2_{01} Q^2_s(Y_A, \mathbf{b})}{4}\right)^k$$

$$\Rightarrow\; F_k(Y = Y_A; \mathbf{x}_{01}, \mathbf{b}) \;=\; \frac{1}{k!} \left(\frac{x^2_{01}\, Q^2_{s0}}{4}\right)^k$$

The equivalence between the two approaches

- In QCD in the LLA, there is an equivalence between the pomeron theory of high energy strong interactions and the dipole approach to QCD (almost always work).

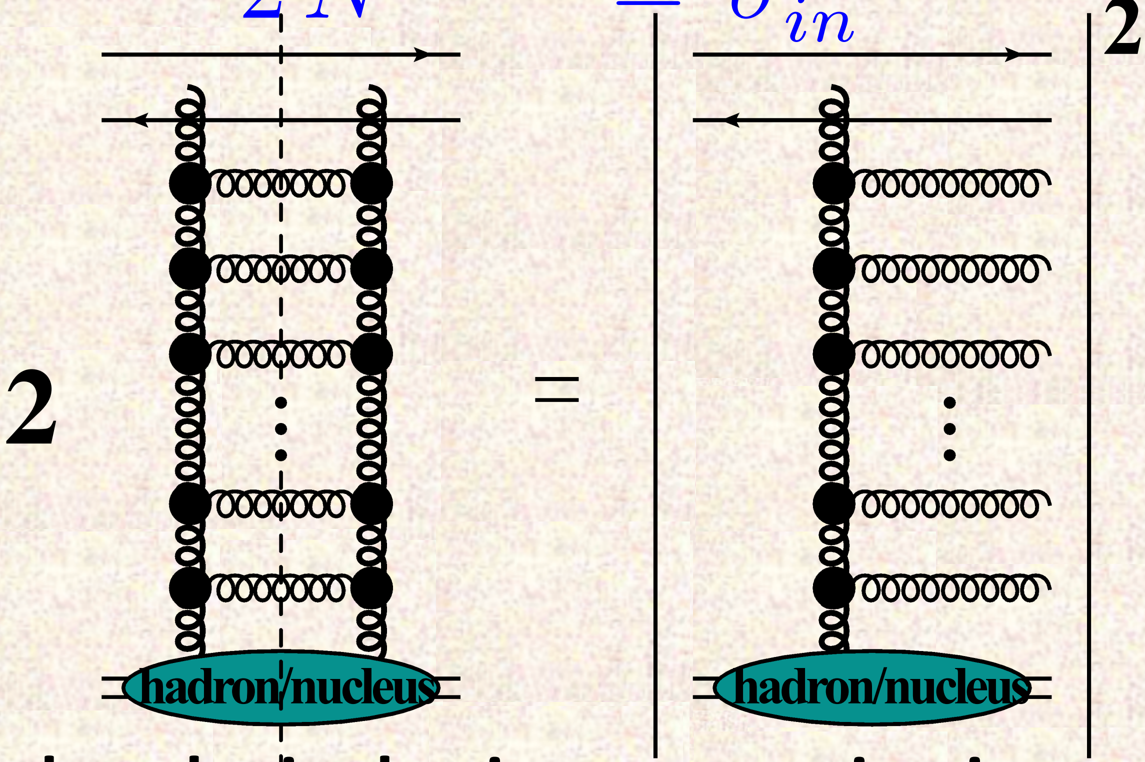
Multiple Pomerons <-> Non-linear evolution

- We learned how to write nonlinear corrections only when the amplitude is the sum of ‘fan’ diagrams, in which we have P-> 2 P vertex.
- In our treatment of the multiplicity distributions, we are going to explore again this equivalence.

The Cut Pomeron

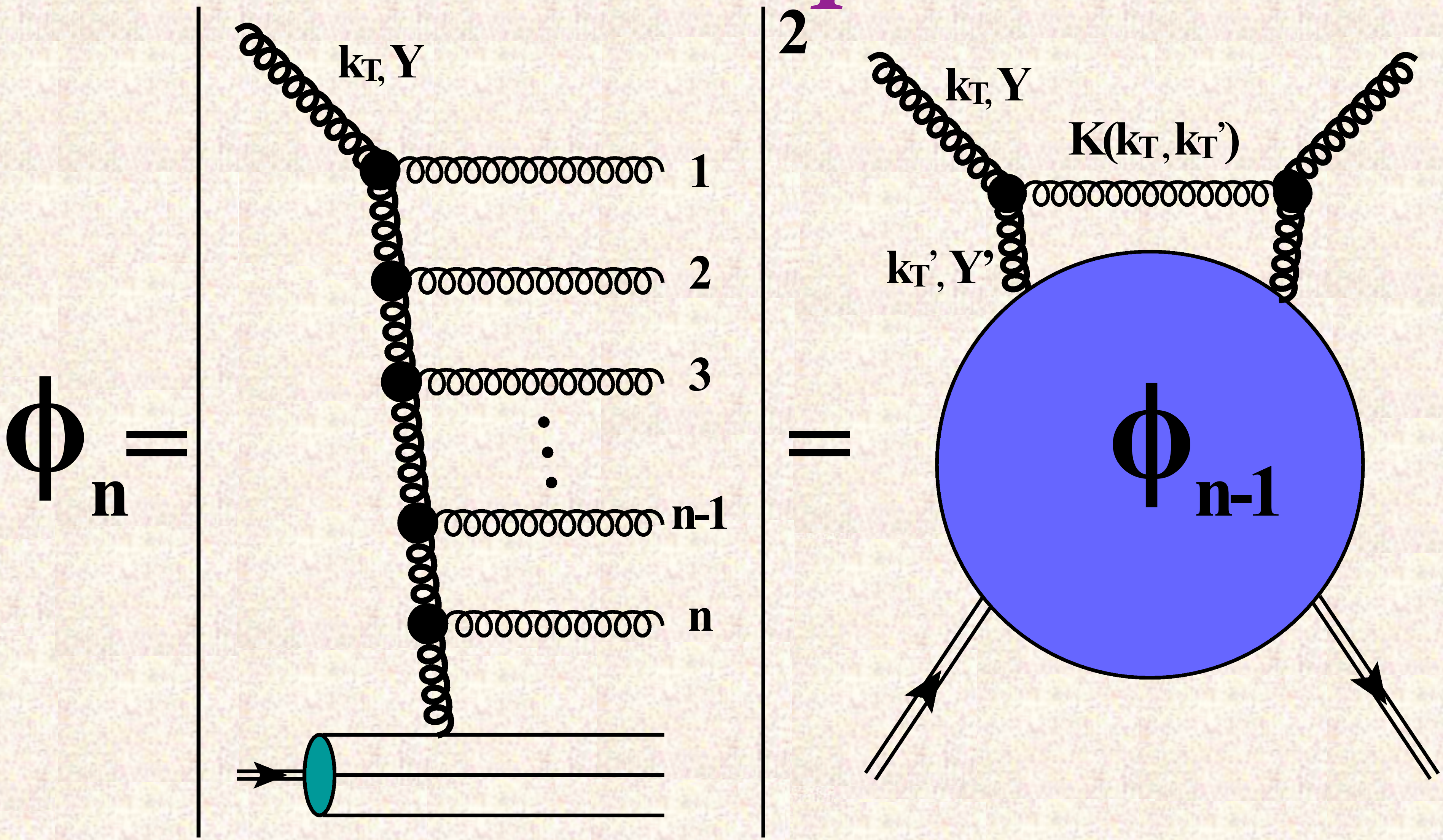
- **In the single-Pomeron approximation, the scattering amplitude is small, N<<1. If N <<1, then the elastic contribution N^2 << N can be neglected:**

$$2\,N^{\mathrm{BFKL}} = \sigma_{in}^{\mathrm{BFKL}}$$



- **It tells us that the inelastic cross section in one pomeron exchange approximation is twice the forward amplitude of the process, since the latter is purely imaginary.**

The BFKL equation



$$\phi_n(Y, \mathbf{k}_T) \; = \; \frac{\alpha_s N_c}{\pi^2} \int dY' \int d^2\mathbf{k}'_T \, K(\mathbf{k}_T, \mathbf{k}'_T) \, \phi_{n-1}(Y', \mathbf{k}'_T)$$

The BFKL equation

- **The BFKL equation**

$$\phi_n(Y,\mathbf{k}_T) \;=\; \frac{\alpha_s N_c}{\pi^2}\int dY' \int d^2\mathbf{k}'_T\, K(\mathbf{k}_T,\mathbf{k}'_T)\,\phi_{n-1}(Y',\mathbf{k}'_T)$$

- **In its differential form**

$$\frac{\partial\,\phi_n(Y,\mathbf{k}_T)}{\partial\,Y} \;=\; \frac{\bar{\alpha}_S}{\pi}\int d^2\mathbf{k}'_T\, K(\mathbf{k}_T,\mathbf{k}'_T)\,\phi_{n-1}(Y',\mathbf{k}'_T)$$

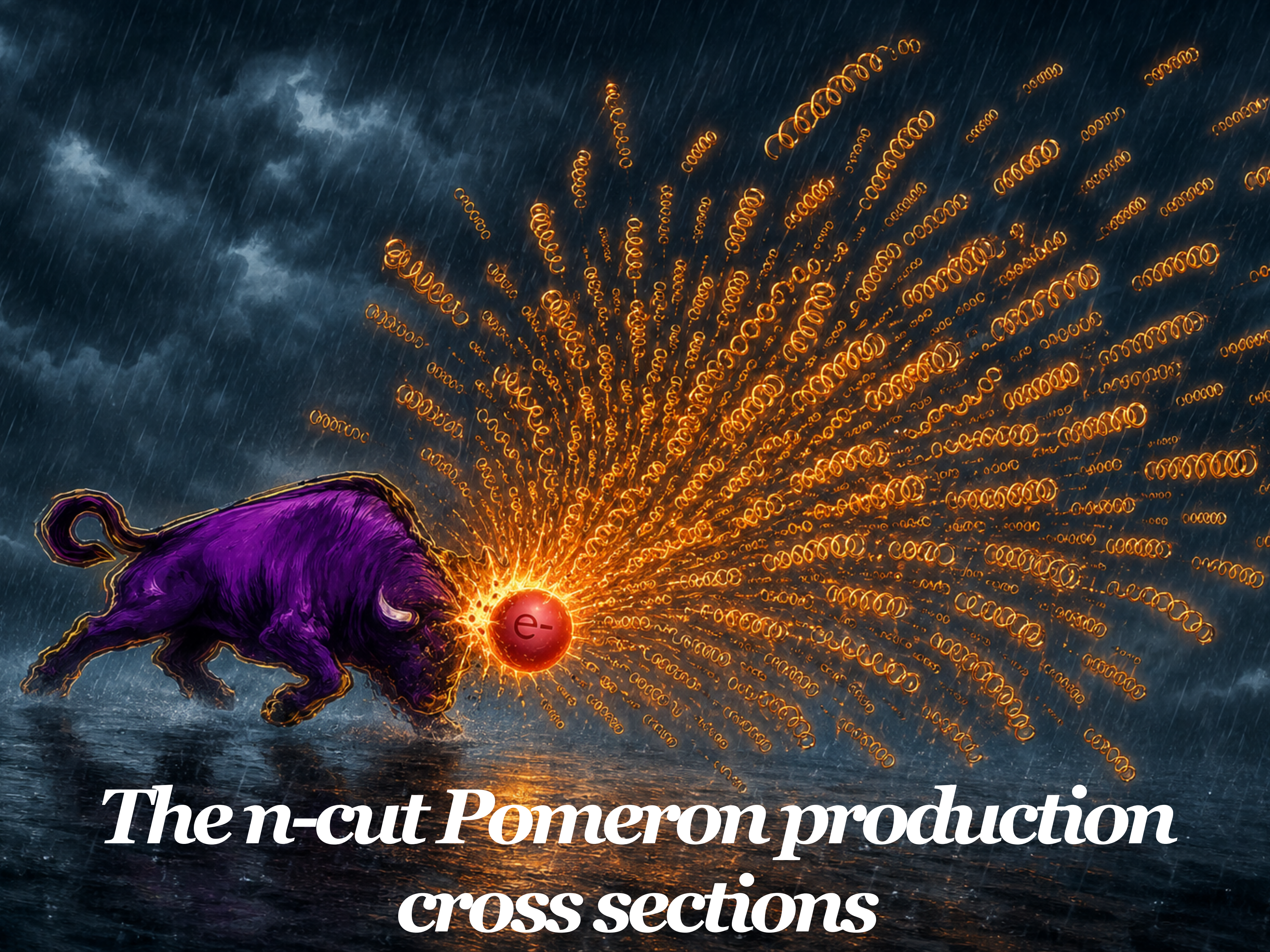
- **Φn it is cross section to find n-gluons in the final state if the energy is Y. Expanding**

$$\phi_n(Y,\mathbf{k}_T) \;=\; \sum_{\gamma} C_n(Y,\gamma)\,(k_T^2)^{\gamma}$$

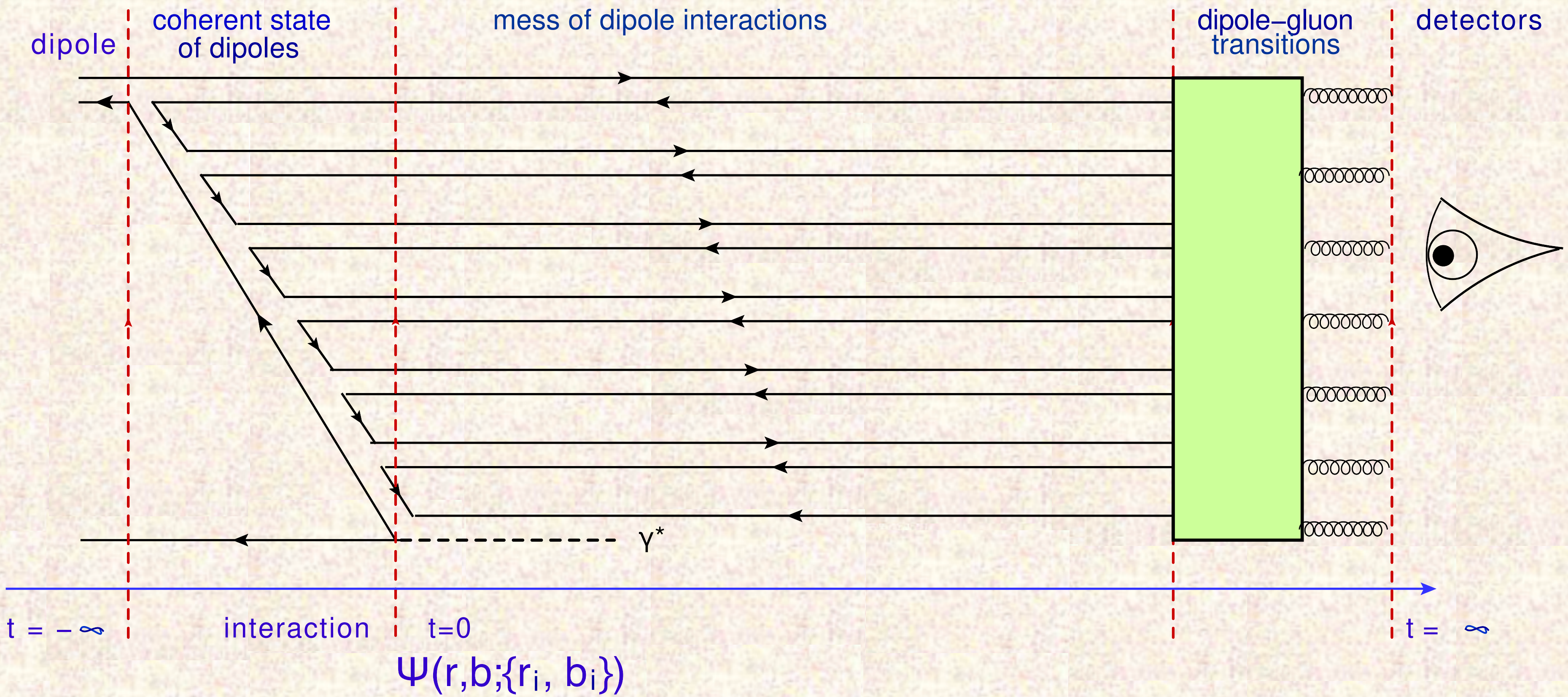
- **we find that the produced gluons in the BFKL follows a Poisson distribution**

$$P_n(Y,\mathbf{k}_T) \;=\; \frac{\phi_n(Y,\mathbf{k}_T)}{\sum_{n=0}^{\infty}\phi_n(Y,\mathbf{k}_T)} \;=\; \frac{(\Delta_{I\!P}\,k_T\,Y)^n}{n!}\,e^{-\Delta_{I\!P}\,k_T\,Y}$$

The n-cut Pomeron production cross sections



Gluon production at small-x



- **The interaction of fast hadron (dipole) with the virtual photon (γ∗). The coherence of the partonic wave function of the fast hadron is destroyed at t = 0, while the gluons can be measured at t = ∞.**

The AGK cutting rules

- The **Abramovsky-Gribov-Kancheli (AGK) cutting rules** allows us to calculate the cross sections for production of n-cut Pomerons if we know F_k: the contribution of the exchange of k-Pomerons to the cross section

$$\sigma_n^{\rm AGK}\left(Y;\mathbf{r},\mathbf{b}\right) \;=\; \sum_{k=n}^{\infty}\sigma_n^k\left(Y;\mathbf{r},\mathbf{b}\right)$$

where

$$\sigma_n^k\left(Y;\mathbf{r},\mathbf{b}\right) \;=\; \begin{cases} (-1)^k\left(2^k \;-\; 2\right)F_k(Y;\mathbf{r},\mathbf{b}) & \text{for } \; n=0 \\ (-1)^{k-n}\dfrac{k!}{(k-n)!\,n!}\,2^k\,F_k(Y;\mathbf{r},\mathbf{b}) & \text{for } \; n\geq 1 \end{cases}$$

and

$$F_k(Y;\mathbf{x}_{01},\mathbf{b}) \;=\; C_k(\mathbf{x}_{01},\mathbf{b})\left(\operatorname{Im} G_{\rm BFKL}(Y;\mathbf{x}_{01},\mathbf{b})\right)^k$$

The AGK cutting rules

- **n=1**

$$\sigma_1^{\mathrm{AGK}} = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{k!}{(k-1)!\,1!} 2^k C_k \left(\mathrm{Im}\, G_{\mathrm{BFKL}}\right)^k$$

$$= C_1\, \sigma_{in}^{\mathrm{BFKL}} + C_2\, 2\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})\, \sigma_{in}^{\mathrm{BFKL}}$$

$$+ C_3\, 3\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})^2\, \sigma_{in}^{\mathrm{BFKL}} + \cdots$$

- **n=2**

$$\sigma_2^{\mathrm{AGK}} = \sum_{k=2}^{\infty} (-1)^{k-2} \frac{k!}{(k-2)!\,2!} 2^k C_k \left(\mathrm{Im}\, G_{\mathrm{BFKL}}\right)^k$$

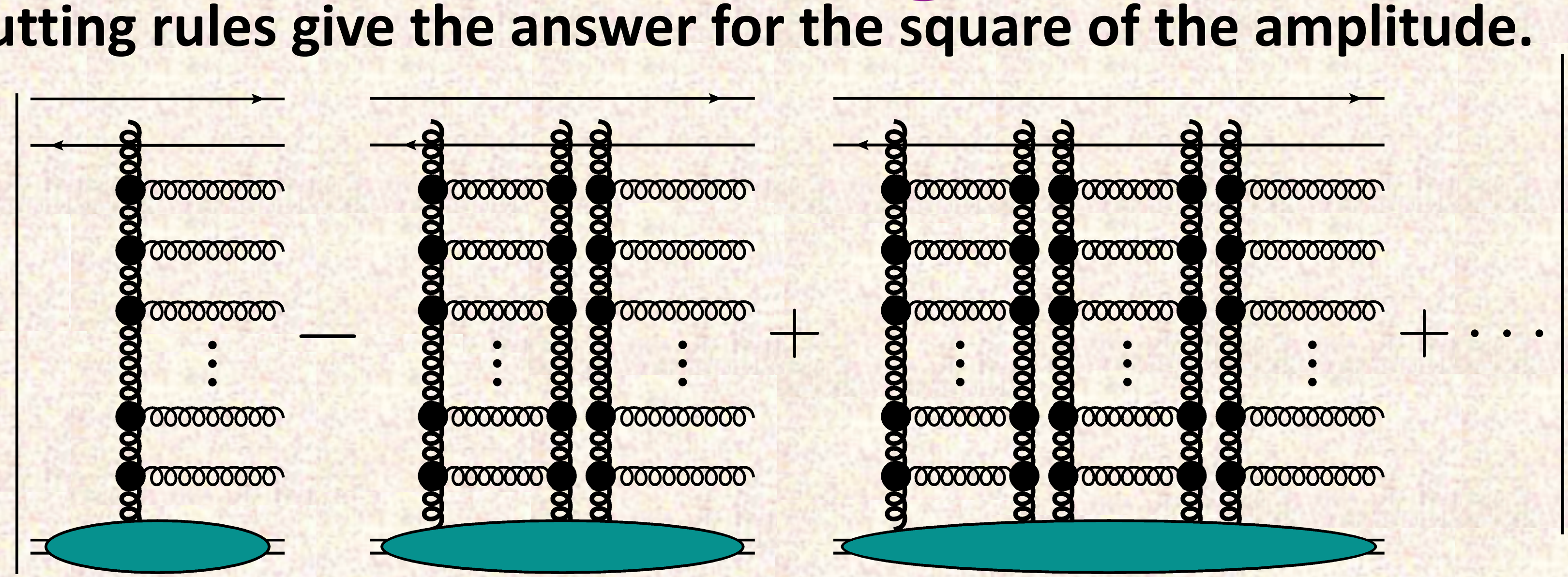
$$= C_2\, (\sigma_{in}^{\mathrm{BFKL}})^2 + C_3\, 3\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})\, (\sigma_{in}^{\mathrm{BFKL}})^2$$

$$+ C_4\, 6\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})^2\, (\sigma_{in}^{\mathrm{BFKL}})^2 + \cdots$$

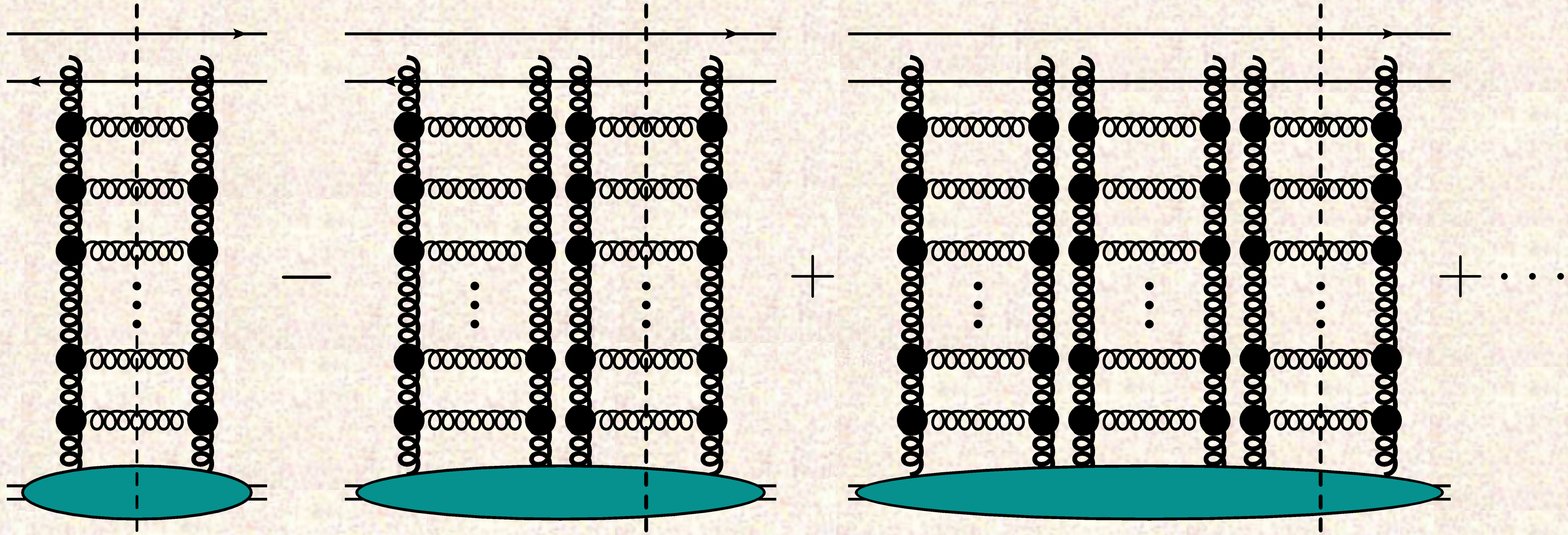
The AGK cutting rules

- **AGK cutting rules give the answer for the square of the amplitude.**

$\sigma_1^{\mathrm{AGK}} =$



$=$



$C_1\, \sigma_{in}^{\mathrm{BFKL}}$ $\quad$ $C_2\, 2\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})\, \sigma_{in}^{\mathrm{BFKL}}$ $\quad$ $C_3\, 2\, (2\, \mathrm{Im}\, G_{\mathrm{BFKL}})^2\, \sigma_{in}^{\mathrm{BFKL}}$

Produced particles

- **To find the multiplicity distribution of particles in the final states one needs to convolute σ_k with the distribution of particles inside k-cut Pomerons**

$$\sigma_n^{f.s.}(Y;\mathbf{x}_{01},\mathbf{b}) = \sum_{k=1}^{\infty} \underbrace{\sigma_k^{\mathrm{AGK}}(Y;\mathbf{x}_{01},\mathbf{b})}_{\propto\ \left(\sigma_{in}^{\mathrm{BFKL}}\right)^k} \underbrace{\mathcal{P}_n^{I\!P}(k\,\Delta_{\mathrm{BFKL}}Y)}_{\text{Poisson distribution}}$$

Poisson distribution

$$\xrightarrow{Y \gg 1} \sigma^{\mathrm{AGK}}_{k=n/(\Delta_{\mathrm{BFKL}}\,Y)}(Y,\mathbf{x}_{01},\mathbf{b})$$

The AGK cutting rules

- **We don't know how to sum this series except for the initial condition:**

$$F_k(Y = Y_A; \mathbf{x}_{01}, \mathbf{b}) \;=\; \frac{1}{k!}\,\left(\frac{x^2_{01}\,Q^2_{s0}}{4}\right)^k$$

$$\sigma^{\rm AGK}_{n,01}(Y = Y_A) \;=\; \sum_{k=n}^{\infty}(-1)^{k-n}\,\frac{k!}{(k\,-\,n)!\,n!}\,2^k\,\frac{1}{k!}\,\left(\frac{x^2_{01}\,Q^2_{s0}}{4}\right)^k$$

$$=\;\frac{\left(\frac{1}{2}\,x^2_{01}\,Q^2_{s0}\right)^n}{n!}\,\exp\left\{-\,\frac{1}{2}\,x^2_{01}\,Q^2_{s0}\right\}$$

- **For** $\xi \;=\; \ln\left(x^2_{01}\,Q^2_{s0}\right)$

$$\sigma^{\rm AGK}_{n}(Y = Y_A; \mathbf{x}_{01}, \mathbf{b}) \;=\; \frac{\left(\frac{1}{2}\,e^{\xi}\right)^n}{n!}\,\exp\left\{-\,\frac{1}{2}\,e^{\xi}\right\}$$

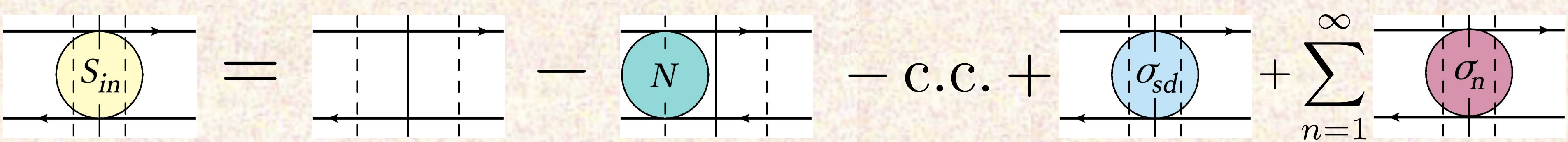
The particle production S-matrix

- **The AGK cutting rules can be replaced by the evolution equations for σ_n. From the s-channel unitarity:**

$$2\,N(Y;\mathbf{x}_{01},\mathbf{b}) \;=\; \sigma_{SD}(Y;\mathbf{x}_{01},\mathbf{b}) \,+\, \sigma_{in}(Y;\mathbf{x}_{01},\mathbf{b})$$

- **We define a new quantity – the particle production S-matrix "S_{in}": it includes σ_n along with the no-interaction term (=1) and all the interaction terms (N and σ_{sd}) on either side (or both sides) of the final state cut:**

$$S_{in} \;=\; 1 \;-\; 2\,N \;+\; \sigma_{SD} \;+\; \sum_{n=1}^{\infty} \sigma_n$$



The particle production S-matrix

- **The evolution equation for S_{in} is**

$$\underbrace{\frac{\partial}{\partial Y} S_{in}(Y;\mathbf{x}_{01},\mathbf{b})}_{\text{select fixed multiplicity terms}} \;=\; \frac{\bar{\alpha}_S}{2\,\pi}\int d^2 x_2\,\frac{x^2_{01}}{x^2_{12}\,x^2_{02}}\,\underbrace{\left[S_{in}(Y;\mathbf{x}_{12},\mathbf{b})\,S_{in}(Y;\mathbf{x}_{02},\mathbf{b})\;-\;S_{in}(Y;\mathbf{x}_{01},\mathbf{b})\right]}_{\text{select fixed multiplicity terms}}$$

- **Replacing $S_{in}=1-2N+\sigma_{SD}+\Sigma_n\sigma_n$ in the equation for the S_{in}-matrix, we obtain the BK, the KL and the AGK/KLP evolution equations:**

$$\frac{\partial}{\partial\,Y}\,\sigma_n\left(Y;\mathbf{x}_{01},\mathbf{b}\right)\;=\;\frac{\bar{\alpha}_S}{2\,\pi}\int d^2 x_2\,\frac{x^2_{01}}{x^2_{02}\,x^2_{12}}$$

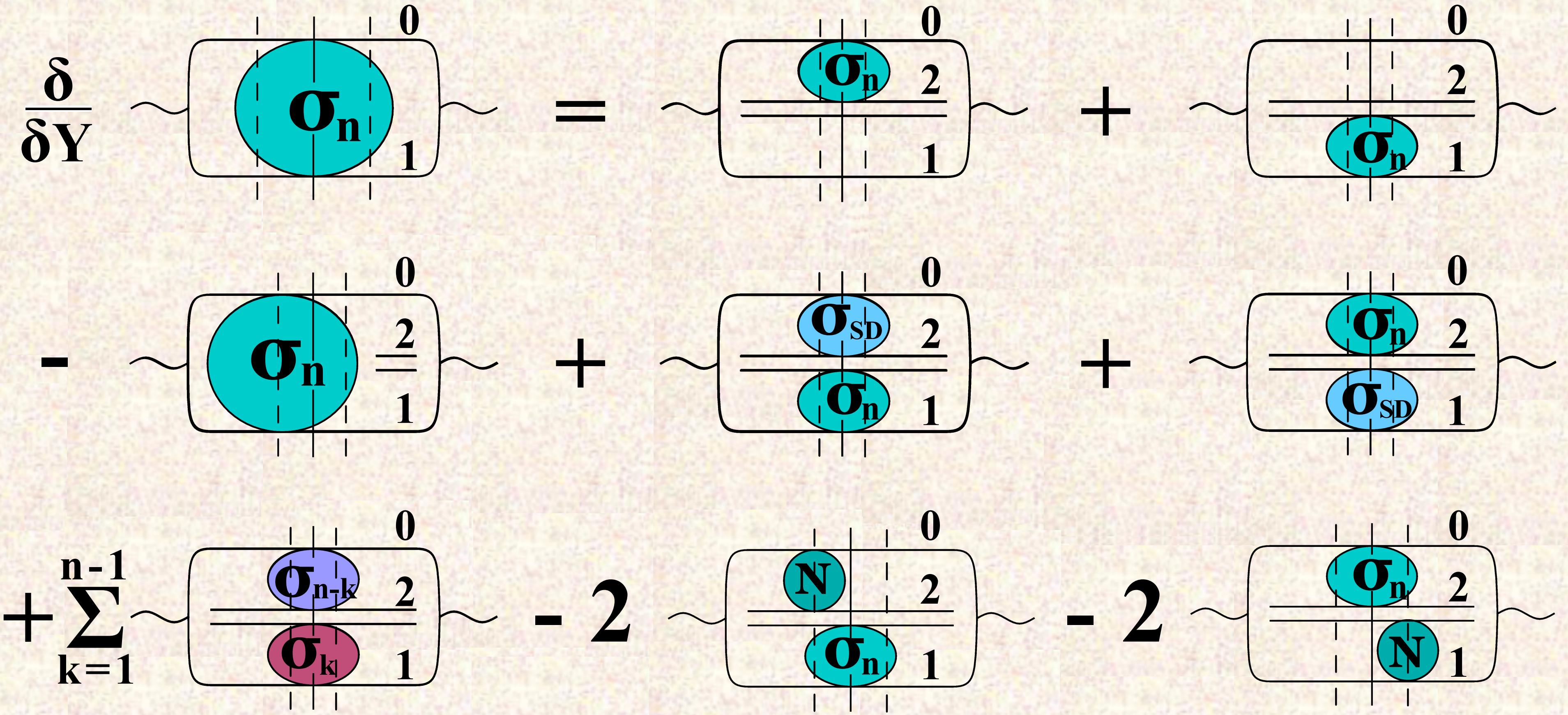
$$\times\left[\sigma_n\left(Y;\mathbf{x}_{12},\mathbf{b}\right)\;+\;\sigma_n\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\;-\;\sigma_n\left(Y;\mathbf{x}_{01},\mathbf{b}\right)\right.$$

$$+\;\sigma_n\left(Y;\mathbf{x}_{12},\mathbf{b}\right)\,\sigma_{SD}\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\;+\;\sigma_n\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\,\sigma_{SD}\left(Y;\mathbf{x}_{12},\mathbf{b}\right)$$

$$+\;\sum_{k=1}^{n-1}\sigma_{n-k}\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\,\sigma_k\left(Y;\mathbf{x}_{12},\mathbf{b}\right)$$

$$\left.-\;2\,\sigma_n\left(Y;\mathbf{x}_{12},\mathbf{b}\right)\,N\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\;-\;2\,\sigma_n\left(Y;\mathbf{x}_{02},\mathbf{b}\right)\,N\left(Y;\mathbf{x}_{12},\mathbf{b}\right)\right]$$

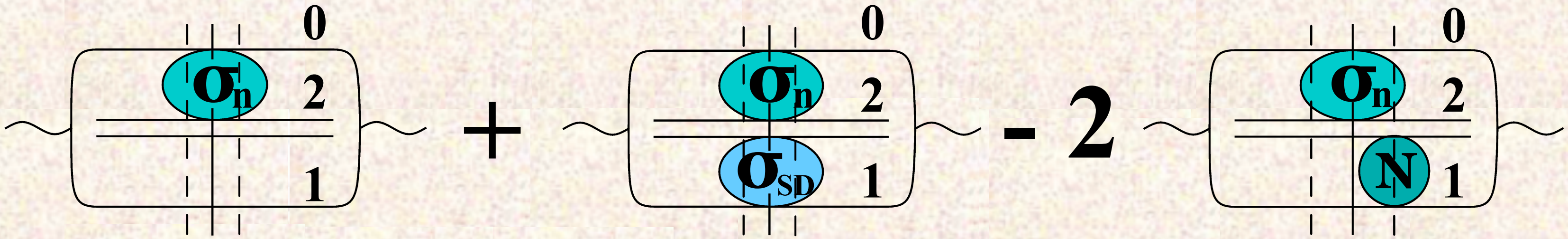
Evolution of σ_n at large N_c



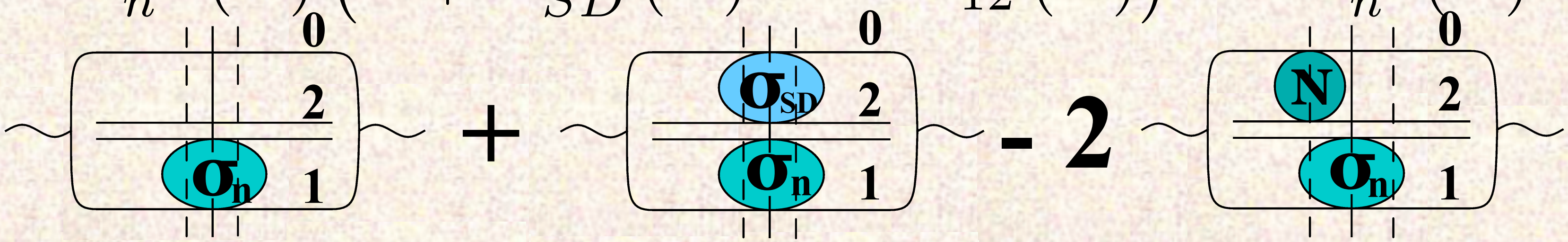
- We can see the equation is linear but with complicated kernel (K_{BFKL} + 2 - 4 N) and non-homogeneous

Evolution of σ_n at large N_c

- **Lets note that**



$$= \sigma_n^{02}(Y)\left(1 + \sigma_{SD}^{12}(Y) - 2\,N_{12}(Y)\right) = \sigma_n^{02}(Y)\,\Delta_{12}(Y)$$



$$= \sigma_n^{12}(Y)\left(1 + \sigma_{SD}^{02}(Y) - 2\,N_{02}(Y)\right) = \sigma_n^{12}(Y)\,\Delta_{02}(Y)$$

- **we reduce to**

$$\Delta = S^D$$

$$\frac{\partial}{\partial Y}\sigma_n(Y,\mathbf{x}_{01},\mathbf{b}) = \frac{\bar{\alpha}_S}{2\pi}\int d^2x_2\,\frac{x_{01}^2}{x_{12}^2\,x_{02}^2}$$
$$\times\left[-\sigma_n(Y,\mathbf{x}_{01},\mathbf{b}) + \sigma_n(Y,\mathbf{x}_{12},\mathbf{b})\,\Delta(Y,\mathbf{x}_{02},\mathbf{b})\right.$$
$$\left.+\,\Delta(Y,\mathbf{x}_{12},\mathbf{b})\,\sigma_n(Y,\mathbf{x}_{02},\mathbf{b}) + \sum_{k=1}^{n-1}\sigma_{n-k}(Y,\mathbf{x}_{02},\mathbf{b})\,\sigma_k(Y,\mathbf{x}_{12},\mathbf{b})\right]$$

$$\sigma_n(Y; \boldsymbol{x}_{01}, \boldsymbol{b})$$

σ_1: region I (GS)

- **For model LT kernel:**

$$\kappa \frac{d\sigma_1(z)}{dz} = -z\,\sigma_1(z) + \sigma_1(z)\int_{\xi_0^A}^{z} dz'\,\Delta(z') + \Delta(z)\int_{\xi_0^A}^{z} dz'\,\sigma_1(z')$$

$$= -\underbrace{\left(z - \int_{\xi_0^A}^{z} dz'\,\Delta(z')\right)}_{T(z)}\sigma_1(z) + \Delta(z)\underbrace{\int_{\xi_0^A}^{z} dz'\,\sigma_1(z')}$$

$$\int_{\xi_0^A}^{z} dz'\,\sigma_1(z') = \underbrace{\int_{\xi_0^A}^{\infty} dz'\,\sigma_1(z')}_{=\sigma_{0,1}} - \underbrace{\int_{z}^{\infty} dz'\,\sigma_1(z')}_{=\tilde{\Sigma}_1(z)}$$

$$= -T(z)\,\sigma_1(z) + \sigma_{0,1}\,\Delta(z) - \Delta(z)\,\tilde{\Sigma}_1(z)$$

$$\Rightarrow \underbrace{\kappa\frac{d\sigma_1(z)}{dz} + T(z)\,\sigma_1(z) - \sigma_{0,1}\,\Delta(z)}_{\mathcal{L}[\sigma_1]} + \underbrace{\Delta(z)\,\tilde{\Sigma}_1(z)}_{\mathcal{N}_{\mathcal{L}}[\sigma_1]} = 0$$

σ_1: region I (GS)

- **We have divided the equation in two parts**

$$\mathcal{L}[\sigma_1] \;=\; \kappa\,\frac{d\sigma_1(z)}{dz} \;+\; T(z)\,\sigma_1(z) \;-\; \sigma_{0,1}\,\Delta(z)$$

$$\mathcal{N}_{\mathcal{L}}[\sigma_1] \;=\; \Delta(z)\,\tilde{\Sigma}_1(z)$$

- **As a solution, we introduce**

$$\sigma_1 \;=\; \sigma_1^{(0)} \;+\; p\,\sigma_1^{(1)} \;+\; p^2\,\sigma_1^{(2)} \;+\;\cdots$$

such that $\mathcal{L}[\sigma_1^{(p)}] + p\,\mathcal{N}_{\mathcal{L}}[\sigma_1^{(p)}] \;=\; 0$

- **The 0th iteration is**

$$\mathcal{L}[\sigma_1^{(0)}];\;\; \kappa\,\frac{d\sigma_1^{(0)}(z)}{dz} \;=\; -\,T(z)\,\sigma_1^{(0)}(z) \;+\; \sigma_{0,1}\,\Delta(z)$$

σ_1: region I (GS)

- **The solution is a sum of the solution to the homogeneous part of the equation and the particular solution of the non-homogeneous one. We obtain**

$$\sigma_1^{(0,I)}(z) = \underbrace{\exp\left(-\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\, T(z')\right)}_{\tilde{\sigma}_1^{(0)}} \left\{\underbrace{\frac{\sigma_{0,1}}{\kappa}\int_{\xi_0^A}^{z} dz'\, \Delta(z')\, \exp\left(\frac{1}{\kappa}\int_{\xi_0^A}^{z'} dz''\, T(z'')\right)}_{\tilde{\sigma}_1^{(0,I)}} + C_\phi^{(1)}\right\}$$

where $C_\phi^{(1)} = \frac{1}{2}e^{\xi_0^A}\exp\left(-\frac{1}{2}e^{\xi_0^A}\right)$

- **For the next homotopy iteration we need to account for the linear terms in p:** $\mathcal{L}[\sigma_1^{(0)} + p\,\sigma_1^{(1)}] + p\mathcal{N}_{\mathcal{L}}[\sigma_1^{(0)}] = 0$
- **The equation takes the form**

$$\kappa\,\frac{d\sigma_1^{(1,I)}(z)}{dz} = -T(z)\,\sigma_1^{(1,I)}(z) - \Delta(z)\,\tilde{\Sigma}_1^{(0)}(z)$$

σ_1: region I (GS)

where $\tilde{\Sigma}_1^{(0)}(z) = \int_z^{\infty} dz' \sigma_1^{(0)}(z')$. The general solution is

$$\sigma_1^{(1,I)}(z) = -\frac{1}{\kappa} \exp\left(-\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\, T(z')\right) \int_{\xi_0^A}^{z} dz' \Delta(z')\, \tilde{\Sigma}_1^{(0)}(z') \exp\left(\frac{1}{\kappa}\int_{\xi_0^A}^{z'} dz''\, T(z'')\right)$$

- In general, the equation for the p-iteration (p≥1) in region I takes the form

$$\kappa \frac{d\sigma_1^{(p,I)}(z)}{dz} = -T(z)\, \sigma_1^{(p,I)}(z) - \Delta(z)\, \tilde{\Sigma}_1^{(p-1)}(z)$$

with solution

$$\sigma_1^{(p,I)}(z) = -\frac{1}{\kappa} \exp\left(-\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\, T(z')\right) \int_{\xi_0^A}^{z} dz'\, \Delta(z')\, \tilde{\Sigma}_1^{(p-1)}(z') \exp\left(\frac{1}{\kappa}\int_{\xi_0^A}^{z'} dz''\, T(z'')\right)$$

σ_1: region II (no GS)

$$\frac{\partial \sigma_1\left(\delta\tilde{Y}, z\right)}{\partial \delta\tilde{Y}} + \kappa \frac{\partial \sigma_1\left(\delta\tilde{Y}, z\right)}{\partial z} =$$

$$- \Bigg(\underbrace{z - \int_{\xi_0^A}^{z} dz' \, \Delta\left(\delta\tilde{Y}, z'\right)}_{T(\delta\tilde{Y}, z)} \Bigg) \sigma_1\left(\delta\tilde{Y}, z\right) + \Delta\left(\delta\tilde{Y}, z\right) \underbrace{\int_{\xi_0^A}^{z} dz' \, \sigma_1\left(\delta\tilde{Y}, z'\right)}$$

$$\int_{\xi_0^A}^{z} dz' \, \sigma_1\left(\delta\tilde{Y}, z'\right) = \underbrace{\int_{\xi_0}^{\infty} dz' \, \sigma_1\left(z'\right)}_{=\sigma_{0,1}} + \underbrace{\int_{\xi_0^A}^{\infty} dz' \left(\sigma_1\left(\delta\tilde{Y}, z'\right) - \sigma_1\left(z'\right)\right)}_{=\delta\Sigma_1\left(\delta\tilde{Y}\right)} - \underbrace{\int_{z}^{\infty} dz' \sigma_1\left(\delta\tilde{Y}, z'\right)}_{=\tilde{\Sigma}_1\left(\delta\tilde{Y}, z\right)}$$

$$\underbrace{\frac{\partial \sigma_1\left(\delta\tilde{Y}, z\right)}{\partial \delta\tilde{Y}} + \kappa \frac{\partial \sigma_1\left(\delta\tilde{Y}, z\right)}{\partial z} + T\left(\delta\tilde{Y}, z\right) \sigma_1\left(\delta\tilde{Y}, z\right) - \sigma_{0,1} \Delta\left(\delta\tilde{Y}, z\right) + \Delta\left(\delta\tilde{Y}, z\right) \delta\Sigma_1\left(\delta\tilde{Y}\right)}_{\mathcal{L}[\sigma_1]}$$

$$\underbrace{- \Delta\left(\delta\tilde{Y}, z\right) \tilde{\Sigma}_1\left(\delta\tilde{Y}, z\right)}_{\mathcal{N}_{\mathcal{L}}[\sigma_1]} = 0$$

σ_1: region II (no GS)

- In this case, the solution is more complex. After some algebra, the general solution takes the form

$$\sigma_1^{(0,II)}\left(\delta\tilde{Y},z\right) \;=\; \Phi_1\left(-\kappa\,\delta\tilde{Y}\;+\;z\right)\sigma_1^{0'}\left(\delta\tilde{Y},z\right)\;+\;\sigma_1^{0'}\left(\delta\tilde{Y},z\right)\tilde{\sigma}_1^{0'}\left(\delta\tilde{Y},z\right)$$

where

$$\sigma_1^{0'}\left(\delta\tilde{Y},z\right)\;=\;\exp\left(-\int\limits_0^{\delta\tilde{Y}} d\delta\tilde{Y}'\,T\left(\delta\tilde{Y}',-\kappa\left(\delta\tilde{Y}\;-\;\delta\tilde{Y}'\right)+\;z\right)\right)$$

$$\tilde{\sigma}_1^{0'}\left(\delta\tilde{Y},z\right)\;=\;\frac{1}{\kappa}\int\limits_{\xi_0^A}^{z} dz'\,\frac{\sigma_{0,1}}{\sigma_1^{0'}\left(\delta\tilde{Y}-\frac{z-z'}{\kappa},z'\right)}\,\Delta\left(\delta\tilde{Y}-\frac{z-z'}{\kappa},z'\right)\;+\;C_{\sigma_1}$$

$$\Phi_1\left(\xi\right)\;=\;\frac{\frac{1}{2}e^{\xi}\exp\left(-\frac{1}{2}e^{\xi}\right)}{\sigma_1^{0'}\left(\delta\tilde{Y}=0,z=\xi\right)}\;-\;\tilde{\sigma}_1^{0'}\left(\delta\tilde{Y}=0,z=\xi\right)$$

- For other iterations, we just modify our definition of the non-homogeneous term by adding the contribution from N_L

σ_n

- **Now we need to add to our definition of L[σ_1] the non-homogeneous term**

$$\mathcal{L}[\sigma_n] = \kappa \frac{d\sigma_n(z)}{dz} + T(z)\,\sigma_n(z) - \underbrace{\sigma_{0,n}\,\Delta(z) - \sum_{k=1}^{n-1} \Sigma_{n-k}(z)\,\sigma_k(z)}_{-U_n(z)}$$

$$\mathcal{N}_{\mathcal{L}}[\sigma_n] = \Delta(z)\,\tilde{\Sigma}_n(z)$$

- **We solve the equation in a similar way to σ_1, yielding**

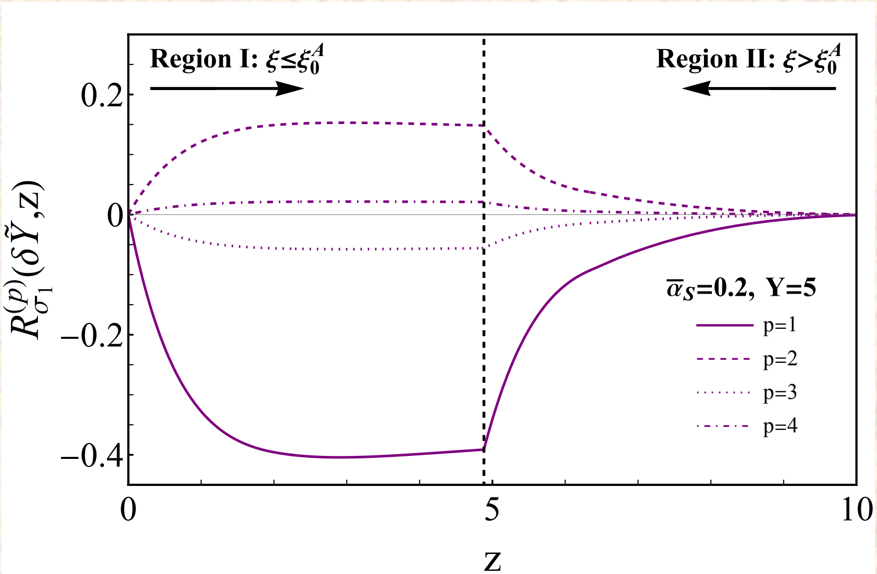
$$\sigma_n^{(0,I)}(z) = \exp\left(-\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\,T(z')\right)\left\{\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\,U_n(z')\exp\left(\frac{1}{\kappa}\int_{\xi_0^A}^{z'} dz''\,T(z'')\right) + C_\phi^{(n)}\right\}$$

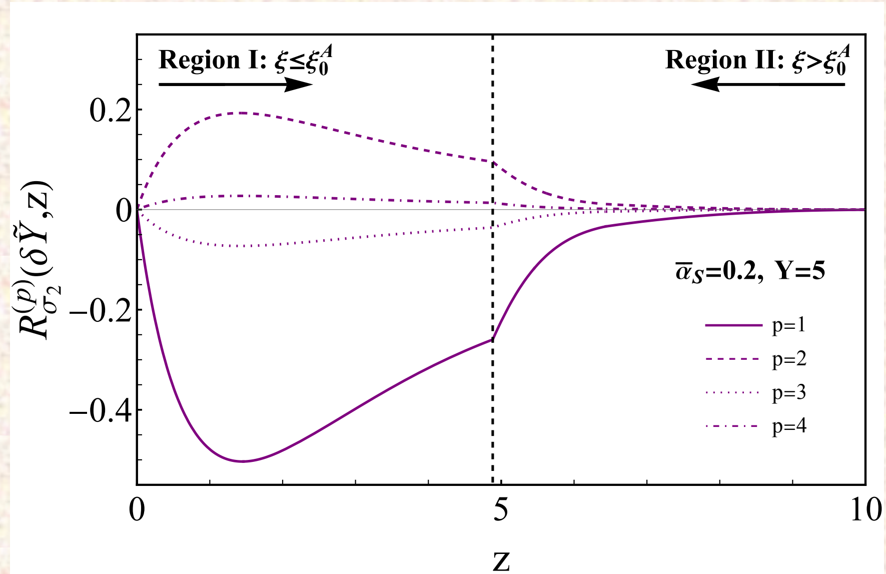
where $C_\phi^{(n)} = \frac{\left(\frac{1}{2}e^{\xi_0^A}\right)^n}{n!}e^{-\frac{1}{2}e^{\xi_0^A}}$**. The formula for the p-iteration is**

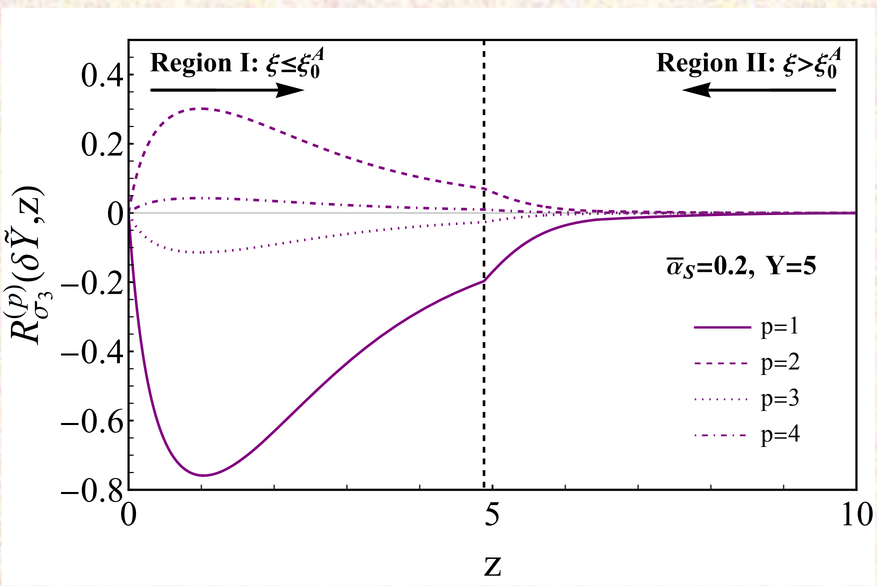
$$\sigma_n^{(p,I)}(z) = \exp\left(-\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\,T(z')\right)\left\{\frac{1}{\kappa}\int_{\xi_0^A}^{z} dz'\,U_n(z')\,\tilde{\Sigma}_n^{(p-1)}(z')\exp\left(\frac{1}{\kappa}\int_{\xi_0^A}^{z'} dz''\,T(z'')\right) + C_\phi^{(n)}\right\}$$

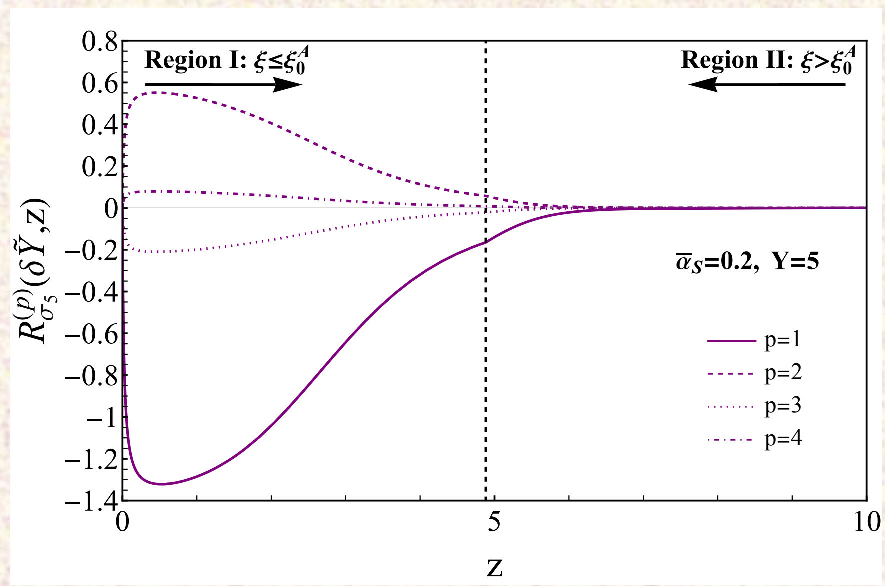
Numerical estimates

- **Ratios of σ_1, σ_2, σ_3 and σ_5 for \xi_A=0, \alpha_S=0.2, z_0=2.**



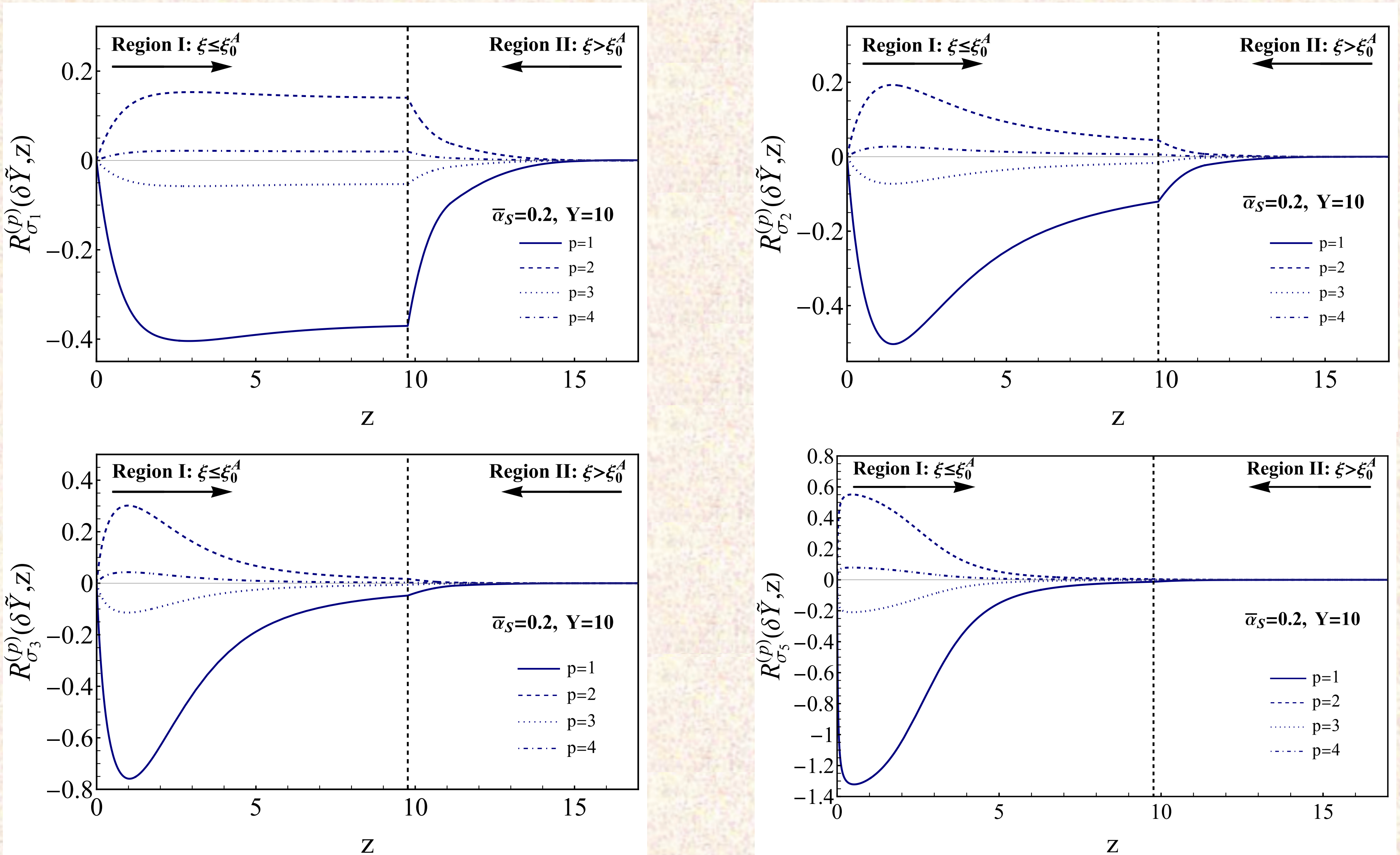






Numerical estimates

- **Ratios of σ_1, σ_2, σ_3 and σ_5 for \xi_A=0, \alpha_S=0.2 , z_0=2.**



Numerical estimates

- One can see that the first iteration turns out to be rather large in the small-z region which corresponds to region I.
- Despite the fact that after an appropriate number of iterations the corrections become small, the approach may break down for a sufficiently large n. For this reason, we use this method only for small values of n.
- We solve the equation in the large-n limit, and study the possible matching with this solution.

The large n≳<n(z)> Solution

- **We start with our equation**

$$\kappa \frac{d\,\sigma_n(z)}{d\,z} \;=\; -\,T(z)\,\sigma_n(z) \;+\; \Delta(z)\,\Sigma_n(z) \;+\; \sum_{k=1}^{n-1} \Sigma_{n-k}(z)\,\sigma_k(z)$$

- **Defining** $\Sigma_n(z) \;=\; \int_{\xi_0^A}^{z} dz'\,\sigma_n(z')$**, we can rewrite it as**

$$\kappa \frac{d^2\,\Sigma_n(z)}{d\,z} \;=\; -\frac{d}{dz}\left(T(z)\,\Sigma_n(z)\right) \;+\; \Sigma_n(z) \;+\; \sum_{k=1}^{n-1} \Sigma_{n-k}(z)\,\sigma_k(z)$$

The large $n \gtrsim \langle n(z) \rangle$ Solution

- **Rewriting the sum as**

$$\sum_{k=1}^{n-1} \Sigma_{n-k}(z)\,\sigma_k(z) \;=\; \frac{1}{2}\,\frac{d}{d\,z}\,\sum_{k=1}^{n-1} \Sigma_{n-k}(z)\Sigma_k(z)$$

- **and introducing** $\mathcal{S}_n\,(z) = \int\limits_{\xi_0^A}^{z} dz'\,\Sigma_n\,(z')$ **, we have**

$$\kappa\,\frac{d^2\,\Sigma_n(z)}{d\,z} \;=\; -\,\frac{d}{dz}\,(T(z)\,\Sigma_n(z)) \;+\; \frac{d\,\mathcal{S}_n(z)}{d\,z}$$

$$+\;\frac{1}{2}\,\frac{d}{d\,z}\left[\sum_{k=1}^{n-1} \Sigma_{n-k}(z)\Sigma_k(z)\right]$$

The large n≳<n(z)> Solution

- **We suggest that the solution has the following form**

$$\Sigma_n(z) \;=\; \phi(z)\,\exp\left(-n\,\Phi(z)\right)$$

- **Functions Φ (z) and φ (z) we will find from the equation. Replacing and taking the large z and n limit we find**

$$\kappa\frac{d\Sigma_n}{dz} \;=\; -(z \,-\, \Sigma_\Delta(z))\Sigma_n(z) + \mathcal{S}_n(z) \;+\; \frac{n-1}{2}\phi(z)\,\Sigma_n\left(z\right)$$

- **We can now solve the equation and find for DIS:**

$$\sigma_n\left(z\right) \;=\; \left(\phi'\left(z\right) - n\phi\left(z\right)\Phi'\left(z\right)\right)\exp\left(-n\Phi\left(z\right)\right)$$

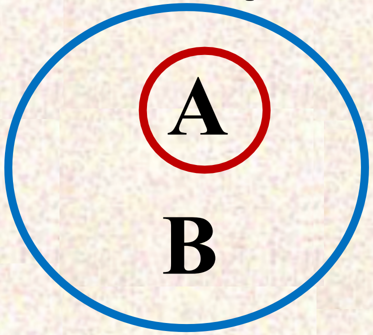
$$\xrightarrow{n\geq\langle n(z)\rangle}\;\frac{2}{\kappa}\frac{z^2}{\langle n\left(z\right)\rangle}\Psi\left(\xi \;=\; \frac{n}{\langle n(z)\rangle}\right)$$

where $\Psi\left(\xi\right) \;=\; \xi\,e^{-\xi}$ **and** $\langle n(z)\rangle \;=\; 2\,n_0\,z\,e^{\frac{(z - z_\Delta)^2}{2\,\kappa}}$

Applications at EIC era

Entropy of produced gluons

- **DIS probes only a part of the proton's wave function (region A).**



$$r^2 \sim 1/Q^2$$

- **The probability to have n-cut BFKL Pomerons in the final state is equal to**

$$P_n^{\rm AGK}(z) \equiv \frac{\sigma_n^{\rm AGK}(z)}{\sum_{n=1}^{\infty}\sigma_n^{\rm AGK}(z)} = \frac{2}{\kappa}\frac{z^2}{\langle n(z)\rangle}\Psi\left(\frac{n}{\langle n(z)\rangle}\right)$$

where $\Psi(\xi) = \xi\, e^{-\xi}$ **and** $\langle n(z)\rangle = 2\, n_0\, z\, e^{\frac{(z-z_\Delta)^2}{2\,\kappa}}$

$$P_n^{\rm gluons}(Y;\mathbf{x}_{01},\mathbf{b}) = \sum_{k=1}^{\infty} P_k^{\rm AGK}(Y;\mathbf{x}_{01},\mathbf{b})\, P_n^{\rm Poisson}(k\,\Delta_{\rm BFKL}Y)$$

Entropy of produced gluons

- **The new observable of DIS — the entanglement entropy — is given by the von Neumann's formula**

$$S_E(z) = -\sum_n P_n(z)\ln(P_n(z))$$
$$= \ln\langle n(z)\rangle$$
$$\xrightarrow{z\gg 1} \frac{z^2}{2\kappa}$$

In agreement with Kharzeev and Levin (2017) predictions.

- **A relation between the S_{hadron} and the initial-state parton entropy, S_{gluon}, due to "parton liberation" and "local parton-hadron duality (LPHD)", is described by:**

$$S_{\text{hadron}} \approx S_{\text{gluons}}$$

Applications at EIC era

- **Having these cross sections, we can calculate the k-th moments of our multiplicity distributions using Mueller's formula**

$$\langle n(n-1)...(n-k+1)\rangle = \langle n^k(z)\rangle$$

$$= \sigma_k^{\mathrm{AGK}}(z)\,\langle n_{\mathrm{BFKL}}(z)\rangle^k$$

where $n_{\mathrm{BFKL}}(z) = \bar{\gamma}\,z$

$$z = \bar{\alpha}_S\,\kappa\,\ln\left(\frac{W^2}{Q^2}\right) + \ln\left(\frac{Q_{s0}^2}{Q^2}\right)$$

- **Do we have**

$$\langle n^k(z)\rangle_{\mathrm{hadrons}} \approx \langle n^k(z)\rangle_{\mathrm{gluons}} \;?$$

Applications at EIC era

- **And therefore, all cumulants can be calculated:**

$$C_1(z) = \langle n(z) \rangle$$

Mean

$$C_2(z) = \langle n^2(z) \rangle - \langle n(z) \rangle^2$$

Variance

$$C_3(z) = \langle n^3(z) \rangle - 3\langle n^2(z) \rangle \langle n(z) \rangle + 2\langle n(z) \rangle^3$$

Third central moment of our distributions

$$C_4(z) = \langle n^4(z) \rangle - 4\langle n^3(z) \rangle \langle n(z) \rangle - 3\langle n^2(z) \rangle^2 + 12\langle n^2(z) \rangle \langle n(z) \rangle^2 - 6\langle n(z) \rangle^4$$

Conclusions and outlook

- We calculated the entropy and the multiplicity distributions directly from QCD evolution equations and the s-channel unitarity constraints. These results are in agreement with the predictions of Kharzeev and Levin.
- σ_1, σ_2 and σ_3 are cross sections for 1, 2 and 3 cut Pomeron production which are closely related to the moments and cumulants of the multiplicity distributions for DIS experiment. Calculations of the moments <n^k> for small k will be subject of next papers.

Thanks/Gracias!

- **Thanks to CFNS (Prof. Abhay Deshpande, Socorro Delquaglio, Melissa Laguerre, and more) for their support, and to my professors (C. Contreras, E. Levin, Y. Kovchegov, and more) for helpful and encouraging discussions.**