Gluons in the pion gravitational form factors

Murilo Pedroso[1,2] Vinícius O. Raimundo[1] Wayne de Paula[1] Tobias Frederico[1]

[1]Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[2]CentraleSupélec, Université Paris-Saclay, France

Light Cone 2026

Outline

Gravitational form factors, and why the pion

- A form factor images a hadron's internal structure. The electric current images the charge; the energy–momentum tensor $T^{\mu\nu}_{\rm QCD}$ images mass, momentum and internal pressure – the "mechanical" structure.
- A spin-0 hadron has three of them: $A(t)$ (momentum), $D(t)$ (the D-term, pressure), $\bar{c}(t)$ (nonzero only for a single sector). They can be accessed via GPDs (DVCS at JLab / the EIC), lattice QCD, perturbative QCD, and model calculations [Tong, JHEP 10 (2022) 046; Liu, PRD 110 (2024) 054022; Freese, PRC 100 (2019) 015201], etc.
- Why the pion? The simplest hadron: spin-0 and a (pseudo-)Goldstone boson with emergent mass; chiral symmetry fixes $D(0) \to -1$ [Polyakov, IJMPA 33 (2018) 1830025].

Our question

How much of the pion's momentum and internal stress do gluons carry? We compute the quark and gluon GFFs in Minkowski space and isolate the gluon contributions.

The EMT form factors and the $\bar{c}$ sum rule

For each species $i = q, g$ [Polyakov, IJMPA 33 (2018) 1830025; Hudson, PRD 96 (2017) 114013]:

$$\boxed{\langle \pi(p') | \hat{T}_i^{\mu\nu} | \pi(p) \rangle = 2P^\mu P^\nu A_i(t) + \tfrac{1}{2}(\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2) D_i(t) + 2m_\pi^2\, \bar{c}_i(t)\, \eta^{\mu\nu}}$$

A: momentum, $A_i(0) = \langle x \rangle_i$; D: D-term (pressure), $\sum_i D_i(0) \to -1$; $\bar{c}$: per-sector only.

Energy–momentum conservation

Conservation $\partial_\mu T^{\mu\nu}_{\text{QCD}} = 0 \;\Rightarrow\; \boxed{\bar{c}_{\text{tot}}(t) \equiv 0}$, with $\bar{c}_q(t) = -\bar{c}_g(t)$ per sector.

From the QCD Lagrangian to the graviton vertices

QCD minimally coupled to weak gravity $(g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},\ \kappa^2 = 32\pi G_N)$ [Plefka, PRD 98 (2018) 026011]: $\mathcal{L} = \sqrt{-g}\,\mathcal{L}_{\rm QCD}(g,\nabla) \to \mathcal{L}_{\rm QCD}(\eta) - \frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}_{\rm QCD}$. The symmetric (Belinfante) EMT generates the vertices [Coriano, arXiv:2606.10993]:

$$T_q^{\mu\nu} = \tfrac{\mathrm{i}}{4}\bar\psi(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu)\psi - \eta^{\mu\nu}(\tfrac{\mathrm{i}}{2}\bar\psi\gamma^\rho\overleftrightarrow{\partial}_\rho\psi - m\bar\psi\psi), \quad T_g^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \tfrac{1}{4}\eta^{\mu\nu}F^2,$$

$$T_{qg}^{\mu\nu} = g_s\,\bar\psi\, T^a\big[\tfrac{1}{2}(\gamma^\mu A^{a\nu} + \gamma^\nu A^{a\mu}) - \eta^{\mu\nu}\not{A}^a\big]\psi \quad \text{(the } g_s A \text{ part of } \overleftrightarrow{D} \text{ in } T_q)$$

$$\mathcal{G}_q^{\mu\nu} = \tfrac{\kappa}{2}\big\{\tfrac{1}{4}[\gamma^\mu(p+p')^\nu + \gamma^\nu(p+p')^\mu] - \eta^{\mu\nu}[\tfrac{1}{2}(\not{p}+\not{p}') - \boxed{M}]\big\} \qquad \text{(quark)}$$

$$\mathcal{G}_2^{\mu\nu,\rho} = g_s\tfrac{\kappa}{2}[\tfrac{1}{2}(\gamma^\mu\eta^{\nu\rho} + \gamma^\nu\eta^{\mu\rho}) - \eta^{\mu\nu}\gamma^\rho] \qquad \text{(seagull)}$$

$$\mathcal{G}_1^{\mu\nu,\alpha\beta} = \tfrac{\kappa}{2}[(q_i\cdot q_f)\,C^{\mu\nu,\alpha\beta} + D^{\mu\nu,\alpha\beta}] \qquad \text{(2-gluon)}$$

$\mathcal{G}_q$: $m \to M$ (constituent); $\quad C^{\mu\nu,\alpha\beta} = \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}$;

$D^{\mu\nu,\alpha\beta} = \eta^{\mu\nu} q_i^\beta q_f^\alpha - [\eta^{\mu\beta} q_i^\nu q_f^\alpha + \eta^{\mu\alpha} q_i^\beta q_f^\nu - \eta^{\alpha\beta} q_i^\mu q_f^\nu + (\mu \leftrightarrow \nu)]$.

Pion vertex, propagators, parameters

Effective pion Bethe–Salpeter vertex (dominant γ_5 component); dressed quark and (Feynman-gauge, massive) gluon propagators [Oliveira, EPJC 79 (2019) 116]:

$$\Gamma_\pi(k) = g_\pi \gamma_5 \, F_\pi(k), \quad F_\pi(k) = \frac{m_\pi^2 - \lambda^2}{k^2 - \lambda^2 + \mathrm{i}\epsilon}, \quad S(p) = \frac{\mathrm{i}(\not{p} + M)}{p^2 - M^2 + \mathrm{i}\epsilon}, \quad D^{\mu\nu}(q) = \frac{-\mathrm{i}\,\eta^{\mu\nu}}{q^2 - \mu_g^2 + \mathrm{i}\epsilon}.$$

- We adopt the Minkowski-space pion vertex inspired by the lattice-QCD running quark mass [Mello, PLB 766 (2017) 86], keeping only its γ_5 component.
- g_π is fixed by the momentum normalization $A(0) = 1$; the axial–pseudoscalar relation then yields f_π, compared to the physical decay constant.

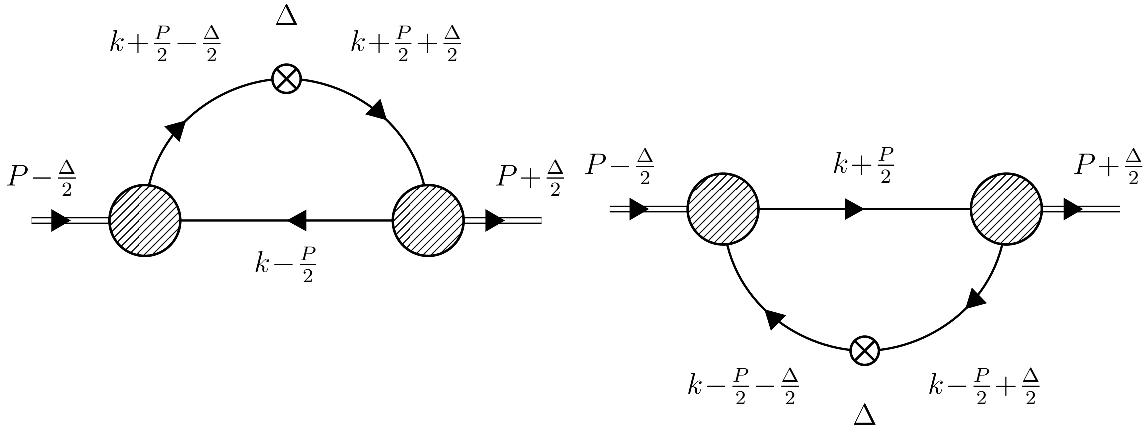
Parameters

pion mass	$m_\pi = 0.14$ GeV
constituent quark mass	$M = 0.23$ GeV
effective gluon mass	$\mu_g = 0.5$ GeV
running-mass pole	$\lambda = 0.86$ GeV

Effective coupling

$\alpha_s = g_s^2/4\pi$ is fixed by the $\bar{c}$ sum rule, not an input.

Quark diagram A_0 (T_q): graviton on quark / antiquark



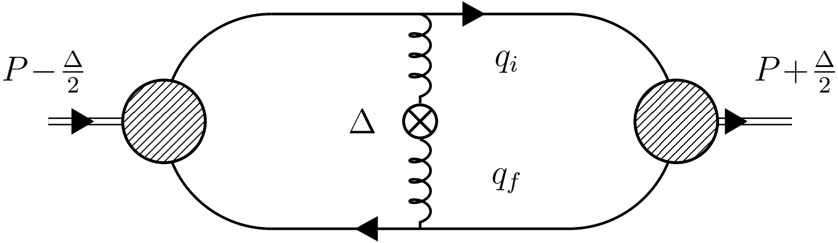
(a) graviton on the quark

(b) graviton on the antiquark (C-conjugate)

$$A_0^{\mu\nu} = -2N_c\, g_\pi^2 \int \frac{d^4k}{(2\pi)^4}\, F_\pi\left(k-\frac{\Delta}{4}\right) F_\pi\left(k+\frac{\Delta}{4}\right) \mathrm{Tr}\Big[S\left(k-\frac{P}{2}\right)\gamma_5\, S\left(k+\frac{P}{2}-\frac{\Delta}{2}\right)\mathcal{G}_q^{\mu\nu}\, S\left(k+\frac{P}{2}+\frac{\Delta}{2}\right)\gamma_5\Big]$$

Gluon diagram A_1 (T_g) and its integral

$$K_i = k + \frac{P}{2} - \frac{\Delta}{4} \qquad K_f = k' + \frac{P}{2} + \frac{\Delta}{4}$$

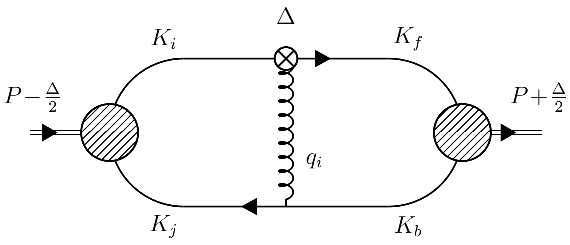


$$K_j = k - \frac{P}{2} + \frac{\Delta}{4} \qquad K_b = k' - \frac{P}{2} - \frac{\Delta}{4}$$

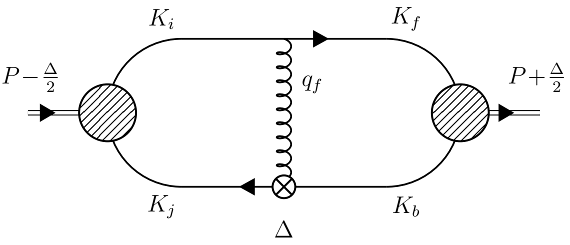
$$A_1^{\mu\nu} = 4 g_s^2 g_\pi^2 \int \frac{\mathrm{d}^4 k \, \mathrm{d}^4 k'}{(2\pi)^8} F_\pi(k) F_\pi(k') \, D(q_i) D(q_f) \, \mathcal{G}_1^{\mu\nu}(q_i, q_f) \, \mathrm{Tr}\big[\gamma_5 S(K_f) \gamma_\alpha S(K_i) \gamma_5 S(K_j) \gamma_\beta S(K_b)\big]$$

$$q_i = k' - k + \frac{\Delta}{2}, \; q_f = k - k' + \frac{\Delta}{2}.$$

Seagull diagram A_2 (T_{qg}): the two insertions



(a) A_{2a}: seagull on the upper vertex



(b) A_{2b}: seagull on the lower vertex (mirror)

$$A_{2a}^{\mu\nu} = 4g_s^2 g_\pi^2 \int \frac{\mathrm{d}^4k\,\mathrm{d}^4k'}{(2\pi)^8} F_\pi(k) F_\pi(k')\, D(q_i)\, \mathrm{Tr}\big[\gamma_5 S(K_f)\, \mathcal{G}_2^{\mu\nu}\, S(K_i)\gamma_5 S(K_j)\gamma_\sigma S(K_b)\big]$$

$A_2 = A_{2a} + A_{2b}$.

The $\bar{c}$ sum rule: the effective coupling

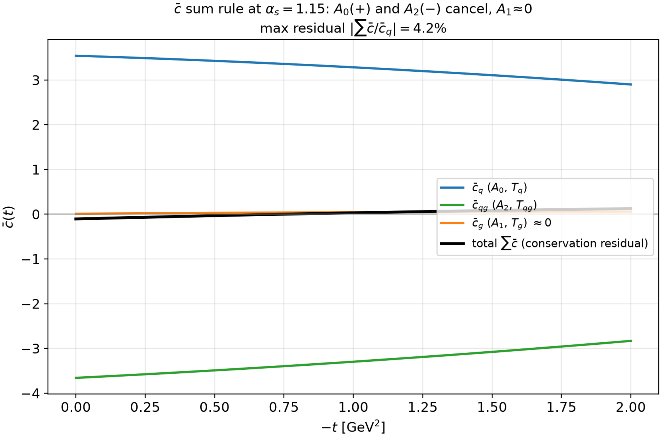
Each sector's $\bar{c}$ carries its own couplings (quark triangle + seagull $\propto g_s^2$, gluon $\propto g_s^2$); conservation turns the sum rule into one equation for the coupling:

$$\boxed{\bar{c}_q(g_s) + \bar{c}_g(g_s) = 0} \implies \boxed{\alpha_s = g_s^2/4\pi = 1.15} \quad \text{(Feynman gauge, } g_s^2 = 14.4\text{).}$$

- The single coupling g_s is chosen to minimize the $\bar{c}$ average over the t-grid (the conservation residual driven down to $\sim 4\%$).

The $\bar{c}$ cancellation

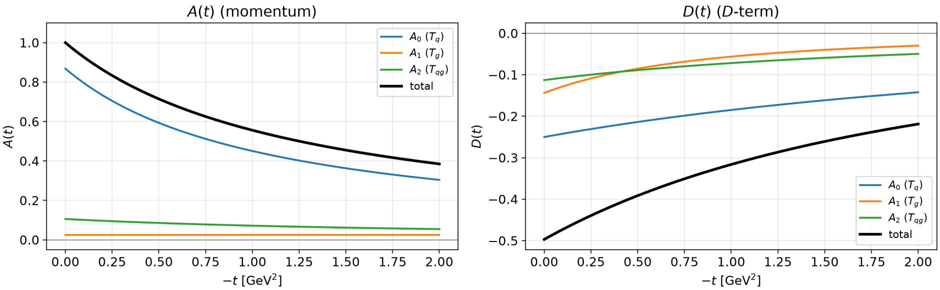
PRELIMINARY



At the tuned α_s, the quark $(A_0, +)$ and interaction $(A_2, -)$ pieces cancel to $\sim 96\%$, the gluon (A_1) is ≈ 0.

PRELIMINARY

Pion GFFs by diagram, $A_{\rm tot}(0) = 1$, $\alpha_s = 1.15$ (Feynman gauge)
A_0=T_q (quark) • A_1=T_g (gluon) • A_2=T_{qg} (interaction)



Diagrams A_0 (T_q), A_1 (T_g), A_2 (T_{qg}) and the total, computed with the tuned effective α_s and $A_{\rm tot}(0) = 1$.

The pion decay constant f_π

With g_π fixed by the normalization $A(0) = 1$, the axial–pseudoscalar relation predicts f_π ($\Psi_\pi = S(k_+)\Gamma_\pi S(k_-)$, $k_\pm = k \pm \frac{P}{2}$) [Mello, PLB 766 (2017) 86]:

$$\boxed{\mathrm{i}\, P^\mu f_\pi = N_c \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \,\mathrm{Tr}\big[\gamma^\mu \gamma_5\, S(k_+)\, \Gamma_\pi(k,P)\, S(k_-)\big]}.$$

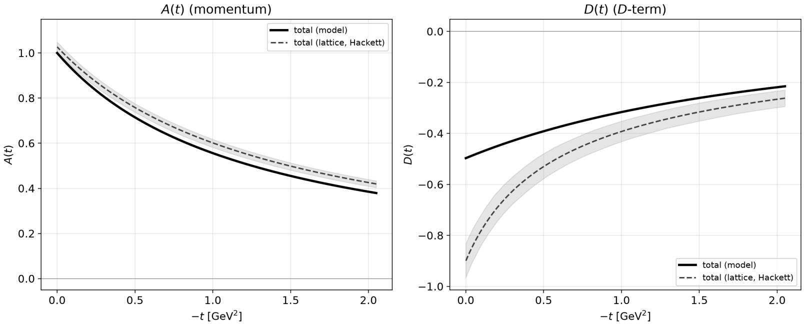
The constituent mass M enters through the propagators $S(k_\pm)$, tying f_π to M (experiment convention $f_\pi(\pi^0) = 130.5$ MeV):

normalization of g_π	f_π [MeV]	experiment
momentum $A(0) = 1$ (model)	**159**	130.5
charge $F_\pi(0) = 1$	150	

- At our $M = 0.23$ GeV, $f_\pi = 150$ (charge) / 159 (A(0)$= 1$) MeV – ~ 15–22% above experiment (constant-mass, single-γ_5 truncation).
- A lighter M reaches 130.5, a heavier M matches the lattice A-slope.

Quark and gluon: the model form factors

PRELIMINARY



	$\langle x \rangle_q$	$\langle x \rangle_g$
model scale	0.97	$\approx \mathbf{0.03}$

Model total GFFs (black) compared to the lattice total (dashed, Hackett [Hackett, PRD 108 (2023) 114504]); the total A, D are RG-invariant. The intrinsic gluon contribution is small at the model scale. $D_{\rm tot}(0) \simeq -0.52$, below the soft-pion limit -1 and the symmetry-preserving DSE benchmark -0.97 [Xu, EPJC 84 (2024) 191]; the deficit might be a *dressing* residual [Freese, PRC 100 (2019) 015201].

The pion mass radius

Light-cone mass radius [Freese, PRC 100 (2019) 015201]:

$$\langle r^2 \rangle^{\rm LC}_{\rm mass} = \frac{4}{A(0)} \frac{{\rm d}A}{{\rm d}t}\Big|_{t=0}.$$

radius	value [fm]
$r^{\rm LC}_{\rm mass}$ (total A, monopole)	0.37

- $r^{\rm LC}_{\rm mass} \approx 0.37$ fm sits slightly *above* the empirical light-cone pion mass radius 0.26–0.32 fm [Freese, PRC 100 (2019) 015201].
- Lattice [Hackett, PRD 108 (2023) 114504] reports $r_{\rm mass} \approx r_{\rm charge}/1.6$, consistent with our ordering $r^{\rm LC}_{\rm mass} < r^{\rm LC}_{\rm charge} \approx 0.54$ fm (light-cone), giving $r_{\rm mass}/r_{\rm charge} \approx 1/1.4$.

Summary and outlook

What we did

Computed the pion gravitational form factors $A, D, \bar{c}$ – quark and gluon – in Minkowski space, with a lattice-constrained pion vertex, in Feynman gauge up to two loops.

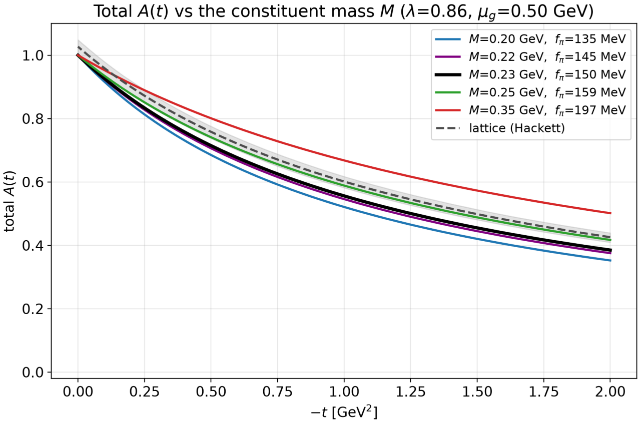
- The gluon contribution at the hadronic scale is small: it carries only $\langle x \rangle_g \approx 0.03$ of the pion momentum, the constituent quarks carrying the rest. This small gluon fraction is consistent with the continuum-QCD picture of a valence-dominated pion at low scales [Cui, EPJC 80 (2020) 1064].
- Outlook: use a running-mass quark propagator; dress the gravitational-coupling vertices; replace the simple γ_5 pion–quark vertex with the full pion Bethe–Salpeter amplitude in Minkowski space; and repeat in other gauges.

Thank you!

Q & A

Appendix: total $A(t)$ vs the constituent mass M

PRELIMINARY



At fixed $\lambda = 0.86$ GeV (Mello) and $\mu_g = 0.50$ GeV the total $A(t)$ hardens with M. The committed $M = 0.23$ (bold black) sits just below the lattice band; $M \gtrsim 0.25$ lies on it, $M = 0.35$ above; the legend gives the corresponding f_π.

Appendix: numerical method (Minkowski Monte Carlo)

For spacelike $t < 0$ no integrand pole is crossed when the loop energy is Wick-rotated ($k^0 \to \mathrm{i}k^4$) with the numerator kept Minkowskian; the $d = 4$ integrals are evaluated by adaptive Monte Carlo.

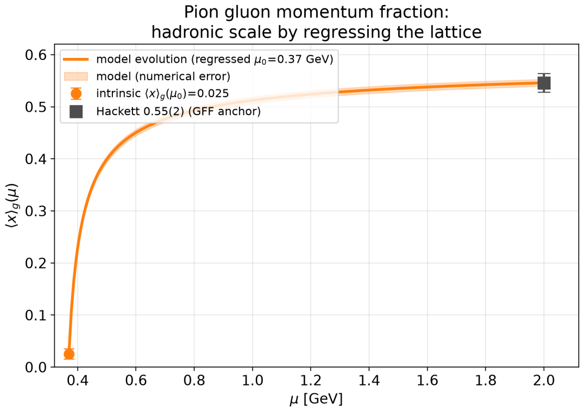
- **Tools**: Dirac traces with FORM [Kuipers, CPC 184 (2013) 1453] and Wolfram FeynCalc [Shtabovenko, CPC 256 (2020) 107478]; adaptive integration with Vegas [Lepage, JCP 439 (2021) 110386].
- Cross-checked against pySecDec [Borowka, CPC 222 (2018) 313] (sector decomposition): agreement at the 0.1–0.3% level.
- **Hardware**: NVIDIA RTX 5060 GPU (8 GB), Intel Core i7-14650HX, 16 GB RAM.

Appendix: DGLAP evolution – the hadronic scale from the lattice

μ_0 was estimated from the Hackett GFF lattice [Hackett, PRD 108 (2023) 114504]: regressing $\langle x \rangle_g(2\,\mathrm{GeV}) = 0.55$ back through LO DGLAP to the model's intrinsic $\langle x \rangle_g(\mu_0) \approx 0.03$ gives a very low scale near Λ_{QCD},

$$\mu_0 \simeq 0.37\ \mathrm{GeV}.$$

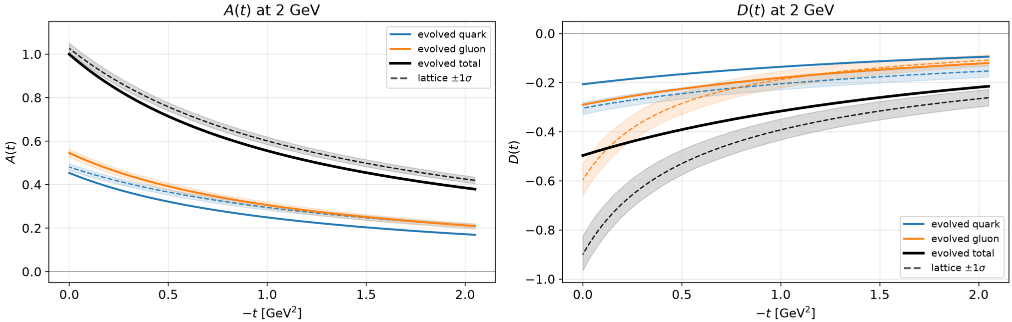
PRELIMINARY



Appendix: evolved to 2 GeV vs lattice QCD

PRELIMINARY

Model evolved to 2 GeV (LO DGLAP, regressed $\mu_0 \simeq 0.37$ GeV) vs lattice (Hackett, 1σ bands). $\langle x \rangle_g \to 0.55$.



Model GFFs forward-evolved from the regressed $\mu_0 \simeq 0.37$ GeV to 2 GeV vs lattice [Hackett, PRD 108 (2023) 114504] with $\pm 1\sigma$ bands; the gluon momentum fraction grows to the Hackett $\langle x \rangle_g \approx 0.55$.