

NLO Heavy Quark Contributions to Polarized DIS Structure Functions in the EIC Era

E. Spezzano^a, T. Ježo^a, M. Klasen^a, I. Schienbein^b

^aInstitut für Theoretische Physik, Universität Münster, Germany

^bLaboratoire de Physique Subatomique et de Cosmologie, Université Grenoble-Alpes, France



Introduction & Motivation

Nucleon Spin Puzzle

How is the nucleon's spin distributed among its constituents?

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma}_{\text{Quark Spin}} + \underbrace{\Delta G}_{\text{Gluon Spin}} + \underbrace{L_q + L_g}_{\text{Orbital AM}}$$

- **Polarized DIS:** Crucial probe to extract polarized PDFs (pPDFs) like $\Delta g(x, Q^2)$.
- **Nucleon Tomography:** 1D pPDFs provide indispensable constraints for multi-dimensional tomography (TMDs, GPDs).
- **Future Facilities:** High-precision measurements upcoming at the **EIC**.

The Heavy Quark Challenge

High-precision fits require moving to **Next-to-Leading Order (NLO)** and incorporating heavy quark masses (c, b):

- **Massive Treatment:** Indispensable near production thresholds ($Q^2 \sim m_Q^2$), where massless (zero-mass) schemes fail.
- **Consistency:** Demands a General-Mass scheme like ACOT to interpolate to high energies.

Heavy Quark Mass Treatment in Perturbative QCD

We need a unified description of two extreme regimes:

1 Fixed Flavor Scheme (FFNS)

- **Massive Quarks:** Generated solely in final states.
- **Threshold region:** Highly accurate ($Q^2 \sim m_Q^2$).
- **High energies:** Fails ($Q^2 \gg m_Q^2$) due to large $\ln(Q^2/m_Q^2)$.

2 Zero-Mass Scheme (ZM-VFNS)

- **Massless Quarks:** Treated as active partons inside PDF.
- **Collinear logs:** Resummed via DGLAP evolution.
- **Threshold region:** Fails due to neglected mass effects.

The GM-VFNS Solution (ACOT Scheme)

General-Mass VFNS (GM-VFNS): Smoothly interpolates between FFNS and ZM-VFNS.

- Retains full mass dependence at low Q^2 and resums collinear logs at high Q^2 .

Aivazis et al., PRD 50 (1994) [hep-ph/9312319]; Collins, PRD 58 (1998) [hep-ph/9806259]

Polarized DIS Formalism

Antisymmetric spin-dependent part of the hadronic tensor:

Spin-Dependent Hadronic Tensor

$$\Delta W_{\mu\nu}^j = \underbrace{i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha}{P \cdot q} \left[S^\beta g_1^j + \left(S^\beta - \frac{S \cdot q}{P \cdot q} P^\beta \right) g_2^j \right]}_{\text{Parity-Conserving } g_1, g_2} + \underbrace{\frac{1}{2P \cdot q} \left[(P_\mu^\perp S_\nu^\perp + S_\mu^\perp P_\nu^\perp) - 2 \frac{S \cdot q}{P \cdot q} P_\mu^\perp P_\nu^\perp \right]}_{\text{Transverse Spin } g_3} g_3^j$$

$$+ \underbrace{\frac{S \cdot q}{P \cdot q} \left[\frac{P_\mu^\perp P_\nu^\perp}{P \cdot q} g_4^j + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \right) g_5^j \right]}_{\text{Parity-Violating } g_4, g_5} + \underbrace{\frac{S \cdot q}{P \cdot q} \left[\frac{q_\mu q_\nu}{Q^2} g_6^j + \frac{q_\mu P_\nu + q_\nu P_\mu}{2P \cdot q} g_7^j \right]}_{\text{Lepton-Mass Suppressed } g_6, g_7}$$

The Polarized Structure Functions

- g_1 : Leading-twist spin structure function.
- g_4, g_5, g_6, g_7 : Parity-violating (weak).

Symmetries & Properties

- **Nucleon Spin:** S^μ (spin four-vector).
- **Twist Suppression:** g_2, g_3 are suppressed by higher twist in the Bjorken limit.
- **Lepton-Mass Suppression:** g_6, g_7 vanish in the incoming massless lepton limit.

Polarized Cross Sections & Observables

In the limit $Q^2 \gg M^2$, the longitudinally polarized cross section simplifies to:

$$\frac{d^2 \Delta \sigma^j}{dx dy} = \frac{2\pi\alpha^2}{xyQ^2} \xi^j \left\{ -[1 + (1-y)^2]g_4^j + y^2 g_L^j + 2x[1 - (1-y)^2]g_1^j \right\}$$

where $g_L^j = g_4^j - 2xg_5^j$ is the longitudinal structure function, and ξ^j is the coupling/propagator factor.

Charged vs. Neutral Current

- **Charged Current (CC):** Probes W^\pm exchange, directly isolating flavor combinations.
- **Neutral Current (NC):** Mixes electromagnetic (γ), weak (Z), and interference (γZ):
 $g_1^{\text{NC}} = g_1^\gamma - g_V^e \eta_{\gamma Z} g_1^{\gamma Z} + (g_V^{e^2} + g_A^{e^2}) \eta_Z g_1^Z$

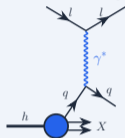
Indices & Channels

- **Index j :** $j \in \{\gamma, \gamma Z, Z\}$ for NC, $\{W^\pm\}$ for CC.
- **Propagator ξ^j :** Coupling factors ($\xi^\gamma = 1$; $\xi^j \propto [Q^2 / (Q^2 + M_{W,Z}^2)]^d$ for weak).
- **Parity:** g_1^j is parity-conserving; g_4^j, g_5^j are parity-violating (never measured!).

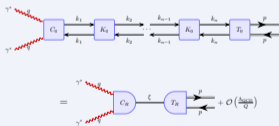
E. C. Aschenauer, T. Burton, M. Stratmann, T. Martini, and H. Spiesberger, PRD 88 (2013) 114025

DIS & Collinear Factorization

1. Deep Inelastic Scattering (DIS)



2. Factorization & Ladder Decomposition



- **Hadronic Tensor ($\Delta W_{\mu\nu}$):** Relates structure functions to forward virtual Compton scattering.
- **Collinear Factorization:** Separates short-distance (perturbative) and long-distance (non-perturbative) physics:

$$\Delta W_{\mu\nu}^j = \sum_i \Delta C_{i,\mu\nu}^j \otimes \Delta f_i + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$$

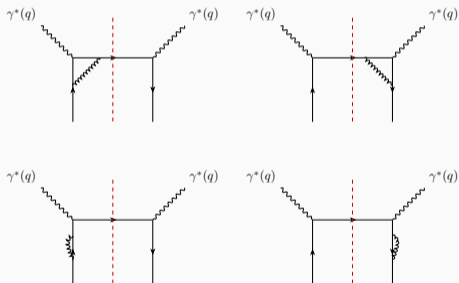
- **Ladder Components:** Organized in terms of 2PI kernels K_0 , bare PDFs T_0 , and hard coefficients C_0 .

Curci, Furmanski, Petronzio, NPB 175 (1980) 27; Collins, PRD 58 (1998) 094002

NLO Feynman Diagrams

Quark-Initiated Contribution: Virtual Corrections

At NLO, the quark-initiated channel receives virtual corrections renormalized on-shell:

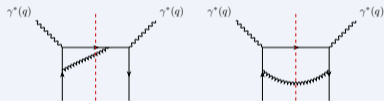


Virtual Corrections

- **Vertex Corrections:** Virtual gluon exchange correcting the electroweak vertex.
- **Self-Energy:** Quark wave-function renormalization.
- **Renormalization:** UV divergences exactly cancelled by counterterms on-shell.

Quark-Initiated Contribution: Real Emission

Real Gluon Emission (ISR and FSR)

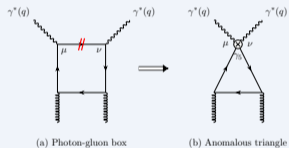


Gluon radiation from initial (ISR) or final struck (FSR) massive quark.

Collinear Regulator & 4D Projection

- **Mass Regulator:** Heavy-quark mass m_Q acts as the physical regulator for collinear singularities.
- **4D Projection:** Avoids complicated D -dimensional axial algebra prescriptions (HVBM/Larin) for γ_5 .
- **Anticommuting γ_5 :** Fully justified strictly in 4-dimensions on open fermion lines.

ABJ Anomaly: Box to Triangle

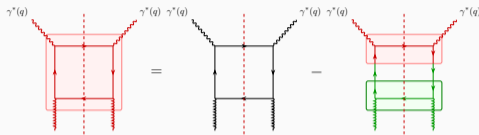


Theoretical Milestones

- **First $\overline{\text{MS}}$ Calcs:** Polarized coefficient functions first computed by Kodaira et al. (1979).
- **Spin Crisis (1987):** Altarelli–Ross & Efremov–Teryaev showed the singlet axial charge is screened by the ABJ anomaly.
- **Modern Re-evaluations:** Massless results recalculated and corrected by Vogelsang et al.
- **NC PV Cancellation:** Singlet channel does not contribute to parity-violating g_4, g_5 functions.

Kodaira et al., PRD 20 (1979) 627; Vogelsang, PRD 54 (1996) 2023; Altarelli, Ross, PLB 212 (1988) 391; Efremov, Teryaev, Sov. J. Nucl. Phys. 50 (1989) 186

The ACOT Scheme & Subtraction Framework



Massive Subtraction & Matching

Matched NLO ACOT coefficient functions ($a \in \{q, g\}$):

$$\Delta C_{i,a}^{\text{ACOT}} = \Delta C_{i,a}^{\text{FFNS}} - \Delta C_{i,a}^{\text{SUB}}$$

with subtraction terms given by:

- **Quark:** $\Delta C_{i,q}^{\text{SUB}} = \Delta C_{i,q}^{\text{LO}} \otimes \Delta P_{qq} \ln \frac{Q^2}{m_Q^2} + \dots$
- **Gluon:** $\Delta C_{i,g}^{\text{SUB}} = \Delta C_{i,q}^{\text{LO}} \otimes \Delta P_{qg} \ln \frac{Q^2}{m_Q^2} + \dots$

Subtraction terms: B. Mele and P. Nason, NPB 361 (1991) 626 (unpol.); E. Spezzano et al. (2026) (pol.)

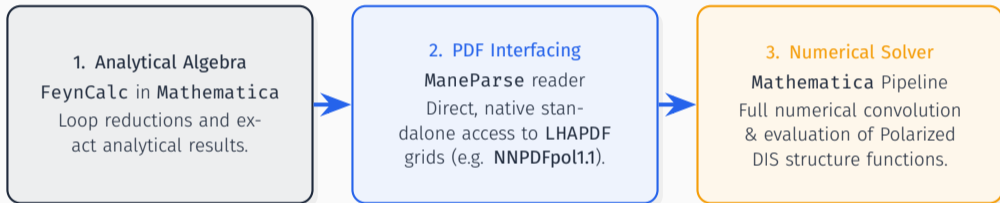
Properties

- **Universal Splitting:** Subtraction terms utilize standard $\overline{\text{MS}}$ splitting functions ($\Delta P_{qq}, \Delta P_{qg}$).
- **Massless Recovery:** For $Q^2 \gg m_Q^2$, subtraction exactly cancels collinear logs, recovering the massless limit.

Methodology & Numerical Results

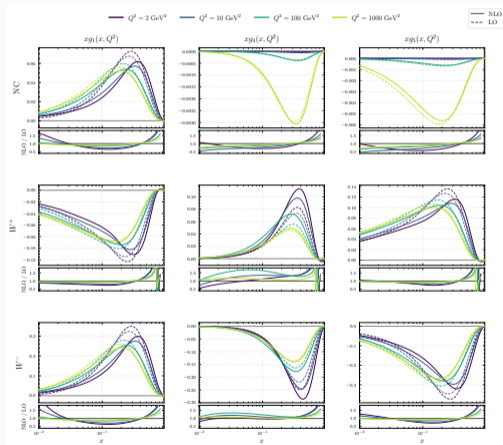
Computational Pipeline & Numerical Stack

The computational pipeline consists of three main stages:



Clark, Godat, Olness, CPC 216 (2017) 126 [arXiv:1605.08012]

Numerical Results: Polarized Structure Functions

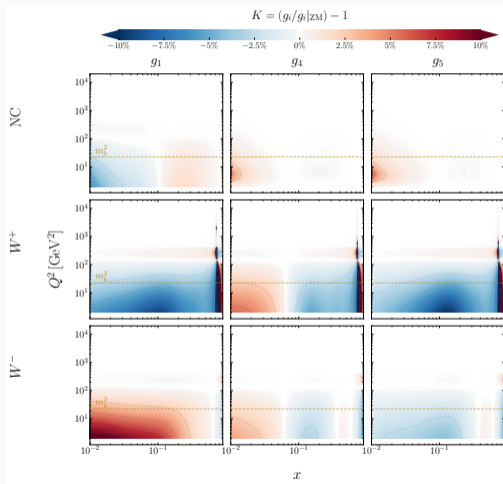


Key Results

- **LO vs NLO:** Substantial NLO corrections for structure functions g_1 , g_4 , and g_5 .
- **Small-x Enhancement:** Gluon-initiated channels drive significant enhancement at small x .
- **Mass Effects:** Highly pronounced in the threshold region ($Q^2 \sim m_Q^2$), respecting physical boundaries.

[Spezzano et al., arXiv:2603.10593 [hep-ph]]

Numerical Results: Mass Effects Heatmaps



Mass Impact Heatmap

Ratio $K = g_i^{\text{ACOT}} / g_i^{\text{ZM-VFNS}}$ measures heavy-quark mass corrections:

- **Low Q^2 :** Significant mass corrections near thresholds.
- **High Q^2 :** Convergence ($K \rightarrow 1$), confirming correct ZM recovery.
- **Large x :** Spikes in K due to zero-crossings of the LO structure functions.

[Spezzano et al., arXiv:2603.10593 [hep-ph]]

Testing Polarized QCD (few of them)

In polarized DIS, there are relations among structure functions analogous to the unpolarized Callan-Gross relation:

The Dicus Relations

Derived by D. Dicus (1972) for polarized lepton-nucleon scattering.

At LO and for the GF channel, Slavnov-Taylor identities imply:

$$g_4 = 2x(g_5 + g_6) \quad \text{and} \quad g_7 = -2g_6$$

In the massless lepton limit ($m_\ell \rightarrow 0$), g_6 and g_7 decouple:

$$g_4 \approx 2xg_5 \quad (\text{i.e. } g_L = g_4 - 2xg_5 \approx 0)$$

NLO Violation & QCD Test

In the Quark Scattering (QS) channel at $\mathcal{O}(\alpha_s)$, the relation is violated:

$$g_4 \neq 2x(g_5 + g_6)$$

even in the massless limit ($m_Q \rightarrow 0$).

- **Polarized F_L Analog:** This breakdown is a clean, physical signature of NLO QCD radiative corrections.
- **Fundamental Constraints:** Together with spin sum rules (Bjorken, Ellis-Jaffe), Dicus relations and their NLO violations provide crucial constraints to test polarized QCD.

D. A. Dicus, Phys. Rev. D 5 (1972) 1367 [doi:10.1103/PhysRevD.5.1367]

Summary & Conclusions

Summary

- **First Massive NLO ACOT Calculation** of polarized structure functions g_1, g_4, g_5, g_6, g_7 .
- **Analytical NLO Coefficients** derived for both quark and gluon initiated channels.
- **Standalone Code** developed in **Mathematica** for numerical evaluation.

Future Outlook

- **Global QCD Fits:** Integrating massive NLO ACOT corrections to constrain polarized PDFs (NNPDFpol, JAM).
- **Higher Orders:** Extending massive GM-VFNS/ACOT formalism to NNLO.

Thank you for your attention!

Questions?

