Nucleon structure in Minkowski space

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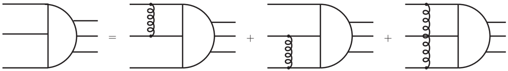
Content

- Minkowski-space Bethe-Salpeter equation for three-particles in ladder approximation: QCD_2 and φ^4_{3+1}
- Projection onto the Light-front: Quasi-Potential expansion
- QCD_2 and φ^4_{3+1}: valence equation @ LO
- Spectrum, PDF, Double PDF, Image, FF
- Summary & Outlook

arXiv: 2605.07095v1 [hep-ph], EPJST235 (2026) 1619, PLB838 (2023) 137732, PRD104 (2021) 114012

BSE for 3q: QCD_2 in LC gauge [arXiv: 2605.07095v1 [hep-ph]]



$$\Psi_M(k_1,k_2,k_3) = -\frac{i\,g^2}{(2\pi)^2}\, S_1(k_1)\otimes S_2(k_2)\otimes S_3(k_3)$$

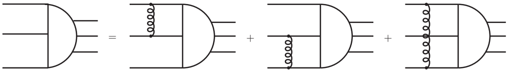
$$\otimes \sum_{ijk} S_i^{-1}(k_i)\otimes \int d^2k_j'\, \frac{\gamma_j^+\otimes\gamma_k^+}{(k_j^+-k_j'^+)^2}\,\Psi_M(k_i,k_j',k_k')\,,$$

where the sum runs over cyclic permutations of $\{i,j,k\}$. The quark propagator is

$$S(k) = i\,\frac{\not{k}_{on}+m}{k^+\left(k^- - k_{on}^- + \frac{i\varepsilon}{k^+}\right)} + i\,\frac{\gamma^+}{2k^+}\,,$$

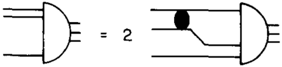
with the second term corresponding to the instantaneous part of the LF propagator. In the light-cone gauge, the quark–gluon vertex reduces to $ig\gamma^+$.

BSE for 3q: φ^4_{3+1} [PLB282 (1992) 409]



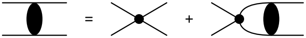
$$i\Phi_M(k_1,k_2,k_3;p) = i^3 \frac{v_M(k_1)+v_M(k_2)+v_M(k_3)}{(k_1^2-m^2+i\epsilon)(k_2^2-m^2+i\epsilon)(k_3^2-m^2+i\epsilon)}$$

- Faddeev eqs. for the vertex function $v_M(q,p)$:

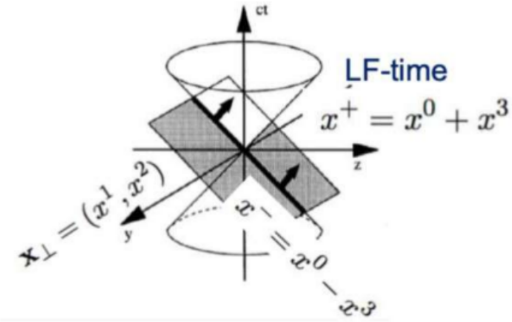


$$v_M(q,p) = 2i\mathcal{F}(M_{12}^2)\int \frac{d^4k}{(2\pi)^4}\frac{i}{[k^2-m^2+i\epsilon]}\frac{i}{[(p-q-k)^2-m^2+i\epsilon]}v_M(k,p)$$

- 2-body amplitude $\mathcal{F}(M_{12}^2)$:



Projection onto the Light-front



- Goal: Eliminate the relative LF-time between the particles
- Tool: QPE applied to the LF projection of the BSE

Review: EPJST235 (2026) 1619

Warming up: Three-Boson Bethe–Salpeter Amplitude

- The three-boson BS amplitude:

$$\Phi^b_M(y_1, y_2, y_3; K) = \langle 0 | T\left(\varphi(y_1)\varphi(y_2)\varphi(y_3)\right) | K \rangle$$

- The corresponding LF valence wave function is obtained through the projection:

$$\psi^b_3(x_1, x_2, x_3) = (K^+)^2 (x_1 x_2 x_3)^{1/2}\, \chi^b_3(x_1, x_2, x_3)$$

with

$$\chi^b_3(x_1, x_2, x_3) = \int dk_1^-\, dk_2^-\, \Phi^b_M(k_1, k_2, k_3; K).$$

LF Projection

Integration over k_1^- & k_2^- eliminates the relative LF time among the constituents

1st ingredient: LF Projection Operator

- For an arbitrary Minkowski-space operator A:

$$|A := \int dk_1^- dk_2^- \langle k_1^-, k_2^- | A \quad \text{and} \quad A| := \int dk_1^- dk_2^- A | k_1^-, k_2^- \rangle .$$

- Valence amplitude: $|\chi_3^b\rangle = |G_0^b|\Gamma_M^b\rangle$ and the vertex BSE: $|\Gamma_M^b\rangle = V^b G_0^b |\Gamma_M^b\rangle$

- Disconnected Green's function:

$$\langle k_1^-, k_2^- | G_0^b | k_1'^-, k_2'^- \rangle = \frac{-i}{(2\pi)^2} \frac{\delta(k_1^- - k_1'^-)\delta(k_2^- - k_2'^-)}{\hat{k}_1^+ \hat{k}_2^+ (K^+ - \hat{k}_1^+ - \hat{k}_2^+)}$$
$$\times \frac{1}{(k_1^- - \hat{k}_{1on}^- + \frac{i\varepsilon}{\hat{k}_1^+})(k_2^- - \hat{k}_{2on}^- + \frac{i\varepsilon}{\hat{k}_2^+})(K^- - k_1^- - k_2^- - \hat{k}_{3on}^- + \frac{i\varepsilon}{\hat{k}_3^+})}$$

with $\hat{k}_{ion}^-$ (on-minus-shell) and $\hat{k}_3^+ = K^+ - \hat{k}_1^+ - \hat{k}_2^+$

2nd ingredient: Three-Quark Green Function

- The disconnected three-quark propagator can be written as

$$G_0 = G_0^b\,(\not{k}_1 + m) \otimes (\not{k}_2 + m)(\not{k}_3 + m)$$

- LF propagating part of the Green's function:

$$\overline{G}_0 = G_0^b\,\Lambda^+(k_1^+) \otimes \Lambda^+(k_2^+) \otimes \Lambda^+(k_3^+) \qquad \text{where} \quad \Lambda^+(k^+) = \frac{\not{k}_{on} + m}{2m}$$

- LF projected propagator: $g_0 = |\overline{G}_0|$

$$g_0(k_1^+, k_2^+, k_3^+) = (2m)^3 g_0^b(k_1^+, k_2^+, k_3^+)\Lambda^+(k_1^+) \otimes \Lambda^+(k_2^+) \otimes \Lambda^+(k_3^+)$$

- The bosonic-like resolvent is

$$g_0^b = \frac{i}{k_1^+ k_2^+ k_3^+}\,\frac{\theta(k_1^+)\theta(k_2^+)\theta(k_3^+)}{K^- - k_{1on}^- - k_{2on}^- - k_{3on}^-}$$

Goal

Reduce the Minkowski BSE to an effective LF equation in the valence sector.

Quasi-Potential Reduction

- Auxiliary propagator

$$\widetilde{G}_0 = \overline{G}_0 | g_0^{-1} | \overline{G}_0$$

- The BSE becomes

$$|\Gamma_M\rangle = W \, \widetilde{G}_0 \, |\Gamma_M\rangle .$$

- The quasi-potential satisfies

$$W = V + V \Delta_0 W , \qquad \Delta_0 = G_0 - \widetilde{G}_0 .$$

- The LF valence amplitude equation:

$$|\chi_3\rangle = g_0 \, w \, |\chi_3\rangle \quad \text{effective interaction} \quad w = g_0^{-1} | \overline{G}_0 W \overline{G}_0 | g_0^{-1}$$

Faddeev Decomposition and QP expansion

- The interaction kernel is

$$V = \sum_{i=1}^{3} V_i = \sum_{i=1}^{3} S_i^{-1} \otimes V_{(2)i}.$$

- The quasi-potential and LF interaction are decomposed as

$$W = \sum_i W_i, \qquad w = \sum_i w_i.$$

- The Faddeev expansion reads

$$W_i = V_i + V_i \Delta_0 (V_i + V_j + V_k) + V_i \Delta_0 (V_i + V_j + V_k) \Delta_0 (V_i + V_j + V_k) + \cdots .$$

- At leading order:

$$w_i^{\mathrm{LO}} = g_0^{-1} | \overline{G}_0 V_i \overline{G}_0 | g_0^{-1}.$$

- Leading-order LF three-particle equation: $|\chi_3\rangle = g_0 \left(\sum_i w_i^{\mathrm{LO}} \right) |\chi_3\rangle$

Derivation of the LO LF effective interaction in QCD$_2$

- For $i = 1$: $|\overline{G}_0 V_1 \overline{G}_0| = \int dk_1^- dk_2^- dk_2'^- (\cdots)$ - Cauchy integration $\cdots$
- Effective interaction in LO: $w_i^{\mathrm{LO}} = \frac{i}{8\pi} \frac{\gamma_i^+ \otimes \gamma_j^+ \otimes \gamma_k^+}{(k_j^+ - k_j'^+)^2}$.
- Leading-order LF baryon equation in QCD$_2$:

$$\chi_3(k_1^+, k_2^+, k_3^+) = i\frac{g^2}{4\pi} g_0(k_1^+, k_2^+, k_3^+)\, \gamma_1^+ \otimes \gamma_2^+ \otimes \gamma_3^+ \times \left\{ \int_0^{K^+ - k_3^+} dk_1'^+ \, \frac{\chi_3(k_1'^+, k_2''^+, k_3^+) - \chi_3(k_1^+, k_2^+, k_3^+)}{(k_1^+ - k_1'^+)^2} + \cdots \right\}$$

- The subtracted terms serve to regularize the integrals analogous to the treatment in the 't Hooft equation.

- Define:

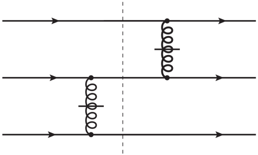
$$\psi(k_1^+, k_2^+, k_3^+) = \gamma_1^+ \otimes \gamma_2^+ \otimes \gamma_3^+ \, \chi_3(k_1^+, k_2^+, k_3^+)$$

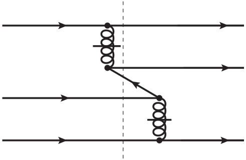
- Eigenvalue equation in LO for the baryon valence wave function ($x_i = k_i^+/K^+$ & $x_1 + x_2 + x_3 = 1$):

$$M^2 \, \psi(x_1, x_2, x_3) = \left(\frac{m^2}{x_1} + \frac{m^2}{x_2} + \frac{m^2}{x_3} \right) \psi(x_1, x_2, x_3)$$
$$- \frac{g^2}{\pi} \left\{ \int_0^{1-x_3} dx_1' \, \frac{\psi(x_1', x_2'', x_3) - \psi(x_1, x_2, x_3)}{(x_1 - x_1')^2} + \cdots \right\}$$

- Equivalent to Bars-Durgut equation!
Bars, PRL36 (1976) 1521, NPB111 (1976) 413; Durgut, NPB 116 (1976) 233

- $w_i^{\mathrm{LO}} = g_0^{-1} | \overline{G}_0 V_i \overline{G}_0 | g_0^{-1}$ $\quad$ $w_i^{\mathrm{NLO}} = w_i^{\mathrm{LO}} + g_0^{-1} | \overline{G}_0 V_i \Delta_0 V_j \overline{G}_0 | g_0^{-1}$





"Nucleon" in φ^4_{3+1} model

- The valence LF wave function and Faddeev vertex components:

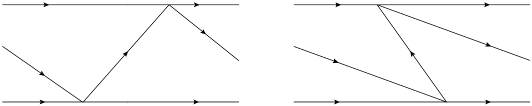
$$\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,k_{1\perp})+\Gamma(x_2,k_{2\perp})+\Gamma(x_3,k_{3\perp},)}{\sqrt{x_1x_2x_3}(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))},$$

where the free 3q mass squared is: $M_0^2 = \sum_{i=1}^3 \frac{\vec{k}_{i\perp}^2+m^2}{x_i}$

$$\Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\int_0^\infty d^2k'_\perp\left(\frac{1}{\widehat{M}_0^2-M_N^2} - \frac{1}{\widehat{M}_0^2-\mu^2}\right)\Gamma(x',k'_\perp)$$

where $\widehat{M}_0^2 = M_0^2(x,\vec{k}_\perp,x',\vec{k}'_\perp,1-x-x',-(\vec{k}_\perp+\vec{k}'_\perp))$

- $w_i^{\mathrm{LO}} = g_0^{-1}|\overline{G}_0 V_i \overline{G}_0|g_0^{-1}$ $\qquad$ $w_i^{\mathrm{NLO}} = w_i^{\mathrm{LO}} + g_0^{-1}|\overline{G}_0 V_i \Delta_0 V_j \overline{G}_0|g_0^{-1}$



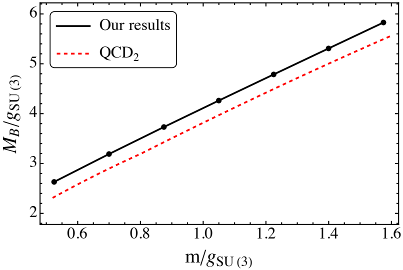
Results for QCD_2

- Endpoint behavior follows 't Hooft analysis: $\psi(x_1, x_2, x_3) \sim (x_1 x_2 x_3)^s$
- Transcendental equation for the exponent: $\frac{m^2}{g^2}\frac{\pi}{2} - 1 + \pi s \cot(\pi s) = 0$
- Relative to the meson case the mass term is modified by a factor of 1/2

- Numerical solution based in a expansion in Jacobi Polynomials:

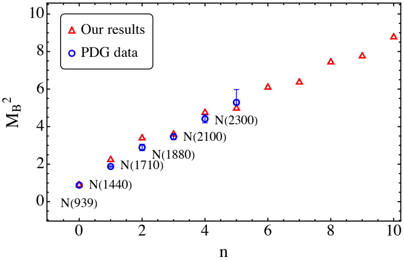
$$\psi(x_1, x_2, x_3) = \sum_k a_k \,(x_1 x_2 x_3)^\beta P_k^{\beta,\beta}(x_1)\, P_k^{\beta,\beta}(x_2)\, P_k^{\beta,\beta}(x_3)\,.$$

- Ground-state baryon mass in SU(3) QCD_2. Dashed curve is an interpolation of the LCQ results from Hornbostel, Brodsky and Pauli (PRD41 (1990) 3814), while the solid curve ours. $g_{\mathrm{SU(N)}} = g\sqrt{2N/(N^2-1)}$

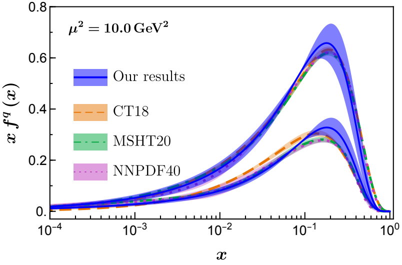


QCD_2: Regge Trajectory and xPDF

- Regge trajectory in QCD2



- Proton xPDF (NNLO DGLAP): fitted $\langle x_u \rangle + \langle x_d \rangle \simeq 0.37 \to \mu_0^2 = 0.23 \pm 0.02\ \mathrm{GeV}^2$



- The parameters: $m = 210$ MeV, $g = 330$ MeV and optimized β are chosen to reproduce the nucleon mass, $M_N = 939$ MeV, and the slope of the Regge trajectory.
- The transcendental equation for the exponent yields $s = 0.413$ and the numerical solution gives $s \simeq 0.420$.

QCD$_2$: Double DA and Image onto the LF

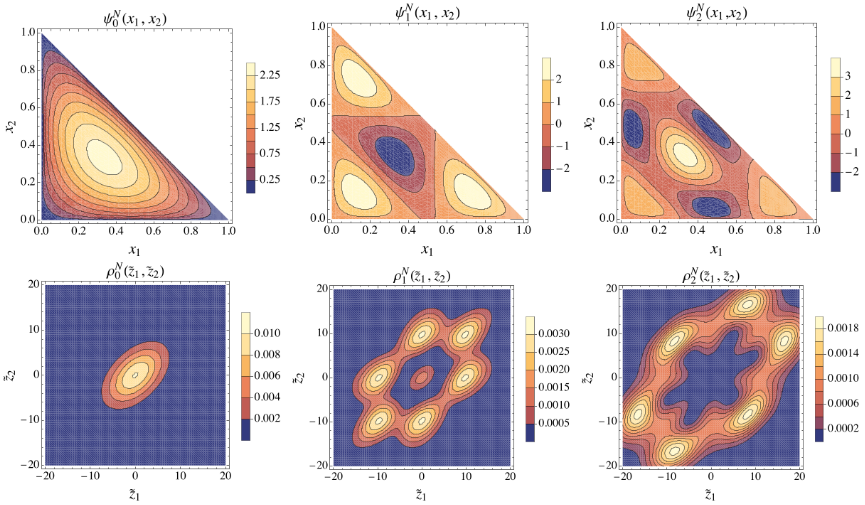


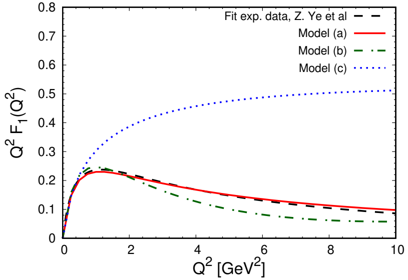
Figure 7: Top: Distribution amplitudes of the nucleon states: N(939) (left), N(1440) (middle) and N(1710) (right). Bottom: Co-ordinate space probability distribution: N(939) (left), N(1440) (middle) and N(1710) (right).

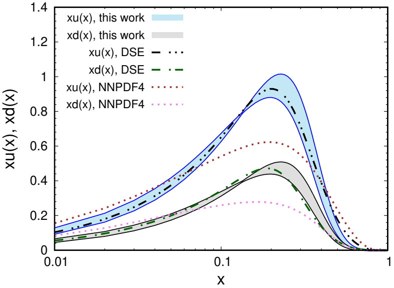
φ^4_{3+1}: Proton Dirac EM FF and xPDF

Model	m [MeV]	$a.m$	μ/m	M_{dq} [MeV]
(a)	366	2.70	1	644
(b)	362	3.60	∞	682
(c)	317	-1.84	∞	-

- Proton Dirac EM Form-Factor

$Q_0 = 0.33 \pm 0.03\,\text{GeV} \rightarrow Q = 3.097\,\text{GeV}$





φ^4_{3+1} TMD, Double DA and Image onto the LF

- Model (b): DDA

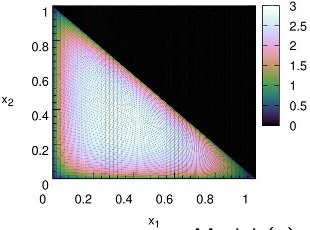
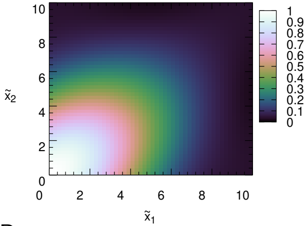
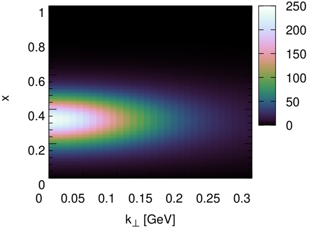


Image onto the LF



- Model (a): TMD



Summary & Outlook

Summary:

- Minkowski BSE in QCD_2 and φ^4_{3+1};
- QPE in LO (Baryon QCD_2 = Bars-Durgut equation);
- QPE in LO φ^4_{3+1};
- NLO $\sim$ attractive 3-body interaction;
- xPDF, DDA, Image and TMD.

Outlook:

- Template to include confinement in 3+1;
- Include NLO interaction;
- Solution in Minkowski space;
- Consider the transverse confinement & QCD_2 [Chabysheva & Hiller, Ann. Phys. 337 (2013) 143] .

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