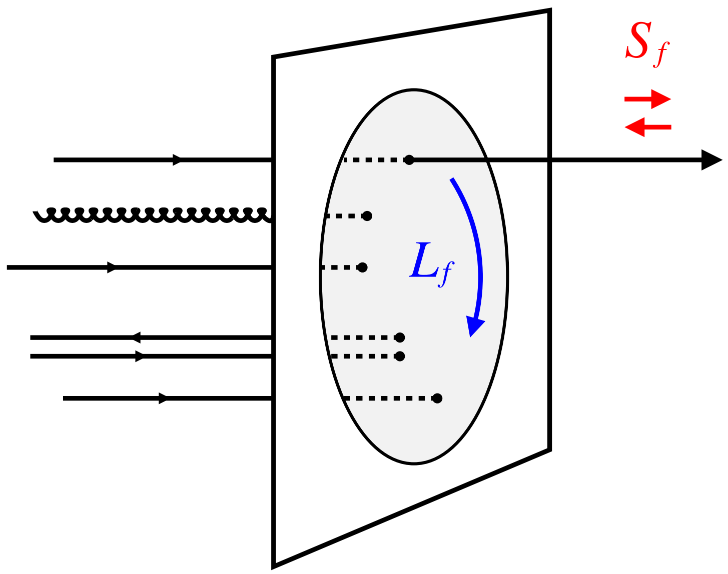
Orbital angular momentum from QCD instantons

C. Weiss (JLab) [weiss@jlab.org], Light Cone 2026, Stony Brook U., 22-26 June 2026 [Webpage]

Jefferson Lab



Explore connection: Chiral symmetry breaking
→ effective spin-flavor dynamics → quark OAM

Employ instanton vacuum and large-N_c mean-field picture of nucleon

Obtain new interpretation of flavor-nonsinglet L_{u-d} as "chiral magnetic effect"

Based on:
J-Y Kim, H-Y Won, C Weiss, 2605.23125 [INSPIRE],
J-Y Kim, C Weiss, PLB 848 (2024) 138387 [INSPIRE],

Angular momentum in QCD

Energy-momentum tensor form factors

Spin and orbital angular momentum

Effective dynamics from ChSB

Instanton vacuum

Effective spin-flavor dynamics from ChSB

Nucleon as mean-field solution

Orbital angular momentum in nucleon

Instanton effect in twist-3 EMT

Large flavor-nonsinglet $L_{u-d} > 0$ in nucleon

Interpretation as "chiral magnetic effect"

Angular momentum: Operators

$$T^{\mu\nu}(x) = \sum_f T_f^{\mu\nu} + T_g^{\mu\nu}$$

Energy-momentum tensor

Obtained from invariance of action under space-time translations

Conserved current $\partial_\mu T^{\mu\nu} = 0$

Describes energy/momentum/stress carried by QCD fields

$$T_f^{\mu\nu} = \bar{\psi}_f \gamma^\mu \overleftrightarrow{\nabla}^\nu \psi_f$$

$$\overleftrightarrow{\nabla}^\nu \equiv \overleftrightarrow{\partial} - igA$$

Contribution of quark fields (flavor f)

Not conserved individually

Contains symmetric/antisymmetric parts with twist-2 and 3

$T_f^{\{\mu\nu\}}$ – trace   twist-2

$T_f^{[\mu\nu]}$ twist-3

Angular momentum of quark fields

Contains orbital and spin angular momentum

Spin-orbital separation can be defined gauge-invariantly
for quarks, not for gluons

Lorce, Leader 2014; Lorce, Mantovani, Pasquini 2017

$$J_f^{\mu\alpha\beta} = x^\alpha T_f^{\mu\beta} - x^\beta T_f^{\mu\alpha}$$

$$\langle N' | T_f^{\mu\nu} | N \rangle = \bar{u}' \Big[\frac{P^\mu P^\nu}{M_N} A_f(t) + \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{4M_N} D_f(t)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\lambda} \Delta_\lambda}{2M_N} J_f(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda} \Delta_\lambda}{2M_N} S_f(t) + M_N g^{\mu\nu} \bar{C}_f(t) \Big] u$$

Nucleon matrix element of quark EMT

Covariant parametrization $P \equiv \frac{1}{2}(p' + p)$, $\Delta \equiv p' - p$

Invariant form factors $A_f, D_f, J_f, S_f(t) \leftrightarrow$ multipoles

$$\langle T_f^{0k} \rangle(\mathbf{x}) \equiv \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\mathbf{x}\cdot\mathbf{\Delta}} \frac{\langle N' | T_f^{0k}(0) | N \rangle}{2P^0}$$

Interpretation in special frames

Here: Use Breit frame $\mathbf{P} = 0$, $\Delta^0 \neq 0$,
alt. use light-front $\mathbf{P}_T = 0$, $\Delta^+ = 0$
Relation understood. See e.g. Lorce, Moutarde, Trawinski 2019

Mechanical properties
Polyakov 2003

$$L_f \equiv \int d^3x \, \epsilon^{3jk} x^j \langle T_f^{0k} \rangle(\mathbf{x}) = J_f(0) - S_f(0)$$

$$S_f \equiv -\frac{1}{2} \int d^3x \, \epsilon^{3jk} x^j \langle T_f^{[0k]} \rangle(\mathbf{x}) = S_f(0)$$

$$J_f \equiv \frac{1}{2} \int d^3x \, \epsilon^{3jk} x^j \langle T_f^{\{0k\}} \rangle(\mathbf{x}) = J_f(0)$$

$$J_f = S_f + L_f$$

Angular momentum in nucleon

Orbital, spin and total AM, natural decomposition

Defined in terms of EMT invariant form factors, only interpretation refers to frame

Here: Accept definitions, explore dynamical content

$$T_f^{[\mu\nu]} = \bar\psi_f \, \gamma^{[\mu} \overleftrightarrow{\nabla}^{\nu]} \psi_f$$

$$= -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar\psi_f(x)\gamma_\beta\gamma_5\psi_f(x)]$$

$$\langle N | \, \bar\psi_f\gamma^\mu\gamma_5\psi_f | N\rangle = \bar u\gamma^\mu\gamma_5 u \; g_{A,f}$$

$$S_f = \frac{1}{2} g_{A,f}$$

Quark spin as axial charge

QCD equations of motion: Antisymmetric part of quark EMT operator equal to total derivative of axial current

Nucleon matrix element of axial current (flavor f)

Quark spin as axial charge

$$T_f^{0i} \; = \; \frac{1}{2}T_f^{\{0i\}} \; + \; \frac{1}{2}T_f^{[\mu\nu]}$$

twist-2 twist-3

Orbital angular momentum

Non-symmetric quark EMT in OAM involves twist-2 and twist-3 parts of tensor

OAM sensitive to interactions - dynamics!

Angular momentum: This study

Goals

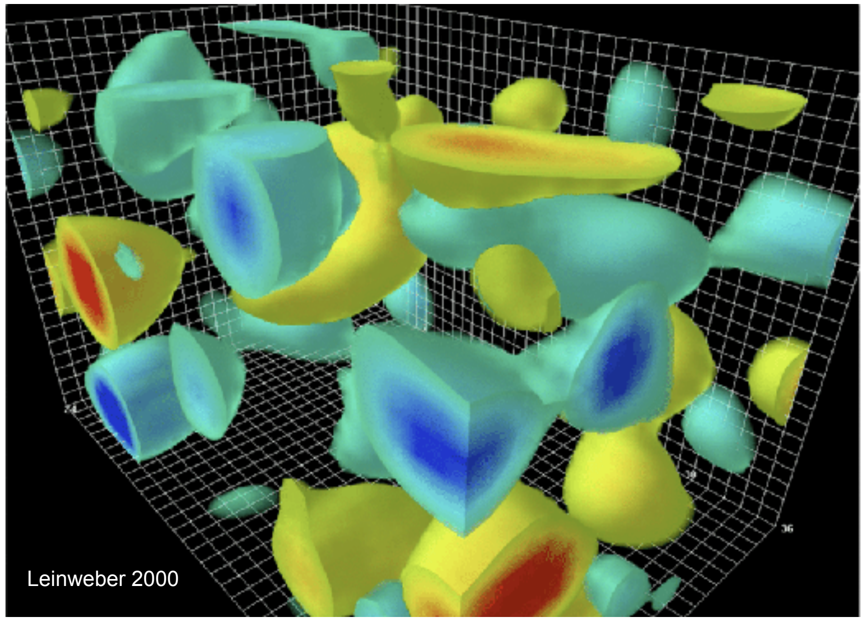
- Compute L_f, S_f, J_f in effective dynamics resulting from chiral symmetry breaking
- Explain interaction effects in $L_f \leftrightarrow$ twist-3
- Understand nucleon spin decomposition in flavor-nonsinglet sector $u - d$

Methods

Instanton vacuum: Chiral symmetry breaking, effective spin-flavor dynamics, effective operators representing QCD gauge interactions

Large-N_c limit: Functional integrals in computable saddle point approximation, bosonization, nucleon as mean-field solution ("soliton")

Effective dynamics: Instanton vacuum



Yellow/blue: Top. charge density positive/negative

QCD vacuum populated by localized gauge fields with topological charge $\frac{1}{32\pi^2}\int F\tilde{F} \approx \pm 1$

Instantons: Classical solutions of YM equations, self-dual fields $\tilde{F} = \pm F$, tunneling trajectories

Typical size $\bar{\rho} \approx$ 0.3 fm, separation $\bar{R} \approx$ 1 fm

Induce localized zero modes of fermion field with definite chirality

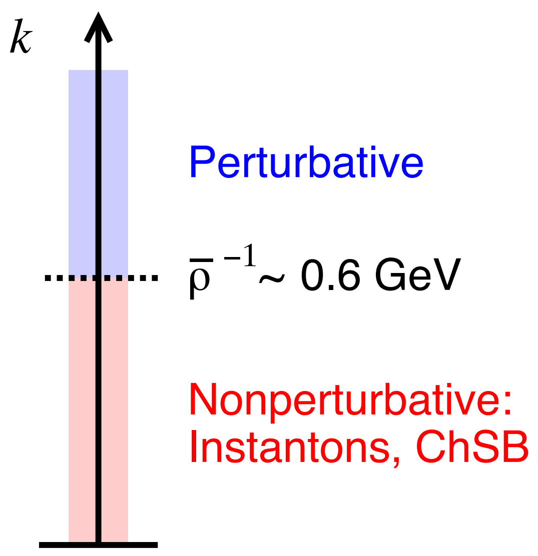
Chiral symmetry breaking: Condensate, mass generation, effective spin-flavor dynamics

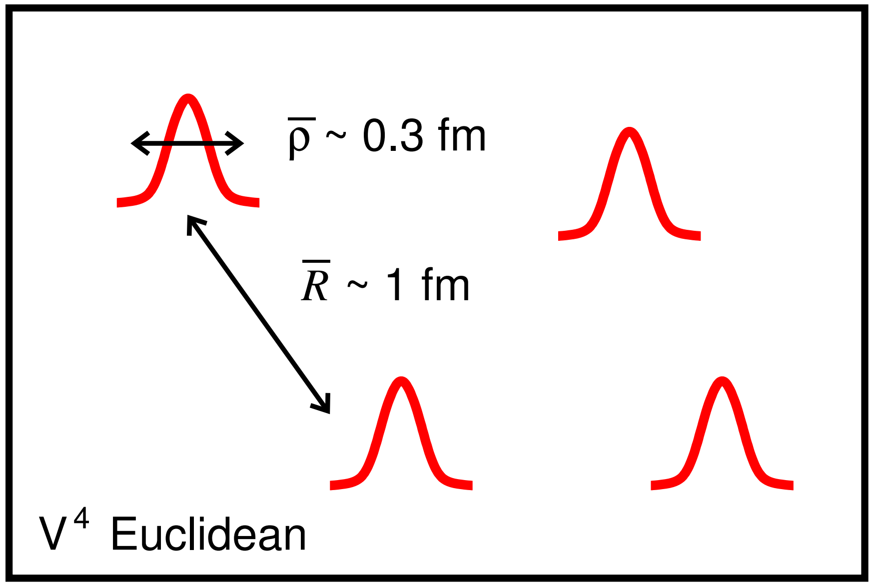
Observed in LQCD: Smooth gauge field configurations from cooling or gradient flow; instanton effects in correlation functions

Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al. 1993; DeGrand et al 1997; de Forcrand et al 1997, …, Athenodorou et al 2018; Alexandrou et al 2020

Effective description: Instanton vacuum

Shuryak 1982; Diakonov, Petrov 1984





Separation of modes

$k > \bar\rho^{-1}$: Integrate perturbatively:
Renormalization, $\bar\rho^{-2} \gg \Lambda^2_{\rm QCD}$

$k < \bar\rho^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions

Instanton ensemble

$$A(x) = \sum_I A_I(x\,|\,z_I, \rho_I, O_I) + \sum_{\bar I} A_{\bar I}(\,..\,)$$

gauge potential → classical top. fields

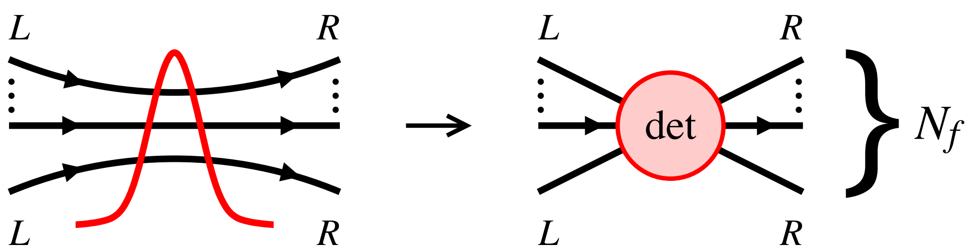
$$\int [DA] \rightarrow \int \prod_{I,\bar I} dz_I\, d\rho_I\, dO_I\; d_0(\rho_I)$$

functional integral → collective coordinates

Stable system emerges due to instanton interactions
Variational principle Diakonov, Petrov 1984; numerical simulations Shuryak 1988+

Small parameter: Packing fraction $\pi^2 \bar\rho^4 / \bar R^4 \approx 0.1$

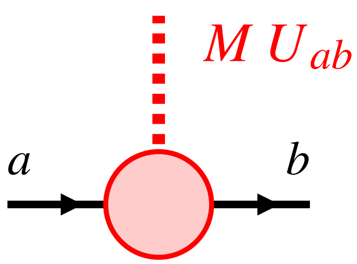
Effective dynamics: Chiral symmetry breaking



Single instanton induces multifermion vertex
$\det_{ab}\, \bar\psi^a_L\, \psi^b_R \;\times$ form factor

Finite instanton density: Dynamical quark mass
$M \approx$ 0.3-0.4 GeV, chiral condensate

Active for momenta $k \lesssim \bar\rho^{-1} \approx$ 0.6 GeV



Large-N_c limit and bosonization

Fermion integral computed in saddle point approximation
Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; …

$M\, \bar\psi_a\, U_{ab}\, \psi_b \;\times$ form factor U chiral boson field

$$Z_{\text{eff}} = \int DU \int D\bar\psi D\psi \, \exp\left(i \int d^4x \, \mathscr{L}_{\text{eff}} \right)$$

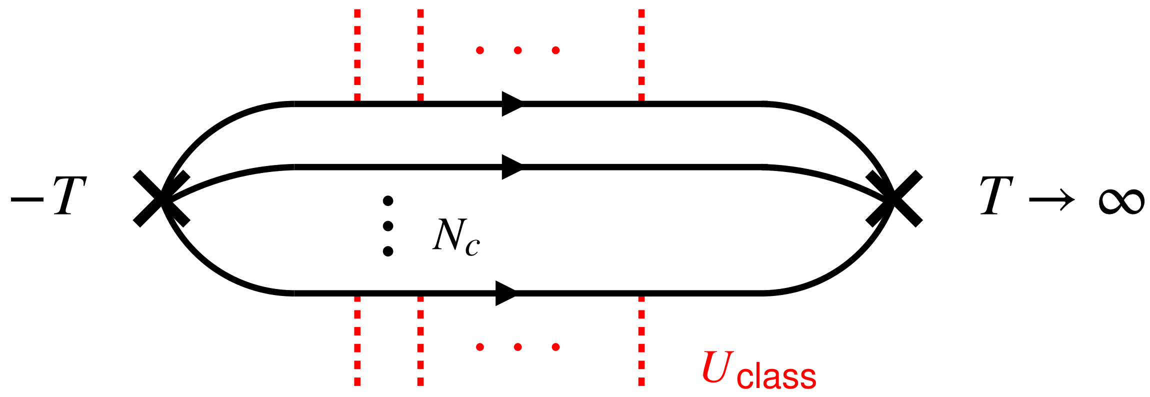
"Bosonized" form of effective dynamics

$$\mathscr{L}_{\text{eff}} = \bar\psi(x) \left[i\gamma\partial - M\, U^{\gamma_5}(x) \right] \psi(x)$$

Spin-flavor interactions of quarks with chiral field

$$U^{\gamma_5} \equiv \frac{1+\gamma_5}{2} U + \frac{1-\gamma_5}{2} U^\dagger$$

Meson correlation functions: Pion pole, η' mass



Baryon correlator develops classical chiral field

$U_{\text{class}} \neq 1$ ("soliton")

Isospin along spatial direction ("hedgehog")

Quarks move in single-particle orbits

Single-particle Hamiltonian $H = \gamma^0(\gamma \mathbf{p} + M U_{\text{class}})$

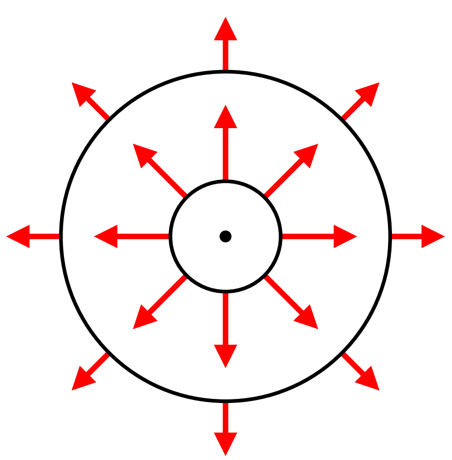
Wave functions $H\Phi_n = E_n \Phi_n$

Spectrum contains discrete level and continuum

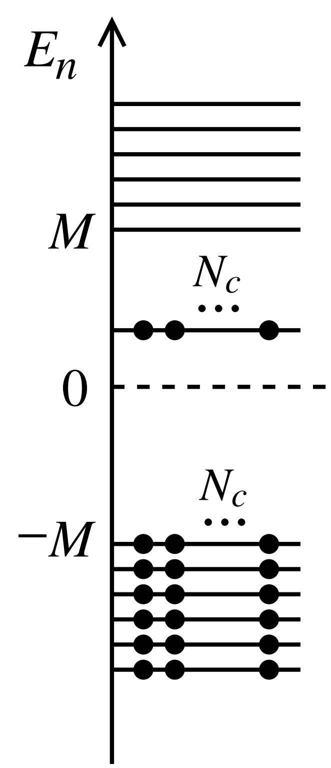
Self-consistent system
Kahana, Ripka1984; Diakonov, Petrov, Pobylitsa 1988

Baryon spin/isospin states from collective rotations

Baryon matrix elements from $1/N_c$ expansion



$U_{\text{class}}(\mathbf{x}) = \exp[i\tau^a \hat{x}^a P(r)]$



Realization of general mean-field picture of baryons at large N_c
Witten 1983

OAM: Evaluating QCD operators with gauge fields

$\mathscr{O}[\bar{\psi}, \psi, A] \longrightarrow \mathscr{O}_{\text{eff}}[\bar{\psi}, \psi, U]$

QCD operator with gauge interactions

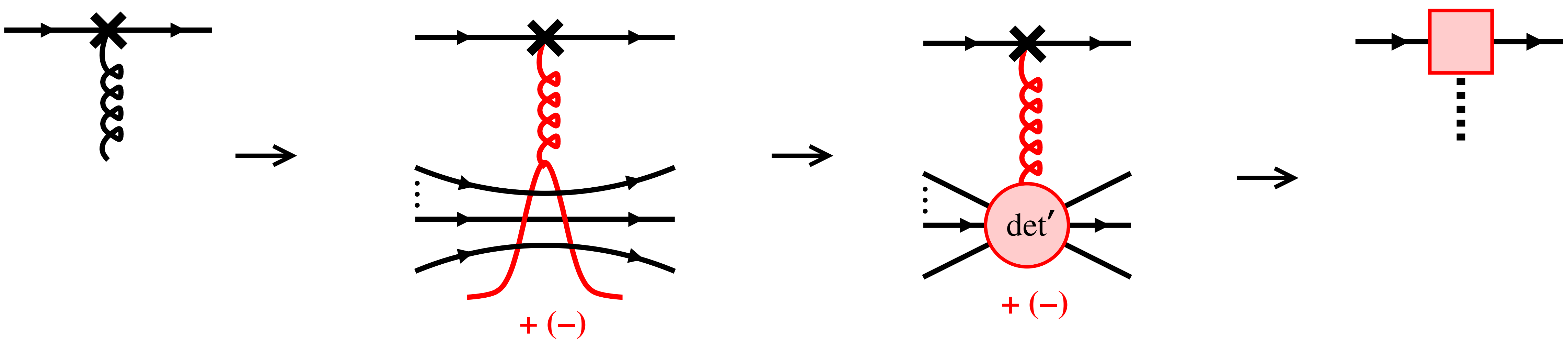
normalized at scale ρ^{-1}

Effective operator with chiral spin-flavor interactions

evaluated in effective theory

Systematic method: Expansion in instanton packing fraction, $1/N_c$

Diakonov, Polyakov, Weiss, 1995



QCD operator with gauge fields

Evaluated in instanton gauge field

Effective operator with multifermion interaction

Effective operator in bosonized form

OAM: Twist-3 quark EMT

$$T^{[\alpha\beta]}_{u-d}(x) = \bar{\psi}(x)\,\gamma^{[\alpha}\nabla^{\beta]}\tau^3\,\psi(x)$$

QCD twist-3 quark EMT, flavor-nonsinglet τ^3

$$\downarrow$$

$$T^{[\alpha\beta]}_{u-d,\,\text{eff}}(x) = \bar{\psi}(x)\left(\gamma^{[\alpha}\partial^{\beta]}\tau + \tfrac{i}{4}M\sigma^{\alpha\beta}[U^{\gamma_5}(x),\tau^3]\right)\psi(x)$$

Effective operator with chiral spin-flavor interaction

$$= \bar{\psi}(x)\left(\ \dots\ - \tfrac{i}{2}M\epsilon^{3bc}\pi^b(x)\,\sigma^{\mu\nu}\gamma_5\,\tau^c\right)\psi(x)$$

Pion field couples to quark spin: Chiral-odd, isovector

with $U^{\gamma_5}(x) = \sigma(x) + i\gamma_5\tau^a\pi^a(x)$

Coupling constant given by dynamical quark mass: Effect of ChSB

Twist-3, flavor nonsinglet $u-d$:	Instanton effect O(1)
Twist-3, flavor singlet $u+d$:	Effect suppressed
Twist-2, any flavor combination:	Effect suppressed

Instanton effect depends on spin and flavor

J-Y Kim, Weiss 2024

OAM: Twist-3 quark EMT

$$T^{[\alpha\beta]}_{u-d,\,\mathrm{eff}}(x) \;=\; \bar\psi(x)\,\Big(\gamma^{[\alpha}\partial^{\beta]}\tau + \frac{i}{4} M\sigma^{\alpha\beta}[U^{\gamma_5}(x), \tau^3]\Big)\,\psi(x)$$

$$= \; -\frac{1}{4}\,\epsilon^{\alpha\beta\gamma\delta}\,\partial_\gamma[\bar\psi(x)\,\gamma_\delta\gamma_5\tau\,\psi(x)]_{\mathrm{eff}}$$

axial current operator in effective theory

Twist-3 effective operator obeys same equation-of-motion relation as QCD operator

Now realized through effective EOM of massive quark field $[i\gamma\partial - MU^{\gamma_5}]\,\psi = 0$

Chiral interaction term essential for EOM relation

Result of consistent treatment of effective dynamics and effective operators

Important for computing AM decomposition in nucleon in effective theory

J-Y Kim, Weiss 2024

OAM: Angular momentum decomposition

T^{0i}_{u-d}, $U(x) \to U_{\rm cl}(\mathbf{x})$ classical chiral field in nucleon

$$L_{u-d} = -\frac{N_c}{3} \sum_{n\,\rm occ} \int d^3x\, \Phi_n^\dagger(\mathbf{x}) \left[L^3 \tau^3 - \frac{1}{2} M \epsilon^{3jk} \epsilon^{3bc} x^j \pi_{\rm cl}^b(\mathbf{x}) \gamma^0 \Sigma^k \tau^c \right] \Phi_n(\mathbf{x})$$

nucleon matrix element of effective operator as sum over quark levels

S_{u-d}, J_{u-d} similar expressions

quarks interact with classical chiral field

AM decomposition

J_{u-d}	0.24	
S_{u-d}	0.49	
L_{u-d}	- 0.25	= - 0.04 - 0.21

Kim, Won, Weiss 2026

Large $L_{u-d} < 0$ obtained due to chiral interaction

Explains LQCD results for J_{u-d} and S_{u-d}

Spin sum rule $S_{u-d} + L_{u-d} = J_{u-d}$ satisfied in $1/N_c$ expansion at LO

NLO corrections can modify individual values

OAM: Chiral magnetic effect

$$L_{u-d}[\text{int}] = \int d^3x \; \epsilon^{3jk} \epsilon^{3bc} B^{jb}(\mathbf{x}) \, S^{kc}(\mathbf{x})$$

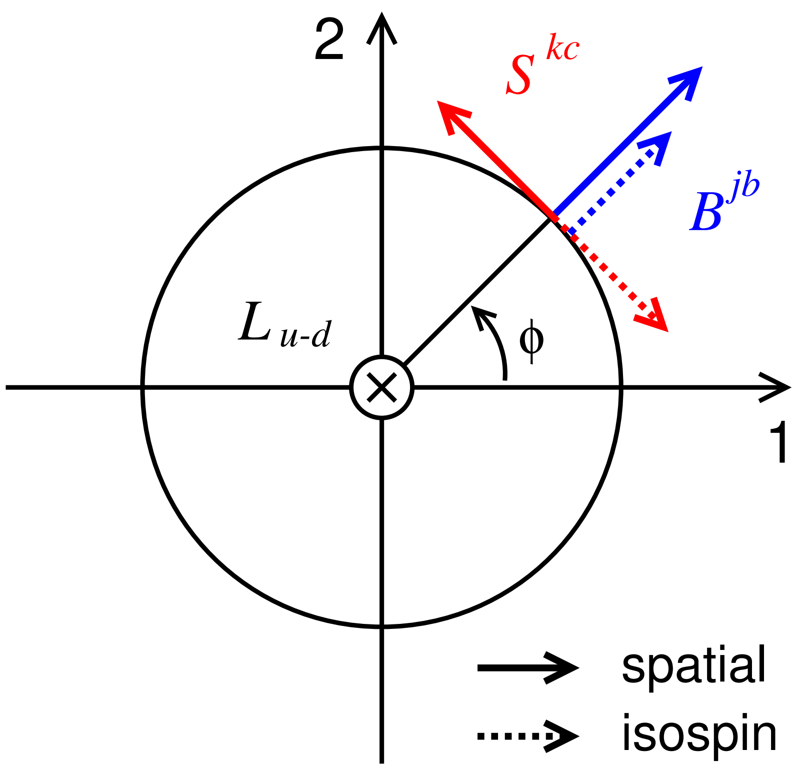
Alt. form of chiral interaction term

$$B^{jb}(\mathbf{x}) \equiv M \, x^j \pi^b_{\text{cl}}(\mathbf{x})$$

"Chiral magnetic field" formed from classical pion field and radius vector

$$S^{kc}(\mathbf{x}) \equiv \frac{N_c}{3} \sum_{n \, \text{occ}} \Phi^\dagger_n(\mathbf{x}) \left[\frac{1}{2} \gamma^0 \Sigma^k \tau^c \right] \Phi_n(\mathbf{x}) \, .$$

Quark spin-isospin current in large-N_c nucleon



Interpretation

L_{u-d} as result of "spin precession" of quarks caused by chiral magnetic field
Kim, Won, Weiss 2026

Directions: Quark spin/isospin tangential in 1,2 plane
Magnetic field radial in 1,2 plane. L_{u-d} in 3-direction

Sign: Quark spin and isospin antialigned, $L_{u-d} < 0$

Magnitude: Large effect, not relativistic correction

Analogy with chiral magnetic effect in hot/dense matter?
Kharzeev 2006; Fukushima, Kharzeev, Warringa 2008

OAM: Summary

- Instantons induce chiral spin-flavor interaction in flavor-nonsinglet twist-3 EMT: Quark spin/isospin couples to pion field

- Effective twist-3 EMT operator obeys same equation-of-motion relation with axial current as original QCD operator

- Interaction of quarks with classical chiral field in nucleon generates large $L_{u-d} < 0$

- Effect explains large difference between J_{u-d} and S_{u-d} observed in LQCD

- Effect can be interpreted as "chiral magnetic effect"

Comment on role of large-N_c limit

In the large-N_c limit the effect can be described as the interaction of quarks with classical chiral field in nucleon.
In $N_c = 3$ nucleon correlation functions the same effect happens due to multifermion interactions.

Extensions

Quark spin-orbit correlations in nucleon

$T^{\mu\nu}_{5,f} = \bar{\psi}_f \gamma^\mu \gamma_5 \overleftrightarrow{\nabla}^\nu \psi_f$ parity-odd analog of EMT Lorce 2014

Large instanton effect in twist-3 tensor operator J-Y Kim, H-C Kim, H-Y Won, C Weiss Phys.Rev.D 110 (2024)

t-dependent gravitational form factors and densities

Nucleon form factors in large-N_c mean field picture Goeke, Grabis, Ossmann, Polyakov, Schweitzer, 2007

$\bar{C}$ form factor of quark and gluon EMT Polyakov, Son 2018

Gluonic GFFs from instanton-antiinstanton molecules Liu, Shuryak, Zahed 2024+

Trace anomaly from instanton density fluctuations Diakonov, Polyakov, Weiss 1996; Liu, Shuryak, Weiss, Zahed 2024

Nucleon matrix elements of QCD operators

Higher-twist operators in DIS processes polarized/unpolarized Balla, Polyakov, Weiss 1998; Dressler, Maul, Weiss, 2000

Higher-dimensional operators in BSM physics Weiss 2021

Recent review: Weiss, *Acta Phys.Polon.B* 56, 3-A7 (2025) [INSPIRE]