Gluon mass bridge between partons and constituents in QCD hadrons

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Computational access to the logarithmically scale-dependent Hamiltonian eigenstate picture of hadrons in the space of virtual quark and gluon states, within the canonical front form of QCD, is impeded by small-x divergences that are stronger than logarithmic. We propose introducing a gluon mass parameter and an auxiliary color-octet scalar field to overcome this barrier, using the renormalization group procedure for effective particles (RGPEP). At the end of the effective Hamiltonian computation, the gluon mass parameter is taken to zero and the auxiliary field decouples, as required in gauge theory. The same method also leads to the cancellation of quadratic ultraviolet transverse divergences in self-interactions. We explain how this approach works in virtual quark and gluon scattering amplitudes, as well as in Hamiltonian eigenvalue problems for bound states, with the discussion focusing on the case of heavy quarkonia. Previously, it was suggested that QCD vacuum effects, such as those appearing in QCD sum rules and those due to instantons, can manifest themselves in the effective dynamics through front-form counterterms to small-x singularities. We use our approach to advance the hypothesis that the corresponding effective interaction terms can instead emerge in finite renormalization-group evolution as the scale parameter is lowered toward $\Lambda_{\rm QCD}$ in the RGPEP scheme.

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Plan

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1. Singularities of the front form Hamiltonian for QCD
2. Regularization using m_g and $\phi = \phi^a T^a$ and cancellation of $\sim 1/x^2$
3. RGPEP for evaluating effective Hamiltonians **R**enormalization **G**roup **P**rocedure for **E**ffective **P**articles
4. Asymptotic freedom and weak-coupling expansion for window Hamiltonians
5. Cancellation of $1/x^2$ in bound-state eigenvalue problems
6. Confinement for $m_g \to 0$
7. Hypothesis concerning extension to light quarks
8. Conclusion - it makes sense to apply the 4th order RGPEP to $H_{\rm QCD}$

Hamiltonian $H \equiv P^-$ $A^+ = 0$

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$$\mathcal{L}_{\rm QCD}^{\rm can} \;=\; \bar\psi\,(i\not\partial - g\not A - m)\,\psi - \frac{1}{2}\;{\rm Tr}\;G_{\mu\nu}G^{\mu\nu}$$

$$P^- \;=\; H_{\rm QCD}^{\rm can} \;=\; \int d^2x^\perp dx^- \;\left[\frac{1}{2}\,\mathcal{T}^{+-} = \mathcal{H}_{\rm QCD}^{\rm can}\right]$$

$$\mathcal{H}_{\rm QCD}^{\rm can} \;=\; \bar\psi\frac{\gamma^+(-\partial^{\perp 2}+m^2)}{2i\partial^+}\psi + \frac{1}{2}\,A^a_\mu\partial^{\perp 2}A^{a\mu} - (J^{a\mu}_\psi + J^{a\mu}_A)A^a_\mu - \frac{1}{4}\,g^2[A_\mu, A_\nu]^a[A^\mu, A^\nu]^a$$
$$+\;\; \frac{1}{2}\,g^2\bar\psi\not A\frac{\gamma^+}{i\partial^+}\not A\psi - \frac{1}{2}\,(J^{a+}_\psi + J^{a+}_A)\frac{1}{\partial^{+2}}(J^{a+}_A + J^{a+}_\psi)$$

$$J^{a\mu}_\psi = -g\bar\psi T^a\gamma^\mu\psi \quad J^{a\mu}_A = ig[\partial^\mu A_\nu, A^\nu]^a$$

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Quanta

$A^+ = 0 \qquad n_f$

.

$$\hat{\psi} = \sum_{c=1}^{3} \sum_{\sigma=1}^{2} \int [p] \left[u_{p\sigma} \chi_c \hat{b}_{p\sigma c} e^{-ipx} + v_{p\sigma} \chi_c \hat{d}^\dagger_{p\sigma c} e^{ipx} \right]_{x^+=0}$$

$$\hat{A}^\mu = \sum_{c=1}^{8} \sum_{\sigma=1}^{2} \int [p] \left[\varepsilon^\mu_{p\sigma} T^c \hat{a}_{p\sigma c} e^{-ipx} + \varepsilon^{\mu *}_{p\sigma} T^c \hat{a}^\dagger_{p\sigma c} e^{ipx} \right]_{x^+=0}$$

$$\int [p] = \int dp^+ d^2 p^\perp / [2p^+ (2\pi)^3] \qquad p^+ \geq \epsilon^+ p^+, p^\perp = (p^1, p^2)$$

$$\left\{ \hat{b}_{p\sigma c}, \hat{b}^\dagger_{p'\sigma' c'} \right\} = \left\{ \hat{d}_{p\sigma c}, \hat{d}^\dagger_{p'\sigma' c'} \right\} = \left[\hat{a}_{p\sigma c}, \hat{a}^\dagger_{p'\sigma' c'} \right] = 2p^+ (2\pi)^3 \delta^3(p - p') \delta_{\sigma\sigma'} \delta_{cc'}$$

cutoff $p^+ \gtrsim \epsilon^+ > 0$ no terms like $a^{\dagger 3}$ in Hamiltonian with $\epsilon^+ \to 0$

$$\boxed{H|0\rangle = 0}$$

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Divergent interactions

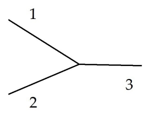
gluons are massless

.

$$\mathcal{H}_{A^3} \; = \; -J^{a\mu}_{Af} A^a_{f\mu} \; = \; -ig[\partial^\mu A_\nu, A^\nu]^a A^a_{f\mu} \quad \sim \quad A^3$$

$$H_{A^3} \; = \; \int dx^- d^2x^\perp \; \mathcal{H}_{A^3} \; = \sum_{\text{discrete}} \int [123] \, a_1^\dagger a_2^\dagger a_3 \; g \, Y_{123} \, \tilde\delta_{12.3} + h.c.$$

$$\tilde\delta_{12.3} = 2(2\pi)^3 \delta^3(p_1 + p_2 - p_3) \; , \quad Y_{123} \; = \; i f^{c_1 c_2 c_3} \left(\varepsilon_1^* \varepsilon_2^* \, \varepsilon_3 k_{12} - \varepsilon_1^* \varepsilon_3 \, \varepsilon_2^* k_{12}/x_2 - \varepsilon_2^* \varepsilon_3 \, \varepsilon_1^* k_{12}/x_1 \right)$$



$$x_1 = p_1^+/p_3^+ \qquad x_2 = p_2^+/p_3^+ \qquad k_{12}^\perp = x_2 p_1^\perp - x_1 p_2^\perp$$

singularities:	$k_{12}^\perp \to \infty, \quad x_i \to 0 \quad k_{12}^\perp/x_i$ both	cutoff:	$\mathcal{M}_{12}^2 = \frac{k^{\perp 2}}{x_1} + \frac{k^{\perp 2}}{x_2} < \Lambda^2$

x is not limited from below for $|k_{12}^\perp| \sim \sqrt{x}$

$\frac{k^{\perp 2}}{x} \longrightarrow \frac{k^{\perp 2} + m_g^2}{x}$ mass is frame independent but $\frac{m_g^2}{k^{\perp 2} + m_g^2} \; \frac{1}{x^2}$

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m_g and ϕ $\longrightarrow$ regularization

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$$\mathcal{H}_{\rm QCD}^{\rm can} \longrightarrow \mathcal{H}_{\rm QCD}^{\rm can} - \frac{1}{2}\, m_g^2 A^a_{f\mu} A^{a\mu}_f + \frac{1}{2}\,\phi^a(-\partial^{\perp 2} + m_g^2)\phi^a + m_g \phi^a \frac{1}{\partial^+}(J^{a+}_{Af} + J^{a+}_{\psi f})$$

$$\mathcal{L}_{\rm QCD}^{m_g} = \mathcal{L}_{\rm QCD} + \frac{1}{2}\,(m_g A^a_\mu + \partial_\mu \phi^a)^2 \qquad A^+ = 0$$

$$\hat\phi = \sum_c \int [p] \left[-iT^c \hat a_{p3c} e^{-ipx} + iT^c \hat a^\dagger_{p3c} e^{ipx} \right] \qquad \phi \, \frac{m_g}{\partial^+} J^+$$

$$H_{A^3} \longrightarrow H^r_{A^3} = \sum_{123} \int [123]\, a_1^\dagger a_2^\dagger a_3 \; g \; r_{12.3} \; Y_{123} \; \tilde\delta_{12.3} + h.c.$$

$$r_{12.3} = e^{-[s(\mathcal{M}^2_{12} - m_g^2)]^2} \qquad \mathcal{M}^2_{12} = \frac{k^{\perp 2} + m_g^2}{x_1} + \frac{k^{\perp 2} + m_g^2}{x_2} < \Lambda^2$$

Cancellation of $1/x^2$ in scattering described using canonical theory $+ m_g + \phi$

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$$T^{(2)} \; = \; H_I^{(1)} \frac{1}{P^- - H_f + i\epsilon} H_I^{(1)} + H_I^{(2)}$$

$$\langle q'\bar{q}'|T^{(2)}|q\bar{q}\rangle \; = \; j_{q\alpha} T^{\alpha\beta} j_{\bar{q}\beta} \qquad\qquad T^{\alpha\beta} \; = \; \frac{1}{k^+(P^- - H_f)} \sum_{\sigma=1}^{3} \epsilon^{\alpha}_{k\sigma} \epsilon^{*\beta}_{k\sigma} + \frac{\eta^\alpha \eta^\beta}{k^{+2}}$$

$$\sum_{\sigma=1}^{3} \epsilon^{\alpha}_{k\sigma} \epsilon^{*\beta}_{k\sigma} \; = \; -g^{\alpha\beta} + (k_0^\alpha \eta^\beta + \eta^\alpha k_0^\beta)/k^+ + m_g^2 \eta^\alpha \eta^\beta / k^{+2}$$

transverse gluons $k_0^- = k^{\perp 2}/k^+$ longitudinal gluons m_g^2/k^{+2}

$$\langle q'\bar{q}'|T^{(2)}|q\bar{q}\rangle \; = \; j_{q\alpha} \left[\frac{-g^{\alpha\beta} + \Pi^{\alpha\beta}}{k^2 - m_g^2} \right] j_{\bar{q}\beta}$$

$$\Pi^{\alpha\beta} \; = \; (k_0^\alpha \eta^\beta + \eta^\alpha k_0^\beta)/k^+ + m_g^2 \eta^\alpha \eta^\beta / k^{+2} + (k^2 - m_g^2)\, \eta^\alpha \eta^\beta / k^{+2} \equiv \Pi \; \eta^\alpha \eta^\beta \; = \; 0$$

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RGPEP for evaluating renormalized scale-dependent Hamiltonians

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$$q_s \; = \mathcal{U}_s^\dagger q \, \mathcal{U}_s$$

$$H(c_s, q_s) \; = \; H(c, q)$$

$$\mathcal{H} \; = \; H(c_s, q) \; = \mathcal{U}_s H(c_s, q_s) \, \mathcal{U}_s^\dagger$$

$$\mathcal{H}' \; = \; [\mathcal{G}, \mathcal{H}] \qquad \mathcal{G} \; = \mathcal{U}' \mathcal{U}^\dagger \;\; \text{generator} \quad \mathcal{G} \; = \; [\mathcal{H}_f, \mathcal{H}_I] \;\; \text{a} \;\; \text{la Wegner}$$

$$\frac{d}{ds^2} \mathcal{H} \; = \; [[\mathcal{H}_f, \mathcal{H}_I], \mathcal{H}]$$

$$\mathcal{H} = \mathcal{H}_f + \mathcal{H}_I \quad \tfrac{d}{ds^2} \mathcal{H}_I^{(1)} = \Big[[\mathcal{H}_f, \mathcal{H}_I^{(1)}], \mathcal{H}_f \Big] \quad \tfrac{d}{ds^2} \mathcal{H}_{Iij}^{(1)} = -(p_i^- - p_j^-)^2 \mathcal{H}_{Iij}^{(1)} \quad \mathcal{H}_{Iij}^{(1)}(s) = e^{-[s(p_i^- - p_j^-)]^2} \mathcal{H}_{Iij}^{(1)}(0)$$

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graphic representation of the evolution problem for quanta, using example of DIS

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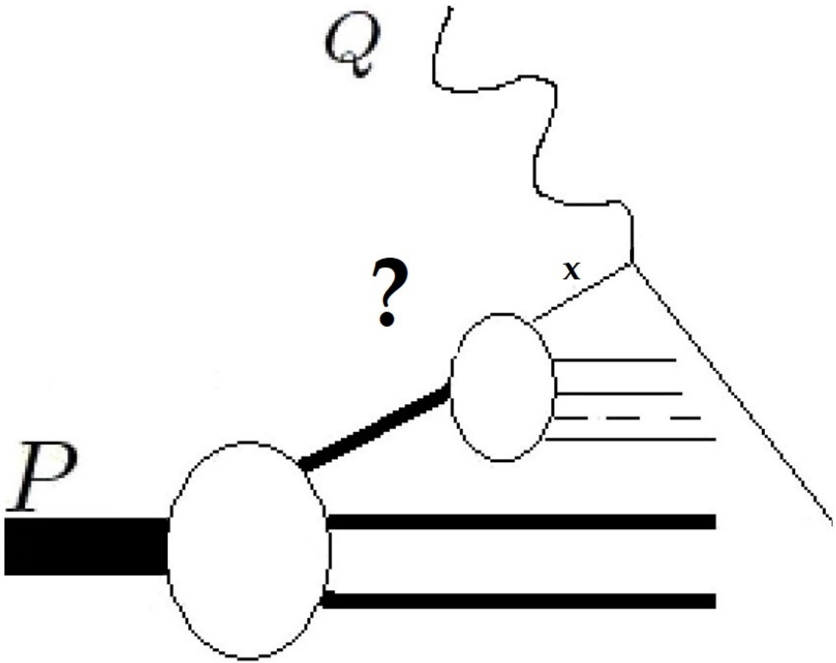

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graphic representation of the RGPEP approach to the problem

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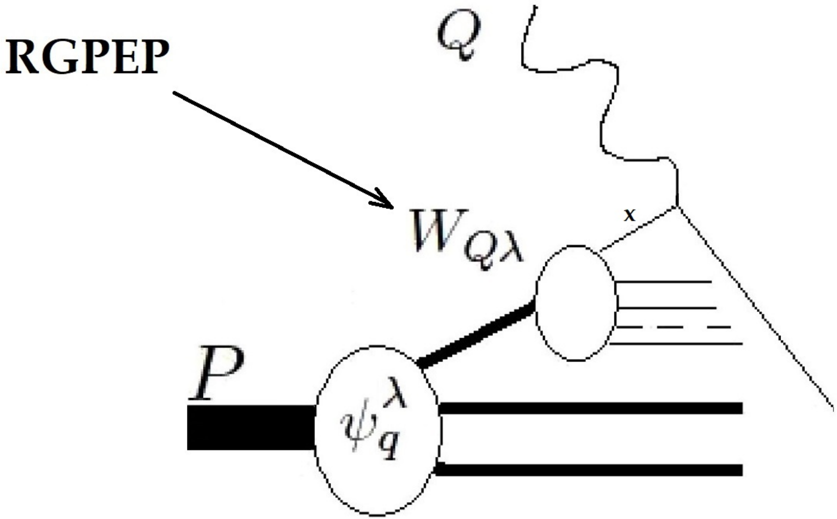

RGPEP: basis in the space of states → basis in the space of operators → new wave functions

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change of basis in the space of operators, $\lambda = 1/s$

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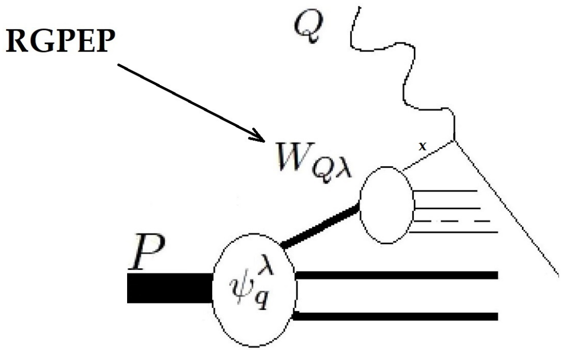
$a \to a_\lambda$ $a^\dagger \to a^\dagger_\lambda$ RGPEP

$a_\lambda = \mathcal{U}^\dagger_\lambda \, a \, \mathcal{U}_\lambda$ $a^\dagger_\lambda = \mathcal{U}^\dagger_\lambda \, a^\dagger \, \mathcal{U}_\lambda$ $\mathcal{H}'_\lambda = [[\mathcal{H}_{\mathbf{f}}, \mathcal{H}_\lambda], \mathcal{H}_\lambda] \longrightarrow \mathcal{U}_\lambda$



$$H_\lambda(c_\lambda, a_\lambda, a^\dagger_\lambda) = H(c, a, a^\dagger) \ , \quad a_Q = W_{Q\lambda} a_\lambda W^\dagger_{Q\lambda} \ , \quad W_{Q\lambda} = \mathcal{U}^\dagger_Q \mathcal{U}_\lambda$$

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Renormalized effective eigenvalue problem

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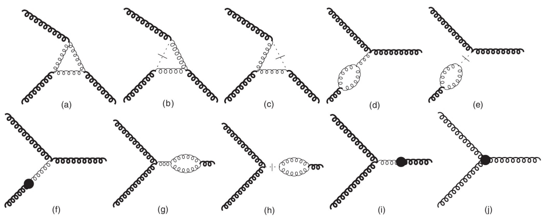
$$H_s \equiv H(c_s, q_s) \; = \mathcal{U}_s^\dagger [\mathcal{H} = H(c_s, q)] \, \mathcal{U}_s$$

eigenvalue problem for hadrons as bound states of effective quarks and gluons

$$H_s|\psi\rangle \; = \; P^-|\psi\rangle \qquad\qquad |\psi\rangle \; = \sum_i \psi_i(s)|i\rangle_s$$

asymptotic freedom



Asymptotic freedom in QCD RGPEP figure from M. Gomez-Rocha, PRD92, 065005 (2015)

$$g \to g_s$$

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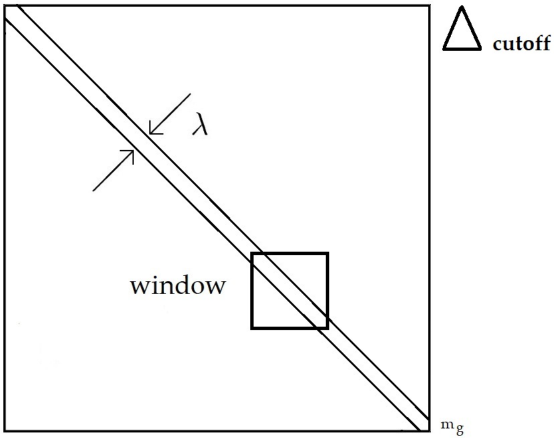
Window eigenvalue problem for hadrons

altered Wegner

$$gY \to g_s f^s Y \qquad f^s \leftrightarrow e^{-[s(p_i^- - p_j^-)]^2}$$



Window computed in weak-coupling expansion (AF) and diagonalized numerically

window middle eigenvalue matches whole matrix eigenvalue

$\mathcal{H}_f$ grows along the diagonal and off-diagonal elements include small g_s

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Bound states in theories with asymptotic freedom (logarithmically altered generator)

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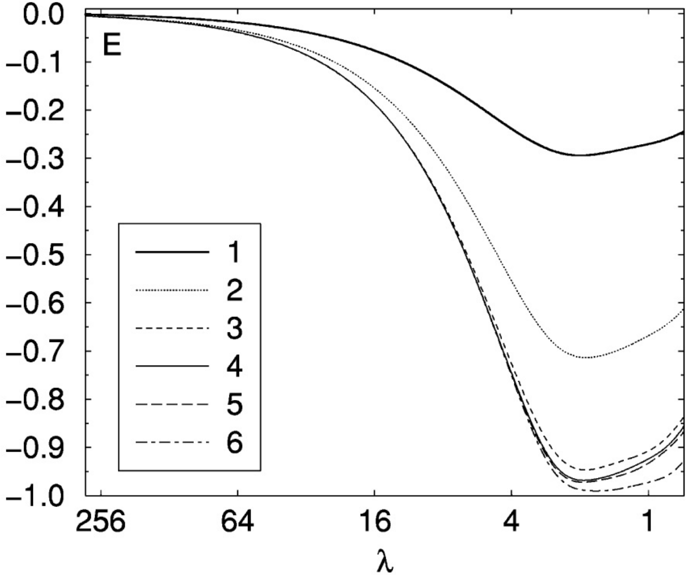

S.D. Glazek, J.Mlynik, Optimization of perturbative similarity renormalization group for Hamiltonians with asymptotic freedom and bound states, Phys. Rev. D 67, 045001 (2003).

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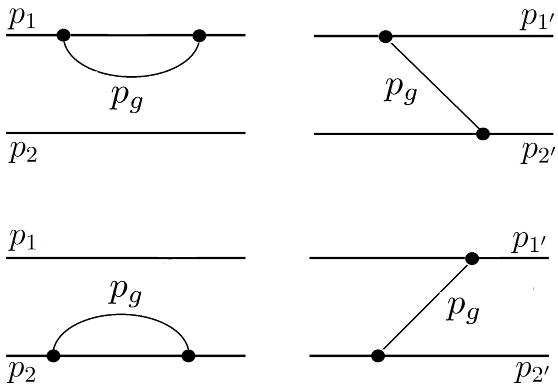
Does $\int dx/x^2$ cancel out in the eigenvalue problem written using effective quanta?

2nd order RGPEP: Yes, like in scattering, “seagulls and sums over 3 polarizations cancel out.”

What about $\int dx/x$?

2nd order RGPEP: Even more interesting, these lead to $\ln m_g$ and diverge for $m_g \to 0$. But $\ln(1/m_g)$ in the self-interaction is canceled by $\ln m_g$ in exchange, iff the quark-antiquark is a singlet. Otherwise the self-interaction wins and the eigenvalue diverges.



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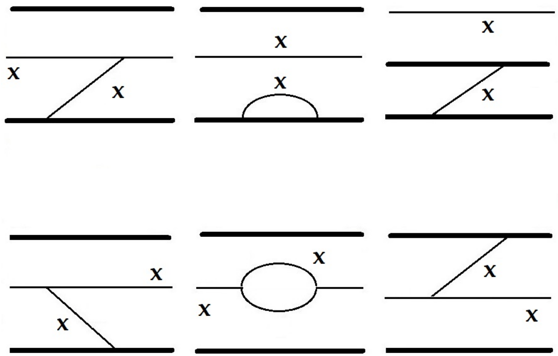
$$|\text{quarkonium}\rangle \;=\; |Q_s\bar{Q}_s\rangle + |Q_s\bar{Q}_sG_s\rangle + |Q_s\bar{Q}_sG_sG_2\rangle + |Q\bar{Q}ggg\rangle + ...$$

Cancellation of $\ln m_g$ in colorless component $|Q_s\bar{Q}_sG_s\rangle$



In colored states $\ln(1/m_g)$ does not cancel.

Mixing of 2 and 3 body components is a correction, f^s, g_s, mas gaps.

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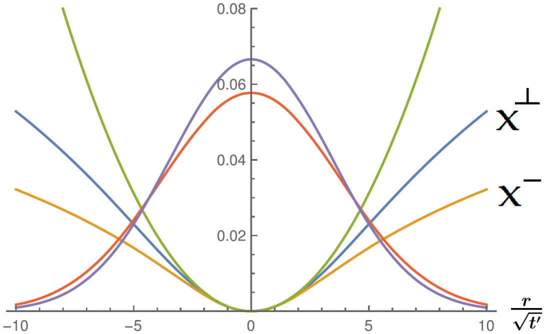
What is left after cancellation of $\ln m_g$? $M = m_1 + m_2 + B$

In component $|Q_s \bar{Q}_s\rangle$, for small r and $P^+ = M \sim m_1 + m_2$ Jacobi relative momentum $\vec{k}$

$$\frac{\vec{k}^{\,2}}{2\mu}\varphi^s_k + \int_{k'} V^C_{kk'}\varphi^s_{k'} - \frac{1}{2}\,\mu\omega^2\Delta_k\varphi^s_k \;=\; B\,\varphi^s_k$$

$$\omega^2 \;=\; \frac{\alpha C_F}{48\sqrt{2\pi}\mu s^3}$$

at large distances logarithmic behavior



M.M.Brisudova, R.J.Perry, K.G.Wilson, Phys.Rev.Lett. 78, 1227 (1997)

K. Serafin et al., Phys. Rev. D 109, 016017 (2024)

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Analytic RGPEP result

valley (m_Q, λ)

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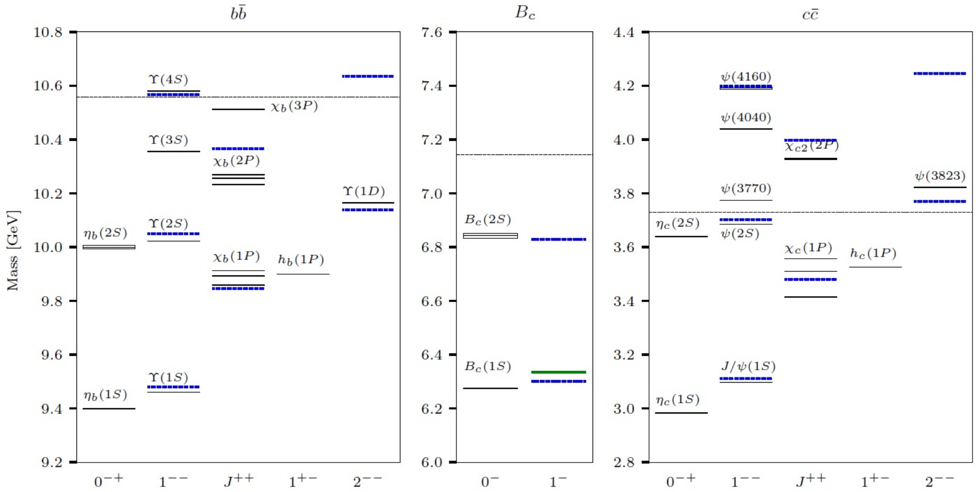

coupling constant adjusted to known evolution from EW scale

quark masses fitted to meson masses K. Serafin et al., Eur. Phys. J. C 78, 964 (2018)

In terms of comparison to lattice results, surprisingly good predictions for ccc baryons K. Serafin

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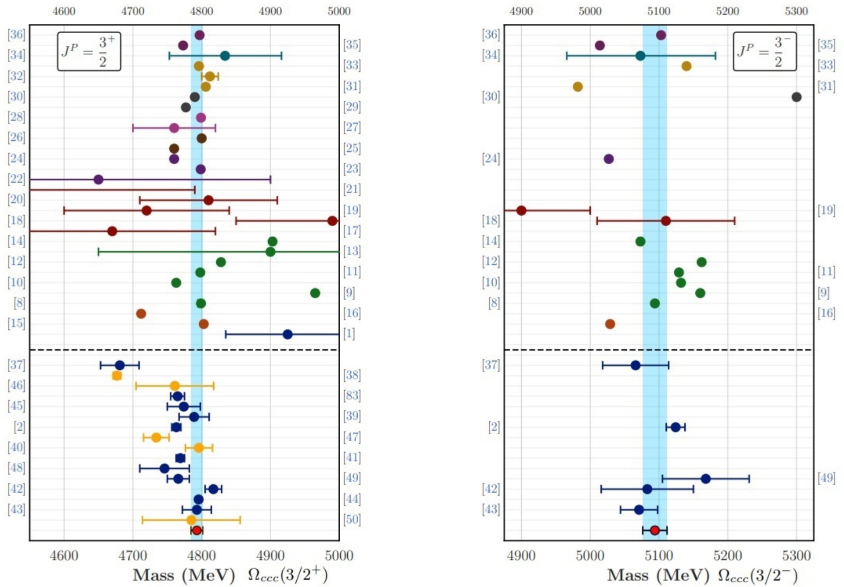

N. Dhindsa et al. Phys. Rev. D 112, L111501 (2025)

[36] = K.Serafin et al., Eur. Phys. J. C 78, 964 (2018)

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Comment concerning CMS, IMF, LaMET, . . .

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The RGPEP vertex form factors in effective Hamiltonian for QCD (any QFT) reads

$$f^s = e^{-[s\Delta p^-)]^2} = e^{-[s(p_i^- - p_j^-)]^2} = e^{-[(s/p^+)(\mathcal{M}_i^2 - \mathcal{M}_j^2)]^2} ,$$

where p^+ is the total + momentum carried by the particles in an interaction term.

It implies that quarks and gluons with large p^+ have interactions evolved only a little. Their interactions closely resemble partonic ones (e.g. DGLAP) understood as of quanta in canonical QCD. By comparison, quarks and gluons with small p^+, on the order of a hadron mass, interact as if they evolved a lot in the RGPEP towards the picture of the constituent quarks. This means, in other words, that the quanta with large p^+, the ones like in the parton model, interact nearly as in the canonical QCD, while the interaction vertices of the quanta of small p^+, such as in the rest frame of a hadron, are tamed by relatively narrow form factors, include terms resembling potentials and are expected to theoretically support the constituent quark models success in the classification of hadrons.

Since the dependence on p^+ and s is only through the ratio s/p^+, one can suspect that the RGPEP not only provides a bridge between currents and constituents in the RG sense, but also between different interpretations of CMS, IMF, and LaMET in terms of the concept of an effective theory.

Comment concerning transition from heavy to light quarks:

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- Include running of g using the RGPEP of 4th order (running coupling in interactions).
- As g_s increases, quark masses increase, oscillator potentials steep up, less motion.
- Gluons may develop masses, perhaps from the seed m_g, or otherwise, from a finite part of the gluon mass counterterm.

Hypothesis:

Maybe this is the effective picture of QCD, one like Gell-Mann had in mind, instead of computer output.

this is a way to ask and answer the question what and how large are the corrections?

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Two late friends of mine worked on similar issues without the RGPEP

Vacuum effects and constituent picture

M. Schaden, PLB 198, 42 (1987)

$\omega_M = \frac{\pi}{3m}\phi \qquad \omega_B = \sqrt{5/8}\,\phi \qquad \langle\Omega|\frac{\alpha}{\pi}G^2|\Omega\rangle$

size and ratio like Karl and Isgur needed using oscillator potentials

QCD vacuum shows up in effective theory

adjusting only masses

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B. L. Ioffe, *Nucl. Phys.* **B188**, 317 (1981), Erratum, *Nucl. Phys.* **B191**, 591 (1981)

$$\underset{I_1}{\gamma^\mu C \times \gamma_\mu \gamma^5}, \quad \underset{I_2}{\frac{-i}{2}\varepsilon_{\alpha\beta\gamma\delta} i\sigma^{\alpha\beta} \times i\sigma^{\gamma\delta}}, \quad \underset{I_3}{\frac{1}{M^2} \not{P} C \times \not{P}\gamma^5}, \quad \frac{1}{M} \not{P} C \times \gamma^5, \quad \frac{1}{M} i\sigma^{\mu\nu} P_\nu C \times \gamma_\mu \gamma^5$$

QCD sum rules with vacuum condensates

J. Namysłowski "Nucleon wave functions with running quark masses", Acta. Phys. Polon. B19, 569 (1988)

$$\psi^{P\lambda}_{\lambda_1\lambda_2\lambda_3}(p_1 p_2 p_3) = N(aI_1 + bI_2 + cI_3) \exp(D_0/6\alpha^2).$$

$$D_0 = M^2 + q_k^2(1-x_k)(x_i x_j)^{-1} + Q_k^2(1-x_k)^{-1} x_k^{-1} - \sum_{n=1}^{3} \frac{m_n^2}{x_n}$$

$\alpha^2 = (1.425 \pm 0.01)\alpha^2_{\text{Isgur-Karl}}$, $m_{\text{constit}} = (0.360 \pm 0.005)\, M$, $b/a = 1.4 \pm 0.1$, $c/a = 2.3 \pm 0.3$,

and denoting the experimental numbers with "exp" we have

$\langle r^2 \rangle_{\text{neutron}} = -0.108 \pm 0.001$, exp. -0.121 ± 0.001,

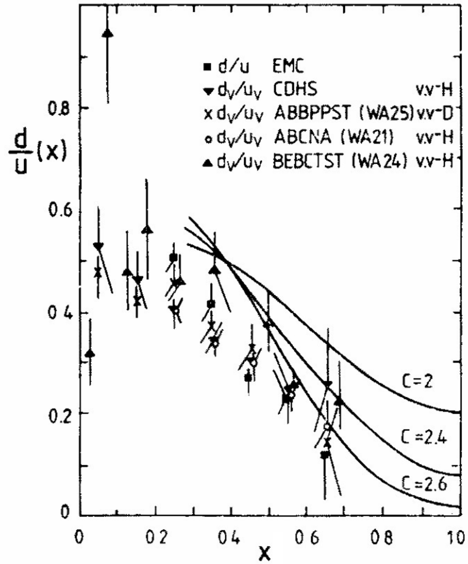
$\mu_{\text{neutron}} = -1.699 \pm 0.01$, exp. -1.913

$\langle r^2 \rangle_{\text{proton}} = 0.735 \pm 0.005$, exp. 0.70 ± 0.03,

$\mu_{\text{proton}} = 2.783 \pm 0.02$, exp. 2.793,

$G_A/G_V = 1.115 \pm 0.003$, exp. 1.251 ± 0.007.

J. Namyslowski, "Nucleon wave functions with running quark masses", Acta. Phys. Polon. B19, 569 (1988)



The *d/u* ratio in proton in the deep inelastic scattering.

$c \equiv c/a$ measures the weight of the I_3 spin structure with respect to the weight of the I_1 structure.

The experimental data are from the review K. Rith, in Proceedings of HEP 83, Intern. Europhys. Conf. on High Energy Physics, Brighton 83, eds. J. Guy, C. Costain, Rutherford Appleton Lab., Chilton, Didcot, UK

Conclusion

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1. Recommendation: apply 4th-order RGPEP in QCD with m_g and ϕ

2. 2nd-order results for heavy quarks in the limit $m_g \to 0$:

   effective potential, first approximation $V_c + V_{ho}$

   analytic formula for *ho* requency, *e.g.* for $P^+ = M \sim m_1 + m_2$ in quarkonium $\omega^2 = \frac{\alpha C_F}{48\sqrt{2\pi}} \frac{1}{\mu s^3}$

   confinement due to $\ln m_g$, infinite masses of colorful states (strings of gluons)

   surprisingly good fits and analytic mass estimates

3. Scale evolution explains relationship of CMS and IMF

4. Heuristic picture for light quarks: AF increase of α_s increases masses and yields the pattern?

5. Gluon puzzle solvable in QCD of heavy quarks ? Then to heavy-light, and only then to only light.

6. Systematic procedure

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