TMDs and Form Factors of the pion and kaon in a self-consistent light-front quark model

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Based on PRD 110, 014006 (with C.-R. Ji)

& arXiv 2512.21642[hep-ph] (with Y. Choi, A. J. Arifi, and C.-R. Ji)

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Outline

1. Motivation
   - Zero modes for Higher-twist TMDs

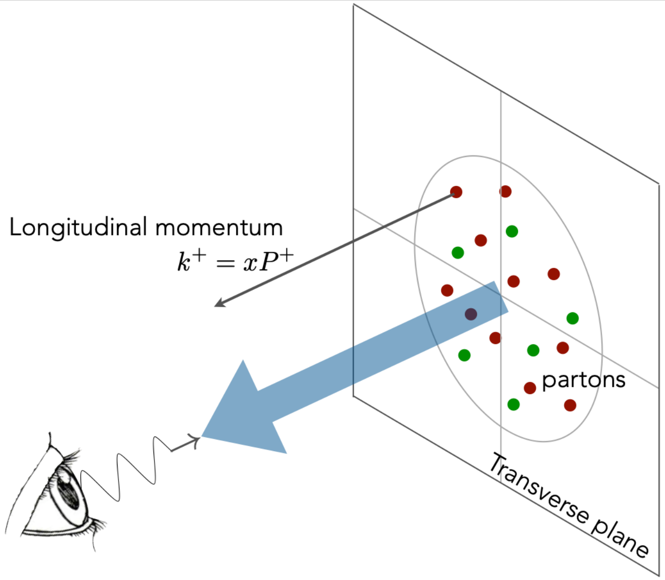
2. LFQM based on Bakamjian-Thomas (BT) construction
   - Covariant Bethe-Salpeter(BS) model vs BT-based LFQM
   - LF zero mode complication and its treatment

3. EM and scalar form factors of the pion and kaon

4. Full T-even TMDs and PDFs of the pion and kaon

5. Summary

1. Motivation: Why TMDs?



Courtesy of A. Bacchetta

- **3D momentum tomography of hadrons**
- **Rich quark dynamics** (spin, chirality, orbital motion)
- **Fundamental building block of hadronic processes** (SIDIS, Drell-Yan, electron-scattering, etc.)
- **Flagship physics program** of EIC, EicC, AMBER

[1] A. Accardi et al., EPJA 52, 268 (2016) — EIC, [2] D. Anderle et al., Front. Phys. 16, 64701 (2021) — EicC , [3] B. Adams et al., CERN-SPSC-2019-022 — AMBER

In this work, we focus on T-even unpolarized TMDs of π and K:

$$f_1^q(x, \boldsymbol{k}_\perp) \text{ (twist-2)}, f^{\perp q}(x, \boldsymbol{k}_\perp), e^q(x, \boldsymbol{k}_\perp) \text{ (twist-3)}, f_4^q(x, \boldsymbol{k}_\perp) \text{ (twist-4)}.$$

Q) How can we extract these TMDs “consistently”?

TMDs and Form Factors

- **TMD** correlator (bilocal, forward): $\langle P|\bar{q}(0)\Gamma q(z)|P\rangle \xrightarrow{\text{forward limit}} f^q(x,\boldsymbol{k}_\perp)$

  (Γ: Dirac structure)

Γ	$\Gamma = \gamma^\mu$	$\Gamma = \mathbf{1}$
twist-2	$f_1^q(x,\boldsymbol{k}_\perp)$ for $\mu = +$	
twist-3	$f^{\perp q}(x,\boldsymbol{k}_\perp)$ for $\mu = \perp$	$e^q(x,\boldsymbol{k}_\perp)$
twist-4	$f_4^q(x,\boldsymbol{k}_\perp)$ for $\mu = -$	

- **Form factor** (local current): $\langle P'|\bar{q}\,(0)\Gamma q\,(0)|P\rangle = \wp^{[\Gamma]}\, F^{[\Gamma]}\,(Q^2)$ ($\wp^{[\Gamma]}$: Lorentz prefactor)

$$\int dx \int d^2\boldsymbol{k}_\perp\, f^q(x,\boldsymbol{k}_\perp) \propto F^{[\Gamma]}\,(Q^2 = 0)$$

In this sense, form factors are the essential building blocks of the TMDs.

→ if one wants the TMDs right, one has to get the form factors right!

Challenge: Zero modes & Higher-Twist TMDs

$$\langle P'|\bar{q}(0)\,\Gamma\,q(0)|P\rangle = \wp^{[\Gamma]}\,F^{[\Gamma]}\,(Q^2)$$

The matrix element built from "bad current" components are sensitive to LF zero modes:

- $\Gamma = \gamma^-$ $\rightarrow F^{[\gamma^-]}\,(Q^2)$: zero-modes $\rightarrow$ f_4^q

- $\Gamma = \mathbf{1} \rightarrow F_s\,(Q^2)$: zero-modes $\rightarrow$ e^q

e.g.) EPJC76, 415(2016) by C. Lorce et al.

We developed a self-consistent LFQM based on Bakamjian-Thomas (BT) construction, that does two things:

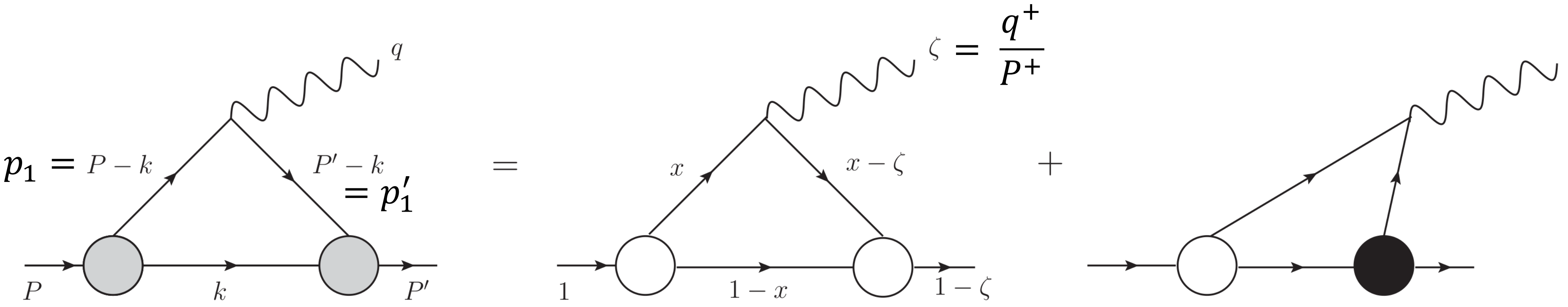
1) current-component-independent extractions

2) zero-mode effects properly incorporated

2. Covariant BS model (reference)

NPA 707, 399 by de Melo, Frederico et al.; PRD 63, 074014 by Bakker, HMC, CRJ

Covariant Bethe-Salpeter(BS) model: $\langle J^\mu \rangle = \langle P' | \bar{q}(0) \gamma^\mu q(0) | P \rangle = (P+P')^\mu F(Q^2)$



(a) (b) LF valence (c) LF nonvalence

$$\langle J^\mu \rangle_{Cov} = i N_c \int \frac{d^4 p_1}{(2\pi)^4} \frac{H_0 \, H_0'}{N_{p_1} N_k N_{p_1'}} S^\mu$$

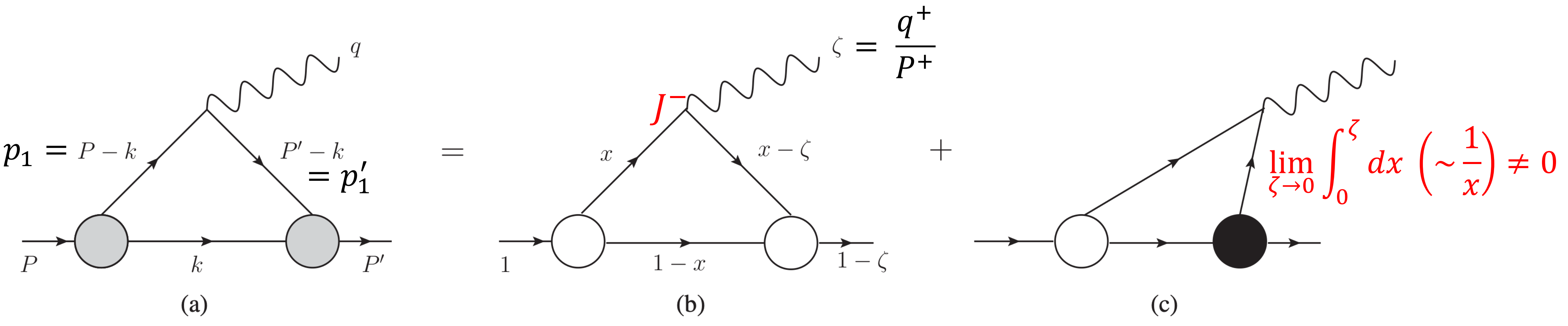
S^μ =spin trace, $N_{p_1} = p_1^2 - m_1^2 + i\epsilon$, *etc.*

- Quarks: off-mass-shell (virtual propagators)
- Vertex: off-LF-energy-shell $P^- \neq p_1^- + k^-$

☞ $\langle J^\mu \rangle_{Cov}$ contains every contribution-including the zero mode.

LF Zero mode for the EM form factor $F_{em}(Q^2)$

$$\langle J^\mu \rangle = \langle P' | \bar{q}(0) \gamma^\mu q(0) | P \rangle = (P + P')^\mu F(Q^2)$$



$$\langle J^\mu \rangle_{Cov} = i N_c \int \frac{d^4 p_1}{(2\pi)^4} \frac{H_0 \, H_0'}{N_{p_1} N_k N_{p_1'}} S^\mu = \langle J^- \rangle_{\text{LF}}^{val} + \langle J^- \rangle_{\text{LF}}^{Z.M.} \quad \text{in } q^+ = 0 \text{ frame}$$

$$\left[F_{em}^{(-)}(Q^2) \right]_{\text{LFBS}}^{\text{full}} = \frac{\langle J^- \rangle_{\text{LF}}^{val} + \langle J^- \rangle_{\text{LF}}^{Z.M.}}{(P + P')^-}$$

Zero mode for scalar form factor $F_S(Q^2)$

2512.21642[hep-ph]: Y. Choi, A. A.J. Arifi, HMC, and CRJ

$$\langle \mathfrak{J}_S \rangle = \langle P' | \bar{q}(0) \mathbf{1}\, q(0) | P \rangle = 2M\, F_S(Q^2)$$

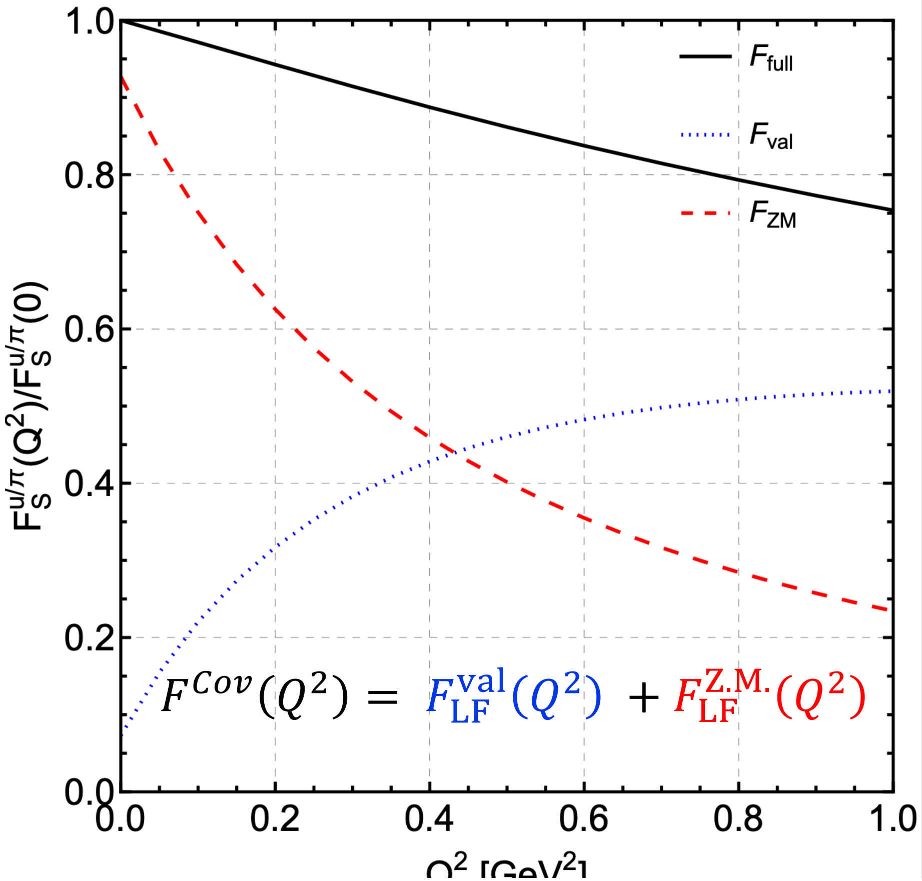
Using the covariant BS model:

$$\langle \mathfrak{J}_S \rangle = i N_c \int \frac{d^4 p_1}{(2\pi)^4} \frac{H_0\, H_0'}{N_{p_1} N_k N_{p_1'}}\ \mathrm{Tr}[S]$$

We obtain the LF result in $q^+ = 0$ frame

$$[F_S(Q^2)]^{\mathrm{full}}_{\mathrm{LF}} = \frac{N_c}{16\pi^2} \int \frac{dx\, d^2\boldsymbol{k}_\perp}{(1-x)} \chi'(x, \boldsymbol{k}'_\perp) \chi(x, \boldsymbol{k}_\perp) \frac{S_{val} + S_{Z.M.}}{2M}$$

$$\chi(x, \boldsymbol{k}_\perp) = \frac{g}{x^2\, (M^2 - M_0^2)(M^2 - M_\Lambda^2)}$$

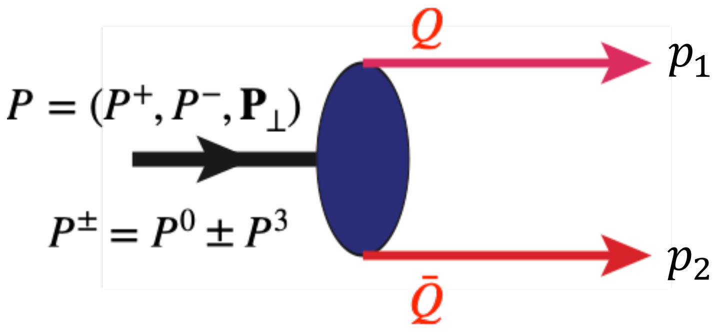


$$[F_S(Q^2)]^{\mathrm{full}}_{\mathrm{LF}} = \frac{\langle \mathfrak{J}_S \rangle_{\mathrm{val}} + \langle \mathfrak{J}_S \rangle_{\mathrm{Z.M.}}}{2M}$$

2. LFQM – BT based construction

PR 92, 1300(1953) by Bakamjian and Thomas;
Adv.Nucl.Phys. 20, 225(1991) by Keister and Polyzou

Meson state: built on a free "on-mass" shell $Q\bar{Q}$ basis



$$P^- = p_1^- + p_2^-$$

$$\frac{M_0^2 + \boldsymbol{P}_\perp^2}{P^+} = \frac{m_1^2 + \boldsymbol{p}_{1\perp}^2}{p_1^+} + \frac{m_2^2 + \boldsymbol{p}_{2\perp}^2}{p_2^+} \xrightarrow{M \to M_0} M_0^2 = \frac{m_1^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_2^2 + \boldsymbol{k}_\perp^2}{1-x}$$

Invariant mass

Interaction enters only through the mass operator $M \coloneqq M_0 + V_{Q\bar{Q}}$

- Kinematics/spin basis = free on-shell quarks

  dynamics carried by $M \coloneqq M_0 + V_{Q\bar{Q}}$

→ Poincare algebra preserved
& internal WF $(x, \boldsymbol{k}_\perp)$ boost-invariant.

Mass eigenvalue problem

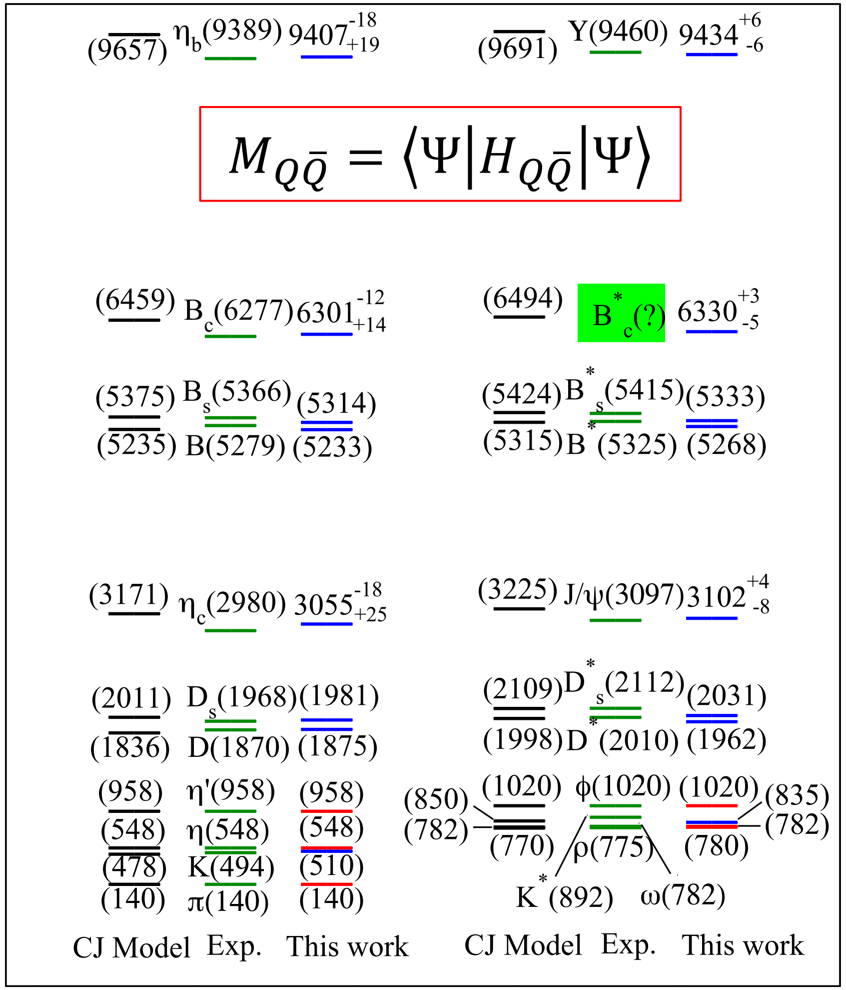
$$H_{Q\bar{Q}} = M_0 + V_{Q\bar{Q}}$$

$$V_{Q\bar{Q}} = a + br(br^2) - c_F \frac{\kappa}{r} + V_{\text{hyp}}$$

$$H_{Q\bar{Q}} \, |\Psi\rangle = M_{Q\bar{Q}} \, |\Psi\rangle$$

Optimized model parameters(in unit of GeV) and 1S state meson mass spectra

Model	m_q	m_s	m_c	m_b	β_{qq}	β_{sq}	β_{ss}	β_{qc}	β_{sc}	β_{cc}	β_{qb}	β_{sb}	β_{cb}	β_{bb}
Linear	0.22	0.45	1.8	5.2	0.366	0.389	0.413	0.468	0.502	0.651	0.527	0.571	0.807	1.145
HO	0.25	0.48	1.8	5.2	0.319	0.342	0.368	0.422	0.469	0.699	0.496	0.574	1.035	1.803



$$M_{Q\bar{Q}} = \langle \Psi | H_{Q\bar{Q}} | \Psi \rangle$$

$$\Psi_{\lambda\bar{\lambda}}(x, \boldsymbol{k}_\perp) = \phi(x, \boldsymbol{k}_\perp)\mathcal{R}_{\lambda\bar{\lambda}}(x, \boldsymbol{k}_\perp)$$

$$\phi(x, \boldsymbol{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\frac{\boldsymbol{k}_\perp^2 + k_z^2}{2\beta^2}\right): \text{radial}$$

$$k_z = \left(x - \frac{1}{2}\right)M_0 + \frac{(m_{\bar{q}}^2 - m_q^2)}{2M_0}$$

$\mathcal{R}_{\lambda\bar{\lambda}}(x, \boldsymbol{k}_\perp)$: spin − orbit

Mass spectroscopy analysis

PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ/ PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu/ PRD 100, 014026(2019) by N. Dhiman, H. Dahiya, HMC, CRJ / PRD106, 014009(2022) by A. J. Arifi, HMC, CRJ, YO

3. How we extract Form factors?

$$\langle P'|\bar{q}\Gamma q|\mathrm{P}\rangle = \wp^{[\Gamma]} F(Q^2), \ (\Gamma = \gamma^\mu, \mathbf{1})$$

- **Matrix element in the BT-based LFQM**: free on-shell $(M \to M_0)$

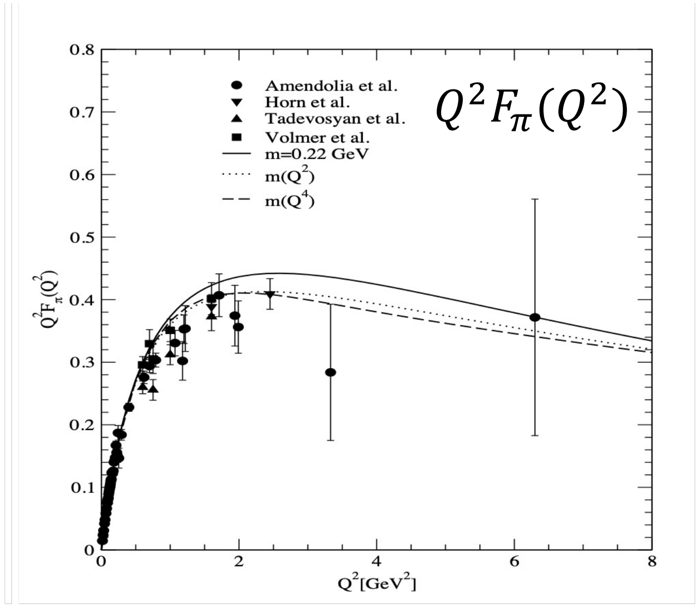
$$\langle P'|\bar{q}\,\Gamma q|\mathrm{P}\rangle_{\mathrm{BT}} = \int [d^3p_1]\phi'(x,\boldsymbol{k}'_\perp)\phi(x,\boldsymbol{k}_\perp)\mathcal{R}^\dagger_{\lambda'_1\lambda_2}\left[\frac{\bar{u}_{\lambda'_1}(p'_1)}{\sqrt{p'^+_1}}\,\Gamma\,\frac{u_{\lambda_2}(p_1)}{\sqrt{p^+_1}}\right]\mathcal{R}_{\lambda_1\lambda_2} \ (\text{in } q^+ = 0 \text{ frame})$$

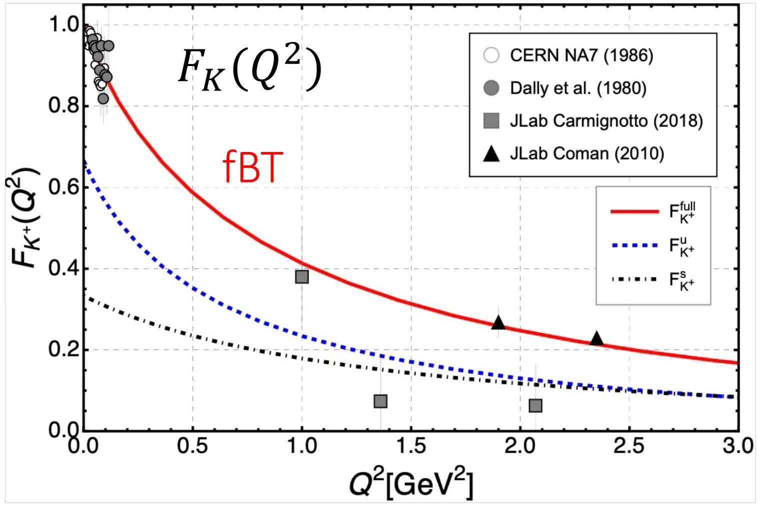
partially BT-based (pBT)-LFQM	full BT-based(fBT)-LFQM
$F^{[\Gamma]}_{\mathrm{pBT}} = \dfrac{\langle P'\vert\bar{q}\,\Gamma q\vert\mathrm{P}\rangle_{\mathrm{BT}}}{\wp^{[\Gamma]}}$	$F^{[\Gamma]}_{\mathrm{fBT}} = \left\langle P'\vert\,\dfrac{\bar{q}\,\Gamma q}{\wp^{[\Gamma]}}\,\vert\mathrm{P}\right\rangle_{\mathrm{BT}}$
BT-construction: "only" in matrix element, (but physical M kept in $\wp^{[\Gamma]}$)	BT-construction: "both" in matrix element and $\wp^{[\Gamma]}$
pBT misses the zero modes for "bad" currents $(\Gamma = \gamma^-, \mathbf{1})$	fBT **captures** the zero modes for bad currents

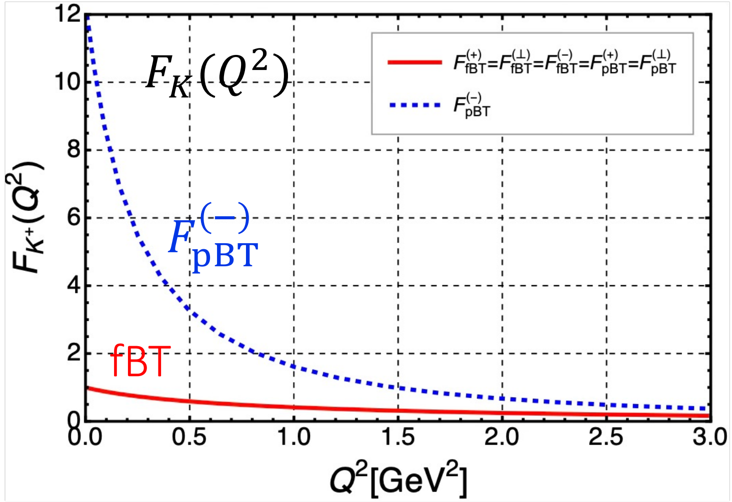
EM and Scalar Form Factors of pion and kaon

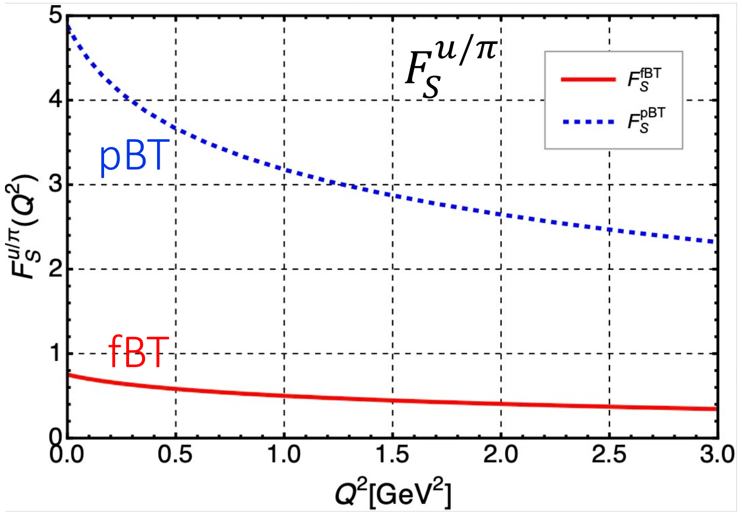
$$F^{(+)}_{\rm fBT} = F^{(\perp)}_{\rm fBT} = F^{(-)}_{\rm fBT}$$

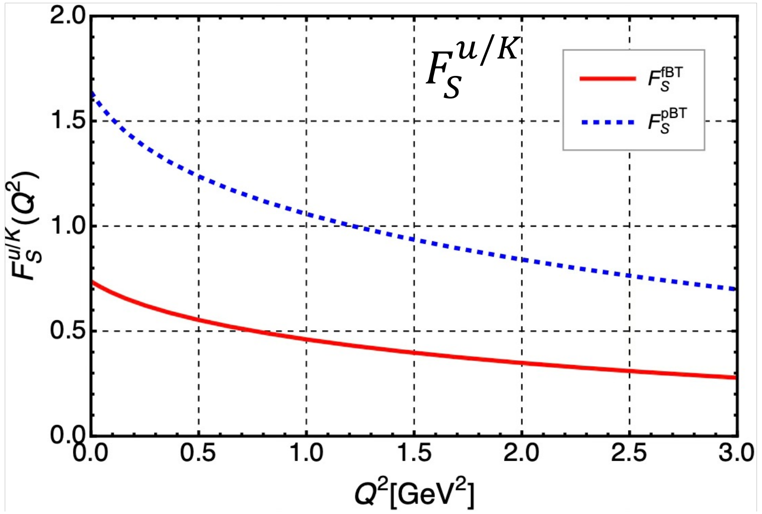
$$F^{(+)}_{\rm pBT} = F^{(\perp)}_{\rm pBT} \neq F^{(-)}_{\rm pBT}$$

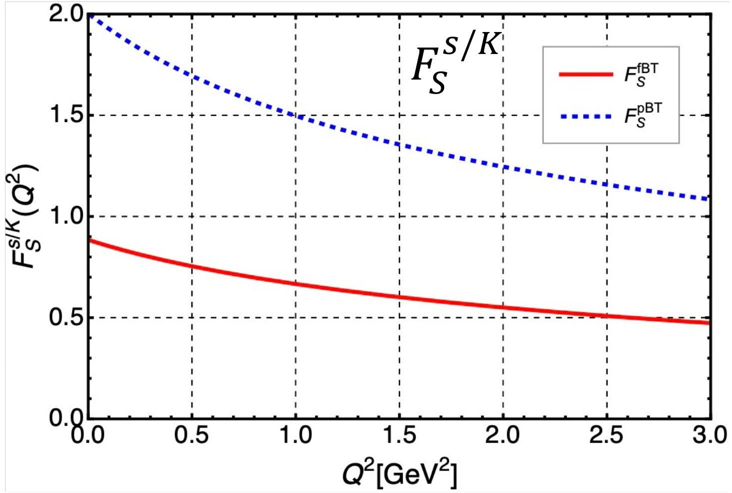












4. Unpolarized TMDs and PDFs

C. Lorce, B. Pasquini, P. Schweitzer; EPJC76,415(2016)

$$\Phi_q^{[\Gamma]}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{dz\, d^2\boldsymbol{z}_\perp}{2(2\pi)^3} e^{ip_1\cdot z} \langle P|\bar{q}(0)\Gamma\, q(z)|P\rangle\Big|_{z^+=0}$$

In conventional(e.g. pBT-LFQM) LF parametrization,

$$\Phi_q^{[\gamma^+]}(x,\boldsymbol{k}_\perp) = f_1^q(x,\boldsymbol{k}_\perp)$$

$$\Phi_q^{[\gamma_\perp^j]}(x,\boldsymbol{k}_\perp) = \frac{\boldsymbol{k}_\perp^j}{P^+} f^{\perp q}(x,\boldsymbol{k}_\perp)\ (j=1,2)$$

Forward limit →

$$\int dx \int d^2\boldsymbol{k}_\perp\, \Phi_q^{[\Gamma]}(x,\boldsymbol{k}_\perp) = \frac{1}{2P^+}\langle P|\bar{q}(0)\Gamma\, q(0)|P\rangle$$

$$\Phi_q^{[\mathbf{1}]}(x,\boldsymbol{k}_\perp) = \frac{M}{P^+} e^q(x,\boldsymbol{k}_\perp)$$

$$\Phi_q^{[\gamma^-]}(x,\boldsymbol{k}_\perp) = \frac{M^2}{(P^+)^2} f_4^q(x,\boldsymbol{k}_\perp)$$

Sum rules for $f_1^q(x)$ & $f_4^q(x)$

pBT-LFQM (e.g. EPJC76, 415(2016) by C. Lorce et al.)

$$\langle P'|\, J^\mu\, |\mathrm{P}\rangle = \wp^\mu_{\mathrm{pBT}}\, F_{em}(q^2)$$

$$1 = F_{em}(Q^2=0) = \lim_{Q\to 0} \frac{\langle P'|\, J^\mu\, |\mathrm{P}\rangle}{\wp^\mu_{\mathrm{pBT}}}$$

$$\lim_{Q\to 0} \frac{\langle P'|\, J^+\, |\mathrm{P}\rangle_{\mathrm{BT}}}{\wp^+_{\mathrm{pBT}}} = \int dx\ f_1^q(x) = 1$$

$$\lim_{Q\to 0} \frac{\langle P'|\, J^-\, |\mathrm{P}\rangle_{\mathrm{BT}}}{\wp^-_{\mathrm{pBT}}} = \int dx\ f_4^q(x) \neq 1$$

fBT-LFQM: This work

$$\langle P'|\, J^\mu\, |\mathrm{P}\rangle = \wp^\mu_{\mathrm{fBT}}\, F_{em}(q^2)$$

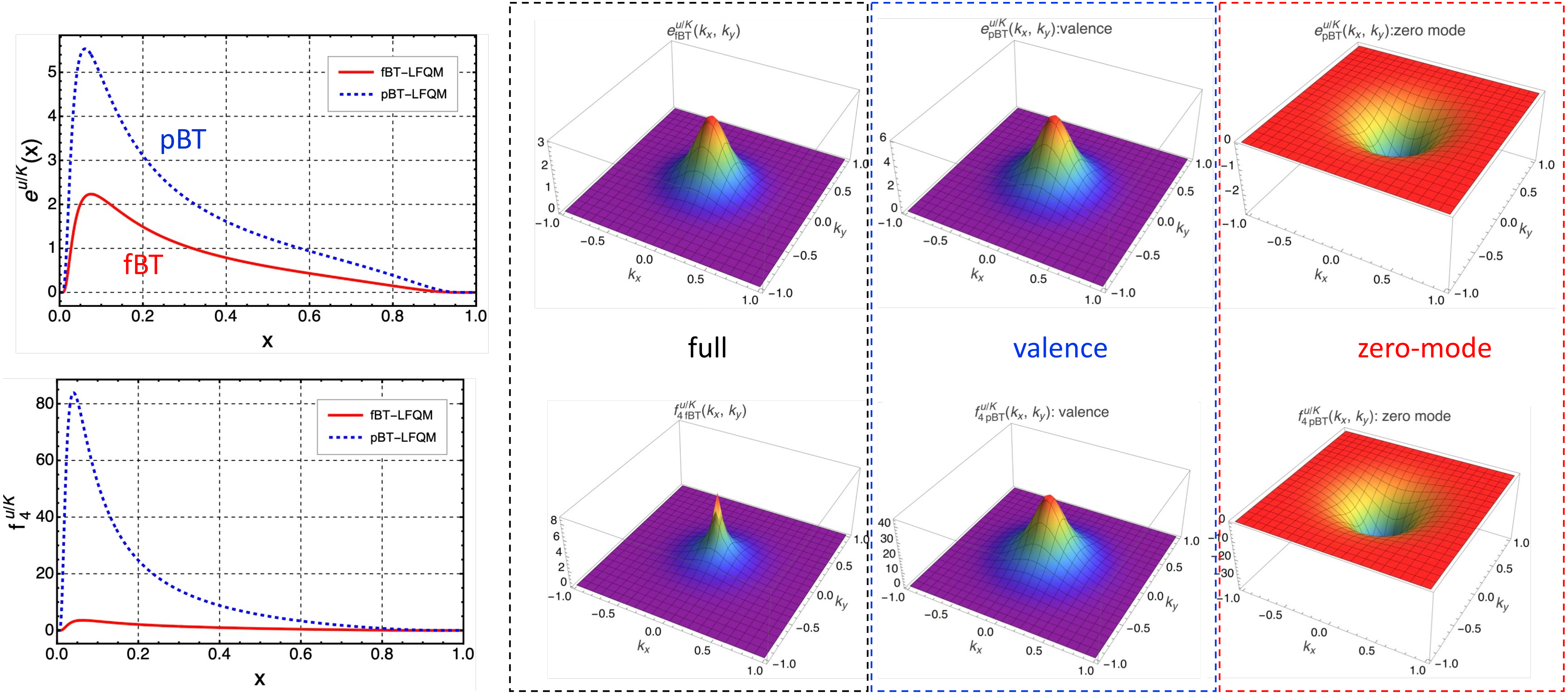
$$1 = F_{em}(Q^2=0) = \lim_{Q\to 0} \left\langle P'\right|\, \frac{J^\mu}{\wp^\mu_{\mathrm{fBT}}}\, \left|\mathrm{P}\right\rangle$$

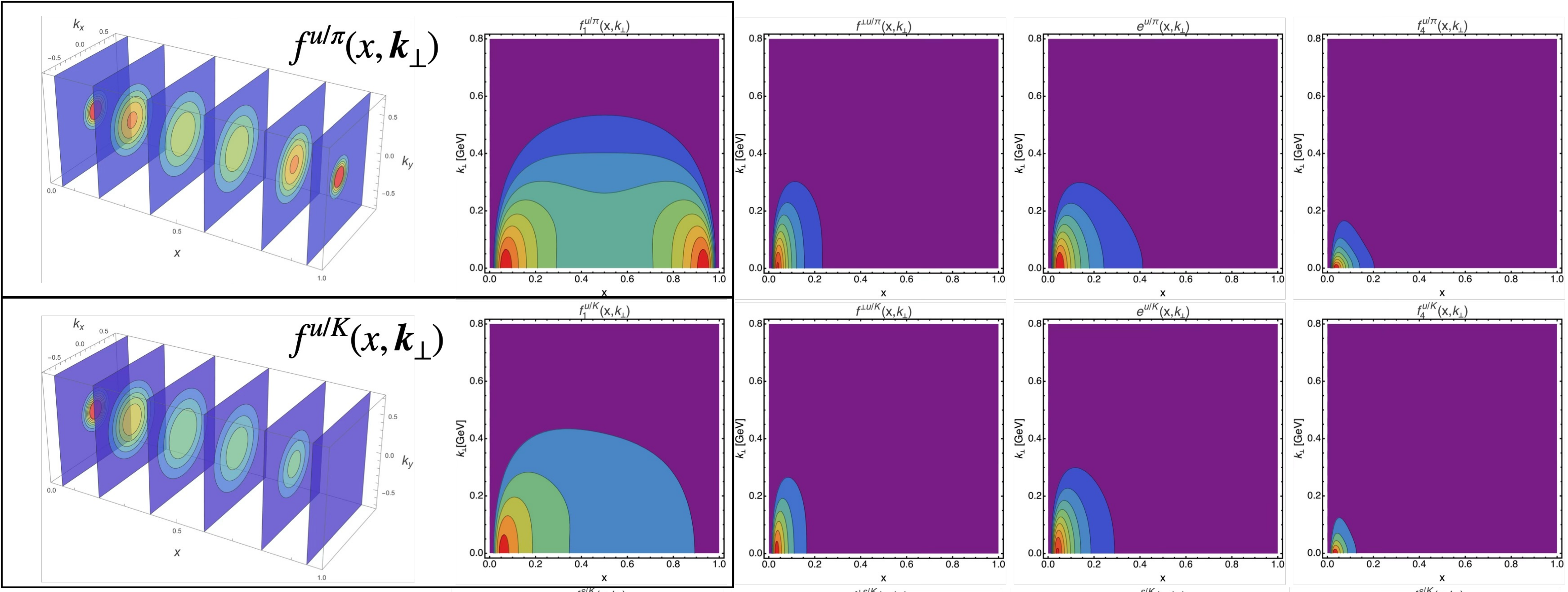
$M \to M_0$

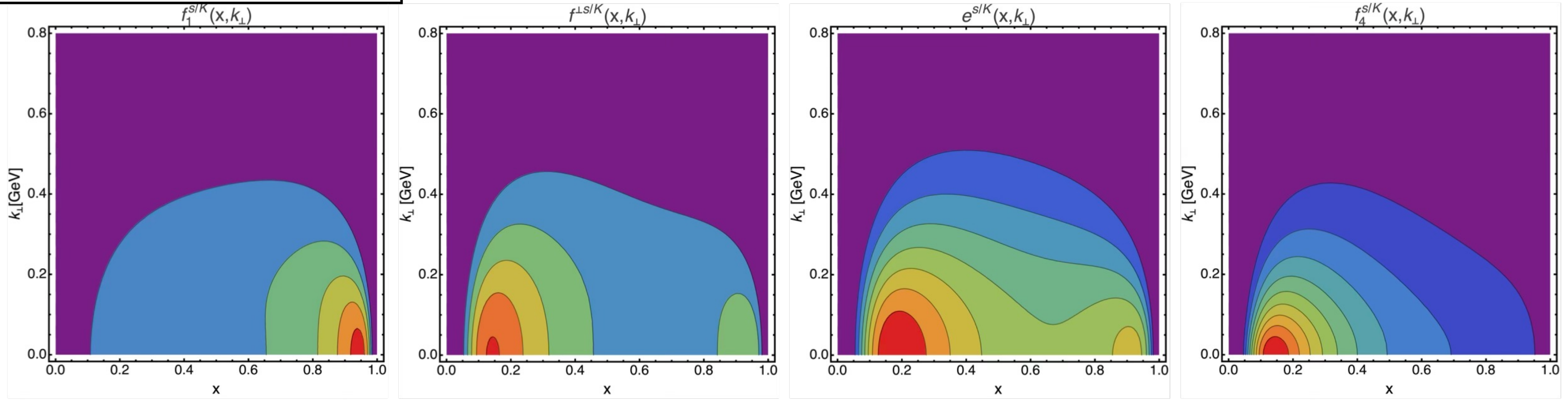
$$\lim_{Q\to 0} \left\langle P'\right|\, \frac{J^+}{\wp^+_{\mathrm{fBT}}}\, \left|\mathrm{P}\right\rangle_{\mathrm{BT}} = \int dx\ f_1^q(x) = 1$$

$$\lim_{Q\to 0} \left\langle P'\right|\, \frac{J^-}{\wp^-_{\mathrm{fBT}}}\, \left|\mathrm{P}\right\rangle_{\mathrm{BT}} = \int dx\ f_4^q(x) = 1$$

Zero mode contributions to $e^{u/K}$ and $f_4^{u/K}$



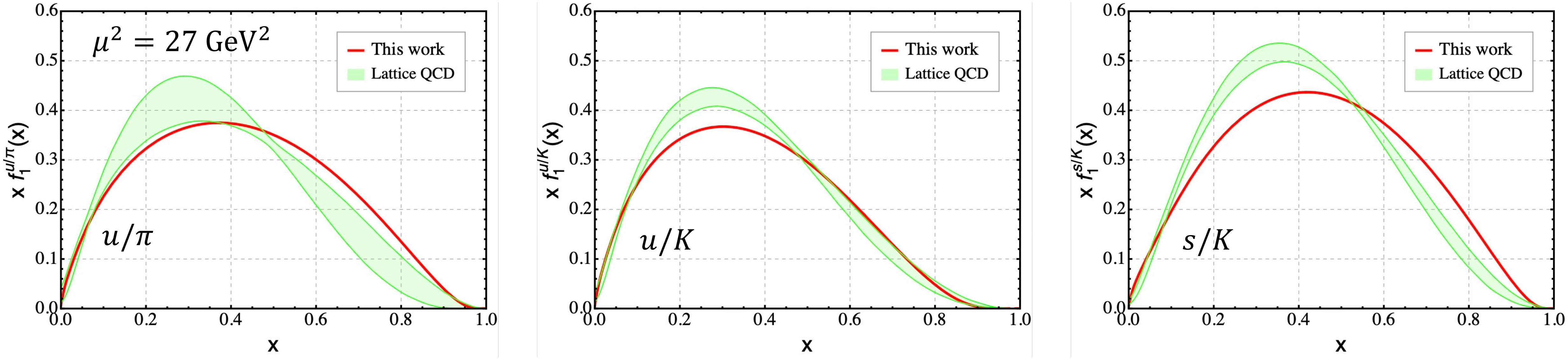




TMDs in the fBT-LFQM

QCD Evolution of twist-2 PDFs

LQCD by C. Alexandrou et al. [PRD 104, 054504 (2021)]



	μ^2 [GeV^2]	$\langle x\rangle^{u/K}_{\text{val}}$	$\langle x^2\rangle^{u/K}_{\text{val}}$	$\langle x^3\rangle^{u/K}_{\text{val}}$	$\langle x^4\rangle^{u/K}_{\text{val}}$	$\langle x\rangle^{s/K}_{\text{val}}$	$\langle x^2\rangle^{s/K}_{\text{val}}$	$\langle x^3\rangle^{s/K}_{\text{val}}$	$\langle x^4\rangle^{s/K}_{\text{val}}$
This work	16	0.213	0.080	0.038	0.021	0.282	0.129	0.072	0.044
	27	0.204	0.075	0.036	0.019	0.271	0.121	0.066	0.041
DSE approach [98]	27	0.28	0.11	0.048		0.36	0.17	0.092	
χCQM [99]		0.23	0.091	0.045		0.24	0.096	0.049	
BLFQ-NJL [16]		0.201	0.077	0.037	0.021	0.228	0.094	0.049	0.029
lattice QCD [52]		0.217	0.079	0.036	0.019	0.279	0.115	0.058	0.033

5. Summary

We developed a self-consistent fBT-LFQM for the PS meson, in which the uniform replacement $M \rightarrow M_0$ at the integrand level ensures current-component-independent extractions, with the LF zero modes properly incorporated.

Main results:

- **EM form factor:** unique across all three projections in the fBT-LFQM.
- **Scalar form factor and e^q:** zero-mode contribution explicitly identified and incorporated.
- **Full T-even TMDs:** SU(3) breaking is manifest; the f_4^q sum rule is satisfied.
- **Valence PDFs and Mellin Moments:** good agreement with LQCD and other models.

Future work will extend to polarized TMDs, GPDs, GTMDs, and comparison with EIC/EicC/AMBER.

Back up

Covariant BS vs fBT-LFQM

Two consistent models — where does the LF zero mode sit? $\langle P'|\mathcal{O}^\mu|P\rangle = \wp^\mu\,\mathcal{F}$

Covariant BS

off-shell · 4D loop → LF projection

$$\mathcal{F}_{\rm BS} = \frac{1}{\wp^\mu}\langle P'|\mathcal{O}^\mu|P\rangle_{\rm full}$$

Zero mode → **in the matrix element**

$\langle\cdots\rangle_{\rm full}$ = valence + zero mode

prefactor: external (physical M)

✓ covariant (reference)

fBT-LFQM

BT-based · on-shell quarks · direct 3D

$$\mathcal{F}_{\rm fBT} = \left\langle P'\left|\frac{\mathcal{O}^\mu}{\wp^\mu}\right|P\right\rangle_{\rm BT}$$

Zero mode → **in the Lorentz prefactor**

$\langle\cdots\rangle_{\rm BT}$ = valence

$M \to M_0\cdot$ prefactor: integrand level

✓ self-consistent

Covariant BS and fBT-LFQM **bookkeep the LF zero mode in different slots** — the **matrix element** (BS) vs the **Lorentz prefactor** (fBT). Two distinct models — each incorporating the zero mode self-consistently in its own slot.

3. Form factors: partially BT-based(pBT) LFQM vs full BT-based (fBT) LFQM

$$\langle P'|\bar{q}\Gamma q|\mathrm{P}\rangle = \wp^{[\Gamma]}F(Q^2),$$

$$\wp^{[\gamma^\mu]} = \left(\bar{P}^\mu - q^\mu \frac{\bar{P}\cdot q}{q^2}\right),$$

$$\bar{P} = P + P', \bar{P}\cdot q = M'^2 - M^2$$

$$\wp\cdot q = 0$$

$$\wp^{[1]} = 2M$$

partially BT-based (pBT)-LFQM	full BT-based(fBT)-LFQM
$F_{\mathrm{pBT}}^{[\Gamma]} = \frac{\langle P'\|\bar{q}\ \Gamma q\|\mathrm{P}\rangle_{\mathrm{BT}}}{\wp^{[\Gamma]}}$	$F_{\mathrm{fBT}}^{[\Gamma]} = \left\langle P'\| \frac{\bar{q}\ \Gamma q}{\wp^{[\Gamma]}}\|\mathrm{P}\right\rangle_{\mathrm{BT}}$
BT-construction: "only" in matrix element, (but physical M kept in $\wp^{[\Gamma]}$)	BT-construction: "both" in matrix element and $\wp^{[\Gamma]}$
pBT misses the zero modes for "bad" currents $(\Gamma = \gamma^-, \mathbf{1})$	fBT **captures** the zero modes for bad currents

$$\wp_{\mathrm{pBT}}^{[\gamma^\mu]} = \bar{P}^\mu$$

$$\wp_{\mathrm{pBT}}^{[\gamma^\mu]} = \left(\bar{P}^\mu - q^\mu \frac{\bar{P}\cdot q}{q^2}\right)$$