# Evidence for Quark Confinement in the Proton

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Confinement : assumed property of QCD

Built in to Lattice QCD calculations

Experimenters asking how  connect confinement to their efforts

Aspects of energy-momentum tensor $T^{\mu\nu}$: accessible -DVCS, J/$\Psi$

Ji gave LBL talk - using old analysis of force density

Needs $\Psi^* \Psi$. PRC112,045204 (2025)

Impossibility of obtaining time-independent,spherically symmetric densities of confined systems of relativistically moving constituents

# Requirements to get force

- Model independence
- Respect relativity -$\Psi^* \Psi$
- Uncertainty principle- define position inside proton

Key result

$$\vec{F}_q(r_\perp) = \frac{M}{4} \frac{\vec{\nabla}_\perp G_{s,q}(r_\perp)}{\rho_q(r_\perp)}$$

Numerator and denominator are observable

# Outline

- Connect $T^{\mu\nu}$ to force
- Relativity & uncertainty principle
- Relevant experiments
- Results

# Theory background- E&M

- Jackson force density $\partial_\alpha \Theta^{\alpha\beta} = -f^\beta$, divergence of symmetric stress tensor $\Theta^{\alpha\beta}$ relates to force density $f^\beta$

- Space component: $\vec{f} = \rho \overrightarrow{E} + \vec{J} \times \overrightarrow{B}$

- Integral of divergence over the volume gives the force on a surface

- Want to do same thing for QCD, Ji group's work

# QCD Energy-Momentum Tensor $T^{\mu\nu}$

- $T^{\mu\nu}$ is symmetric rank-2 tensor

- Space-time translation invariance $\rightarrow \partial\mu T^{\mu\nu} = 0$

- Decompose $T^{\mu\nu}$ into traceless $\bar{T}^{\mu\nu}$ and trace $\hat{T}^{\mu\nu}$ terms

- $T^{\mu\nu}(x) = \bar{T}^{\mu\nu}(x) + \hat{T}^{\mu\nu}(x)$ , irreducible spin 2 and spin 0 representations of Lorentz group

# Energy momentum tensor and force density

- $\bar{T}_q^{\mu\nu} = 1/2\bar{\psi}i\overleftrightarrow{D^{\mu}}\gamma^{\nu}\psi$ Symmetrized in $\mu, \nu$

- $\bar{T}_g^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F_{\alpha}^{\nu}.$ $F^{\mu\nu}$ is field-strength tensor

Force density $\mathscr{F}_q^j$

- $\mathscr{F}_q^j = \partial_{\mu}\bar{T}_q^{\mu j} = g\bar{\psi}\gamma_{\mu}F^{\mu j}\psi = \sum_{c=1}^{8} g(\rho_c E_c^j + (\vec{j}_c \times \overrightarrow{B}_c)^j)$

Divergence of quark EMT is color Lorentz force

# To get $\mathscr{F}^j_q$ need matrix elements $T^{\mu\nu}$

- Use Lorentz covariant description of EMT with 0 divergence,
  $q = P' - P, \bar{P} = \frac{1}{2}(P + P')$

- $\langle P' | T^{\mu\nu} | P \rangle = \bar{U}(P')[A(q^2)\gamma^\mu \bar{P}^\nu + B(q^2) i \frac{\bar{P}^\mu}{2M} \sigma^{\nu\alpha} q_\alpha + C(q^2)(q^\mu q^\nu - q^2 g^{\mu\nu})]U(P)$

- The ``Gravitational form factors" $A, B, C$ are measurable, $M$ is proton mass

- The quark & gluon terms of $T^{\mu\nu}$ have non-zero divergent terms proportional to $g^{\mu\nu}$

- A is a measure of momentum content, B is related to angular momentum, C is the new thing to measure

# Quark term

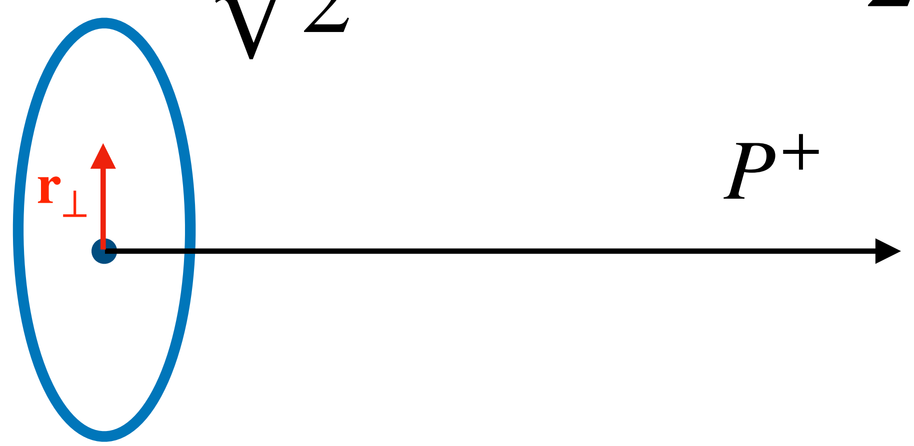
- $\langle P' | \bar{T}_q^{\mu\nu} | P \rangle = \bar{U}(P')[A_q(q^2)\gamma^\mu \bar{P}^\nu + B_q(q^2) i \frac{\bar{P}^\mu}{2M} \sigma^{\nu\alpha} q_\alpha + C_q(q^2)(q^\mu q^\nu - q^2 g^{\mu\nu}) + g^{\mu\nu}\bar{C}]U(P)$
- Choose $\bar{C}$ to make $\bar{T}_q^{\mu\nu}$ traceless: $\bar{C} = -\frac{M}{4} G_{s,q}(q^2)$
- $G_{s,q}(q^2) = A_q(q^2) - C_q(q^2)\frac{3q^2}{M^2}$
- Force on quarks: $\partial_\mu \bar{T}_q^{\mu\nu} = -\frac{M}{4}\partial^\nu G_{s,q}$ =force density

# Relativistic Definition -Proton Position

- Light front variables

$$P^\mu = (P^-, P^+, \mathbf{P}_\perp), P^\pm = \frac{P^0 \pm P^3}{\sqrt{2}}, P^- = \frac{\mathbf{P}_\perp{}^2 + M^2}{2P^+}$$

- $r^\mu = (r^-, r^+, \mathbf{r}_\perp)$




- $|\Psi\rangle = \int d^2P_\perp |P^+, \mathbf{P}_\perp\rangle$, localizes $\mathbf{R}_\perp$ to **0**, $P^+ \to \infty$

Infinite momentum frame

- All proton states in the integral have same internal structure

# Coordinate space densities


- $\rho_{\mathcal{O}}(r^-, r^+, \mathbf{r}_\perp) = \lim_{P^+ \to \infty} \int dr^- \langle \Psi | \mathcal{O}(r^+, r^-, \mathbf{r}_\perp) | \Psi \rangle$
- $\mathcal{O} = \partial_\mu T^{\mu\nu} = \dfrac{\partial T^{+\nu}}{\partial x^+} + \dfrac{\partial T^{-\nu}}{\partial x^-} + \partial_i T^{i\nu}$

$$= -\frac{M}{4} \partial^\nu G_{s,q}$$

- first two terms vanish because $P^+$ is fixed and infinite
- $\langle \Psi | \partial_\pm T^{\pm,\mu} | \Psi \rangle = \langle \Psi | [\partial_\pm, T^{\pm,\mu}] | \Psi \rangle = \langle \Psi | P^\pm T^{\pm,\mu} - T^{\pm,\mu} P^\pm | \Psi \rangle = 0$
- After integration over the perpendicular momenta

$$\mathcal{F}^i_q = \partial^i \int d^2 q e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} G_{s,q}(q^2_\perp) \equiv \partial^i G_{s,q}(r_\perp)$$

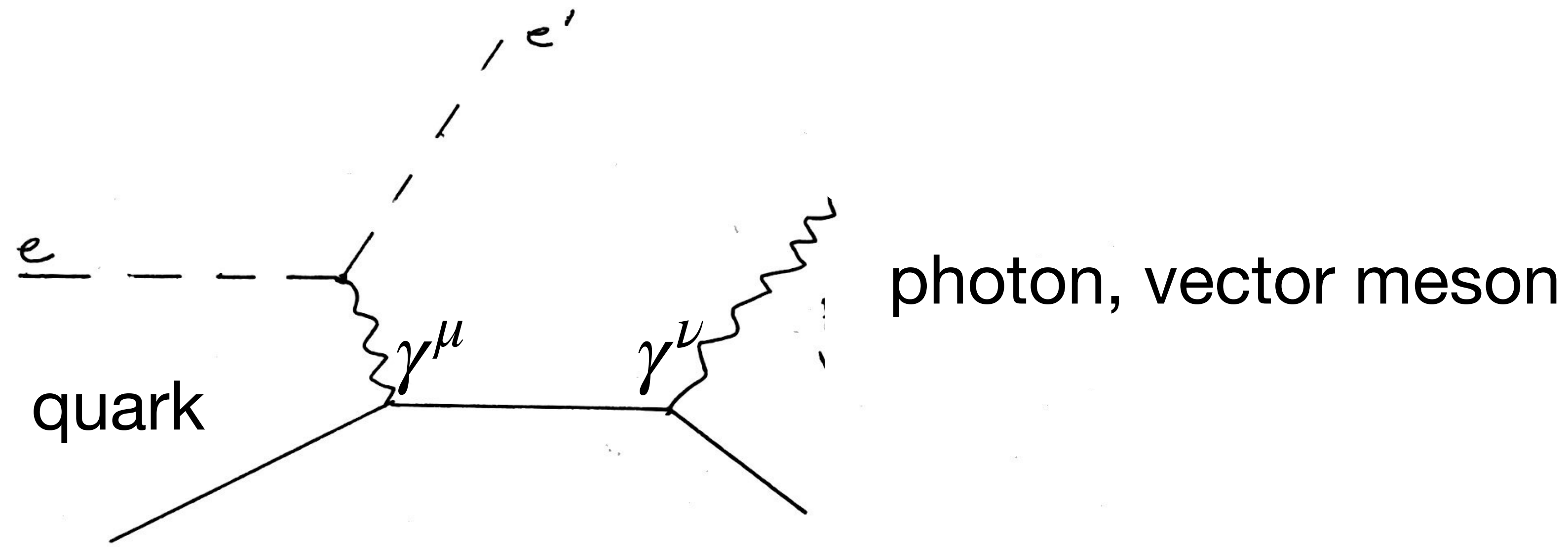
We have the force density !

# From force density to force

- $\text{force} = \dfrac{\text{force density}}{\text{quark density}}, \quad \rho_q(r_\perp) = 3(F_1^p(r_\perp) + F_1^n(r_\perp))$
- $\overrightarrow{F}_q(r_\perp) = \dfrac{M}{4} \dfrac{\overrightarrow{\nabla}_\perp G_{s,q}(r_\perp)}{\rho_q(r_\perp)}$
- Denominator known from electron scattering
- Numerator systematically improvable by more measurements

# Electron scattering accesses gravitational form factors A,C

$T^{\mu\nu}$ second rank tensor




Deeply Virtual Compton Scattering DVCS, or DV meson production

Electron scattering measures Generalized Parton Distributions GPDs

# Results- $G_{s,q}$

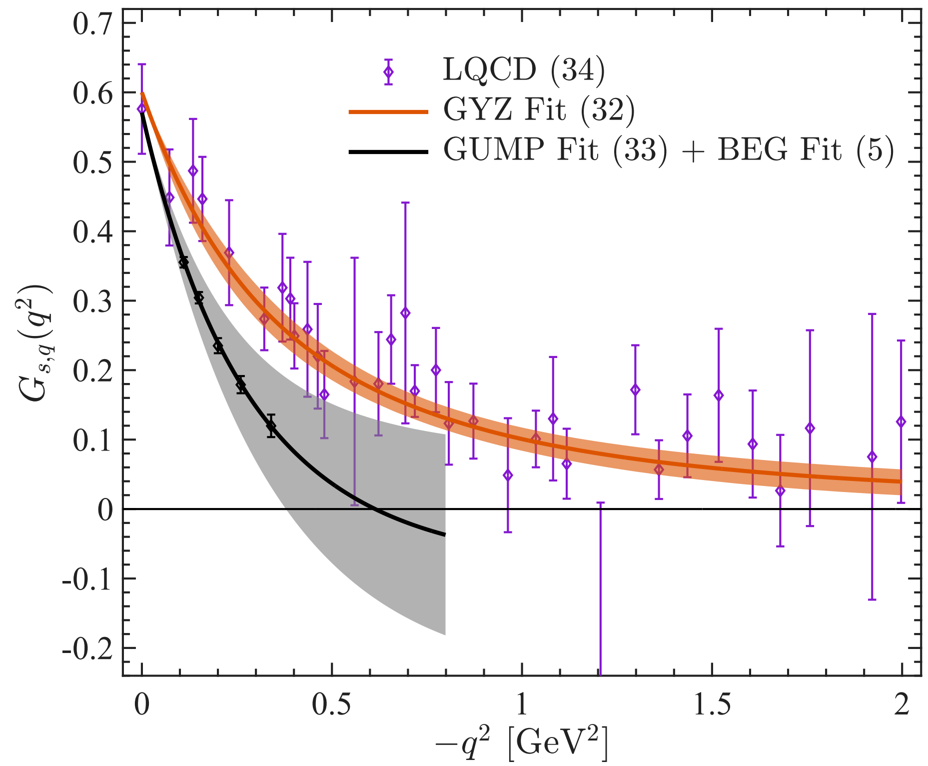


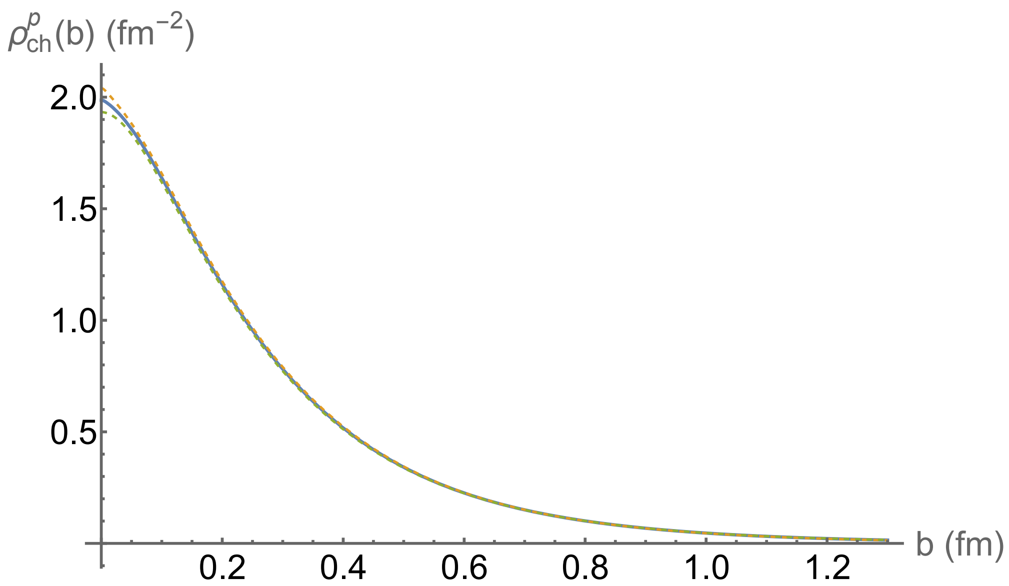

**Figure 1**: **The quark scalar form factor $G_{s,q}(q^2)$ obtained from LQCD calculations and experimental data.** Direct LQCD calculations are shown as purple error bars (*34*). The GYZ fit (orange curve and band) represents the Bayesian inference of LQCD (main) and $J/\psi$ photoproduction data (*32*). The black curve and band exhibits the experimental information from GUMP fit and BEG's dispersive analysis of DVCS data (*5, 33*).

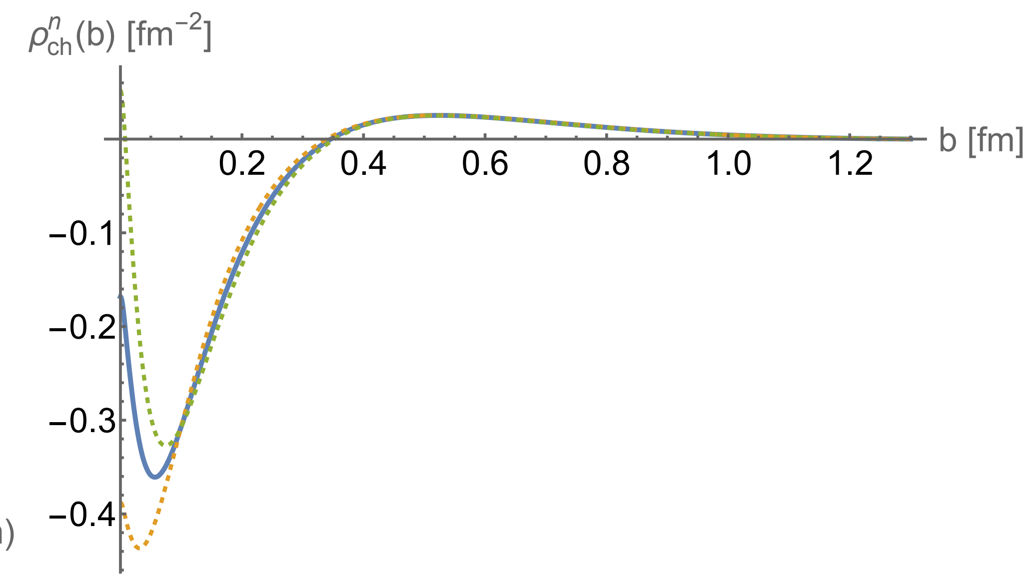
GUMP- GPDs through universal moment parameterization
Combines DVCS, DVMP,Pdfs, nucleon electromagnetic form factor

Lattice calculations , BEG is DVCS

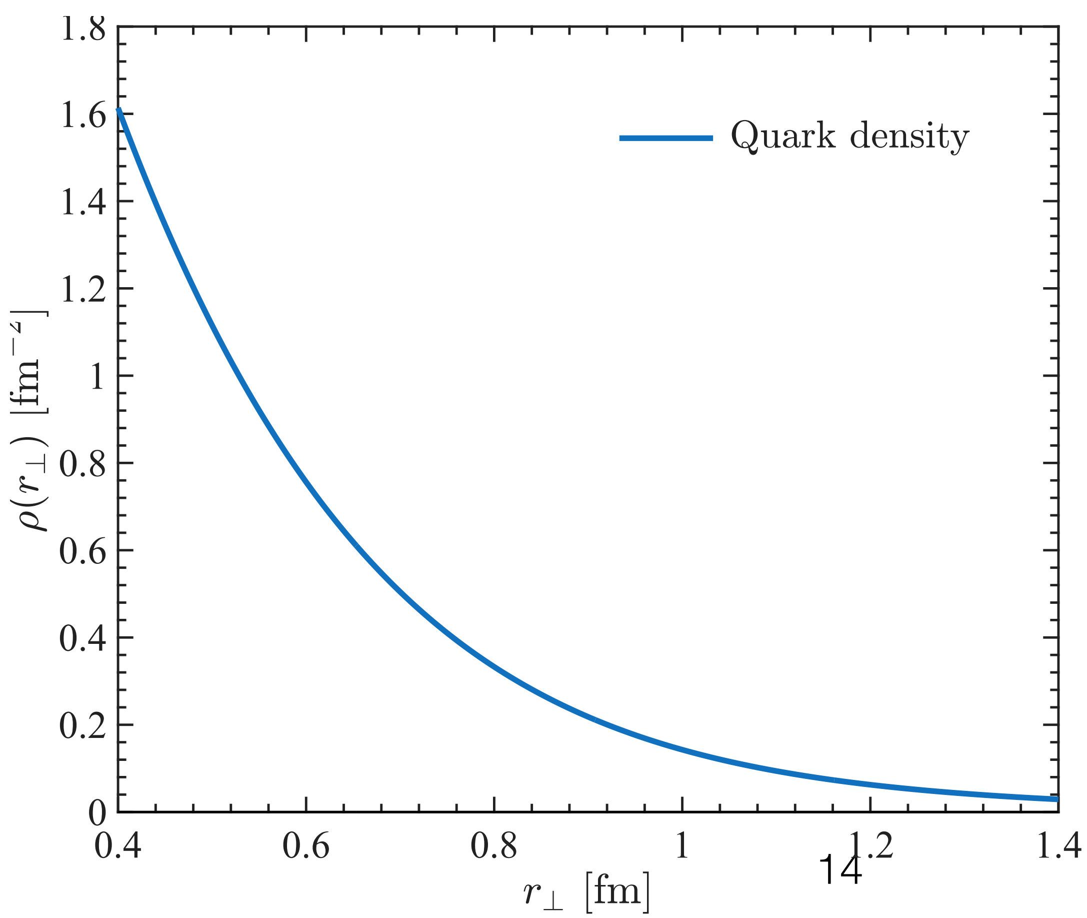
# Quark density

## Charge density




From Hague,Arrington et al 2501.18443- all from data



## Results- $\overrightarrow{F}_q(r_\perp) = \frac{M}{4} \frac{\overrightarrow{\nabla}_\perp G_{s,q}(r_\perp)}{\rho_q(r_\perp)}$

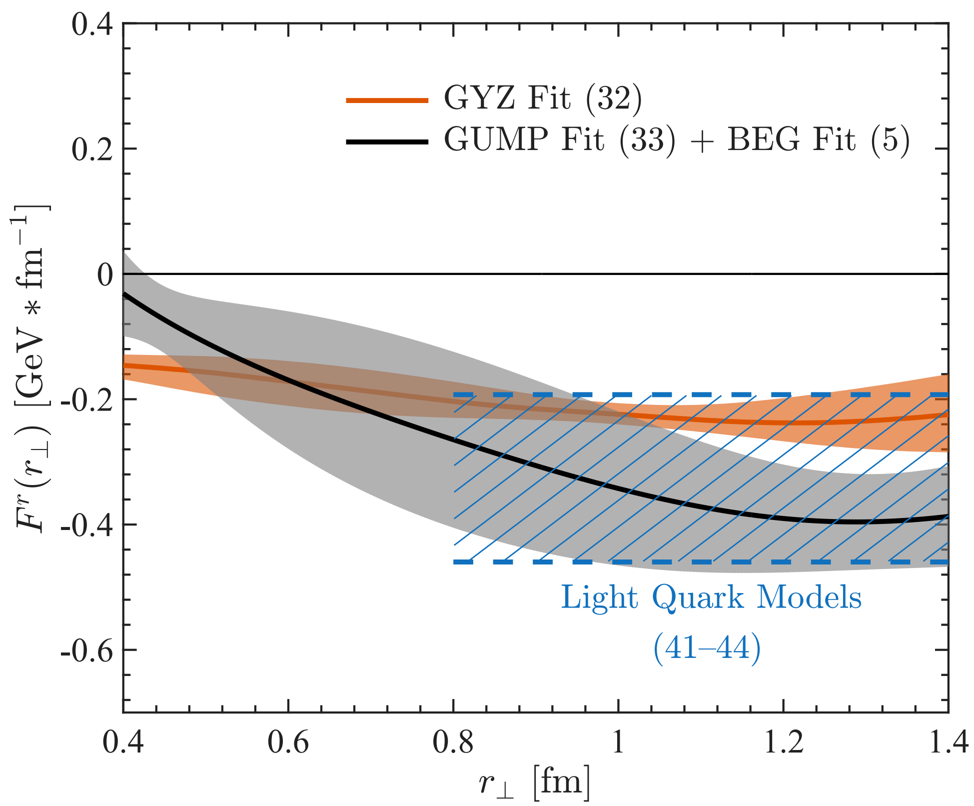



**Figure 2**: **Strong force on quarks in the proton in the transverse plane of the IMF.** The forces are obtained using the latest global analyses of the quark momentum current form factors (*5, 32, 33*). The figure shows the total force from experimental data (black curve and band) and from the fit dominated by LQCD calculations (orange curve and band). The blue hatched band shows the string tensions from several relativistic quark models (*41–44*).

Constant negative force means attractive linear potential

# Summary

- Found evidence for quark confinement in the proton
- Presented force as a ratio of two quantities that can be determined from experiments
- Force is consistent with linear potential, magnitude consistent with phenomenology
- Experimental uncertainties can be reduced through further measurements