

Electromagnetic Form Factors of the Nucleon at Large Momentum

Sergey Syritsyn (Stony Brook University)
with M.Engelhardt, J.Green, S.Krieg, S.Meinel,
A.Pochinsky, J.Negele, M.Rodekamp
(Lattice Hadron Physics collaboration)

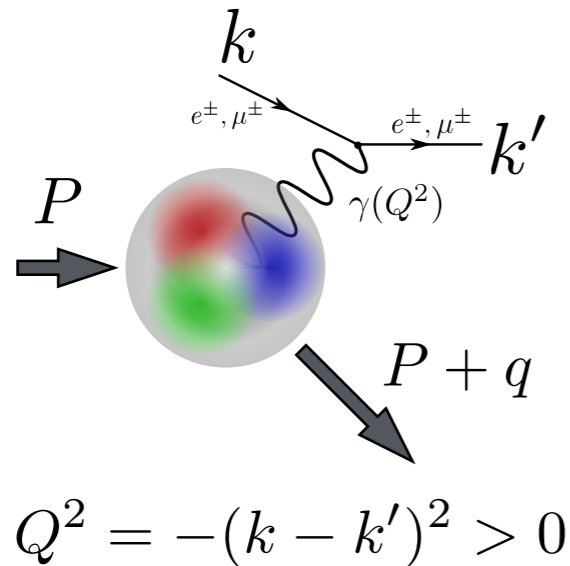
Light Cone 2026
CFNS, Stony Brook, Jun 22–26, 2026



Outline

- Large- Q^2 Nucleon vector form factors
 - *Motivation*
 - *Methodology & challenges on a lattice*
- Preview of results
 - *Form factors and ratios*
 - *Light-Flavor contributions*
- Examining systematic effects
 - *G_E/G_M ratio*
 - *Disconnected diagrams*
- Summary & Outlook

Nucleon Elastic E&M Form Factors



Elastic e^-p amplitude

$$\langle P + q | \bar{q} \gamma^\mu q | P \rangle = \bar{U}_{P+q} \left[\overset{\text{(Dirac)}}{F_1(Q^2)} \gamma^\mu + \overset{\text{(Pauli)}}{F_2(Q^2)} \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] U_P$$

Sachs Electric $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$

Magnetic $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ -dep. at fixed $\tau(Q^2)$
("Rosenbluth separation")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1 \text{ GeV}^2$

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1 + \tau} \left[G_E^2 + \frac{\tau}{\epsilon} G_M^2 \right]$$

$$\tau = \frac{Q^2}{4M_N^2} \quad \epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1}$$

Polarization transfer: polarized e^- beam

+ detect polarization of recoil nucleon

(alt.: transverse asymmetry on pol. target)

- G_E/G_M ratio (only small radiative corrections)

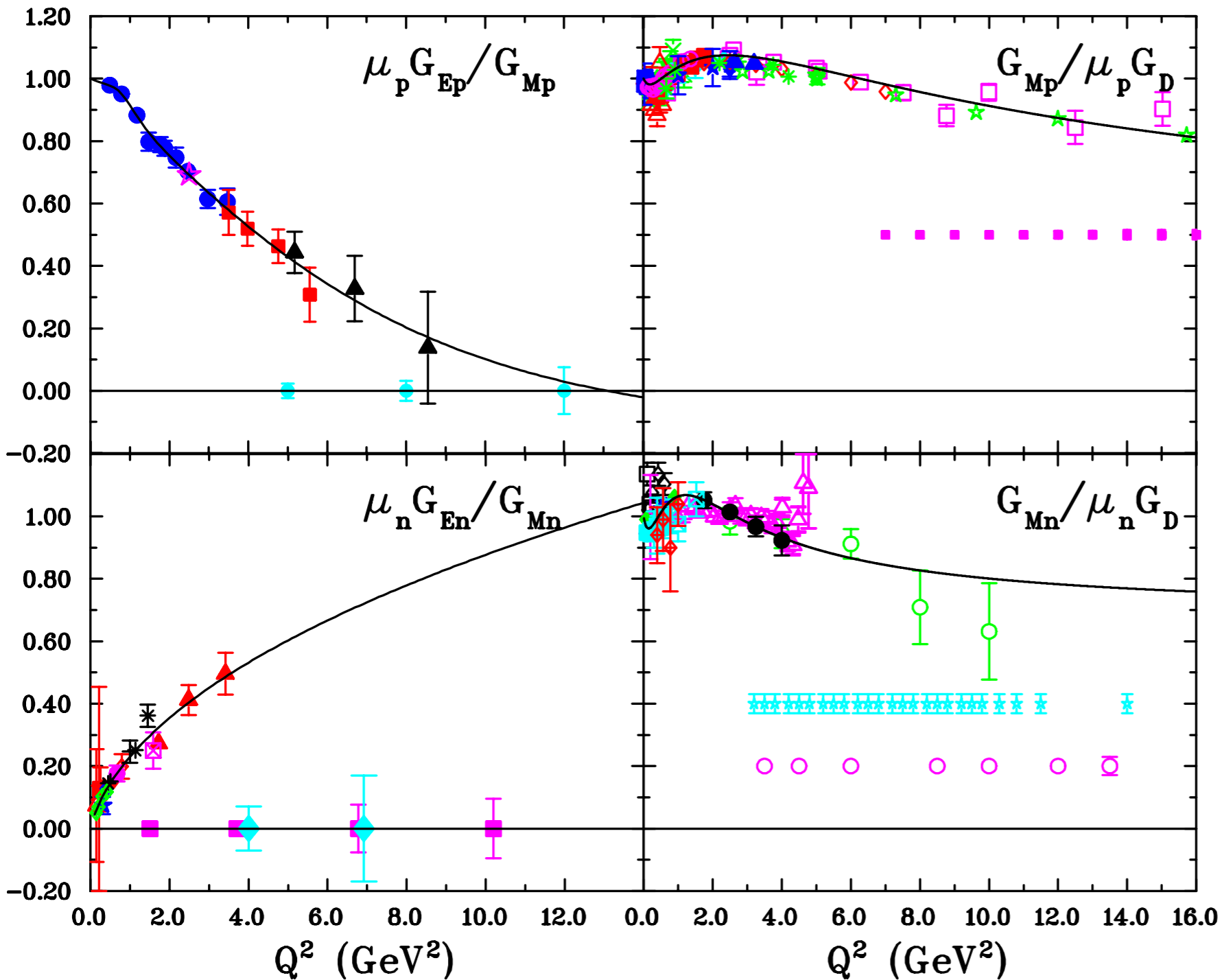
$$P_t/P_l \propto G_E/G_M$$

Recent Experiments @JLab : Projections



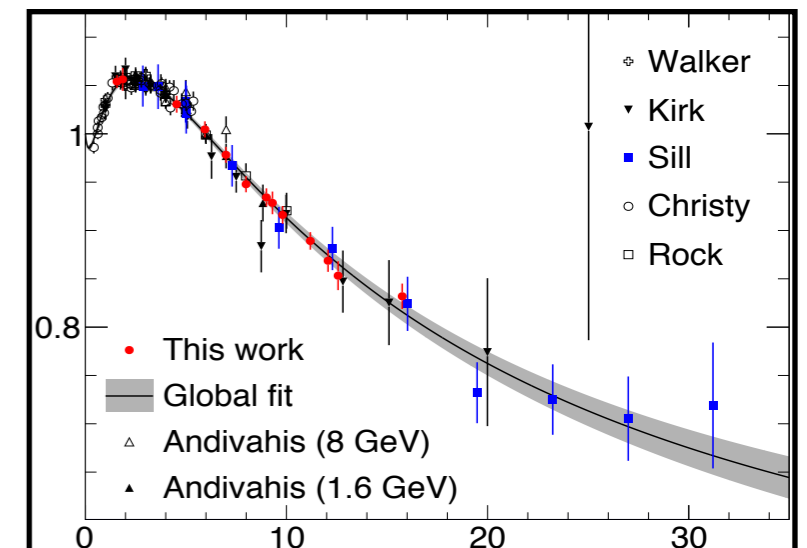
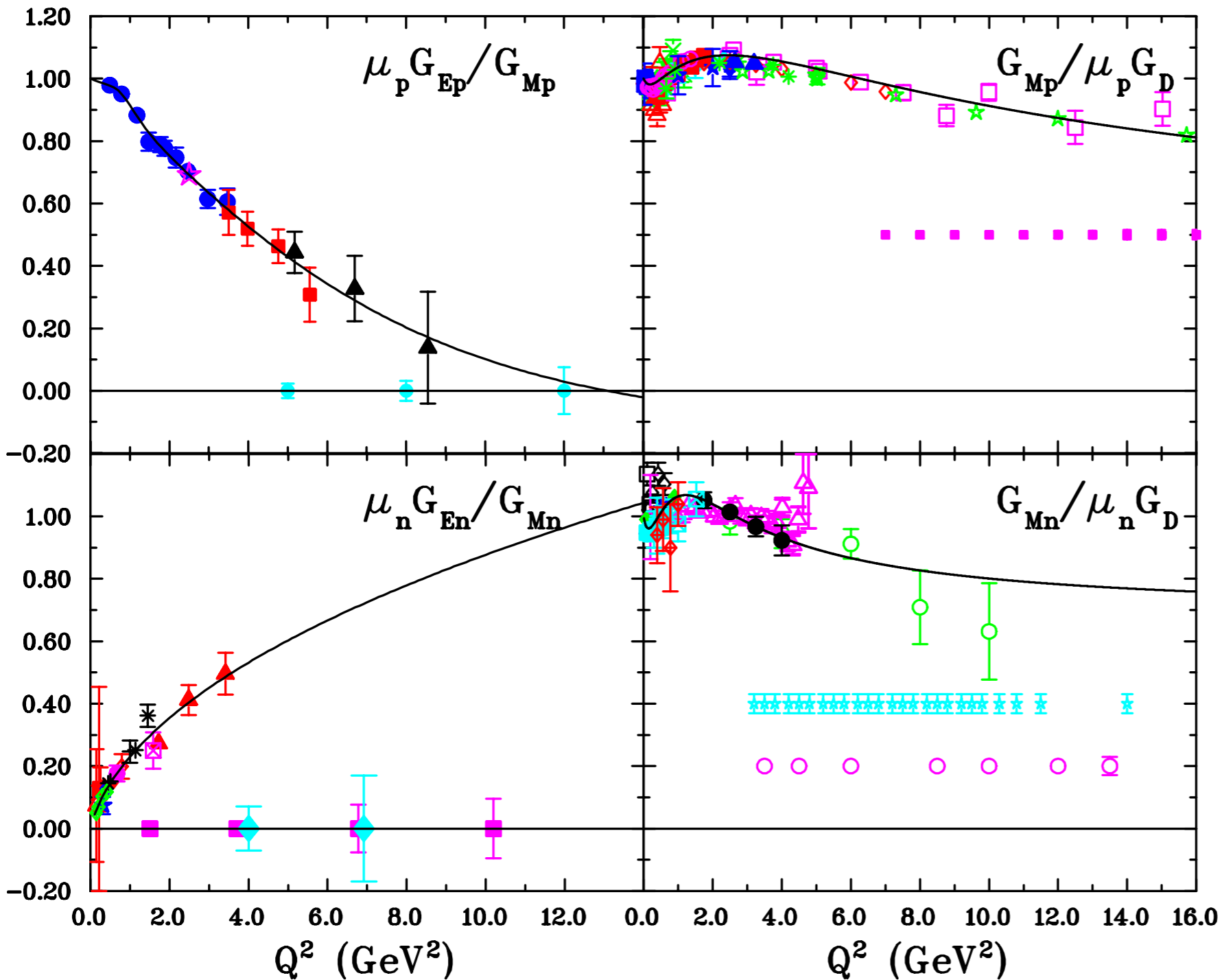
Experiments at JLab@12GeV

- Hall A (HRS, SBS):
 - G_{Mp} @ $Q^2 \approx 17.5 \text{ GeV}^2$
 - G_{Ep}/G_{Mp} @ $Q^2 \approx 15 \text{ GeV}^2$;
 - G_{Mn} @ $Q^2 \approx 18 \text{ GeV}^2$
 - G_{En}/G_{Mn} @ $Q^2 \approx 10.2 \text{ GeV}^2$;
- Hall B (CLAS12):
 - G_{Mn} @ $Q^2 \approx 14 \text{ GeV}^2$
- Hall C :
 - G_{En}/G_{Mn} @ $Q^2 \approx 6.9 \text{ GeV}^2$



Projected new precision on proton & neutron form factors
 [V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Recent Experiments: Projections + G_{Mp} Result

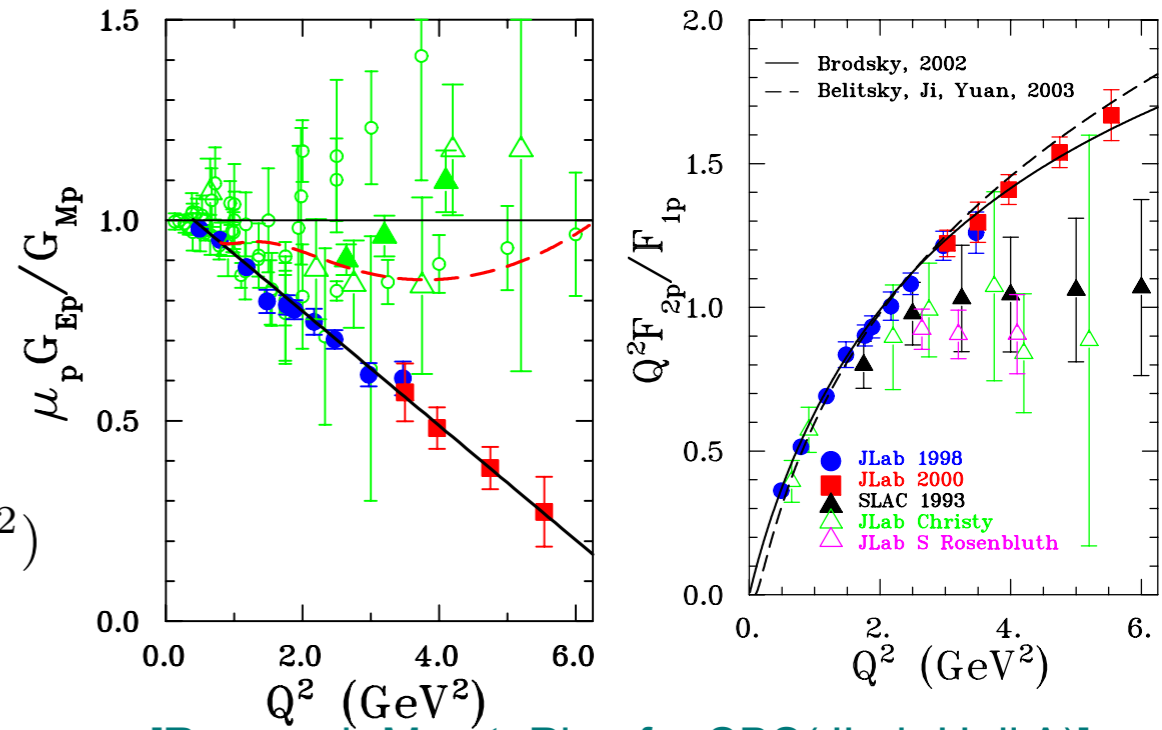


New G_{Mp} results (JLab Hall A)
[Christy et al, PRL'22]

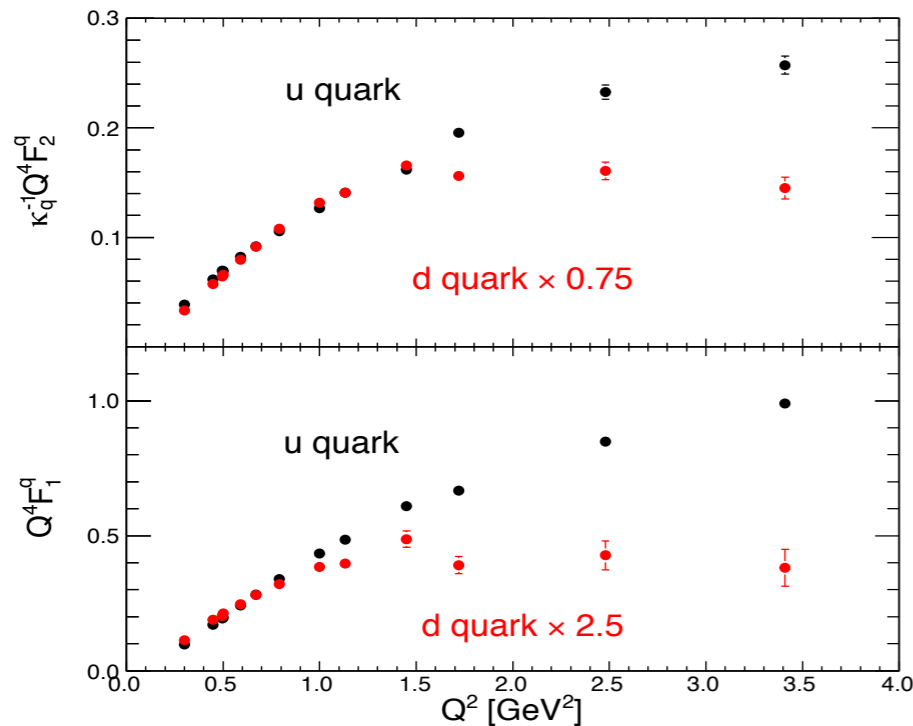
Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Why Large- Q^2 Nucleon Form Factors?

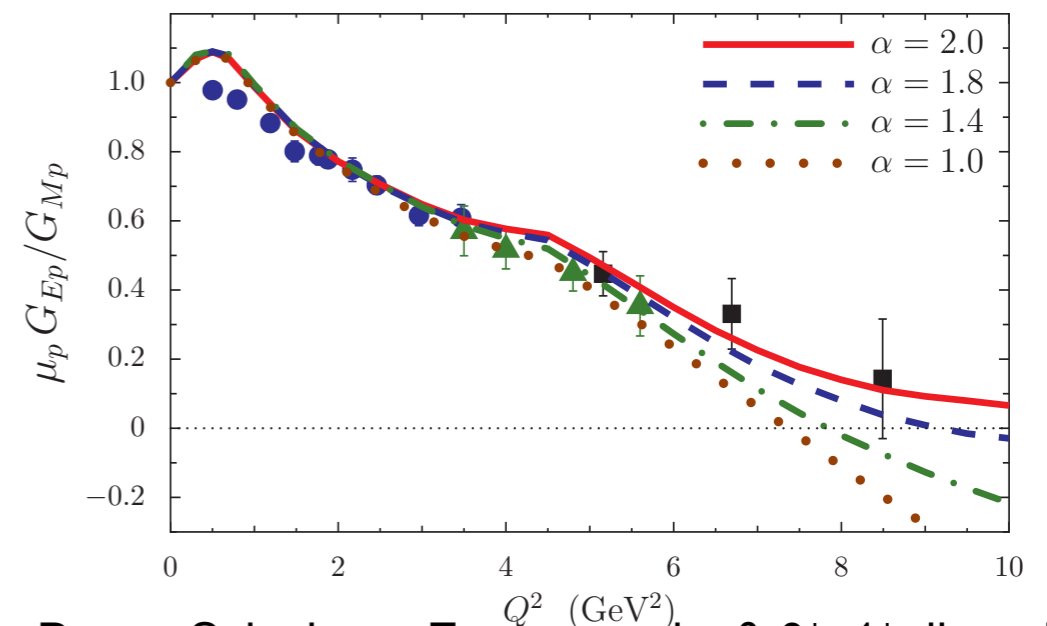
- *Rosenbluth / Pol.transfer discrepancy in G_E/G_M ?*
Input from lattice QCD may help
- *Role of diquark correlations in elastic scattering?*
Neutron & proton G_E/G_M at/above $Q^2 = 8 \text{ GeV}^2$
- *Scale of transition to perturbative QCD?*
(F_2/F_1) scaling at large Q^2 : $Q^2 F_{2p}/F_{1p} \stackrel{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
Proton and neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



Dyson-Schwinger Eqns : quarks & 0^+ , 1^+ diquarks
($\alpha \approx$ rate of transition const. quarks \rightarrow pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Nucleon Structure from Lattice QCD (in a Nutshell)

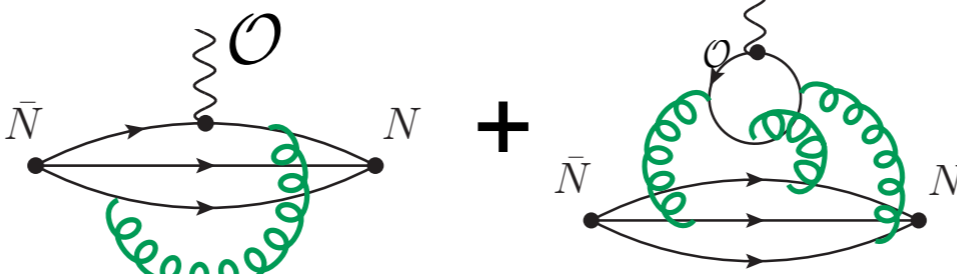
- Compute nucleon correlation functions

Hadron Spectrum:

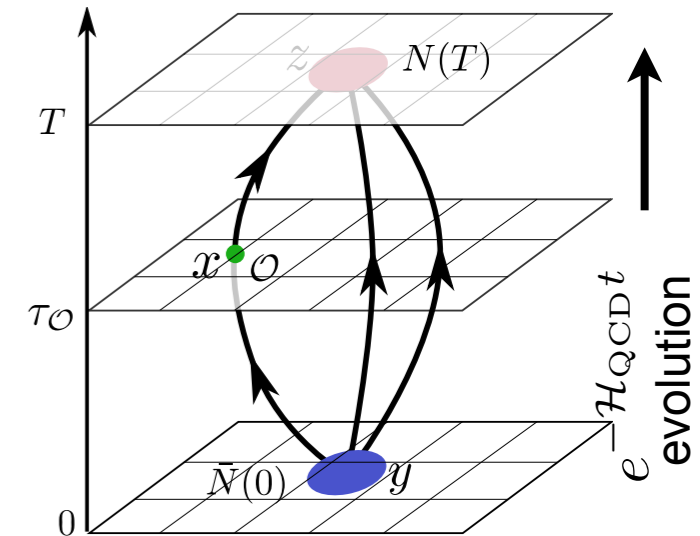
$$\langle N(\vec{p}, T) \bar{N}(0) \rangle = \sum_{\vec{y}} \langle N(\vec{y}, T) \bar{N}(0) \rangle$$

$$\sim |Z_0|^2 e^{-E_0 T} [1 + e^{-\Delta E_{10} T} + \dots]$$

Hadron Matrix Elements:

$$\langle N(T) \mathcal{O}(\tau) \bar{N}(0) \rangle =$$


quark-connected quark-disconnected



$$\text{Quark lines} = (\not{D} + m)^{-1} \cdot \psi$$

- Extract ground state matrix elements

$$\langle N(T) \mathcal{O}(\tau) N(0) \rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} \langle n | \mathcal{O} | m \rangle e^{-E_m \tau} Z_n^*$$

$$\xrightarrow{T \rightarrow \infty} Z_0 Z_0^* e^{-E_0 \tau - E_0'(T-\tau)} \left[\langle 0' | \mathcal{O} | 0 \rangle + \underbrace{O(e^{-\Delta E \{T, \tau, (T-\tau)\}})}_{\text{excited states}} \right]$$

Ground state form factors

$$[\bar{u}' \gamma^\mu u] F_1 + [\bar{u}' \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} u] F_2$$

Fit and discard

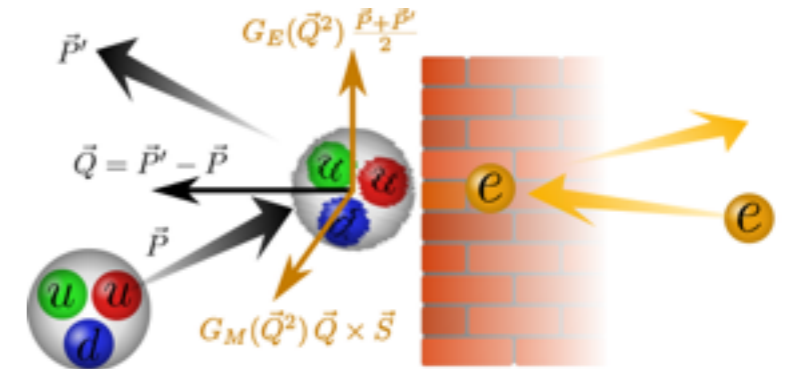
Systematic uncertainties

- discretization effects
- unphysical-heavy pion mass
- finite volume
- excited state contributions
- quark-disconnected diagrams

Challenges at Large Q^2

- Excited-states gaps shrink

- $E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$
- $N(\sim 1500): p_N \rightarrow 1.5 \text{ GeV} \Rightarrow \Delta E = 500 \rightarrow 300 \text{ MeV}$



- Stochastic noise grows faster with T [Lepage'89]:

Signal $\langle N(T)\bar{N}(0) \rangle$

Noise $\langle |N(T)\bar{N}(0)|^2 \rangle - |\langle N(T)\bar{N}(0) \rangle|^2$

Signal/Noise

$$\sim e^{-E_N T}$$

$$\sim e^{-3m_\pi T}$$

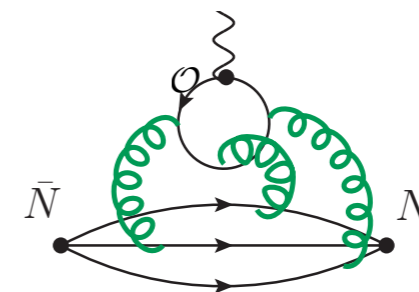
$$\sim e^{-(E_N - \frac{3}{2}m_\pi)T}$$

**SNR decay $\sim O(10^{-4})$
at 1 fm/c
(phys. quarks, $Q^2 \approx 12 \text{ GeV}^2$)**

- Discretization effects:
O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V a \underbrace{\partial_\nu [\bar{q}i\sigma_{\mu\nu}q]}_{\propto Q}$$

- Quark-disconnected contributions:
negligible ($\approx 1\%$) at $Q^2 \leq 1 \text{ GeV}^2$, unknown at large Q^2



- No reliable EFT/ChPT for m_{π^-} , lattice-volume extrapolation at large- $p_{Nucleon}$

*Many of the same challenges as for parton physics on a lattice
(LaMET, Quasi-/Pseudo-PDFs/DAs/TMDs)*

Present QCD Calculation Parameters

QCD on lattice with $N_F = 2+1$ $O(a^2)$ dynamic quarks

Many thanks to [JLab / W&M / LANL / MIT]

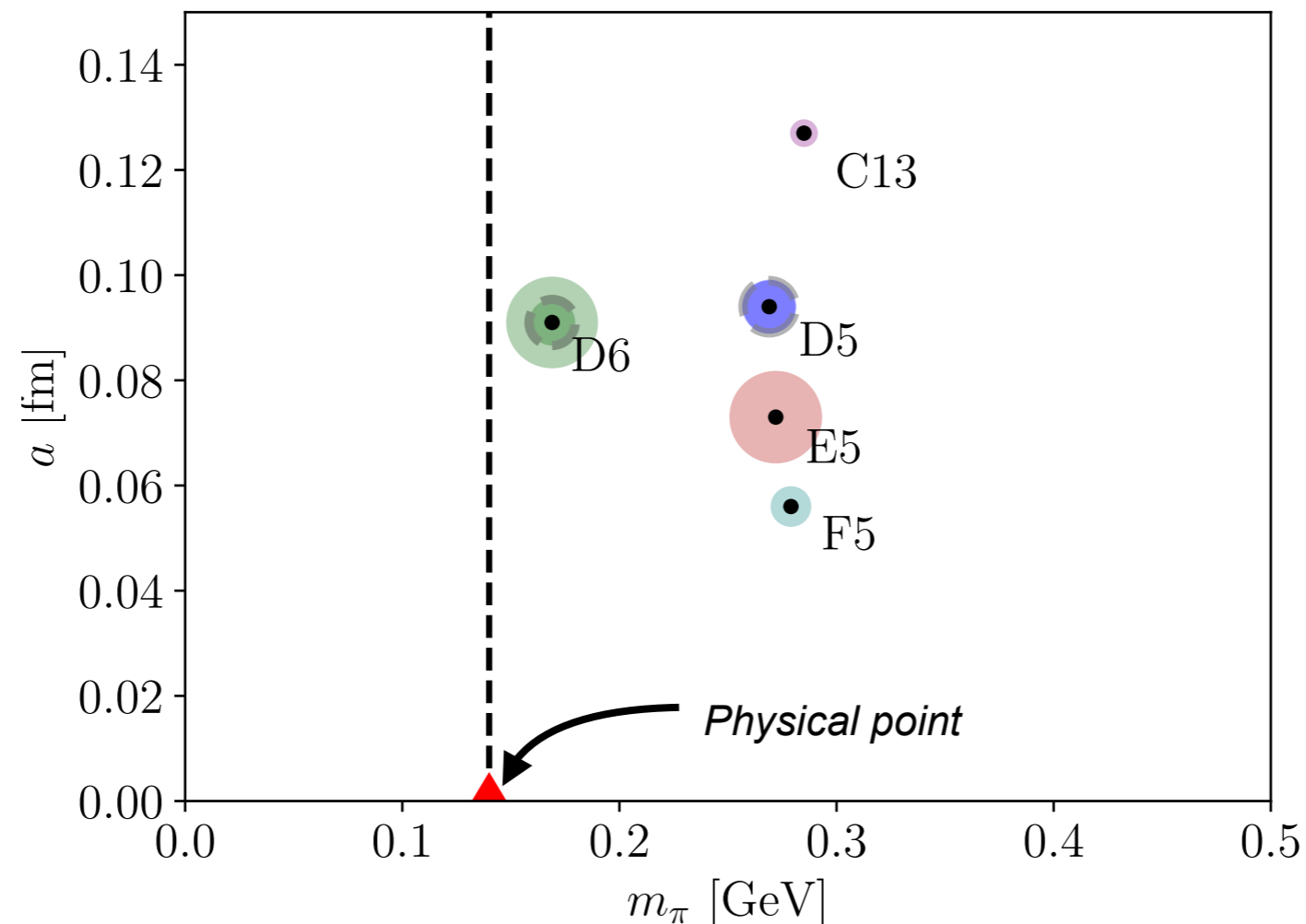
- Lattice spacing $a \approx 0.056 \div 0.091$ fm
- $SU(2)_f$ -symmetric + strange quarks
- Pion mass $m_\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7 (m_\pi)^{-1}$
- Euclidean time $t_{sep} = 0.5 \div 1.1$ fm

2022/25:

- Large Statistics ($\gtrsim 250k$) on D6 ($m_\pi = 170$ MeV), E5 ($m_\pi = 280$ MeV, $a=0.073$ fm)
- Disconnected contractions on D5, D6

2026:

- initial Statistics $\gtrsim 47k$ on F5, ($m_\pi = 280$ MeV, $a=0.056$ fm)



Present QCD Calculation Parameters

QCD on lattice with $N_F = 2+1$ $O(a^2)$ dynamic quarks

Many thanks to [JLab / W&M / LANL / MIT]

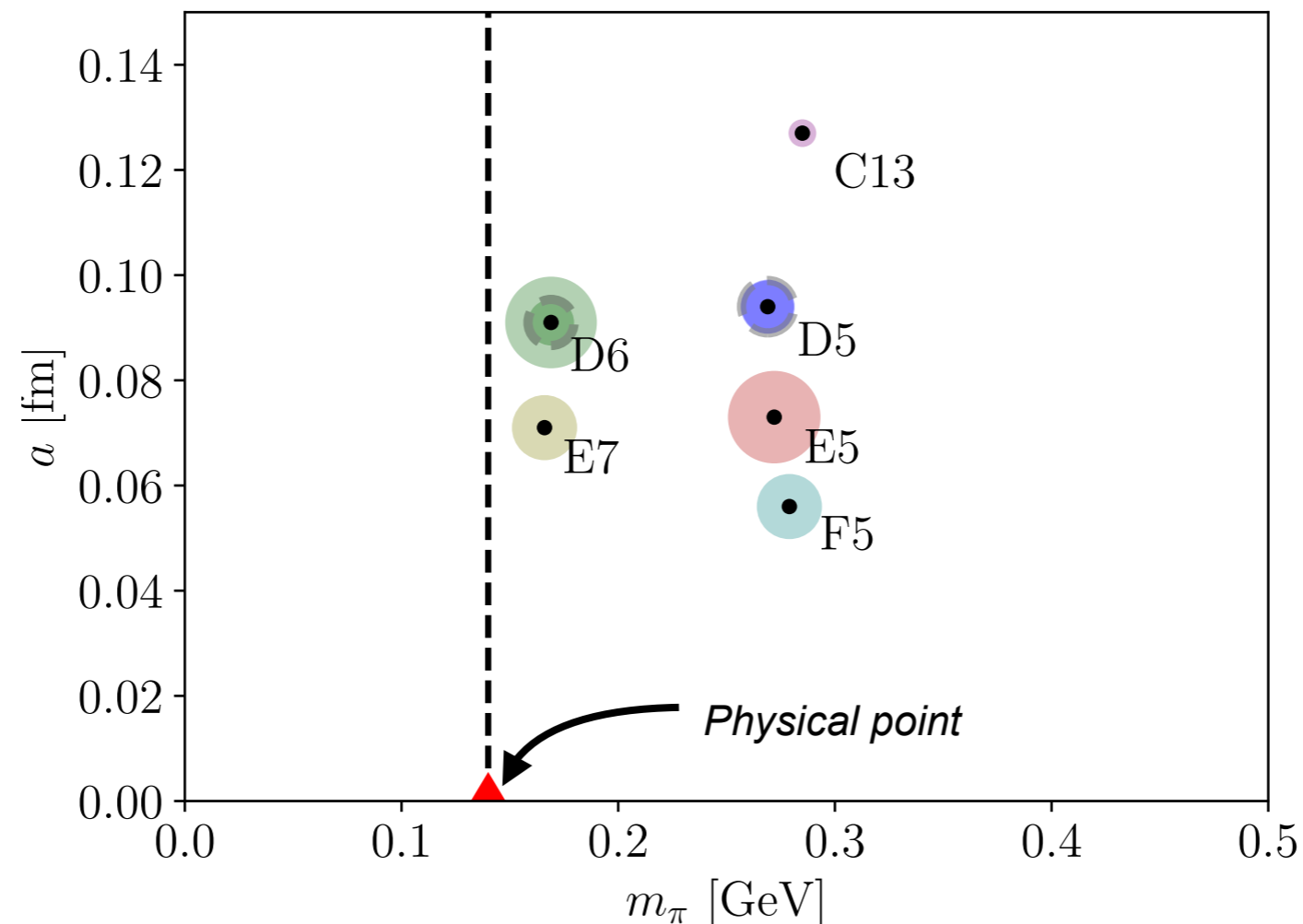
- Lattice spacing $a \approx 0.056 \div 0.091$ fm
- $SU(2)_f$ -symmetric + strange quarks
- Pion mass $m_\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7 (m_\pi)^{-1}$
- Euclidean time $t_{sep} = 0.5 \div 1.1$ fm

2022/25:

- Large Statistics ($\gtrsim 250k$) on D6 ($m_\pi = 170$ MeV), E5 ($m_\pi = 280$ MeV, $a=0.073$ fm)
- Disconnected contractions on D5, D6

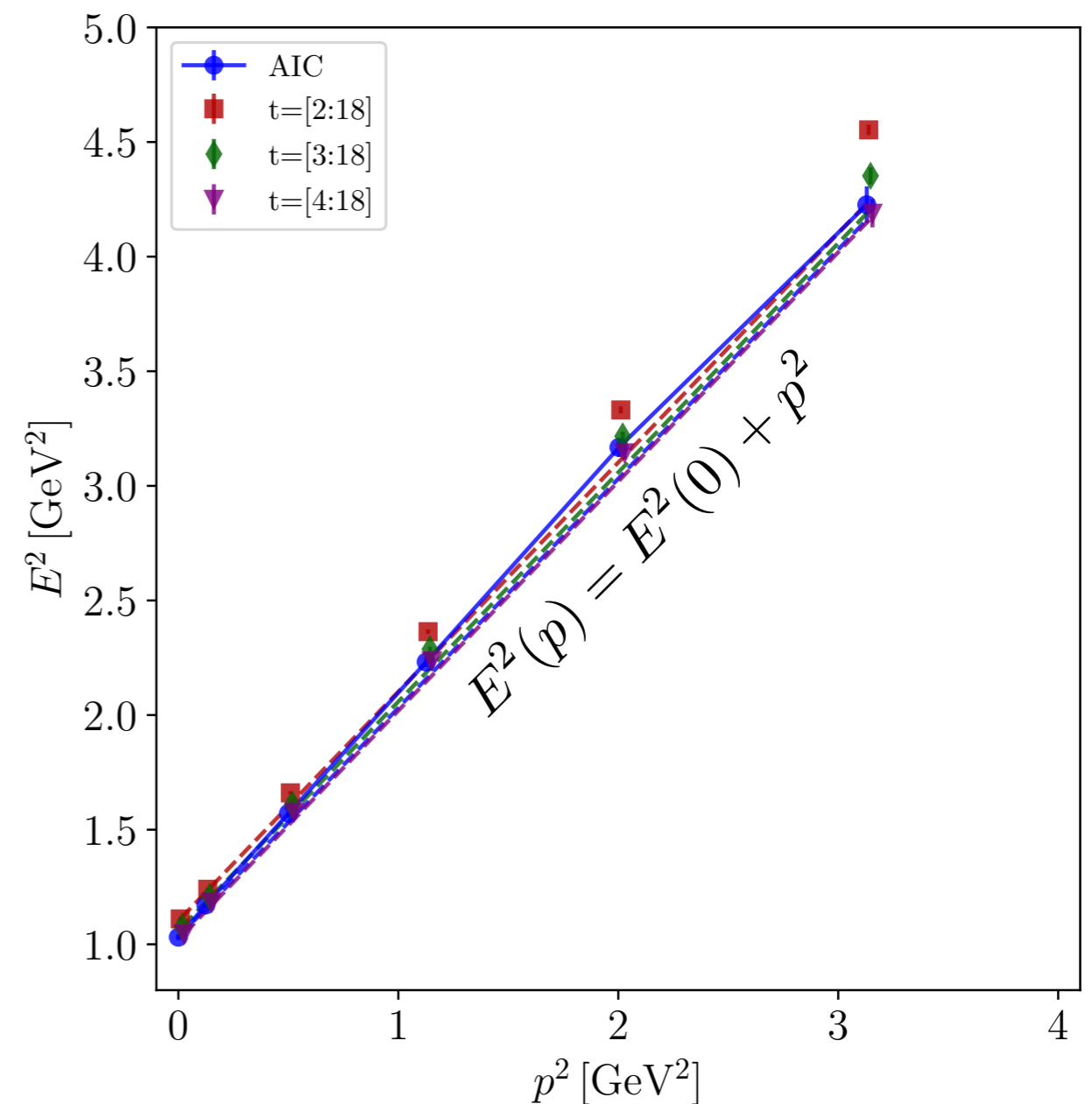
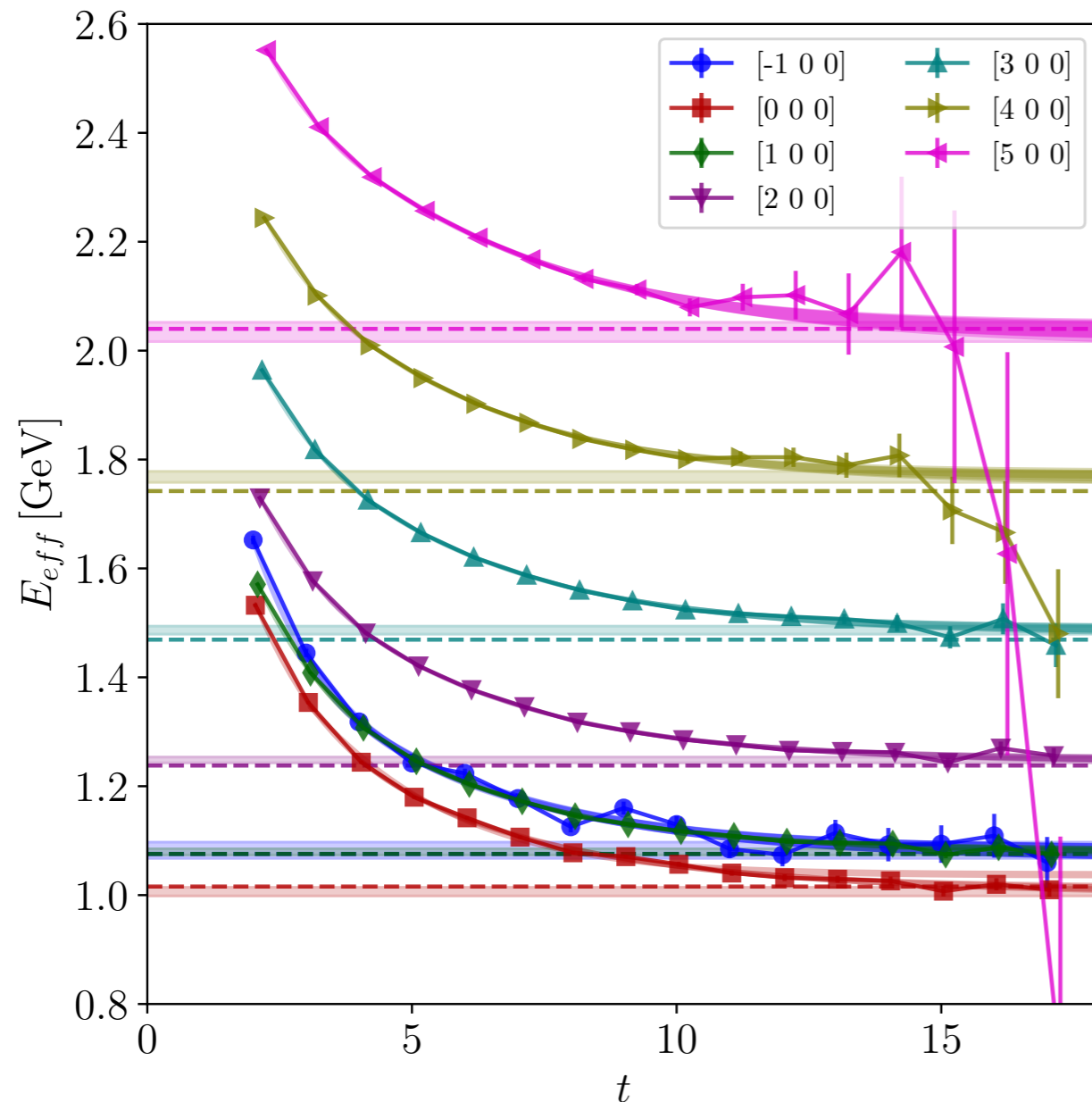
2026:

- initial Statistics $\gtrsim 47k$ on F5 ($m_\pi = 280$ MeV, $a=0.056$ fm)



Lattice Nucleon Energy & Dispersion Relation (E5)

● E5 : $m\pi = 272 \text{ MeV}$, spacing $a = 0.073 \text{ fm}$, 266k MC samples



● 2-state fits compared to Effective energy

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

● Lattice energies vs. continuum dispersion relation (dashed)

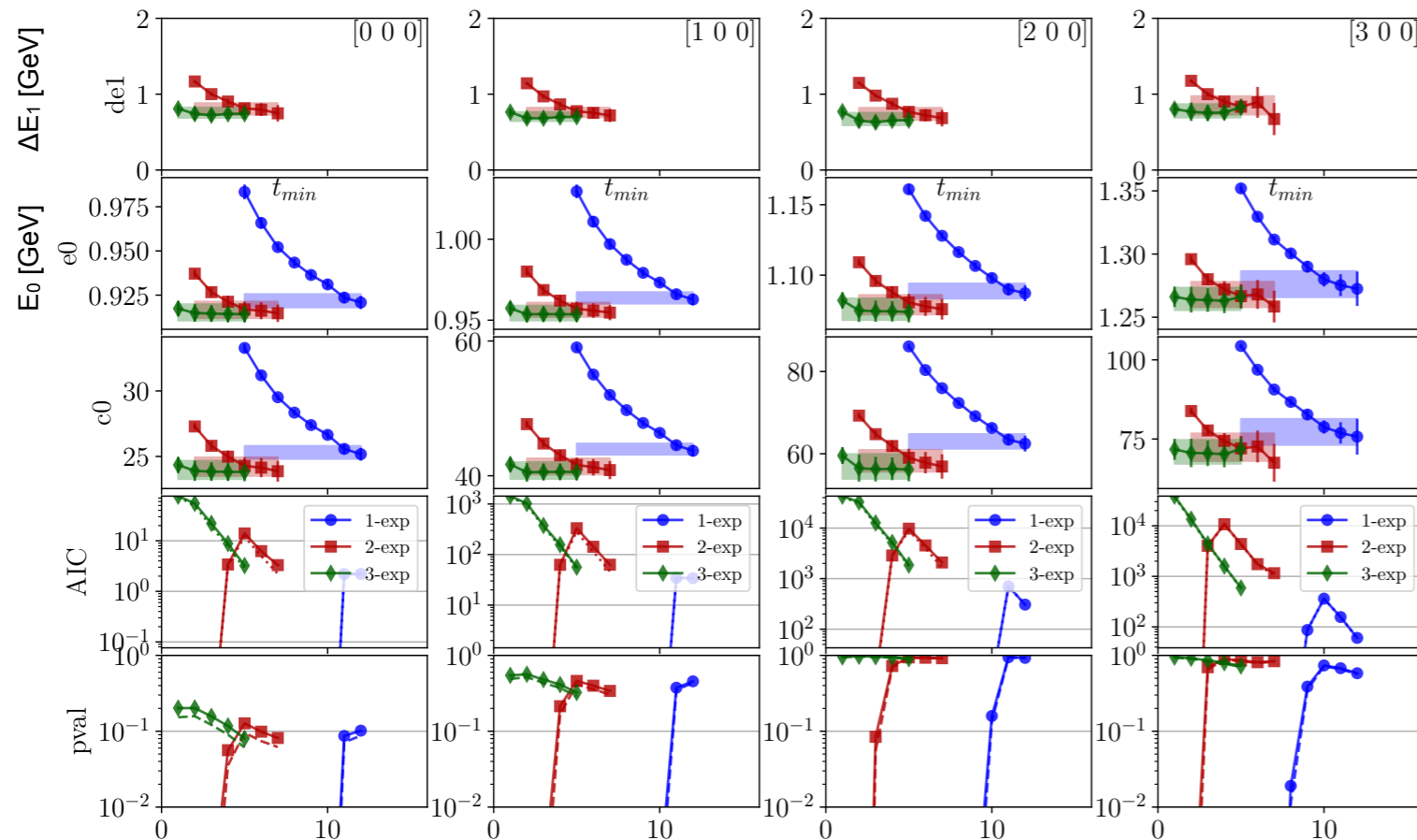
Analysis Minutiae

- Filter outliers: large deviation from median (~2% at $a=0.09$ fm, ~0.15% at $a=0.07$ fm)
- suppress noise in covariance estimates [A.Agadjanov et al, Phys.Rev.Lett. 131 (2023) 261902]

$$|y - y_{median}| \geq 25 \cdot \underbrace{\Delta y}_{\text{(median abs. deviation)}}$$

energies and ground-state amplitudes from $\langle N \bar{N} \rangle$:

$$\langle N(t) \bar{N}(0) \rangle \sim c_0 e^{-E_0 t} + c_1 e^{-(E_0 + \Delta E_1) t} + \dots$$



- Varying range fits Averaged with AIC

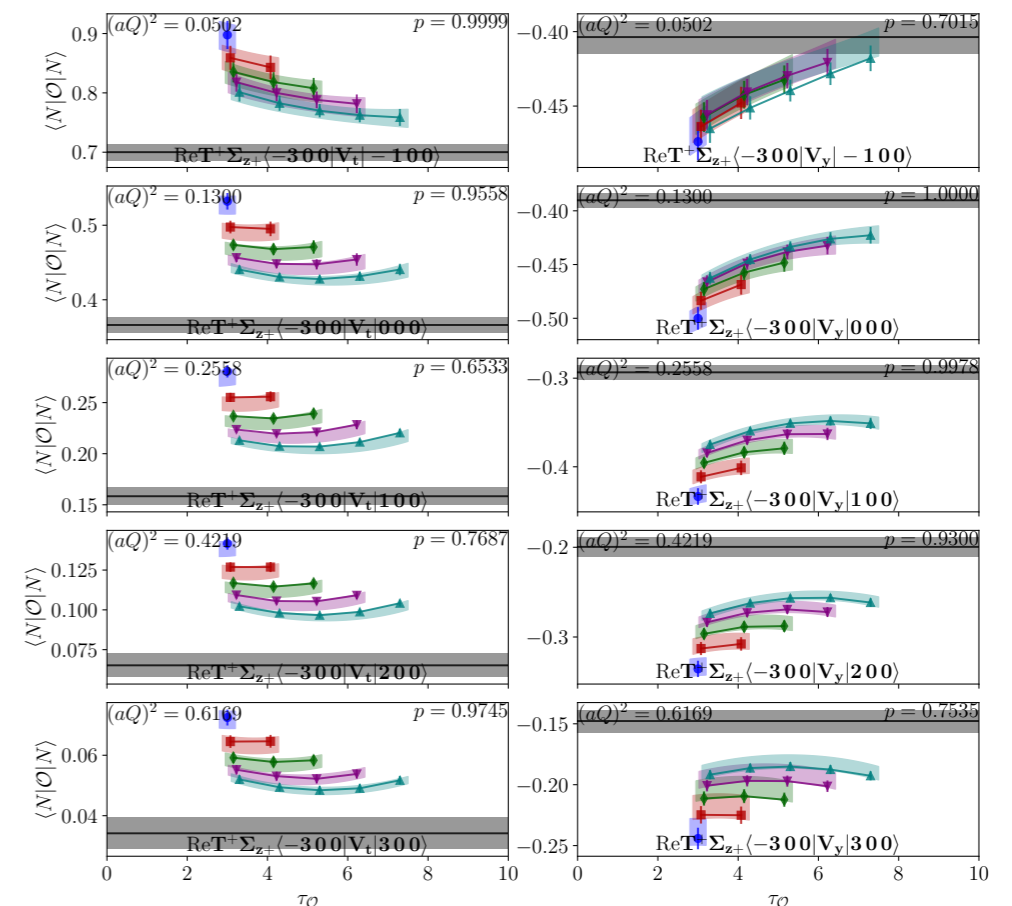
[W.Jay, E.Neil PRD 103:114502(2021); E.Neil, J.W.Sitison, 109:014510(2024)].

$$w_i \propto \exp(-\chi_i^2 + N_i^{\text{dof}})$$

current matrix elements from $\langle N J \bar{N} \rangle$:

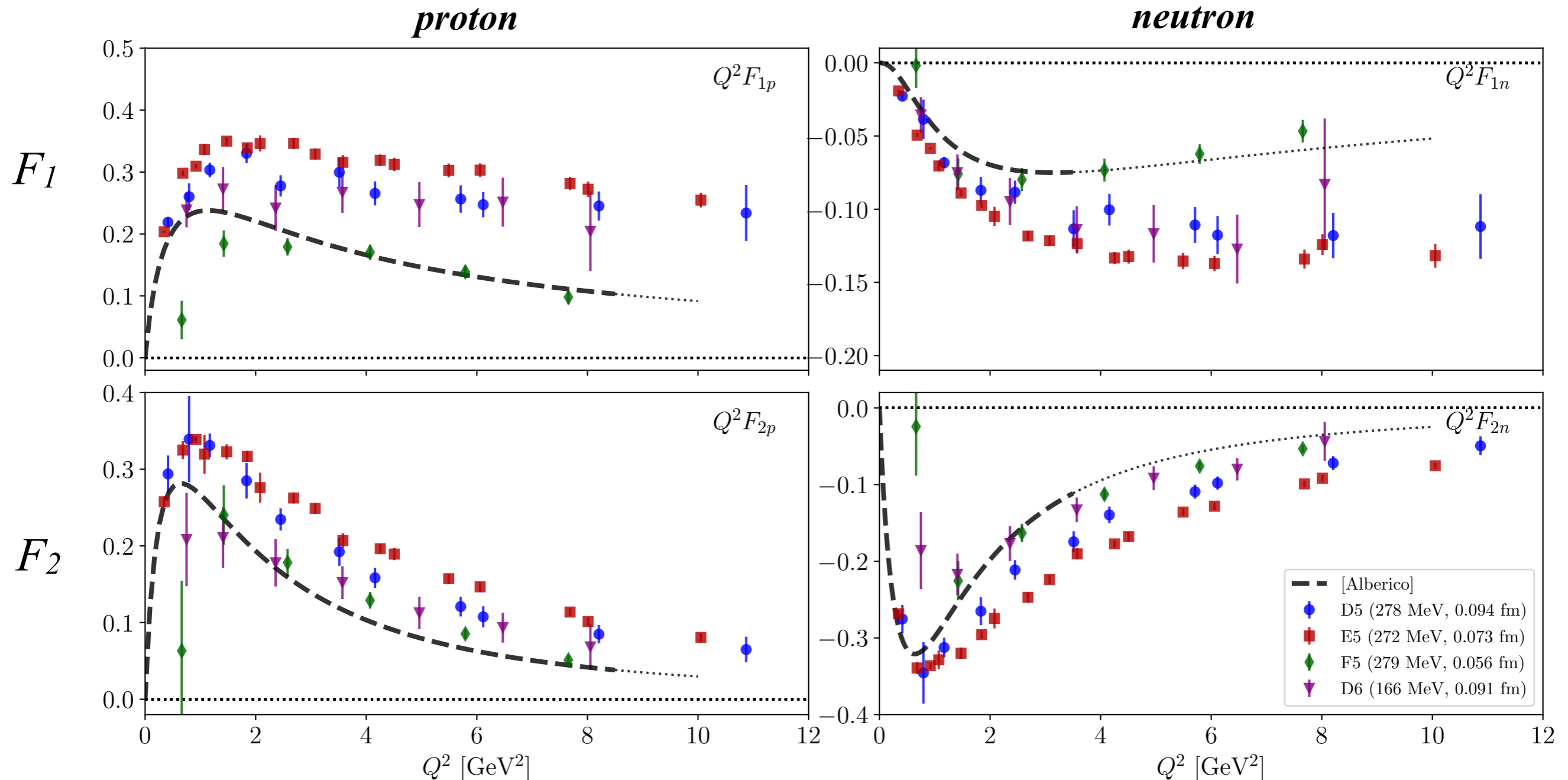
$$\begin{cases} \langle J_t \rangle & \propto G_E \\ \langle J_{\vec{q} \times \vec{S}} \rangle & \propto G_M \end{cases}$$

$$\langle N' | J(\tau) | N \rangle \sim \mathcal{A}_{00} \left[1 + O(e^{-\Delta E_1^{(')} t} + \dots)_{\text{exc}} \right]$$



Nucleon Form Factors: Dirac and Pauli

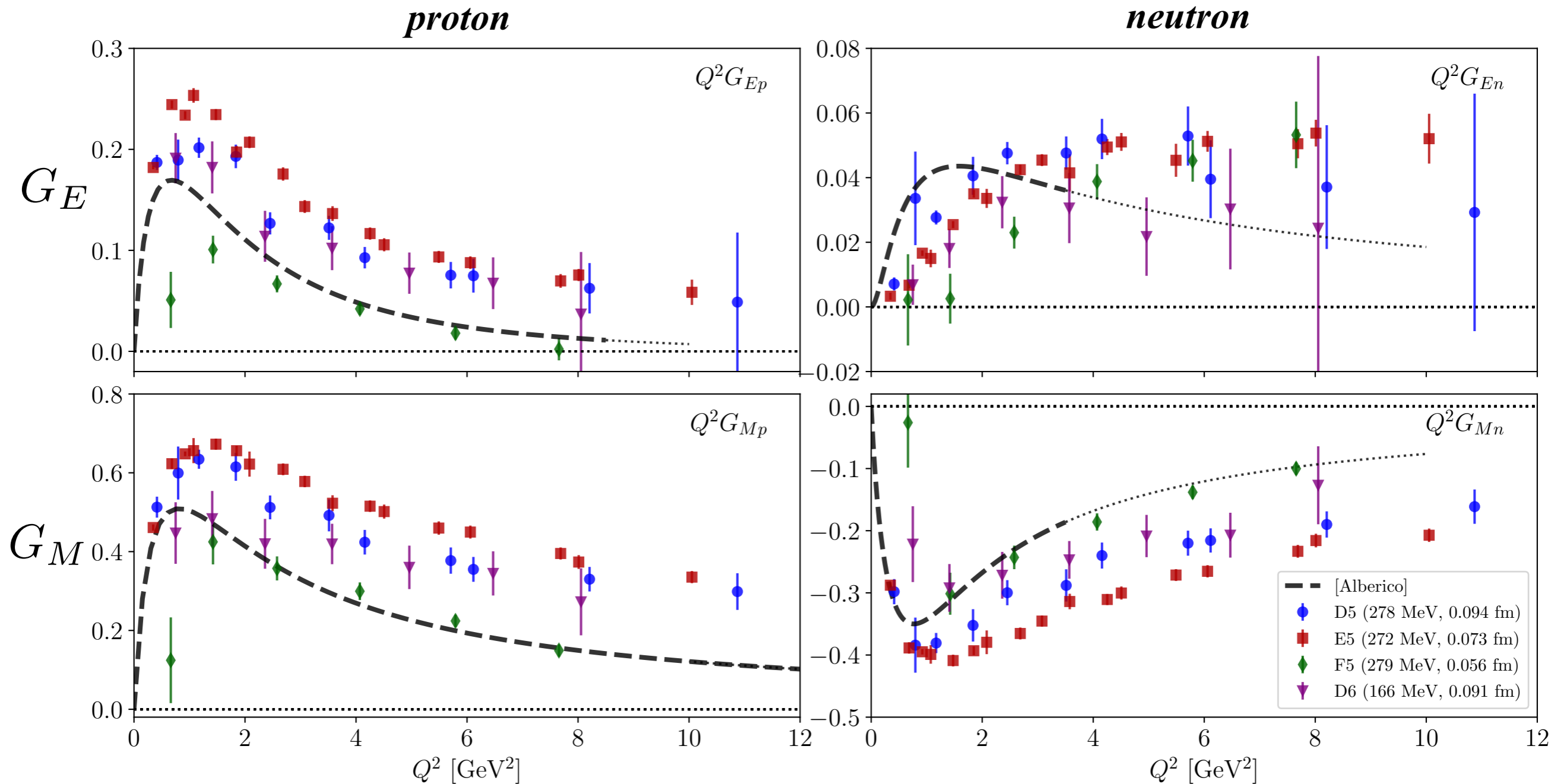
- Comparison of 4 ensembles (D5 : 86k, E5 : 266k, F5 : 47k, D6 : 261k)
- Lattice error bars: stat. + sys[high exc. states]
- (dashed line) Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



• No disconnected diagrams

Nucleon Form Factors: Sachs Electric and Magn.

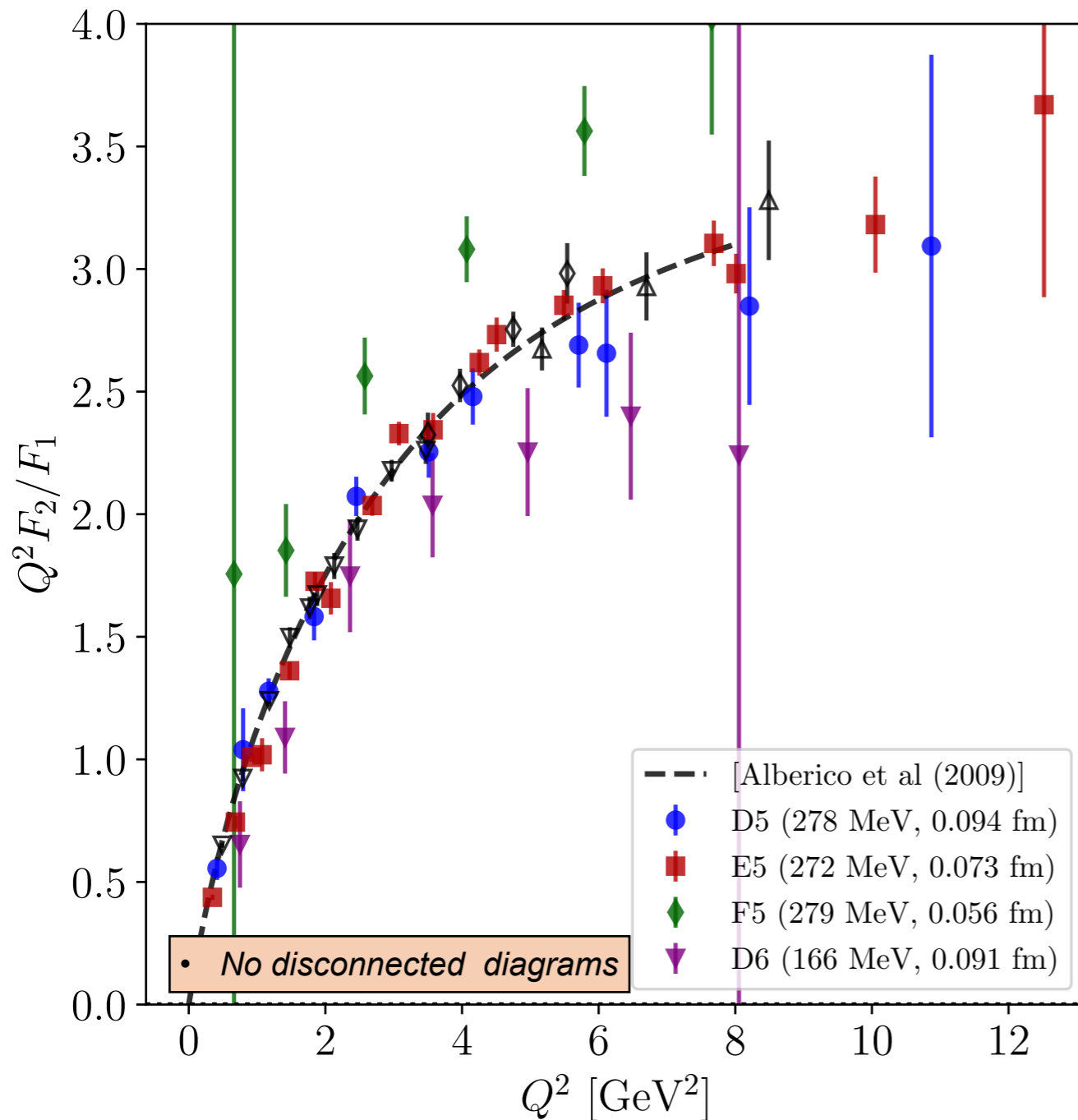
- Comparison of 4 ensembles (D5 : 86k, E5 : 266k, F5 : 47k, D6 : 261k)
- Lattice error bars: stat. + sys[high exc. states]
- (dashed line) Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



• No disconnected diagrams

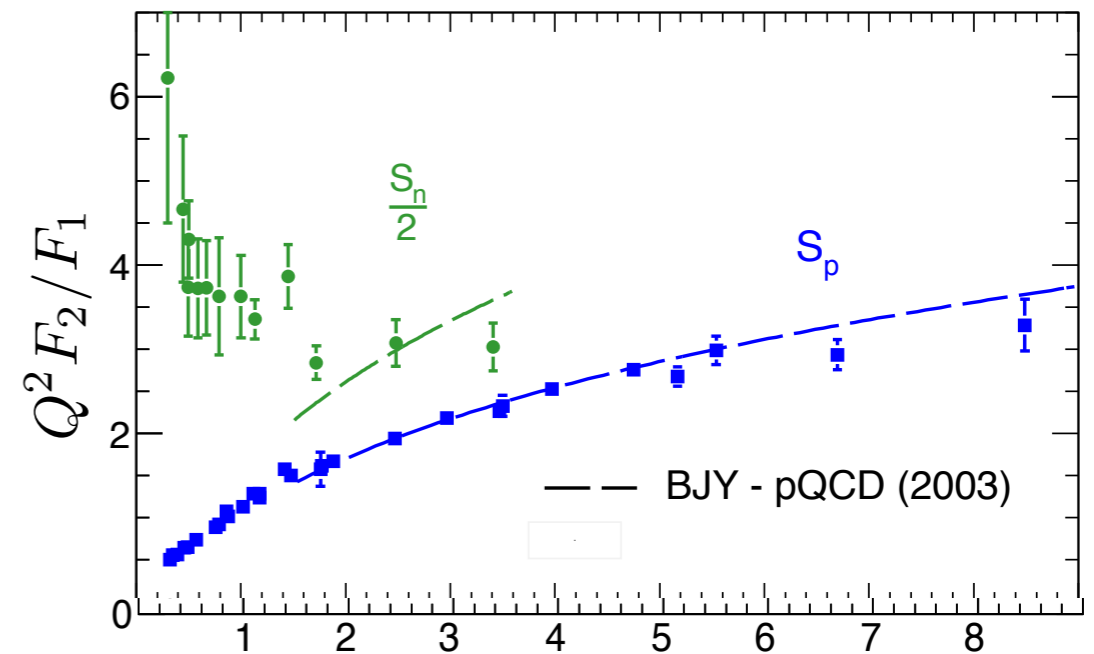
Proton F_2/F_1 Ratio

- Comparison of 4 ensembles (D5 : 86k, E5 : 266k, F5 : 47k, D6 : 261k)
- (dashed line) Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- (black points) Experimental data for proton ($Q^2 \lesssim 8.5 \text{ GeV}^2$)



- Prediction from pQCD + quark OAM [Balitsky, Ji, Yuan (2003)]

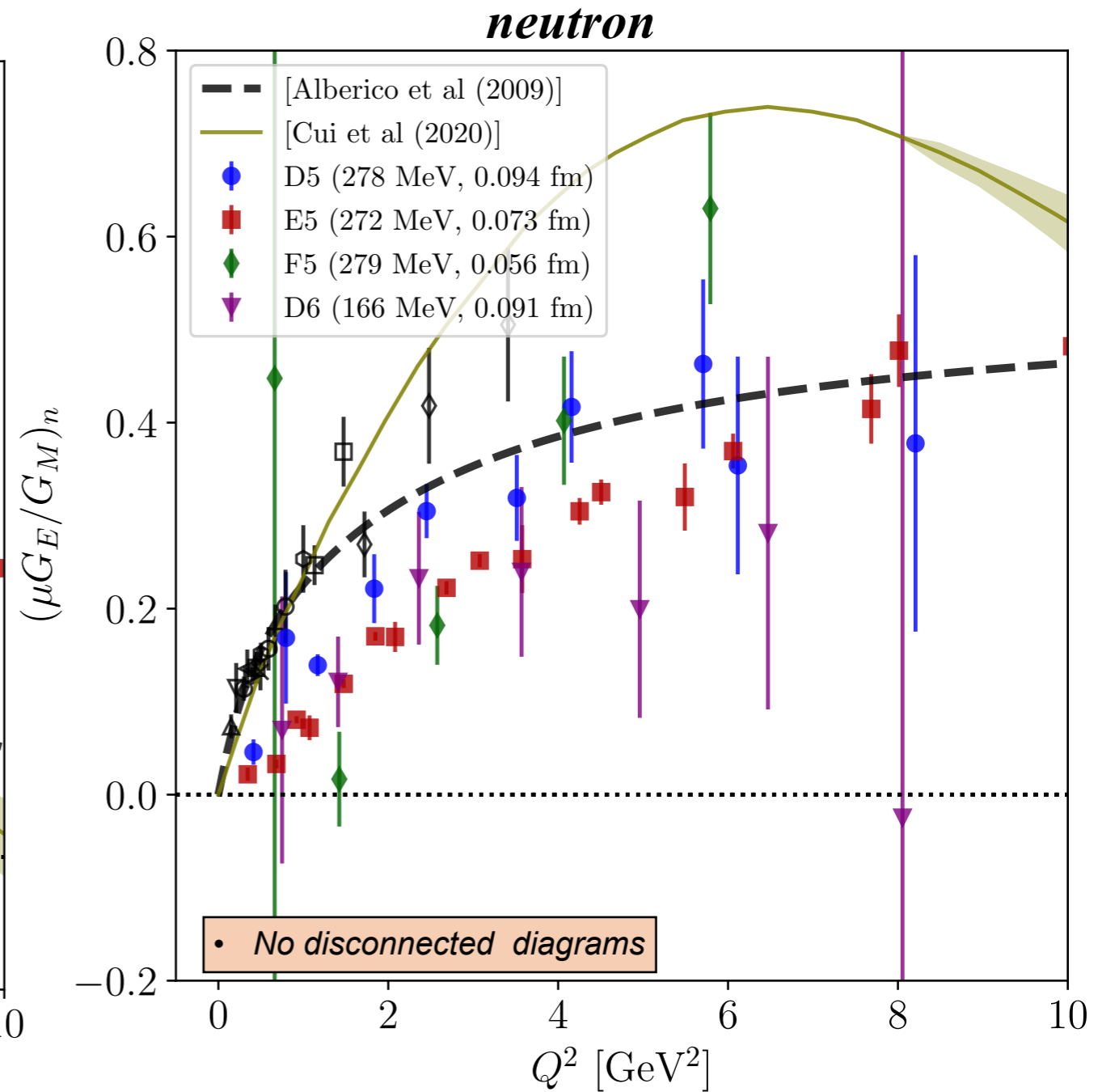
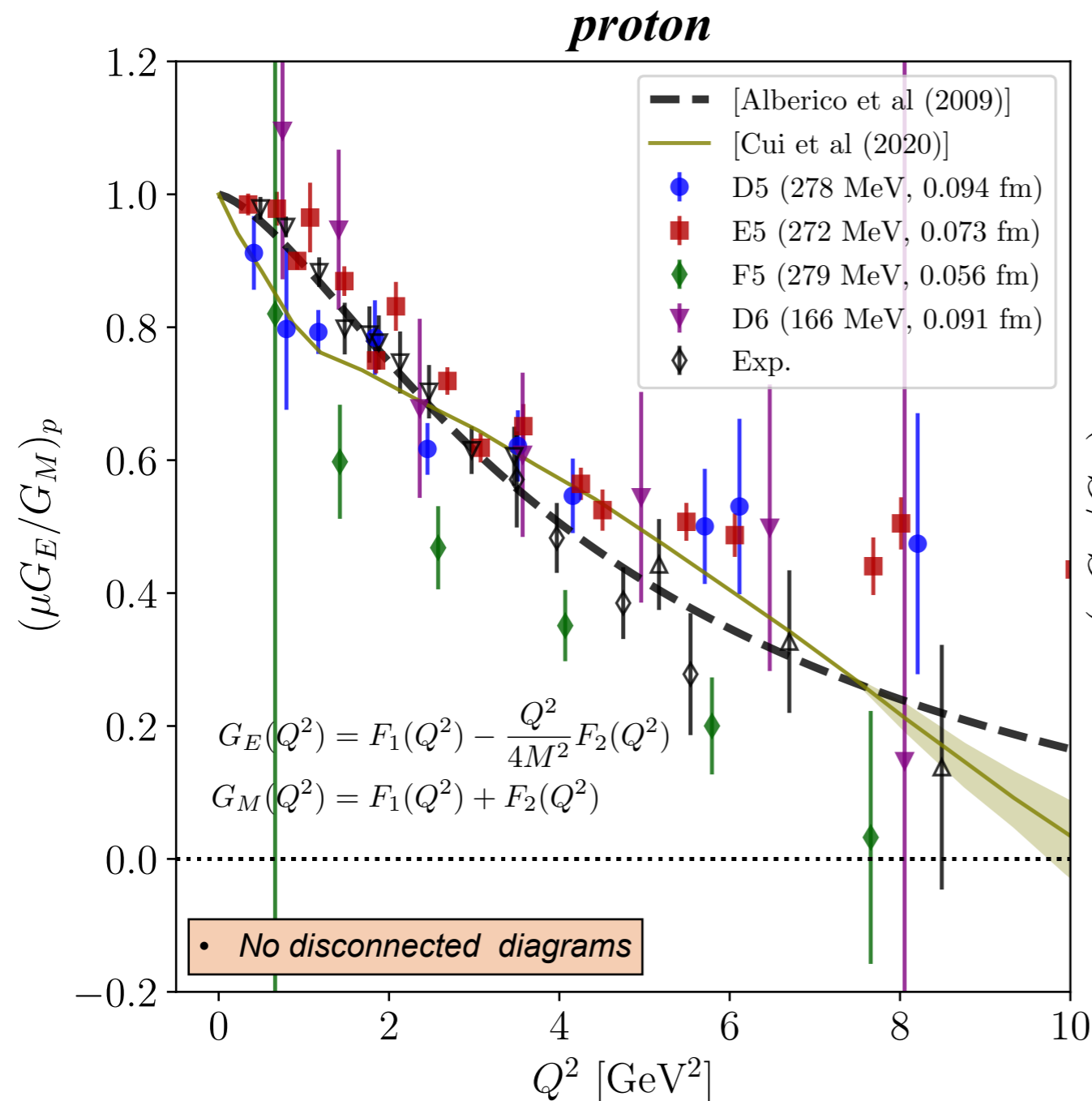
$$Q^2 F_{2p}/F_{1p} \stackrel{?}{\propto} \log^2(Q^2/\Lambda^2)$$



[G.D.Cates, et al, PRL106:252003 (2011)]

Proton & Neutron G_E/G_M Ratio

- Comparison of 4 ensembles (D5 : 86k, E5 : 266k, F5 : 47k, D6 : 261k)
- (dashed line) Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- (black points) Experimental data for proton ($Q^2 \lesssim 8.5 \text{ GeV}^2$) and neutron ($Q^2 \lesssim 3.4 \text{ GeV}^2$)

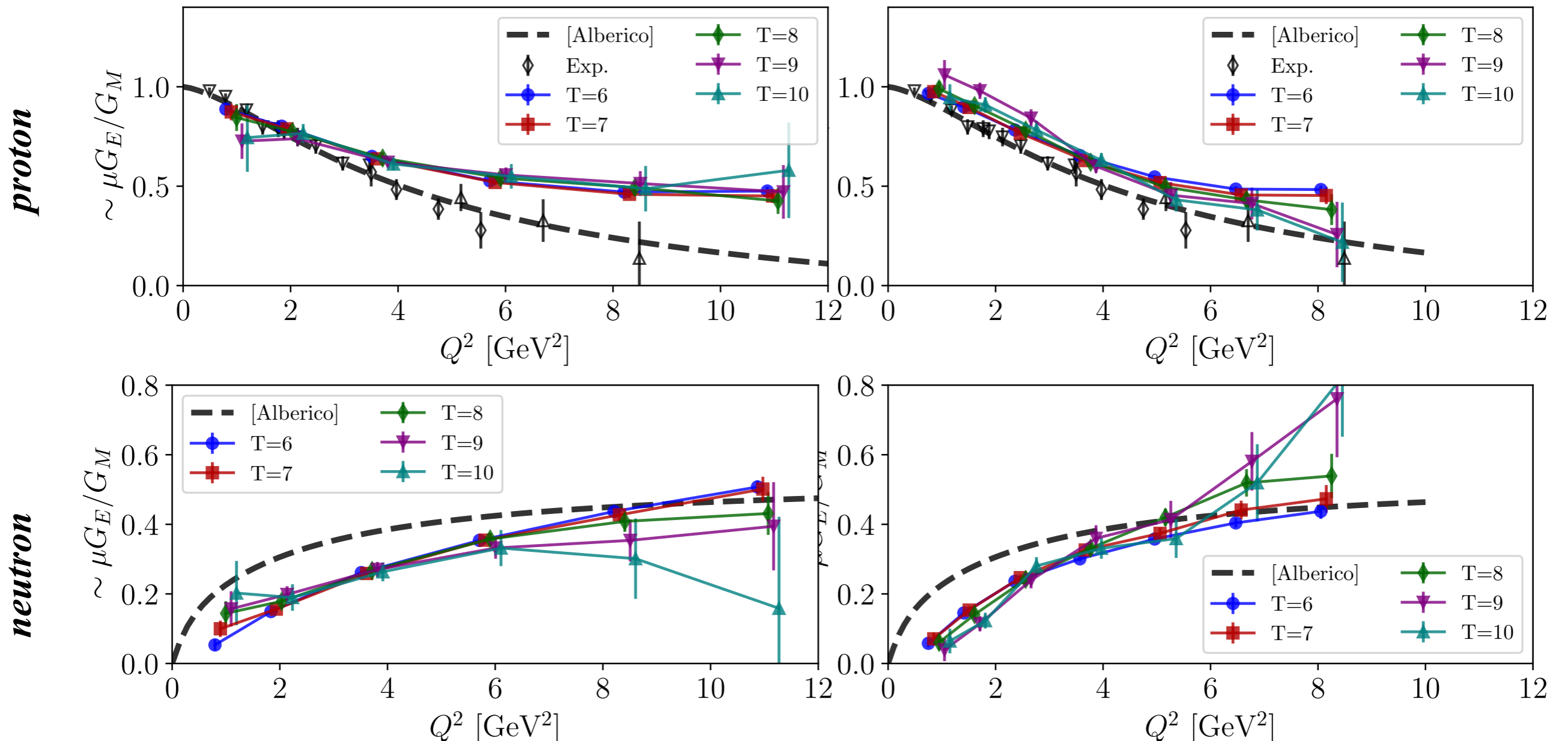


Proton&Neutron G_E/G_M from $\langle N\bar{J}\bar{N} \rangle$ Correlators

$$\left(\frac{\sinh \frac{\lambda' - \lambda}{2}}{\cosh \frac{\lambda' + \lambda}{2}} \right) \frac{\text{Re} \langle N_{\uparrow}(p'_x, T) J_t(T/2) \bar{N}_{\uparrow}(p_x, 0) \rangle}{\text{Re} \langle N_{\uparrow}(p'_x, T) J_y(T/2) \bar{N}_{\uparrow}(p_x, 0) \rangle} \stackrel{T \rightarrow \infty}{=} G_E/G_M \quad \text{with} \quad \begin{pmatrix} p^{(\prime)} & = m_N \sinh \lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh \lambda^{(\prime)} \end{pmatrix}$$

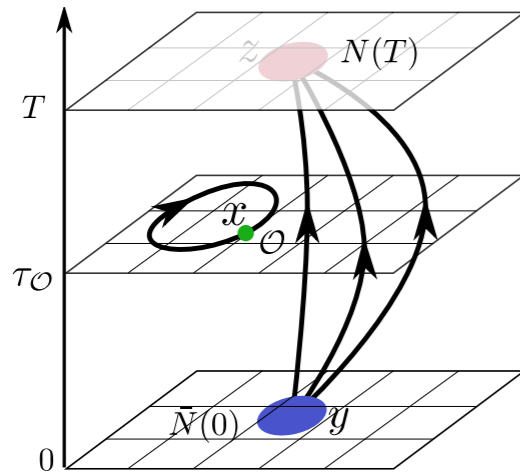
D5($m\pi = 278$ MeV, $a = 0.094$ fm)

D6($m\pi = 170$ MeV, $a = 0.094$ fm)



- Comparison of 4 ensembles (**D5** : 86k, **D6** : 261k); varying tsep = amount of exc.states
- *Nucleon excited state(s) have very similar G_E/G_M at large Q^2 ?*

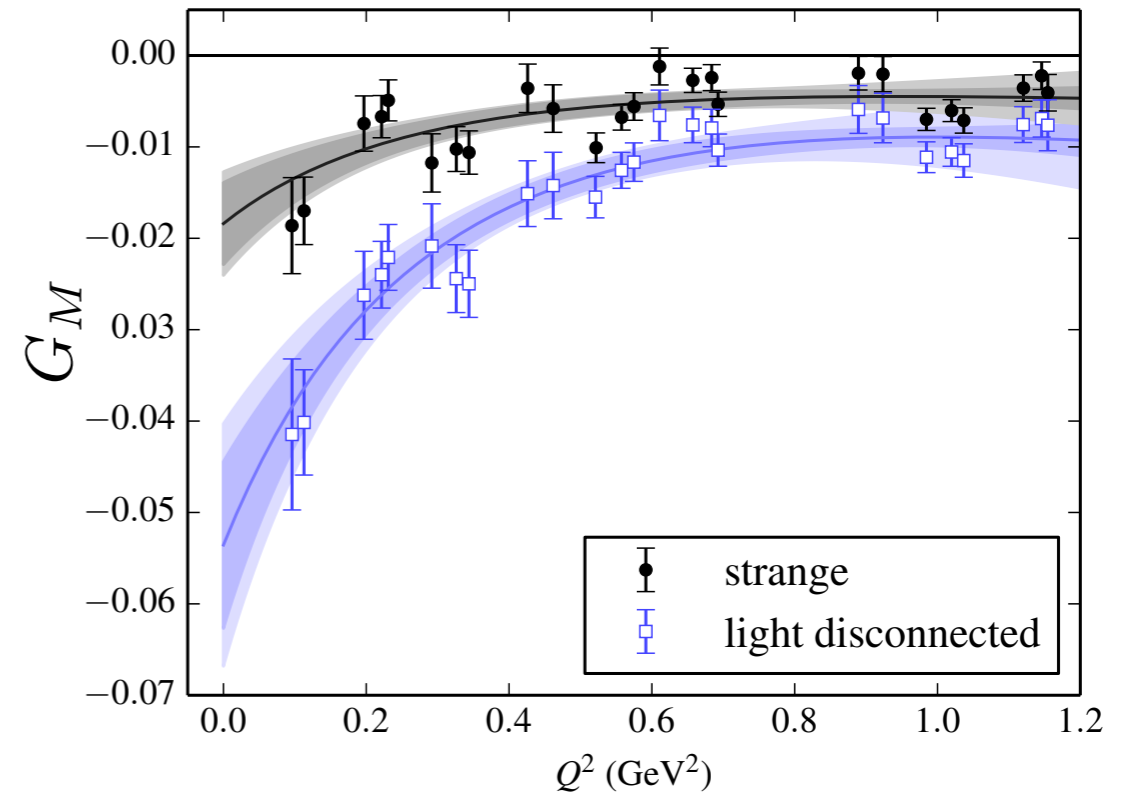
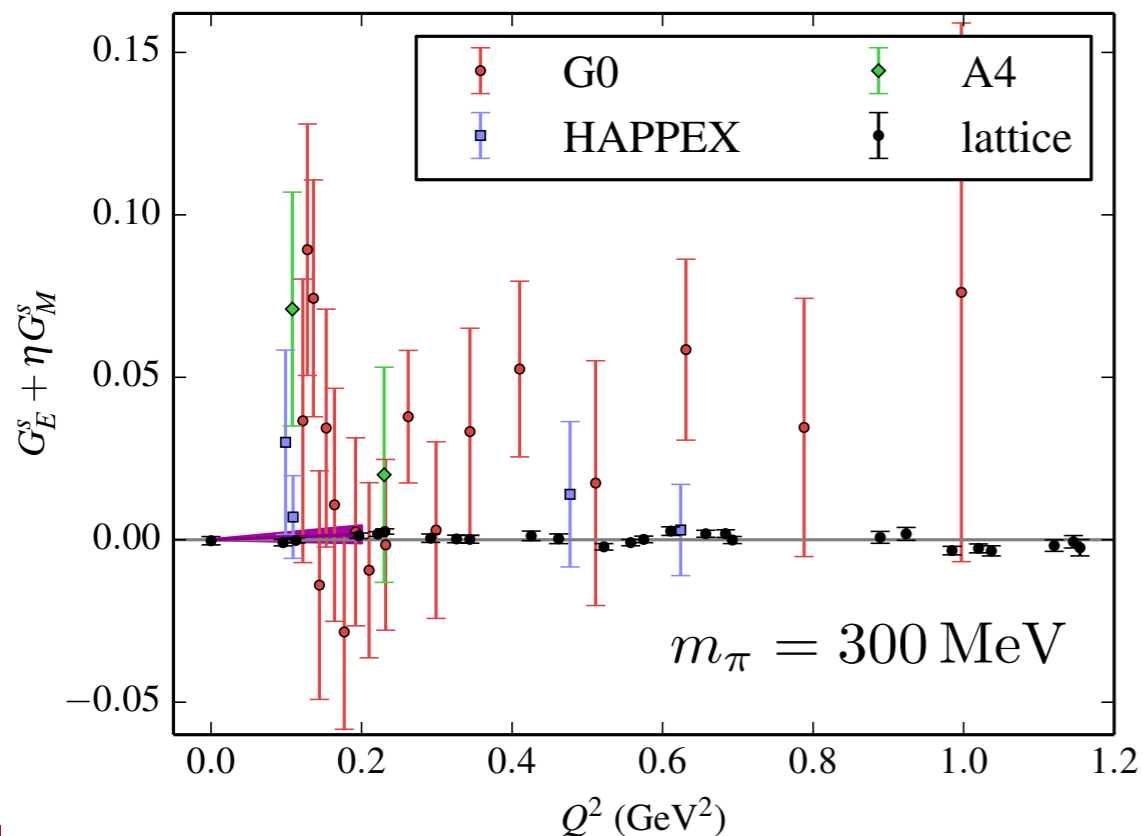
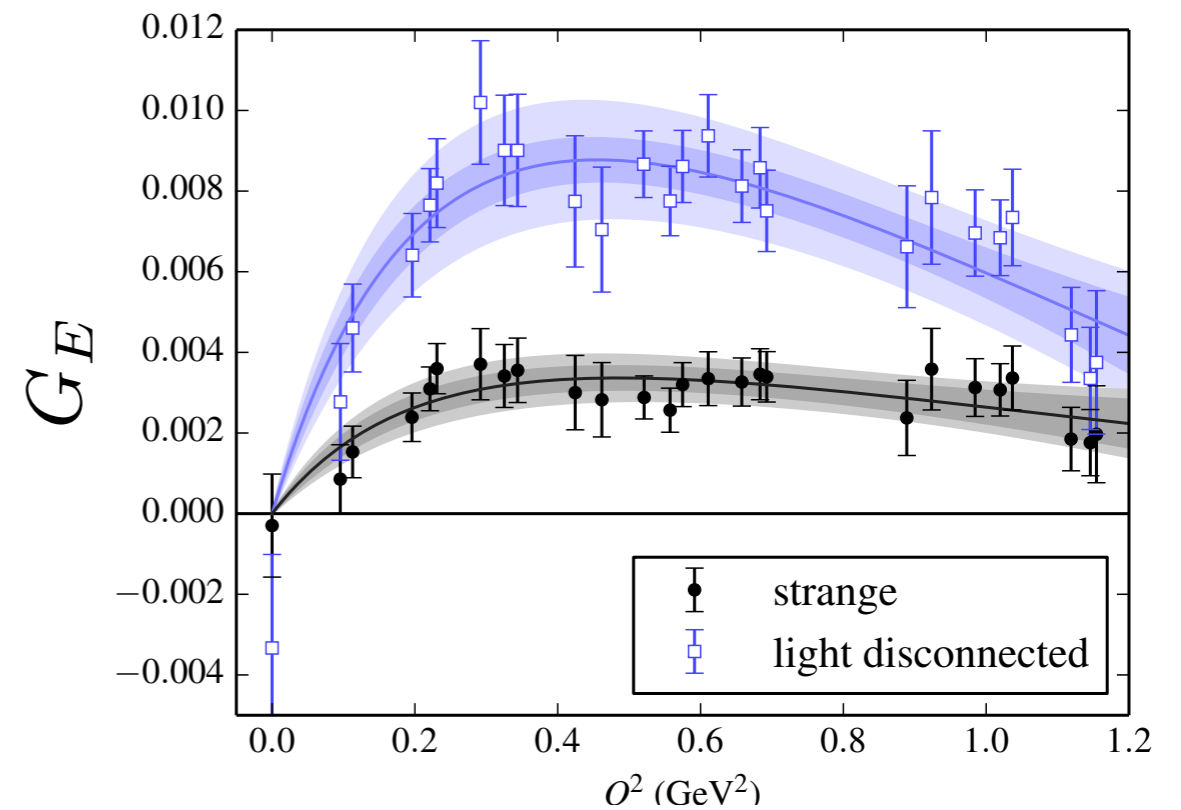
Disconnected Contributions to Vector FFs?



Disconnected-U,D and S:

$\approx 1\%$ of nucleon FFs at $Q^2 \approx 1 \text{ GeV}^2$

[J. Green, S. Meinel, S.S. et al; PRD92:031501 (2015)]

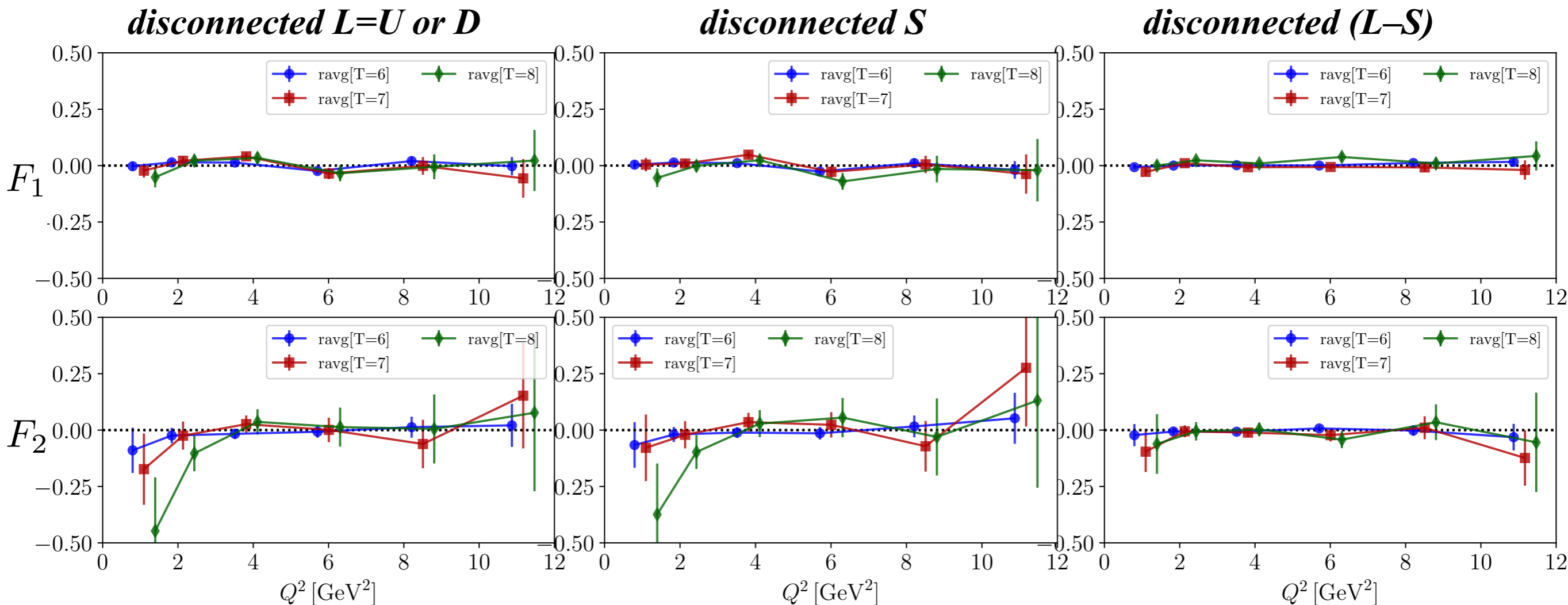


Disconnected-U,D & Strange vs. Connected (D5)

- relative correction $|\delta F_{1,2}^{\text{disc}} / F_{1,2}^{\text{conn}}| \approx 10\%$ but quickly grows with Euclidean time
- partial noise cancellation in (L-S) contributing to proton & neutron

$$P = \frac{1}{3} [2U - D]_{\text{conn}} + \frac{1}{3} [L - S]_{\text{disc}}$$

$$N = \frac{1}{3} [2D - U]_{\text{conn}} + \frac{1}{3} [L - S]_{\text{disc}}$$



D5 ensemble ($m_\pi=280$ MeV, $a=0.094$ fm), 86k samples of $\langle N\bar{N} \rangle$

G_E/G_M : Systematics from Disconnected U,D & S

- partial noise cancellation in (L-S) contributing to proton & neutron

$$P = \frac{1}{3} [2U - D]_{conn} + \frac{1}{3} [L - S]_{disc}$$

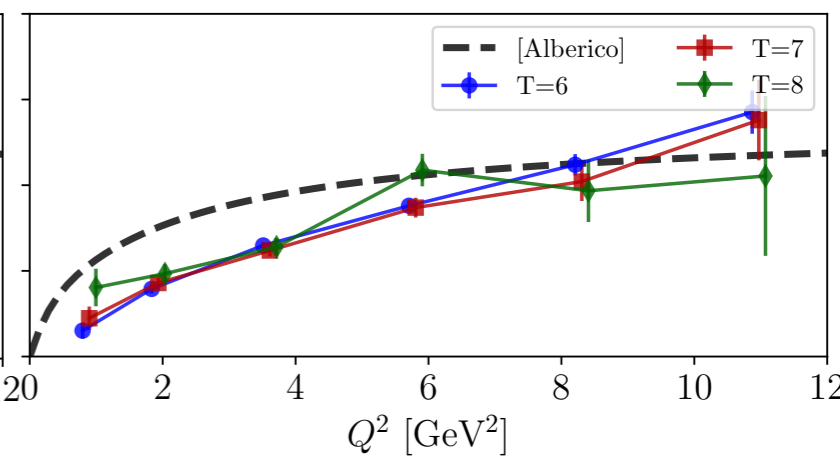
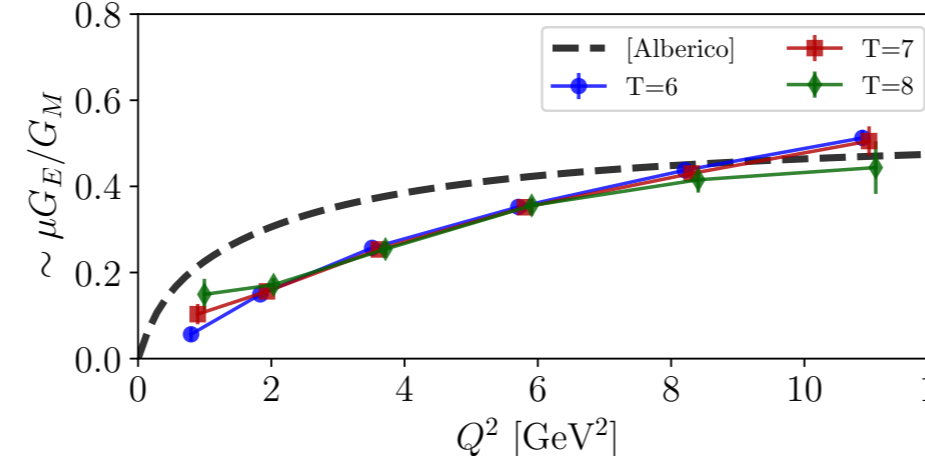
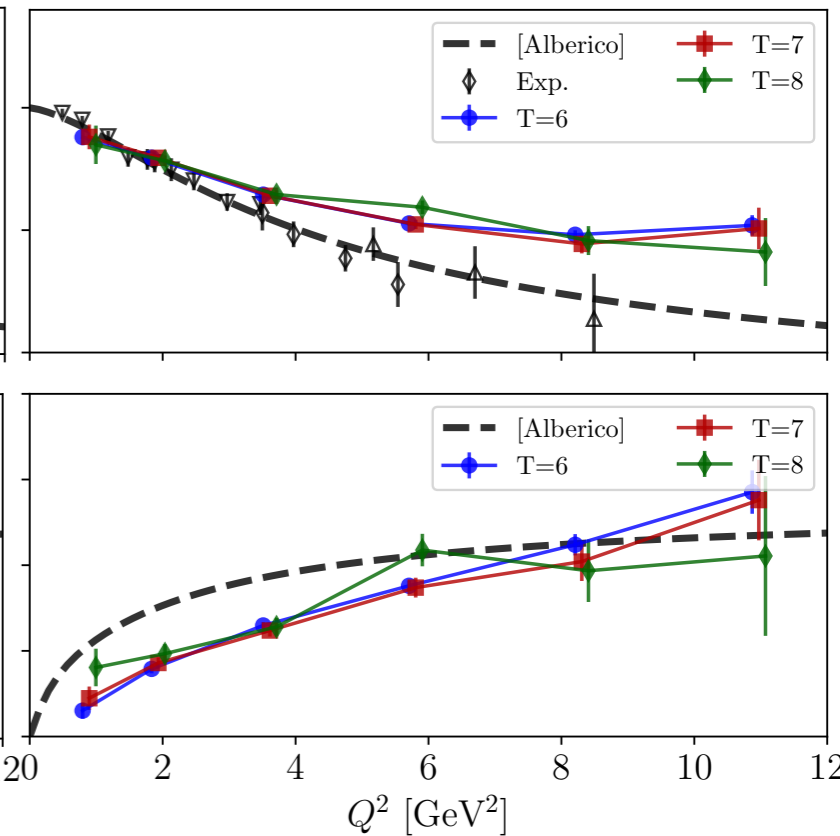
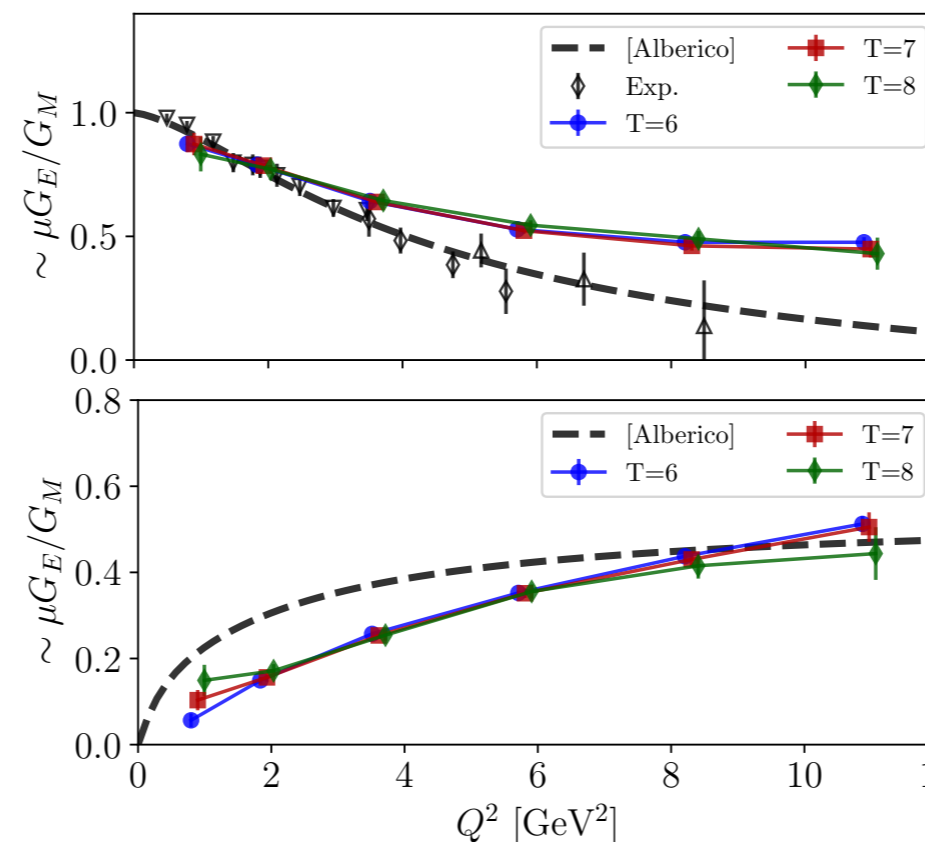
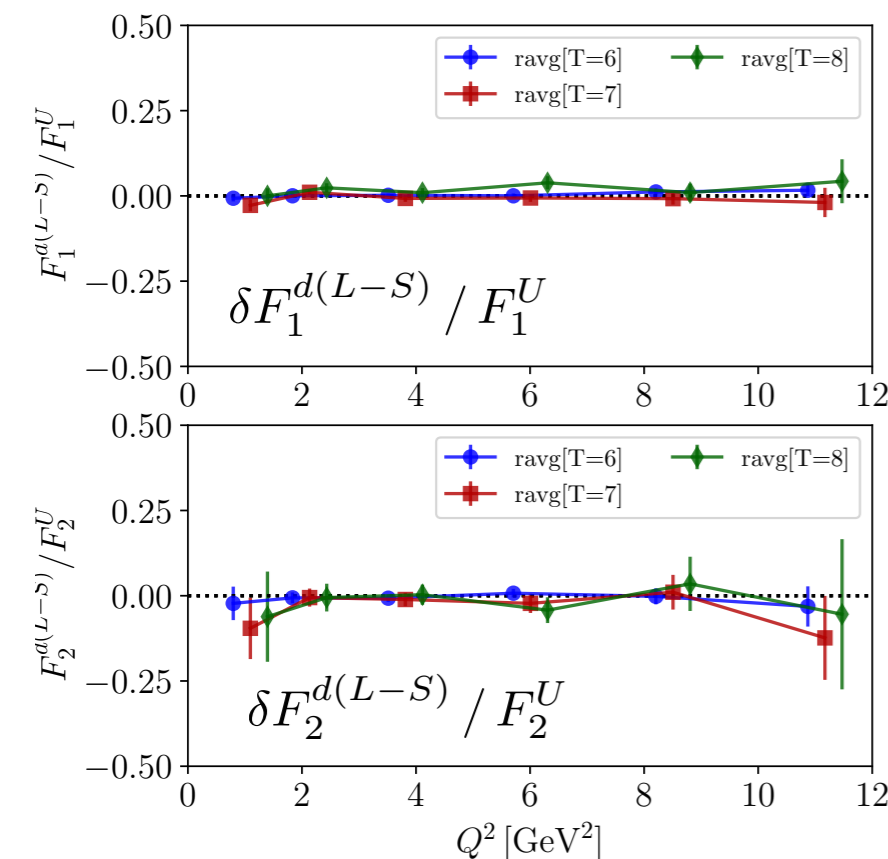
$$N = \frac{1}{3} [2D - U]_{conn} + \frac{1}{3} [L - S]_{disc}$$

- limited effect on G_E/G_M (within stat. noise)
- larger effect on flavor-decomposition of $F_{1,2}$

*disconnected (L-S),
relative to conn.*

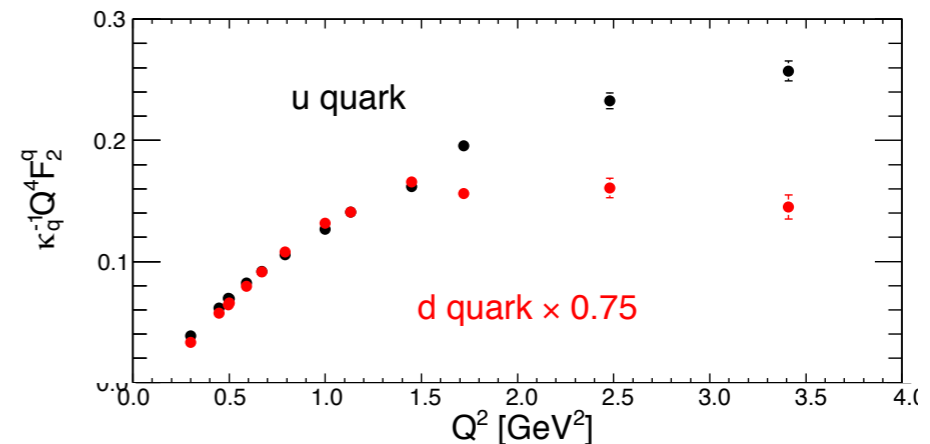
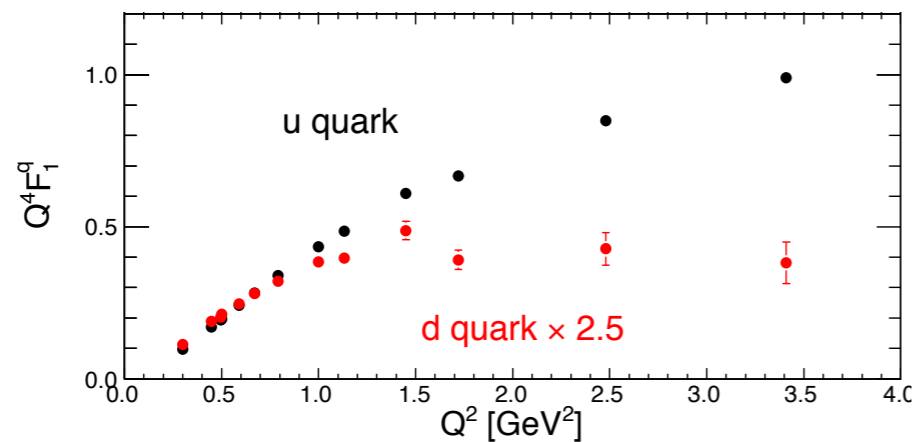
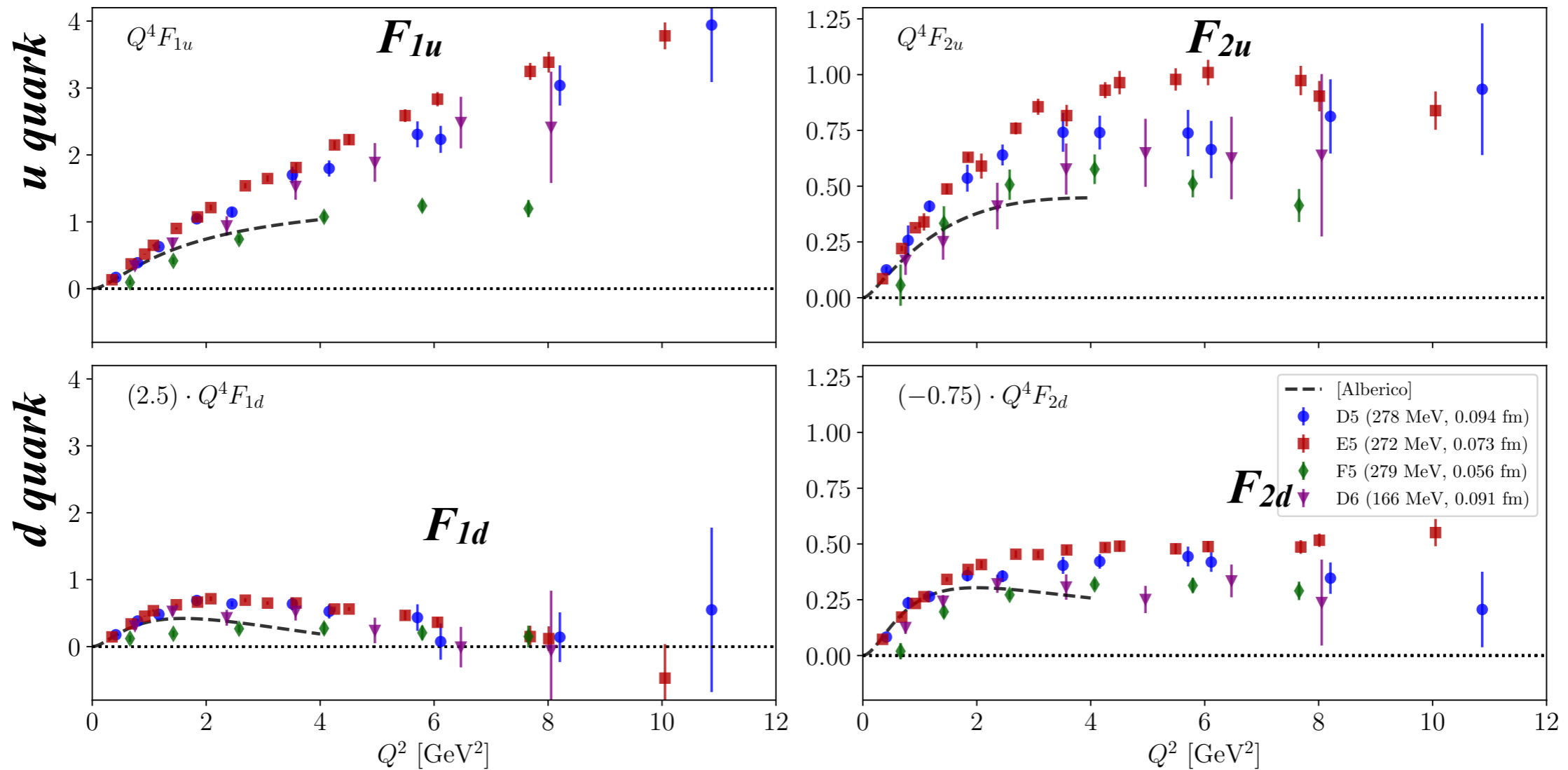
G_E/G_M , conn. only

G_E/G_M , conn.+disc.



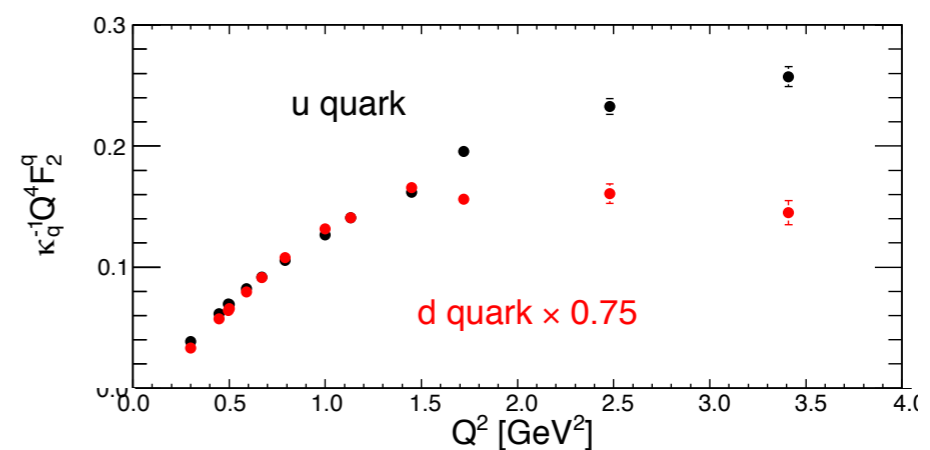
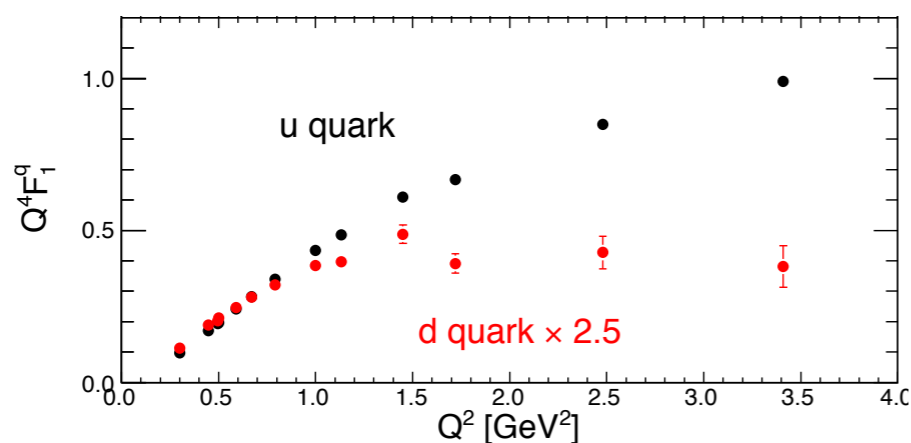
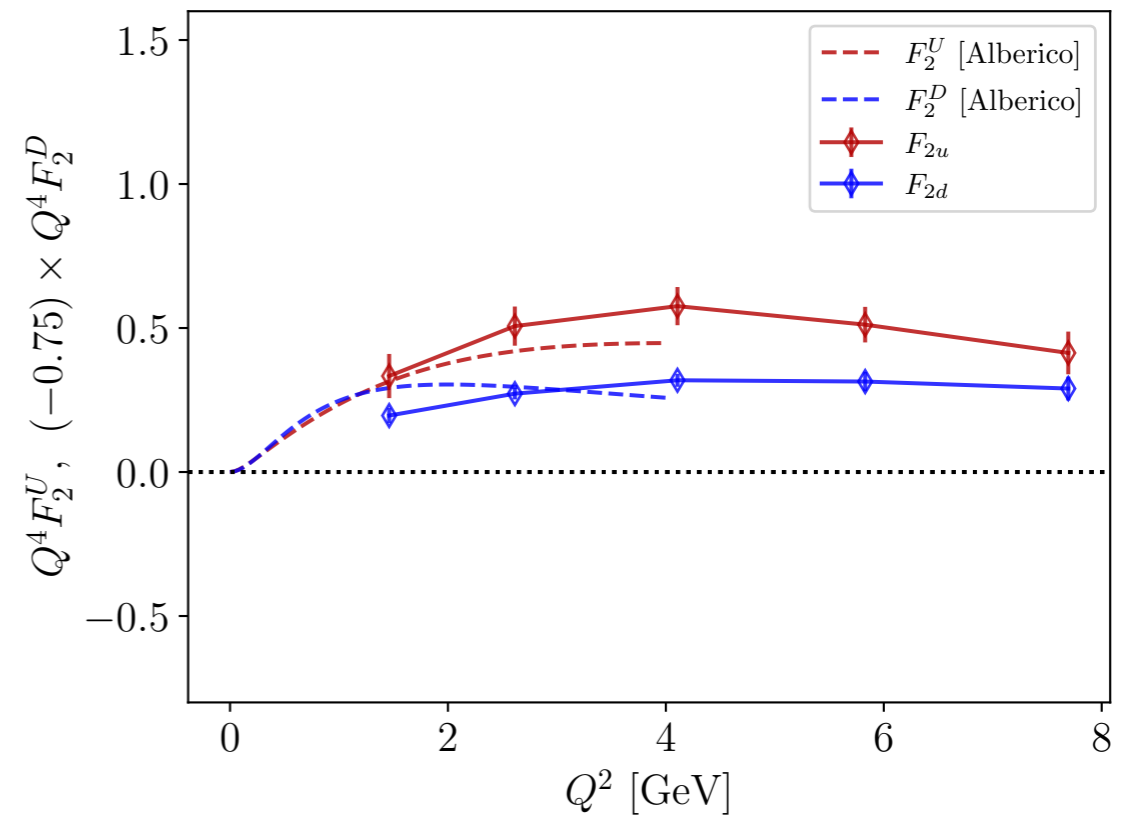
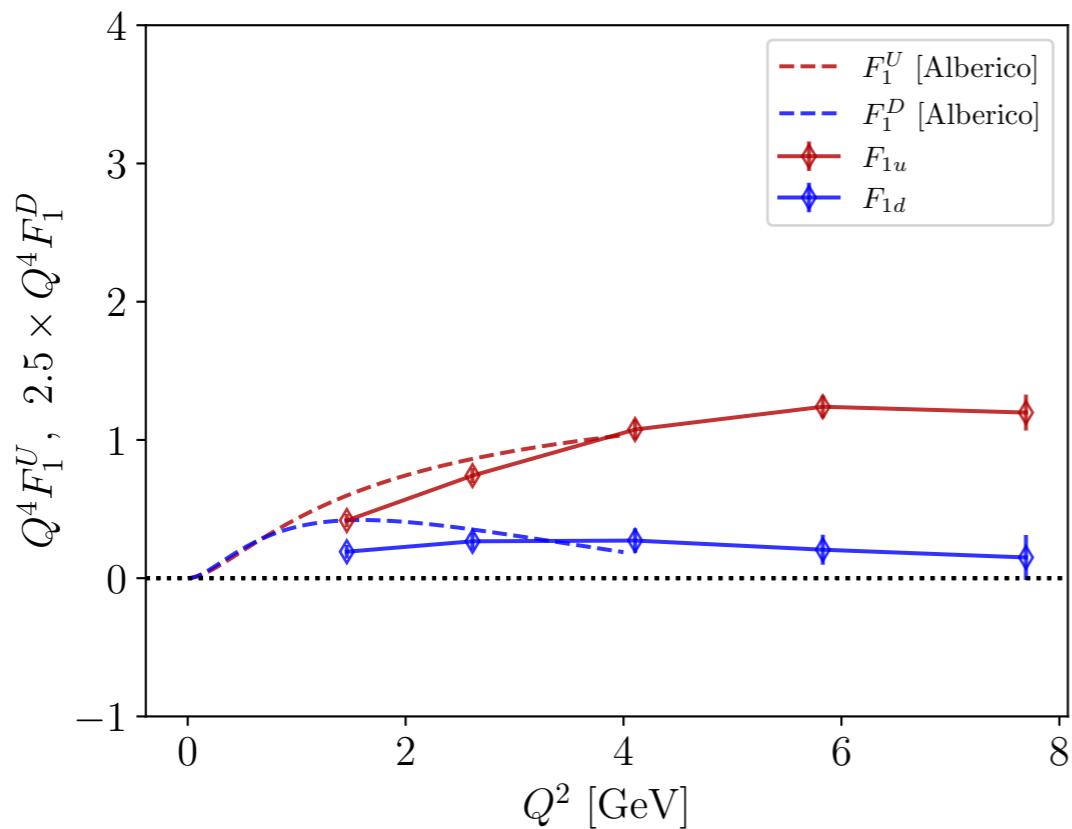
D5 ensemble ($m_\pi=280$ MeV, $a=0.094$ fm), $t_{sep}=0.50\div 0.75$ fm

Light-flavor Contributions to Proton Form Factors



- Comparison of 4 ensembles (D5 : 86k, E5 : 266k, F5 : 47k, D6 : 261k)
- (dashed lines) flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

Light-flavor Contrib's to Proton FFs ($a=0.056\text{fm}$)



● Initial statistics on **F5** : 47k)

● (dashed lines) flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

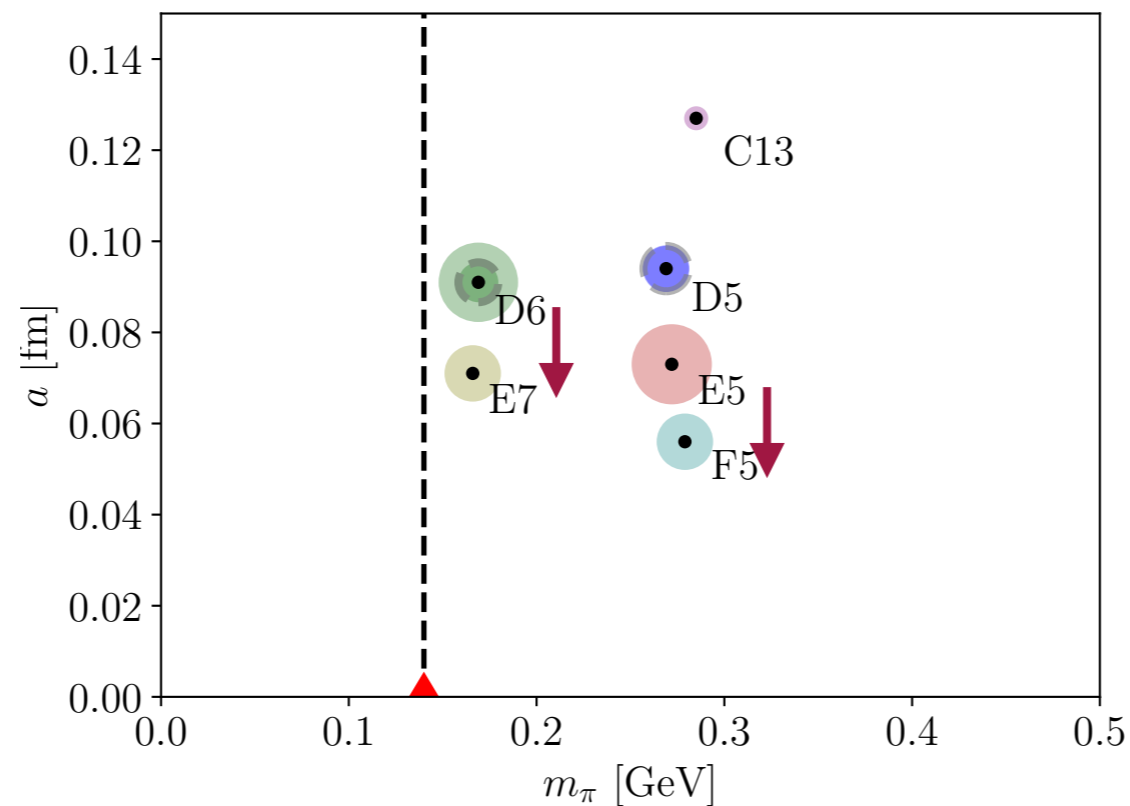
Summary

- High-statistics lattice calculations of nucleon form factors at high-momentum
 - up to $Q^2 \lesssim 10 \text{ GeV}^2$
 - two lattice spacings $a \gtrsim 0.056 \text{ fm}$
 - two pion masses $m_\pi \gtrsim 170 \text{ MeV}$
- Quark-disconnected & strange contributions at $a \approx 0.09 \text{ fm}$, m_π down to 170 MeV
 - little impact below $Q^2 \lesssim 6 \text{ GeV}^2$
 - still major source of uncertainty (esp. above $Q^2 \gtrsim 8 \text{ GeV}^2$)
 - large stoch. uncertainty, more stats. needed
- Form factor results overshoot experimental data $x(2 \dots 2.5)$;
 G_E/G_M ratios in qualitative agreement
 - Non-physical quarks masses?
 - Discretization?
 - Excited states (most likely) — *improved/variational nucleon operators may help*

*Important cross-check with experiments,
relevant for calculations of relativistic nucleon matrix elements
as well as TMDs, PDFs, DAs ...*

Outlook

- Next-gen computing facilities (nVidia GH200 clusters, e.g. Jupiter at Jülich SC)
 - More statistics (x10) to control excited state systematics
 - Smaller lattice spacings (0.09 → 0.07 → 0.05 fm), physical and 2x physical pion mass
 - Improved lattice methods for nucleon structure

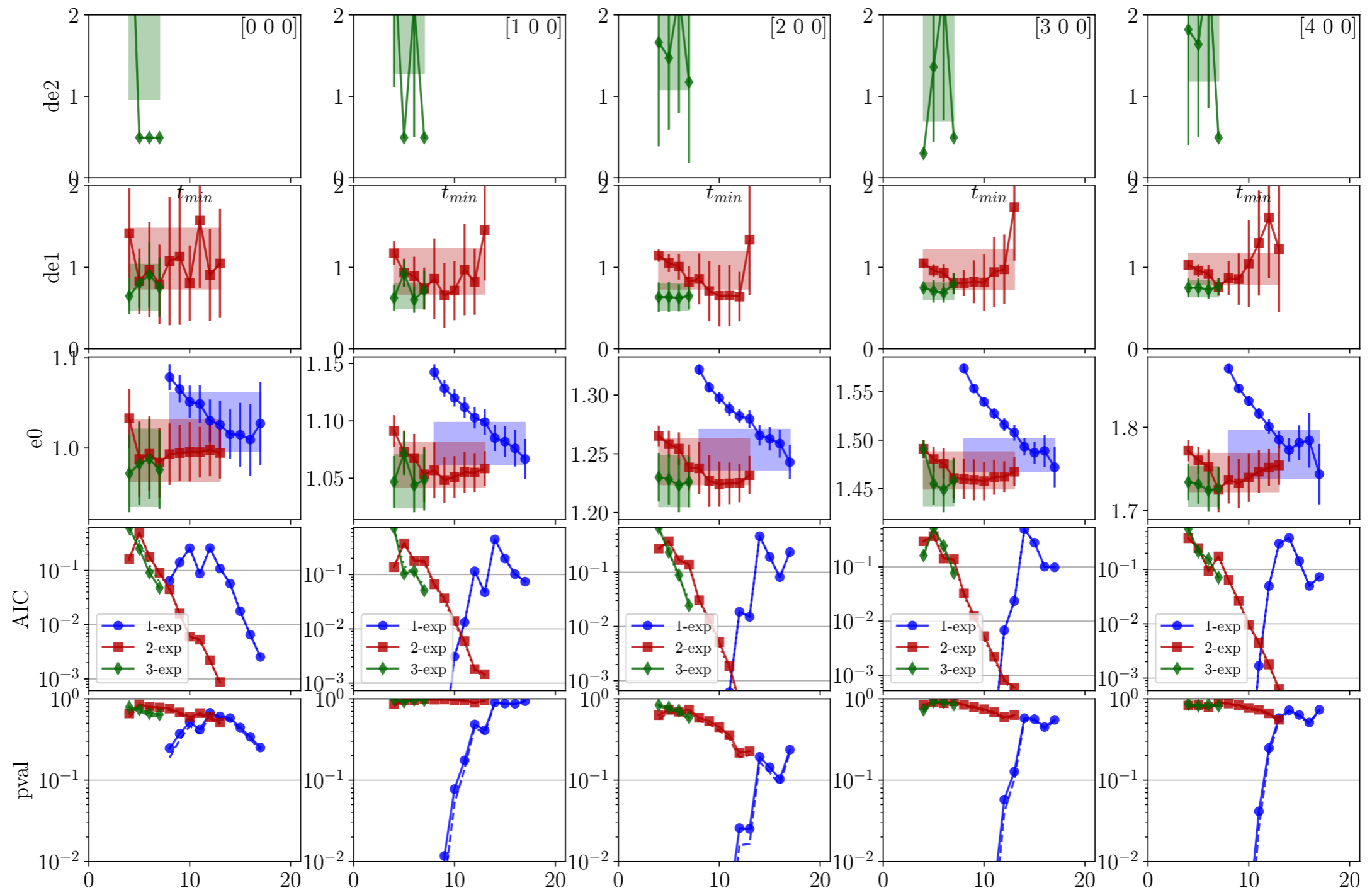


- Effective theory for large-momentum form factors of nucleon-like states?
- Any hypothesis for universal G_E/G_M behavior?

BACKUP

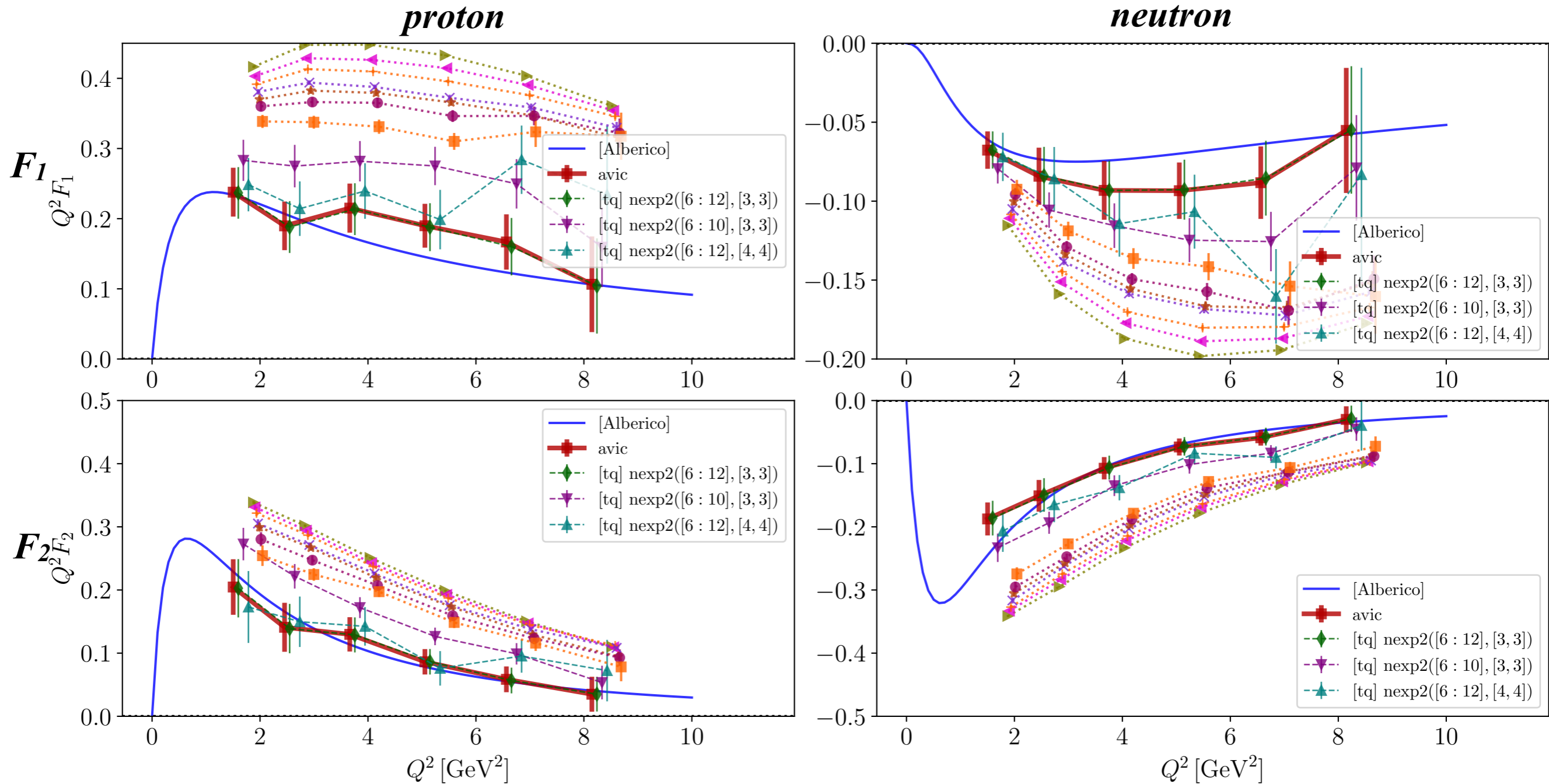
Nucleon Excited States

- F5 ensemble (47k samples)
- 1-,2-,3-state fits
- excited state: $\Delta E \approx 0.7-0.8$ GeV



Form Factors: Excited State Effects (D6)

- D6 ensemble (260k samples) : fits with $t_{sep} = 0.5 \div 1.1 \text{ fm}$
- Solid line : AIC average
- Phenomenology (dashed) : [Alberico et al, PRC79:065204 (2008)]



• No disconnected diagrams

Form Factors: Excited State Effects (D6)

- F5 ensemble (47k samples) : fits with $t_{sep} = 0.7 \div 1.1 \text{ fm}$
- Solid line : AIC average
- Phenomenology (dashed) : [Alberico et al, PRC79:065204 (2008)]

