

Recent lattice results in nucleon PDF from LaMET

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- Introduction: lattice matrix elements, LaMET and short distance factorization
- Moments of helicity PDF
X. Gao, A. Hanlon, S. Mukherjee, PP, H.-T. Shu, F. Yao, R. Zhang, Y. Zhao, arXiv:2604.00143
- Moment of twist-3 transversity function, d_2
X. Gao, A. Hanlon, S. Mukherjee, PP, H.-T. Shu, F. Yao, R. Zhang, Y. Zhao, arXiv:2604.00143
- Helicity PDF from LaMET
X. Gao, A. Hanlon, S. Mukherjee, PP, H.-T. Shu, F. Yao, R. Zhang, Y. Zhao, arXiv:2604.00143
- Unpolarized, helicity and transversity PDF from LaMET in Coulomb gauge
X. Gao, J. He, J. Lin, S. Mukherjee, PP, R. Zhang, Y. Zhao, arXiv:2602.11283
- Summary

Nucleon Matrix Elements from Lattice QCD

$$\mathcal{O}_\Gamma(z) \equiv \bar{\psi}(z\hat{z}) \Gamma W(z\hat{z}, 0) \psi(0).$$

$$\Gamma = \gamma_t, \quad i\gamma_5\gamma_z, \quad i\gamma_t\gamma_x\gamma_5, \quad i\gamma_5\gamma_x$$

↑
↑
↑
↑

unpolarized, helicity, transversity, g_T

$$\Gamma = i\gamma_5\gamma_z \quad \tilde{h}_1^B(z, P_z, a) \equiv \frac{1}{2E} \langle P, S | \bar{\psi}(z) \Gamma W(z, 0) \psi(0) | P, S \rangle.$$

$$\Gamma = i\gamma_5\gamma_x \quad \tilde{h}_T^B(z, P_z, a) \equiv \frac{1}{2m_N} \langle P, S | \bar{\psi}(z) \Gamma W(z, 0) \psi(0) | P, S \rangle$$

$$C_{3\text{pt}}^\Gamma(\vec{P}, t_{\text{sep}}, \tau, z) = \sum_{\vec{y}, \vec{z}_0} e^{-i\vec{P}\cdot(\vec{y}-\vec{x}_0)} \mathcal{P}_{3\text{pt}} \times \langle N(\vec{y}, t_{\text{sep}} + t_0) \mathcal{O}_\Gamma(\vec{z}_0 + z\hat{z}, \tau + t_0) \bar{N}(\vec{x}_0, t_0) \rangle$$

$$C_{2\text{pt}}(\vec{p}, t_{\text{sep}}) = \sum_{\vec{y}} e^{-i\vec{p}\cdot(\vec{y}-\vec{x}_0)} \mathcal{P}_{2\text{pt}} \times \langle N(\vec{y}, t_{\text{sep}} + t_0) \bar{N}(\vec{x}_0, t_0) \rangle$$

$$N(x) = \epsilon_{abc} (d_{\vec{k}}^T(x) C \gamma_5 u_{\vec{k}}(x)) u_{\vec{k}}(x),$$

$$R(\tau, t_{\text{sep}}, P_z; z) \equiv \frac{C_{3\text{pt}}^\Gamma(\vec{p}, t_{\text{sep}}, \tau, z)}{C_{2\text{pt}}(\vec{p}, t_{\text{sep}})}$$

$$\lim_{t_{\text{sep}} \gg \tau \gg 0} R_1(\tau, t_{\text{sep}}, P_z; z) = \tilde{h}_1^B(z, P_z, a),$$

$$\lim_{t_{\text{sep}} \gg \tau \gg 0} R_T(\tau, t_{\text{sep}}, P_z; z) = \frac{m_N}{E} \tilde{h}_T^B(z, P_z, a)$$

LaMET and PDF

1) Quasi-PDF approach (x-space matching):

$$\tilde{f}_v(x, P^z, \mu) = \int \frac{dz}{2\pi} e^{ixP^z z} \tilde{h}(z, P^z, \mu) \quad \tilde{h}(z, P^z, \mu) \equiv \langle P | O_{\gamma^t}(z) | P \rangle / (2P^t), \quad P^\mu = (P^t, 0, 0, P^z)$$

calculable in perturbation theory,
known to NNLO

$$C = \delta\left(\frac{x}{y} - 1\right) + \alpha_s C^{(1)}\left(\frac{x}{y}, \frac{\mu}{P_z y}\right) + \dots$$

$$\tilde{f}_v(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}_v(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)P^z)^2}\right)$$

Light cone PDF Access the x -dependence for PDF, except for $x \rightarrow 0$ and $x \rightarrow 1$

2) Short distance factorization:

Wilson coefficients $C_n = 1 + \alpha_s C_n^{(1)}(\mu^2 z^2) + \dots$

$$\tilde{h}(z, P_z, \mu) = \sum_n C_n(\mu^2 z^2) \langle x^n \rangle_\mu \frac{(-izP_z)^n}{n!} + z^2 \Lambda_{\text{QCD}}^2, \quad \langle x^n \rangle_\mu = \int_{-1}^{+1} dx x^n f_v(x, \mu)$$

$\lambda = zP_z$ - Ioffe time; one needs large λ to probe higher moments of PDF

1) And 2) are equivalent in the infinite momentum limit $P_z \rightarrow \infty$

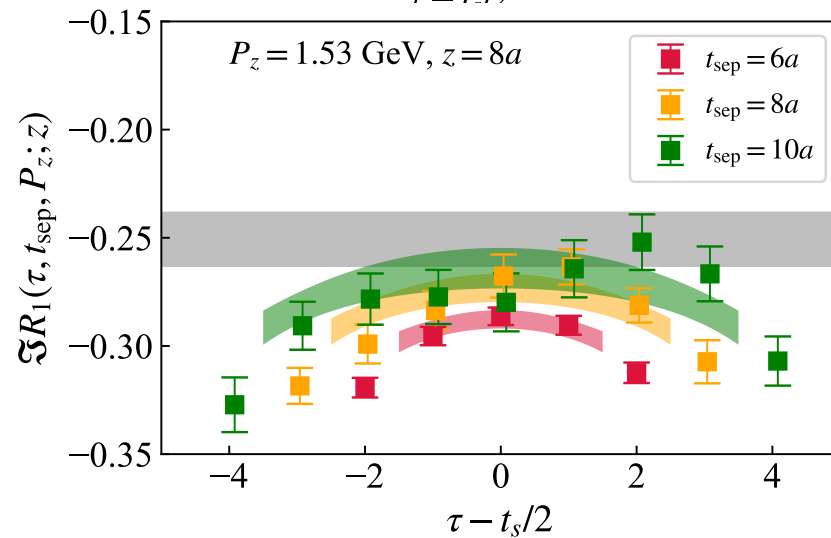
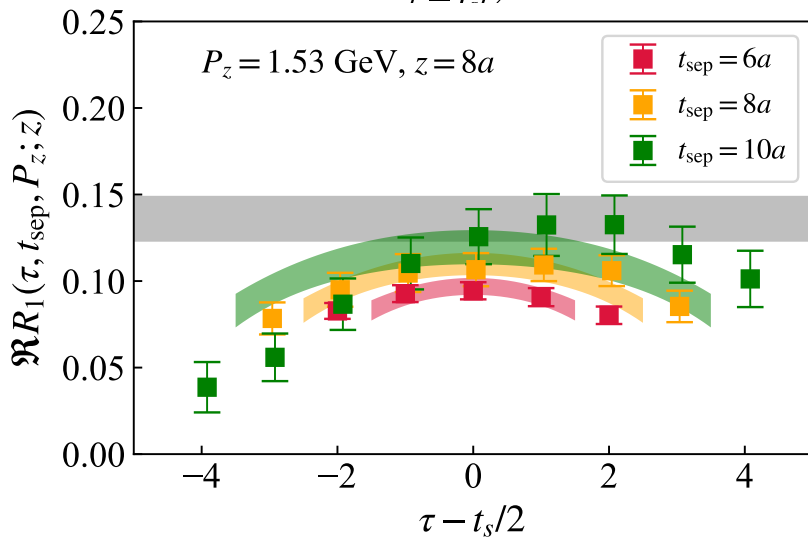
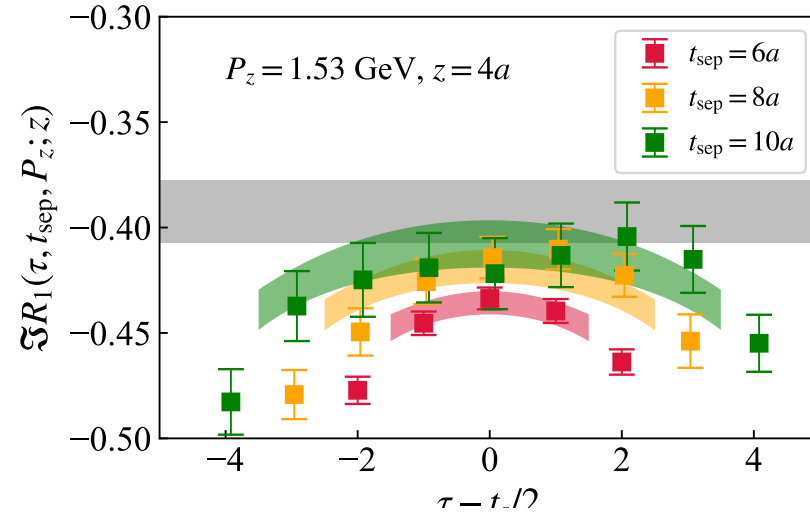
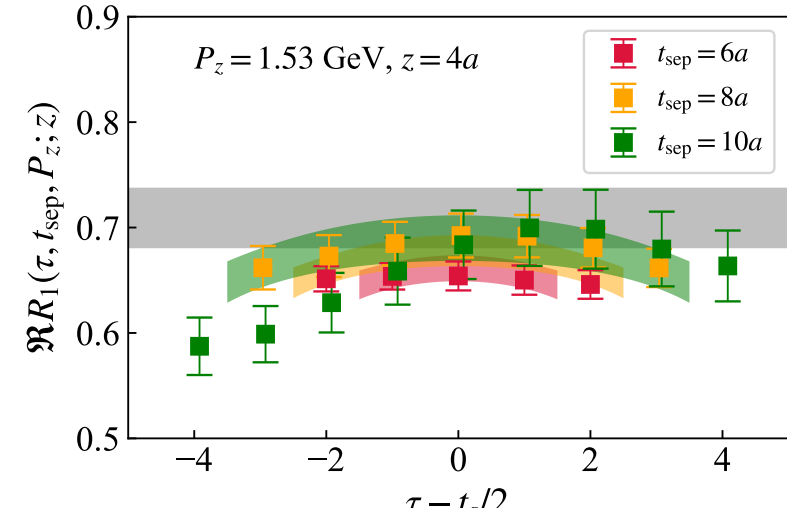
Renormalization: RI-MOM, ratio scheme, hybrid scheme
linear self-energy ($1/a$) divergence in the Wilson line

Lattice QCD setup

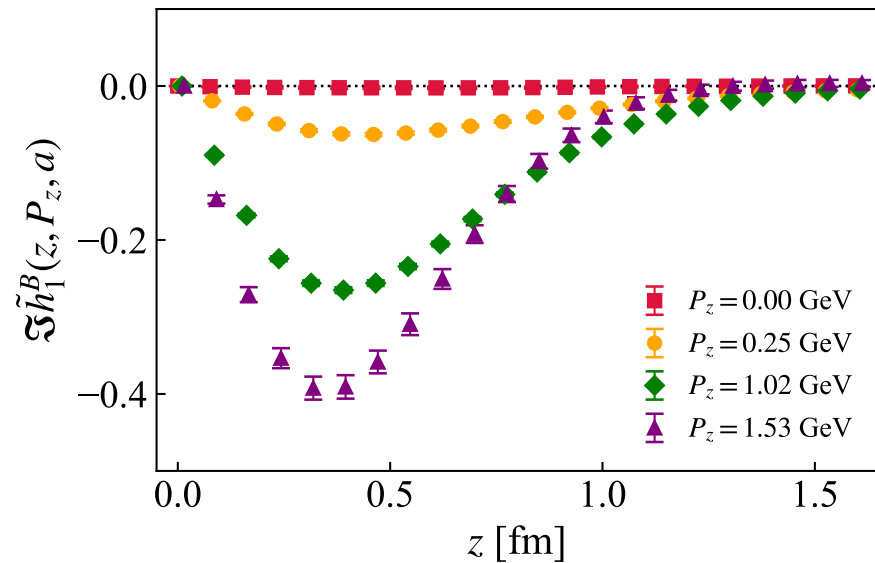
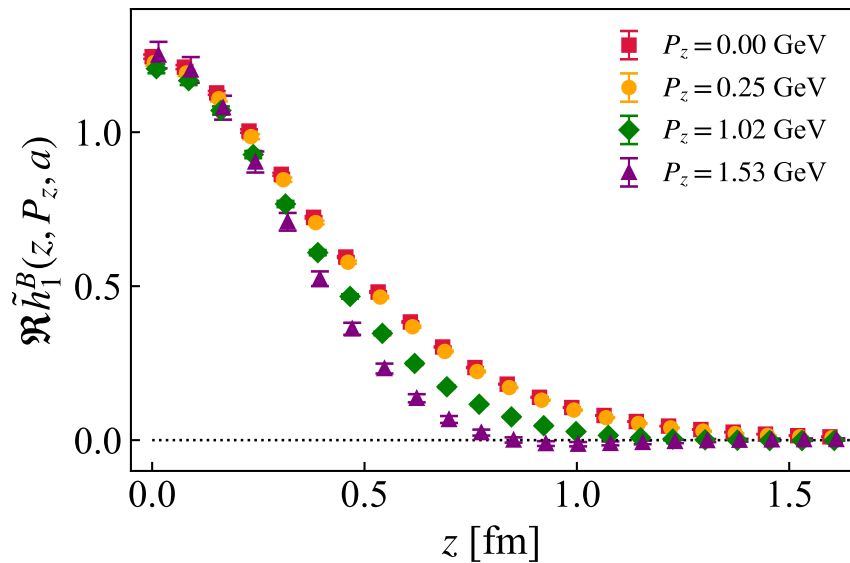
HISQ quark sea + HYP smeared tadpole improved clover Wilson action

β	a [fm]	m_π [GeV]	N_σ	N_τ	# conf.	$P_z = \frac{2\pi}{aN_\sigma} \times \{1, 4, 6\} = \{0.25, 1.02, 1.53\}$ GeV.
7.13	0.076	0.14	64	64	350	$t_{\text{sep}}/a = 6, 8, 10, 12.$

CG: $a = 0.06$ fm, $m_\pi = 300$ MeV, $P_z = 2.43, 3.04$ GeV

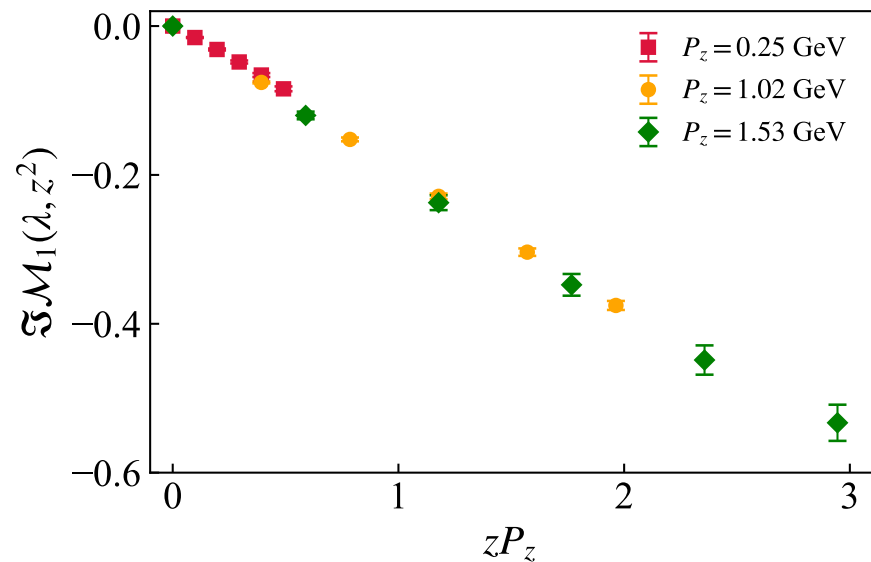
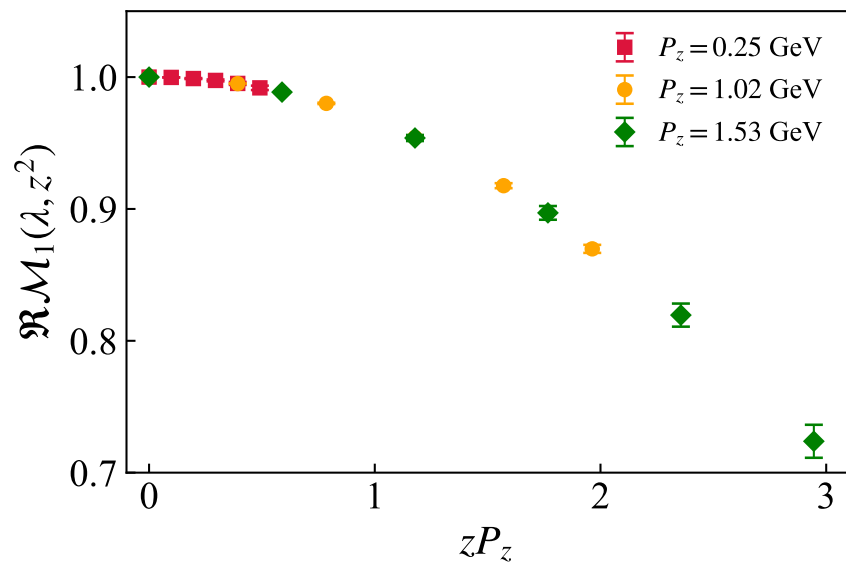


Helicity PDF : renormalization via ratio scheme



Ratio scheme:

$$\lambda = zP_z$$



$$\mathcal{M}(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

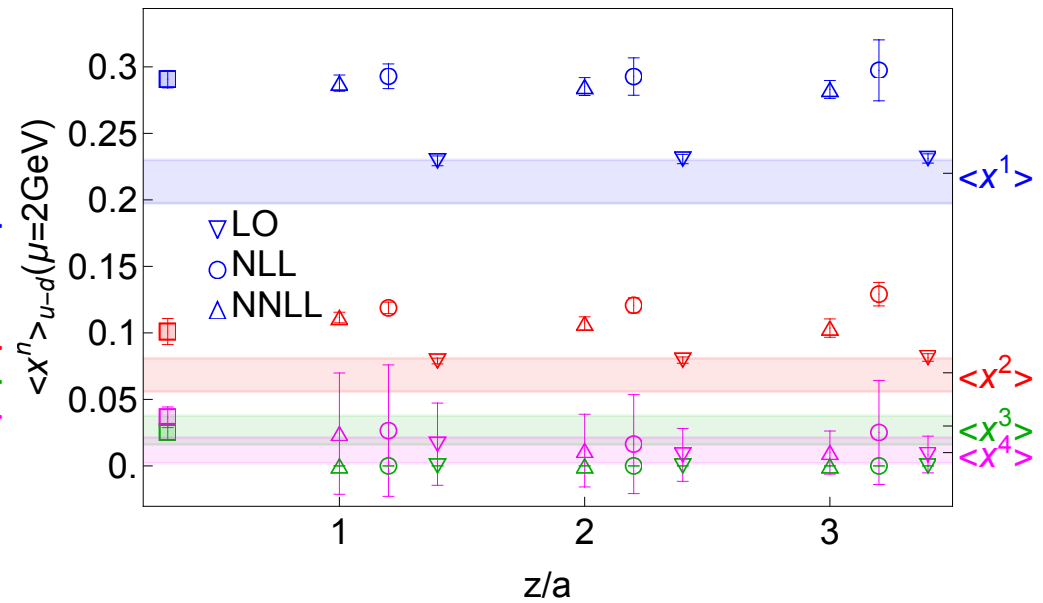
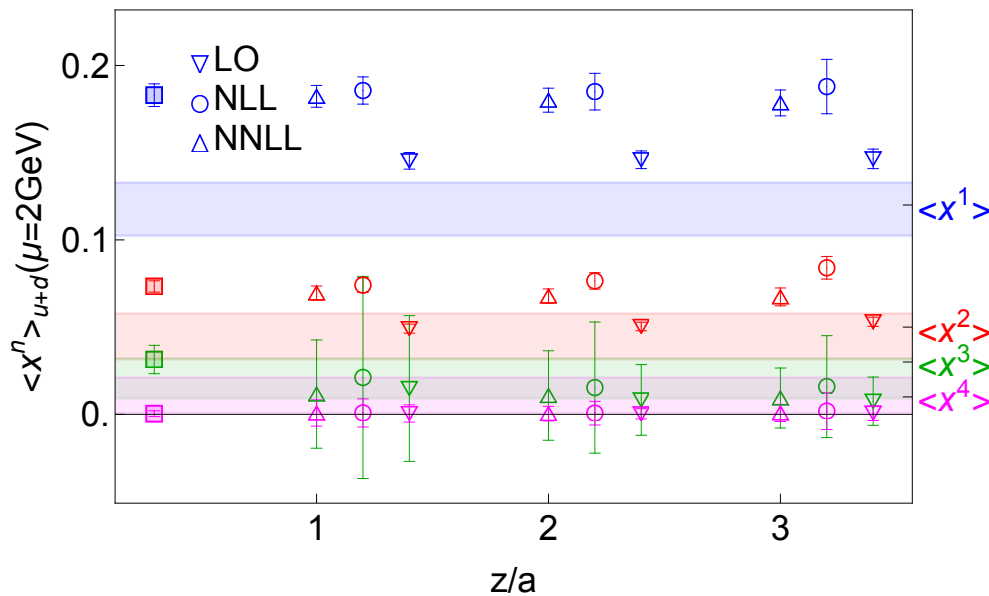
Short distance factorization: moments of helicity PDF

$$\mathcal{M}(z, P_z, P_z^0) =$$

$$\frac{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}$$



$$\langle x^n \rangle_\mu$$



The first two moments are larger than the one obtained from the global analysis

Moments of twist-3 transversity

$$h_T(\lambda) = h_T^{\text{tw}2}(\lambda) + h_T^{\text{tw}3}(\lambda) \quad h_T^{\text{tw}2}(\lambda) = \int_0^1 d\xi h_1(\xi\lambda) \longleftrightarrow g_T^{\text{tw}2}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$

$$g_T(x) = g_1(x) + g_2(x)$$

WW relation

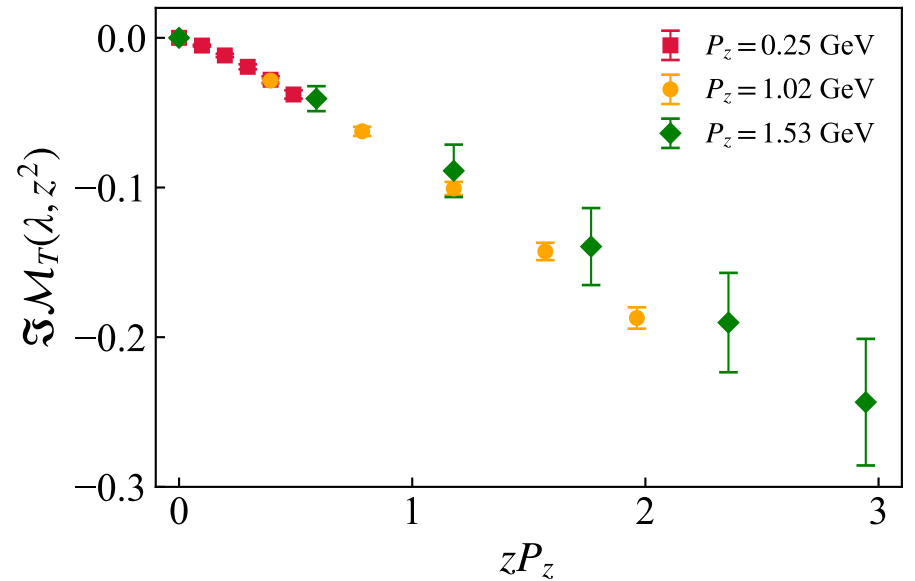
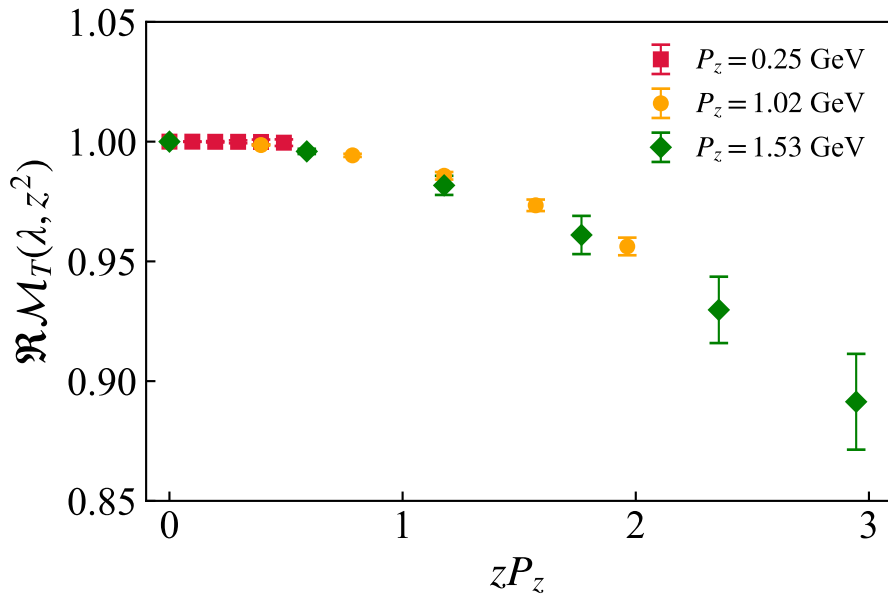
$$\int_{-1}^1 dx x^n g_1(x) = a_n,$$

$$\tilde{d}_2 \equiv \int_{-1}^1 dx x^2 [3g_T(x) - g_1(x)] = \int [dx] S^-(x_1, x_2, x_3),$$

$$\int_{-1}^1 dx x^n g_2(x) = \frac{n}{n+1} (d_n - a_n)$$

related averaged Lorentz force of the nucleus action
on the struck quark, M. Burkardt, [PRD 88 \(2013\) 114502](#)

$$\mathcal{M}_T(\lambda, z^2) \equiv \frac{\tilde{h}_T^B(z, P_z)}{\tilde{h}_T^B(z, 0)} \frac{\tilde{h}_T^B(0, 0)}{\tilde{h}_T^B(0, P_z)} = \frac{\tilde{h}_T^{\overline{\text{MS}}}(\lambda, z^2 \mu^2)}{\tilde{h}_T^{\overline{\text{MS}}}(0, z^2 \mu^2)}$$



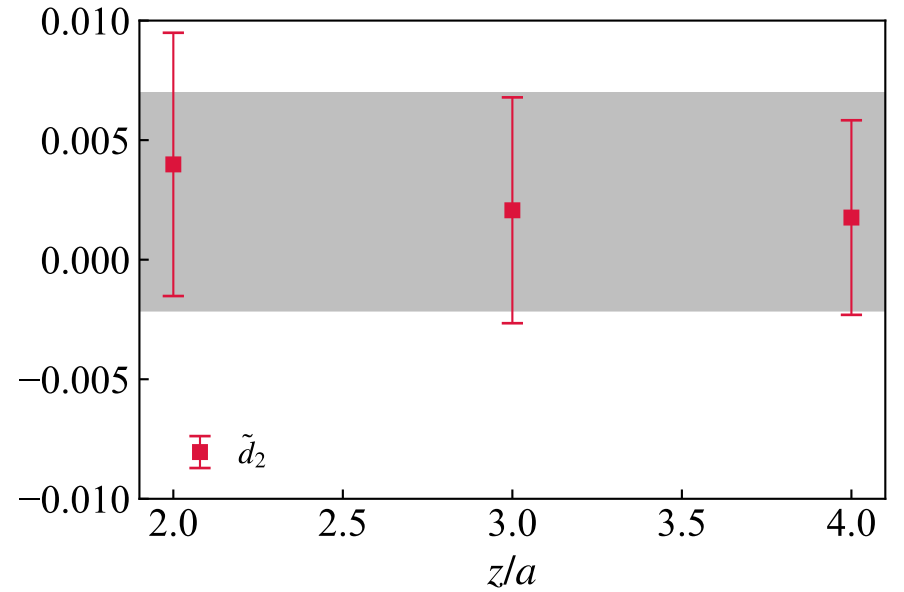
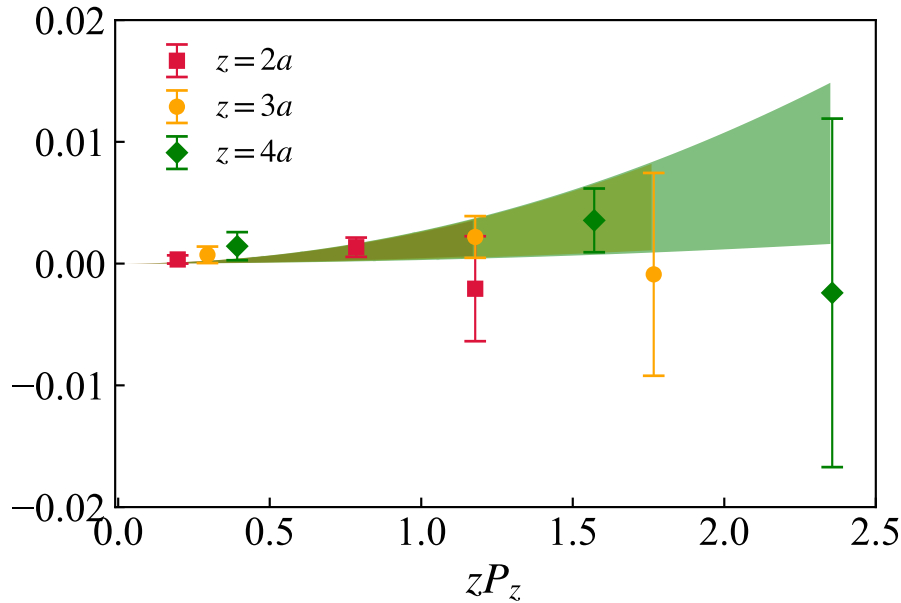
Problem: $\tilde{h}_T^{\text{tw}2}(\lambda, z^2) - \int_0^1 d\alpha \tilde{h}_1(\alpha\lambda, z^2) \neq 0$

Determination of d_2 on the lattice

$$\tilde{d}_2 \equiv \int_{-1}^1 dx x^2 [3g_T(x) - g_1(x)] = \int [dx] S^-(x_1, x_2, x_3).$$

$$\begin{aligned} \tilde{h}_T^{\overline{\text{MS}}}(\lambda, z^2 \mu^2) &= \int_0^1 d\alpha \tilde{h}_1^{\overline{\text{MS}}}(\alpha\lambda, z^2 \mu^2) - \frac{\alpha_s C_F}{\pi} a_0 + i \lambda \frac{\alpha_s C_F}{3\pi} a_1 && \text{V. Braun, Y. Ji, A. Vladimirov, JHEP 05 (2021) 086} \\ &+ \frac{\lambda^2}{3} \left\{ \tilde{d}_2 + \frac{7\alpha_s C_F}{24\pi} a_2 + \frac{\alpha_s}{4\pi} \tilde{d}_2 \left[\left(\frac{43C_F}{12} + \frac{9}{4N_c} + \frac{3N_c}{4} \right) L_z - \frac{13C_F}{4} + \frac{3}{4N_c} - \frac{3N_c}{4} \right] \right\} + \mathcal{O}(\lambda^3) && \lambda = zP_z \end{aligned}$$

$$\mathcal{M}_T^{\text{tw}3}(\lambda, z^2 \mu^2) \equiv \mathcal{M}_T(\lambda, z^2 \mu^2) - \frac{\tilde{h}_1^{\overline{\text{MS}}}(0, z^2 \mu^2)}{\tilde{h}_T^{\overline{\text{MS}}}(0, z^2 \mu^2)} \int_0^1 d\alpha \mathcal{M}_1(\alpha\lambda, z^2 \mu^2) - \frac{1}{\tilde{h}_T^{\overline{\text{MS}}}(0, z^2 \mu^2)} \left(-\frac{\alpha_s C_F}{\pi} a_0 + i \lambda \frac{\alpha_s C_F}{3\pi} a_1 \right) \sim \lambda^2$$



$$\tilde{d}_2^{u-d}(\mu = 2 \text{ GeV}) = 0.0024(46)$$

Determination of helicity PDF in LaMET

Hybrid renormalization scheme

$$\tilde{h}_1^R(z, P_z) = \begin{cases} \frac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z, 0, a)} & |z| \leq z_s \\ \frac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z_s, 0, a)} e^{(\delta m + m_0)(z - z_s)} & |z| \geq z_s \end{cases}$$

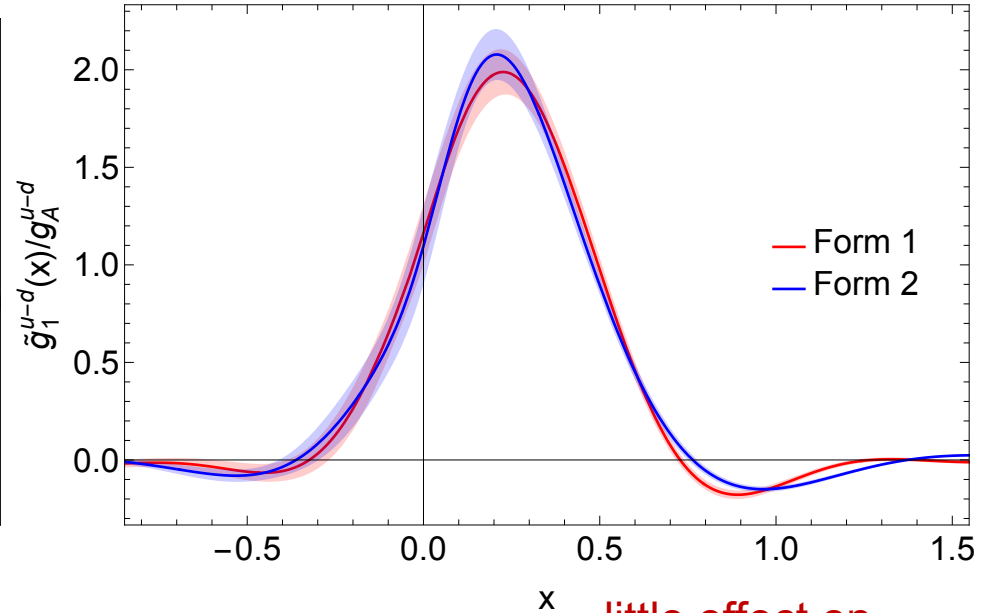
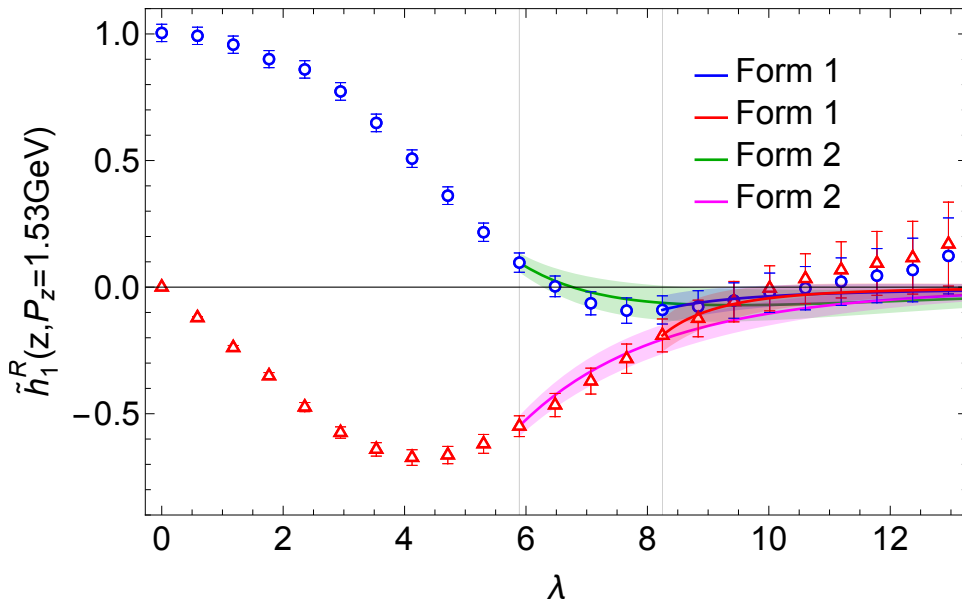
m_0 determined by matching condition:

$$\frac{\tilde{h}_1^B(z, 0, a)}{\tilde{h}_1^B(z', 0, a)} e^{|z - z'|(\delta m(a) + m_0)} = \frac{C_0(z, \mu_0) e^{-\mathcal{I}(\mu_0)}}{C_0(z', \mu'_0) e^{-\mathcal{I}(\mu'_0)}}$$

$$\mathcal{I}(\mu) = \int d\alpha \frac{\gamma(\alpha)}{\beta(\alpha)} \Big|_{\alpha = \alpha_s(\mu)}$$

$$\mu_0 = 2\kappa e^{-\gamma_E}/z \quad \mu'_0 = 2\kappa e^{-\gamma_E}/z'$$

C_0 from LRR with PV prescription

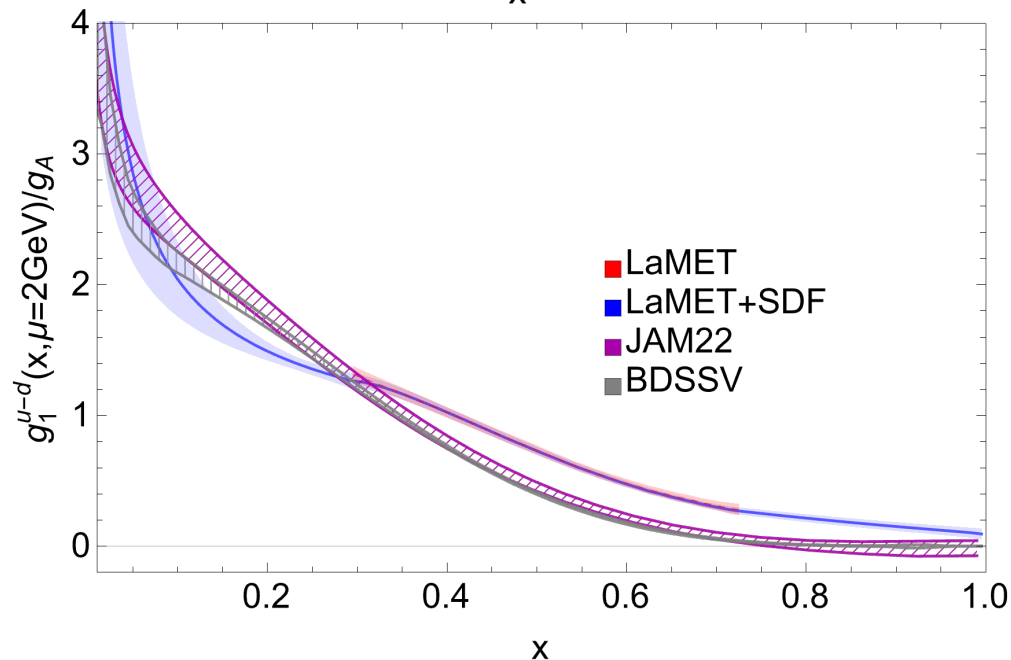
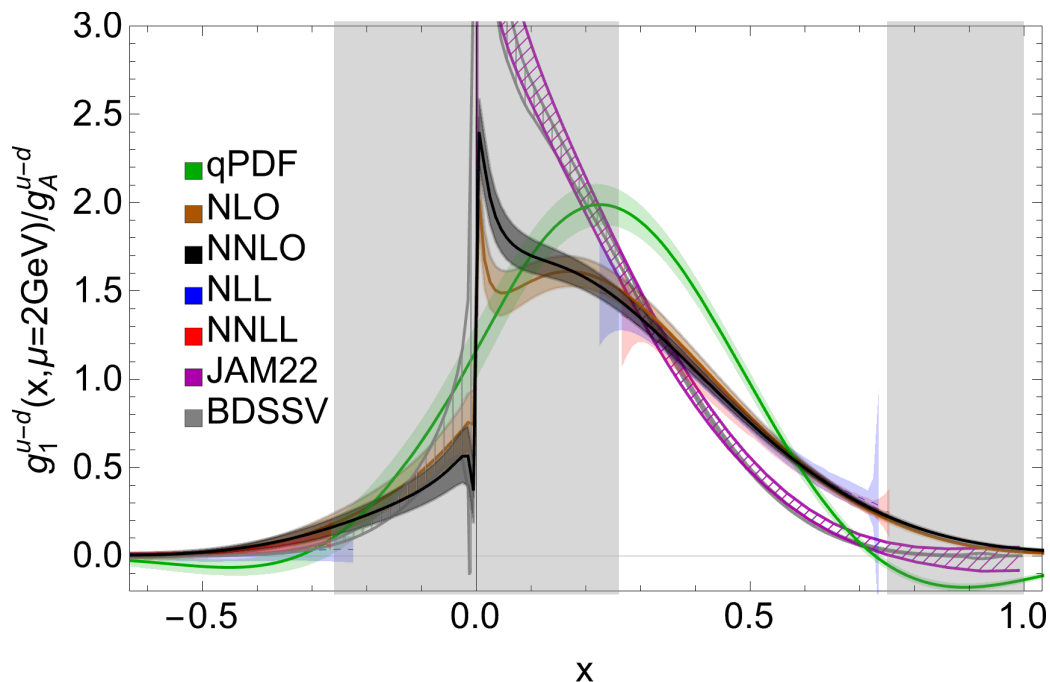


little effect on
qPDF from fit forms

Use fits to physically motivated at large loffe time

$$\tilde{h}_1^{\text{asy}}(\lambda, P_z) = \frac{A e^{-m_{\text{eff}} \lambda / P_z}}{|\lambda|^d} \quad \tilde{h}_1^{\text{asy}}(z, P_z) = \left(A e^{i\phi \text{sgn}(z)} + \frac{A' e^{i\phi' \text{sgn}(z)}}{|z|} \right) e^{-|z|\Lambda}$$

Determination of helicity PDF in LaMET



$C^{-1}(x/y, \mu/P_z)$ is calculated up to NNLO with and without threshold resummation.

Y. Su, J. Holligan, X. Ji, F. Yao, J.H. Zhang, R. Zhang, NPB 991 (2023) 1162901

good control over uncertainties for intermediate x but disagreement with global analysis $x > 0.4$

Combine LaMET with SDF for large and small x

$$h(z^2, z \cdot P) = \int_{-1}^1 K(x\omega, z^2, \mu) g_1(x, \mu) dx,$$

$$g_1(x) = \begin{cases} f(x_0) \frac{x^a}{x_0^a}, & x < x_0 \\ f(x), & x_0 < x < 1 - x_0 \\ f(1 - x_0) \frac{(1-x)^b}{(1-x_0)^b}, & x > 1 - x_0 \end{cases}$$

Nucleon PDF from LaMET in Coulomb Gauge

$$\tilde{f}_{\tilde{\Gamma}}(x, P^z; \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \tilde{h}_{\tilde{\Gamma}}(z, P^z, \mu),$$

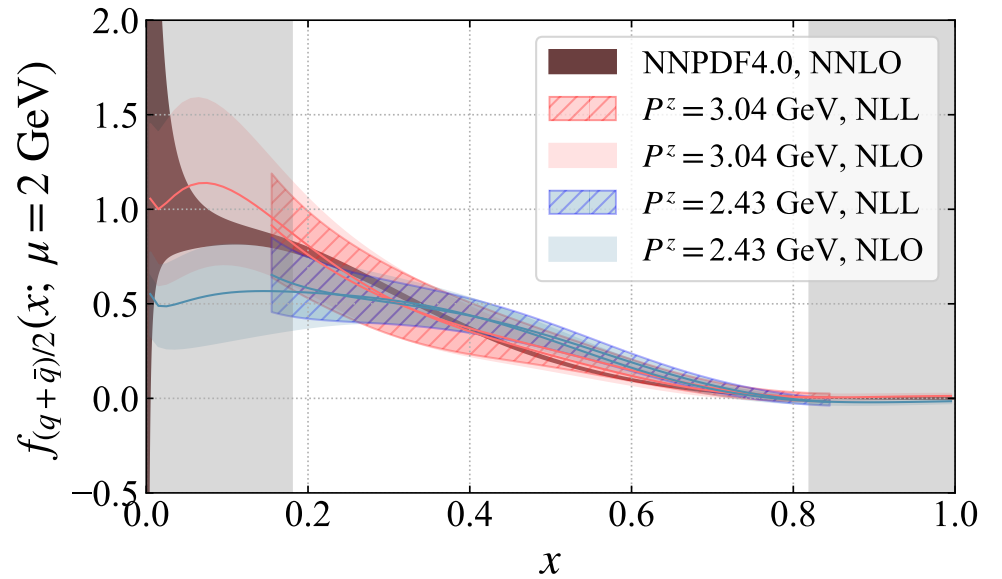
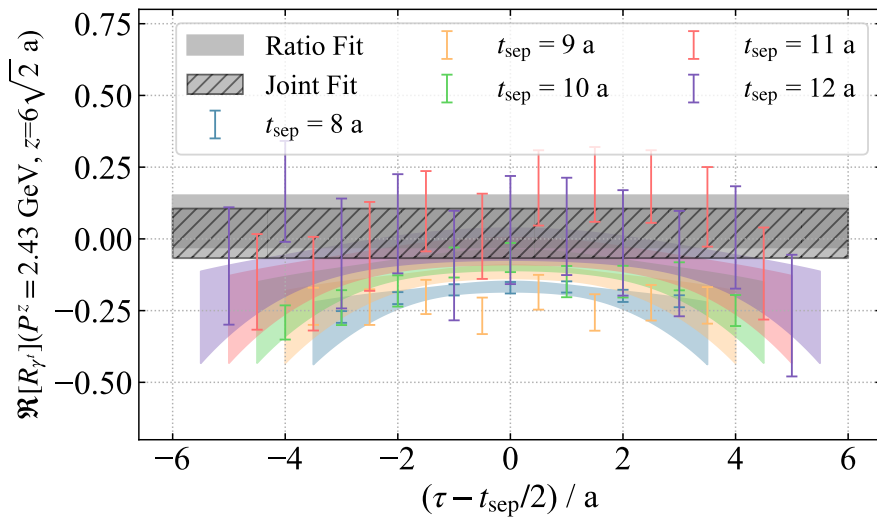
No gauge links \Rightarrow less noise
 \Rightarrow larger P_z

$$\tilde{h}_{\tilde{\Gamma}}(z, P^z; \mu) \equiv \frac{1}{2N_{\tilde{\Gamma}}} \langle PS | \bar{\psi}(z) \tilde{\Gamma} \psi(0) |_{\vec{\nabla} \cdot \vec{A}=0} | PS \rangle$$

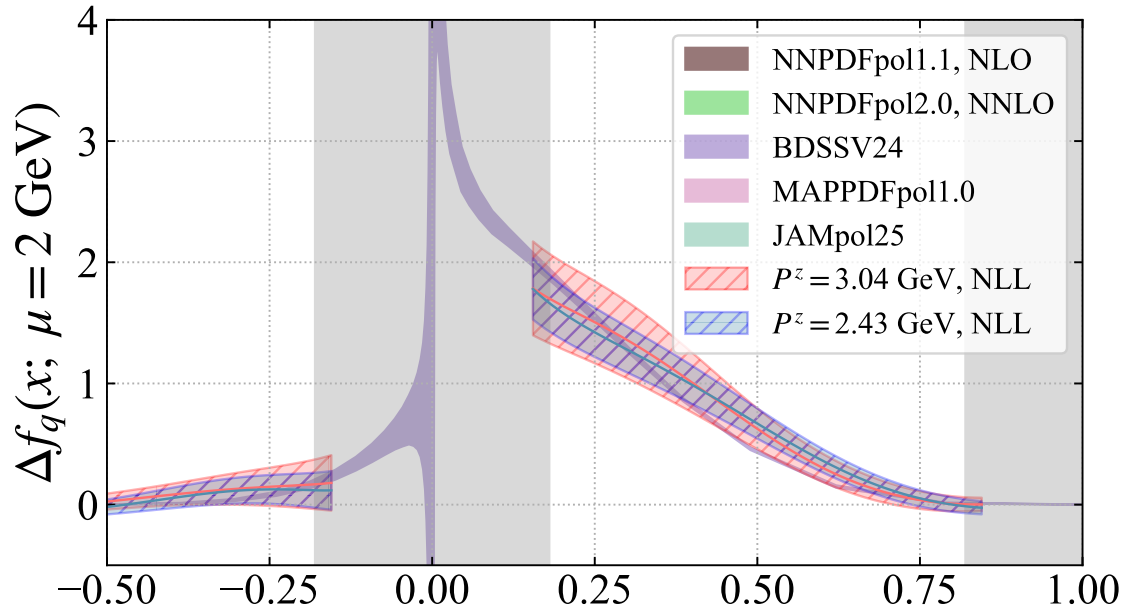
CG: $a = 0.06$ fm, $m_\pi = 300$ MeV, $P_z = 2.43, 3.04$ GeV

Matching is known up to NLO:

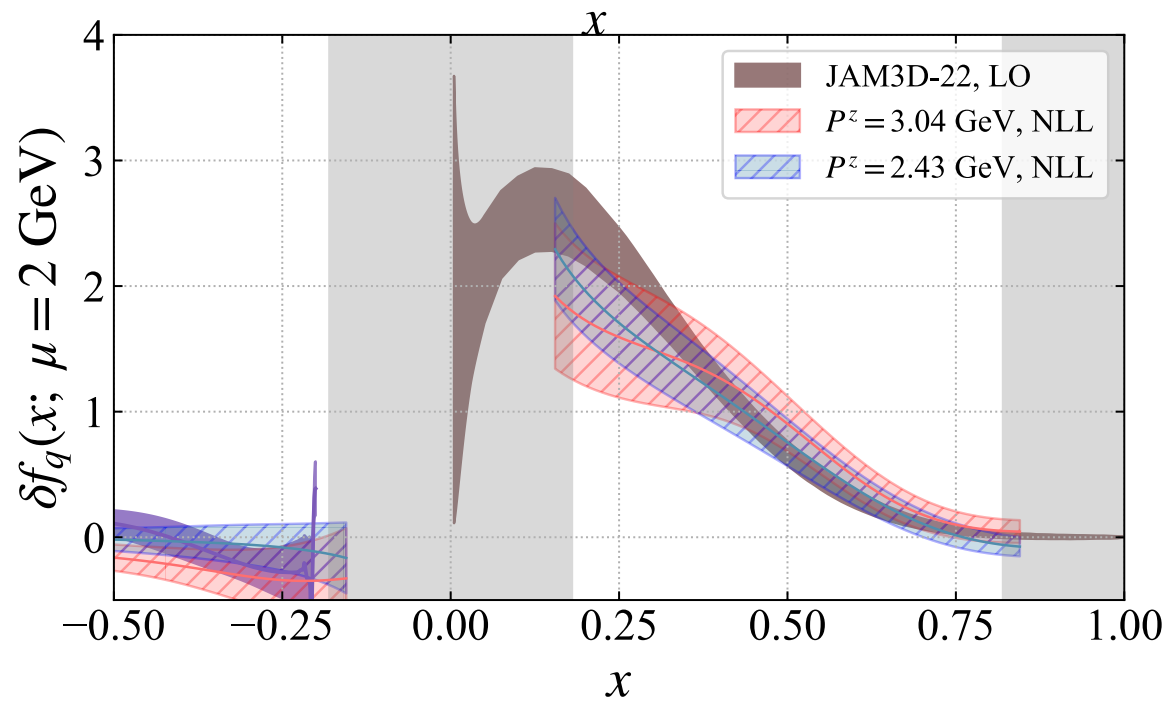
Hybrid scheme:
$$\tilde{h}_{\tilde{\Gamma}}^{\text{hyb.}}(z, P^z; z_s) = \frac{\tilde{h}_{\tilde{\Gamma}}^0(z, P^z; a)}{\tilde{h}_{\tilde{\Gamma}}^0(z, 0; a)} \theta(z_s - |z|) + \frac{\tilde{h}_{\tilde{\Gamma}}^0(z, P^z; a)}{\tilde{h}_{\tilde{\Gamma}}^0(z_s, 0; a)} \theta(|z| - z_s)$$



Helicity and Transversity PDF in Coulomb Gauge



No P_z dependence within estimated uncertainties



Reasonably good agreement with global analysis

Summary

- Nucleon PDF can be studied on the lattice using LaMET approach down to physical light quark masses
 - The first two moment of helicity PDF are larger than the global analysis and helicity PDF is larger than global analysis result for $x > 0.4$ when momentum boost of 1.5 GeV
 - The moment of genuine twist-3 contribution to g_T has been determined to be very small on the lattice through short distance factorization
 - The use of quasi-PDF operators in Coulomb gauge allows to go to much larger momentum boost leading to better agreement with the global analysis:
- => while not all systematic uncertainties in the LaMET calculations are under control the smallness of momentum boost could be the most important sources of systematic errors.