Light-Cone 2026: Applications at EIC era (LC2026)

Relativistic centers and spatial distributions of angular momentum in the nucleon

Ho-Yeon Won

Supervisor: Cédric Lorcé

CPHT, CNRS, École Polytechnique, France

Based on:

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

Lorcé, Mukherjee, Singh, and Won, in preparation

Contents

operator level

distribution level

Introduction

Nucleon Spin

Citation: F. Takahashi *et al.* (Particle Data Group), Int. J. Mod. Phys. A **41**, 2630011 (2026)

N BARYONS
$(S = 0,\ I = 1/2)$

$p,\ N^+ = uud; \quad n,\ N^0 = udd$

p

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.0072764665789 \pm 0.0000000000083$ u

Mass $m = 938.27208943 \pm 0.00000029$ MeV [a]

$\left|m_p - m_{\overline{p}}\right|/m_p < 7 \times 10^{-10}$, CL = 90% [b]

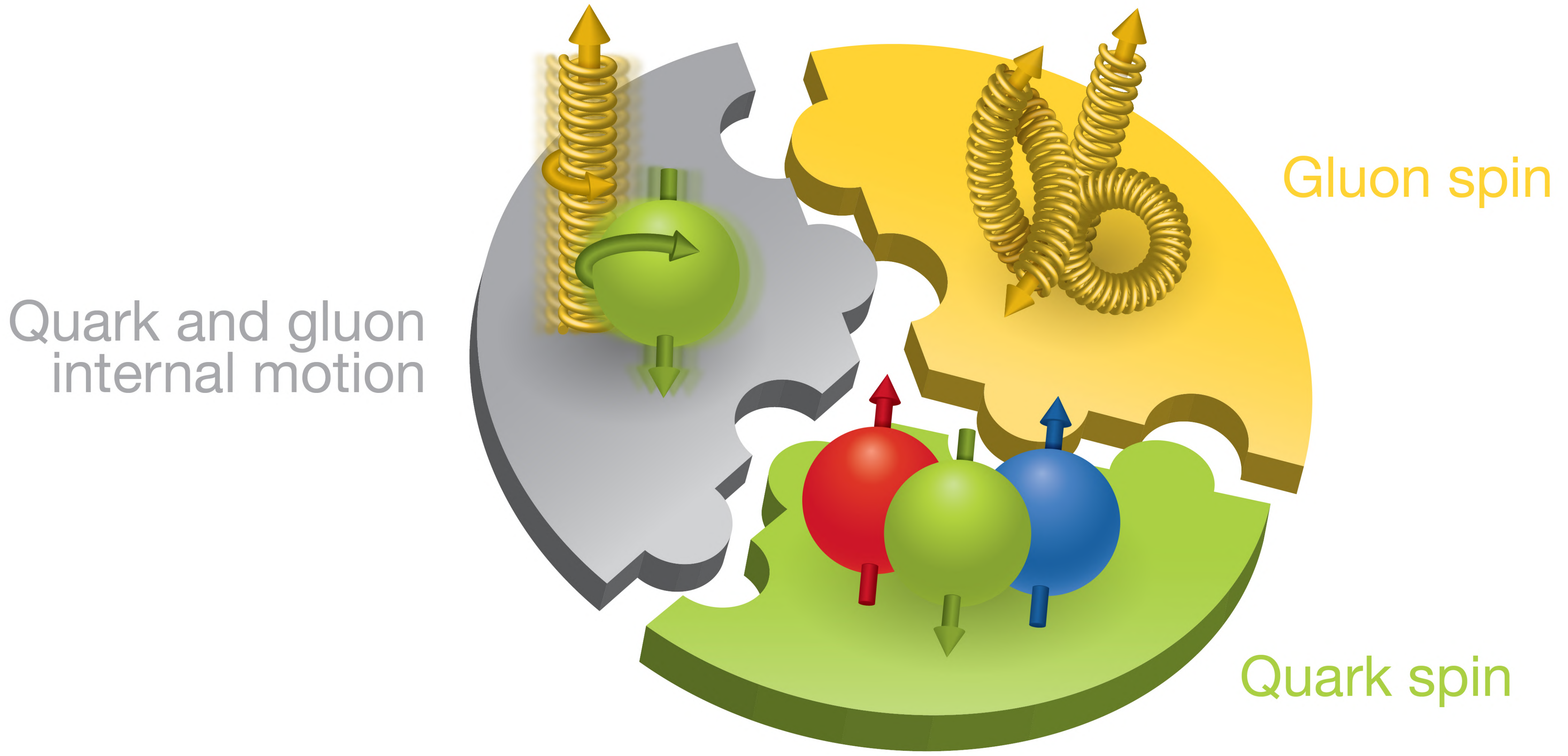
$\left|\frac{q_{\overline{p}}}{m_{\overline{p}}}\right|/(\frac{q_p}{m_p}) = 1.000000000003 \pm 0.000000000016$

$\left|q_p + q_{\overline{p}}\right|/e < 7 \times 10^{-10}$, CL = 90% [b]

How does the nucleon get spin-1/2?

Electron-Ion Collider (EIC) project

Images: BNL



- The origin of the nucleon spin
- Quarks and gluons distributions in position space

Abdul Khalek et al., NPA 1026 (2022)

Transverse Spin Sum Rule

“Local” operator level

Ji, PRL 78 (1997)

Gauge-invariant spin decomposition

$$\boldsymbol{J} = \boldsymbol{L}_q + \boldsymbol{S}_q + \boldsymbol{J}_G$$

quark orbital angular momentum (OAM)

quark intrinsic spin

gluon total angular momentum (TAM, $J = L + S$)

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Hadronic level

Longitudinal component $\because [J^a, K^3] = -i\epsilon^{ab3}K^b$

$$\langle N, s'|J^z|N, s\rangle = \frac{\sigma^z_{s's}}{2}$$

Transverse components

$$\langle N, s'|J^a|N, s\rangle = \begin{cases} \dfrac{\sigma^a_{s's}}{2} \\ \dfrac{E_P}{M}\dfrac{\sigma^a_{s's}}{2} \end{cases}$$

Bakker, Leader, and Trueman, PRD 70 (2004)
Leader, PRD 85 (2012)

Ji and Yuan, PLB 810 (2020)

with $E_P = \sqrt{M^2 + (P^z)^2}$

Boost-dependence

Transverse Spin Sum Rule

"Local" operator level

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gluon total angular momentum (TAM, $J = L + S$)

*We impose the symmetric operator product, i.e., AB:=(AB+BA)/2.

Hadronic level

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$$\langle N, s' | J^z | N, s \rangle = \frac{\sigma^z_{s's}}{2}$$

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Boost-dependence

Internal TAM & relativistic centers

Lorcé, EPJC 78 (2018)
Lorcé, EPJC 81 (2021)

$$\boldsymbol{J}_X = \boldsymbol{J} - \boldsymbol{R}_X \times \boldsymbol{P}^*$$

relativistic centers of energy, mass, and spin

Transverse components

$$\langle N, s' | J^a_X | N, s \rangle = \begin{cases} \dfrac{\sigma^a_{s's}}{2} & \text{for } R_c \\ \dfrac{E_P}{M} \dfrac{\sigma^a_{s's}}{2} & \text{for } R_M \\ \dfrac{M}{E_P} \dfrac{\sigma^a_{s's}}{2} & \text{for } R_E \end{cases}$$

New sum rule

Today's subject: position mapping

Internal Angular Momentum

Angular Momentum

Leader and Lorcé, PR 541 (2014)
Lorcé, Mantovani, and Pasquini, PLB 776 (2018)

Generalized angular momentum tensor densities

$$J^{\mu\alpha\beta}(x) = L^{\mu\alpha\beta}(x) + S^{\mu\alpha\beta}(x),$$

Generalized OAM tensor

$$L^{\mu\alpha\beta} = x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha},$$

asymmetric energy-momentum tensor (EMT)

Generalized OAM tensor

$$S^{\mu\alpha\beta} = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\bar{\psi}\gamma_\lambda\gamma_5\psi.$$

Angular momentum operator

$$J^{\alpha\beta} = \int_\Sigma d\Sigma_\mu \, J^{\mu\alpha\beta}(x),$$

$L^{\alpha\beta}, S^{\alpha\beta}$ same as $J^{\alpha\beta}$

$$d\Sigma_\mu = n_\mu \, d\Sigma,$$

quantization surface and its normal vector

On the hypersurface at x^0 = constant

Total angular momentum (TAM)

$$J^i = \frac{1}{2}\epsilon^{ijk}J^{jk} = L^i + S^i$$

Today: transverse components

Boost

$$K^i = \int d^3x \, J^{00i}(x)$$

Kogut and Soper, PRD 1 (1970)

On the hypersurface at x^+ = constant

$$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

Light-Front (LF) TAM

$$\mathcal{J}^a = \epsilon_\perp^{ab} J^{-b} = \mathcal{L}^a + \mathcal{S}^a,$$ transverse

$$\mathcal{J}^+ = \frac{1}{2}\epsilon_\perp^{ab} J^{ab} = \mathcal{L}^+ + \mathcal{S}^+$$ longitudinal

dynamical

LF boost

$$\mathcal{B}^a = J^{+a}$$ transverse

$$\mathcal{B}^+ = J^{+-},$$ longitudinal

Other LF AM operators

Harindranath, Mukherjee, and Ratabole, PRD 63 (2001)

Center of Rotation and Angular Momentum

Orbital Angular Momentum

$$L^i = \frac{1}{2}\epsilon^{ijk}\int d^3x\, L^{0jk}(x) = \epsilon^{ijk}\int d^3x\, x^j T^{0k}(x),$$

Gauge-invariant and asymmetric EMT

$$T^{\mu\nu} = \bar{\psi}\gamma^\mu \frac{i}{2}\overleftrightarrow{D}^\nu\psi + F^{a,\mu\lambda}F^{a,}{}_\lambda{}^\nu - \frac{1}{4}g^{\mu\nu}F^{a,\lambda\rho}F^a{}_{\rho\lambda}$$

Conventionally, the position vector defined from the origin of the coordinate system

The "relative" position vector relative to the pivot (center of rotation) & arbitrary choice, not necessarily the origin

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Conventionally, the position vector defined from the origin of the coordinate system

The “relative” position vector relative to the pivot (center of rotation) & arbitrary choice, not necessarily the origin

Lorcé, EPJC 81 (2021)

Internal Angular Momentum

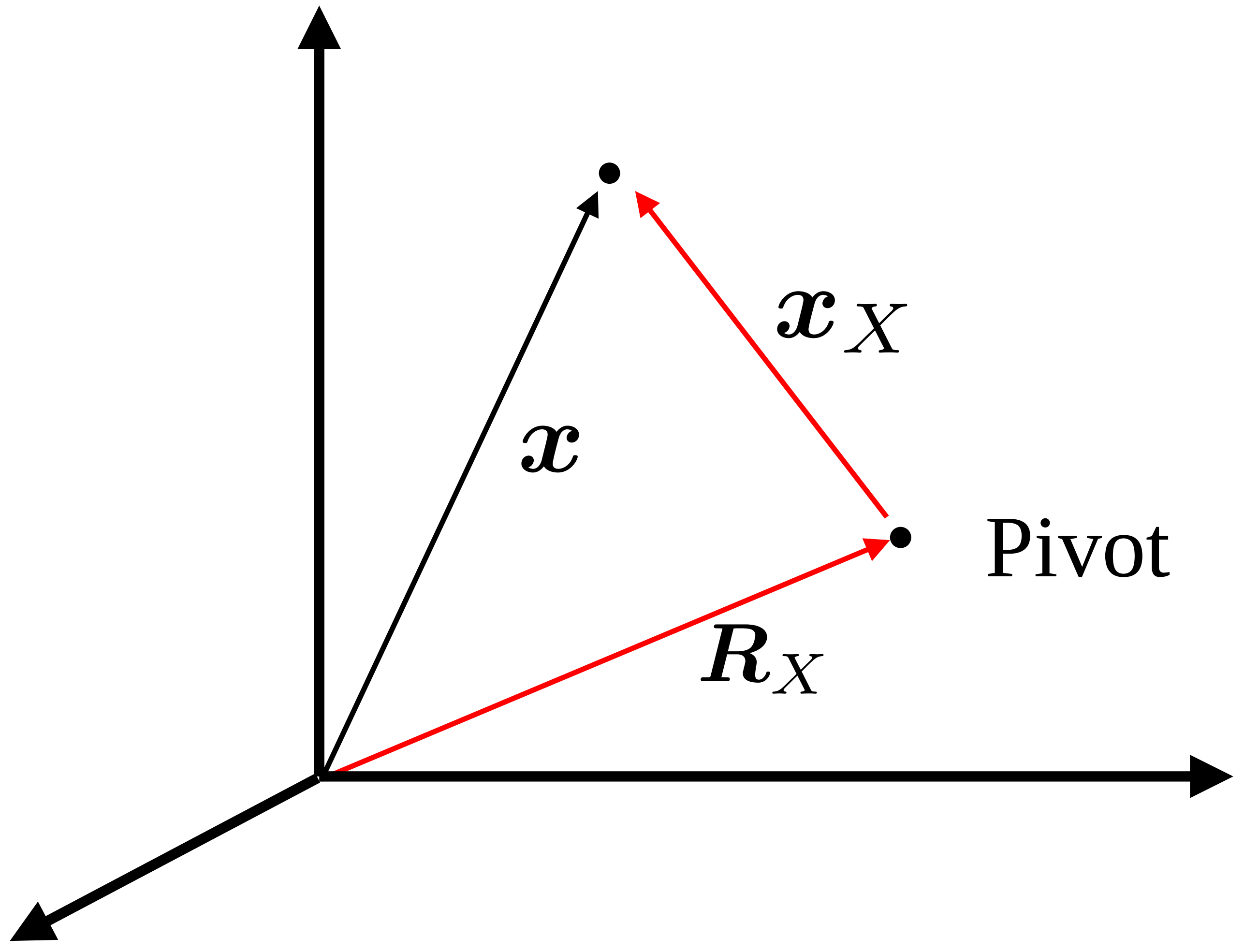
$$\boldsymbol{J}_X = \boldsymbol{J} - \boldsymbol{R}_X \times \boldsymbol{P}$$

origin — external TAM

$$\boldsymbol{K}_X = \boldsymbol{K} - (R_X^0\boldsymbol{P} - \boldsymbol{R}_X P^0)$$

origin — external boost

kind of parallel-axis theorem



Relativistic Centers

Table: Cédric Lorcé

Pauli-Lubanski pseudo-vector: $J^i_E = W^i/P^0$

Qualifications of the position operator (Canonical relation, Vector under rotation); Qualification of the localized state in Newton-Wigner sense (Compatibility of components); Qualification of the internal AM operator (su(2) spin algebra)

Relativistic center	Position operator	Canonical relation $[R^i_X, P^j] = i\delta^{ij}$	Vector under rotation $[R^i_X, J^j_X] = i\epsilon^{ijk} R^k_X$	Compatibility of components $[R^i_X, R^j_X] = 0$	su(2) spin algebra $[J^i_X, J^j_X] = i\epsilon^{ijk} J^k_X$	
Energy	$R^i_E \overset{r^0=0}{=} \frac{1}{P^0}\int d^3r\, r^i T^{00} = -\frac{K^i}{P^0}$	✓	✓	✗	✗	
Mass	$R^\mu_M = \Lambda^\mu{}_\nu R^\nu_E\vert_{\text{rest}}$	✓	✓	✗	✗	3D
Spin	$R^i_c = \frac{P^0 R^i_E + M R^i_M}{P^0 + M}$	✓	✓	✓	✓	canonical center
Light-Front momentum	$R^a_n \overset{r^+=0}{=} \frac{1}{P^+}\int d^2r_\perp dr^-\, r^a_\perp T^{++} = -\frac{B^a}{P^+}$	✓	✓	✓	✗	2D

$\boldsymbol{R}_c\,|\boldsymbol{r}\rangle = \boldsymbol{r}\,|\boldsymbol{r}\rangle$ & $\boldsymbol{R}_n\,|\boldsymbol{r}_\perp\rangle = \boldsymbol{r}_\perp\,|\boldsymbol{r}_\perp\rangle$

In the infinite-momentum frame (IMF)

$$R^\mu_n = \lim_{|\boldsymbol{P}|\to\infty} R^\mu_E = \lim_{|\boldsymbol{P}|\to\infty} R^\mu_c$$

Pryce, PRSLA 195 (1948)
Newton, and Wigner, RMP 21 (1949)
Fleming, PR 137 (1965)
Murkardt, PRD 72 (2005)
Lorcé, EPJC 78 (2018)
Lorcé, EPJC 81 (2021)

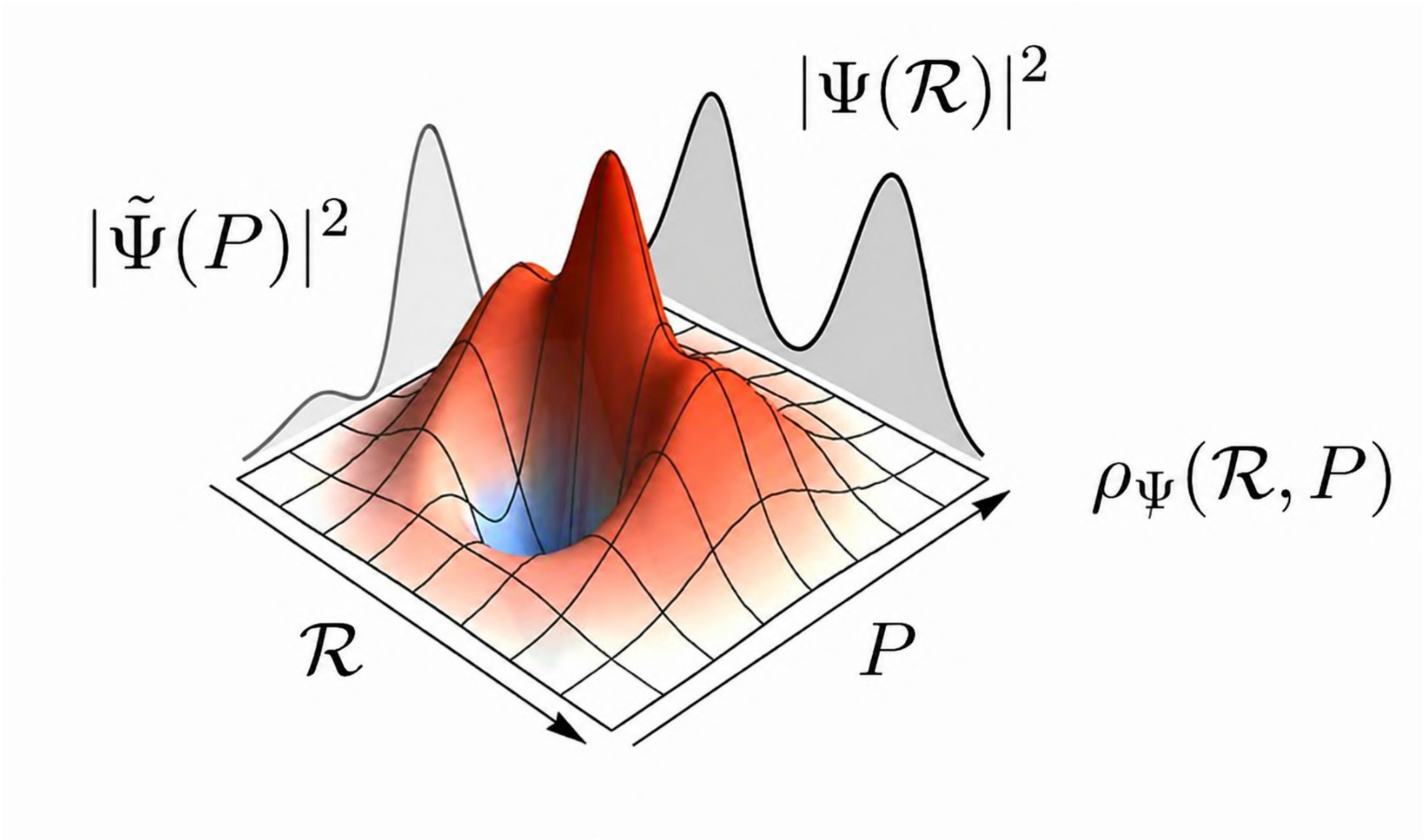
Spatial Distributions

Quantum Phase-Space Formalism

Hadronic matrix element

In quantum phase-space formalism

$$\langle\Psi|O(\boldsymbol{x})|\Psi\rangle = \sum_{s',s}\int\frac{d^3P}{(2\pi)^3}\int d^3\mathcal{R}\,\rho_\Psi^{s's}(\boldsymbol{\mathcal{R}},\boldsymbol{P})\,\langle O(\boldsymbol{x})\rangle_{\boldsymbol{\mathcal{R}},\boldsymbol{P}}^{s's},$$



Wigner, PR40 (1932)
Hillery, O'Connell, Scully, and Wigner, PR106 (1984)
Bialynicki-Birula, Gornicki, and Rafelski, PRD 44 (1991)

I. Wigner distribution (hadronic wave packet)

$$\rho_\Psi^{s's}(\boldsymbol{\mathcal{R}},\boldsymbol{P}) = \int\frac{d^3q}{(2\pi)^3}\,e^{i\boldsymbol{q}\cdot\boldsymbol{\mathcal{R}}}\widetilde{\Psi}^*\left(\boldsymbol{P}+\frac{\boldsymbol{q}}{2},s'\right)\widetilde{\Psi}\left(\boldsymbol{P}-\frac{\boldsymbol{q}}{2},s\right),$$

$$= \int d^3Y\,e^{i\boldsymbol{P}\cdot\boldsymbol{Y}}\,\Psi^*\left(\boldsymbol{\mathcal{R}}-\frac{\boldsymbol{Y}}{2},s'\right)\Psi\left(\boldsymbol{\mathcal{R}}+\frac{\boldsymbol{Y}}{2},s\right),$$

quasi-density

$$\int d^3\mathcal{R}\,\rho_\Psi^{ss}(\boldsymbol{\mathcal{R}},\boldsymbol{P}) = \left|\widetilde{\Psi}(\boldsymbol{P},s)\right|^2,\quad \int\frac{d^3P}{(2\pi)^3}\,\rho_\Psi^{ss}(\boldsymbol{\mathcal{R}},\boldsymbol{P}) = \left|\Psi(\boldsymbol{\mathcal{R}},s)\right|^2$$

II. Phase-space amplitude (internal distribution)

$$\langle O(\boldsymbol{x})\rangle_{\boldsymbol{\mathcal{R}},\boldsymbol{P}}^{s's} = \int\frac{d^3\Delta}{(2\pi)^3}\,e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\frac{\langle P+\frac{\Delta}{2},s'|O(0)|P-\frac{\Delta}{2},s\rangle}{\sqrt{2(P^0+\Delta^0/2)}\sqrt{2(P^0-\Delta^0/2)}},$$

3D Spatial distribution in the relative coordinate $\boldsymbol{X}=\boldsymbol{x}-\boldsymbol{\mathcal{R}}$

$$O(\boldsymbol{X};s',s) := \langle O(\boldsymbol{x})\rangle_{\boldsymbol{\mathcal{R}},\boldsymbol{P}}^{s's}$$

2D spatial distribution

$$O(\boldsymbol{X}_\perp;s',s) := \int dX^3 O(\boldsymbol{X};s',s)$$

Lorcé, Moutarde, and Trawiński, EPJC 79 (2019)
Lorcé, PRL 125 (2020)
Chen and Lorcé, PRD 106 (2022)
Chen and Lorcé, PRD 107 (2023)
Won and Lorcé, PRD 111 (2025)
Chen, JHEP 04 (2025)
Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)
Won and Lorcé, PRD XXX (2026) (accepted)

Image: Cédric Lorcé

Generic Frame

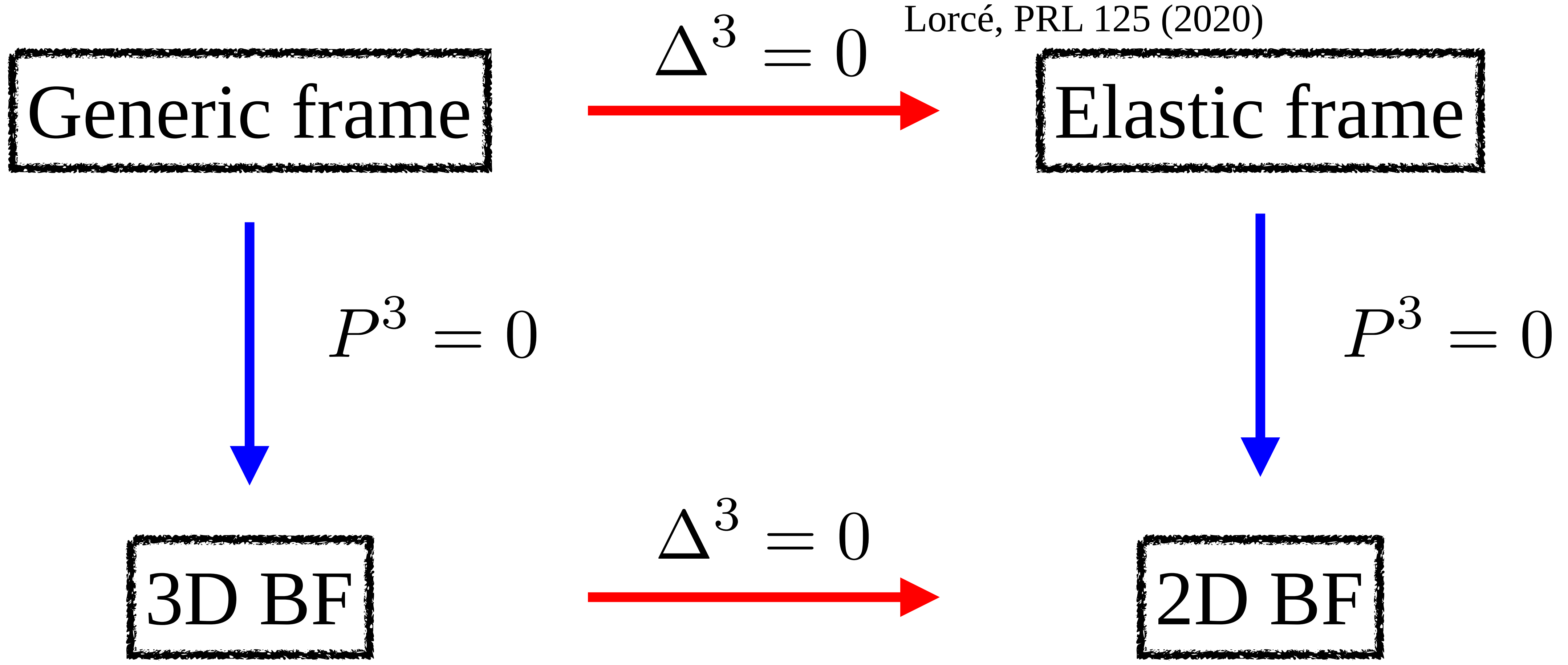
I. A generic Lorentz frame in the 3D space

Connection between the rest frame and the infinite-momentum frame

$$P = (P^0, \mathbf{0}_\perp, P^3), \qquad \Delta = (\Delta^0, \mathbf{\Delta}_\perp, \Delta^3)$$

Frame hierarchy

Lorcé, Mantovani, and Pasquini, PLB 776 (2018)
Lorcé, PRL 125 (2020)



Lorcé, Moutarde, and Trawiński, EPJC 79 (2019)

Non-zero energy transfer despite the on-shell

$$p^2 = p'^2 = M^2 \quad \rightarrow \quad P \cdot \Delta = 0 \quad \rightarrow \quad \Delta^0 = \boldsymbol{P} \cdot \mathbf{\Delta} / P^0 \neq 0$$

Generic Frame

Durand, DeCelles, and Marr, PR 126 (1962)
Polyzou, Glökle, and Witala, FBS 54 (2013)

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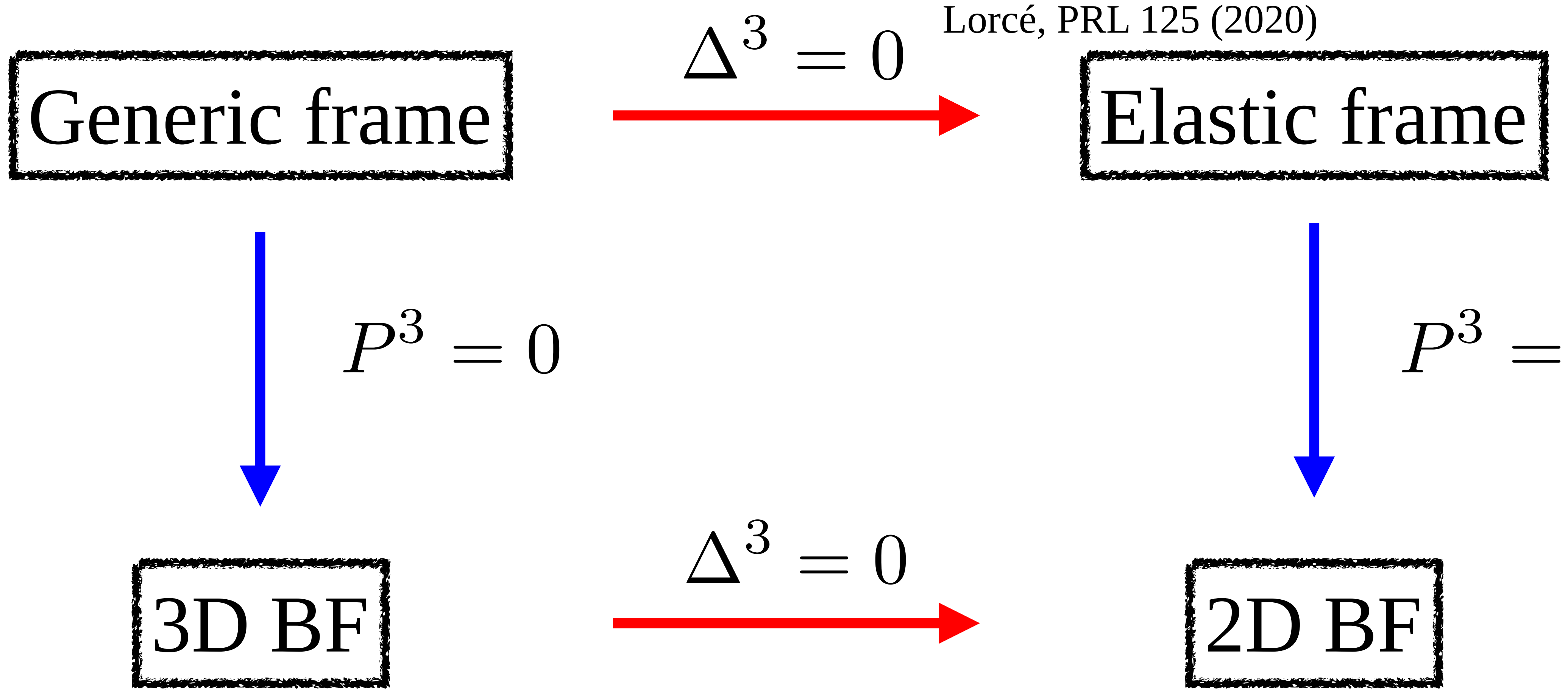

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$$p^2 = p'^2 = M^2 \quad \to \quad P \cdot \Delta = 0 \quad \to \quad \Delta^0 = \boldsymbol{P} \cdot \boldsymbol{\Delta} / P^0 \neq 0$$

II. Lorentz transformation of matrix elements

For spin-j target $\qquad \because [K^i, K^j] = -i\varepsilon^{ijk} J^k$

$$\langle p', s' | O^{\mu_1 \cdots \mu_n} | p, s \rangle = \sum_{s'_{\mathrm{BF}}, s_{\mathrm{BF}}} \boxed{D^{(j)}_{s_{\mathrm{BF}} s}(p_{\mathrm{BF}}, \Lambda)\, D^{*(j')}_{s'_{\mathrm{BF}} s'}(p'_{\mathrm{BF}}, \Lambda)} \\ \times \Lambda^{\mu_1}{}_{\alpha_1} \cdots \Lambda^{\mu_n}{}_{\alpha_n} \langle p'_{\mathrm{BF}}, s'_{\mathrm{BF}} | \hat{O}^{\alpha_1 \cdots \alpha_n} | p_{\mathrm{BF}}, s_{\mathrm{BF}} \rangle$$

Wigner rotation

Wigner rotation and Melosh rotation $\quad p_f = \Lambda p_i$

$$|p_f, s_f\rangle = \sum_{s_i} U(p_i)\, |p_i, s_i\rangle\, D_{s_i s_f}(p_i, \Lambda), \qquad |p, \lambda\rangle_{\mathrm{LF}} = \sum_s |p, s\rangle_{\mathrm{IF}}\, \mathcal{M}_{s\lambda}(p),$$

* IMF Wigner rotation and BF Melosh rotation

$$\lim_{p^3 \to \infty} D(p, \Lambda) = \lim_{p^3 \to 0} \mathcal{M}(p)$$

* IMF Melosh rotation

$$\lim_{p^3 \to \infty} \mathcal{M}_{s\lambda}(p) = \delta_{s\lambda}$$

Since $\Delta^3 \neq 0$, Wigner angles θ and θ' for initial and final states, respectively

$$\cos\frac{\theta}{2} \neq \cos\frac{\theta'}{2}, \qquad \sin\frac{\theta}{2} \neq \sin\frac{\theta'}{2}$$

Parametrization

Lorcé, Mantovani, and Pasquini, PLB 776 (2018)
V.D. Burkert et al., RMP 95 (2023)
Won, and Lorcé, PRD 111 (2025)

Matrix elements of the axial-vector current and EMT

$$\langle p', s' | O(0) | p, s \rangle = \bar{u}(p', s') \Gamma[O] u(p, s),$$

$$\Gamma[V_5^\mu] = \gamma^\mu \gamma_5 G_A(t) + \frac{\gamma_5 \Delta^\mu}{2M} G_P(t), \qquad V_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi,$$

$$\Gamma[T_a^{\mu\nu}] = \frac{P^\mu P^\nu}{M} A^a + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D^a + M g^{\mu\nu} \bar{C}^a + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} J^a - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M} S^a, \qquad a = q,\ G$$

Teryaev, hep-ph/9904376, (1999)
Leader and Lorcé, PR 541, (2014)
Lowdon, Chiu, and Brodsky, PLB 774 (2017)
Cotogno, Lorcé, and Lowdon, PRD 100 (2019)

Poincaré symmetry

$$A^q(0) + A^G(0) = 1, \quad J^q(0) + J^G(0) = 1/2, \quad \bar{C}^q(t) + \bar{C}^G(t) = 0$$

Parametrization

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Matrix elements of the axial-vector current and EMT

$$\langle p', s'|O\,(0)|p, s\rangle = \bar{u}\,(p', s')\,\Gamma[O]u\,(p, s)\,,$$

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$$\Gamma[T_a^{\mu\nu}] = \frac{P^\mu P^\nu}{M}A^a + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}D^a + Mg^{\mu\nu}\bar{C}^a + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M}J^a - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M}S^a, \qquad a = q,\ G$$

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Disclaimer: The following analysis is model-independent and concerns only the kinematical structure

Hackett, Pefkou, and Shanahan, PRL 132 (2024)

- Multipole fitting for $A, J, D, \bar{C}$ based on lattice data
- Multipole fitting for S   Won and Lorcé, PRD 111 (2025)

- QCD equation of motion for $G_A(t)$

$$T_q^{[\mu\nu]} = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha V_{5,\beta} \longrightarrow S^q(t) = \frac{1}{2}G_A(t)$$

- Imposing pion-pole dominance for $G_P(t)$

$$G_P(t) = \frac{4M^2}{M_\pi^2 - t}G_A(t)$$

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

"Transverse" components of internal TAM distribution in the transverse plane

$$J_X^a(\boldsymbol{b}_\perp, P^z; s', s) = J^a(\boldsymbol{b}_\perp, P^z; s', s) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} R_X^b(s', s'')P^3(\boldsymbol{b}_\perp, P^z; s'', s) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} P^3(\boldsymbol{b}_\perp, P^z; s', s'')R_X^b(s'', s),$$

$\boldsymbol{b}_\perp = \boldsymbol{x}_\perp - \boldsymbol{\mathcal{R}}_\perp$

Origin External

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

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$\boldsymbol{b}_\perp = \boldsymbol{x}_\perp - \boldsymbol{\mathcal{R}}_\perp$

TAM relative to the origin

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

$$J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = L^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) + S^a\left(\boldsymbol{b}_\perp, P^z; s', s\right)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\epsilon^{ab}_\perp \sqrt{\tau} X^b_1 \tilde{J}_1 + \sigma^a_{s's}\tilde{J}_0 + \sigma^b_{s's}\tau X^{ab}_2 \tilde{J}_2\right],$$

spin-dependent: monopole

spin-independent dipole

quadrupole

$$\tilde{J}_0 = -4MP_z\frac{d}{dt}\left[\frac{P^0}{2M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(A-2J\right)\right] - \frac{1}{4}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau D + \frac{1}{2}\frac{1}{\gamma}\cos\theta\left(J - S + \frac{1}{2}G_A\right) + \frac{M}{4P^0}\left(G_A - \tau G_P\right)$$
$$- 2M^2\frac{d}{dt}\left[\tau\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{\tau}{\gamma}\cos\theta S\right],$$
$$\tilde{J}_1 = 4MP^z\frac{d}{dt}\left(\frac{P^0}{M}\frac{1}{\gamma}\cos\theta A + \frac{1}{\beta}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}2\tau J\right) + \frac{1}{2}\frac{\beta}{\gamma}\cos\theta D + \frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\frac{1}{2}\left(J - S + G_A\right)$$
$$+ 4M^2\frac{d}{dt}\left[\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(J+S\right)\right],$$
$$\tilde{J}_2 = 4MP^z\frac{d}{dt}\left[\frac{P^0}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\left(A-2J\right)\right] + \frac{1}{2}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}D + \frac{M}{2P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}G_A - \frac{M}{2P^0}G_P$$
$$+ 4M^2\frac{d}{dt}\left[\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{1}{\gamma}\cos\theta S\right]$$

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

"Transverse" components of internal TAM distribution in the transverse plane

$$J_X^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} R_X^b(s', s'')P^3\left(\boldsymbol{b}_\perp, P^z; s'', s\right) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} P^3(\boldsymbol{b}_\perp, P^z; s', s'')R_X^b\left(s'', s\right),$$

$\boldsymbol{b}_\perp = \boldsymbol{x}_\perp - \boldsymbol{\mathcal{R}}_\perp$

TAM relative to the origin

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

$$J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = L^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) + S^a\left(\boldsymbol{b}_\perp, P^z; s', s\right)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\epsilon_\perp^{ab}\sqrt{\tau}X_1^b\tilde{J}_1 + \sigma_{s's}^a\tilde{J}_0 + \sigma_{s's}^b\tau X_2^{ab}\tilde{J}_2\right],$$

spin-independent dipole; spin-dependent: monopole; quadrupole

$$\tilde{J}_0 = -4MP_z\frac{d}{dt}\left[\frac{P^0}{2M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(A-2J\right)\right] - \frac{1}{4}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau D + \frac{1}{2}\frac{1}{\gamma}\cos\theta\left(J - S + \frac{1}{2}G_A\right) + \frac{M}{4P^0}\left(G_A - \tau G_P\right)$$
$$- 2M^2\frac{d}{dt}\left[\tau\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{\tau}{\gamma}\cos\theta S\right],$$

$$\tilde{J}_1 = 4MP^z\frac{d}{dt}\left(\frac{P^0}{M}\frac{1}{\gamma}\cos\theta A + \frac{1}{\beta}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}2\tau J\right) + \frac{1}{2}\frac{\beta}{\gamma}\cos\theta D + \frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\frac{1}{2}\left(J - S + G_A\right)$$
$$+ 4M^2\frac{d}{dt}\left[\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(J+S\right)\right],$$

$$\tilde{J}_2 = 4MP^z\frac{d}{dt}\left[\frac{P^0}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\left(A-2J\right)\right] + \frac{1}{2}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}D + \frac{M}{2P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}G_A - \frac{M}{2P^0}G_P$$
$$+ 4M^2\frac{d}{dt}\left[\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{1}{\gamma}\cos\theta S\right]$$

Relativistic momentum distribution

$$P^k\left(\boldsymbol{b}_\perp, P^z; s'', s\right) = \int dx^3\left[\int \frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\left.\frac{\langle p', s''|T^{0k}(0)|p, s\rangle}{2\sqrt{p'^0p^0}}\right|_{\mathrm{GF}}\right]$$

Won and Lorcé, PRD 111 (2025)

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

"Transverse" components of internal TAM distribution in the transverse plane

$$J_X^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} R_X^b(s', s'')P^3\left(\boldsymbol{b}_\perp, P^z; s'', s\right) - \frac{1}{2}\epsilon_\perp^{ab}\sum_{s''} P^3(\boldsymbol{b}_\perp, P^z; s', s'')R_X^b\left(s'', s\right),$$

$\boldsymbol{b}_\perp = \boldsymbol{x}_\perp - \boldsymbol{\mathcal{R}}_\perp$

TAM relative to the origin

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

$$J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = L^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) + S^a\left(\boldsymbol{b}_\perp, P^z; s', s\right)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\epsilon_\perp^{ab}\sqrt{\tau}X_1^b\tilde{J}_1 + \sigma_{s's}^a\tilde{J}_0 + \sigma_{s's}^b\tau X_2^{ab}\tilde{J}_2\right],$$

spin-dependent: monopole

spin-independent dipole

quadrupole

Relativistic centers

Lorcé, EPJC 78 (2018)
Lorcé, EPJC 81 (2021)

$$\boldsymbol{R}_E(s', s) = \mathbb{1}_{s's}\boldsymbol{\mathcal{R}} + \frac{\boldsymbol{P}\times\boldsymbol{\sigma}_{s's}}{2E_P\left(E_P + M\right)},$$

$$\boldsymbol{R}_M(s', s) = \mathbb{1}_{s's}\boldsymbol{\mathcal{R}} - \frac{\boldsymbol{P}\times\boldsymbol{\sigma}_{s's}}{2M\left(E_P + M\right)},$$

$$\boldsymbol{R}_c(s', s) = \mathbb{1}_{s's}\boldsymbol{\mathcal{R}},$$

Choice of the origin of coordinate system $\boldsymbol{\mathcal{R}} = 0$

$$\tilde{J}_0 = -4MP_z\frac{d}{dt}\left[\frac{P^0}{2M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(A - 2J\right)\right] - \frac{1}{4}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau D + \frac{1}{2}\frac{1}{\gamma}\cos\theta\left(J - S + \frac{1}{2}G_A\right) + \frac{M}{4P^0}\left(G_A - \tau G_P\right)$$
$$- 2M^2\frac{d}{dt}\left[\tau\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{\tau}{\gamma}\cos\theta S\right],$$

$$\tilde{J}_1 = 4MP^z\frac{d}{dt}\left(\frac{P^0}{M}\frac{1}{\gamma}\cos\theta A + \frac{1}{\beta}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}2\tau J\right) + \frac{1}{2}\frac{\beta}{\gamma}\cos\theta D + \frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\frac{1}{2}\left(J - S + G_A\right)$$
$$+ 4M^2\frac{d}{dt}\left[\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(J + S\right)\right],$$

$$\tilde{J}_2 = 4MP^z\frac{d}{dt}\left[\frac{P^0}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\left(A - 2J\right)\right] + \frac{1}{2}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}D + \frac{M}{2P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}G_A - \frac{M}{2P^0}G_P$$
$$+ 4M^2\frac{d}{dt}\left[\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{1}{\gamma}\cos\theta S\right]$$

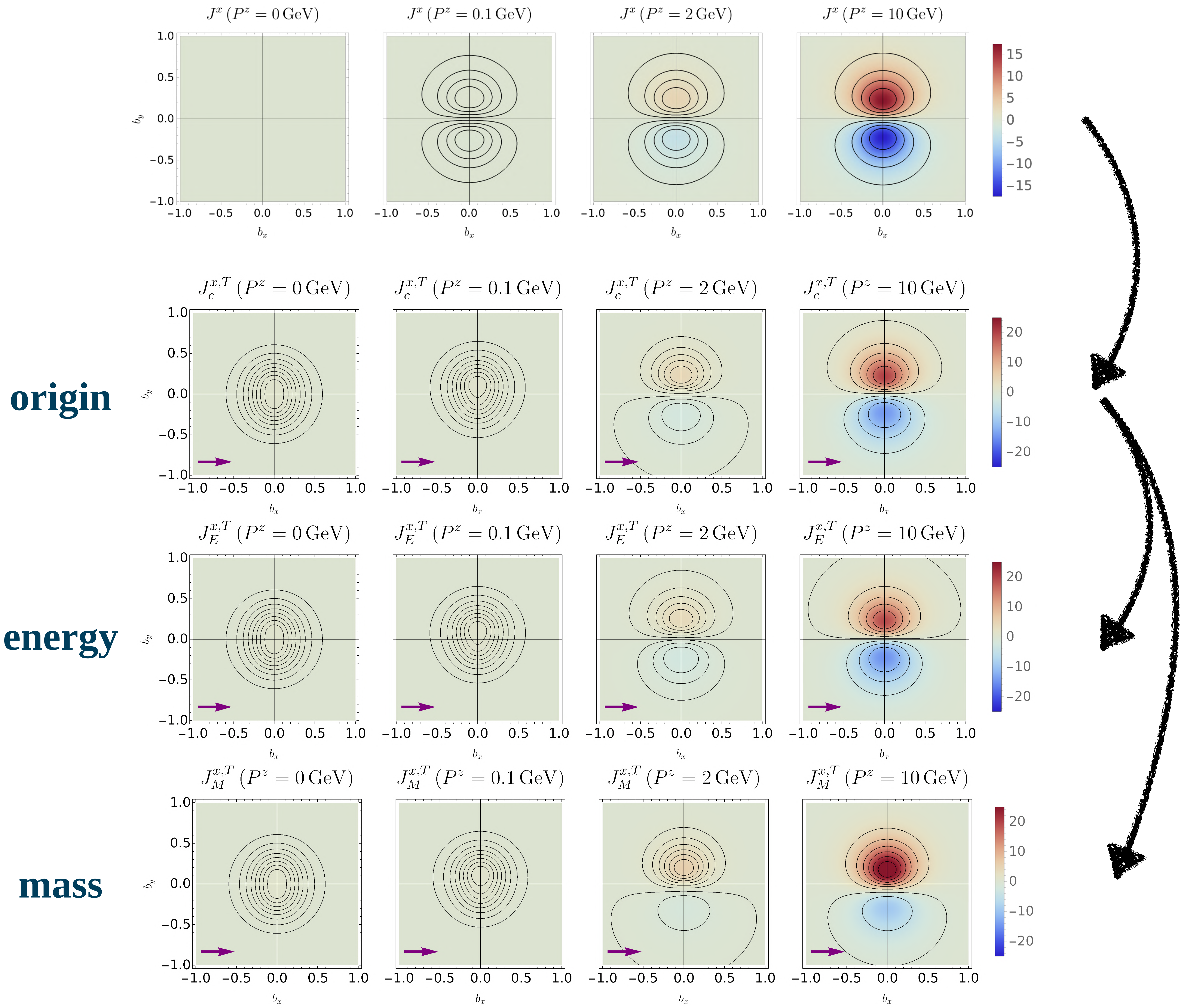
Relativistic momentum distribution

$$P^k\left(\boldsymbol{b}_\perp, P^z; s'', s\right) = \int dx^3\left[\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\left.\frac{\langle p', s''|T^{0k}(0)|p, s\rangle}{2\sqrt{p'^0p^0}}\right|_{\mathrm{GF}}\right]$$

Won and Lorcé, PRD 111 (2025)

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)
Lorcé, Mukherjee, Singh, and Won, in preparation

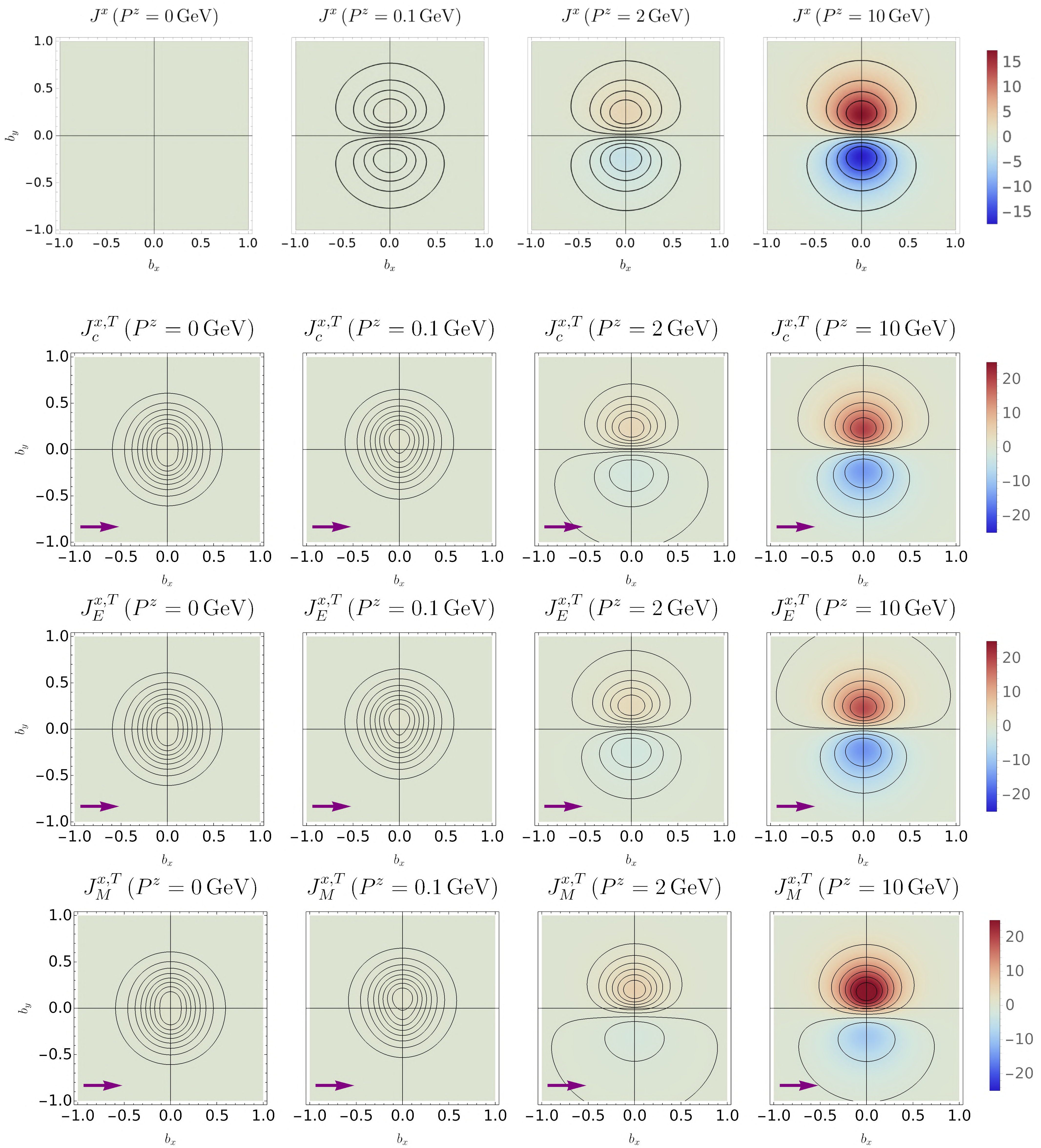


The nucleon spin polarization

pivot choice

Transverse Internal TAM Distribution

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)
Lorcé, Mukherjee, Singh, and Won, in preparation



origin

energy

mass

unpolarized nucleon

spin-independent (dipole)

$$\int d^2b_\perp\, J^a(\boldsymbol{b}_\perp, P^z) = 0$$

transversely polarized nucleon along the *x*-axis

dipole+monopole+quadrupole

$$\int d^2b_\perp\, J_c^a(\boldsymbol{b}_\perp, P^z; s', s) = \frac{\sigma^a_{s's}}{2}$$

$$\int d^2b_\perp\, J_E^a(\boldsymbol{b}_\perp, P^z; s', s) = \frac{M}{E_P}\frac{\sigma^a_{s's}}{2}$$

dipole+monopole+quadrupole
+pivot (dipole+monopole)

$$\int d^2b_\perp\, J_M^a(\boldsymbol{b}_\perp, P^z; s', s) = \frac{E_P}{M}\frac{\sigma^a_{s's}}{2}$$

Transverse Internal Boost Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

"Transverse" components of internal boost distribution in the transverse plane

$$K_X^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = K^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) + \frac{1}{2}\sum_{s''} R_X^a(s', s'')E\left(\boldsymbol{b}_\perp, P^z; s'', s\right) + \frac{1}{2}\sum_{s''} E(\boldsymbol{b}_\perp, P^z; s', s'')R_X^a\left(s'', s\right),$$

Relativistic centers

Boost relative to the origin

Lorcé, Mukherjee, Singh, and Won, in preparation

spin-dependent: monopole

$$K^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\sqrt{\tau} X_1^a \tilde{K}_1 + \epsilon_\perp^{ab}\sigma_{s's}^b \tilde{K}_0 + \epsilon_\perp^{bc}\sigma_{s's}^b \tau X_2^{ca}\tilde{K}_2\right]$$

spin-independent dipole — quadrupole

c.f. $J^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) \sim \left[\delta_{s's} i\epsilon_\perp^{ab}\sqrt{\tau} X_1^b \tilde{J}_1 + \sigma_{s's}^a \tilde{J}_0 + \sigma_{s's}^b \tau X_2^{ab}\tilde{J}_2\right],$

$$K^i\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \int dx^3 \left[\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\, i\frac{\partial}{\partial\Delta^i} \frac{\langle p', s'|T^{00}(0)|p, s\rangle}{2\sqrt{p'^0 p^0}}\Bigg|_{\text{GF}}\right],$$

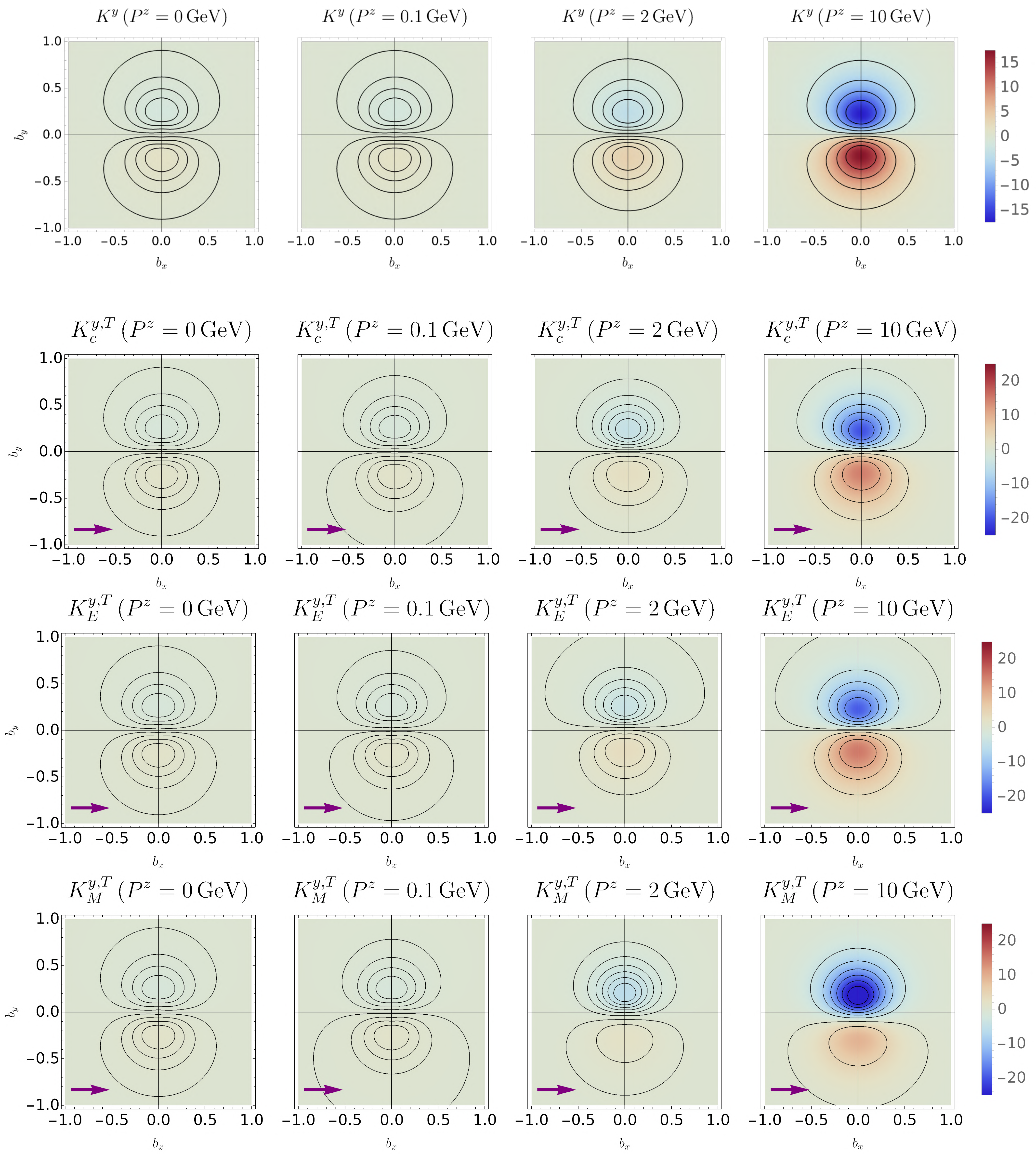
Relativistic energy distribution

Won and Lorcé, PRD 111 (2025)

$$E\left(\boldsymbol{b}_\perp, P^z; s'', s\right) = \int dx^3 \left[\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})} \frac{\langle p', s''|T^{00}(0)|p, s\rangle}{2\sqrt{p'^0 p^0}}\Bigg|_{\text{GF}}\right]$$

Transverse Internal Boost Distribution

Lorcé, Mukherjee, Singh, and Won, in preparation



origin

energy

mass

unpolarized nucleon

spin-independent (dipole)

$$\int d^2b_\perp K^a(\boldsymbol{b}_\perp, P^z) = 0$$

transversely polarized nucleon along the *x*-axis

dipole+monopole+quadrupole

$$\int d^2b_\perp K_c^a(\boldsymbol{b}_\perp, P^z; s', s) = \frac{(\boldsymbol{\sigma}_{s's} \times \boldsymbol{P})^a}{2\,(E_P + M)}$$

$$\int d^2b_\perp K_E^a(\boldsymbol{b}_\perp, P^z; s', s) = 0$$

∵by definition of the center of energy

dipole+monopole+quadrupole
+pivot (dipole+monopole)

$$\int d^2b_\perp K_M^a(\boldsymbol{b}_\perp, P^z; s', s) = \frac{(\boldsymbol{\sigma}_{s's} \times \boldsymbol{P})^a}{2M}$$

Light-Front Formalism

Center of LF momentum

Choice of the origin of coordinate system $R_n^\mu = 0 \leftrightarrow \mathcal{R} = 0$

Light-Front Formalism

Center of LF momentum

Choice of the origin of coordinate system $R_n^\mu = 0 \leftrightarrow \mathcal{R} = 0$

2D spatial distribution in relative coordinate $X^\mu := x^\mu - R_n^\mu$

$d^3\Delta = d^2\Delta_\perp d\Delta^+$

λ, λ': LF helicity polarization

$$O^i\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \int dX^- \left[\int \frac{d^3\Delta}{(2\pi)^3} e^{i\Delta^+ X^- - i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{X}_\perp} \frac{\langle p', \lambda' | O\,(0) | p, \lambda\rangle}{2\sqrt{p'^+ p^+}}\bigg|_{\mathrm{GLF}}\right],$$

with $\boldsymbol{b}_\perp = \boldsymbol{X}_\perp$

Light-Front Formalism

Center of LF momentum

Choice of the origin of coordinate system $R_n^\mu = 0 \leftrightarrow \mathcal{R} = 0$

2D spatial distribution in relative coordinate $X^\mu := x^\mu - R_n^\mu$

$d^3\Delta = d^2\Delta_\perp d\Delta^+$

λ, λ': LF helicity polarization

$$O^i\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \int dX^- \left[\int \frac{d^3\Delta}{(2\pi)^3} e^{i\Delta^+ X^- - i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{X}_\perp} \frac{\langle p', \lambda' | O\,(0) | p, \lambda\rangle}{2\sqrt{p'^+ p^+}}\bigg|_{\mathrm{GLF}}\right],$$

with $\boldsymbol{b}_\perp = \boldsymbol{X}_\perp$

Generic LF frame (GLF)

$P = (P^+, P^-, \mathbf{0}_\perp), \quad \Delta = (\Delta^+, \Delta^-, \boldsymbol{\Delta}_\perp),$

$p'^2 = p^2 = M^2 \to \Delta \cdot P = 0 \to \Delta^- = -\Delta^+ P^- / P^+$

$\Delta^+ = 0$ → Drell-Yan frame

Transverse LF TAM Distribution

Transverse LF TAM distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

$$\mathcal{L}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \int dx^- \left[\epsilon_\perp^{ab} \int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\Delta^+ x^- - i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left(i\frac{\partial}{\partial\Delta^+} \frac{\langle p',\lambda'|T^{+b}|p,\lambda\rangle}{2\sqrt{p'^+p^+}} + i\frac{\partial}{\partial\Delta_\perp^b} \frac{\langle p',\lambda'|T^{+-}|p,\lambda\rangle}{2\sqrt{p'^+p^+}} \right)\right],$$

$$\mathcal{S}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \int dx^- \left[\frac{1}{2} \int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\Delta^+ x^- - i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \frac{\langle p',\lambda'|V_5^a|p,\lambda\rangle}{2\sqrt{p'^+p^+}}\right]$$

$$\mathcal{J}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \mathcal{L}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) + \mathcal{S}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right)$$

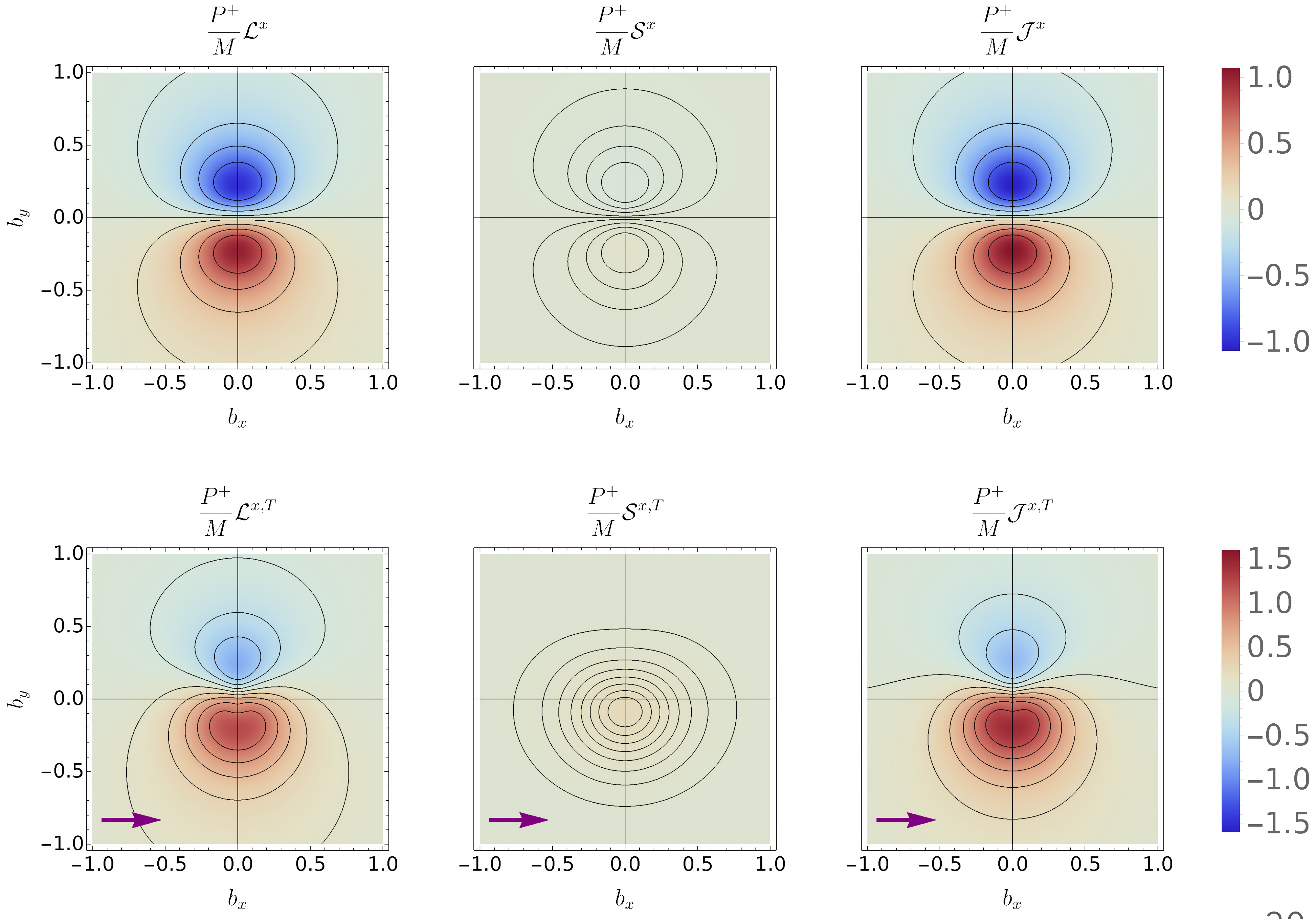
$$= \frac{M}{P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[i\delta_{\lambda'\lambda}\epsilon_\perp^{ab}\sqrt{\tau} X_1^b \tilde{\mathcal{J}}_1 + \sigma^a_{\lambda'\lambda}\tilde{\mathcal{J}}_0 + \sigma^b_{\lambda'\lambda}\tau X_2^{ba}\tilde{\mathcal{J}}_2 \right]$$

P^+-independent amplitudes

inverse LF-boost factor

The same multipole structure as the IF formalism

Orbital | Intrinsic spin | TAM



unpolarized nucleon

spin-independent (dipole)

$$\int d^2b_\perp \mathcal{J}^a(\boldsymbol{b}_\perp, P^+) = 0$$

transversely polarized nucleon along the x-axis

dipole+monopole+quadrupole

$$\int d^2b_\perp \mathcal{J}^a(\boldsymbol{b}_\perp, P^+; \lambda', \lambda) = \frac{M}{P^+}\frac{\sigma^a_{\lambda'\lambda}}{2}$$

Transverse LF Boost Distribution

Transverse LF boost distribution

Lorcé, Mukherjee, Singh, and Won, in preparation

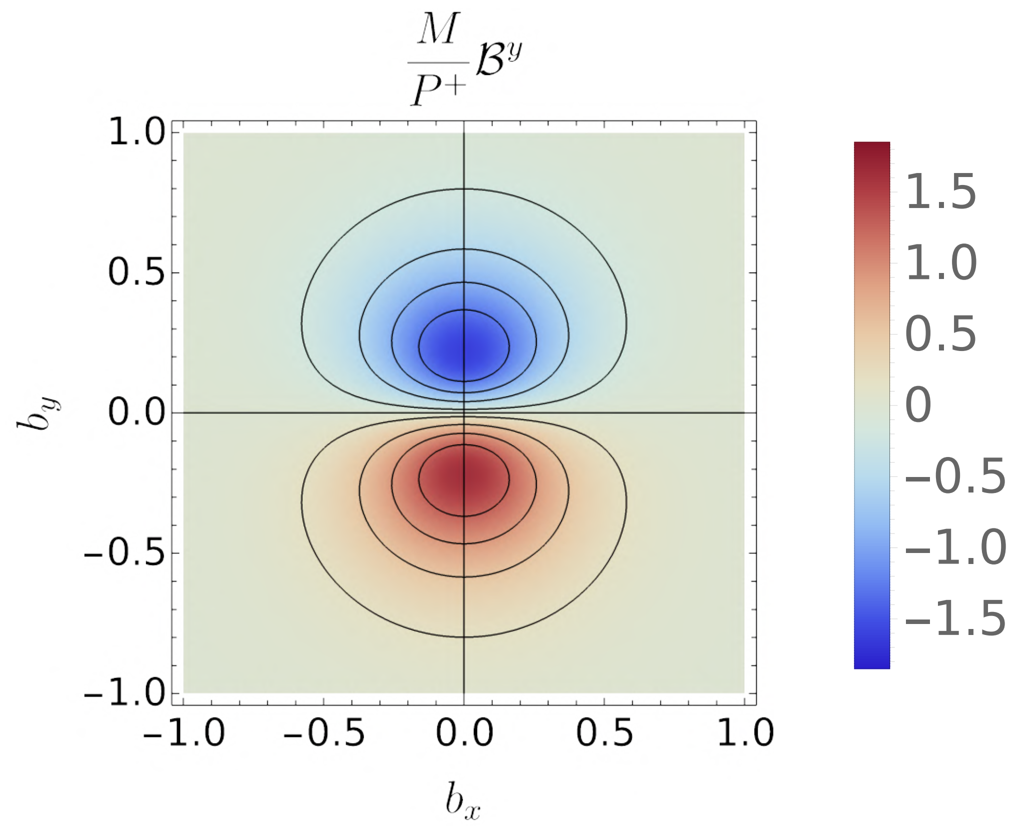
$$\mathcal{B}^a\left(\boldsymbol{b}_\perp, P^+;\lambda',\lambda\right) = \int dx^- \left[\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\Delta^+ x^- - i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} i\frac{\partial}{\partial\Delta^a_\perp}\frac{\langle p',\lambda'|T^{++}|p,\lambda\rangle}{2\sqrt{p'^+p^+}}\right]$$

$$\mathcal{B}^a\left(\boldsymbol{b}_\perp, P^+;\lambda',\lambda\right) = \frac{P^+}{M}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[i\delta_{\lambda'\lambda}\sqrt{\tau}X^a_1\mathcal{B}_1 + \epsilon^{ab}_\perp\sigma^b_{\lambda'\lambda}\mathcal{B}_0 + \epsilon^{bc}_\perp\sigma^b_{\lambda'\lambda}\tau X^{ca}_2\mathcal{B}_2\right],$$

P^+-independent amplitudes

LF-boost factor

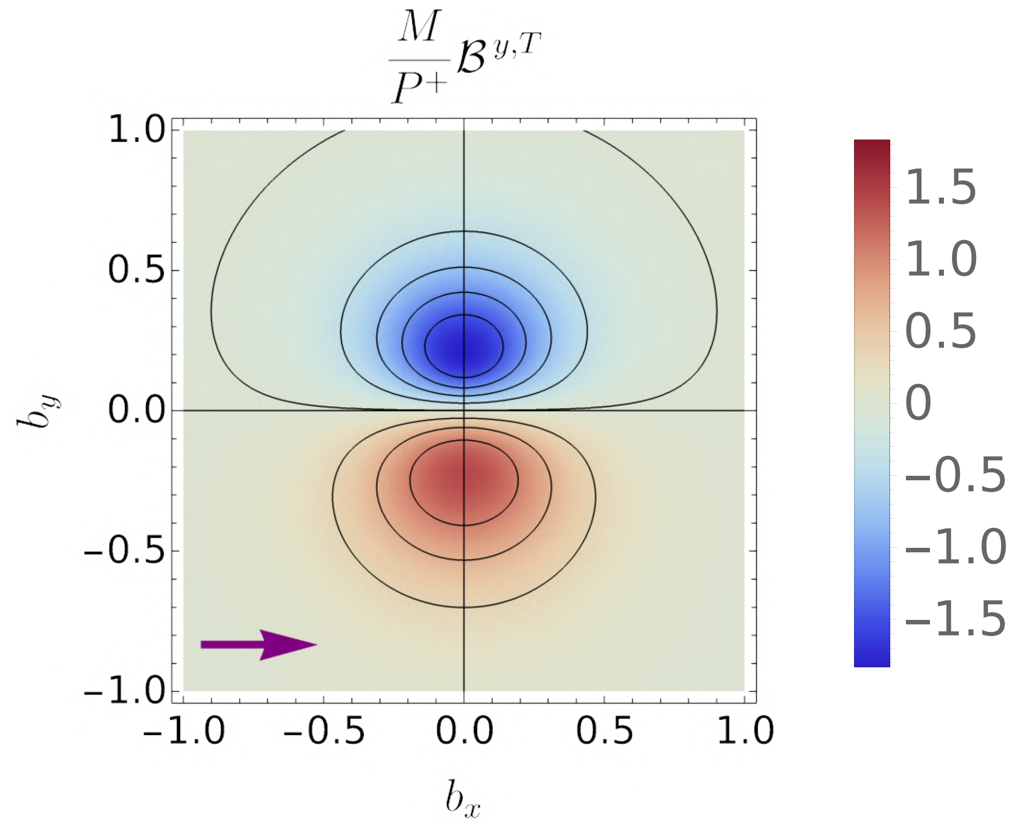
The same multipole structure as the IF formalism



unpolarized nucleon

spin-independent (dipole)

$$\int d^2b_\perp \mathcal{B}^a(\boldsymbol{b}_\perp, P^+) = 0$$



transversely polarized nucleon along the *x*-axis

dipole+monopole+quadrupole

$$\int d^2b_\perp \mathcal{B}^a(\boldsymbol{b}_\perp, P^+;\lambda',\lambda) = 0$$

∵by definition of the center of LF momentum

Interpolation between the IF and LF Formalism

Transverse LF boost: "good" component

$$\frac{M}{P^+}\mathcal{B}^a\left(\boldsymbol{b}_\perp, P^+;\lambda',\lambda\right) = \lim_{P^z\to\infty}\frac{M}{P^z}K^a_c\left(\boldsymbol{b}_\perp, P^z; s', s\right)$$

inverse LF-boost factor

inverse Lorentz-boost factor

IMF Wigner angle = Melosh angle

Transverse LF TAM: "bad" component (inspired by Lorentz mixing)

$$\frac{P^+}{M}\mathcal{J}^a\left(\boldsymbol{b}_\perp, P^+;\lambda',\lambda\right) = \lim_{P^z\to\infty}\frac{P^z}{M}\left[J^a_{E/c}\left(\boldsymbol{b}_\perp, P^z; s', s\right) + \epsilon^{ab}_\perp K^b_{E/c}\left(\boldsymbol{b}_\perp, P^z; s', s\right)\right]$$

$$\neq \lim_{P^z\to\infty}\frac{P^z}{M}\left[J^a_M\left(\boldsymbol{b}_\perp, P^z; s', s\right) + \epsilon^{ab}_\perp K^b_M\left(\boldsymbol{b}_\perp, P^z; s', s\right)\right] \qquad \because \lim_{P^z\to\infty}\boldsymbol{R}_{E/c} = O\left(\frac{M}{P^z}\right), \qquad \lim_{P^z\to\infty}\boldsymbol{R}_M = O\left(\frac{P^z}{M}\right),$$

LF-boost factor

Lorentz-boost factor

Conclusion and Summary

Conclusion and Summary

hoyeon.won@polytechnique.edu

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)
Lorcé, Mukherjee, Singh, and Won, in preparation

- We reviewed **internal AM (TAM + boost)** and **relativistic centers** in QFT.

- Using the **quantum phase-space** formalism, we defined 3D spatial distributions and obtained 2D transverse distributions by integrating over the longitudinal coordinate.

- We showed the spatial distributions of **transverse internal AM** for an unpolarized nucleon and for a transversely polarized nucleon in the transverse plane. The main result is that the choice of pivot, i.e., **centers of spin, energy, and mass**, is directly visible in the spatial distributions.

- We also studied the **transverse LF AM** distributions and compared them with the IF results in the IMF limit.

- Overall, this work provides a model-independent framework to connect transverse sum rules with relativistic spatial distributions of AM and boost.

Thank you for listening

Appendix

Definition of Spatial Distribution in Transverse Plane

For comparison with 2D light-front distributions → 2D projection

Two ways: direct and indirect

$$O(\boldsymbol{X}_\perp; s', s) := \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{X}_\perp\cdot\boldsymbol{\Delta}_\perp} \frac{\langle p', s'|O\,(0)|p, s\rangle}{2P^0}\bigg|_{\rm 2D} \qquad \text{Direct}$$

$$O(\boldsymbol{X}_\perp; s', s) := \int dx^3 O(\boldsymbol{X}; s', s) \qquad \text{Indirect}$$

$$p' = P + \frac{\Delta}{2}, \qquad p = P - \frac{\Delta}{2}$$

$$p^2 = p'^2 = M^2$$

Both coincide for electromagnetic, axial-vector, energy-momentum tensor, etc.

Compound operators: explicit position factors, e.g., $L^i_\perp(x) = \epsilon^{ijk}x^jT^{0k}(x)$,

cannot be defined in the 2D framework, "directly"

$$L^i_\perp(\boldsymbol{X}_\perp; s', s) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{X}_\perp\cdot\boldsymbol{\Delta}_\perp}\left[-\epsilon^{i3k} i\frac{\partial}{\partial\Delta^3}\frac{\langle p', s'|T^{0k}\,(0)|p, s\rangle}{2P^0}\bigg|_{\rm 2D} - \epsilon^{ij3} i\frac{\partial}{\partial\Delta^j}\frac{\langle p', s'|T^{03}\,(0)|p, s\rangle}{2P^0}\bigg|_{\rm 2D}\right],$$

"Indirect" way is needed

Wigner Rotation and Melosh Rotation

For spin-j particle

$$\langle p', s'|O^{\mu_1\cdots\mu_n}|p, s\rangle = \sum_{s'_{\rm BF}, s_{\rm BF}} D^{(j)}_{s_{\rm BF}s}(p_{\rm BF}, \Lambda)\, D^{*(j')}_{s'_{\rm BF}s'}(p'_{\rm BF}, \Lambda) \times \Lambda^{\mu_1}{}_{\alpha_1}\cdots\Lambda^{\mu_n}{}_{\alpha_n}\,\langle p'_{\rm BF}, s'_{\rm BF}|\hat{O}^{\alpha_1\cdots\alpha_n}|p_{\rm BF}, s_{\rm BF}\rangle$$

Wigner rotation matrix for spin-1/2 targets

$$D^{(1/2)}_{ss'}(p, \Lambda) = \cos\frac{\theta}{2}\delta_{ss'} + i\sin\frac{\theta}{2}\frac{(\boldsymbol{p}_\perp \times \boldsymbol{\sigma}_{ss'})_z}{|\boldsymbol{p}_\perp|},$$

$$D^{(1/2)}_{ss'}(p', \Lambda) = \cos\frac{\theta'}{2}\delta_{ss'} + i\sin\frac{\theta'}{2}\frac{(\boldsymbol{p}'_\perp \times \boldsymbol{\sigma}_{ss'})_z}{|\boldsymbol{p}'_\perp|}$$

$$\lim_{p_z\to 0}\cos\frac{\theta_M}{2} = \lim_{p_z\to\infty}\cos\frac{\theta}{2}, \quad \lim_{p_z\to 0}\sin\frac{\theta_M}{2} = \lim_{p_z\to\infty}\sin\frac{\theta}{2}$$

Melosh rotation matrix for spin-1/2 targets

$$|p, \lambda\rangle_{\rm LF} = \sum_s |p, s\rangle_{\rm IF}\, \mathcal{M}_{s\lambda}(p)$$

$$\mathcal{M}_{s\lambda}(p) = \cos\frac{\theta_M}{2}\,\delta_{s\lambda} + i\sin\frac{\theta_M}{2}\,\frac{(\boldsymbol{p}_\perp \times \boldsymbol{\sigma}_{s\lambda})_z}{|\boldsymbol{p}_\perp|}$$

Chen, and Lorcé, PRD 106 (2022)

Wigner Rotation and Melosh Rotation Angles

In the GF, $\Delta^3 \neq 0 \to \theta \neq \theta'$

$$\gamma = \frac{P^0}{\sqrt{P^2}} = \frac{P^0}{M\sqrt{1+\tau}}, \qquad \beta = \frac{P^z}{P^0}$$

$$\cos\frac{\theta+\theta'}{2} = \frac{P^0 + M(1+\tau)}{\mathcal{N}_s\sqrt{1+\tau}}, \qquad \sin\frac{\theta+\theta'}{2} = -\frac{\delta_\perp P_z}{\mathcal{N}_s\sqrt{1+\tau}},$$

$$\cos\frac{\theta-\theta'}{2} = \omega_\perp\frac{P^0+M}{\mathcal{N}_s} + \omega_\parallel\frac{P^0 + M(1+\tau)}{\mathcal{N}_s\sqrt{1+\tau}},$$

$$\sin\frac{\theta-\theta'}{2} = \operatorname{sgn}(\delta_\parallel)\sqrt{\omega_\perp\omega_\parallel}\left[\frac{P^0 + M(1+\tau)}{\mathcal{N}_s\sqrt{1+\tau}} - \frac{P^0+M}{\mathcal{N}_s}\right],$$

$$\delta\perp := \frac{|\boldsymbol{\Delta}_\perp|}{2M}, \quad \delta_\parallel := \frac{\Delta^3/\gamma}{2M}, \quad \omega_\perp := \frac{\delta_\perp^2}{\tau}, \quad \omega_\parallel := \frac{\delta_\parallel^2}{\tau}, \quad \mathcal{N}_s := \sqrt{(p^0+M)(p'^0+M)},$$

$\Delta^3 = 0$ →

2D projection $\Delta^3 \to 0$

$$\lim_{\Delta_z\to 0}\cos\frac{\theta+\theta'}{2} = \frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}, \qquad \lim_{\Delta_z\to 0}\sin\frac{\theta+\theta'}{2} = -\frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}},$$

$$\lim_{\Delta_z\to 0}\cos\frac{\theta-\theta'}{2} = 1, \qquad \lim_{\Delta_z\to 0}\sin\frac{\theta-\theta'}{2} = 0.$$

↓ $P^3 = 0$ (left) ↓ $P^3 = 0$ (right)

IMF limit

$$\lim_{P_z\to\infty}\cos\frac{\theta^{(\prime)}}{2} = \sqrt{\frac{2\left(1+\sqrt{1+\tau}\right) + \delta_\perp^2\left(2+\sqrt{1+\tau}\mp\delta_\parallel\right)}{2\left(1+\sqrt{1+\tau}\right)\left(1+\delta_\perp^2\right)}},$$

$$\lim_{P_z\to\infty}\sin\frac{\theta^{(\prime)}}{2} = -\delta_\perp\sqrt{\frac{\sqrt{1+\tau}\pm\delta_\parallel}{2\left(1+\sqrt{1+\tau}\right)\left(1+\delta_\perp^2\right)}}$$

$\Delta^3 = 0$ →

IMF limit & 2D projection

$$\lim_{\substack{\Delta_z\to 0\\ P_z\to\infty}}\cos\frac{\theta'}{2} = \lim_{\substack{\Delta_z\to 0\\ P_z\to\infty}}\cos\frac{\theta}{2} = \sqrt{\frac{1+\sqrt{1+\tau}}{2\sqrt{1+\tau}}},$$

$$\lim_{\substack{\Delta_z\to 0\\ P_z\to\infty}}\sin\frac{\theta'}{2} = \lim_{\substack{\Delta_z\to 0\\ P_z\to\infty}}\sin\frac{\theta}{2} = -\sqrt{\frac{\tau}{2\sqrt{1+\tau}\left(1+\sqrt{1+\tau}\right)}}$$

Amplitudes - Transverse TAM

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

2D spatial distribution of the transverse TAM relative to the canonical center

$$J_c^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\epsilon_\perp^{ab}\sqrt{\tau} X_1^b \tilde{J}_1 + \sigma_{s's}^a \tilde{J}_0 + \sigma_{s's}^b \tau X_2^{ab} \tilde{J}_2\right],$$

multipole amplitudes

$$\begin{aligned}
\tilde{J}_0 &= -4MP_z\frac{d}{dt}\left[\frac{P^0}{2M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(A-2J\right)\right] - \frac{1}{4}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau D + \frac{1}{2}\frac{1}{\gamma}\cos\theta\left(J - S + \frac{1}{2}G_A\right) + \frac{M}{4P^0}\left(G_A - \tau G_P\right) \\
&\quad - 2M^2\frac{d}{dt}\left[\tau\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{\tau}{\gamma}\cos\theta S\right], \\
\tilde{J}_1 &= 4MP^z\frac{d}{dt}\left(\frac{P^0}{M}\frac{1}{\gamma}\cos\theta A + \frac{1}{\beta}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}2\tau J\right) + \frac{1}{2}\frac{\beta}{\gamma}\cos\theta D + \frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\frac{1}{2}\left(J - S + G_A\right) \\
&\quad + 4M^2\frac{d}{dt}\left[\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau\left(J+S\right)\right], \\
\tilde{J}_2 &= 4MP^z\frac{d}{dt}\left[\frac{P^0}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\left(A-2J\right)\right] + \frac{1}{2}\frac{\beta}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}D + \frac{M}{2P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}G_A - \frac{M}{2P^0}G_P \\
&\quad + 4M^2\frac{d}{dt}\left[\left(1 - \frac{P_z}{M}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\right)J - \frac{1}{\gamma}\cos\theta S\right]
\end{aligned}$$

$$\begin{aligned}
E &= A + \overline{C} + \tau\left(A - 2J + D\right), \\
J &= \frac{A+B}{2}, \\
F &= -\tau D - \overline{C},
\end{aligned}$$

$$L^i\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \int dx^3\left[-i\epsilon^{ijk}\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\frac{\partial}{\partial\Delta^j}\frac{\langle p', s'|T^{0k}(0)|p, s\rangle}{2\sqrt{p'^0p^0}}\bigg|_{\text{GF}}\right],$$

$$S^i\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \frac{1}{2}\int dx^3\left[\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{\mathcal{R}})}\frac{\langle p', s'|V_5^i(0)|p, s\rangle}{2\sqrt{p'^0p^0}}\bigg|_{\text{GF}}\right]$$

Amplitudes - Transverse Boost

Lorcé, Mukherjee, Singh, and Won, PLB 868 (2025)

2D spatial distribution of the transverse boost relative to the canonical center

$$K_c^a\left(\boldsymbol{b}_\perp, P^z; s', s\right) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's} i\sqrt{\tau} X_1^a \tilde{K}_1 + \epsilon_\perp^{ab}\sigma^b_{s's}\tilde{K}_0 + \epsilon_\perp^{bc}\sigma^b_{s's}\tau X_2^{ca}\tilde{K}_2\right]$$

multipole amplitudes

$$\tilde{K}_0 = -4MP^z \frac{d}{dt}\left[\frac{\left(P^0\right)^2}{2MP^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau A + \tau J - \frac{M}{2P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}}\tau F\right],$$

$$\tilde{K}_1 = 4M^2 \frac{d}{dt}\left[-\frac{\left(P^0\right)^2}{M^2}\frac{1}{\gamma}\cos\theta A + 2\tau J + \frac{1}{\gamma}\cos\theta F\right],$$

$$\tilde{K}_2 = 4MP^z \frac{d}{dt}\left[\frac{\left(P^0\right)^2}{MP^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}} A + 2J - \frac{M}{P^z}\frac{1}{\gamma}\frac{\sin\theta}{\sqrt{\tau}} F\right],$$

$$E = A + \overline{C} + \tau\left(A - 2J + D\right),$$

$$J = \frac{A+B}{2},$$

$$F = -\tau D - \overline{C},$$

Amplitudes - Transverse LF TAM

2D spatial distribution of the transverse LF boost relative to the center of LF momentum

$$\mathcal{J}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \mathcal{L}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) + \mathcal{S}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right)$$

2D spatial distribution of the transverse LF OAM

$$\mathcal{L}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \frac{M}{P^+}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[i\delta_{\lambda'\lambda}\epsilon_\perp^{ab}\sqrt{\tau}X_1^b\tilde{\mathcal{L}}_1 + \sigma^a_{\lambda'\lambda}\tilde{\mathcal{L}}_0 + \sigma^b_{\lambda'\lambda}\tau X_2^{ba}\tilde{\mathcal{L}}_2\right]$$

multipole amplitudes

$$\tilde{\mathcal{L}}_0 = -2M^2\frac{d}{dt}\left[\frac{P^2}{2M^2}\tau(A-2J) + \tau(L-F)\right] + \frac{\tau}{4}D + \frac{1}{2}L,$$

$$\tilde{\mathcal{L}}_1 = -4M^2\frac{d}{dt}\left(\frac{P^2}{2M^2}A - \tau L - F\right) + \frac{1}{2}D - \frac{1}{2}L,$$

$$\tilde{\mathcal{L}}_2 = 4M^2\frac{d}{dt}\left[\frac{P^2}{2M^2}(A-2J) + L - F\right] - \frac{1}{2}D$$

$$\tilde{\mathcal{S}}_0 = \frac{1}{2}G_A - \frac{\tau}{4}G_P,$$

$$\tilde{\mathcal{S}}_1 = -\frac{1}{2}G_A,$$

$$\tilde{\mathcal{S}}_2 = -\frac{1}{2}G_P$$

$$E = A + \overline{C} + \tau(A - 2J + D),$$

$$J = \frac{A+B}{2},$$

$$F = -\tau D - \overline{C},$$

Amplitudes - Transverse LF boost

2D spatial distribution of the transverse LF boost relative to the center of LF momentum

$$\mathcal{B}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \frac{P^+}{M}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[i\delta_{\lambda'\lambda}\sqrt{\tau}X_1^a\mathcal{B}_1 + \epsilon_\perp^{ab}\sigma^b_{\lambda'\lambda}\mathcal{B}_0 + \epsilon_\perp^{bc}\sigma^b_{\lambda'\lambda}\tau X_2^{ca}\mathcal{B}_2\right]$$

multipole amplitudes

$$\tilde{\mathcal{B}}_0 = 2M^2\frac{d}{dt}\left[\tau\left(A - 2J\right)\right],$$

$$\tilde{\mathcal{B}}_1 = -4M^2\frac{dA}{dt},$$

$$\tilde{\mathcal{B}}_2 = -4M^2\frac{d}{dt}\left(A - 2J\right),$$

$$E = A + \overline{C} + \tau\left(A - 2J + D\right),$$

$$J = \frac{A+B}{2},$$

$$F = -\tau D - \overline{C},$$

Amplitudes - Energy and Longitudinal Momentum

2D spatial distribution of the energy and longitudinal momentum

$$h\left(\boldsymbol{b}_\perp, P^z; s', s\right) = M \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left(\delta_{s's}\tilde{h}_0 + i\epsilon_\perp^{ij}\sigma^i_{s's}\sqrt{\tau}X_1^j\tilde{h}_1\right)$$

Monopole; spin-independent   Dipole; spin-dependent → Rank: spin=position in multipole order

with $h = E, P^z$

$$E = A + \overline{C} + \tau\left(A - 2J + D\right),$$
$$J = \frac{A+B}{2},$$
$$F = -\tau D - \overline{C},$$

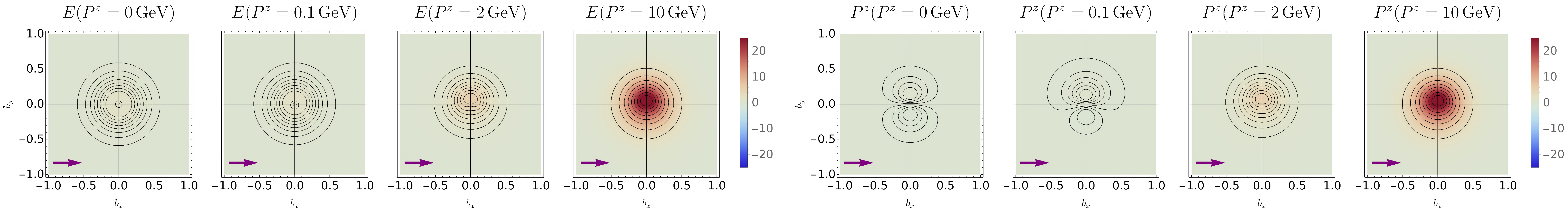
Multipole amplitudes

$$\tilde{E}_0 = \gamma\cos\theta\left(E + \beta^2 F\right) - \gamma\beta\frac{\sin\theta}{\sqrt{\tau}}2\tau J,$$

$$\tilde{E}_1 = \gamma\frac{\sin\theta}{\sqrt{\tau}}\left(E + \beta^2 F\right) + \gamma\beta\cos\theta 2J,$$

$$\tilde{P}^z_0 = \gamma\beta\cos\theta\left(E + F\right) - \gamma\frac{\sin\theta}{\sqrt{\tau}}\tau\left[L + \beta^2\left(J + S\right)\right],$$

$$\tilde{P}^z_1 = \gamma\beta\frac{\sin\theta}{\sqrt{\tau}}\left(E + F\right) + \gamma\cos\theta\left[L + \beta^2\left(J + S\right)\right]$$

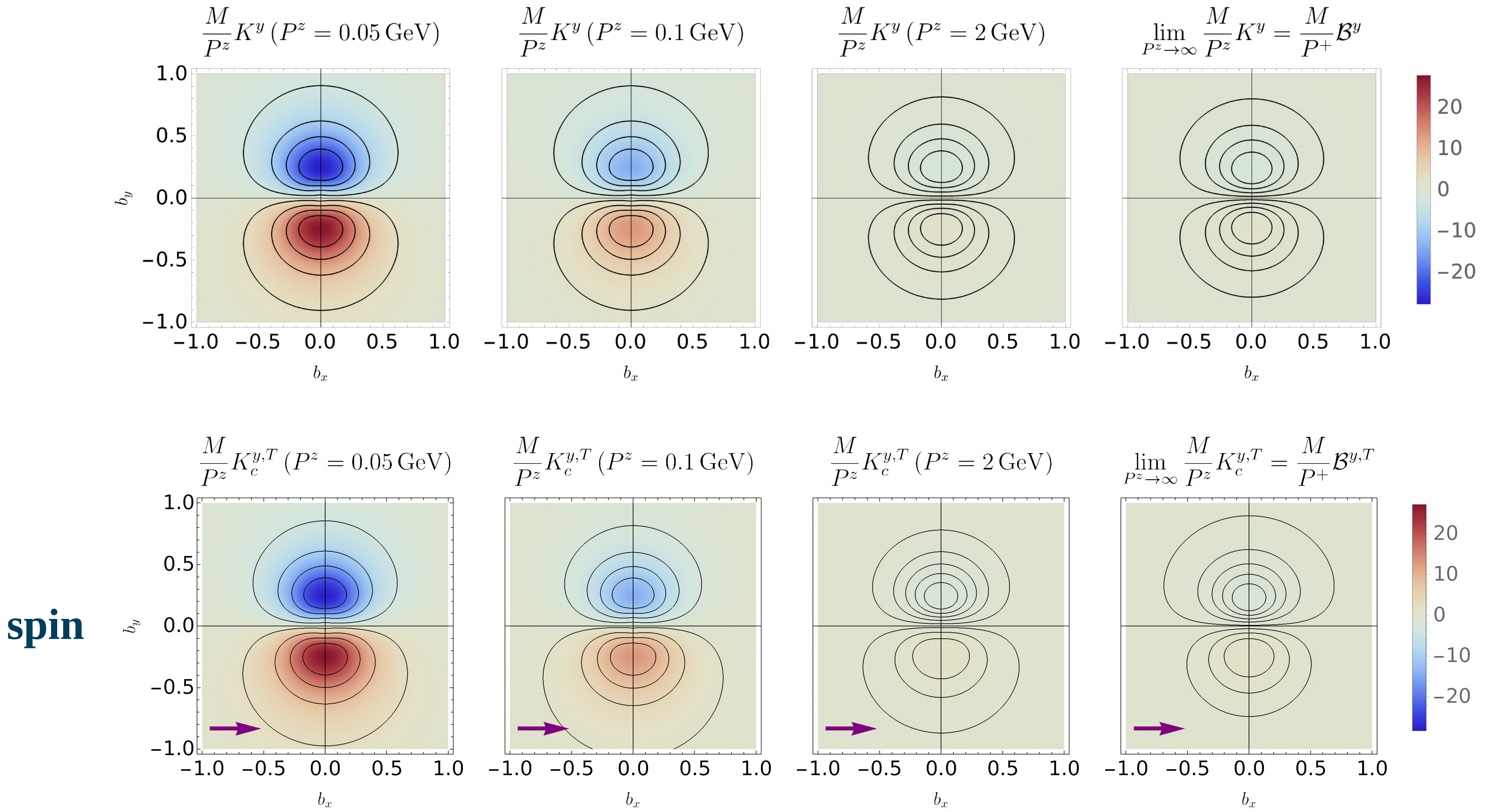


Interpolation between the IF and LF Formalism

Transverse LF boost: "good" component

$$\frac{M}{P^+}\mathcal{B}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \lim_{P^z\to\infty}\frac{M}{P^z}K^a_c\left(\boldsymbol{b}_\perp, P^z; s', s\right)$$

IMF Wigner angle = Melosh angle



unpolarized nucleon

dipole

transversely polarized nucleon along the *x*-axis

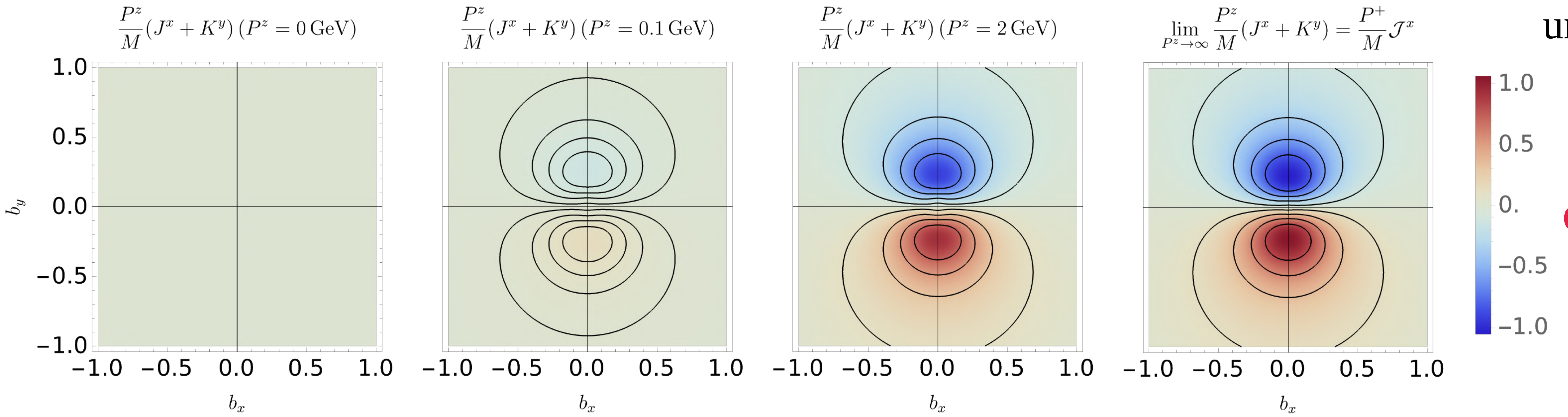
dipole (spin-independent)
+
monopole + quadrupole
(spin-dependent)

Interpolation between the IF and LF Formalism

Transverse LF TAM: "bad" component

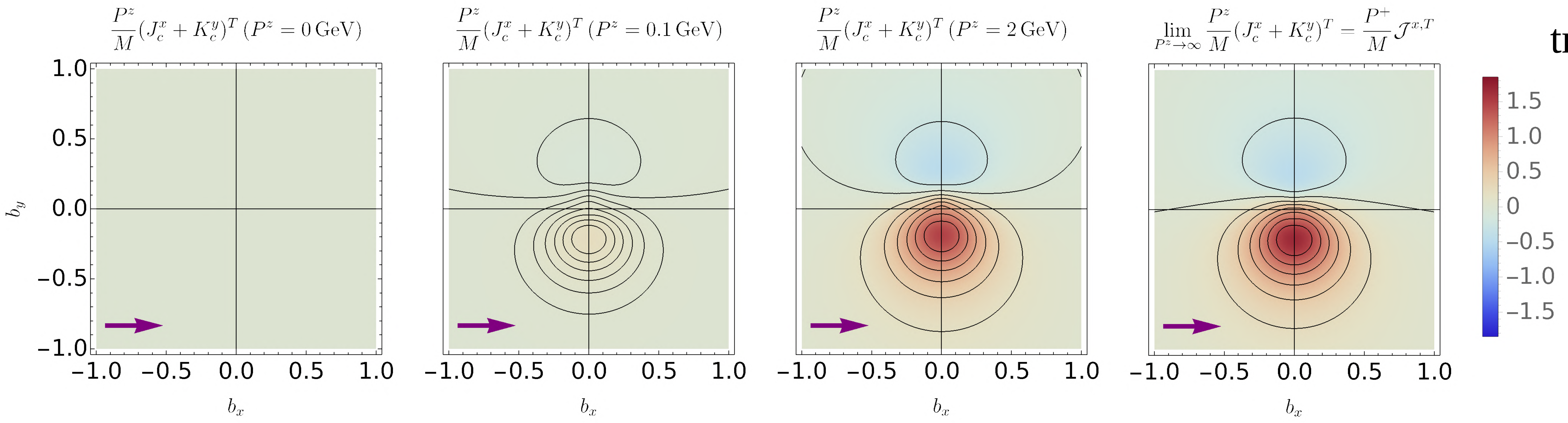
$$\frac{P^+}{M}\mathcal{J}^a\left(\boldsymbol{b}_\perp, P^+; \lambda', \lambda\right) = \lim_{P^z\to\infty}\frac{P^z}{M}\left[J^a_{E/c}\left(\boldsymbol{b}_\perp, P^z; s', s\right) + \epsilon^{ab}_\perp K^b_{E/c}\left(\boldsymbol{b}_\perp, P^z; s', s\right)\right]$$

IMF Wigner angle = Melosh angle



unpolarized nucleon

dipole



spin

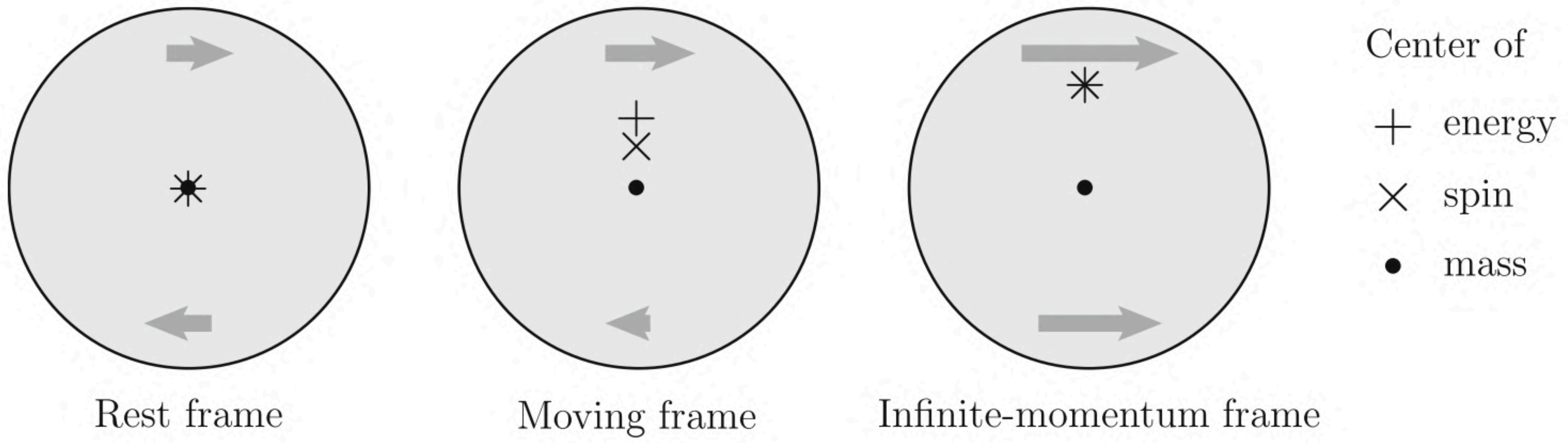
transversely polarized nucleon along the *x*-axis

dipole (spin-independent)
+
monopole + quadrupole
(spin-dependent)

Relativistic Position Vectors in QFT

Lorcé, EPJC 81 (2023)

$$[J^i, K^j] = i\epsilon^{ijk} K^k$$



Chen, Lorcé, PRD 107 (2023)