Toward quantum simulations of lattice field theory in 2+1 dimensions

Felix Ringer

Stony Brook University

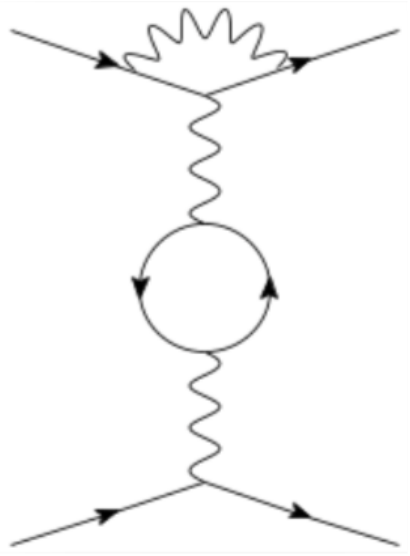
Light Cone Conference 2026

Stony Brook University

Quantum chromodynamics

1. Perturbative QCD at high energies

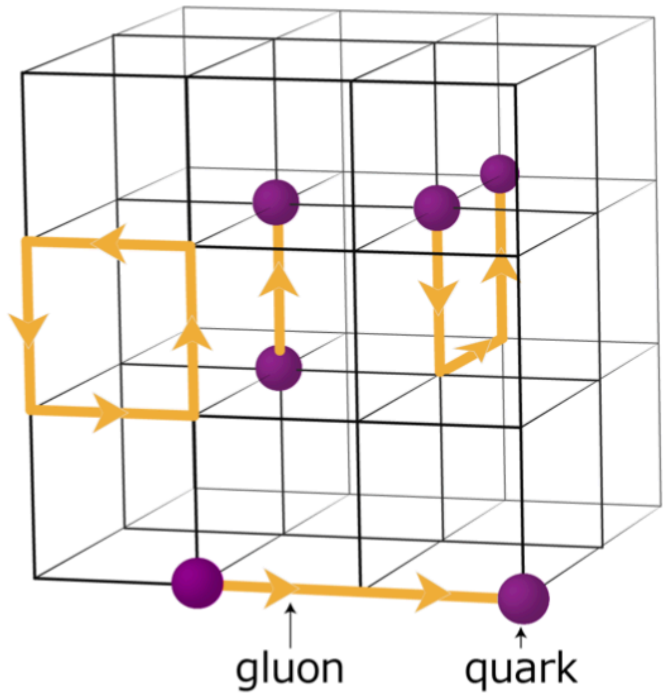
$$\sigma = \sigma^{(0)} + \alpha_s \, \sigma^{(1)} + \alpha_s^2 \, \sigma^{(2)} + \ldots$$

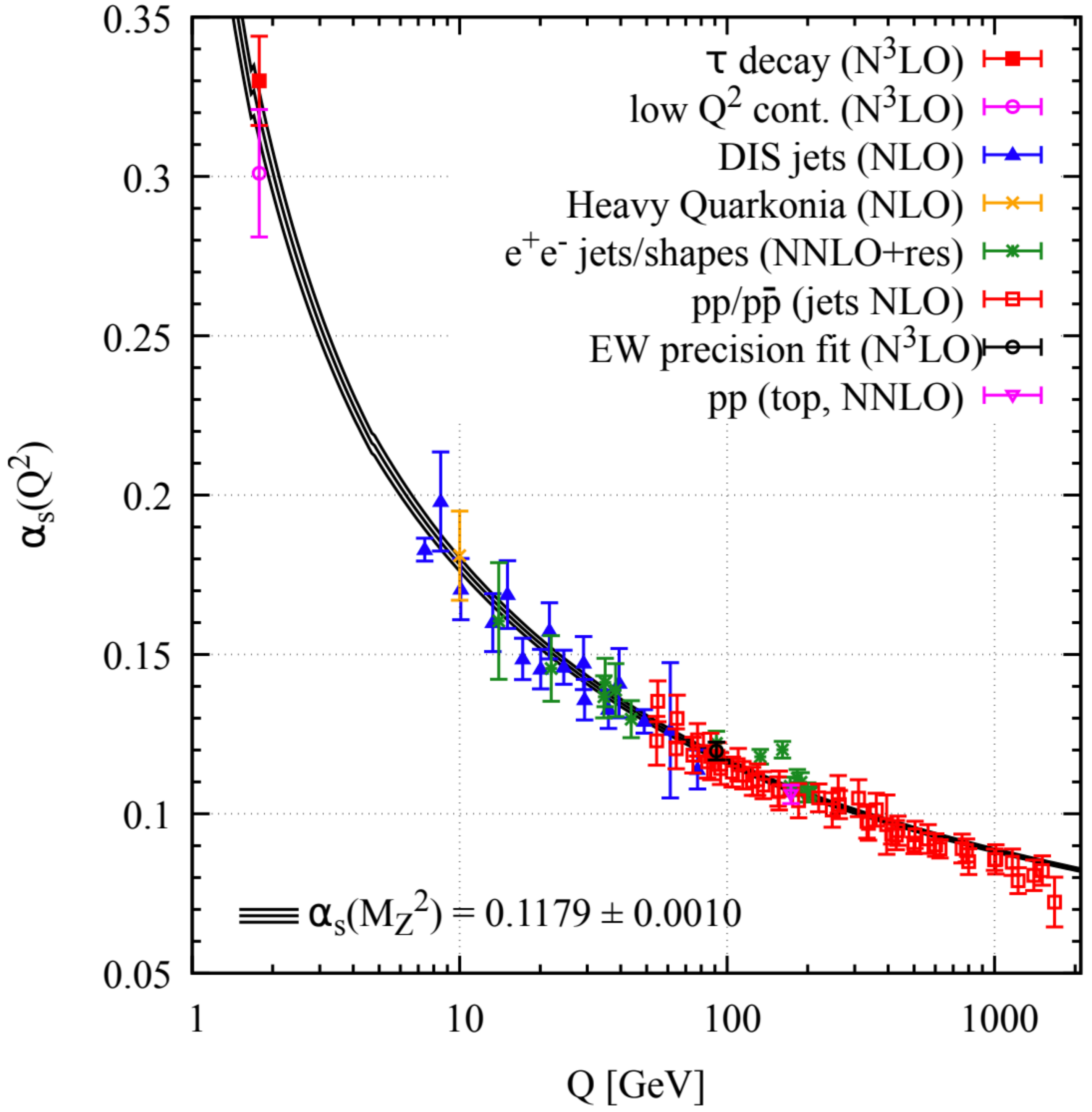


2. Nonperturbative at low energies

- Simulations using a lattice discretization
- Path integral and Hamiltonian formulation

Wilson; Kogut, Susskind `70s





Huston, Rabbertz, Zanderighi (PDG)

Hadron structure and hadronization

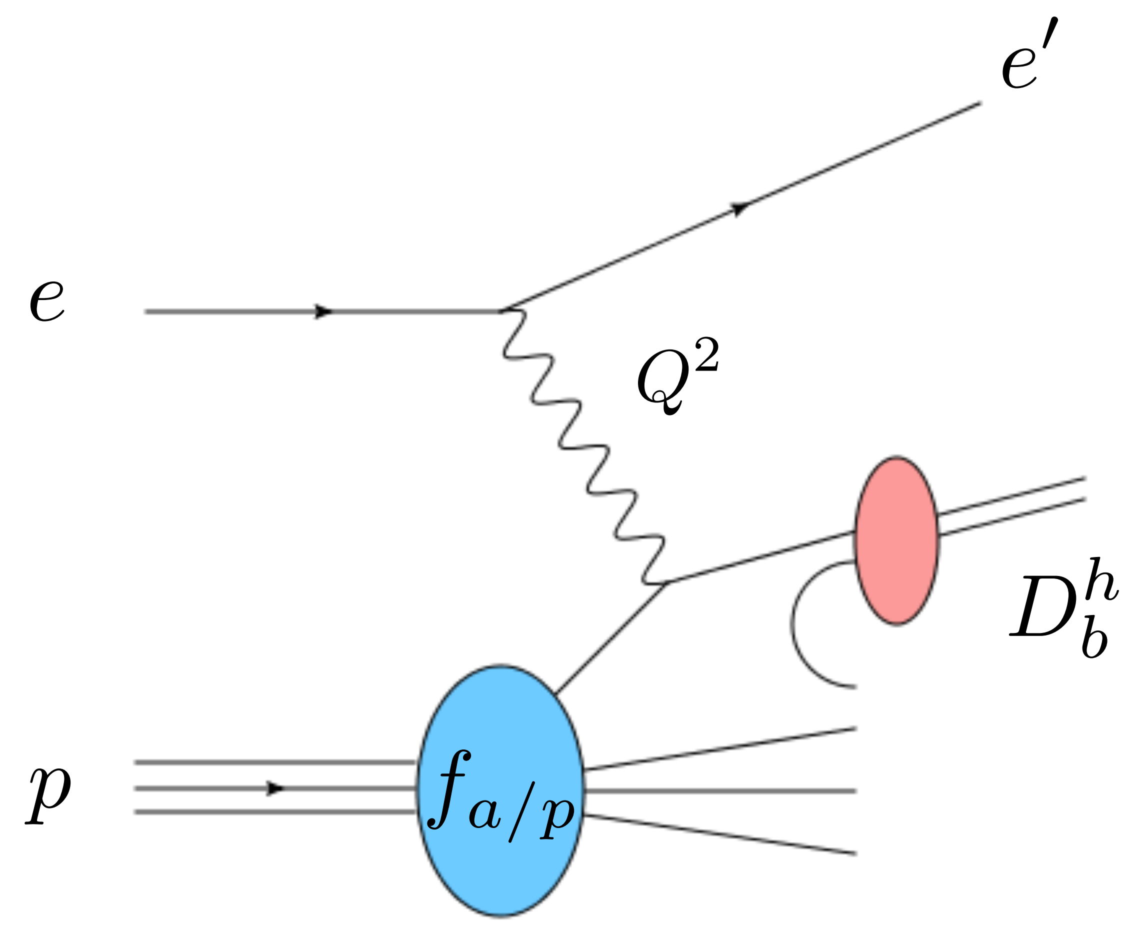
- QCD factorization

Collins, Soper, Sterman et al.

Perturbative

$$\mathrm{d}\sigma^{ep\to h+X} = \sum_{ab} f_{a/p} \otimes H_{ab} \otimes D_b^h$$

Non-perturbative

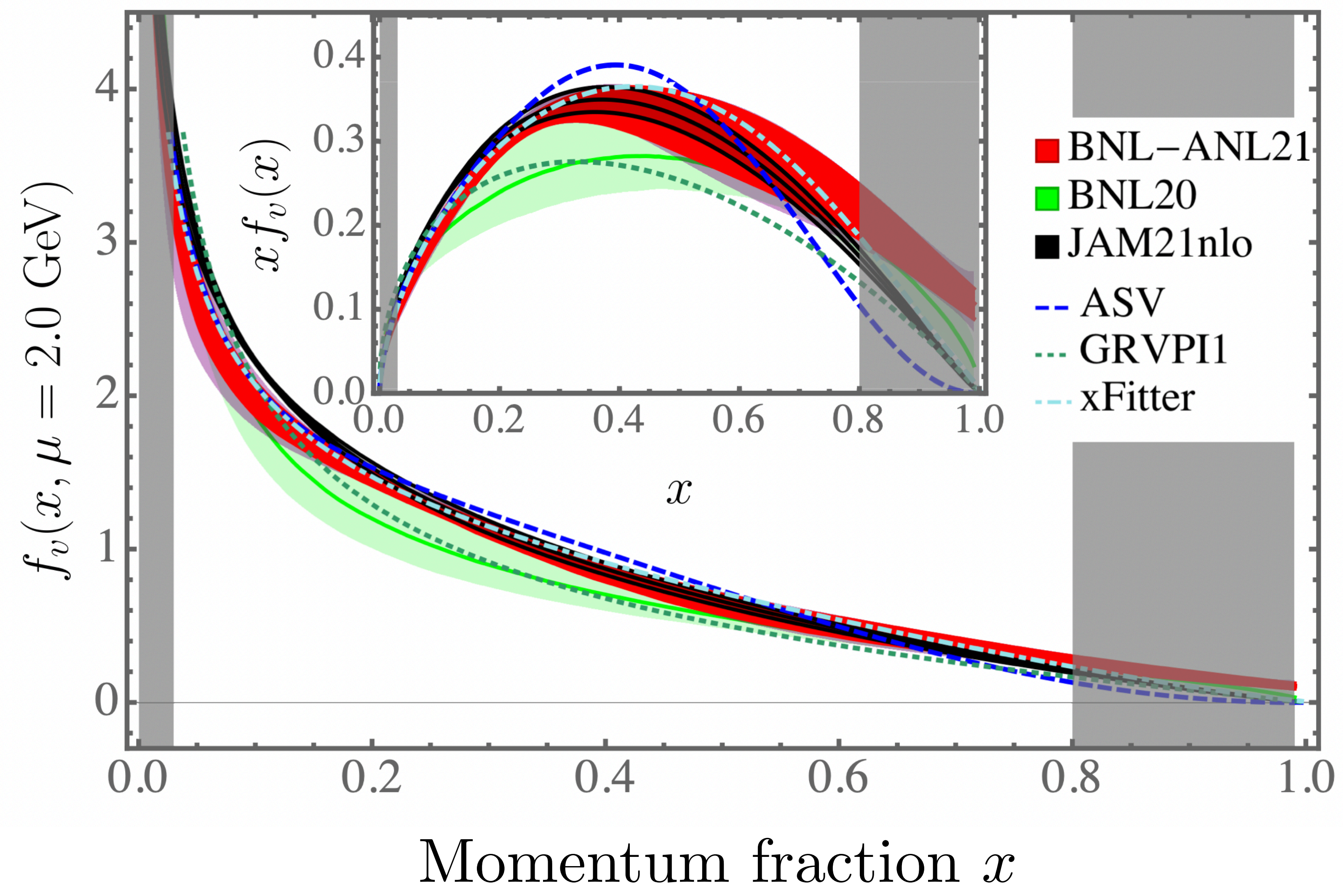


- Non-perturbative dynamics at low energies

$$\sim \mathrm{F.T.}\, \langle p|\mathcal{O}(t,\vec{x})\mathcal{O}(0)|p\rangle$$

N-point lightcone correlation functions $t^2 - \vec{x}^{\,2} = 0$

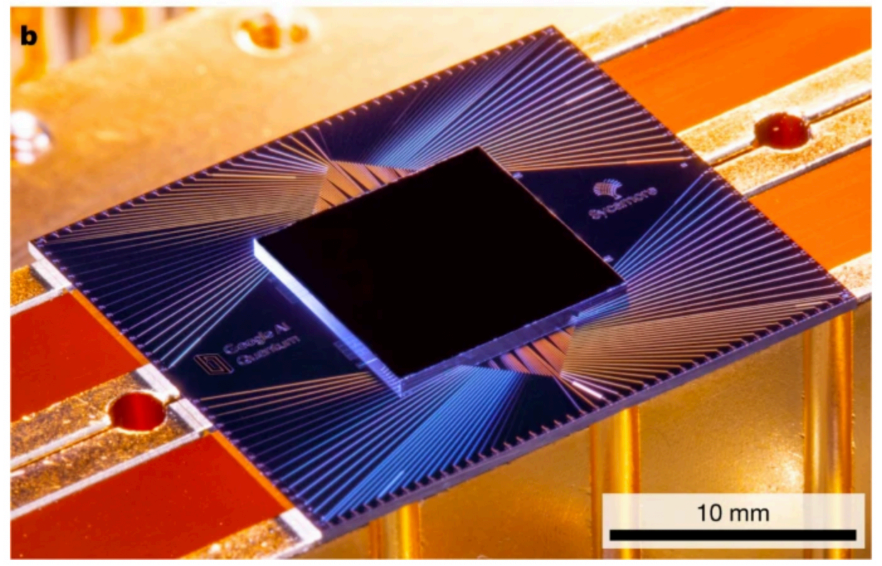
Quasi-PDF/FF, see Jake Montgomery's talk



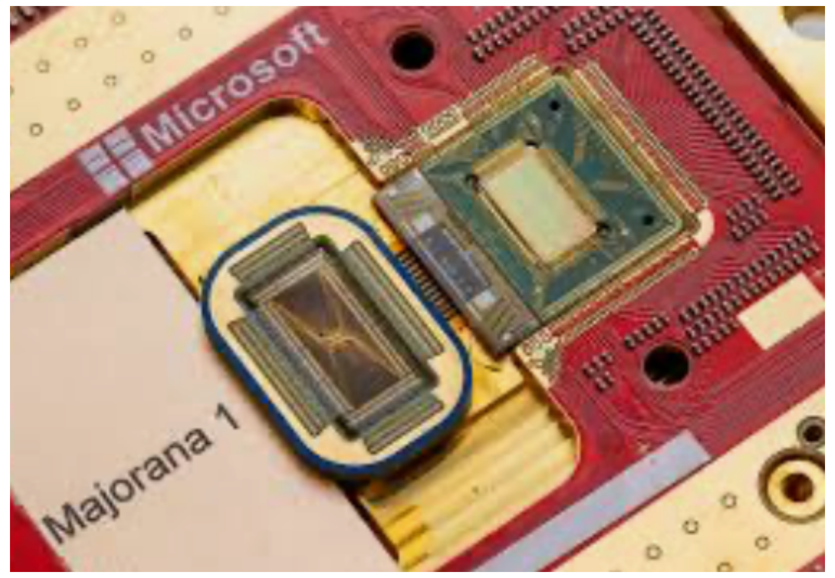
Zhao et al. `22

Quantum computing for nuclear physics

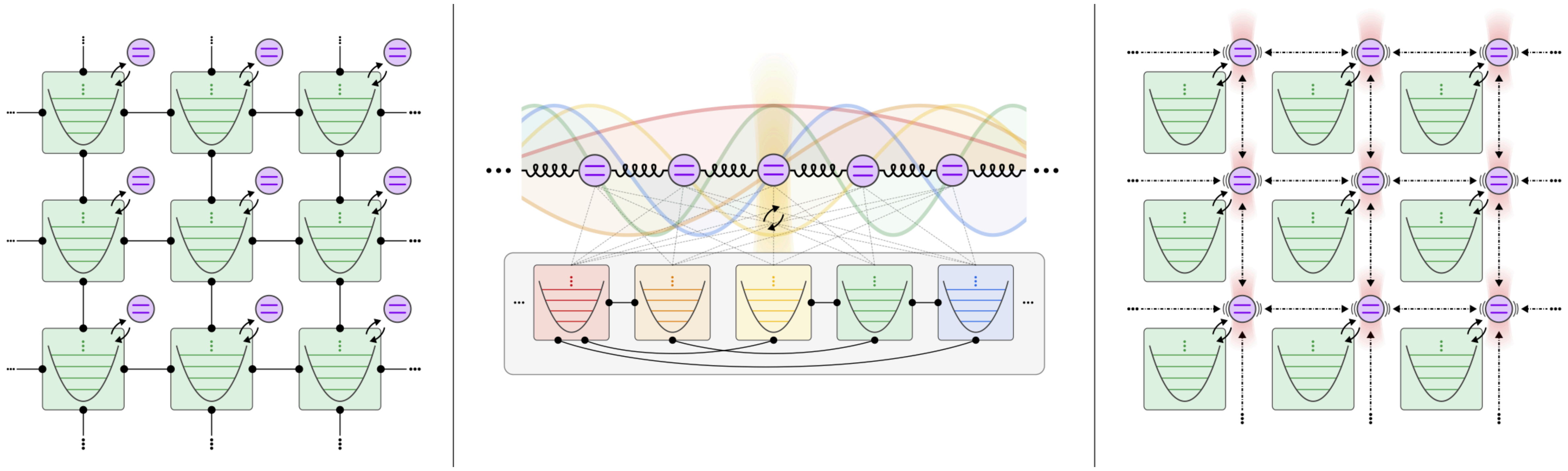
- Quantum advantage for random circuit sampling (2019)

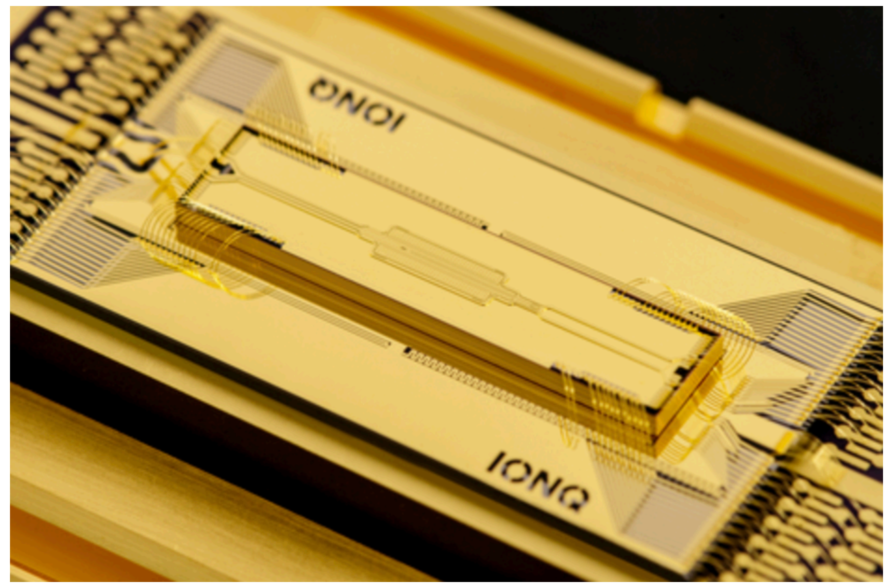


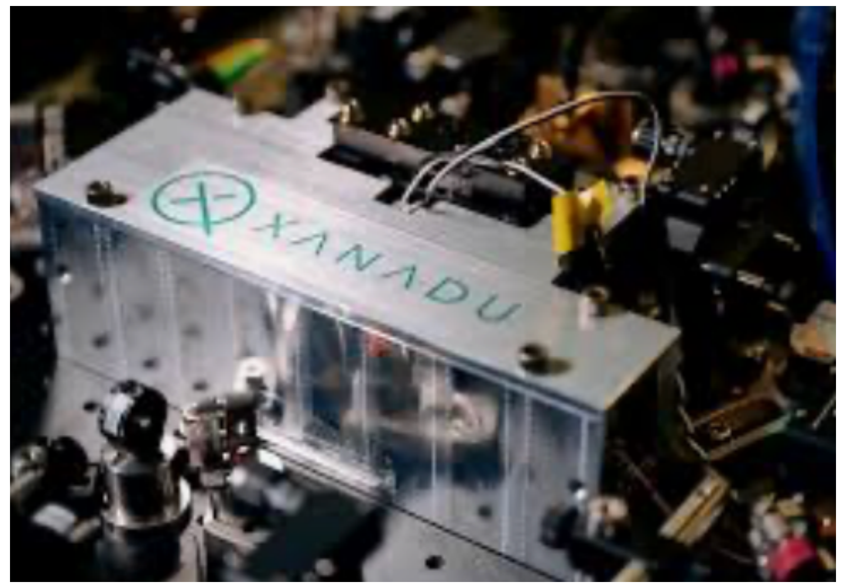
- Progress toward long-lived logical qubits using quantum error correction



- Hybrid quantum hardware



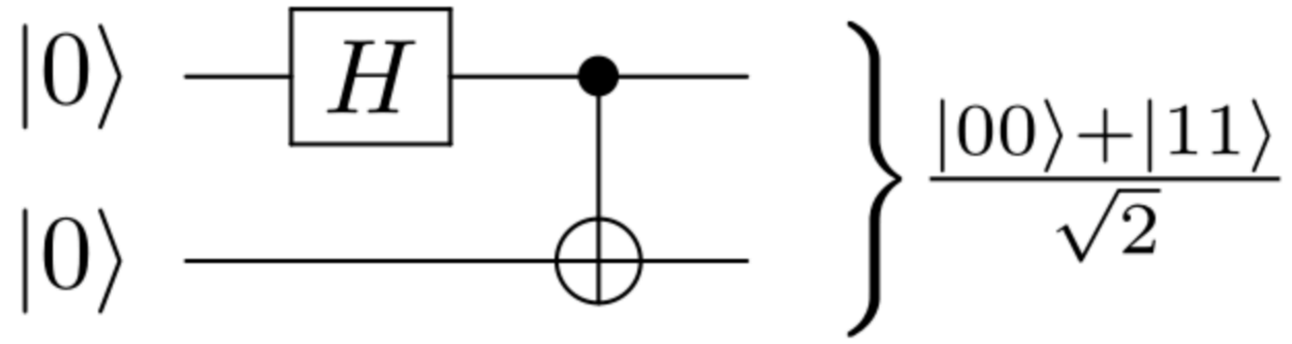


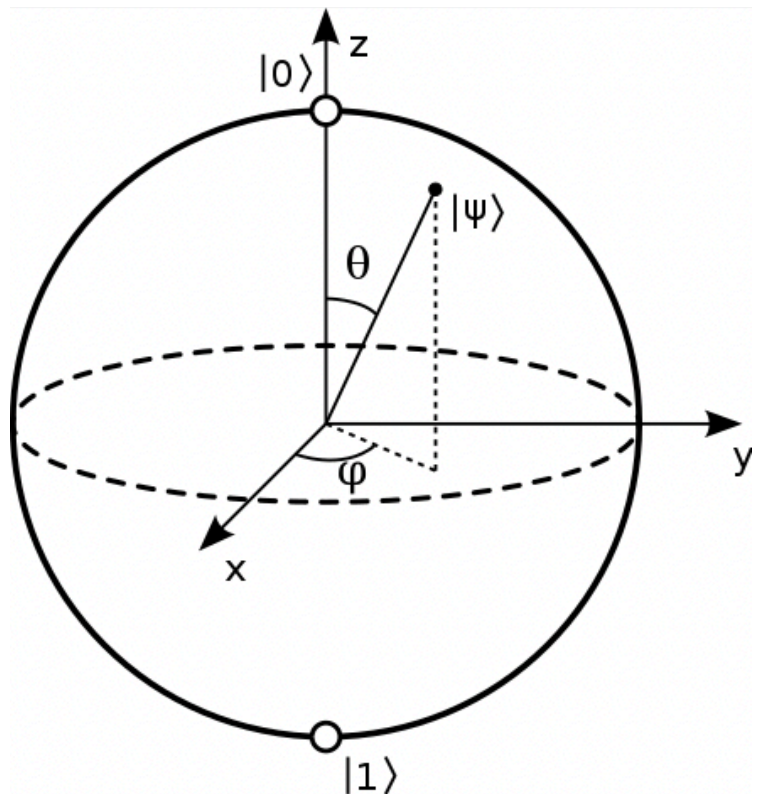


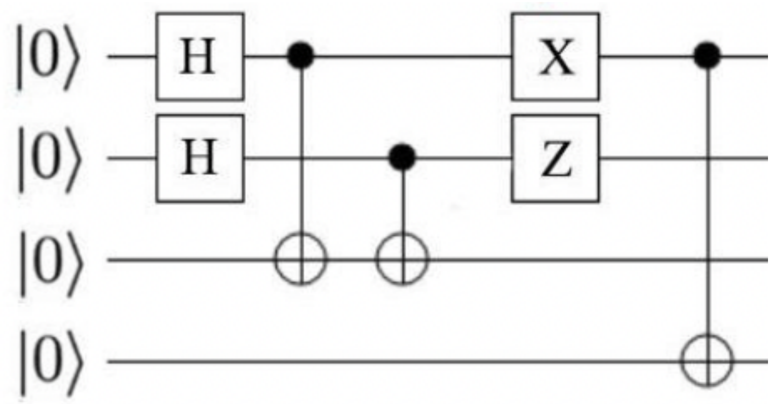
Qubits, qudits and qumodes

Elementary units for computing

- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Superposition $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$
  - Digital gate-based computing
  - Entanglement



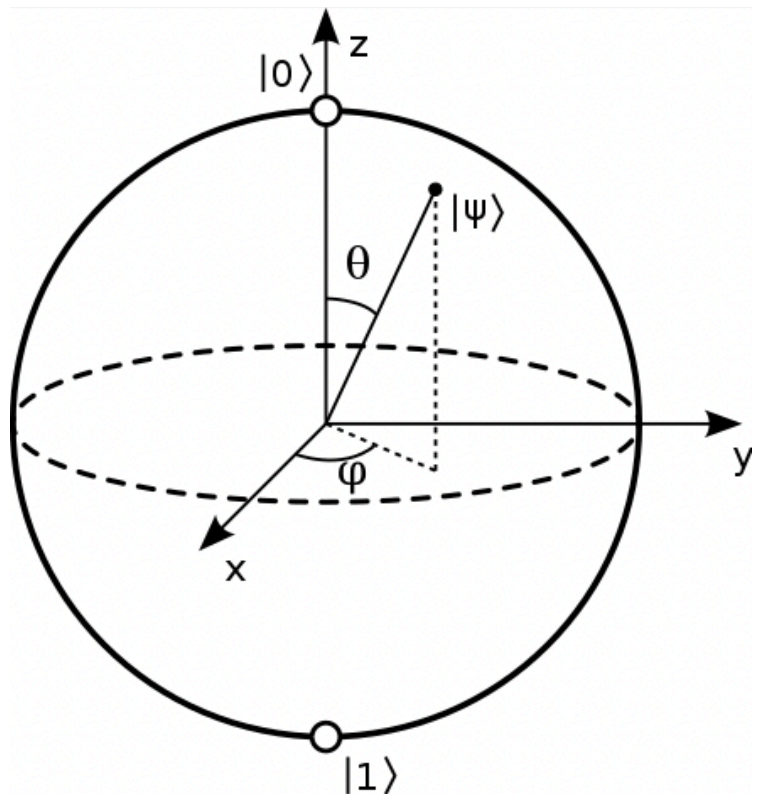


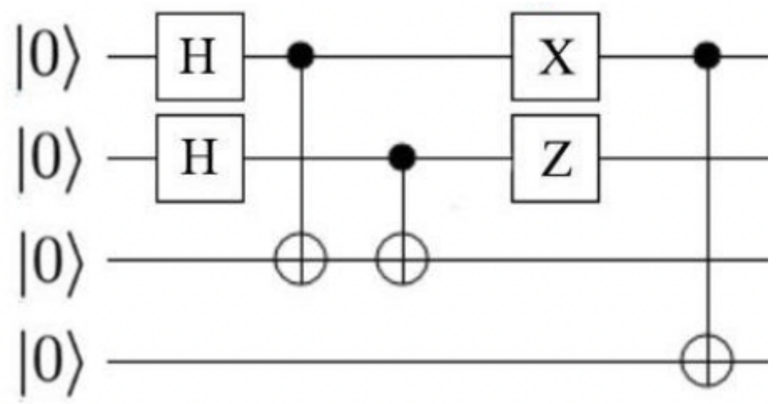


Qubits, qudits and qumodes

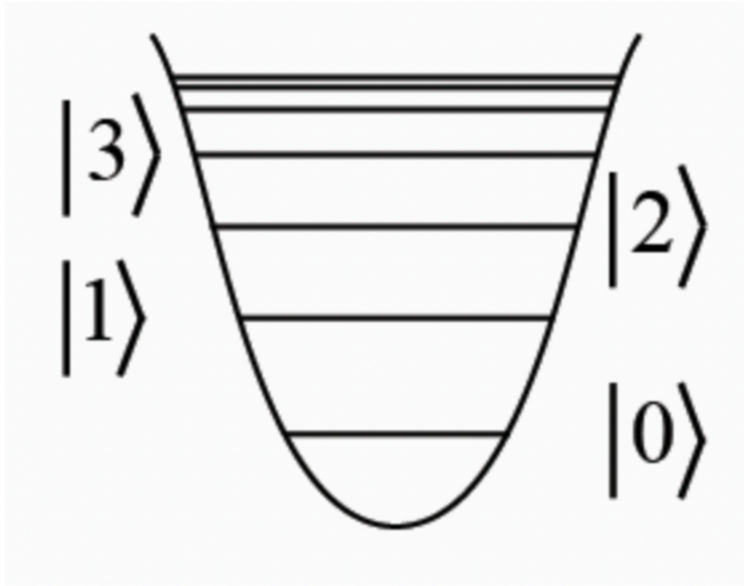
Elementary units for computing

- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Digital gate-based computing





- **Qudits**
  - Multi-level $d > 2$ computational units
  - Gates e.g. $X_d,\ Z_d,\ C_2[R_d]$
  - Various hardware platforms

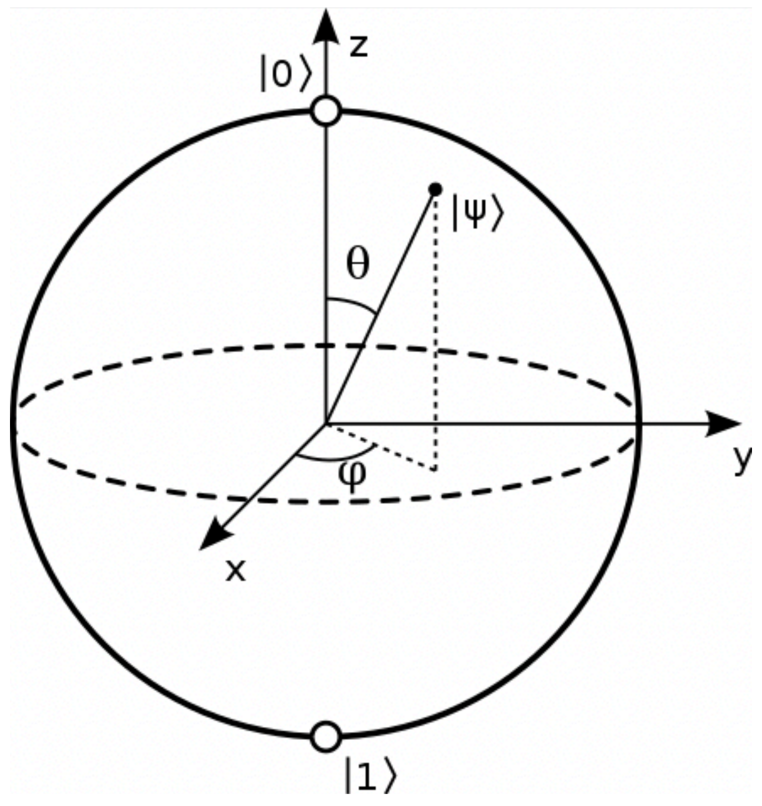


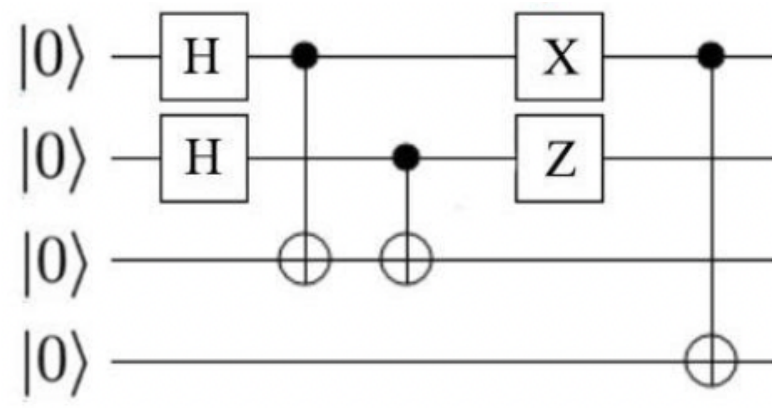
see e.g. Savage et al., Zoller et al.

Qubits, qudits and qumodes

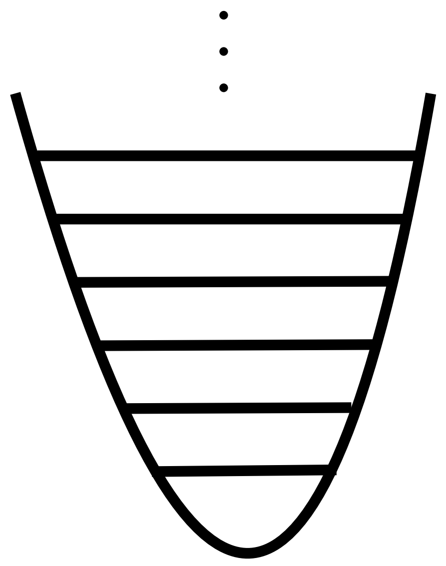
Elementary units for computing

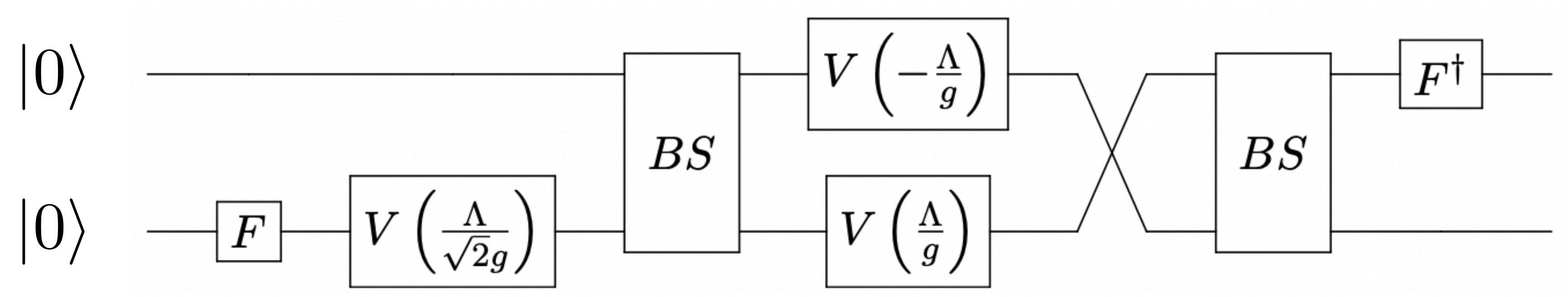
- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Digital gate-based computing





- **Qumodes**
  - Photonics, trapped ions, superconducting circuits
  - Infinite-dimensional Hilbert space
  - Gate based but with continuous variables





Quantum simulations with qumodes

see Lloyd, Braunstein `99

- Bosonic raising/lowering operators $\hat{a}, \hat{a}^\dagger$ and Fock states $|n\rangle \sim \left(a^\dagger\right)^n |0\rangle$

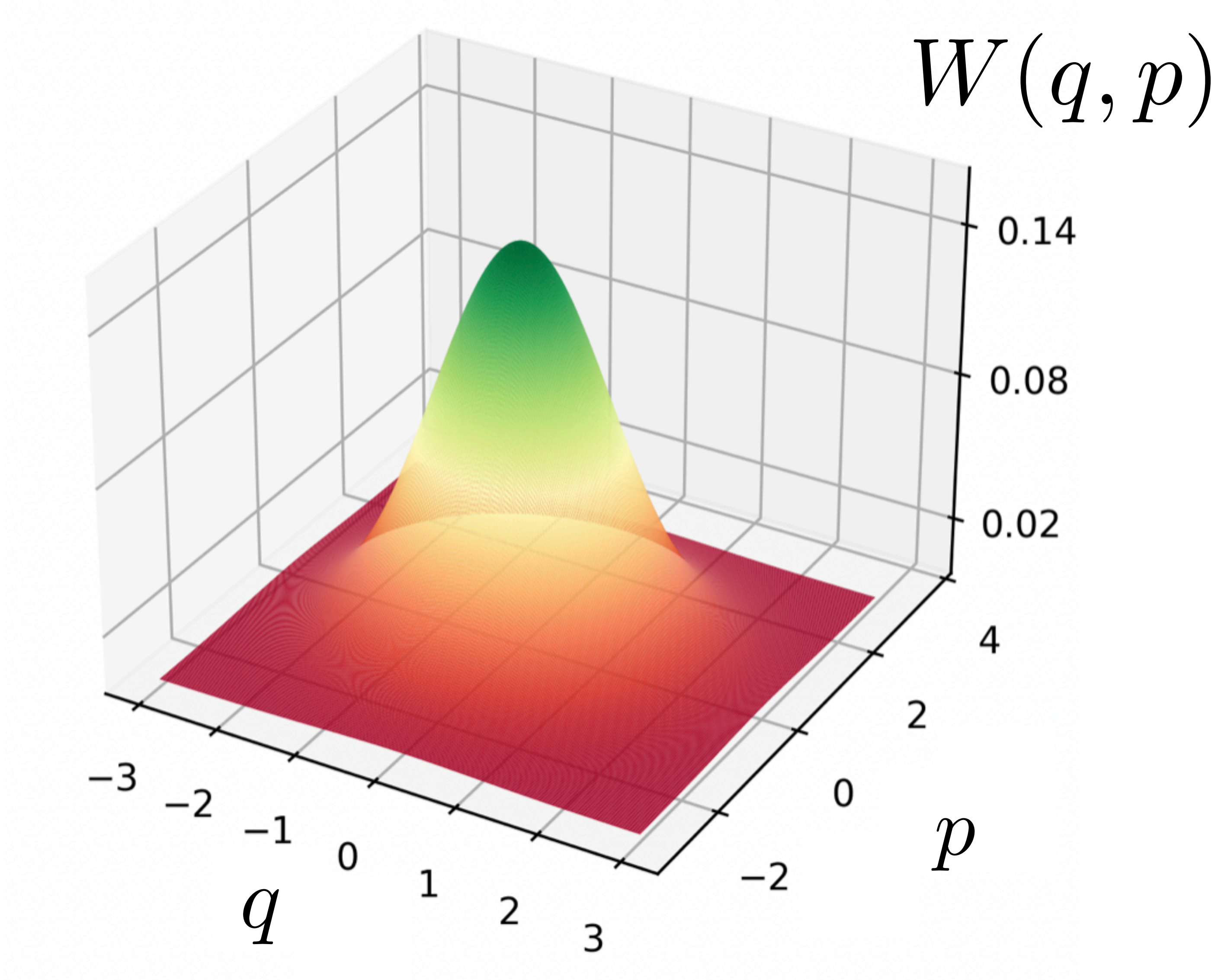
- Quadratures $\hat{q}, \hat{p}$ with $[\hat{q}, \hat{p}] = i$ and state $|\psi\rangle = \int_{\mathbb{R}} \psi(q)\,|q\rangle\,\mathrm{d}q$

- Visualize using Wigner functions

$$W(q,p) \sim \ \text{F.T. } \psi^*(q+q')\psi(q-q')$$

Ground state $|0\rangle$

Phase space

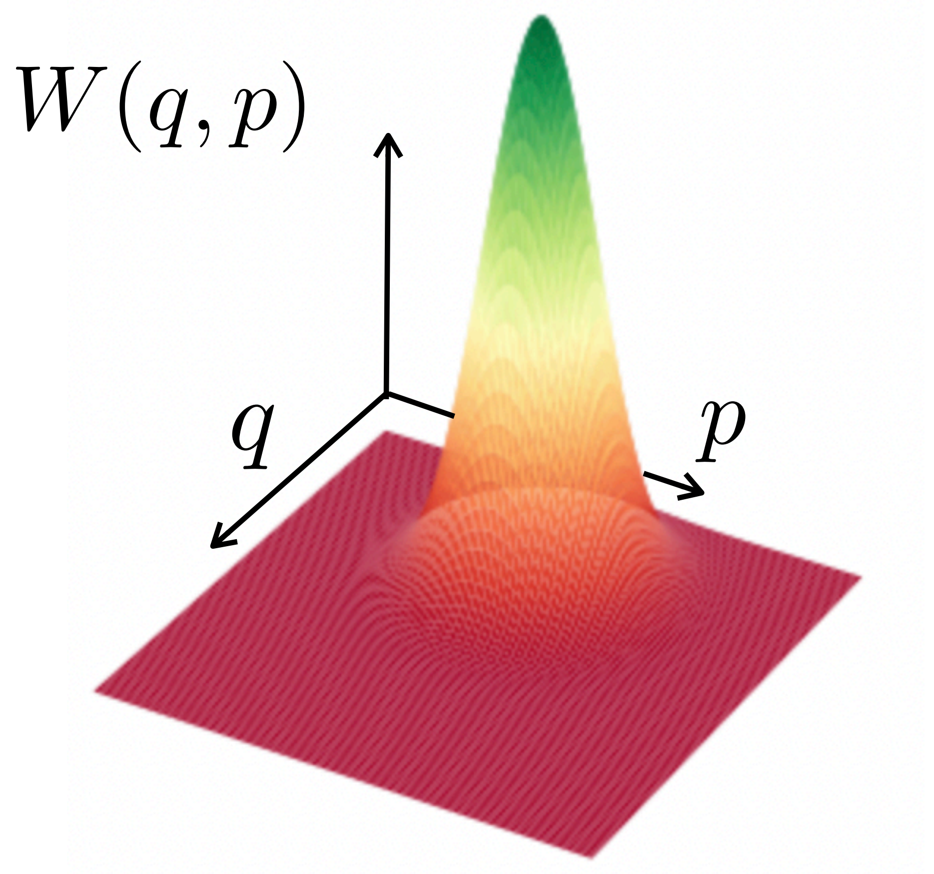


Quantum simulations with qumodes

see Lloyd, Braunstein `99

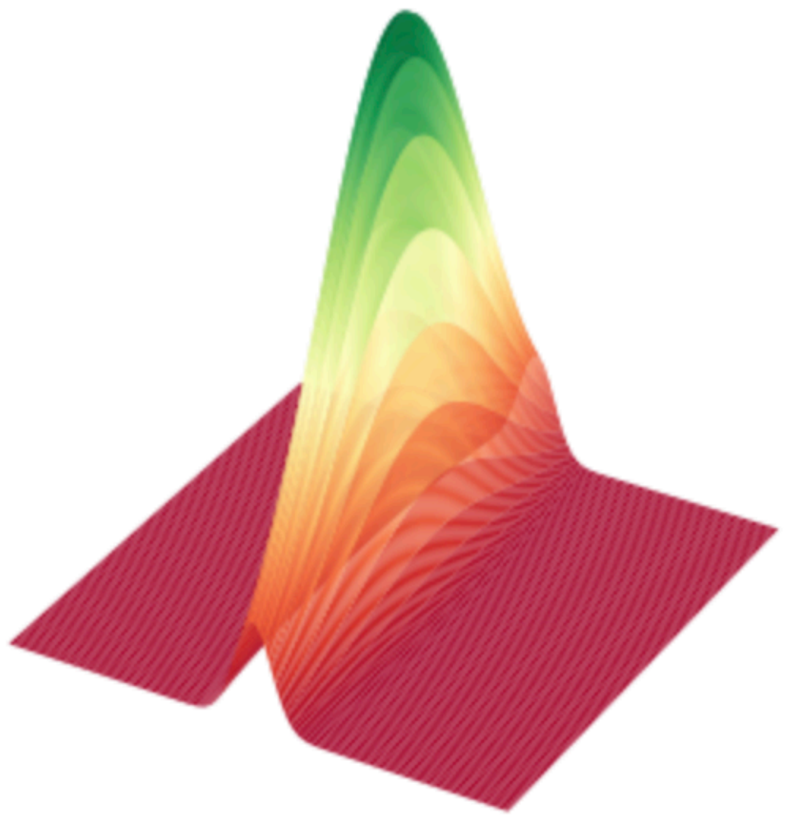
Universal gate set

- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$



- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$

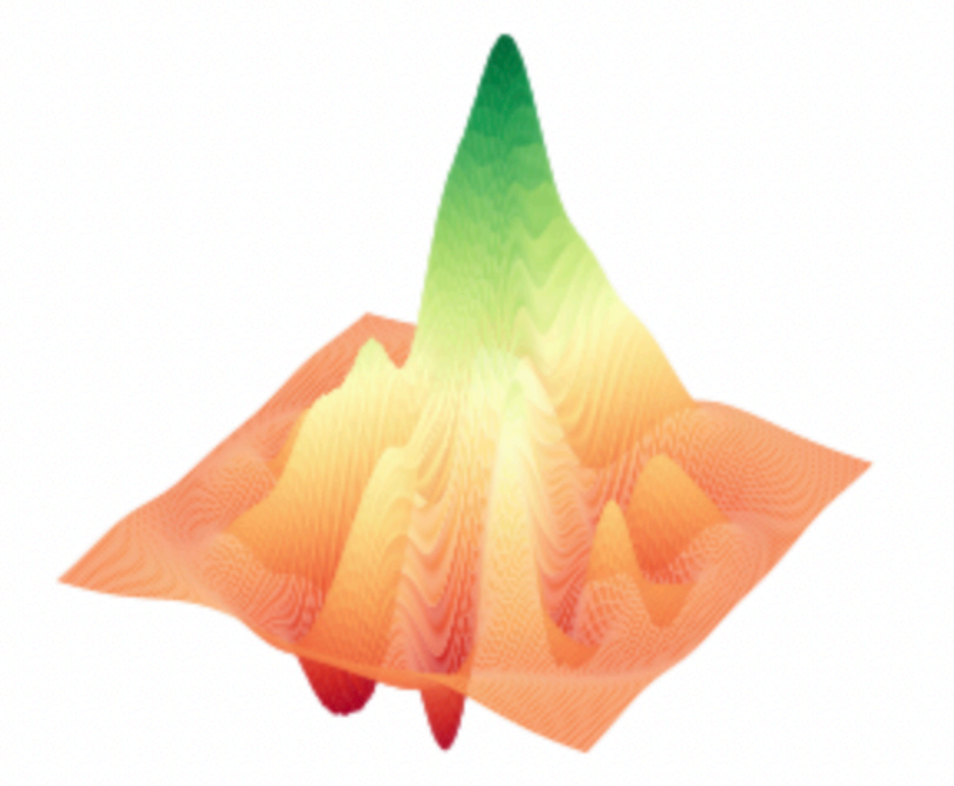


- Two-qumode beam splitter

$$U_{\rm bs}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}} \qquad z = \theta e^{i\phi}$$

- Non-Gaussian operations

  e.g. Kerr gate



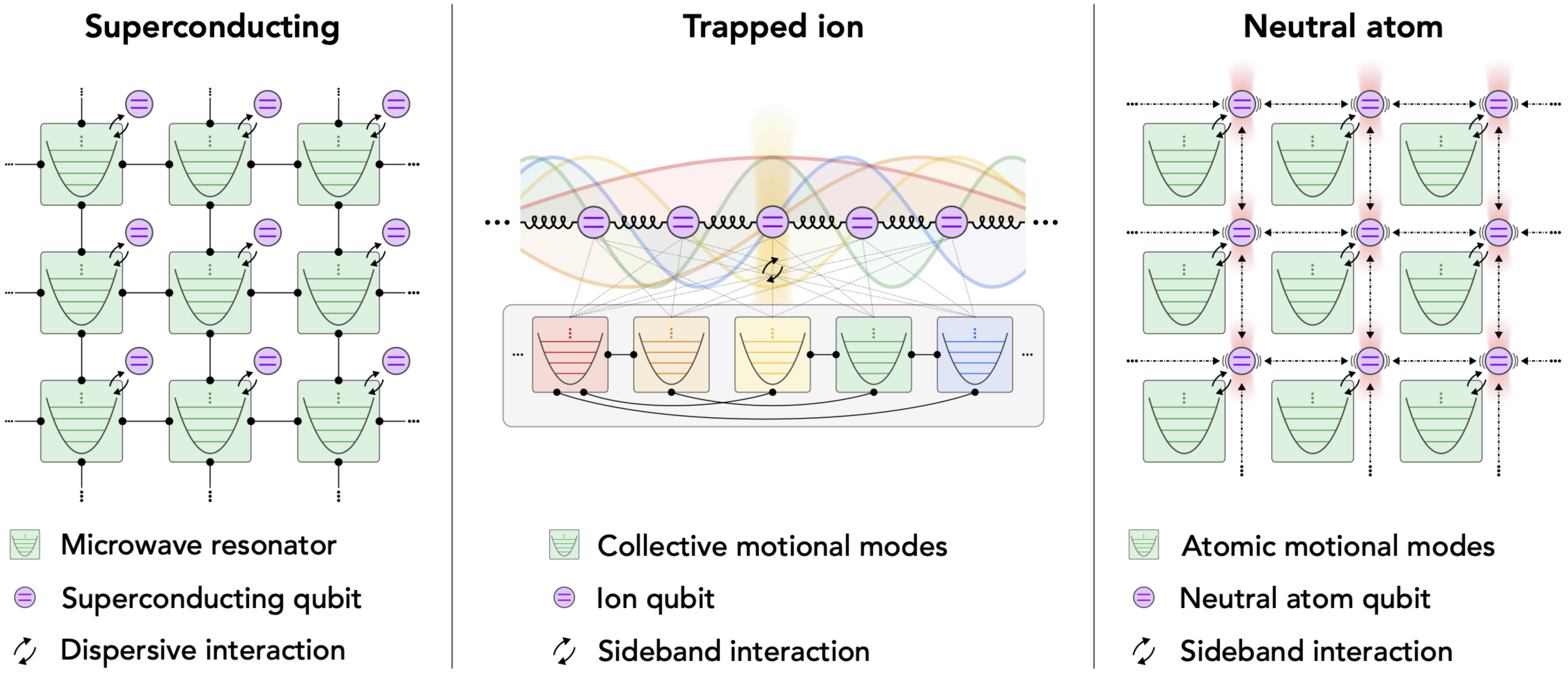
Qubit & qumode universality

- Minimal gate sets

Qubits	Hybrid	Qumodes
Universal: Clifford ($S = \sigma_z^{1/2}$, H, CNOT); Magic $T = e^{-i\frac{\pi}{8}\sigma_z}$	Universal: $\mathrm{CD}(\theta) = e^{\sigma_z(\theta\hat{a}^\dagger - \theta^*\hat{a})}$, $R_i(\theta)$, $\mathrm{BS}(\theta,\phi)$	Universal: Gaussian ($\mathrm{S}(\theta)$, $\mathrm{R}(\theta)$, $\mathrm{D}(\theta)$, $\mathrm{BS}(\theta,\phi)$); Non-Gaussian $V(\theta) = e^{-i\theta\hat{q}^3}$
Pauli string σ_i	$\sigma_i \otimes P(\hat{q},\hat{p})$	Polynomial $P(\hat{q},\hat{p})$
Barenco et al., Gottesman, Knill	*Girvin, Wiebe et al. `24*	*Lloyd, Braunstein `97*

Qubit & qumodes hardware platforms

Chuang, Girvin, Wiebe, et al. `24



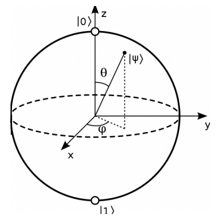
Platform dependence: • Native gate sets • Coherence times • Connectivity

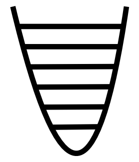
Quantum chromodynamics

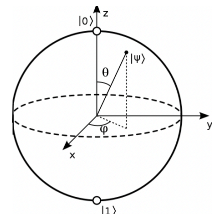
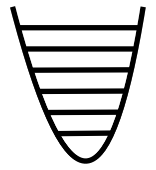
- Simulate quarks and gluons with different quantum resources?

- QCD Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\partial^{\mu}\gamma_{\mu} - m)\,\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A^a_{\mu} - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$$

Qubits 

Qumodes 

Qubit-qumode coupling  

→ **Develop a hybrid qubit/qumode approach**

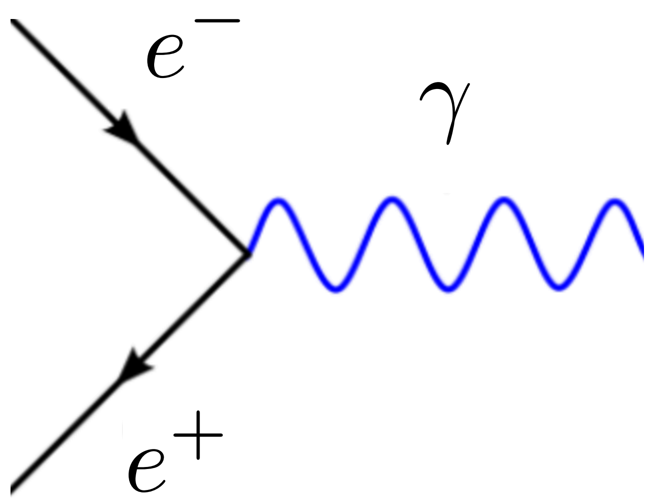
$$\mathcal{H}^m_{\text{qumode}} \otimes \mathcal{H}^n_{\text{qubit}}$$

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- QED Lagrangian

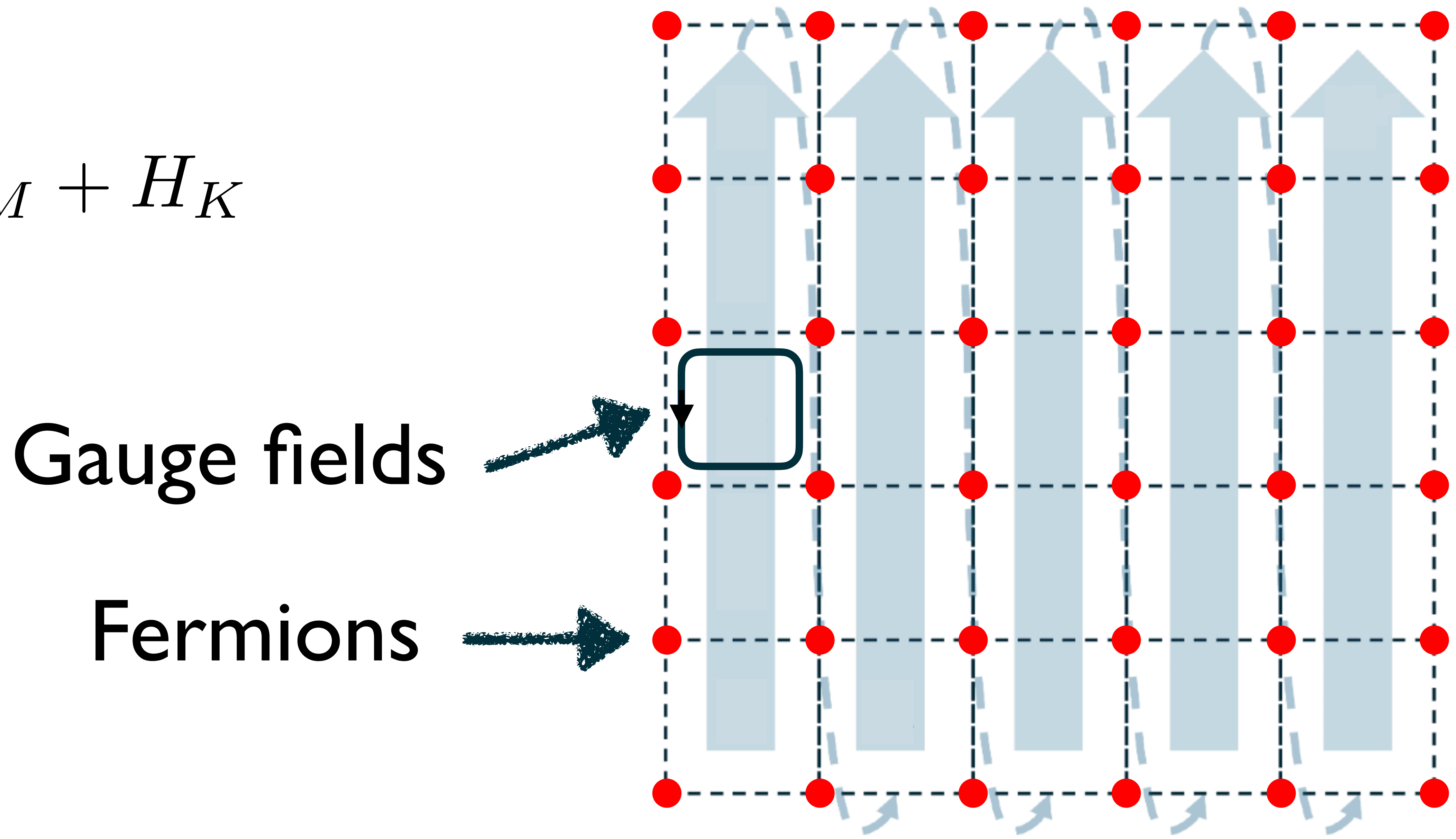
$$\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$



- Discretize on a spatial lattice $H = H_E + H_B + H_M + H_K$

$$H_E = \frac{g^2}{2}\sum_{\boldsymbol{n}} E^2_{\boldsymbol{n},i}, \quad H_B = -\frac{1}{2g^2a^2}\sum_{\boldsymbol{n}}\left(P_{\boldsymbol{n}} + P^\dagger_{\boldsymbol{n}} - 2\right)$$

with $P_{\boldsymbol{n}} \equiv U_{\boldsymbol{n},x}U_{\boldsymbol{n}+\boldsymbol{e}_x,y}U^\dagger_{\boldsymbol{n}+\boldsymbol{e}_y,x}U^\dagger_{\boldsymbol{n},y}$



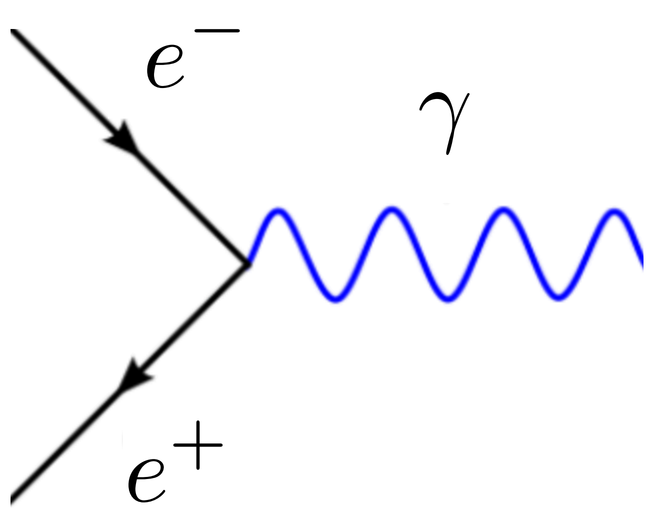
See also e.g. Girvin, Wiebe et al., Zoller et al., Savage et al.

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- QED Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

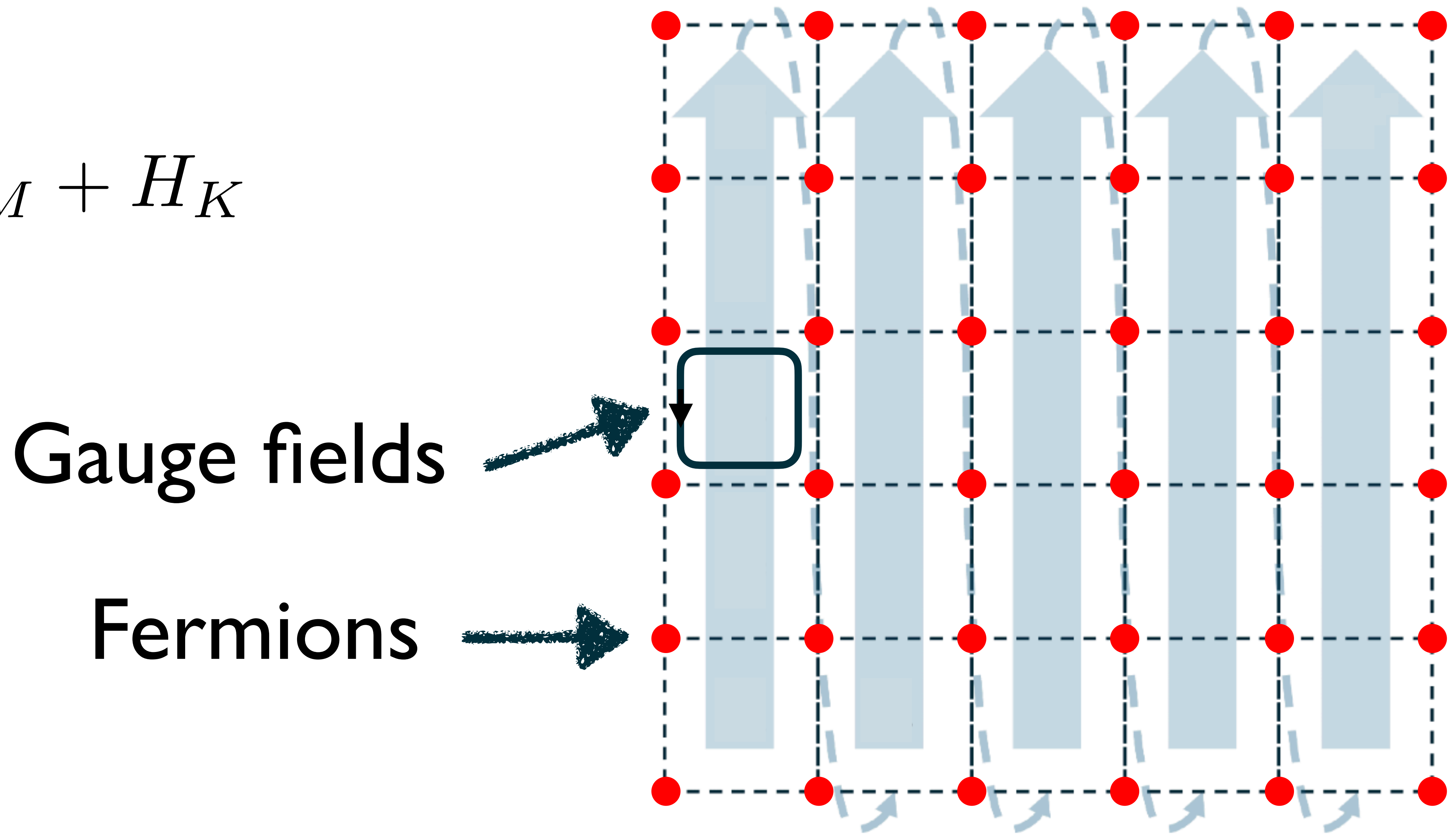


- Discretize on a spatial lattice $H = H_E + H_B + H_M + H_K$

$$H_E = \frac{g^2}{2}\sum_{\boldsymbol{n}} E^2_{\boldsymbol{n},i}, \quad H_B = -\frac{1}{2g^2a^2}\sum_{\boldsymbol{n}}\left(P_{\boldsymbol{n}} + P^\dagger_{\boldsymbol{n}} - 2\right)$$

with $P_{\boldsymbol{n}} \equiv U_{\boldsymbol{n},x}U_{\boldsymbol{n}+\boldsymbol{e}_x,y}U^\dagger_{\boldsymbol{n}+\boldsymbol{e}_y,x}U^\dagger_{\boldsymbol{n},y}$

$$H_M = m_0\sum_{\boldsymbol{n}}(-)^{n_x+n_y}\Psi^\dagger_{\boldsymbol{n}}\Psi_{\boldsymbol{n}}, \quad H_K = \frac{1}{2}\sum_{\boldsymbol{n}}\sum_{i\in\{x,y\}}\left[\Psi^\dagger_{\boldsymbol{n}}U^\dagger_{\boldsymbol{n},\boldsymbol{e}_i}\Psi_{\boldsymbol{n}+\boldsymbol{e}_i} + \text{ h.c. }\right]$$



See also e.g. Girvin, Wiebe et al., Zoller et al., Savage et al.

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- Qumodes are non-compact $q \in (-\infty, \infty)$
  - Scalar field theory *Marshall, Pooser, Siopsis, Weedbrock `15*

Quantum electrodynamics in 2+1d

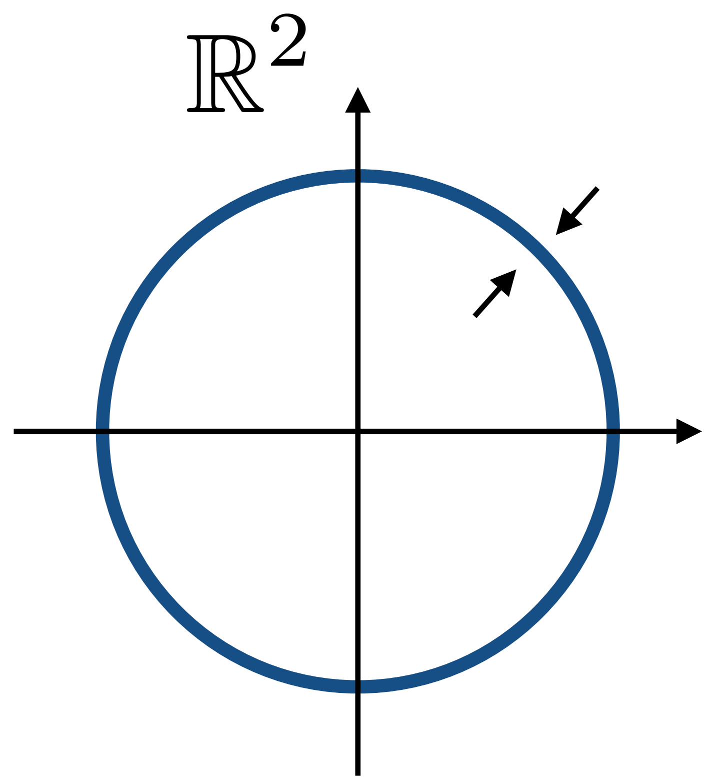
Ale, Rainaldi, Rico, FR, Siopsis `25

- Qumodes are non-compact $q \in (-\infty, \infty)$
  - Scalar field theory *Marshall, Pooser, Siopsis, Weedbrock `15*
- Gauge theories typically involve compact variables
  - U(1) link variable $U_{\boldsymbol{n}} = e^{i\theta_{\boldsymbol{n}}}, \ \theta_{\boldsymbol{n}} \in [0, 2\pi)$
  - With qubits typically use either finite representation or finite subgroup

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- Qumodes are non-compact $q \in (-\infty, \infty)$
  - Scalar field theory *Marshall, Pooser, Siopsis, Weedbrock `15*



- Gauge theories typically involve compact variables
  - U(1) link variable $U_{\boldsymbol{n}} = e^{i\theta_{\boldsymbol{n}}}, \; \theta_{\boldsymbol{n}} \in [0, 2\pi)$

Start with two qumodes $U_{\boldsymbol{n}} = q_0 + iq_1, \; q_{0,1} \in (-\infty, \infty)$

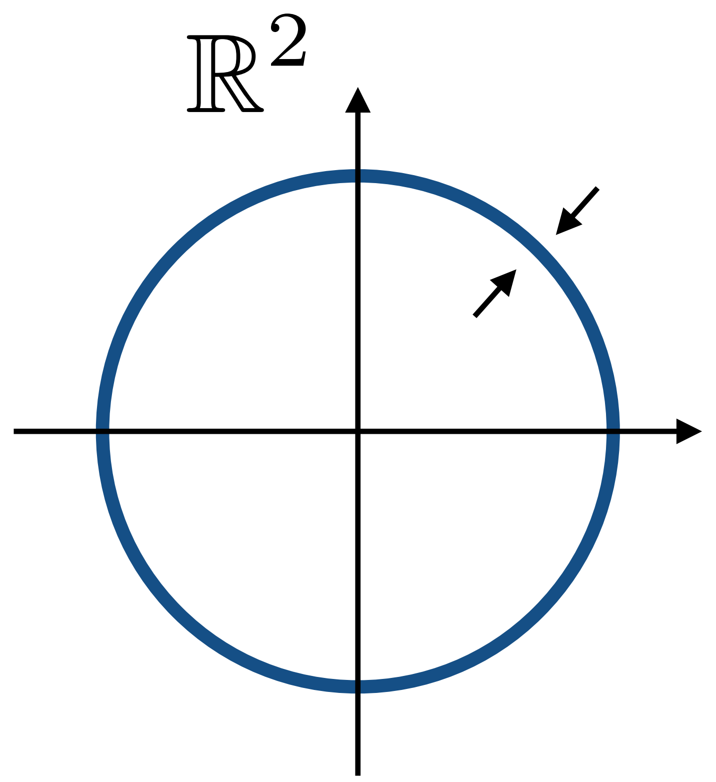
with the conjugate electric field $E_{\boldsymbol{n}} = q_0 p_1 - q_1 p_0$

and enforce the constraint $q_0^2 + q_1^2 = 1$ → continuous and compact variable

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- Enforcing the constraint per link variable
  - Penalty term in the Hamiltonian $H_\mu = \frac{\mu}{2} \sum_{\boldsymbol{n},i} \left(\boldsymbol{q}_{\boldsymbol{n},i}^2 - 1 \right)^2$
  - Modifying the inner product $|\psi\rangle \propto \int d^2 q \, \delta\left(\boldsymbol{q}^2 - 1 \right) \psi(\boldsymbol{q}) |\boldsymbol{q}\rangle$



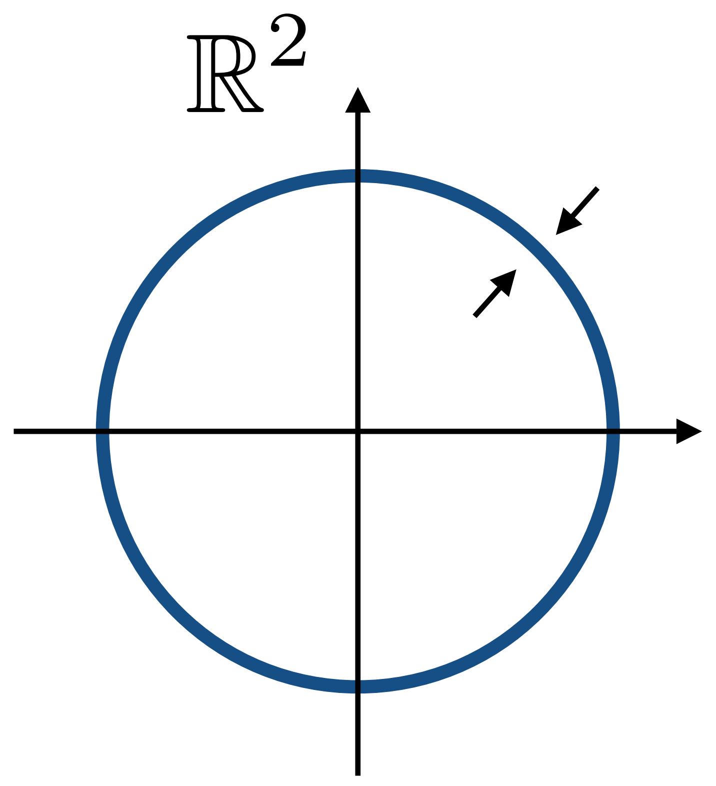
→ Without discretization of the gauge group

→ Direct extension to SU(2) possible

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- Enforcing the constraint per link variable
- Full Hamiltonian



$$H_E^{(Q)} = g^2 \sum_{\boldsymbol{n},\boldsymbol{n}'} \left(\mathcal{H}^{(2)}_{\boldsymbol{n}\boldsymbol{n}'} J_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(1)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(0)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} Q_{\boldsymbol{n}'} \right),$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \left(\boldsymbol{q}_{\boldsymbol{n}+e_y} - \boldsymbol{q}_{\boldsymbol{n}} \right)^2,$$

$$H_M^{(Q)} = m_0 \sum_{\boldsymbol{n}} (-)^{n_x+n_y} Q_{\boldsymbol{n}},$$

$$H_K^{(Q)} = \frac{1}{2} \sum_{\boldsymbol{n}} \left(q^0_{\boldsymbol{n}} - i q^1_{\boldsymbol{n}} \right) X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_x} + \frac{1}{2} \sum_{\boldsymbol{n}} P_{L(\boldsymbol{n},\boldsymbol{n}+e_y)} X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_y} + \text{h.c.}$$

$$Q_{\mathbf{n}} = \Psi^\dagger_{\boldsymbol{n}} \Psi_{\boldsymbol{n}} - \frac{1-(-)^{n_x+n_y}}{2} \mathbb{1}$$

$$J_{\boldsymbol{n}} = q^0_{\boldsymbol{n}} p^1_{\boldsymbol{n}} - q^1_{\boldsymbol{n}} p^0_{\boldsymbol{n}}$$

Qumode — Qubit/qumode

see also Wiebe, Girvin et al.

Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- Enforcing the constraint per link variable
- Full Hamiltonian

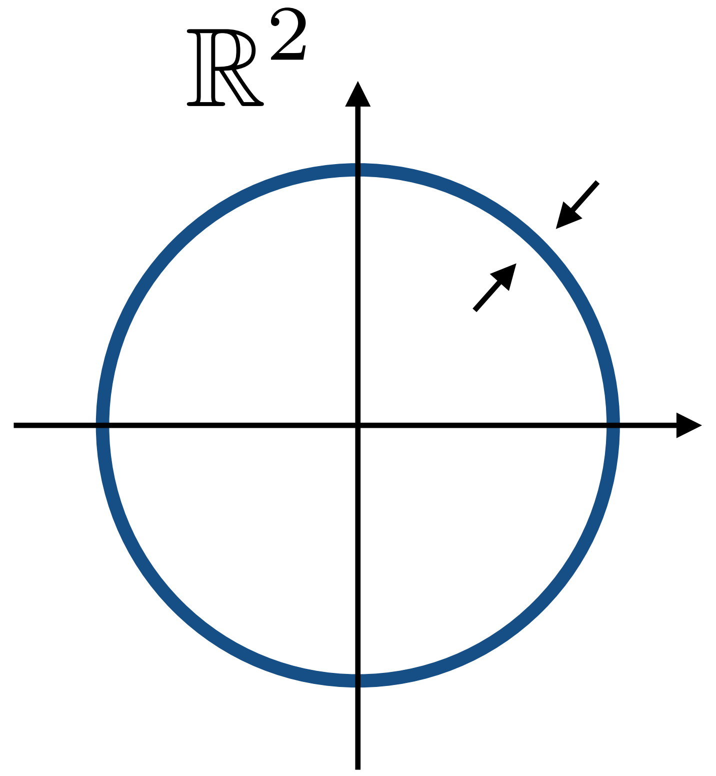
Due to Gauss's law

$$H_E^{(Q)} = g^2 \sum_{\boldsymbol{n},\boldsymbol{n}'} \left(\mathcal{H}^{(2)}_{\boldsymbol{n}\boldsymbol{n}'} J_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(1)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(0)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} Q_{\boldsymbol{n}'} \right),$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \left(\boldsymbol{q}_{\boldsymbol{n}+e_y} - \boldsymbol{q}_{\boldsymbol{n}} \right)^2,$$

$$H_M^{(Q)} = m_0 \sum_{\boldsymbol{n}} (-)^{n_x+n_y} Q_{\boldsymbol{n}},$$

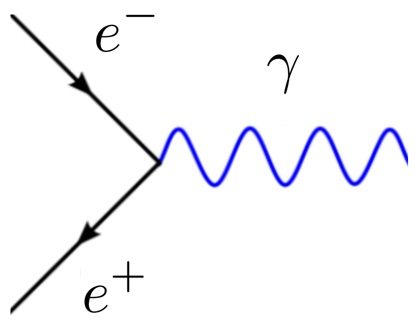
$$H_K^{(Q)} = \frac{1}{2} \sum_{\boldsymbol{n}} \left(q^0_{\boldsymbol{n}} - i q^1_{\boldsymbol{n}} \right) X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_x} + \frac{1}{2} \sum_{\boldsymbol{n}} P_{L(\boldsymbol{n},\boldsymbol{n}+e_y)} X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_y} + \text{h.c.}$$



$$Q_{\mathbf{n}} = \Psi^\dagger_{\boldsymbol{n}} \Psi_{\boldsymbol{n}} - \frac{1-(-)^{n_x+n_y}}{2} \mathbb{1}$$

$$J_{\boldsymbol{n}} = q^0_{\boldsymbol{n}} p^1_{\boldsymbol{n}} - q^1_{\boldsymbol{n}} p^0_{\boldsymbol{n}}$$

Electron/photon coupling



Qubit/qumode Qumode

Quantum electrodynamics in 2+1d

- More efficient formulation involving trigonometric gates

$$Q_i = \Psi_i^\dagger \Psi_i - \frac{1-(-1)^i}{2}\mathbb{I}$$

- Electric & magnetic part

$$\hat{H}_E = \frac{g^2}{2}\sum_{ij}\left(\hat{p}_i\mathcal{H}^{(2)}_{ij}\hat{p}_j + \hat{p}_i\mathcal{H}^{(1)}_{ij}Q_j + Q_i\mathcal{H}^{(0)}_{ij}Q_j\right)$$

$$\hat{H}_B = -\frac{1}{g^2}\sum_j \cos\left(\hat{q}_j\right)$$

  - Non-compact $(\hat{q}_i, \hat{p}_i)$
  - Non-polynomial trigonometric gate

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis, in preparation

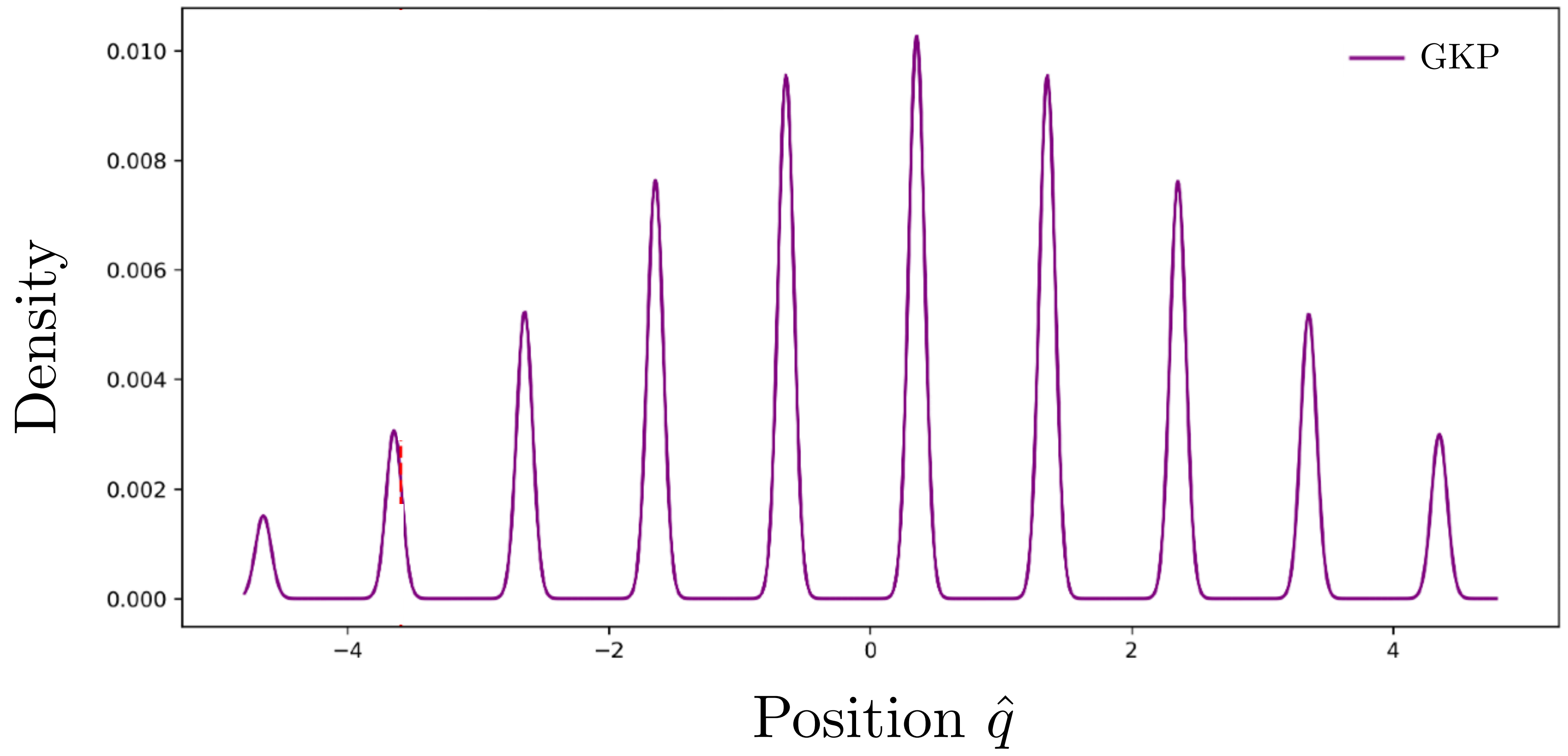
Quantum electrodynamics in 2+1d

- More efficient formulation involving trigonometric gates

- Electric & magnetic part

$$\hat{H}_E = \frac{g^2}{2}\sum_{ij}\left(\hat{p}_i\mathcal{H}^{(2)}_{ij}\hat{p}_j + \hat{p}_i\mathcal{H}^{(1)}_{ij}Q_j + Q_i\mathcal{H}^{(0)}_{ij}Q_j\right)$$

$$\hat{H}_B = -\frac{1}{g^2}\sum_j \cos\left(\hat{q}_j\right)$$



$$\psi_\beta(\chi) = \mathcal{N}\sum_{n\in\mathbb{Z}}\exp\left[-\frac{\beta}{2}(\chi+2\pi n)^2\right], \quad \chi\in[-\pi,\pi)$$

- Non-compact $(\hat{q}_i, \hat{p}_i)$
- Non-polynomial trigonometric gate

$\longrightarrow$ $L^2(\mathbb{R}) \to L^2(S^1),\ p\in\mathbb{Z}$

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis, in preparation

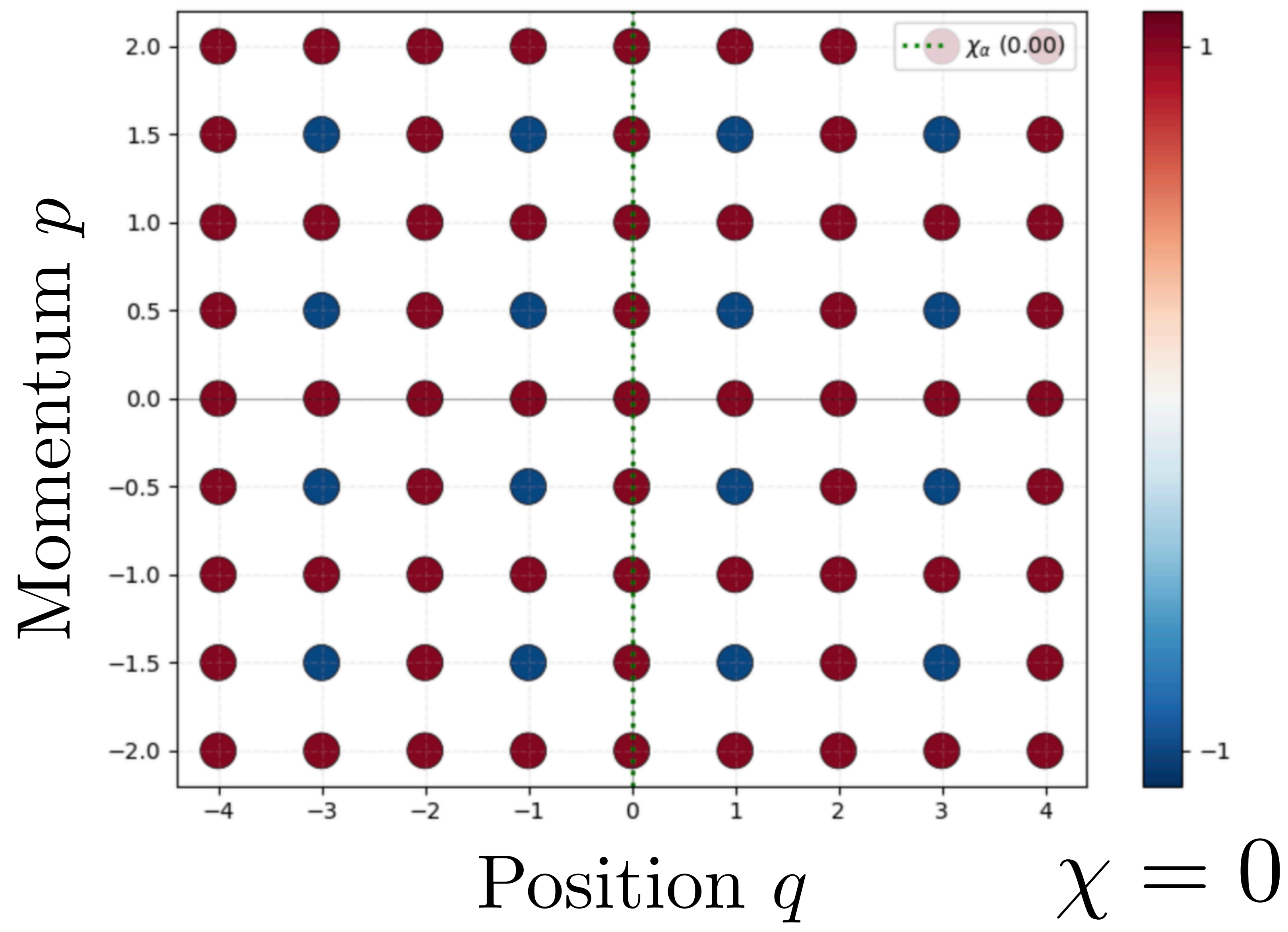
Quantum electrodynamics in 2+1d

- Electric & magnetic part

$$\hat{H}_E = \frac{g^2}{2}\sum_{ij}\left(\hat{p}_i \mathcal{H}^{(2)}_{ij}\hat{p}_j + \hat{p}_i \mathcal{H}^{(1)}_{ij} Q_j + Q_i \mathcal{H}^{(0)}_{ij} Q_j\right)$$

$$\hat{H}_B = -\frac{1}{g^2}\sum_j \cos\left(\hat{q}_j\right)$$

$$\hat{H}_J = -J\sum_j \cos\left(2\pi\hat{p}_j\right)$$



$\chi = 0$

GKP stabilizers converting non-compact to compact variables

+ quantum error correction

GKP/qunaught Wigner function

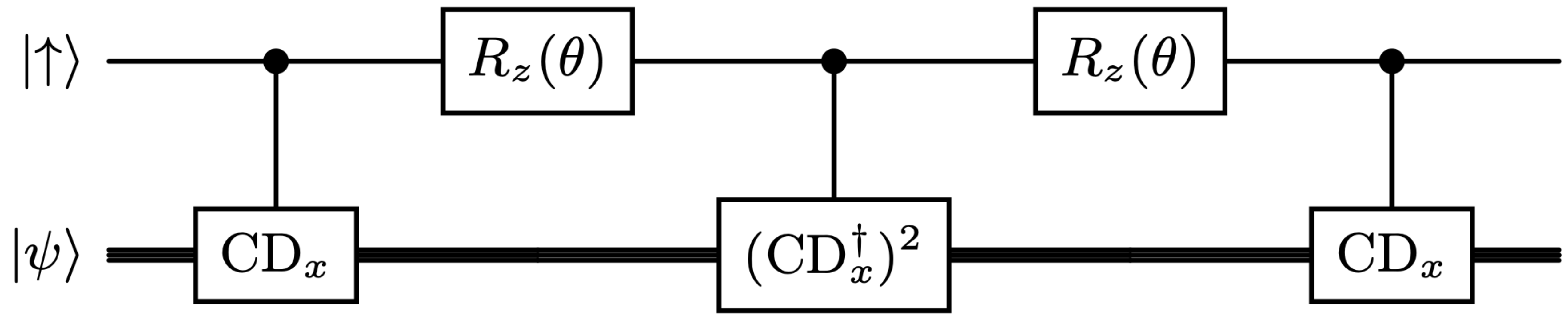
$$W(q,p) = \frac{1}{\pi}\int \mathrm{d}q'\, \psi^*(q+q')\psi(q-q')e^{2ipy}$$

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis, in preparation

Trigonometric gates

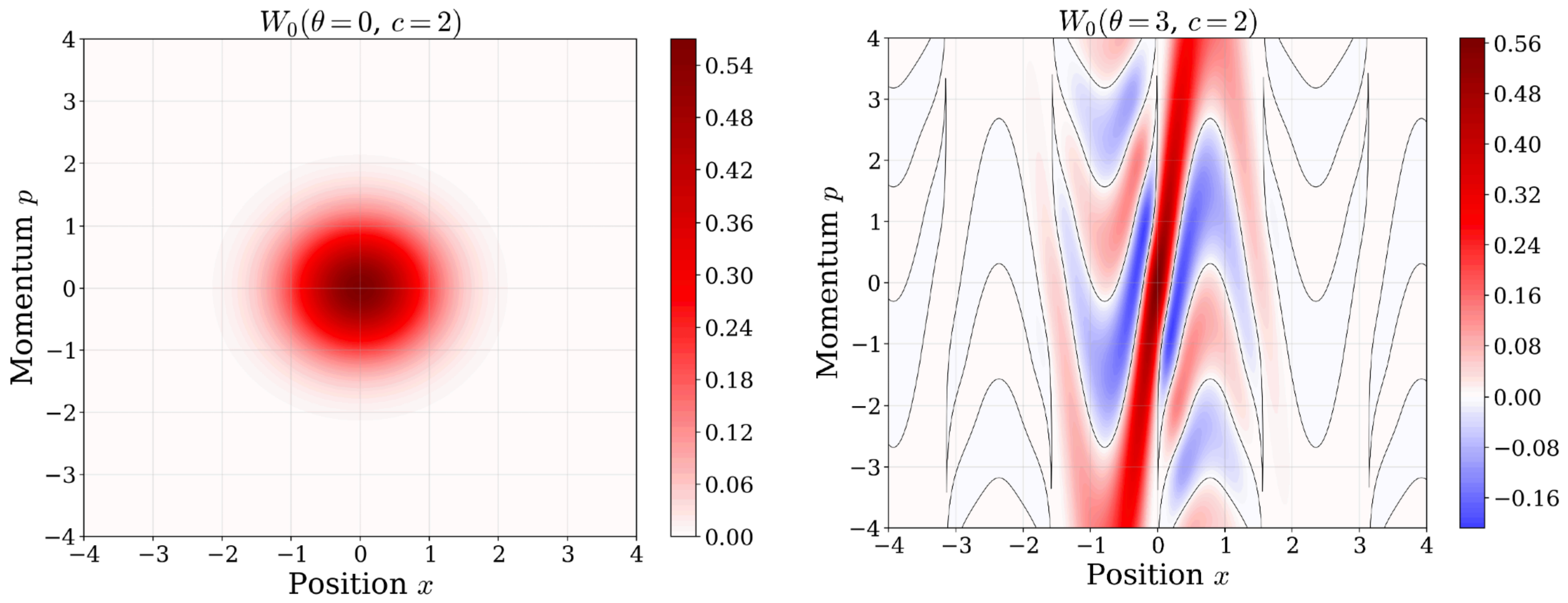
Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25
Kassel, Tan et al. `15

- Qubit/qumode circuit for $e^{-i\theta\cos(c\hat{q})}$



← Qubit decouples

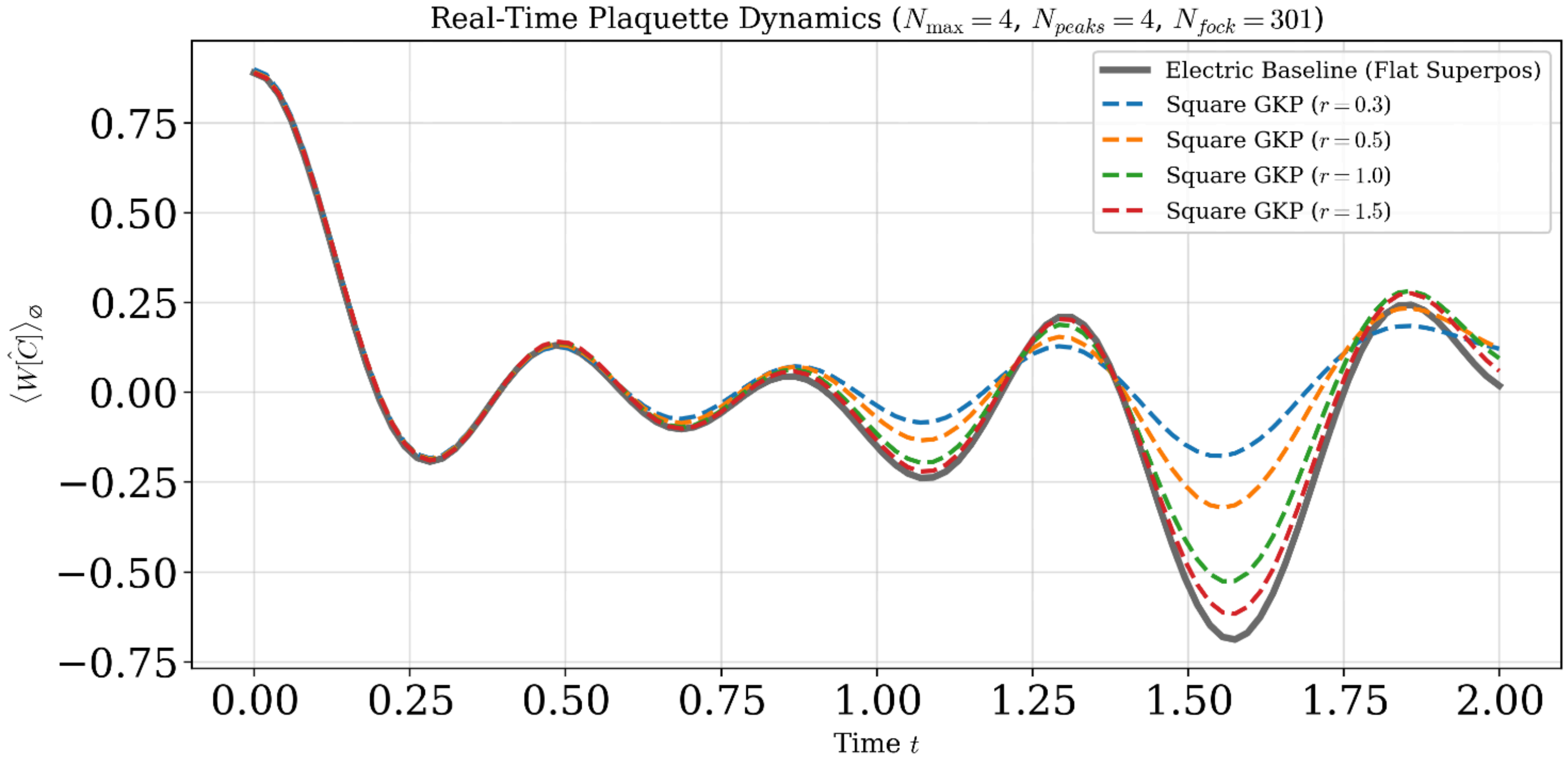
- Using unitary & hermitian Pauli operator
- First order Trotter approximation



Quantum electrodynamics in 2+1d

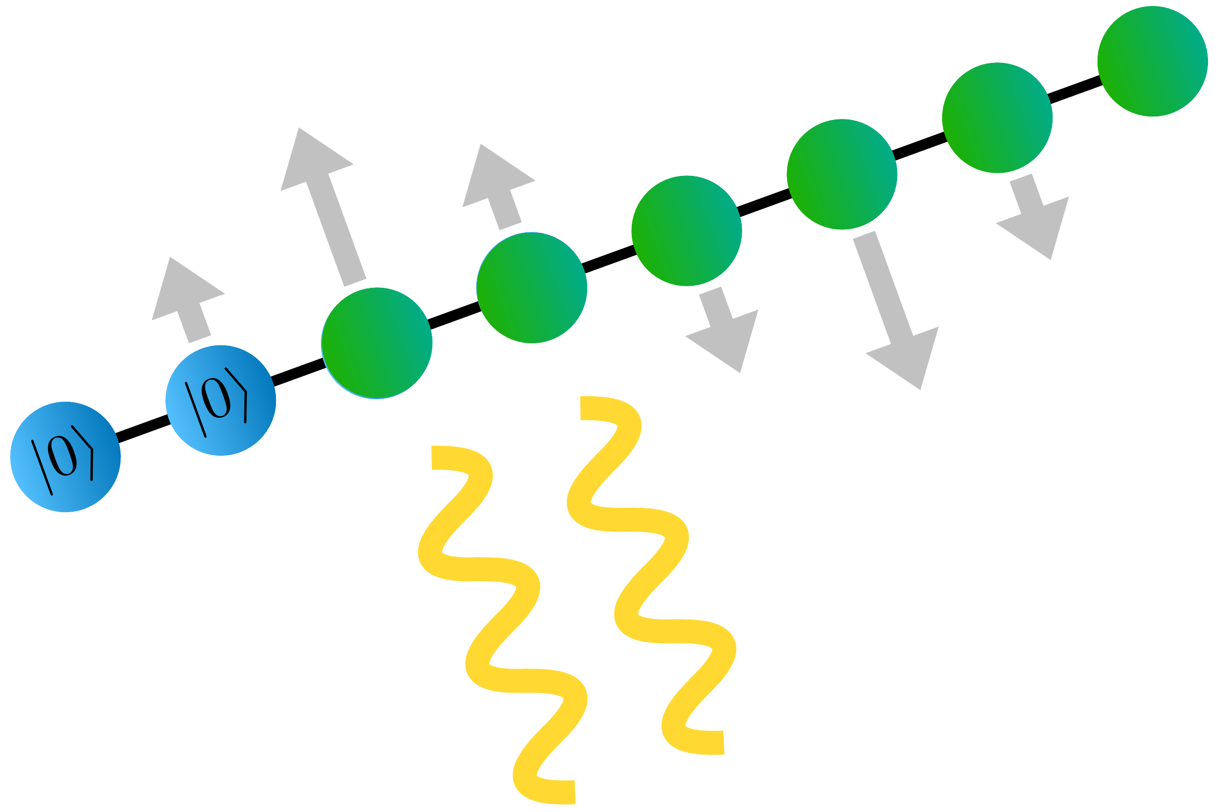
Ale, Rainaldi, Rico, FR, Siopsis, in preparation

- Real-time evolution
- E.g. single plaquette, one-point function

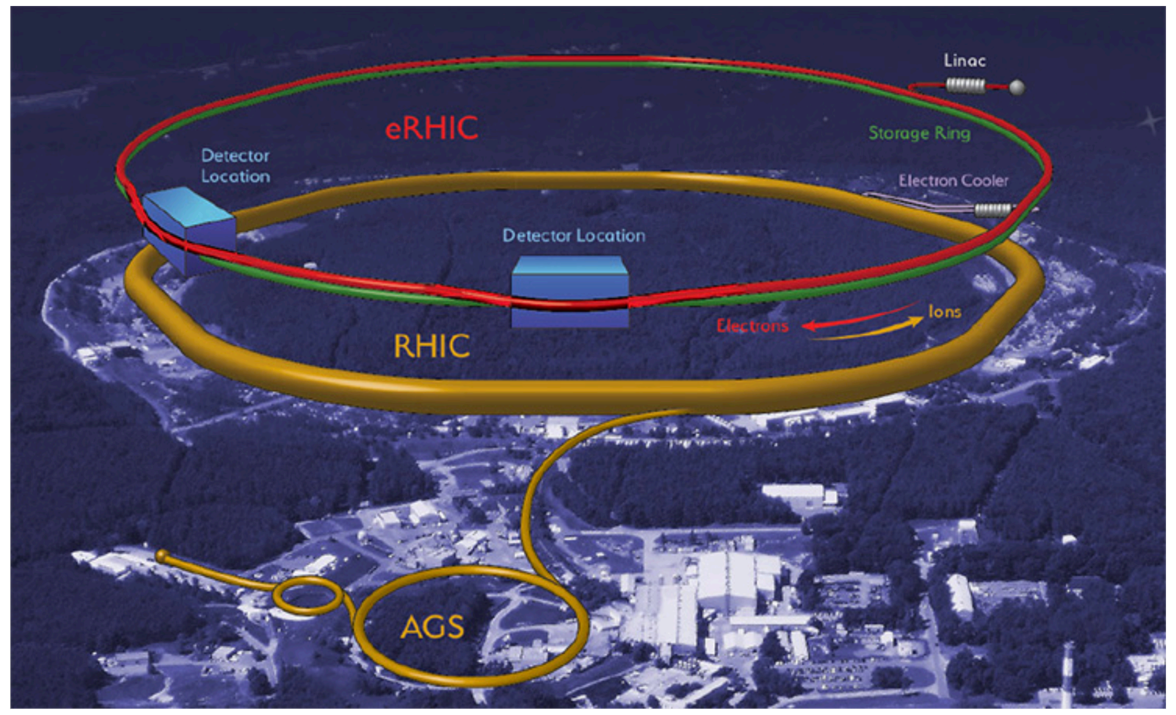


Conclusions

- Quantum computing is well-suited to address challenges in fundamental physics
- Exploration of qubits, qudits, qumodes



- Study lightcone correlators in 2+1d
- Build up toward quantum utility in nuclear physics



U.S. DEPARTMENT OF ENERGY

NERSC

QSCOUT