Can we discover the Odderon(s)* at the EIC ?

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" Gluon Saturation, Diffraction and Multihadron Production at the EIC:
Theory, Simulations and Feasibility of Measurements"
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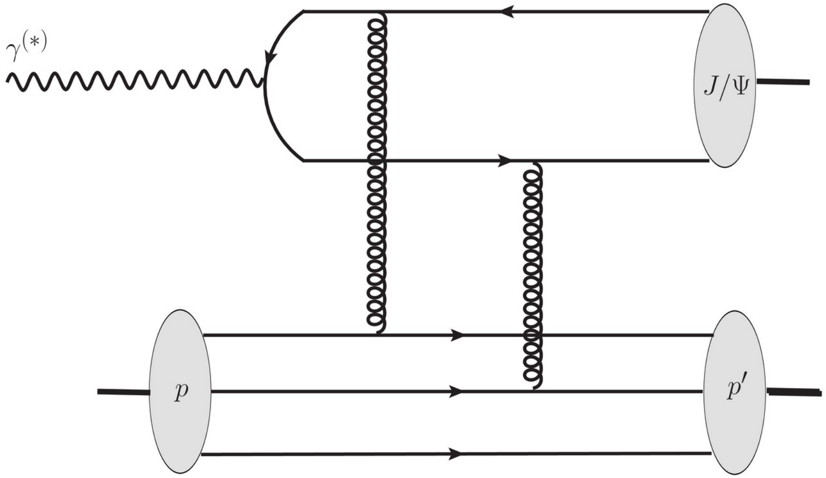
(*) This talk is about the perturbative / hard Odderon(s) ~ t-channel ggg exchange, different from the (awesome) Odderon discovered by TOTEM in pp vs $p\bar{p}$ scattering

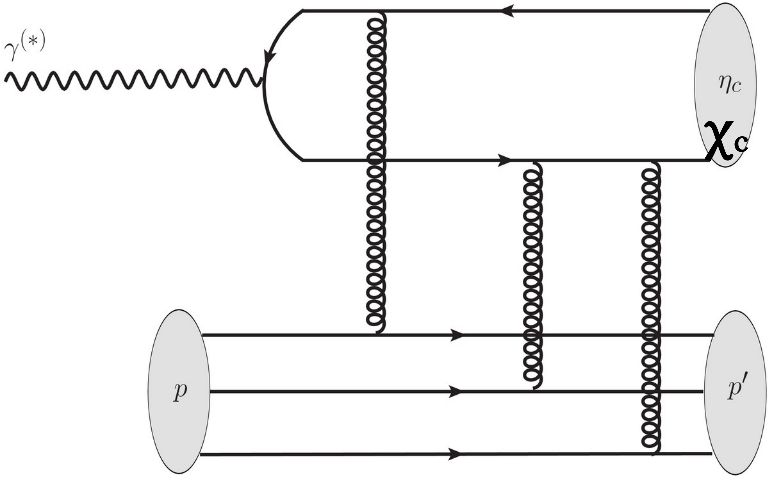
Exclusive charmonium production processes



$$\delta^{ab}\ \langle A^{+a} A^{+b} \rangle$$

C and P even

eikonal fields



$$d^{abc}\ \langle A^{+a} A^{+b} A^{+c} \rangle$$

C and P odd

proportional to cubic Casimir / anomaly coefficient of fund. rep.

* hard scale(s): m_c, $|t|$, Q^2

* J/Ψ vector meson production observed & well measured at HERA (differential cross section dσ/dt)

* C-odd exchange related to cubic Casimir has not been seen (yet; except GlueX @ threshold & TOTEM in non-pert. regime)

Motivation, C-odd ggg / Odderon exchange :

* not "generic" for color-SU(Nc): exists only in Nc ≥ 3, not for SU(2); requires non-vanishing anomaly coefficient / cubic Casimir for N-repr

$$\frac{1}{2}\mathrm{tr}\,\{t^a,t^b\}t^c = A_F d^{abc}$$

* coupling to proton depends critically on its structure!
  - non-Gaussian color field fluctuations, $\left\langle A^{+a}(\vec{q}_1)\, A^{+b}(\vec{q}_2)\, A^{+c}(\vec{q}_3)\right\rangle \neq 0$
  - magnitude ~ 100-1000 smaller than gg exchange (prediction of light-front |qqq (g)⟩ models)
  - t-dependence much weaker than gg exchange, proton much more likely to "survive" high $t \gtrsim 1\ \mathrm{GeV}^2$
  - spin dependent Odderon / gluon Sivers: spin-orbit entanglement

* high-energy evolution very different from gg / Pomeron exchange, $O(x_\perp,y_\perp) \longrightarrow 1$ is not a fixed point of small-x evolution

Odderon amplitude O(**r**,**b**) ~ **r.b** $a_1(r,b)$:

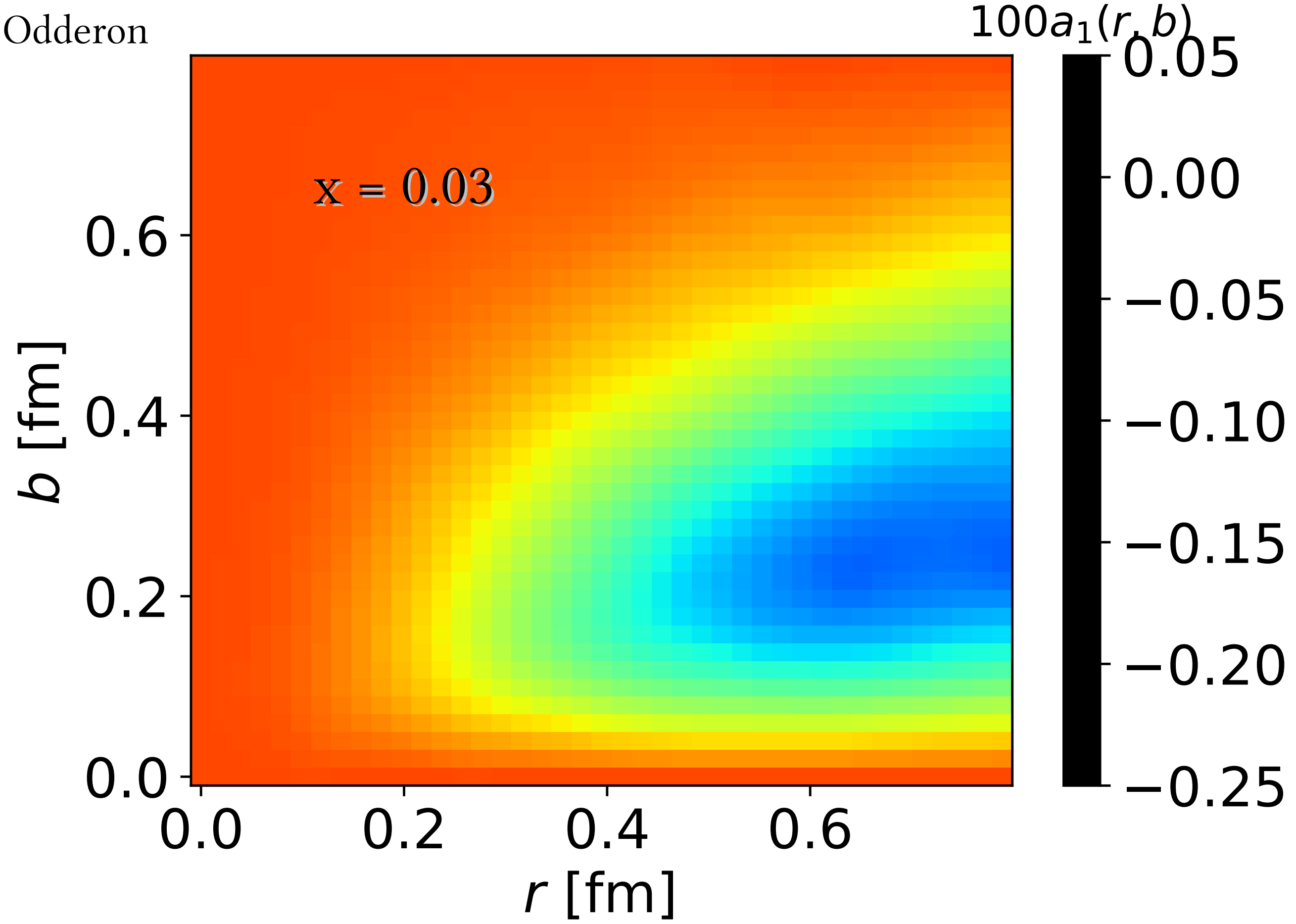
the "usual, b-dependent, spin independent", Odderon

from Erratum to
A.D., H. Mantysaari, R. Paatelainen,
PRD 107 (2023) L011501

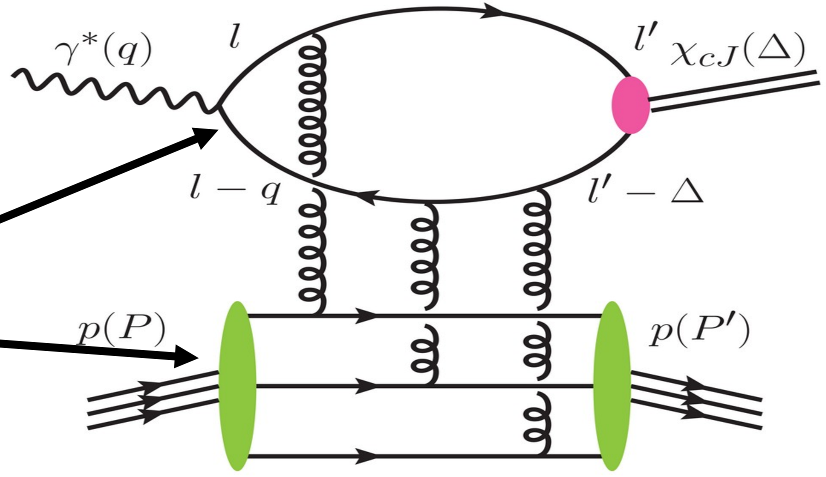
* |p> ≈ |qqq⟩ + |qqqg⟩

* note: for say r = 0.25 fm, max amplitude at rather <u>small b ~ 0.1 – 0.2 fm</u>

* vanishes for b ⟶ 0 though (P odd)



Eikonal amplitude for $\gamma^{(*)} \to \chi_{cJ}$



$$\gamma^*(q)p(P) \to \mathcal{H}(\Delta)p(P')$$

$$\langle \mathcal{M}_{\lambda\bar{\lambda}} \rangle = 2q^- N_c \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \boxed{\mathrm{i}\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)} \mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp)$$

reduced
amplitude

$$\mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp) = \int_z \int_{\boldsymbol{l}_\perp \boldsymbol{l}'_\perp} \sum_{h\bar{h}} \Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{l}_\perp, z) \Psi^{\mathcal{H}*}_{\bar{\lambda},h\bar{h}}(\boldsymbol{l}'_\perp - z\boldsymbol{\Delta}_\perp, z) \mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp + \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \boldsymbol{r}_\perp}$$

These amplitudes for J = 0, 1, 2 have been computed in arXiv:2402.19134

Primakoff photon exchange amplitude

$$\langle \mathcal{M}_{\lambda\lambda'}(\gamma^* p \to \mathcal{H} p)\rangle = -\frac{eF_1(\ell_\perp)}{\ell_\perp^2} n_\mu \mathcal{M}^\mu_{\lambda\lambda'}(\gamma^*\gamma^* \to \mathcal{H})$$

main uncertainty: quarkonium wave function

Important constraint from γγ decay width:

$$\lim_{|t|\to 0} |t| \frac{\mathrm{d}\sigma(\gamma p \to \mathcal{H} p)}{\mathrm{d}|t|} = \frac{8\pi\alpha(2J+1)\ \Gamma(\mathcal{H} \to \gamma\gamma)}{M_{\mathcal{H}}^3}$$

* model independent (i.e., no assumption about quarkonium LCwf)
* all orders in QCD coupling
* proved via "Collins-Ellis trick" (backup slides)
* for axial vector, Γγγ = 0 (Landau-Yang theorem), so t dσ/dt –> 0

Primakoff for χ_{cJ} at $Q^2 \sim 0$, high $|t| \gg m_c^2$

Cross sections scale

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} \sim \frac{F_1^2(t) + \frac{|t|}{4M_p^2} F_2^2(t)}{|t|^3}$$

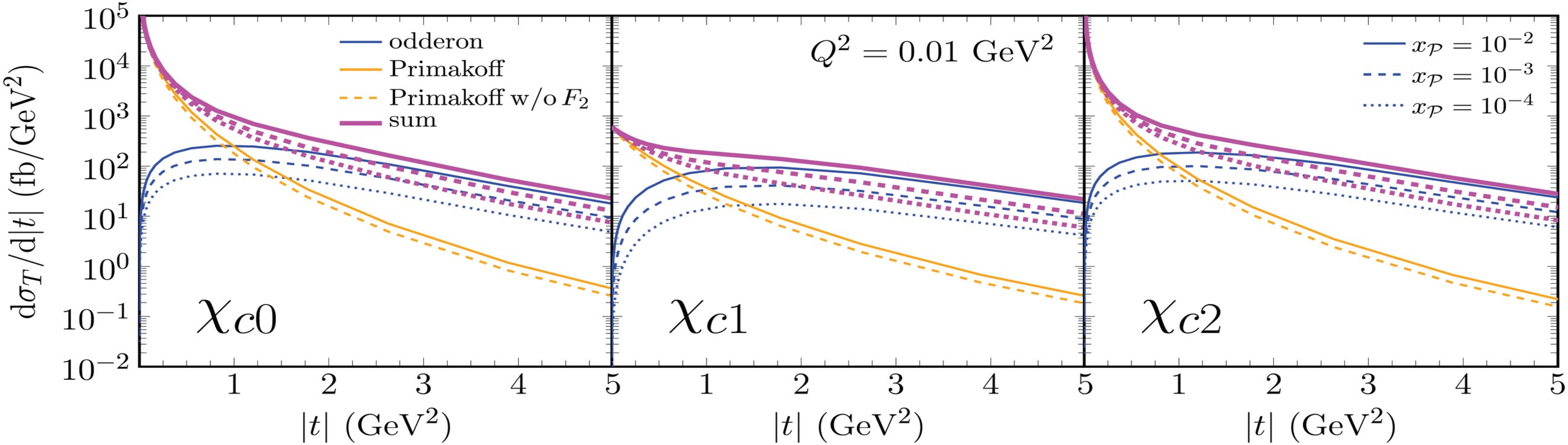
F1(t), F2(t): Dirac and Pauli form factors of the proton

The coefficient involves the meson LC-DA (and depends on J) but the above scaling with |t| is universal !

If this scaling at high |t| is broken then there's another process at work !

Numerical results, differential X-sections

arXiv:2402.19134



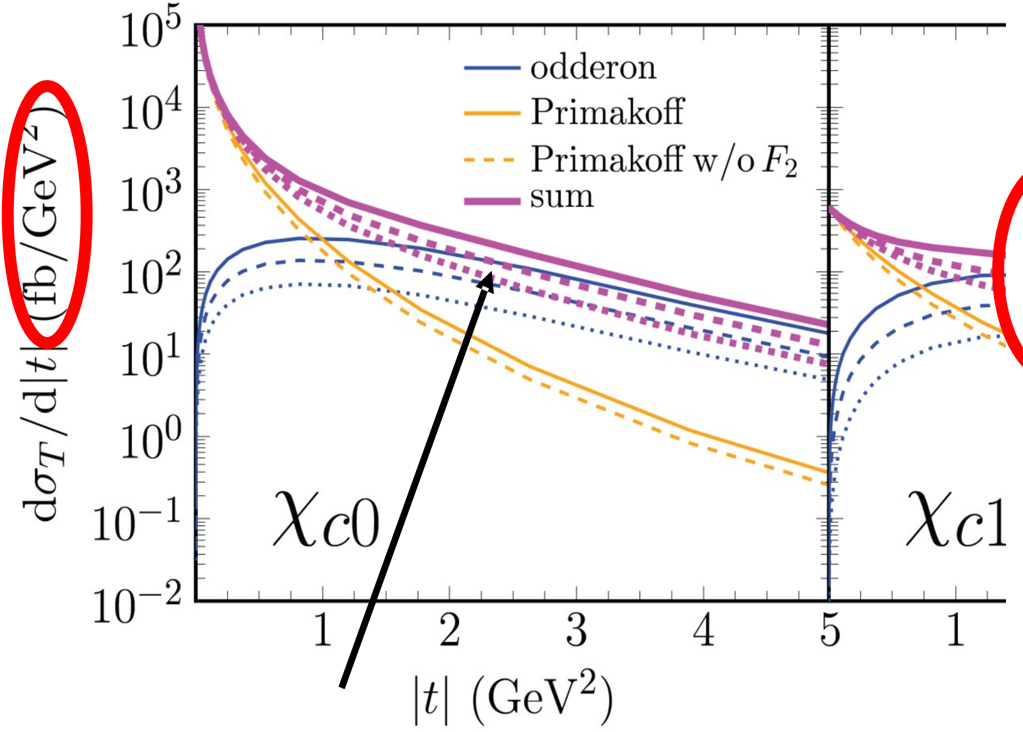
* low |t| dominated by Primakoff (also: large background from $\psi(2S) \longrightarrow \chi_c + \gamma$)
  ggg exchange important from $|t| \sim 1$ GeV^2, slow fall-off with increasing |t| !

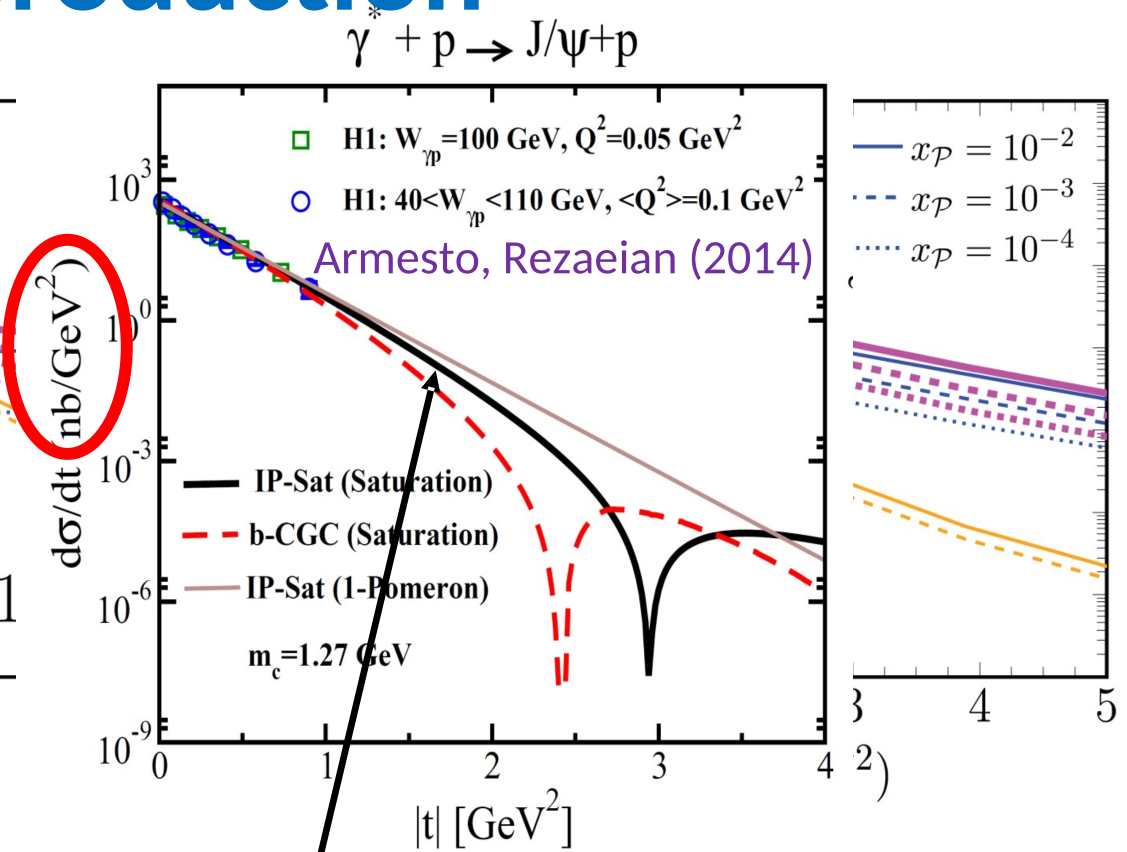
* cross-section at $|t| \sim 1$ GeV^2 is enhanced by factor ~4 due to constructive interference

* X-section for χc1 is finite as t –> 0 (due to Landau-Yang)

* small X-section at $|t| \sim 1$ GeV^2 and beyond (due to P-odd ggg), high lumin. critical for discovery

Contrast with J/ψ production





Odderon: **weak t-dependence**

$d\sigma/d|t| \propto |t|*\exp(-B|t|)$, B≈1 GeV^{-2}

Pomeron: **strong t-dependence**

$d\sigma/d|t| \propto \exp(-B|t|)$, B≈4 GeV^{-2}

Odderon in the dipole/GTMD framework

Boussarie, Hatta, Szymanowski, Wallon (2019)
Zhou (2013)
Boer, Echevarria, Mulders, Zhou (2016)
also, S. Benic et al, 2603.06092

Odderon = imaginary part of eikonal dipole

$$\mathcal{D}(\mathbf{x}_\perp, \mathbf{y}_\perp) = 1 - \frac{1}{N_c}\left\langle \mathrm{tr}\, V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp)\right\rangle$$

decomposition into GTMDs at small-x

$$\mathcal{D}_{SS'}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \approx \frac{(2\pi)^3 g^2}{4M_p N_c}\frac{1}{\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}}\bar{u}(P',S')\left[\boxed{F_{1,1}} + \mathrm{i}\frac{\sigma^{i+}}{P^+}k_\perp^i \boxed{F_{1,2}} + \mathrm{i}\frac{\sigma^{i+}}{P^+}\Delta_\perp^i F_{1,3}\right]u(P,S)$$

$$f_{1,1}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) + \underbrace{\boxed{\mathrm{i}\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp}{M_p^2} g_{1,1}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)}}_{\text{spin-independent Odderon}}$$

$$\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp}{M_p^2} f_{1,2}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) + \underbrace{\boxed{\mathrm{i} g_{1,2}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)}}_{\text{spin-dependent Odderon}}$$

spin-independent Odderon
(previous slides),
no TMD

spin-*dependent* Odderon
gluon Sivers TMD at |t|->0

$$g_{1,2}(\boldsymbol{k}_\perp, 0) = -\frac{1}{2} x f_{1T}^{\perp g}(x, \boldsymbol{k}_\perp)$$

Eikonal / small-x (dipole) gluon Sivers function of the proton:

S. Benic, A.D., F. Hechenberger, T. Stebel, arXiv:2603.09630

$$\mathcal{O}_{\Lambda'\Lambda}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \frac{1}{N_c}\,\mathrm{Im}\langle P'\Lambda'|\,\mathrm{tr}\,V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)|P\Lambda\rangle$$

Helicity flip matrix element

$$\begin{aligned}\mathcal{O}_{\Lambda'\Lambda}(\vec{k}_\perp) &= -\frac{g^6(N_c^2-4)(N_c^2-1)}{16N_c^2}\int \mathrm{d}^2 q_\perp\, \frac{G_{3\Lambda'\Lambda}(\vec{k}_\perp, \vec{q}_\perp, -\vec{k}_\perp - \vec{q}_\perp)}{\vec{k}_\perp^2\, \vec{q}_\perp^2\, (\vec{k}_\perp + \vec{q}_\perp)^2} \\ &\sim x f_{1T}^{\perp g}(x, k_\perp)\end{aligned}$$

$$d^{abc}\langle P'\Lambda'|J^{+a}(\vec{q}_{1\perp})J^{+b}(\vec{q}_{2\perp})J^{+c}(\vec{q}_{3\perp})|P\Lambda\rangle = N_c C_{3F}\, G_{3\Lambda'\Lambda}(\vec{q}_{1\perp}, \vec{q}_{2\perp}, \vec{q}_{3\perp})$$

explicit demonstration in |qqq> LF constituent quark model :

$$|P,\Lambda\rangle = \int [dx_i] \int [d^2k_i] \sum_{\{\lambda_i\}} \Phi_\Lambda(\lambda_i, x_i, \vec{k}_i)\, \Psi(x_i, \vec{k}_i) \sum_{i_1,i_2,i_3} \frac{1}{\sqrt{6}} \epsilon_{i_1 i_2 i_3} \,|x_1 P^+, \vec{k}_1, \lambda_1, i_1; \cdots\rangle$$

NR quark model / collinear quarks ($R_p \longrightarrow \infty$): *separable* spin-spatial wave fcts.

$$\Phi_+(++-)|++-\rangle = \frac{2}{\sqrt{6}}\,|\uparrow\uparrow\downarrow\rangle\ ,\quad \Phi_+(+-+)|+-+\rangle = \frac{-1}{\sqrt{6}}\,|\uparrow\downarrow\uparrow\rangle\ ,\quad \Phi_+(-++)|-++\rangle = \frac{-1}{\sqrt{6}}\,|\downarrow\uparrow\uparrow\rangle$$

LF helicity spinors

Pauli spinors

and $\Phi_+(\lambda_i)=0$ for any other combination of quark helicities

$$\Phi_-(\lambda_i) = -\Phi_+(-\lambda_i) \ \rightarrow\ \Phi^*_{\Lambda'}(\lambda_i)\,\Phi_\Lambda(\lambda_i) \sim \delta_{\Lambda\Lambda'}$$

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non-zero k_i: Melosh transformation of Pauli spins ⟶ LF helicity

(for details, e.g. Pasquini, Cazzaniga, Boffi, 0806.2298)

$$\Phi_+(++-) = \frac{\sqrt{\frac{2}{3}}a_1 a_2 a_3 + \sqrt{\frac{1}{6}}a_1 k_{2L} k_{3R} + \sqrt{\frac{1}{6}}a_2 k_{1L} k_{3R}}{\sqrt{N_1 N_2 N_3}}$$

a_i, N_i are functions of m, x_i, k_i ;
$a_i^2 = N_i$ when $k_i \to 0$

L_z = -1 →

$$\Phi_-(++-) = \frac{\sqrt{\frac{2}{3}}k_{1L} k_{2L} k_{3R} + \sqrt{\frac{1}{6}}k_{1L} a_2 a_3 + \sqrt{\frac{1}{6}}a_1 a_3 k_{2L}}{\sqrt{N_1 N_2 N_3}}$$

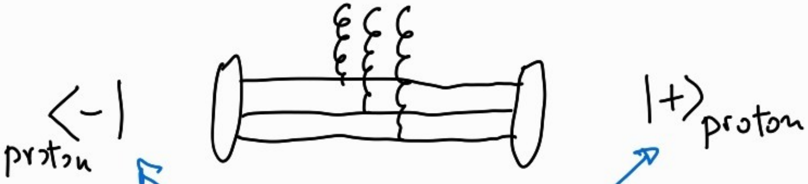
The matrix element for the Sivers fct. is diagonal in the transverse spin basis :

$$\begin{aligned}\mathcal{O}_{\vec{S}_\perp \vec{S}_\perp}(\vec{r}, \vec{\Delta}_\perp) &= \sum_{\Lambda', \Lambda} \langle \vec{S}_\perp | \Lambda' \rangle \, \mathcal{O}_{\Lambda' \Lambda} \, \langle \Lambda | \vec{S}_\perp \rangle \\ &= \cos(\phi_{r\Delta}) i \mathcal{O}(\vec{r}, \vec{\Delta}_\perp) - \vec{S}_\perp \times \hat{\vec{r}} \, \mathcal{O}_S(\vec{r}, \vec{\Delta}_\perp) - \vec{S}_\perp \times \hat{\vec{\Delta}}_\perp \cos(\phi_{r\Delta}) \mathcal{O}_S^\perp(\vec{r}, \vec{\Delta}_\perp)\end{aligned}$$

However, it does involve a helicity flip

$$\begin{aligned}\mathcal{O}_{\Lambda' \Lambda}(\vec{r}, \vec{\Delta}_\perp) &= \delta_{\Lambda \Lambda'} i \cos(\phi_{r\Delta}) \mathcal{O}(\vec{r}, \vec{\Delta}_\perp) \\ &+ \boxed{\delta_{\Lambda, -\Lambda'} i \Lambda e^{i\Lambda\phi_r} \mathcal{O}_S(\vec{r}, \vec{\Delta}_\perp)} + \delta_{\Lambda, -\Lambda'} i \Lambda e^{i\Lambda\phi_\Delta} \cos(\phi_{r\Delta}) \mathcal{O}_S^\perp(\vec{r}, \vec{\Delta}_\perp)\end{aligned}$$

Microscopic level (3 quark state, say) :



$$\langle ++-, L_z = -1 | \widehat{\text{Odderon}} | ++-, L_z = 0 \rangle \neq 0$$

quark helicities

note: spin-orbit entanglement !

see, also,
Pasquini, Cazzaniga, Boffi, 0806.2298 and
Lorce, Pasquini, Xiong, Yuan, 1111.4827

arXiv:2603.09630

BFKL power-law tail: $k_\perp^{-4} \longrightarrow k_\perp^{-3.3}$ at $\alpha_s Y \sim 1$

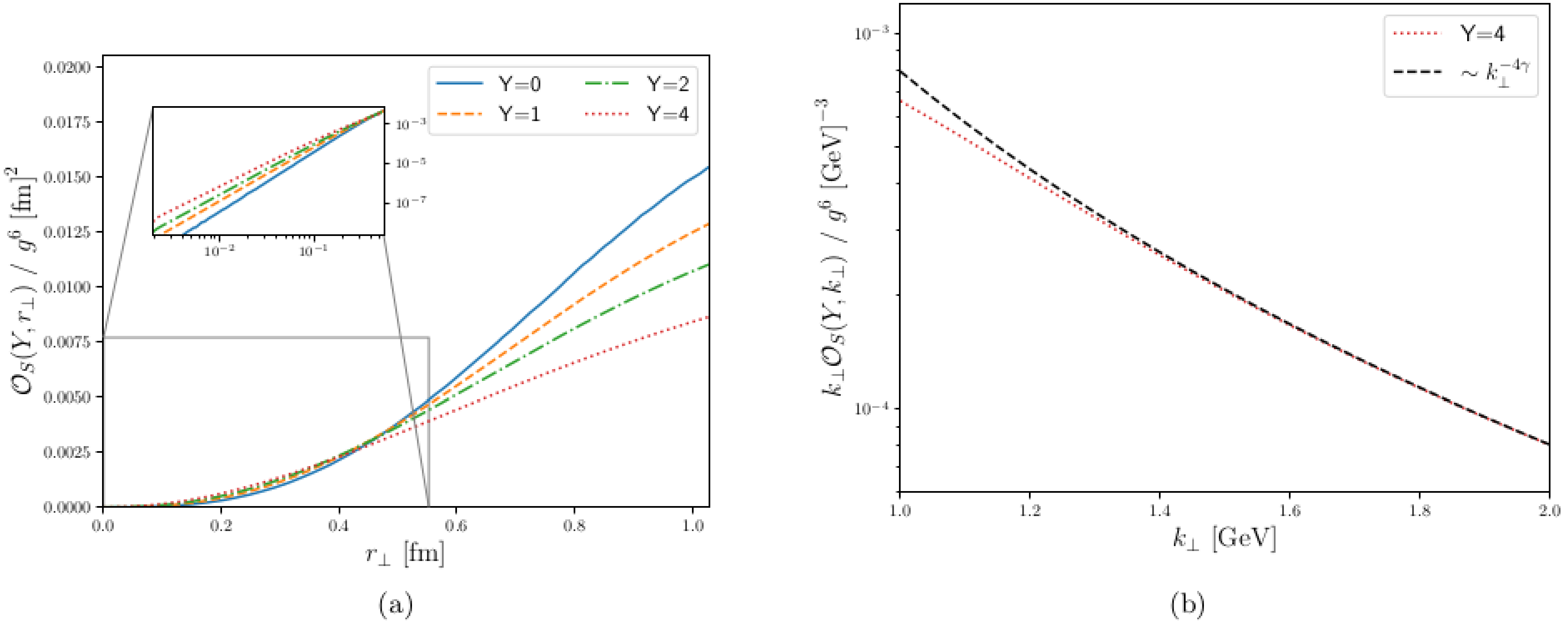
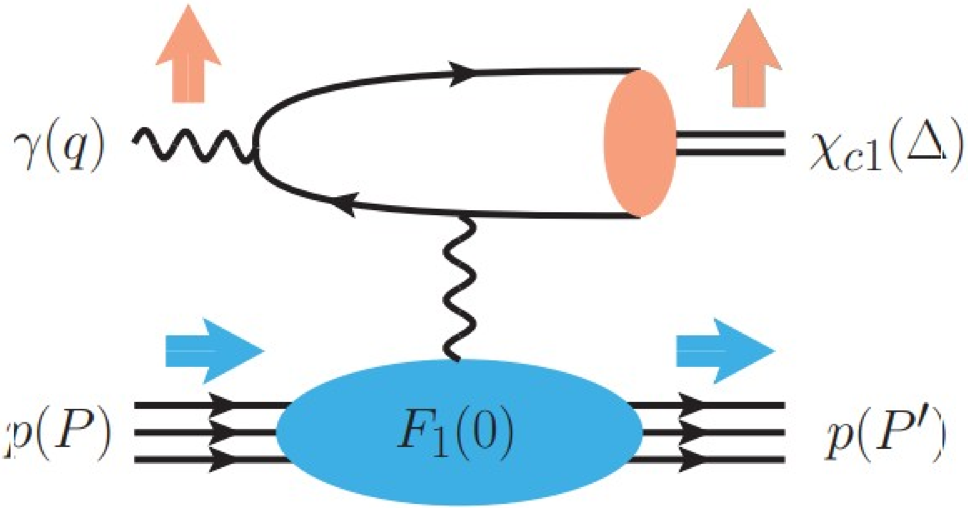


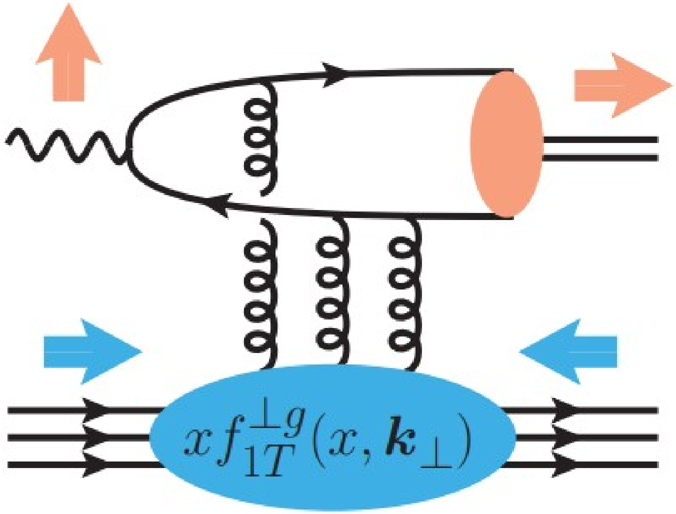
FIG. 3: (a) Radial part of the helicity flip Odderon amplitude at different rapidities, $Y = 0, 1, 2, 4$. The inset is a double logarithmic version of the plot to better show the small-$r_\perp$ regime. (b): high-$k_\perp$ tail of the momentum-space Odderon amplitude $k_\perp \mathcal{O}_S(k_\perp)$, which is proportional to $x f_{1T}^{\perp g}(x, k_\perp^2)$, at $Y = 4$. The dashed line indicates the $\sim 1/k_\perp^{3.3}$ power-law asymptotics.



eikonal current:

$$
\begin{aligned}
\bar{u}(P',\Lambda')\gamma^\mu u(P,\Lambda) &\rightarrow \bar{u}(P',\Lambda')\gamma^+ u(P,\Lambda) \\
&= 2\sqrt{P^+P'^+}\,\delta_{\Lambda\Lambda'}
\end{aligned}
$$

→ s-channel helicity conservation (SCHC)



But: in ggg exchange,

- each gluon carries $\perp$ momentum even if $-t = (\vec{q}_{\perp 1} + \vec{q}_{\perp 2} + \vec{q}_{\perp 3})^2 \rightarrow 0$
- → may change orbital angular momentum of quarks, gluons
- → changes helicity of proton ! (→ χ_{c1} is long. polarized)

Exclusive χ_{c1} axial vector production (on unpolarized protons !) at $t \longrightarrow 0$, NRQCD limit (arXiv:2407.04968):

$$\lim_{t\to 0} \frac{d\sigma_{\text{Siv}}}{dt} \sim \frac{\alpha\alpha_s^2\,|R'(0)|^2\,|xf_{1T}^{\perp(1)g}(x)|^2}{m_c^{11}}$$

R'(0): deriv. of wf @ origin
$xf_{1T}^{\perp(1)}(x)$: k_T^2 moment of Siv. fct.

[Compare to "Ryskin formula" for vector meson production, $\left.\frac{d\sigma_{J/\psi}}{dt}\right|_{t=0} \sim [xg(x)]^2$.]

$$\lim_{t\to 0} \frac{d\sigma_{\text{Prim}}}{dt} \sim \frac{\alpha^3\,|R'(0)|^2}{m_c^9}$$

Ratio of Sivers/Primakoff for χ_{c1} production at $t \longrightarrow 0$, NRQCD limit :

$$r(x) = \frac{4\pi^2}{q_c^2 N_c^2}\frac{\alpha_S^2}{\alpha^2}\frac{M_p^2}{M_\chi^2}|xf_{1T}^{\perp(1)g}(x)|^2$$

Summary

* Eikonal Odderon exchange is an intriguing prediction of QCD with Nc $\geq$ 3 colors !
Proton helicity flip possible even in the eikonal limit $\longrightarrow$
eikonal gluon Sivers fct. requires partonic spin-orbit entanglement

* $\langle A^{+\,3} \rangle \neq 0$ $\longrightarrow$ evidence for non-Gaussian color fluctuations
in the proton at sub-femtometer scales !

* magnitude, t-dependence, and phase (relative to γ exchange) offer
most valuable insight into the structure of the proton

* high luminosity EIC may be able to discover “hard Odderon(s)”
(C, P odd eikonal gluon exchanges)

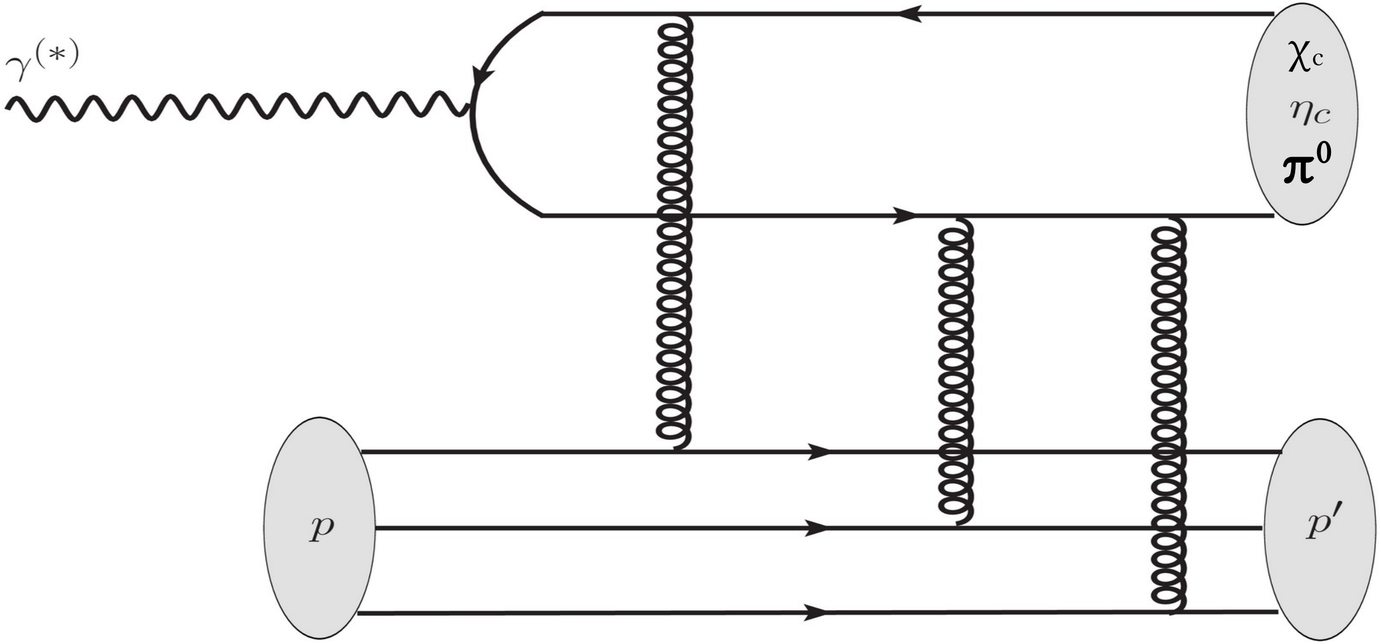
Thank you !

Backup Slides

Processes involving the "hard" Odderon :

kinematic regime: hard ggg exchange at x ~ 0.01 – 0.1

* exclusive production of C-even mesons in $\gamma^* p \longrightarrow M\, p$, large $|t|$
  (and high Q^2 in case of π^0 production)



χ_{cJ}: S. Benic et al, 2402.19134

Czyzewski, Kwiecinski, L. Motyka, M. Sadzikowski, PLB 398 (1997);
Engel, Ivanov, Kirschner, Szymanowski, Eur.Phys.J.C4 (1998);
Kilian, Nachtmann, Eur.Phys.J.C5 (1998); A.D, T. Stebel, Phys.Rev.D 99 (2019); S. Benic et al, PRD 2023

Collins-Ellis "trick" for γp –> Hp:

$$\mathcal{M}_{\lambda\lambda'}(\gamma p \to \mathcal{H} p) = P_\mu \, \mathcal{M}^\mu_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \frac{eF_1(\ell^2)}{\ell^2} \frac{1}{q \cdot P} \bar{u}(P') \not{q} \, u(P)$$

In high-E limit:

$$P_\mu \, \mathcal{M}^\mu_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \simeq P^+ \mathcal{M}^-_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \simeq -\frac{1}{x} \ell_i \mathcal{M}^i_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H})$$

since $\ell_\mu \, \mathcal{M}^\mu_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) = 0$ by QED gauge invariance, and $\ell^\mu \simeq (xP^+, 0, \vec{\ell}_\perp)$

This then leads to (appendix C.1 in arXiv:2402.19134)

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{H} p)}{\mathrm{d}|t|} = \frac{8\pi\alpha(2J+1)F_1^2(t) \; \Gamma(\mathcal{H} \to \gamma\gamma)}{|t| \, M_{\mathcal{H}}^3}$$

at high E and |t| ≪ m_p.

Primakoff in the NRQCD limit at Q^2, t --> 0

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{S} p)}{\mathrm{d}|t|} \to \frac{9\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^7 |t|}$$

t –> 0, m_c –> ∞

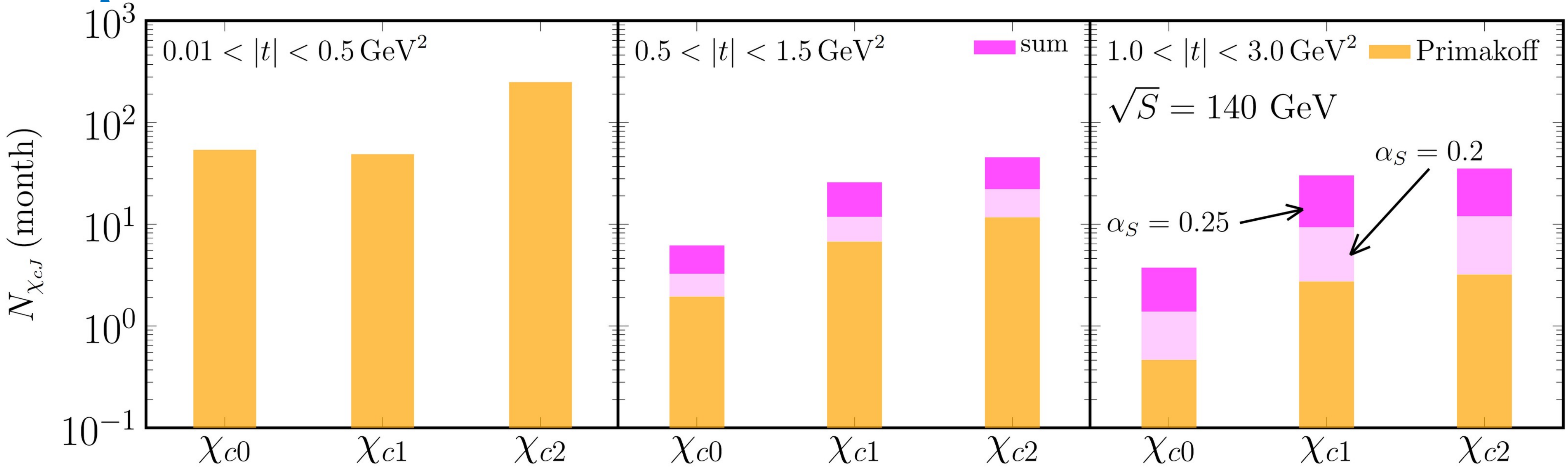
$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{A} p)}{\mathrm{d}|t|} \to \frac{3\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^9}$$

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{T} p)}{\mathrm{d}|t|} \to \frac{12\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^7 |t|}$$

* NR limits agree with literature   Jia, Mo, Pan, Zhang,   2207.14171

* Note the absence of a Coulomb tail for axial quarkonia
* For χ_{c0}, χ_{c2} parameter free (thanks to constraint for t dσ/dt;
  note $m_c = M_H/2$ in NR limit)

Expected number of events at the EIC



* detection channel: $\chi_{cJ} \rightarrow J/\psi\gamma$, $J/\psi \rightarrow e^+e^- / \mu^+\mu^-$
* we predict **excess** odderon events over Primakoff background
* for χ_{c1} (30% BR to $J/\psi + \gamma$): with EIC luminosity 10^{34} cm^{-2} s^{-1} expect ~20 events/month (only Primakoff: ~7 events/month) in $0.5 < |t| < 1.5$ window