Resummation in High Energy Scattering
-- BFKL vs Sudakov

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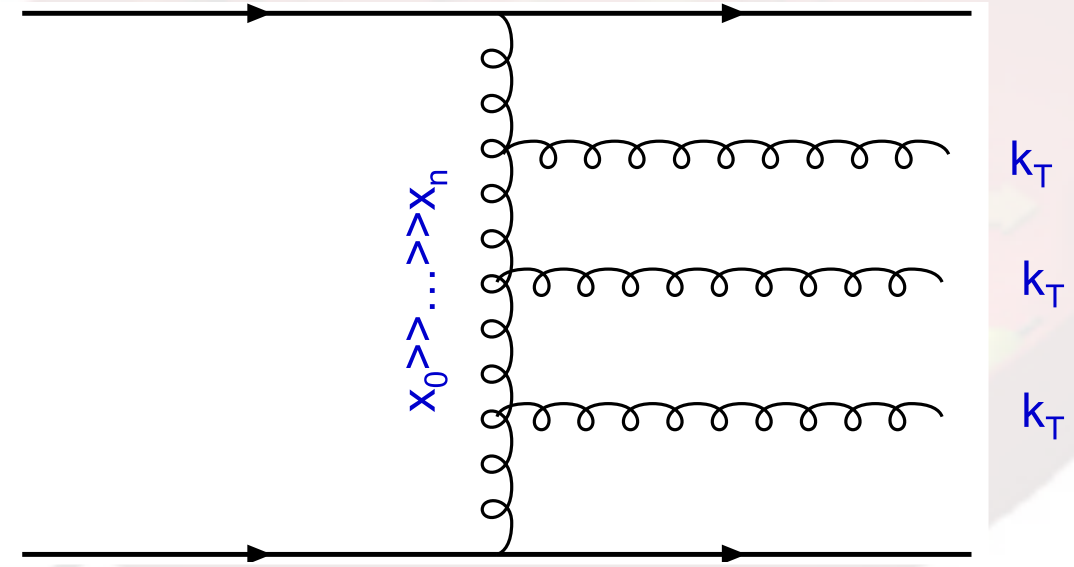
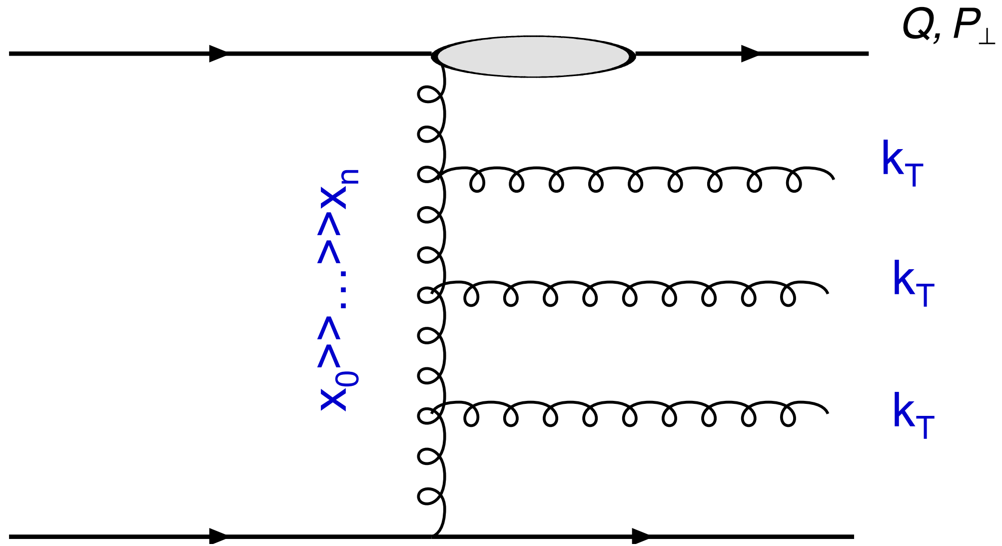
Refs: Mueller, Xiao, Yuan, PRL110, 082301 (2013); Phys.Rev. D88 (2013) 114010; Mueller, Szymanowski, Wallon, Xiao, Yuan, JHEP 03(2016)096; Stasto, Strikman, Yuan, et al., to appear

High energy scattering



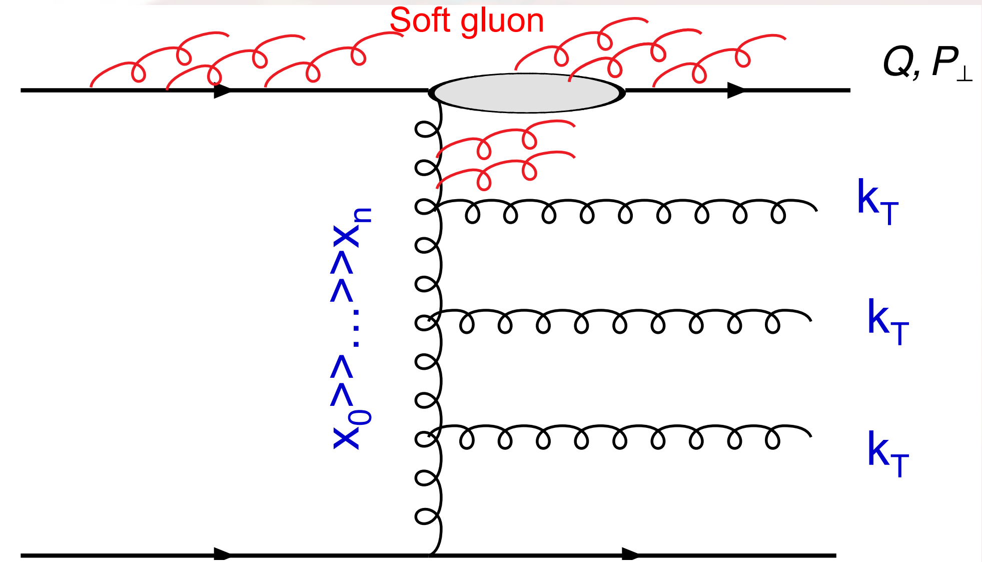
BFKL: $\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{K} \bigotimes \mathcal{F}$ Un-integrated gluon distribution

Hard processes at small-x



- Manifest dependence on un-integrated gluon distributions
  - Dominguiz-Marquet-Xiao-Yuan, 2010

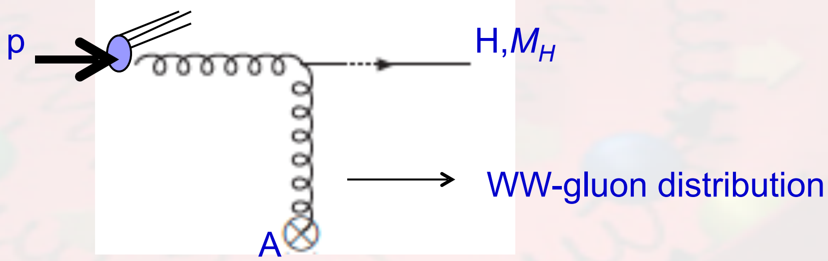
Additional dynamics comes in



- BFKL vs Sudakov resummations (LL)

Sudakov resummation at small-x

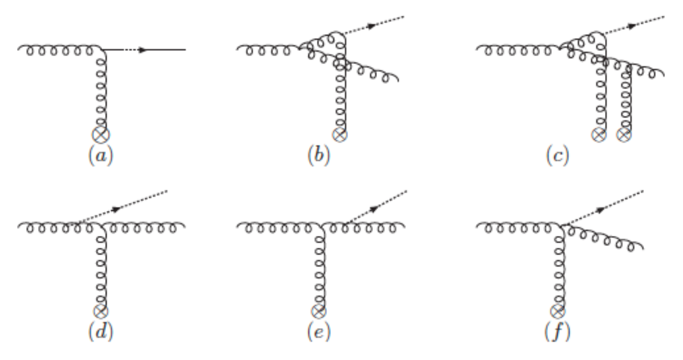
- Take massive scalar particle production p+A->H+X as an example to demonstrate the double logarithms, and resummation



$$\frac{d\sigma^{(\mathrm{LO})}}{dyd^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$$

Explicit one-loop calculations



$$x_0 g_p(x_0) \int \frac{d\xi}{\xi} \mathbf{K}_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left(-\frac{1}{\epsilon}\right) S^{WW}(x_\perp, y_\perp) \mathcal{P}_{g/g} \otimes x_0 g(x_0) \ ,$$

- Collinear divergence → DGLAP evolution
- Small-x divergence → BK-type evolution

Dominguiz-Mueller-Munier-Xiao, 2011

Soft vs Collinear gluons

- Radiated gluon momentum
  - $$k_g = \alpha_g P_1 + \beta_g P_2 + k_{g\perp}$$
- Soft gluon, α~β<<1
- Collinear gluon, α~1, β<<1
- Small-x collinear gluon, 1-β<<1, α→0
  - Rapidity divergence

Final result

- Double logs at one-loop order

$$\frac{d\sigma^{(\mathrm{LO+NLO})}}{dyd^2k_\perp}\Big|_{k_\perp\ll Q}=\sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot r_\perp}S^{WW}_{Y=\ln 1/x_a}(x_\perp,x'_\perp)xg_p(x,\mu^2=\frac{c_0^2}{r_\perp^2})$$
$$\left\{1+\frac{\alpha_s}{\pi}C_A\left[\beta_0\ln\frac{Q^2r_\perp^2}{c_0^2}-\frac{1}{2}\left(\ln\frac{Q^2r_\perp^2}{c_0^2}\right)^2+\frac{\pi^2}{2}\right]\right\}\ ,$$

- Include both BFKL (BK) and Sudakov

$$\frac{d\sigma^{(\mathrm{resum})}}{dyd^2k_\perp}\Big|_{k_\perp\ll Q}=\sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot r_\perp}e^{-S_{sud}(Q^2,r_\perp^2)}S^{WW}_{Y=\ln 1/x_a}(x_\perp,x'_\perp)$$
$$\times xg_p(x,\mu^2=c_0^2/r_\perp^2)\left[1+\frac{\alpha_s}{\pi}\frac{\pi^2}{2}N_c\right]\ ,$$

Mueller-Xiao-Yuan 2012

Small-x gluon distribution with TMD resummation

Mueller-Xiao-Yuan, 2012; Xiao-Yuan-Zhou 2016
Caucal-Salazar-Schenke-Venugopalan 2022-23
Taels-Altinoluk-Beuf-Marquet 2022
Mukherjee-Skokov-Tarasov-Tiwari, 2023
Altinoluk-Jalilian-Marian-Marquet, 2024
Caucal-Iancu, Caucal-Iancu-Mueller-Yuan, 2024
Duan-Kovner-Lublinsky, 2024

$$xG^{(1)}(x,k_\perp,\zeta_c=\mu_F=Q)$$

Hard scale entering TMD Factorization, e.g., dijet in DIS

$$-\frac{2}{\alpha_S}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^4}e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-S_{sud}(Q^2,r_\perp^2)}$$
$$\times\mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp,y_\perp)\ ,$$

Small-x evolution

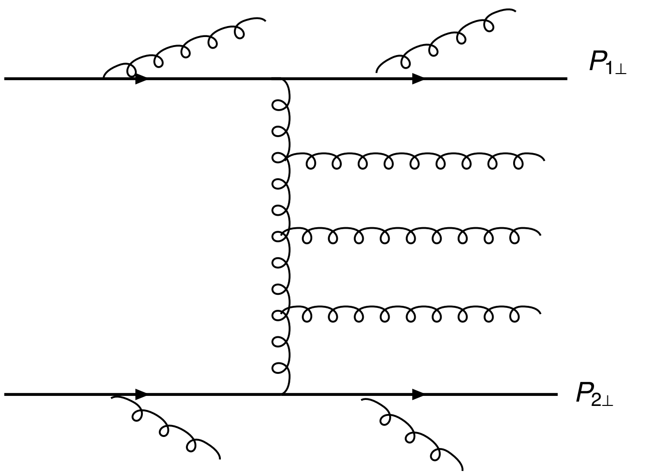
Pert. corrections

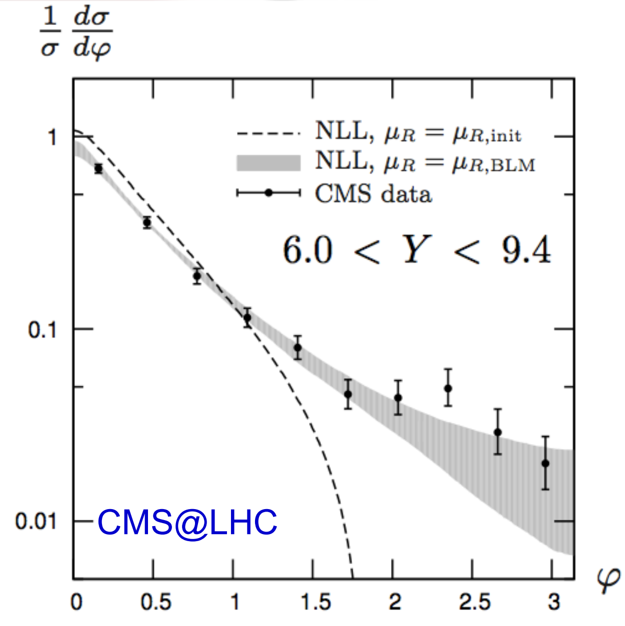
Sudakov resum.

Prediction Power!!

See, Edmond's talk

Dijet with large rapidity difference (Mueller –Navelet)



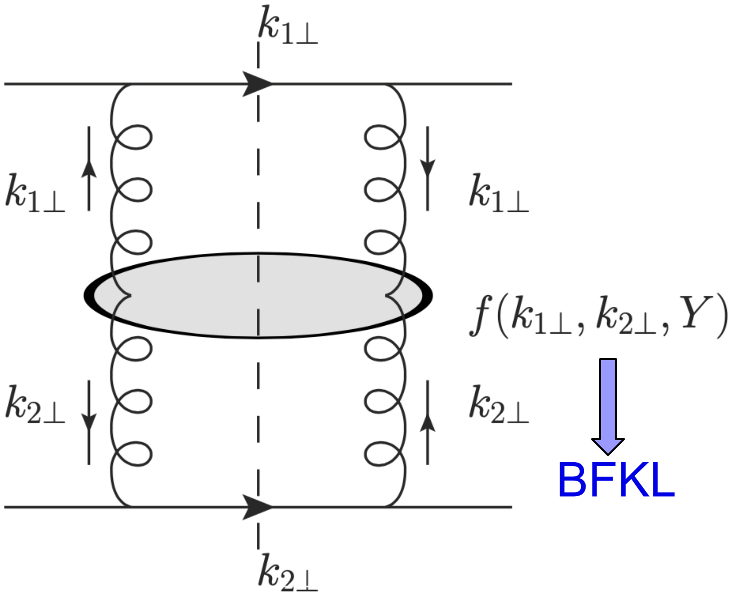


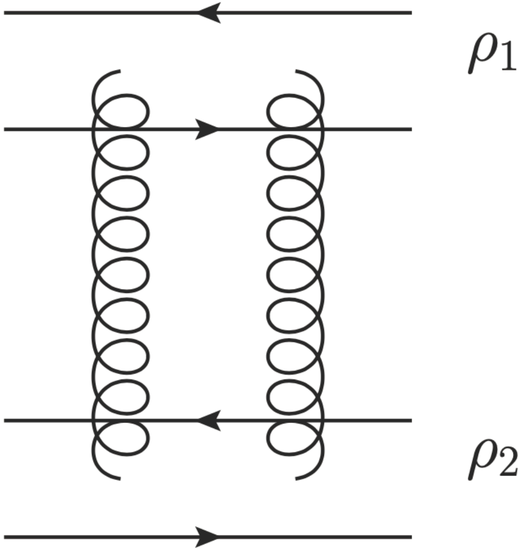
Sudakov resummation will dominate
Small angle distribution (Mueller-Xiao-Yuan)

Ducloue, Szymanowski, Wallon, 1309.3229
only take into account BFKL

BFKL and Sudakov are both present:
Mueller, Szymanowski, Wallon, Xiao, Yuan, JHEP 03(2016)096

Leading order

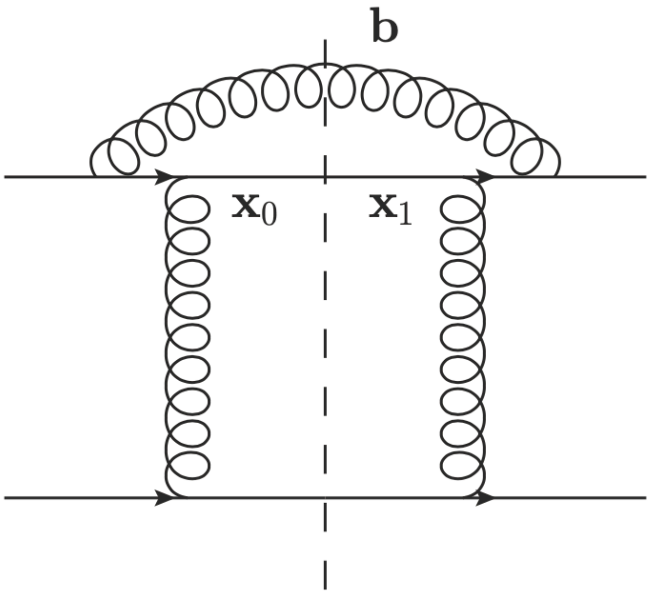


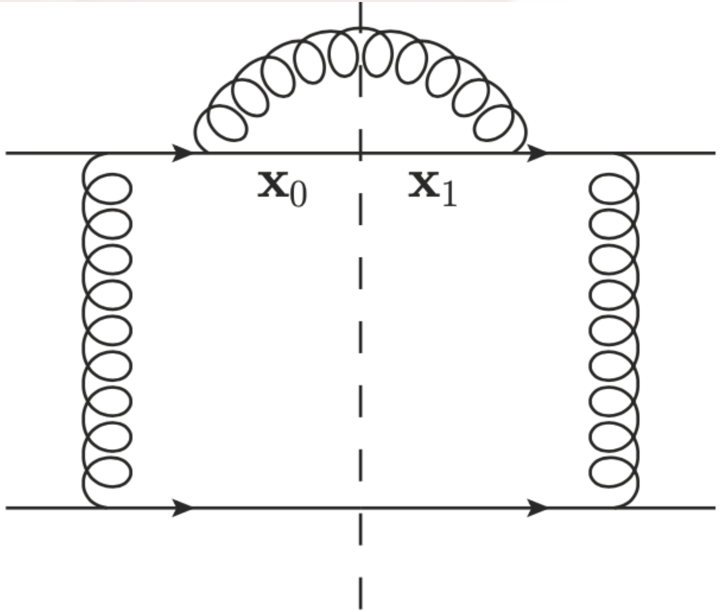


Transverse momentum space

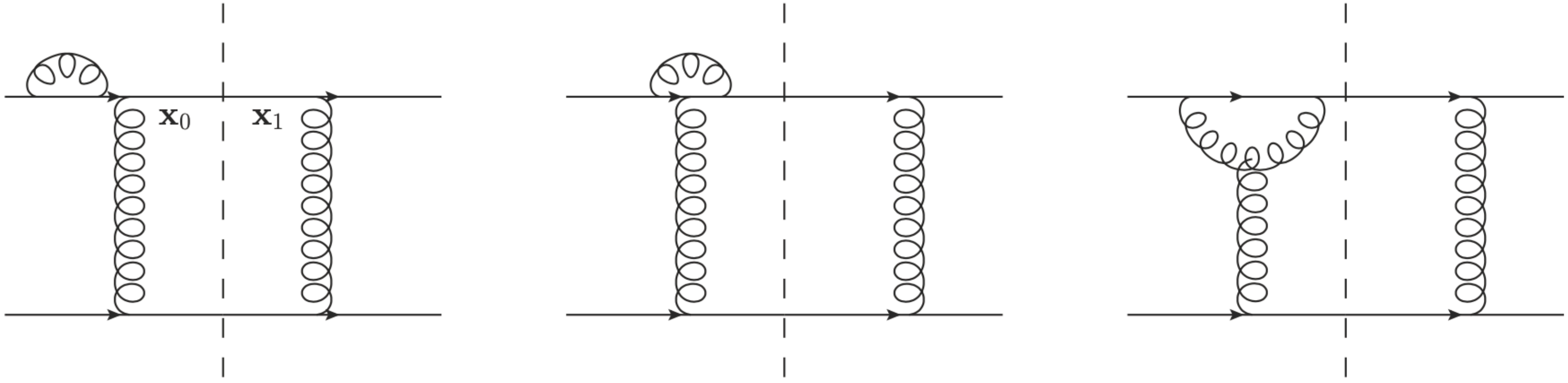
Dipole frame

Gluon radiations: real gluons



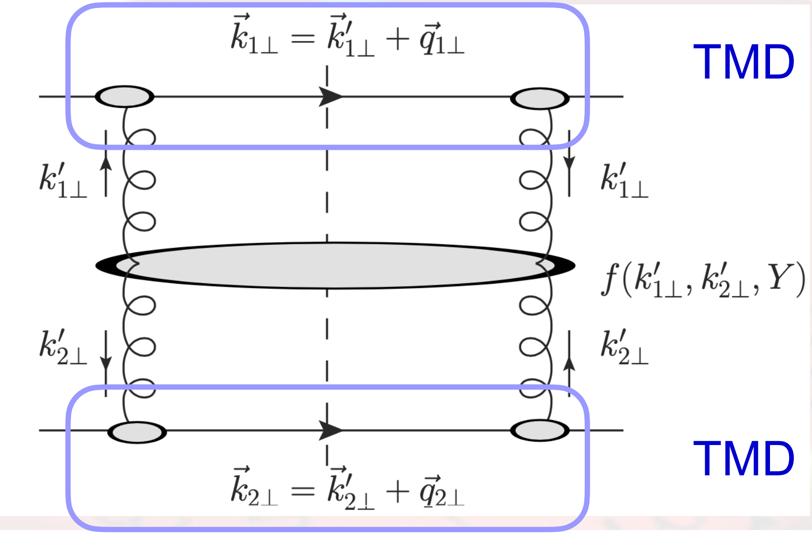


Gluon radiations: virtual gluons



(a) (b) (c)

Factorization: $k_{1,2} \gg q_{1,2}$



TMD <— Sudakov

<— BFKL

TMD <— Sudakov

$$\int d^2q_{1\perp} d^2q_{2\perp} \mathcal{F}_a(x_1, q_{1\perp}; \mu = k_{1\perp}) \mathcal{F}_b(x_2, q_{2\perp}; \mu = k_{2\perp})$$

$$\times\, \hat{\sigma}_{ab}(k_{1\perp}, k_{2\perp}; \mu) f_{BFKL}(\vec{k}_{1\perp} - \vec{q}_{1\perp}, \vec{k}_{2\perp} - \vec{q}_{2\perp}; Y)\,,$$

TMDs and Sudakov Resummation

$$\mathcal{F}_q(x, q_\perp; \mu_F = k_\perp) = x \int \frac{d^2 R_\perp}{(2\pi)^2} e^{iq_\perp \cdot R_\perp} e^{-\mathcal{S}^q_{sud}(\mu_F = k_\perp, R_\perp)} C \otimes f_q(x, \mu_b)\,,$$

$$\mathcal{F}_g(x, q_\perp; \mu_F = k_\perp) = x \int \frac{d^2 R_\perp}{(2\pi)^2} e^{iq_\perp \cdot R_\perp} e^{-\mathcal{S}^g_{sud}(\mu_F = k_\perp, R_\perp)} C \otimes f_g(x, \mu_b)$$

$$\mathcal{S}^a_{sud} = \int_{c_0^2/R_\perp^2}^{k_\perp^2} \frac{d\mu^2}{\mu^2} \left(A_a \ln \frac{k_\perp^2}{\mu^2} + B_a + D_a \ln \frac{1}{R^2} \right)$$

Usual TMD resummation for the quark/gluon distributions
Additional R-dependence comes from jet in the final state

See also dijet in general kinematics: Sun, Yuan, Yuan, 2014, 2015

One-loop Corrections to the Hard Factor

Universal structure:

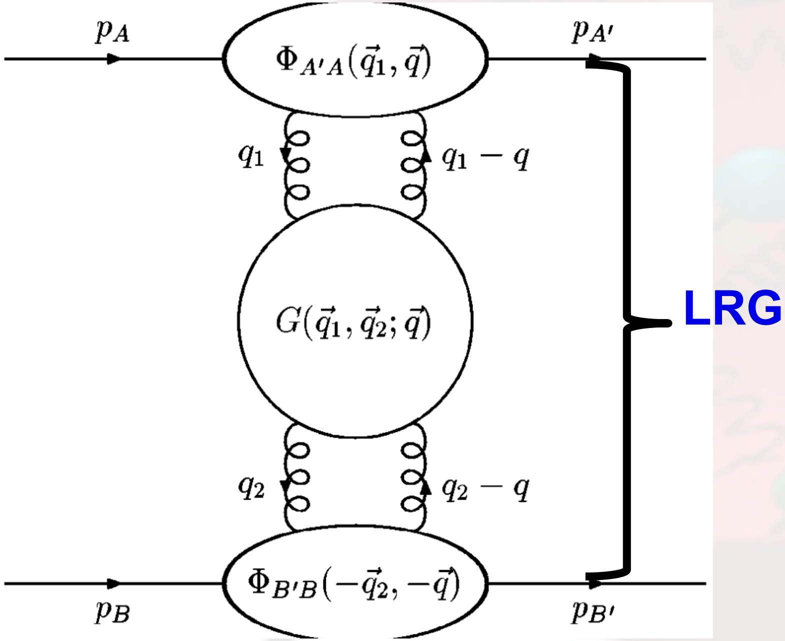
$$\hat{\sigma}^{(0)}_{qq'\to qq'}\left\{1+\frac{\alpha_s}{2\pi}\left[2\mathcal{K}+2\Delta I_q\right]\right\},$$
$$\hat{\sigma}^{(0)}_{qg\to qg}\left\{1+\frac{\alpha_s}{2\pi}\left[2\mathcal{K}+\Delta I_q+\Delta I_g\right]\right\},$$
$$\hat{\sigma}^{(0)}_{gg\to gg}\left\{1+\frac{\alpha_s}{2\pi}\left[2\mathcal{K}+2\Delta I_g\right]\right\},$$

$$\mathcal{K}=C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5N_f}{9},$$
$$\Delta I_q=C_F\left[\frac{3}{2}\ln\frac{1}{R^2}+\frac{3}{4}+\frac{2}{3}\pi^2\right],$$
$$\Delta I_g=C_A\left(2\beta_0\ln\frac{1}{R^2}-\frac{\pi^2}{6}\right)-\frac{N_f}{6}$$

K: Ciafaloni and Colferai, Nucl. Phys. B 538 (1999) 187

$\Delta I_{q,g}$: Jet related coefficients

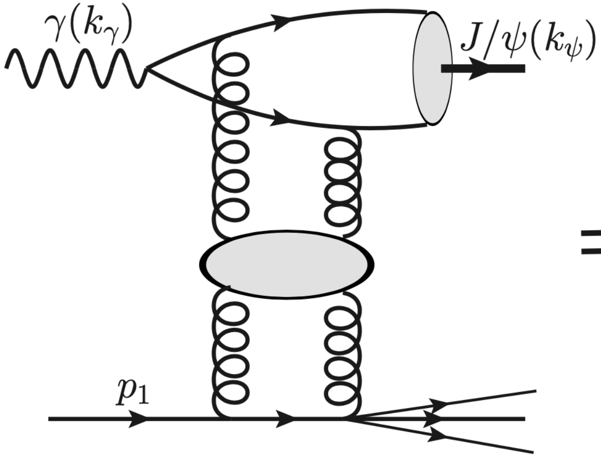
What happens for dijet with Large Rapidity Gap: Mueller-Tang dijet

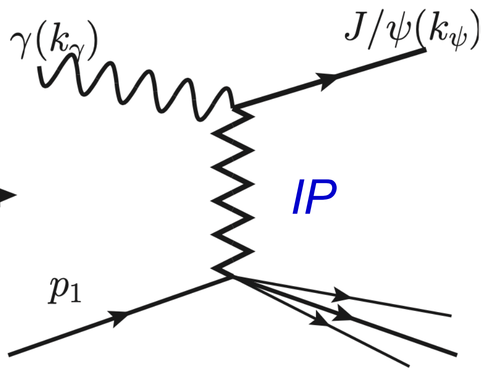


- Non-forward BFKL (amplitude level) + quark/gluon impact factors
  - Mueller-Tang, 1992
  - Fadin, Fiore, Papa, 1999
  - Fadin, Fiore, Kotsky, Papa, 2000
  - Hentschinski, Madrigal, Murdaca, Sabio Vera, 2014
  - Colferai1, Deganutti, Raben, Royon, 2023
- Soft gluon radiation will introduce Sudakov double logs as well

Simpler process: Jpsi+Jet with LRG

Stasto, Strikman, Yuan, et al., to appear



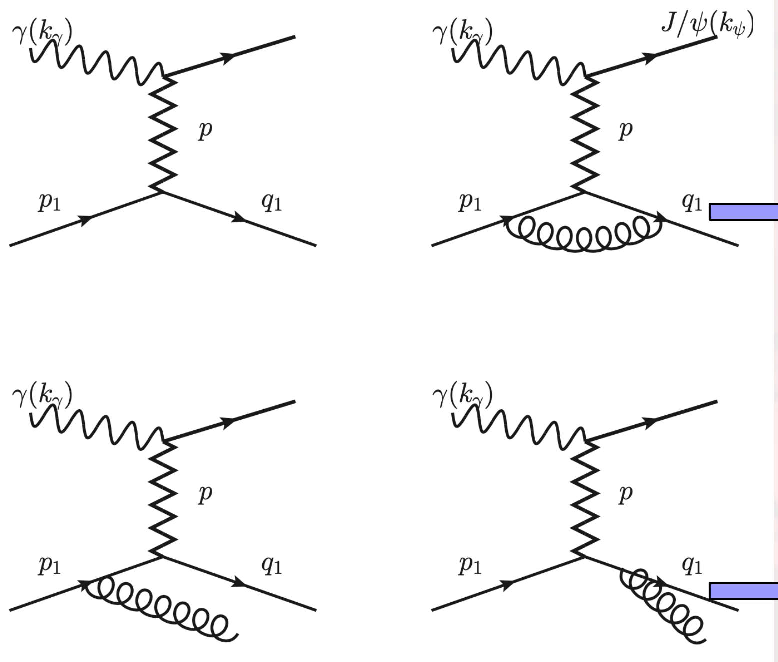


Large-t diffractive Jpsi:

Forshaw, Ryskin,
Z. Phys. C 68, 137(1995)
Forshaw, Poludniowski,
EPJC 26, 411 (2003)
Kotko, Motyka, Sadzikowski,
Stasto, JHEP 07, 129 (2019)

$$\frac{d\sigma(\gamma p \to J/\psi + Jet + X)}{dy_\psi dy_J dk_{\psi\perp}^2 d^2k_{J\perp}} = \int dx \sum_i f_i(x_i,\mu) C_i(-t,\mu) \times |\mathcal{A}^i_{BFKL}(\hat{s},t)|^2 \ , \qquad (2)$$

Gluon radiations: no crossing between



Impact factor calculations: BFKL kernel included, soft divergences

Gluon radiation associated with PDF splitting, jet production, and Soft gluon radiation

Soft and collinear gluons

- Real gluon

Collinear

$$\int \frac{d\xi}{\xi} x' f_q(x') \frac{\alpha_s}{2\pi^2} C_F \frac{1}{k_{g\perp}^2} \left\{ \frac{1+\xi^2}{(1-\xi)_+} + \epsilon(1-\xi) \right.$$
$$\left. +\delta(1-\xi)\left[\ln\frac{-t}{k_{g\perp}^2} + \ln\frac{1}{R^2} + \epsilon\left(\frac{1}{2}\ln^2\frac{1}{R^2}\right)\right]\right\} ,$$

Soft

Jet identification term

Virtual gluon

$$\frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \cdots \right.$$

Plus BFKL terms

Fadin, Fiore, Papa, 1999

Fadin, Fiore, Kotsky, Papa, 2000

Hentschinski, Madrigal, Murdaca

Sabio Vera, 2014

Sudakov double logs and resummation

$$\frac{\alpha_s}{2\pi}C_F\int\frac{d\xi}{\xi}xf_q(x')\left\{\left(-\frac{1}{\epsilon}+\ln\frac{\mu_b^2}{\mu^2}\right)\right.$$
$$\times\mathcal{P}_{q\to q}(\xi)+(1-\xi)+\delta(1-\xi)\left[\left(\frac{3}{2}-\ln\frac{1}{R^2}\right)\ln\frac{-t}{\mu_b^2}\right.$$
$$\left.\left.-\frac{1}{2}\left(\ln\frac{Q^2}{\mu_b^2}\right)^2+\frac{3}{2}\ln\frac{1}{R^2}-\frac{3}{2}-\frac{2\pi^2}{3}\right]\right\}, \qquad (11)$$

Double log. term

Factorized in terms of TMDs + BFKL (squared):

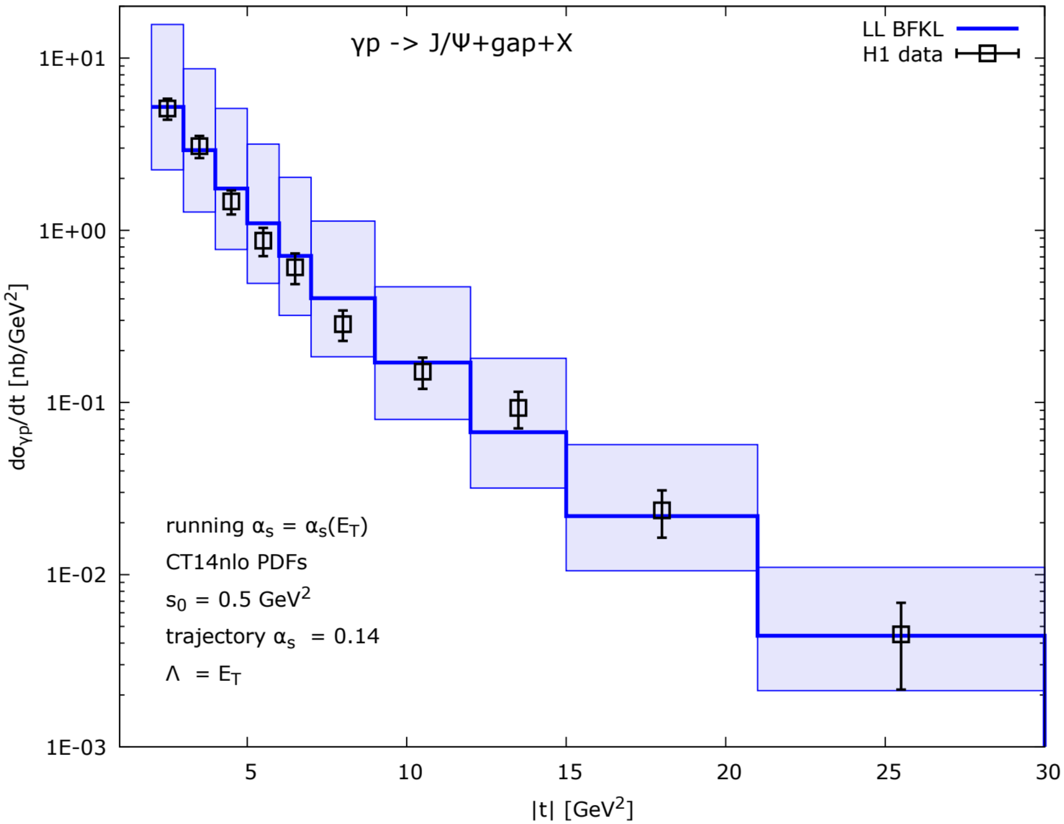
$$\frac{d\sigma(\gamma p\to J/\psi+Jet+X)}{dy_\psi dy_J dk_{\psi\perp}^2 d^2k_{J\perp}}=\int\frac{d^2b_\perp}{(2\pi)^2}e^{i(\vec{k}_{\psi\perp}+\vec{k}_{J\perp})\cdot b_\perp}$$
$$\times\sum_i e^{-\mathcal{S}_i(-t,b_\perp)}C\otimes f_i(x_i,\mu_b)|\mathcal{A}^i_{BFKL}(\hat{s},t)|^2, (12)$$

The same Sudakov form factor in the resummation

Conclusion

- It is important to understand soft gluon radiations in the BFKL dominated processes
  - Sudakov double logs and resummation for Muller-Navelet and Mueller-Tang-type jet processes in hadronic collisions
- Jpsi+Jet production with LRG will provide a unique channel to explore BFKL dynamics
  - Theoretically simpler in terms of soft gluon radiation, etc.
- Phenomenology/experiment studies should be pursued for LHC and future EIC as well

Large-t diffractive Jpsi photo production at HERA



Kotko, Motyka, Sadzikowski, Stasto, JHEP 07, 129 (2019)

Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T)$$

- Balitsky-Kovchegov: Non-linear term, 98

$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T) - \alpha_s [N(x, r_T)]^2.$$