

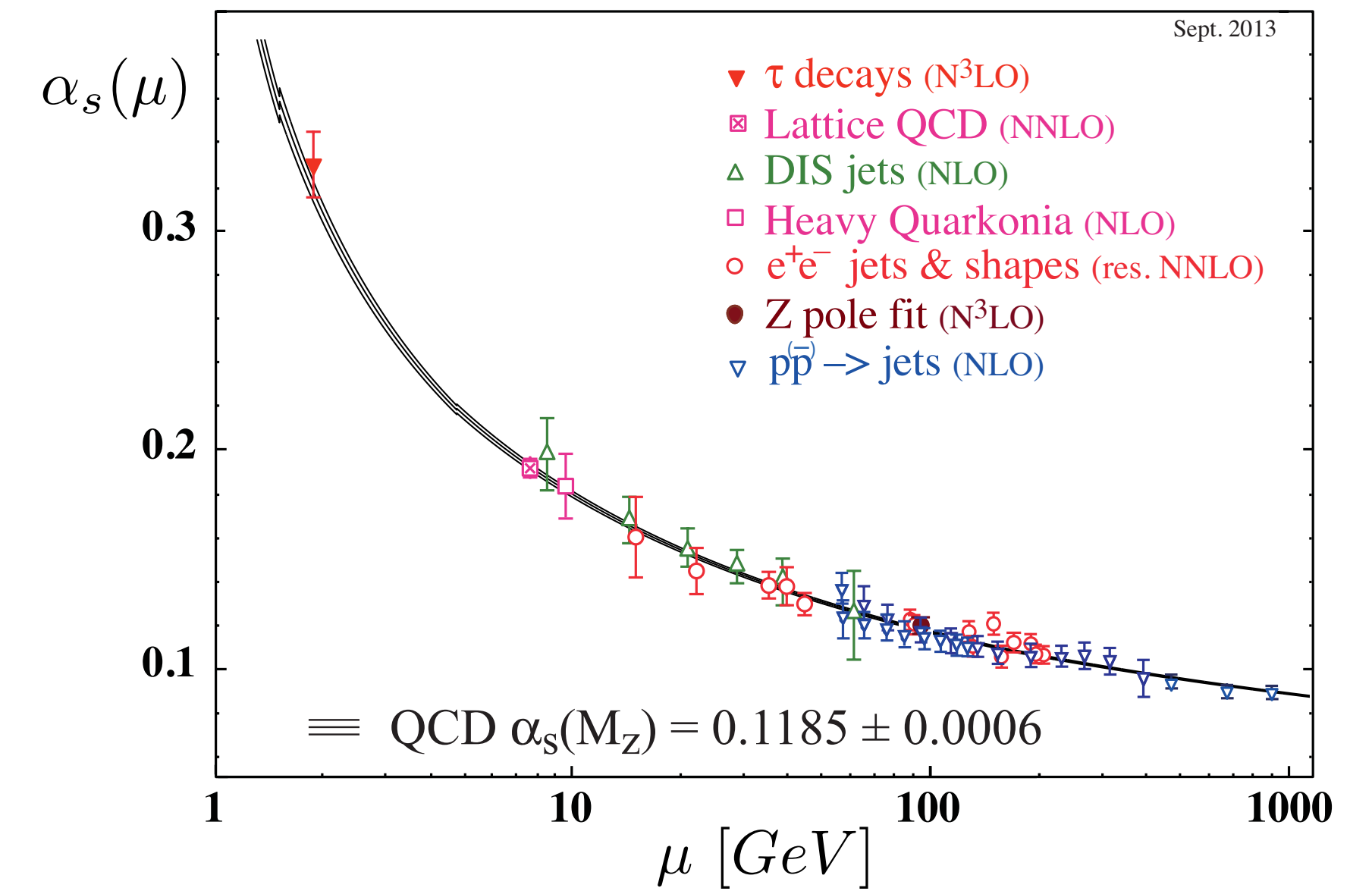
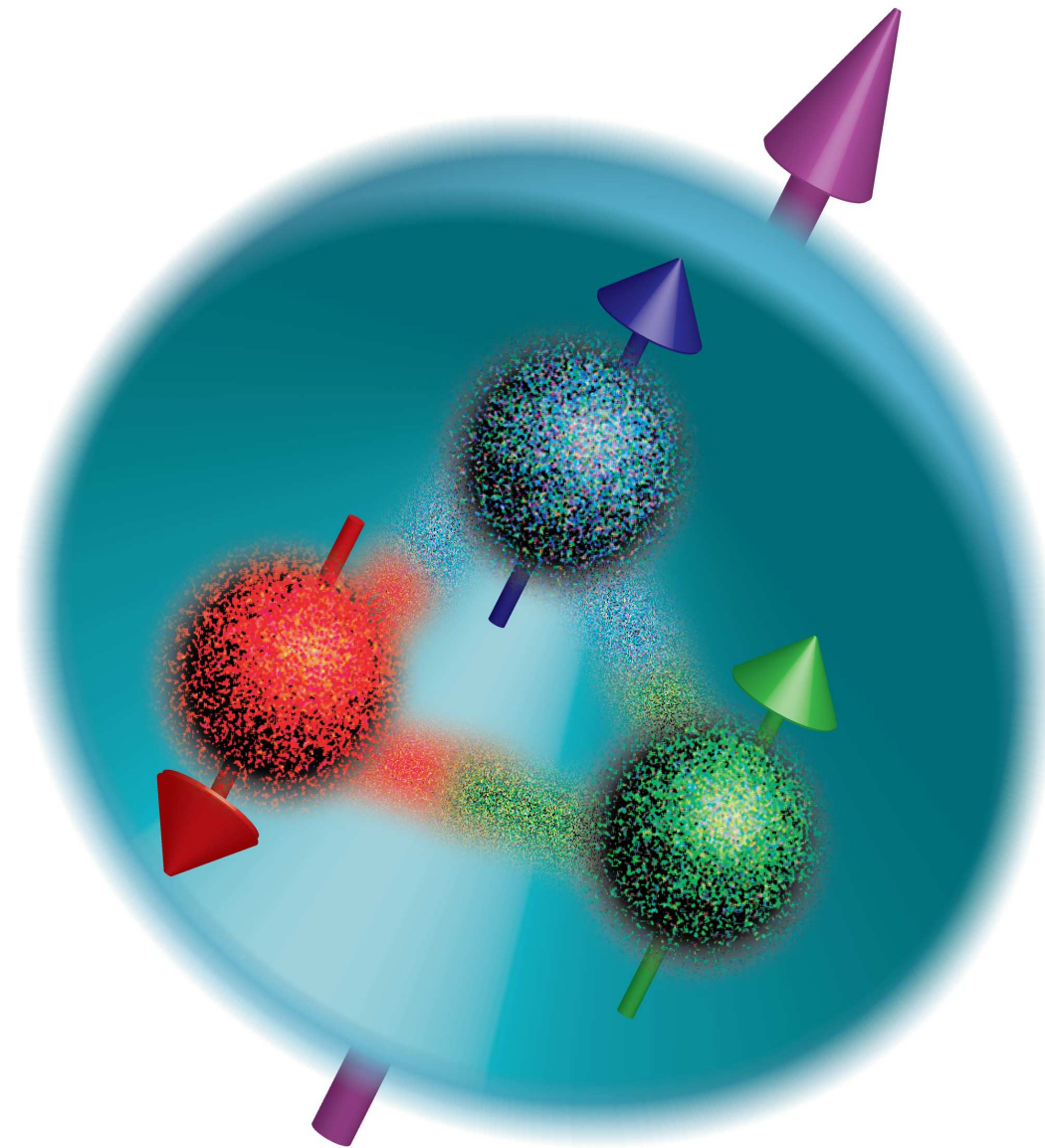
DIS dijet production and the search for saturation at the EIC

Andrey Tarasov

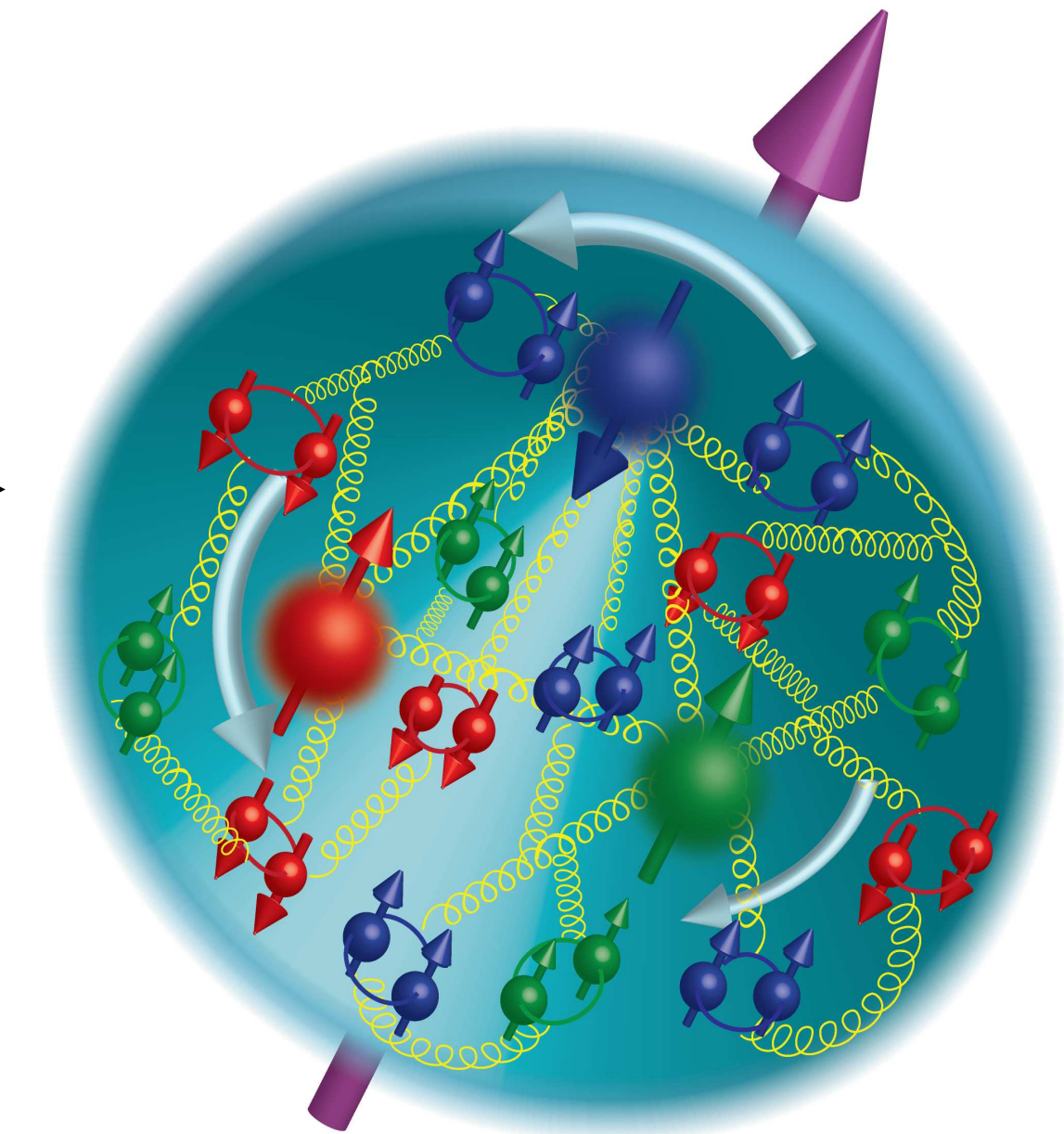
Based on Tiyasa Kar, AT, Vladimir Skokov “DIS dijet production in Background Field Approach: General formalism and methods”, arXiv:2603.08805 and in preparation

Gluon Saturation, Diffraction and Multihadron Production at the EIC, 16th July 2026

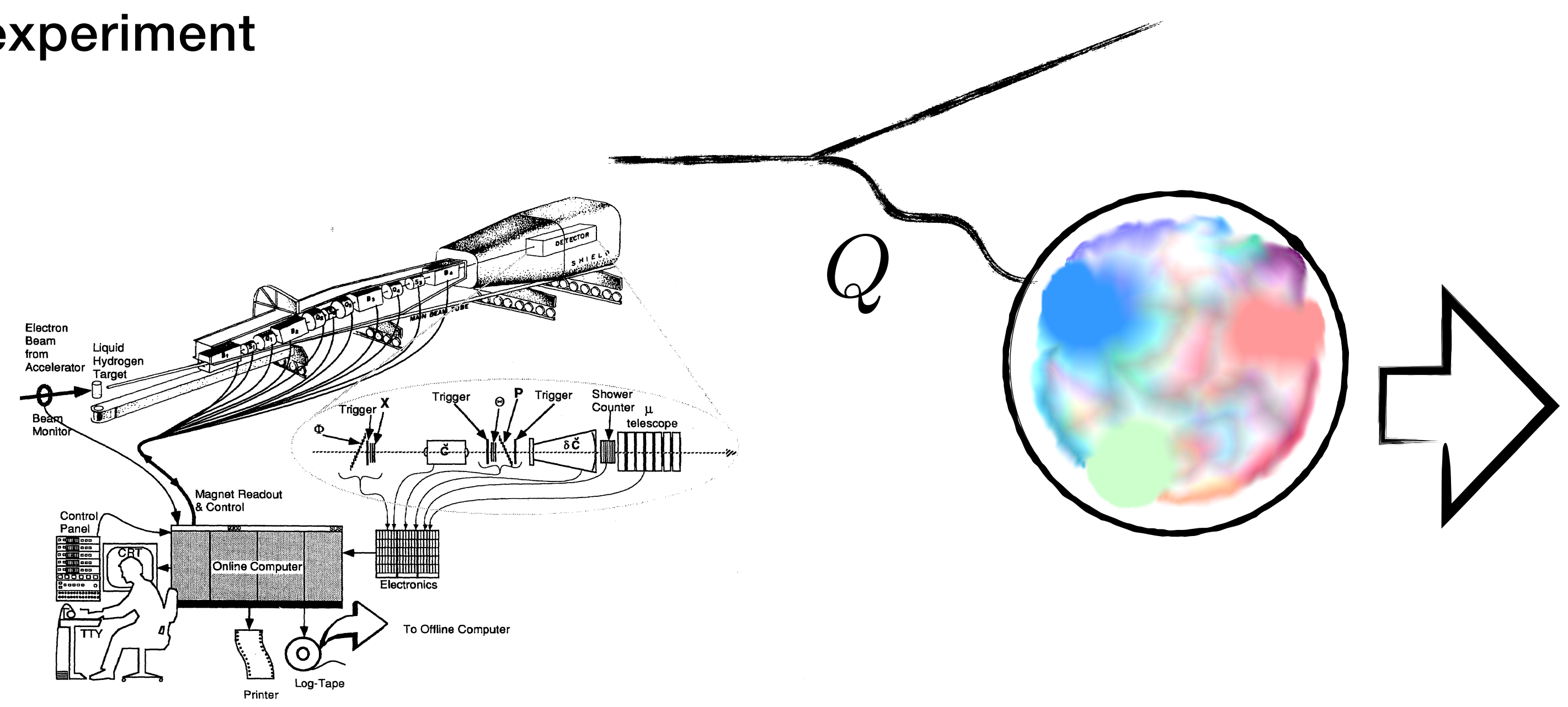
How do the characteristic properties of hadrons arise from the interactions of their most elementary constituents — quarks and gluons?



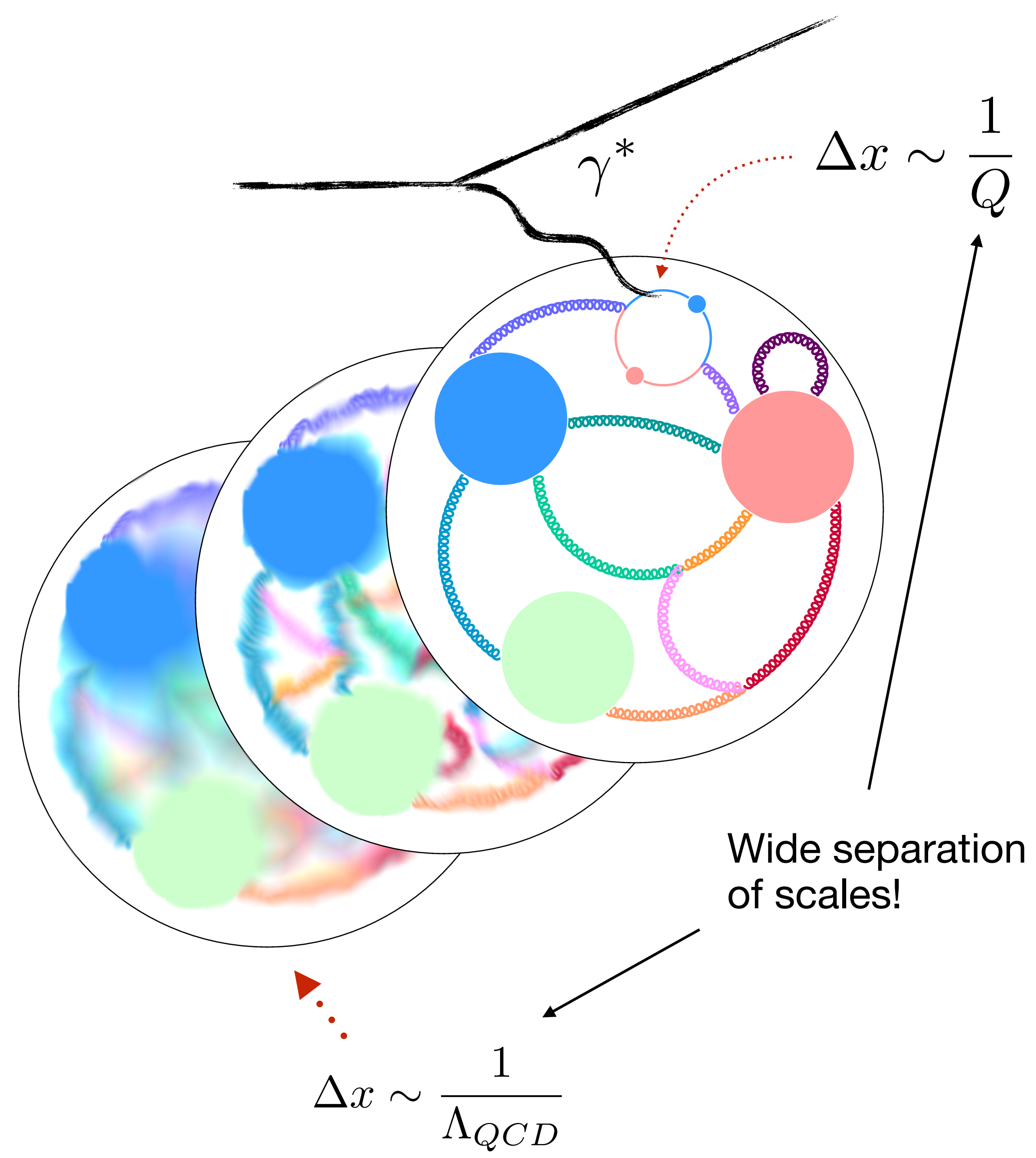
- The fundamental complication is the phenomenon of confinement: hadron structure is governed by the dynamics of a **dense** QCD medium



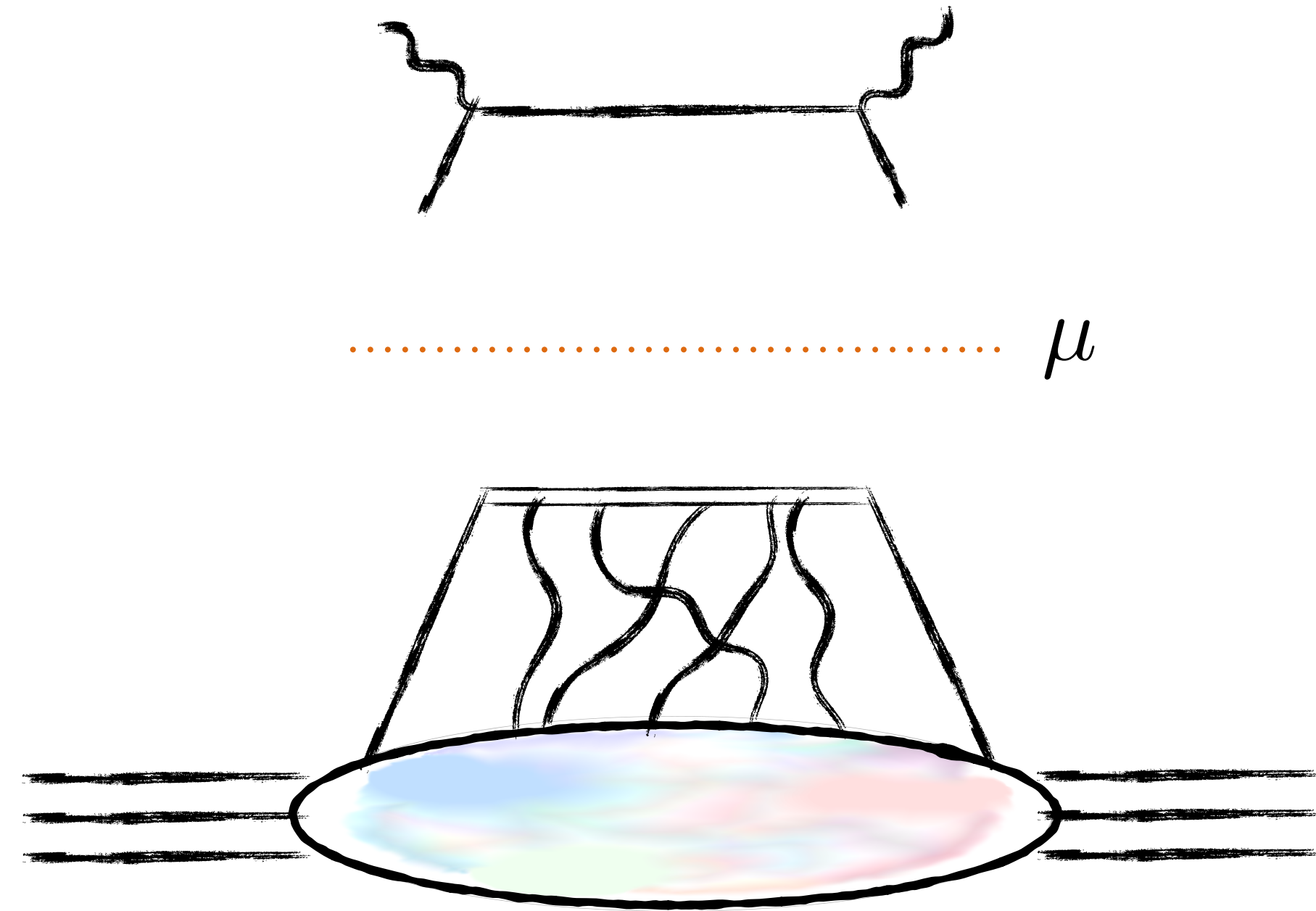
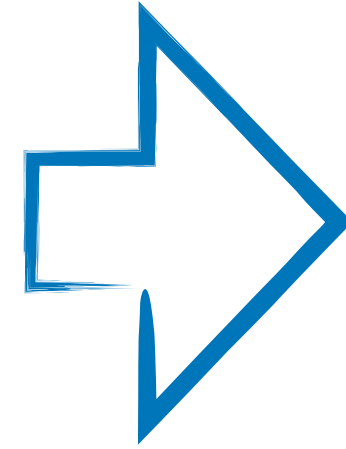
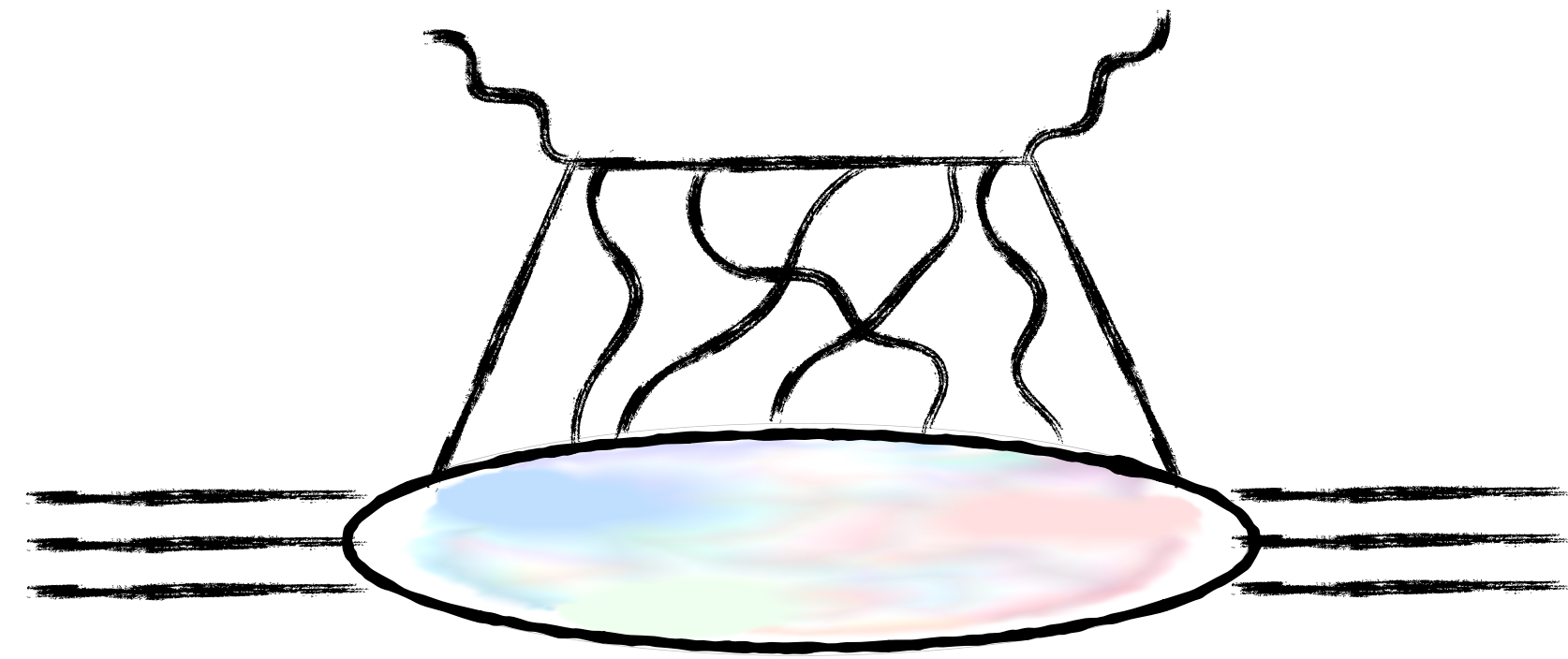
The structure of the dense QCD medium can be probed in high-energy scattering experiment



- At high energies, large external scales, the QCD medium admits a factorization description: different partonic interaction modes, can be identified
- The “quantum” field modes can be computed $\Rightarrow$ theoretical constraints on the structure of the medium



Factorization



collinear factorization in DIS

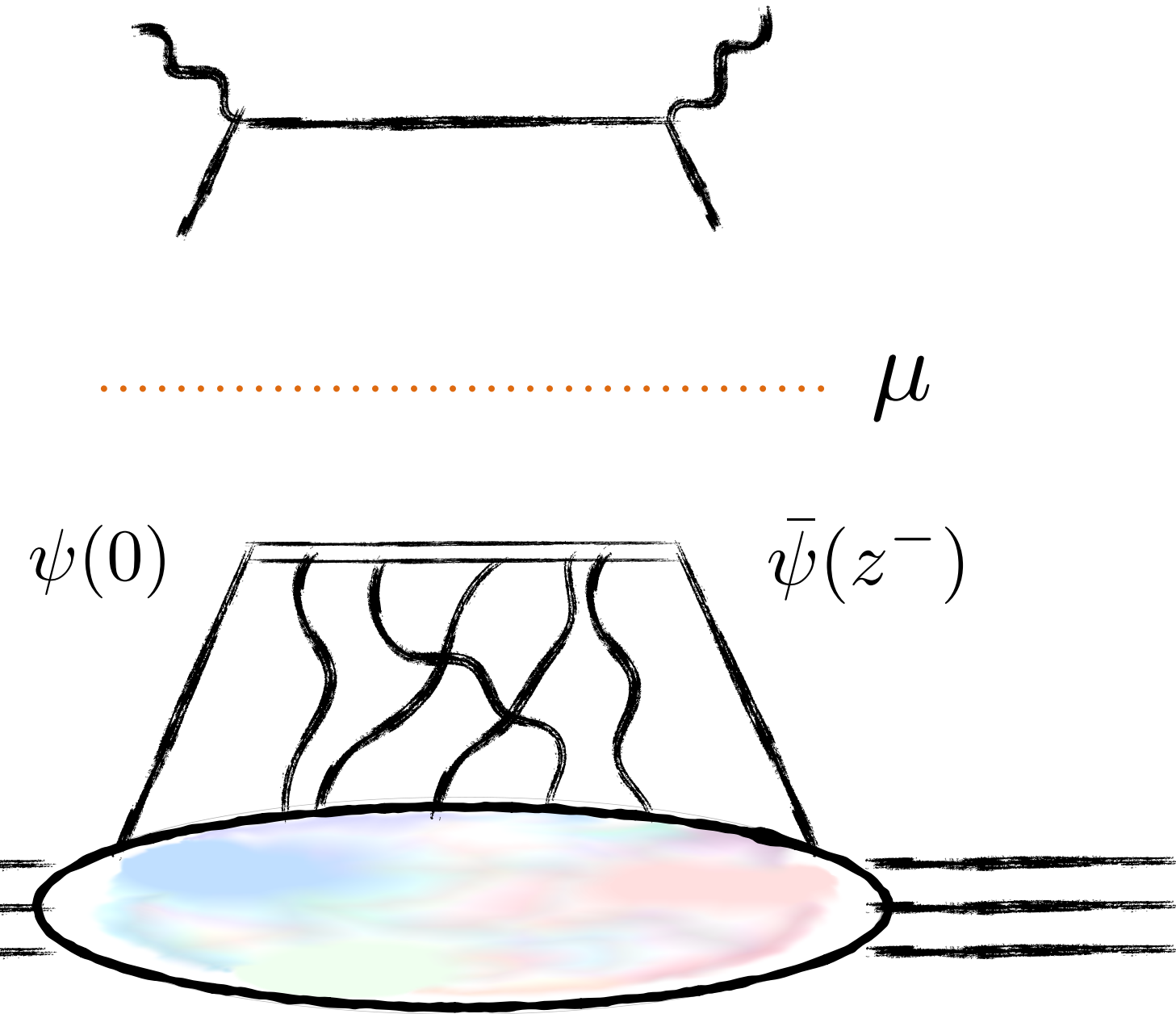
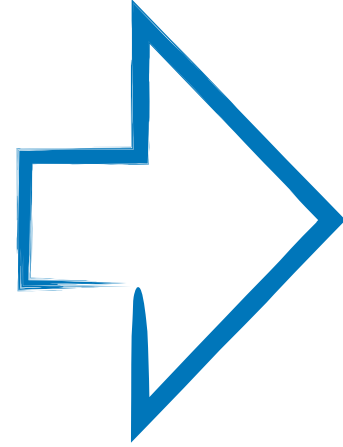
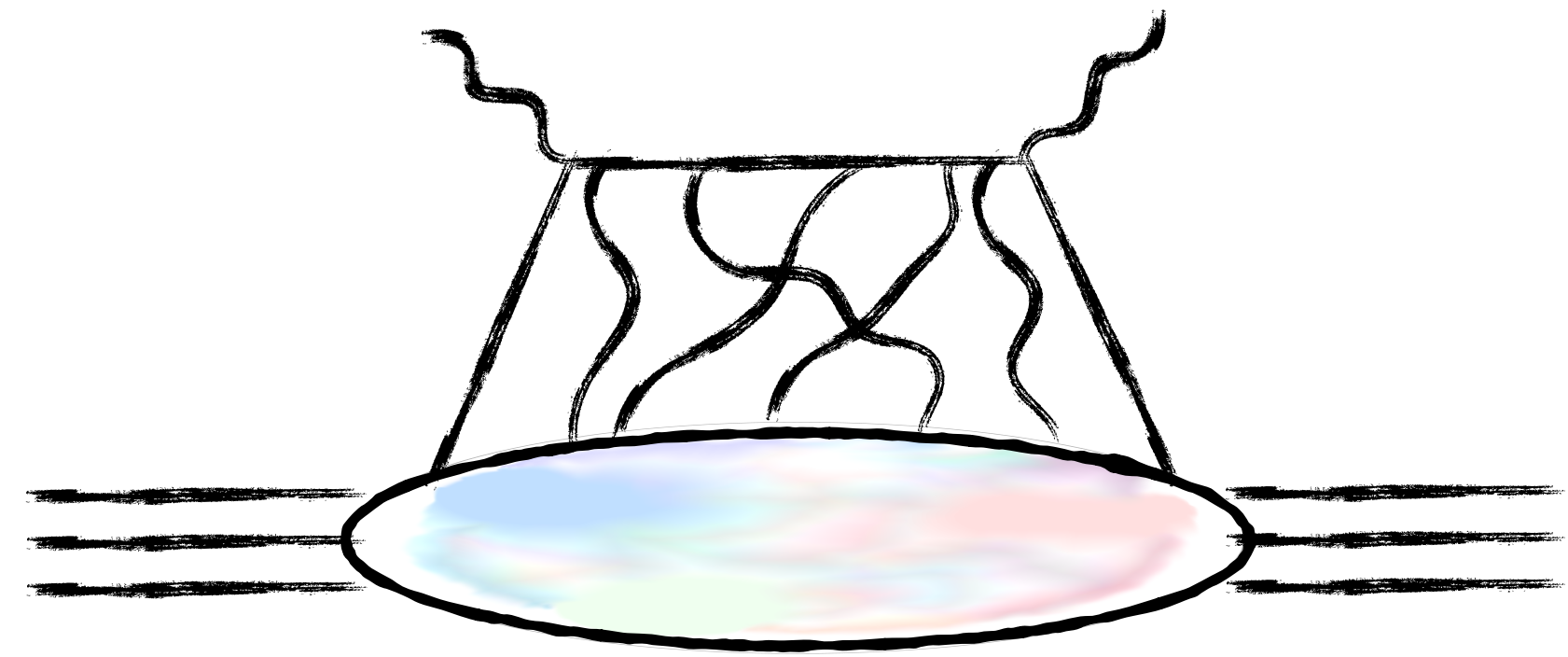
$$p_{\perp} \gg \mu$$

$$p_{\perp} \ll \mu$$

ordering of the emission
in transverse momentum

- The phenomenon can be formalized $\rightarrow$ factorization theorems
- Separation of modes is characterized by a set of factorization scale(s) σ (e.g. a single scale $\sigma = \mu$ in col. fact.)
- The structure of the factorized modes depends on the scattering reaction and observable

Factorization



collinear factorization in DIS

$$p_{\perp} \gg \mu$$

$$p_{\perp} \ll \mu$$

$$\int \frac{dz^-}{2\pi} e^{-ixP^+z^-} \langle P | \mathcal{O}(z^-) | P \rangle = f_1(x)$$

ordering of the emission
in transverse momentum

- The structure of the dense QCD medium can be formally defined in terms of matrix elements of **gauge invariant QCD operators (quark and gluon fields)** → distribution functions.
- The structure of these operators is defined by the **structure of interaction between the perturbative and non-perturbative mode**
- The operators fully describe the dense QCD medium

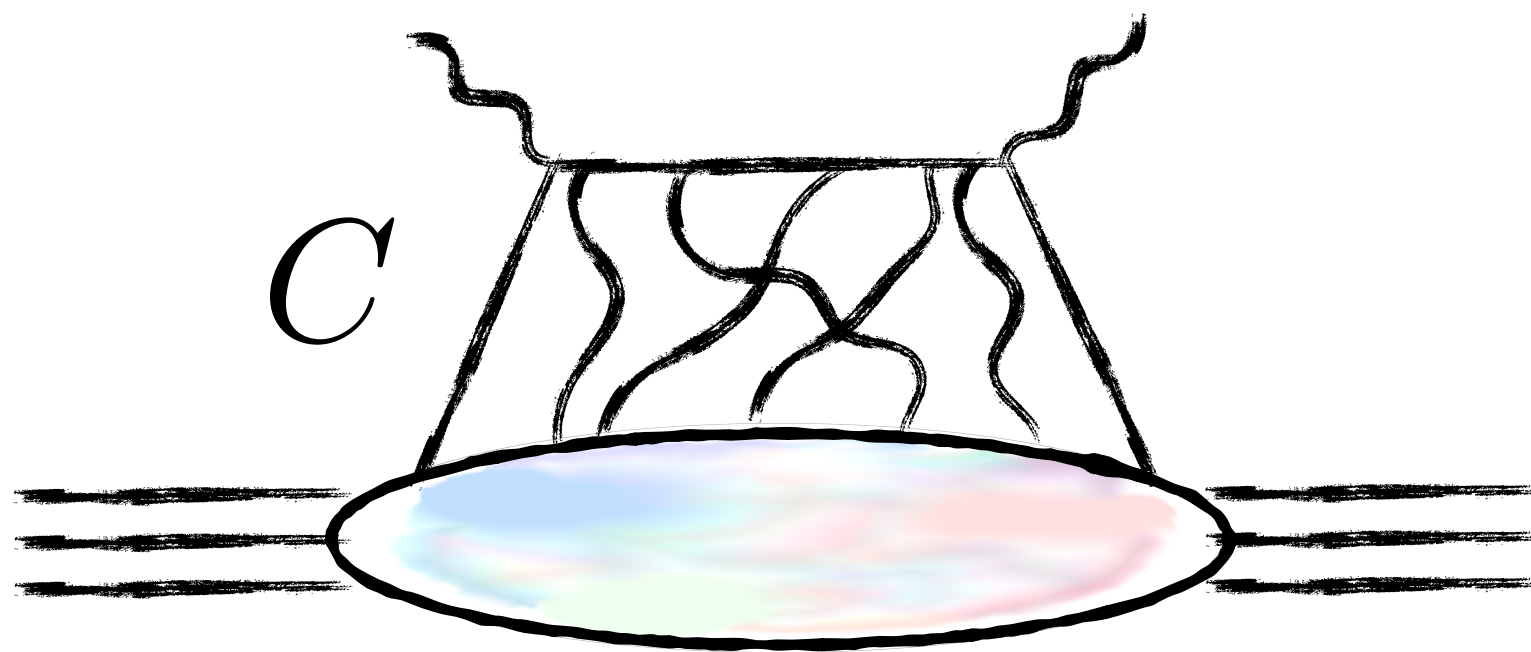
The background field approach provides an efficient formulation of the QCD factorization

Functional integral representation of a general physical observable:

$$\mathcal{X} \propto \int \mathcal{D}C \, \Psi_{P_f}^* (\vec{C}(t_f)) \mathcal{H}(C) \Psi_{P_i} (\vec{C}(t_i)) e^{iS_{\text{QCD}}(C)}$$

↑
operator defining the observable, e.g. product
of electromagnetic currents

Evaluation of this functional integral is **hard in general**, so, following the logic of the background field method, we split the field into different field components



The background field approach provides an efficient formulation of the QCD factorization

Functional integral representation of a general physical observable:

$$\mathcal{X} \propto \int \mathcal{D}C \, \Psi_{P_f}^* (\vec{C}(t_f)) \mathcal{H}(C) \Psi_{P_i} (\vec{C}(t_i)) e^{iS_{\text{QCD}}(C)}$$

↑
 operator defining the observable, e.g. product
 of electromagnetic currents

Evaluation of this functional integral is hard in general, so, following the logic of the background field method, we split the field into different field components

$$C_\mu \rightarrow A_\mu + B_\mu$$

↑
 perturbative “quantum” fields

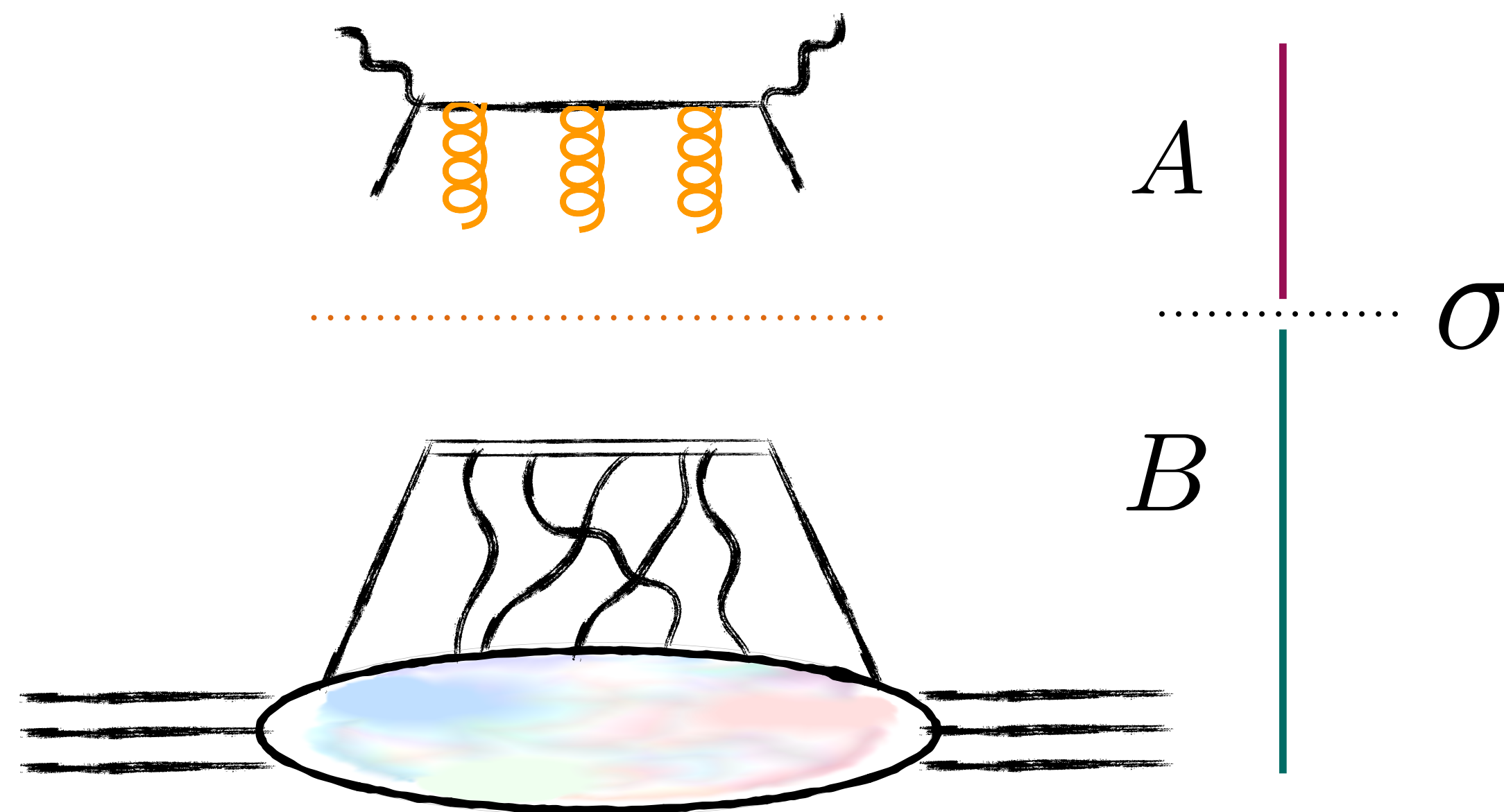
↑
 non-perturbative background fields

$$\int \mathcal{D}C e^{iS_{\text{QCD}}(C)} \rightarrow \int \mathcal{D}B e^{iS_{\text{QCD}}(B)} \int \mathcal{D}A e^{iS_{\text{bQCD}}(A,B)}$$

↑
QCD functional
integral

↑
functional integral over background
fields, gives rise to the matrix element
of the operators of the B fields

↑
functional integral over quantum field,
to be integrated by means of the
perturbative method, coefficient
functions, computation in the non-
perturbative background!



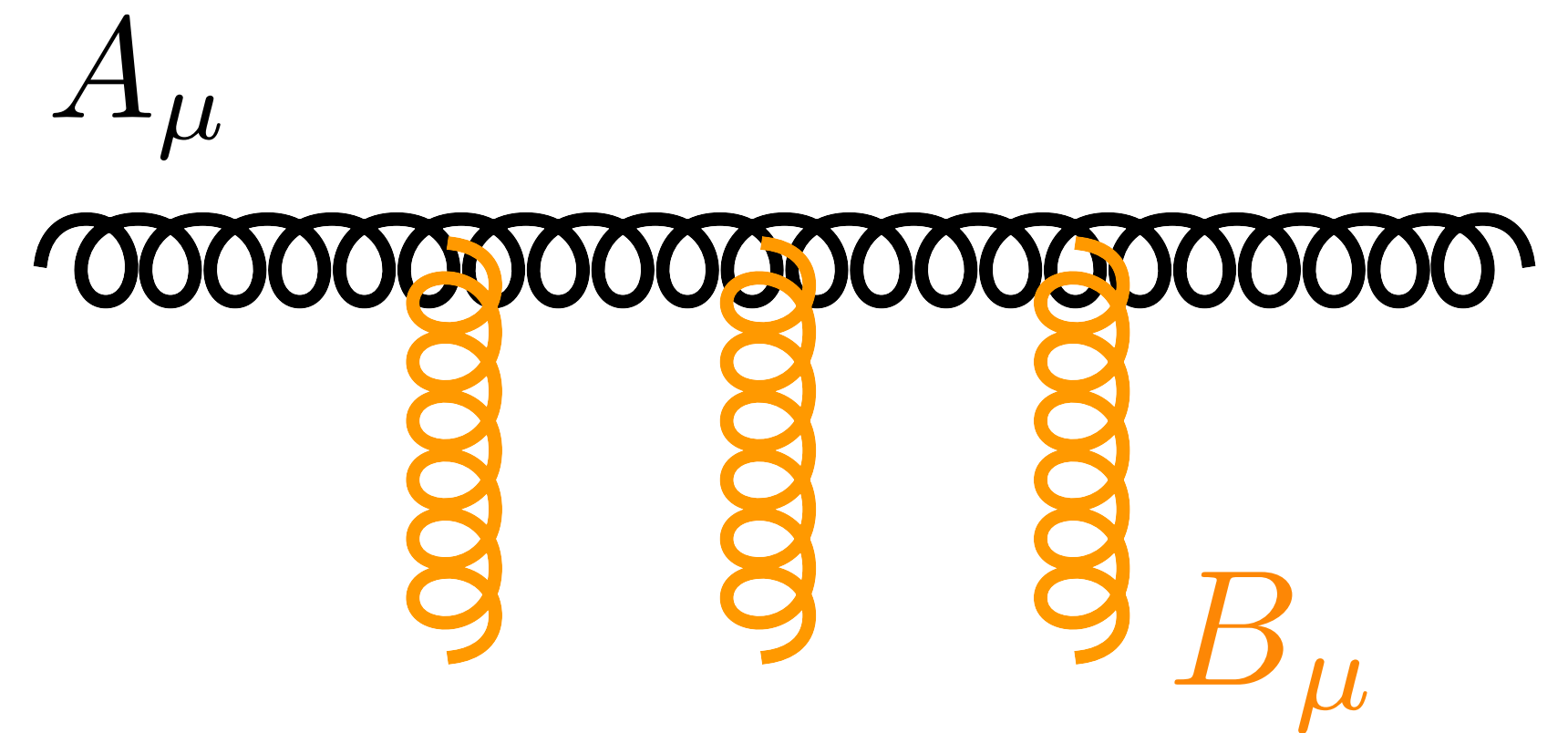
$$\int \mathcal{D}C e^{iS_{\text{QCD}}(C)} \rightarrow \int \mathcal{D}B e^{iS_{\text{QCD}}(B)} \int \mathcal{D}A e^{iS_{\text{bQCD}}(A,B)}$$

↑
Computation of the A mode is not in the vacuum but in the background of the B fields

background-QCD action describes propagation of a “quantum” particle (A_μ) in a background field B_μ

$$S_{\text{bQCD}}(A, B) = S_{\text{QCD}}(A + B) - S_{\text{QCD}}(B)$$

We can systematically study the interaction between the field modes, unambiguously derive the structure of the QCD operators, distribution functions etc.



We look for the following form of the result of integration over A fields:

$$\int \mathcal{D}A e^{iS_{\text{bQCD}}(A,B)} \mathcal{H}(A+B) = \sum_k H_k(\sigma) \otimes \mathcal{O}_k(B, \sigma)$$

where operators $\mathcal{O}$ are constructed from the background fields B . As a result we get (the matrix element corresponds to the functional integral over background fields B)

$$\mathcal{X} \propto \sum_k H_k(\sigma) \otimes \langle P_f | \mathcal{O}_k(\hat{B}) | P_i \rangle^\sigma$$


Factorization formula

$$\mathcal{X} \propto \sum_k H_k(\sigma) \otimes \langle P_f | \mathcal{O}_k(\hat{B}) | P_i \rangle^\sigma$$

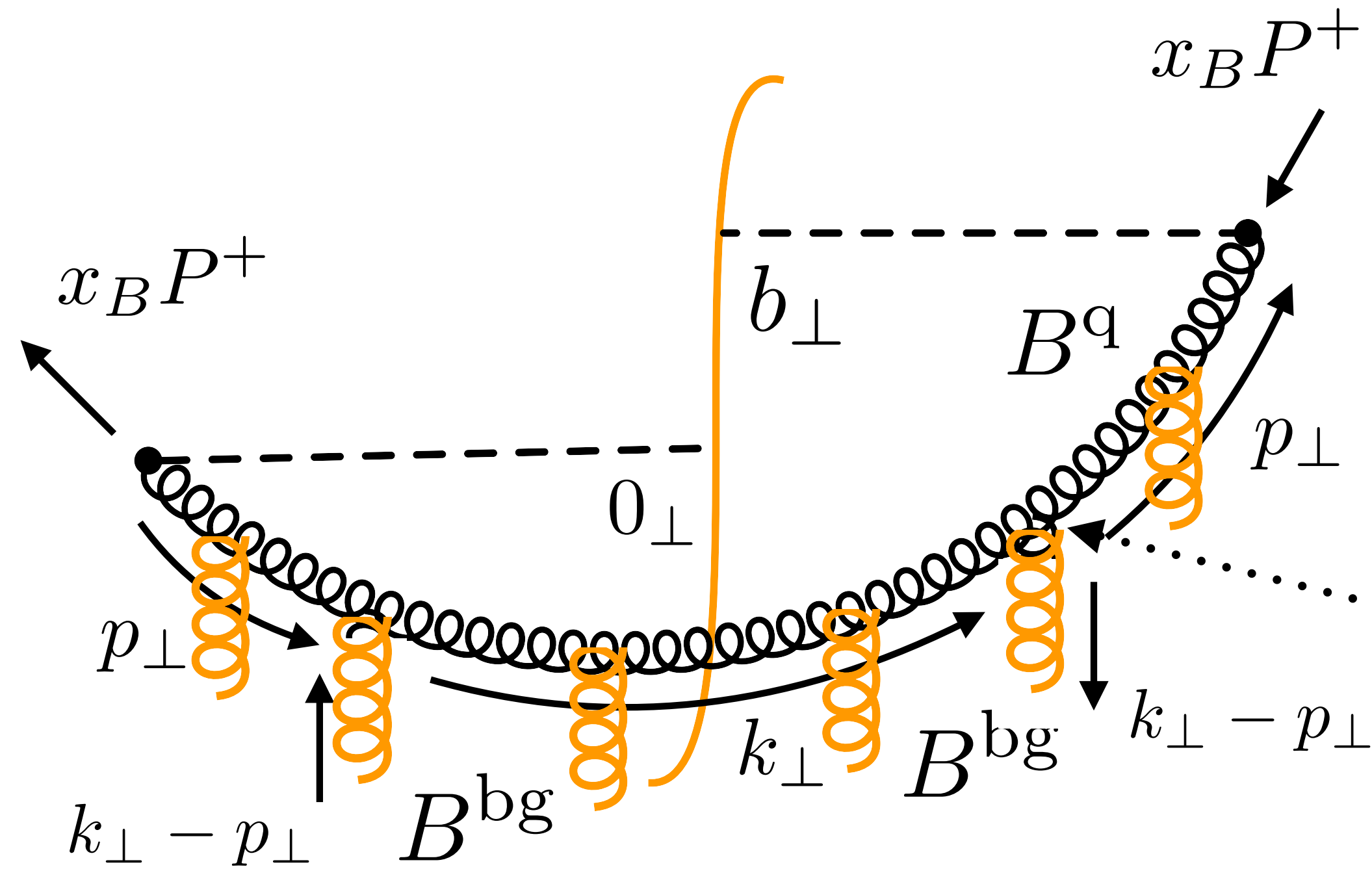
Result of the integration
over the A field modes

Contribution of the B
field modes

- Contribution of the A and B field modes are disentangled, the result has the factorized form
- The operators of the background B fields are gauge invariant
- It is not obvious that these conditions can be attained (higher twist effects, TMD factorization, sub-eikonal corrections, spin effects at small-x etc.)

$\propto$ quantum $\otimes$ background

Perturbative computation in the non-perturbative background



- Entanglement of the quantum momentum flow and the background-field interactions
- The background fields B don't enter the computation in the gauge covariant form

$$g^{\rho\sigma} (p_\alpha B^\alpha + B^\alpha k_\alpha) + 2i(\partial^\rho B^\sigma - \partial^\sigma B^\rho) - p^\rho B^\sigma - B^\rho k^\sigma$$

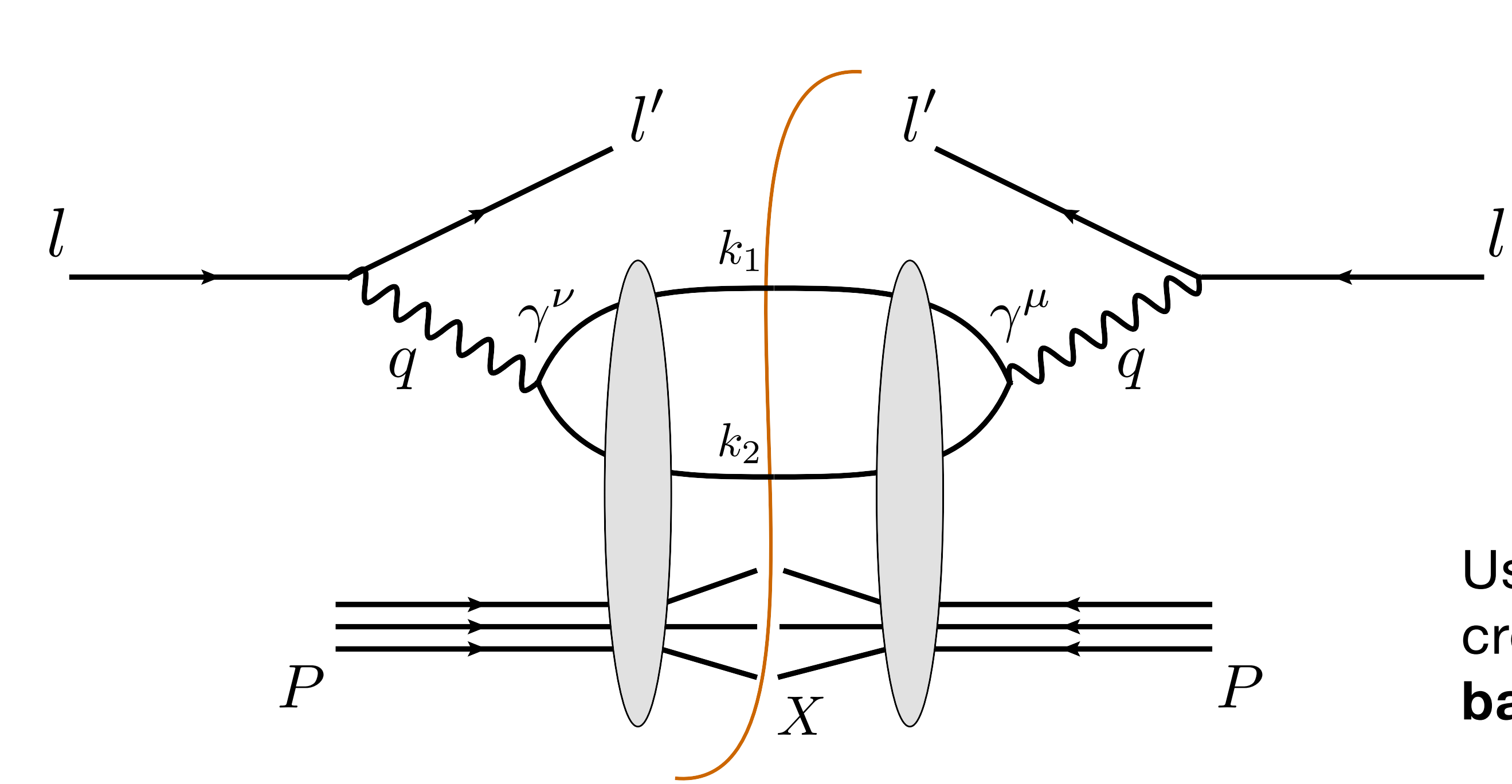
Can we formulate a systematic framework for **gauge-covariant factorization computations**?

Kar, Skokov, Tarasov, arXiv:2603.08805

Mukherjee, Skokov, Tarasov, Tiwari, Yao, 2602.15137

Mukherjee, Skokov, Tarasov, Tiwari (2025)

Computation of the dijet production in the background field approach

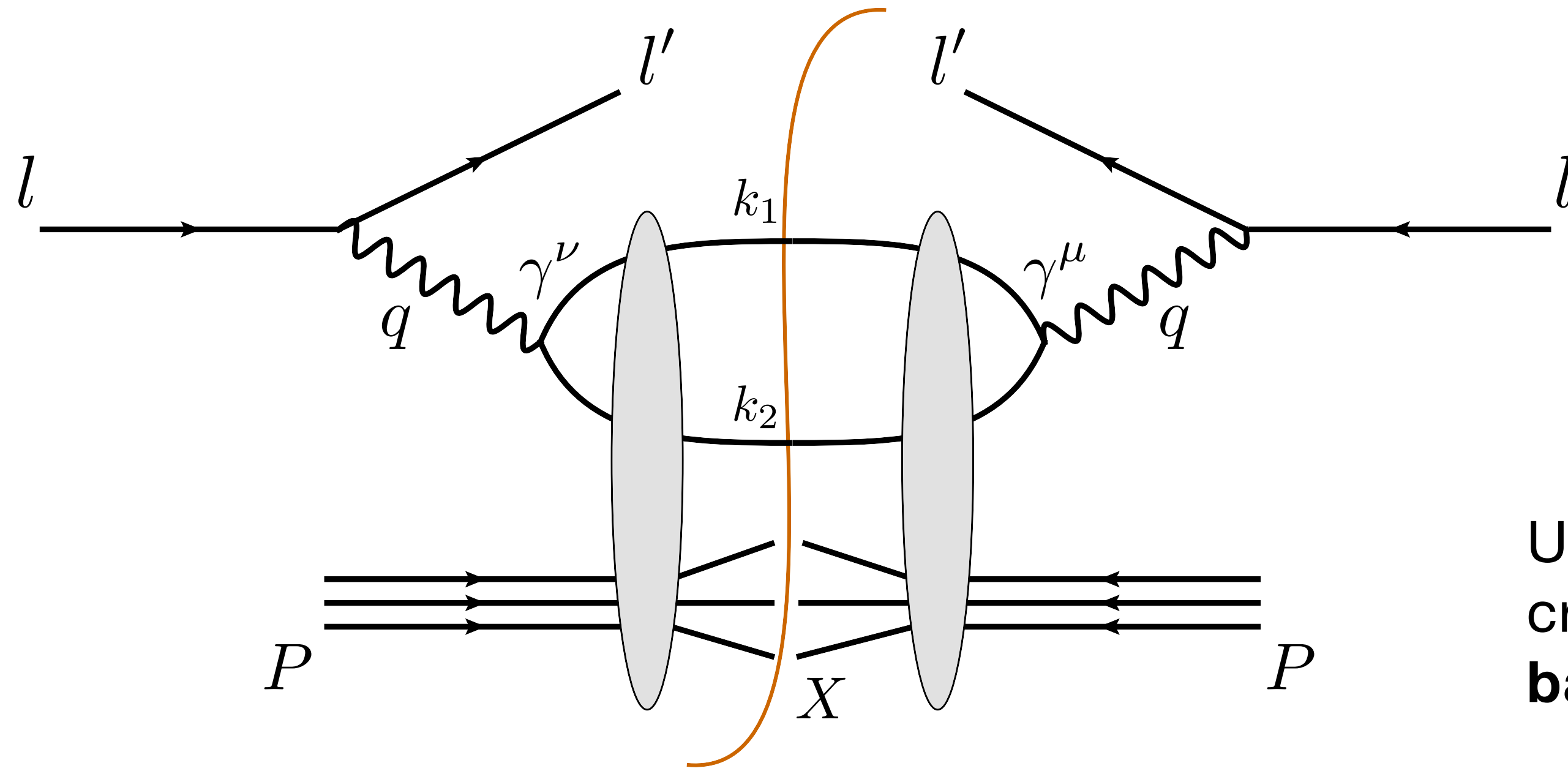


$$E' \frac{d\sigma}{d^3l'} = \frac{\alpha_{\text{EM}} y}{4\pi^2 Q^4} \sum_{\lambda, \lambda' = -1}^1 (-1)^{\lambda + \lambda'} L^{\lambda \lambda'} \sigma_{\lambda \lambda'}^{\gamma^* p}$$

The scattering cross section for a virtual photon off the target

Using the background field method we can write the cross section in terms of **quark propagators in the background field**

Computation of the dijet production in the background field approach



$$E' \frac{d\sigma}{d^3l'} = \frac{\alpha_{\text{EM}} y}{4\pi^2 Q^4} \sum_{\lambda, \lambda'=-1}^1 (-1)^{\lambda+\lambda'} L^{\lambda\lambda'} \sigma_{\lambda\lambda'}^{\gamma^* p}$$

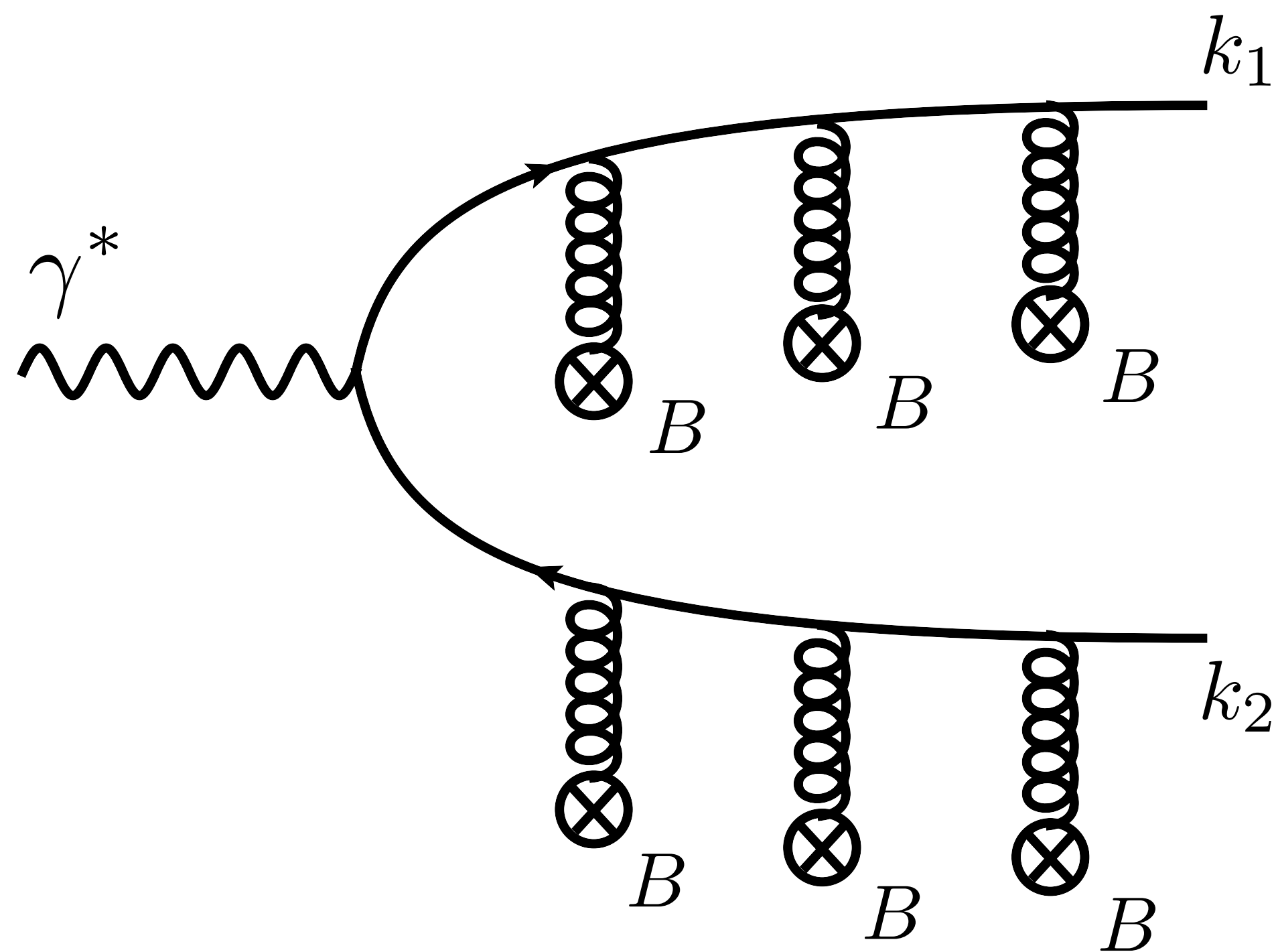
The scattering cross section for a virtual photon off the target

Using the background field method we can write the cross section in terms of **quark propagators in the background field**

$$\begin{aligned} d\sigma_{\lambda\lambda'}^{\gamma^* p} = & \sum_{s_1, s_2} \frac{2\pi\alpha_{\text{EM}} x}{V_4 Q^2} \epsilon_{\mu}^{\lambda*} \epsilon_{\nu}^{\lambda'} \frac{dk_1^- d^2 k_{1\perp}}{(2\pi)^3 2k_1^-} \Big|_{k_1^2=0} \frac{dk_2^- d^2 k_{2\perp}}{(2\pi)^3 2k_2^-} \Big|_{k_2^2=0} Z_2^{-1} \int d^4 x e^{iqx} \\ & \times \int d^4 x_1 e^{-ik_1 x_1} \int d^4 x_2 e^{-ik_2 x_2} \bar{v}_{s_2}(k_2) \not{k}_2 \langle P, S | \tilde{T} \{ \psi(x_2) j^{\mu}(x) \bar{\psi}(x_1) \} \tilde{B} | P, S \rangle \not{k}_1 u_{s_1}(k_1) Z_2^{-1} \\ & \times \int d^4 y e^{-iqy} \int d^4 y_1 e^{ik_1 y_1} \int d^4 y_2 e^{ik_2 y_2} \bar{u}_{s_1}(k_1) \not{k}_1 T \{ \psi(y_1) j^{\nu}(y) \bar{\psi}(y_2) \} B | P, S \rangle \not{k}_2 v_{s_2}(k_2) \end{aligned}$$

$$T\{\psi(y_1)j^\nu(y)\bar{\psi}(y_2)\}_B = e(y_1|\frac{i\hat{p}}{\hat{p}^2+i\epsilon} + \frac{i\hat{p}}{\hat{p}^2+i\epsilon}(ig\hat{B})\frac{i\hat{p}}{\hat{p}^2+i\epsilon} + \dots|y)\gamma^\nu$$

$$\times (y|\frac{i\hat{p}}{\hat{p}^2+i\epsilon} + \frac{i\hat{p}}{\hat{p}^2+i\epsilon}(ig\hat{B})\frac{i\hat{p}}{\hat{p}^2+i\epsilon} + \dots|y_2) - (B=0),$$



∴ Infinite number of interactions with the background field B dressed with quantum propagators. The standard quark-gluon vertex

Can we represent the background fields B in terms of the gauge covariant operators?

Can we disentangle quantum momenta $\hat{p}$ and the background field B insertions?

One can construct gauge covariant form of the quark propagators in the background field

$$\int d^4 y_1 e^{i k_1 y_1} \int d^4 y_2 e^{i k_2 y_2} \bar{u}_{s_1}(k_1) \not{k}_1 T \{ \psi(y_1) j^\nu(y) \bar{\psi}(y_2) \}_B \not{k}_2 v_{s_2}(k_2)$$

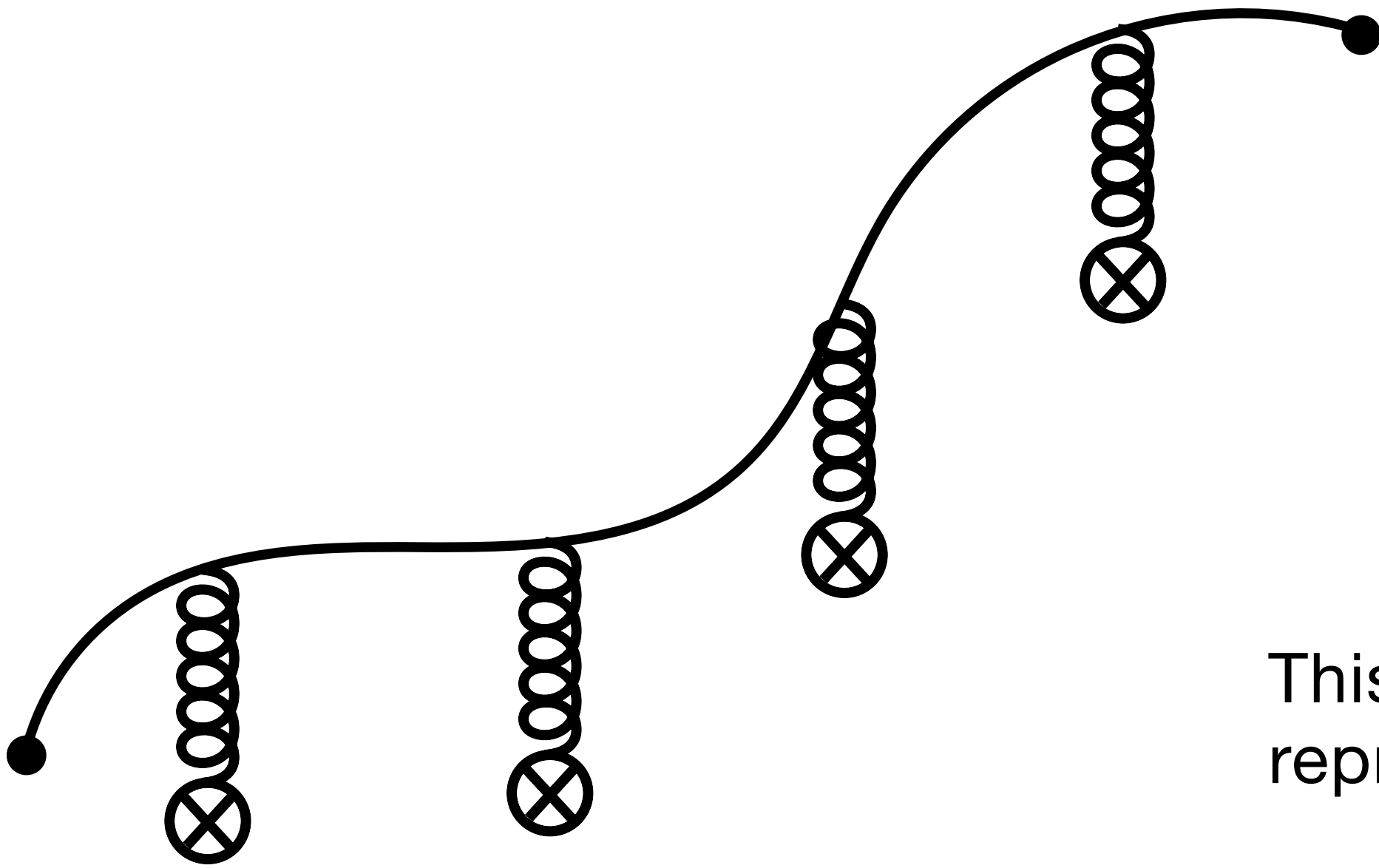
$$= e \bar{u}_{s_1}(k_1) k_1^2 (k_1 | \frac{i}{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y) \gamma^\nu (y | \frac{i}{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | -k_2) (-k_2^2) v_{s_2}(k_2) - (B=0)$$

The quark propagation is represented in terms of the covariant derivatives and strength tensors in the background field:

$$P_\mu \equiv p_\mu + g B_\mu$$

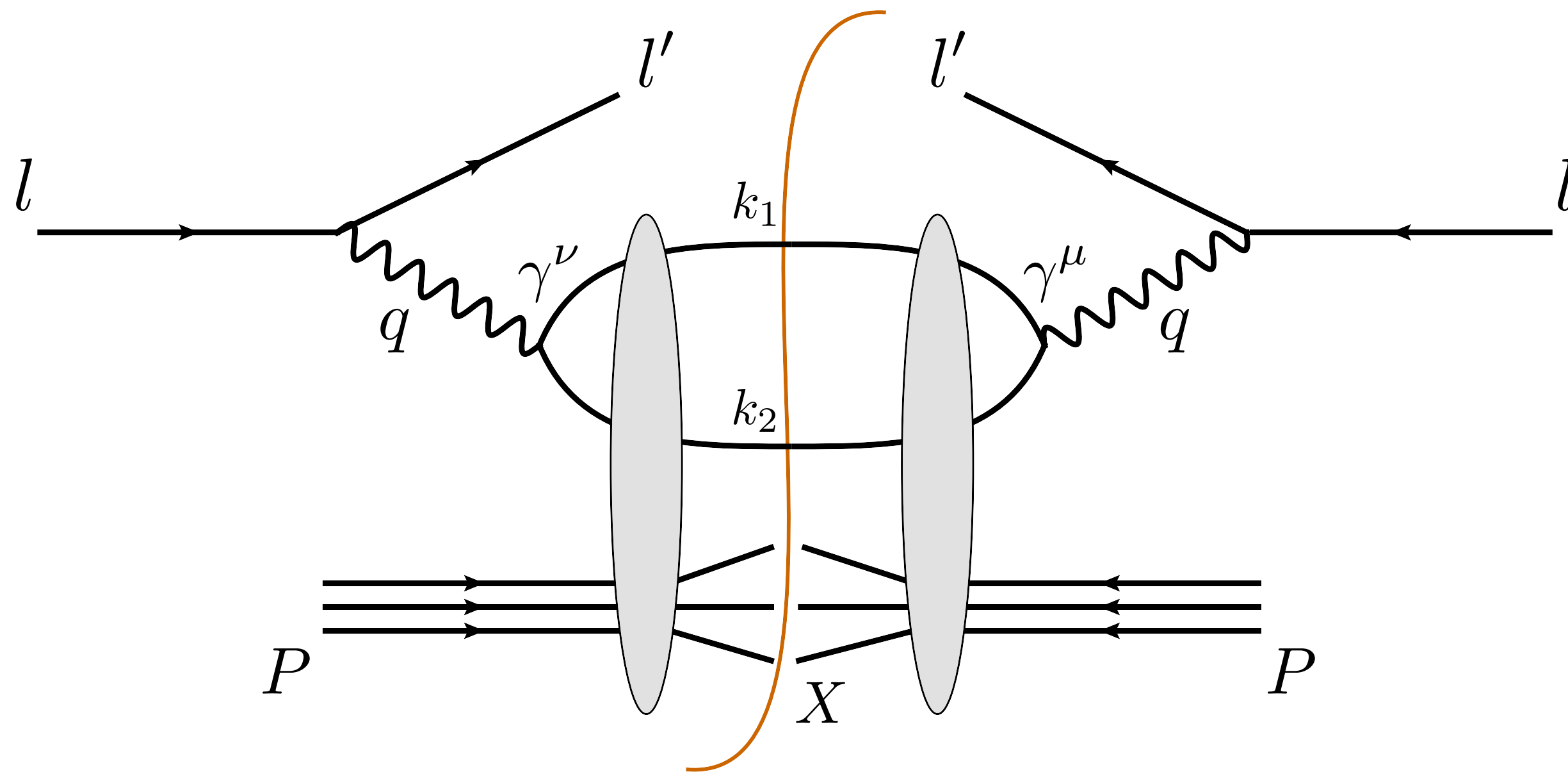
$$F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu - ig [B_\mu, B_\nu]$$

This form of the propagator matches the worldliness representation of the quark propagator [Tarasov, Venugopalan \(2019\)](#)



The building block of our computation of the dijet production is the quark propagator in the background field

$$\sigma_{\lambda\lambda'}^{\gamma^* p} \propto \left| \frac{i}{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} \right|^4$$



The dijet production cross section can be represented in terms of these propagators in the most general form of the background field

This representation is quite general. The main issue is that it entangles quantum (quark momentum) and the background field degrees of freedom. We need to find a way to disentangle the quantum and background effects

$$P_\mu \equiv p_\mu + gB_\mu$$

$$\mathcal{X} \propto \sum_k H_k(\sigma) \otimes \langle P_f | \mathcal{O}_k(\hat{B}) | P_i \rangle^\sigma$$

The gauge invariant operators are organized along a particular piecewise contour

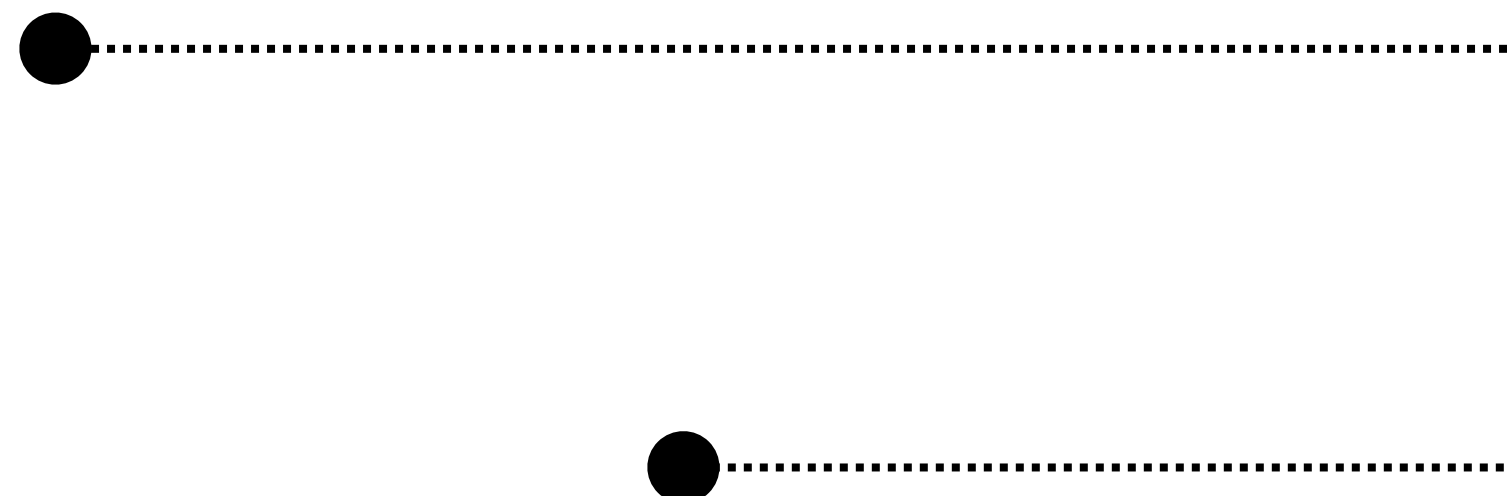
Straight line counter for the collinear operators:



Dipole operators at small-x



Staple contour for TMD operators



i

$$\overline{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon}$$

$\propto$ quantum $\otimes$ background

To disentangle “quantum” and background fields of freedom we need to expand background fields of the propagator onto a given contour

$\Rightarrow$ such expansion can be constructed using the path-ordered exponent representation of the propagator

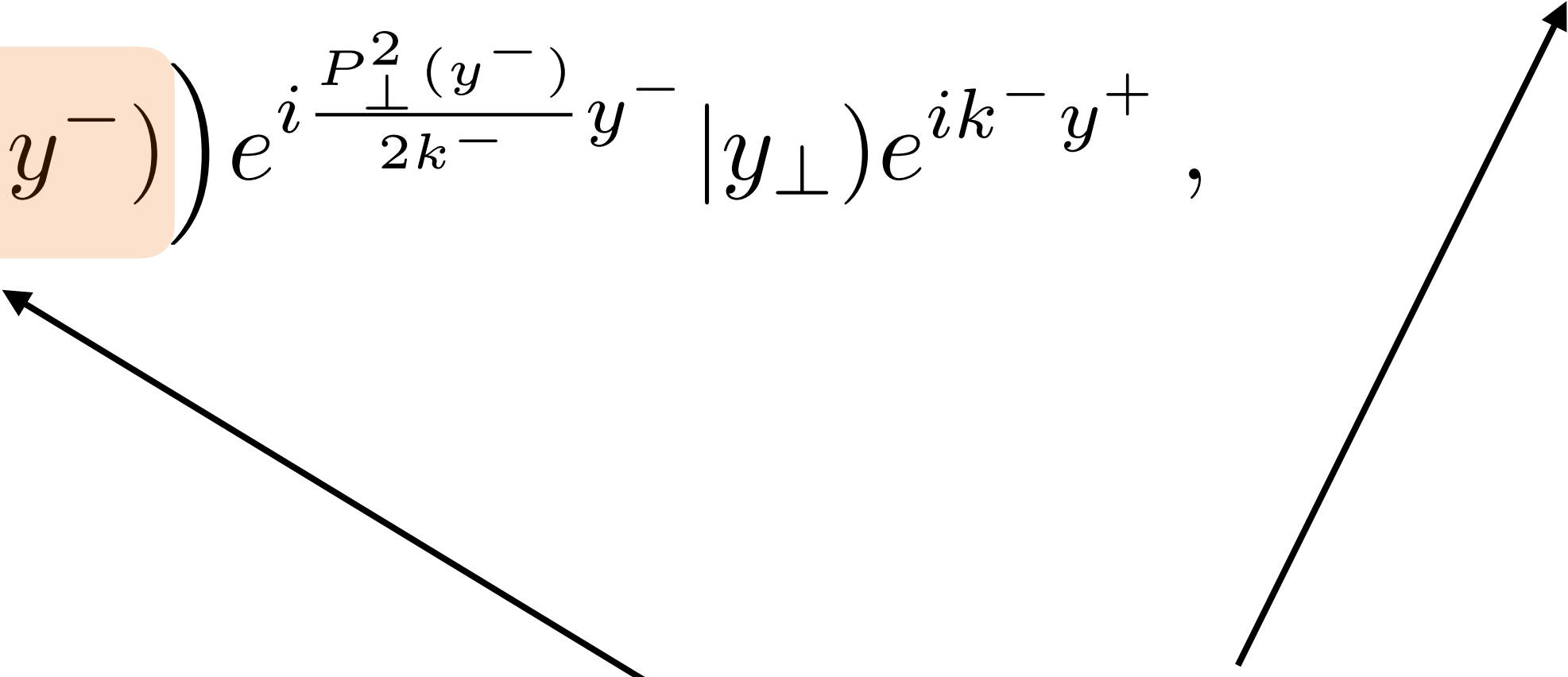
Kar, Skokov, Tarasov, [arXiv:2603.08805](https://arxiv.org/abs/2603.08805)

Path-ordered exponent representation of the quark propagator in the background field

$$B_\mu(x) = B_\mu(x^-, x_\perp)$$

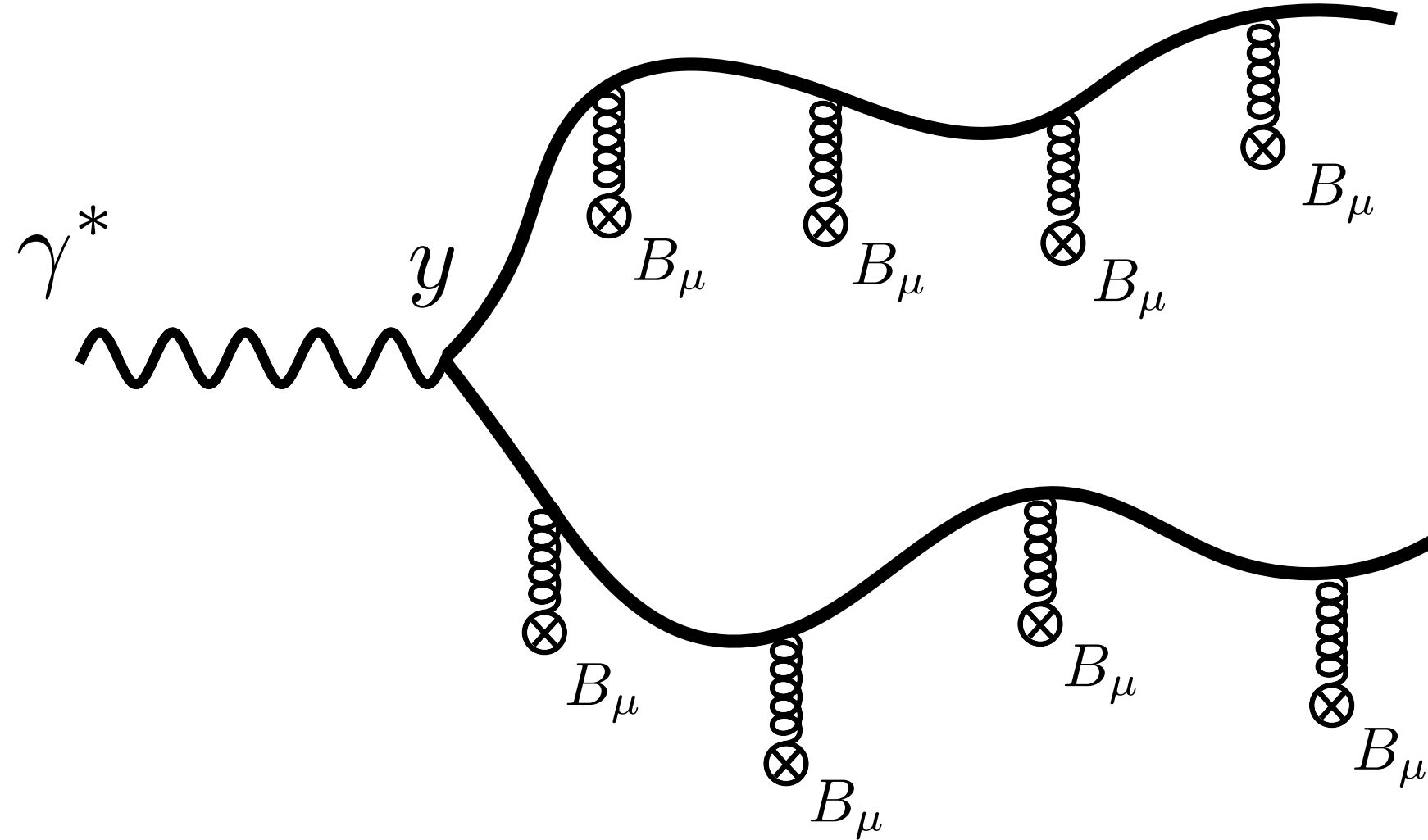
$$\begin{aligned} \lim_{k^2 \rightarrow 0} k^2 (k | \frac{1}{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y) &= \lim_{k^2 \rightarrow 0} (k_\perp | \left(\theta(k^-) e^{i \frac{k_\perp^2}{2k^-} z^-} e^{-i \frac{P_\perp^2(z^-)}{2k^-} z^-} \Big|^\infty \mathcal{G}(\infty, y^-) \right. \\ &\left. + \theta(-k^-) e^{i \frac{k_\perp^2}{2k^-} z^-} e^{-i \frac{P_\perp^2(z^-)}{2k^-} z^-} \Big|^{-\infty} \mathcal{G}(-\infty, y^-) \right) e^{i \frac{P_\perp^2(y^-)}{2k^-} y^-} | y_\perp) e^{ik^- y^+}, \end{aligned}$$

Path-ordered exponents



The exponents describe quark's propagation along an arbitrary trajectory in the background field B

$$B_\mu(x) = B_\mu(x^-, x_\perp)$$



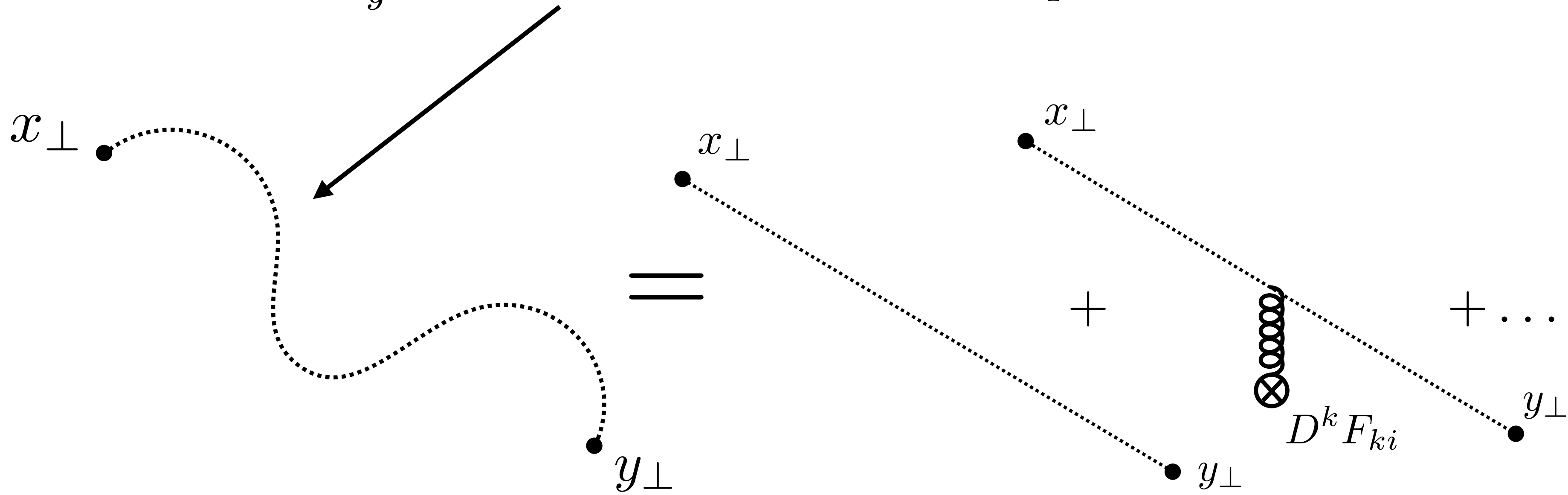
Gauge covariant formulation. This result reproduces all known limits, e.g. Wilson lines of the dipole operators in the CGC computations etc.

$$\begin{aligned} & (x_\perp | \mathcal{G}(x^-, y^-) | y_\perp) \\ &= (x_\perp | \mathcal{P} \exp \left\{ i \int_{y^-}^{x^-} dz^- \left(e^{i \frac{P_\perp^2(z^-)}{2p^-}} z^- \left(iD_- + \frac{g}{4p^-} \sigma^{\mu\nu} F_{\mu\nu} \right) e^{-i \frac{P_\perp^2(z^-)}{2p^-}} z^- - \frac{P_\perp^2(z^-)}{2p^-} \right) \right\} | y_\perp) \end{aligned}$$

Contains contribution of the transverse gauge links

$$(x_{\perp} | \mathcal{G}(x^-, y^-) | y_{\perp})$$

$$= (x_{\perp} | \mathcal{P} \exp \left\{ i \int_{y^-}^{x^-} dz^- \left(e^{i \frac{P_{\perp}^2(z^-)}{2p^-} z^-} \left(i D_- + \frac{g}{4p^-} \sigma^{\mu\nu} F_{\mu\nu} \right) e^{-i \frac{P_{\perp}^2(z^-)}{2p^-} z^-} - \frac{P_{\perp}^2(z^-)}{2p^-} \right) \right\} | y_{\perp})$$



$$(x_{\perp} | e^{isP_{\perp}^2} | y_{\perp}) = (x_{\perp} | e^{isp_{\perp}^2} | y_{\perp}) \left\{ U(x_{\perp}, y_{\perp}) + s \int_0^1 du \, u \bar{u} \, (x - y)^k U(x_{\perp}, \xi_u) D_m F_{mk}(\xi_u) U(\xi_u, y_{\perp}) \right.$$

$$\left. + 2is \int_0^1 du \, \bar{u} \int_0^u dv \, v \, (x - y)^k (x - y)^m U(x_{\perp}, \xi_u) F_{ks}(\xi_u) U(\xi_u, \xi_v) F_{ms}(\xi_v) U(\xi_v, y_{\perp}) + \dots \right\},$$

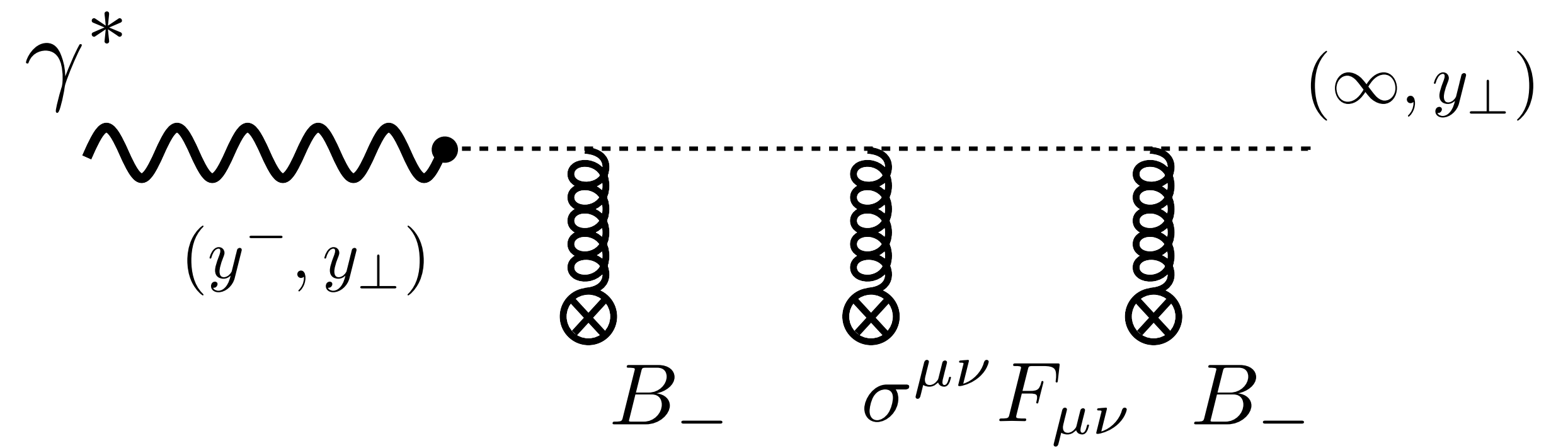
Expansion on an arbitrary piecewise contour is straightforward using simple computation relations (exact)

$$\begin{aligned} & P_k(x^-)U(x^-, y^-) - U(x^-, y^-)P_k(y^-) \\ &= g \int_{y^-}^{x^-} dz^- U(x^-, z^-) F_{-k}(z^-) U(z^-, y^-) \end{aligned}$$

$$U(x^-, y^-) = \mathcal{P} \exp \left\{ ig \int_{y^-}^{x^-} dz^- B_-(z^-) \right\}$$

This kind of relations allow to construct the expansion of the propagators in the background field in a gauge covariant way to any necessary order. The higher terms of the expansion have a structure similar to the leading terms $\Rightarrow$ allows automatization

- 1) Choose a contour
- 2) Construct an expansion using the commutation relations



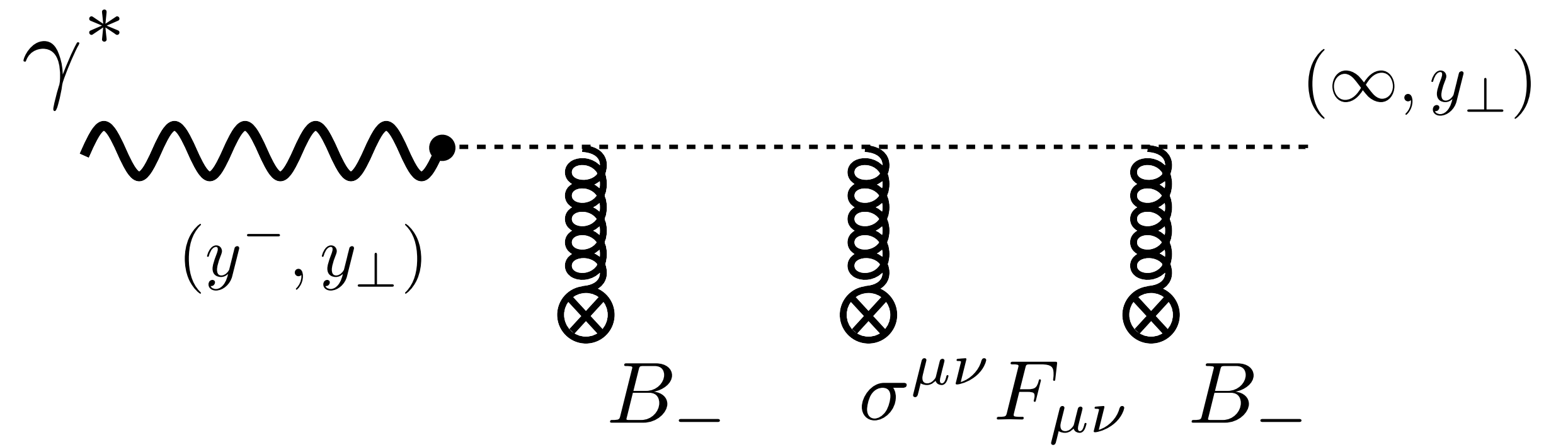
$$\lim_{k^2 \rightarrow 0} k^2 (k | \frac{1}{P^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y) \quad \longleftarrow \quad \text{quantum and background degrees of freedom are mixed}$$

$$= e^{iky} \left(U(\infty, y^-; y_\perp) + \frac{ig}{4k^-} \int_{y^-}^{\infty} dz^- U(\infty, z^-; y_\perp) \sigma^{\mu\nu} F_{\mu\nu}(z^-, y_\perp) U(z^-, y^-; y_\perp) \right. \\ \left. + \frac{igk_k}{k^-} \int_{y^-}^{\infty} dz^- (z^- - y^-) U(\infty, z^-; y_\perp) F_{-k}(z^-, y_\perp) U(z^-, y^-; y_\perp) + \dots \right)$$

Mukherjee, Skokov, Tarasov, Tiwari, Yao, 2602.15137

$$\propto \text{quantum} \otimes \text{background}$$

- 1) Choose a contour
- 2) Construct an expansion using the commutation relations

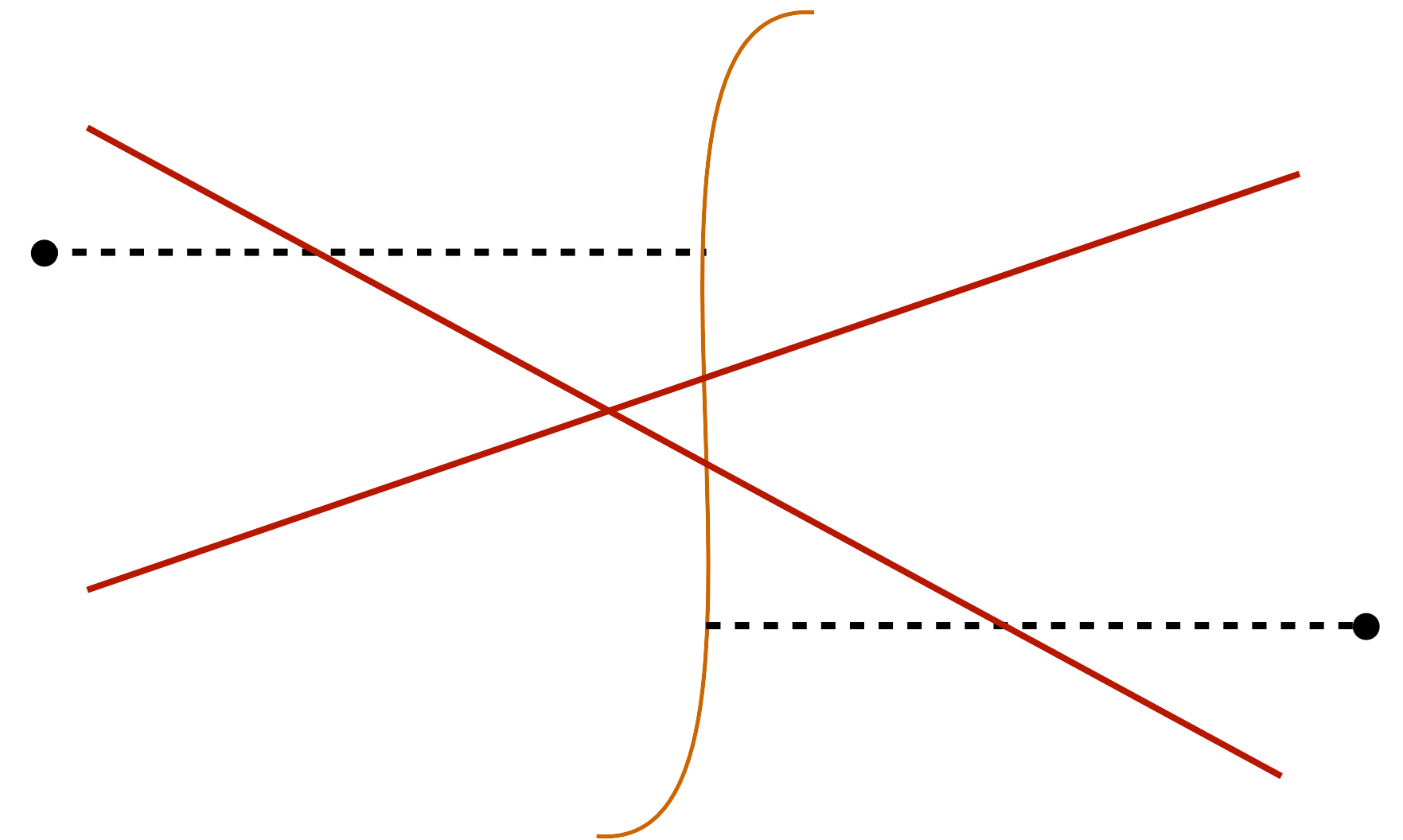
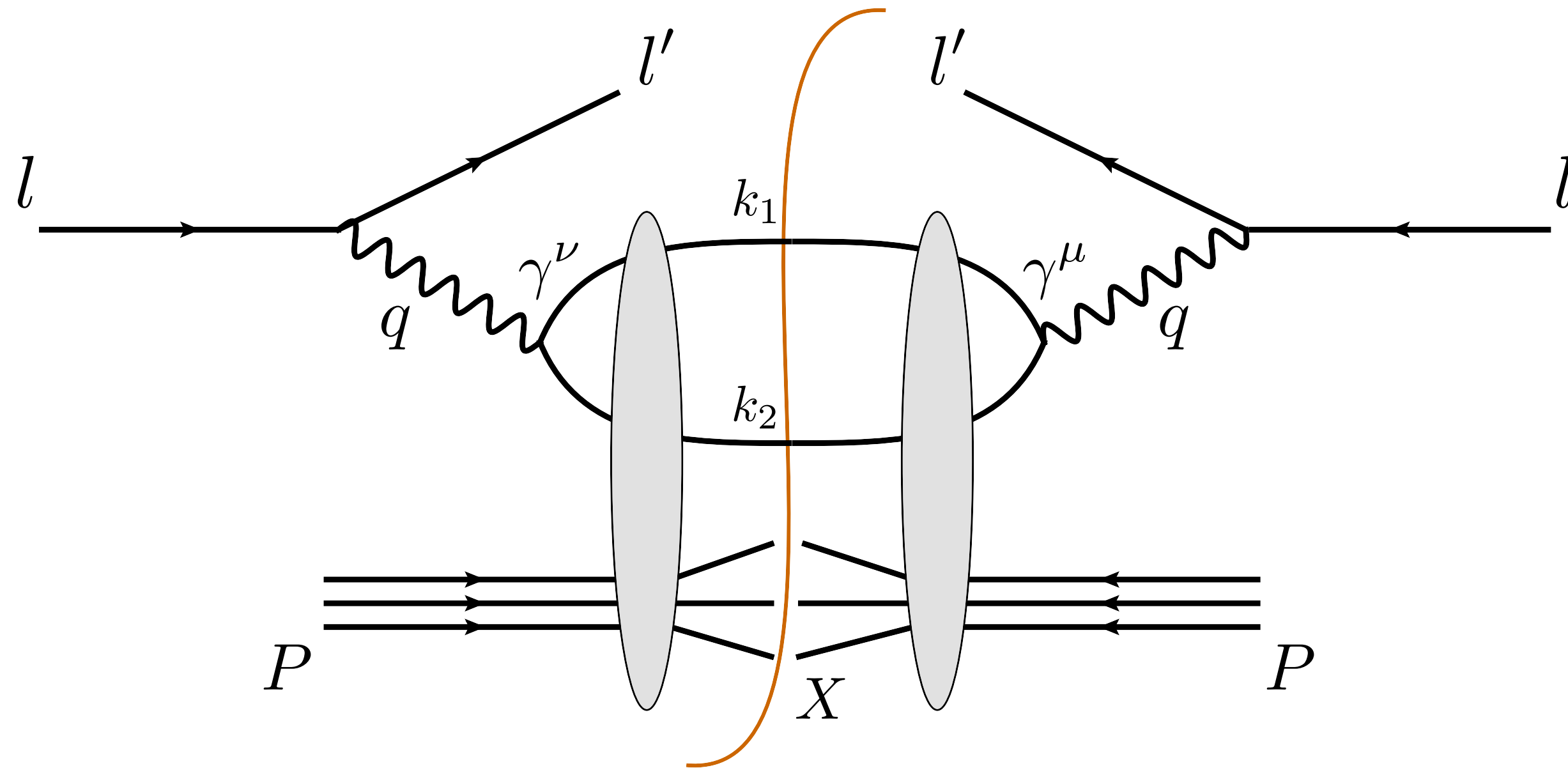


$$\begin{aligned}
 d\sigma_{00}^{\gamma^* p} \Big|_{\text{lead. power}}^{\text{back-to-back}} &= \sum_f \frac{2\pi e_f^2 g^2 \alpha_{\text{EM}} x}{Q^2} \left(\frac{Q}{q^-} \right)^2 \frac{dk_1^- d^2 k_{1\perp}}{(2\pi)^3 2k_1^-} \Big|_{k_1^2=0} \frac{dk_2^- d^2 k_{2\perp}}{(2\pi)^3 2k_2^-} \Big|_{k_2^2=0} \\
 &\times 2\pi \delta(q^- - k_1^- - k_2^-) 2\bar{P}_k \bar{P}_m \frac{32z^3 \bar{z}^3 (q^-)^4}{(\bar{P}_\perp^2 + z\bar{z}Q^2)^4} \int d^2 x_\perp e^{i(k_{1\perp} + k_{2\perp} - q_\perp)x_\perp} \int_{-\infty}^{\infty} dz^- e^{-i(k_1^+ + k_2^+ - q^+)z^-} \\
 &\times \langle P, S | F_{-k}^a(z^-, x_\perp) U^{ac}(z^-, \infty; x_\perp) U^{cd}(x_\perp, 0_\perp; \infty) U^{db}(\infty, 0^-; 0_\perp) F_{-m}^b(0^-, 0_\perp) | P, S \rangle .
 \end{aligned}$$

$$\propto \text{quantum} \otimes \text{background}$$

Suppose we have a CGC background. $B_-(x^+, x^-, x_\perp) \sim \lambda \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp)$

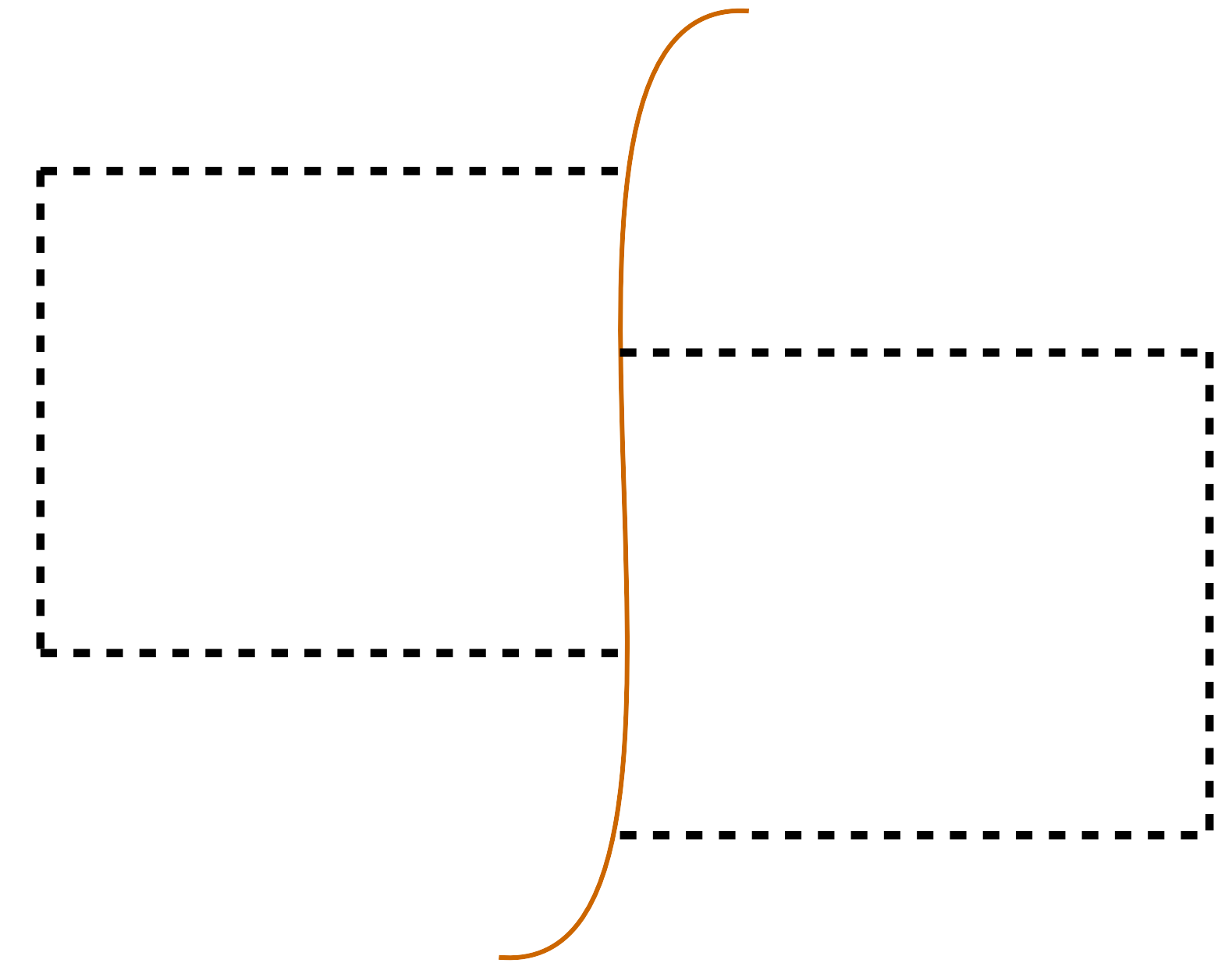
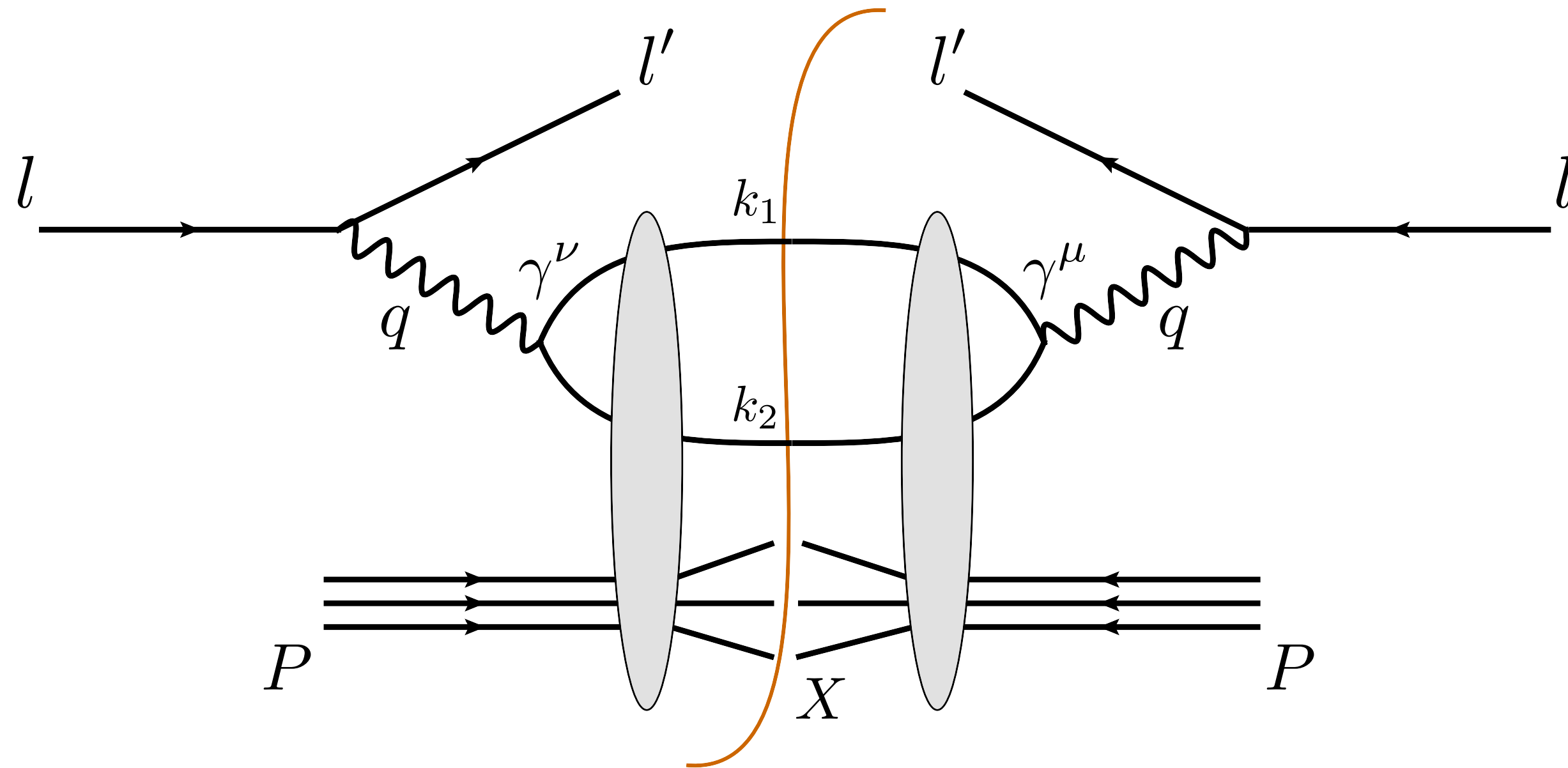
$$\lambda \rightarrow \infty$$



Expansion onto the TMD contour requires an infinite number of terms (all collinear terms do contribute)

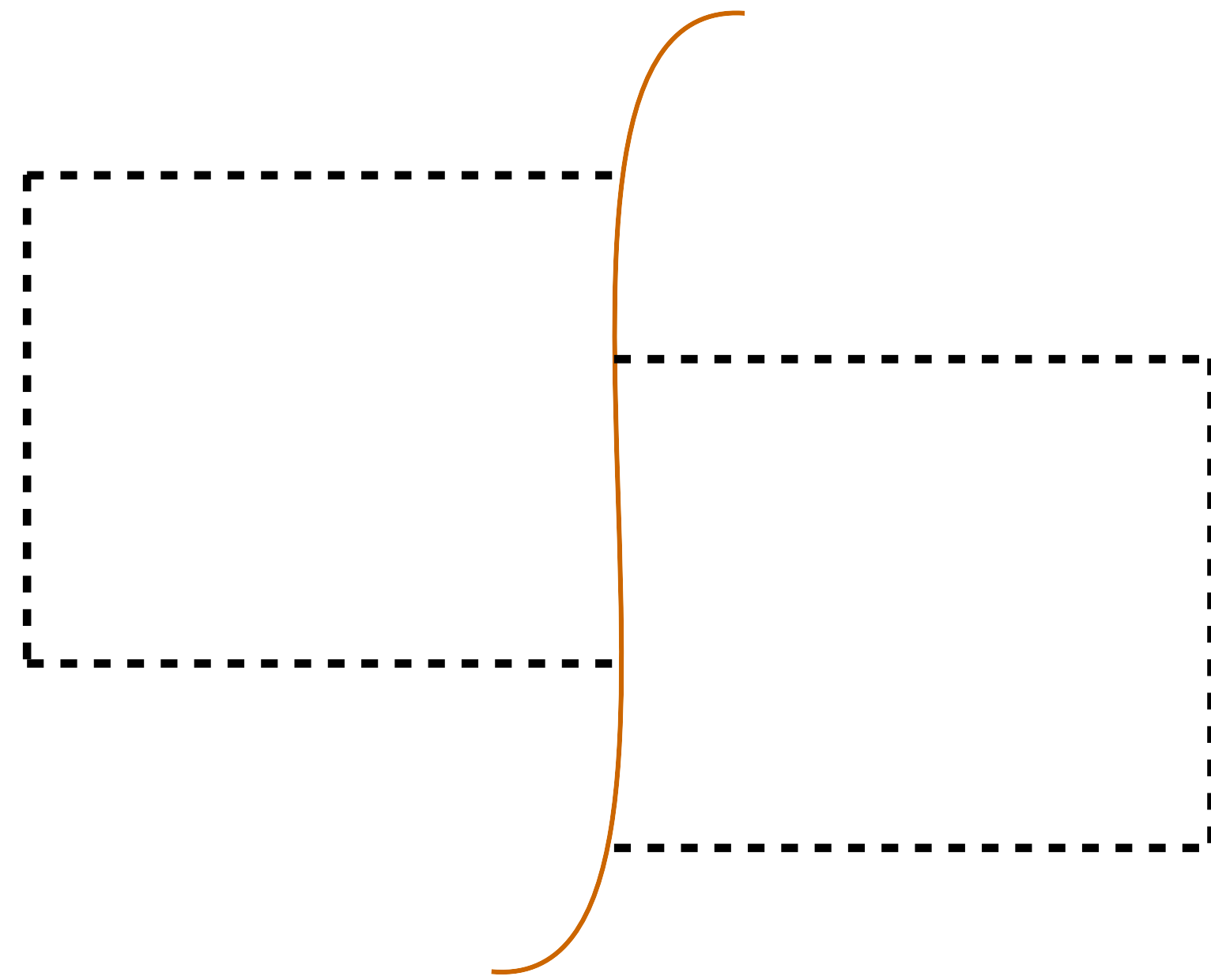
Suppose we have the CGC background. $B_-(x^+, x^-, x_\perp) \sim \lambda \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp)$

$$\lambda \rightarrow \infty$$



The higher terms of the expansion are suppressed by the higher orders of the expansion parameter λ^{-1}

We construct expansion for the background field
beyond the CGC approximation

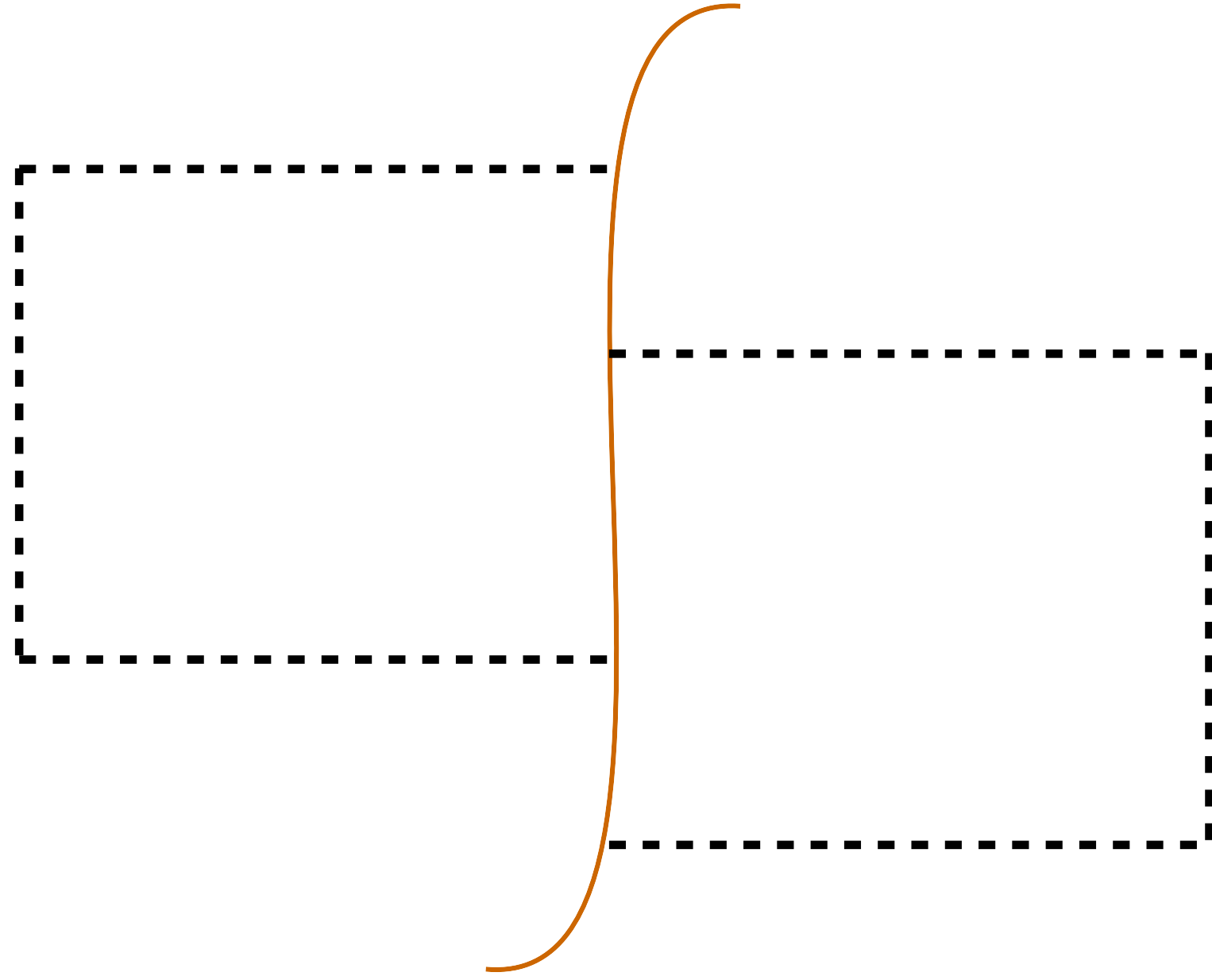


CGC field $\cdots \rightarrow B_{-}(x^{+}, x^{-}, x_{\perp}) \sim \lambda \tilde{B}_{-}(\lambda^{-1} x^{+}, \lambda x^{-}, x_{\perp});$
 $\cdots \rightarrow B_i(x^{+}, x^{-}, x_{\perp}) \sim \tilde{B}_i(\lambda^{-1} x^{+}, \lambda x^{-}, x_{\perp});$
 $B_{+}(x^{+}, x^{-}, x_{\perp}) \sim \lambda^{-1} \tilde{B}_{-}(\lambda^{-1} x^{+}, \lambda x^{-}, x_{\perp}),$

extension of the model, spin effects,
sub-eikonal corrections

The transverse field component of the background
field at small x starts to contribute already at the
eikonal order of the computation through
contribution of the transverse gauge links at the
spatial infinity

We construct expansion for the background field
beyond the CGC approximation



$$\begin{aligned}
 d\sigma_{00}^{\gamma^* p} \Big|_{\text{eik.}} &= \sum_f \frac{2\pi e_f^2 \alpha_{\text{EM}} x}{V_3 Q^2} \left(\frac{Q}{q^-} \right)^2 \frac{dk_1^- d^2 k_{1\perp}}{(2\pi)^3 2k_1^-} \Big|_{k_1^2=0} \frac{dk_2^- d^2 k_{2\perp}}{(2\pi)^3 2k_2^-} \Big|_{k_2^2=0} \delta(q^- - k_1^- - k_2^-) 32 z^3 \bar{z}^3 (q^-)^4 \\
 &\times \int d^2 \omega_\perp e^{ik_{1\perp} \omega_\perp} \int d^2 \omega'_\perp e^{ik_{2\perp} \omega'_\perp} K_0(\sqrt{z\bar{z}Q^2} |\omega_\perp - \omega'_\perp|) \int d^2 z_\perp e^{-ik_{1\perp} z_\perp} \int d^2 z'_\perp e^{-ik_{2\perp} z'_\perp} \\
 &\times K_0(\sqrt{z\bar{z}Q^2} |z_\perp - z'_\perp|) \mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp).
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp) &\equiv \langle P, S | \text{tr}_c \left(U(\infty, -\infty; \omega'_\perp) U(\omega'_\perp, \omega_\perp; -\infty) U(-\infty, \infty; \omega_\perp) - 1 \right) U(\omega_\perp, z_\perp; \infty) \\
 &\times \left(U(\infty, -\infty; z_\perp) U(z_\perp, z'_\perp; -\infty) U(-\infty, \infty; z'_\perp) - 1 \right) U(z'_\perp, \omega'_\perp; \infty) | P, S \rangle.
 \end{aligned}$$

The transverse field component in the quark propagator in the small-x background

$$\frac{i}{P^2 + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}$$

$$F_{-i} = -\partial_i B_- + \partial_- B_i - ig[B_-, B_i]$$

$$\propto \lambda \quad \propto \lambda \quad \propto \lambda$$

eikonal order

$$\begin{aligned} B_-(x^+, x^-, x_\perp) &\sim \lambda \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp); \\ B_i(x^+, x^-, x_\perp) &\sim \tilde{B}_i(\lambda^{-1}x^+, \lambda x^-, x_\perp); \\ B_+(x^+, x^-, x_\perp) &\sim \lambda^{-1} \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp), \end{aligned}$$

δ -like form of a small field component, the derivative is enhanced. Transverse component contribution in the eikonal small-x simulations?

Thank you for your attention!