Feasibility of Extracting OAM Distributions via Dijet/Dihadron Production at EIC

Brandon Manley
Ohio State University

Based on:
Kovchegov, **BM**, JHEP 02 (2024) 060
Kovchegov, **BM**, PRD 111 (2025) 5
JAM small-x, in preparation

Saturation, Diffraction, and Multihadron Production at EIC, July 16 , 2026

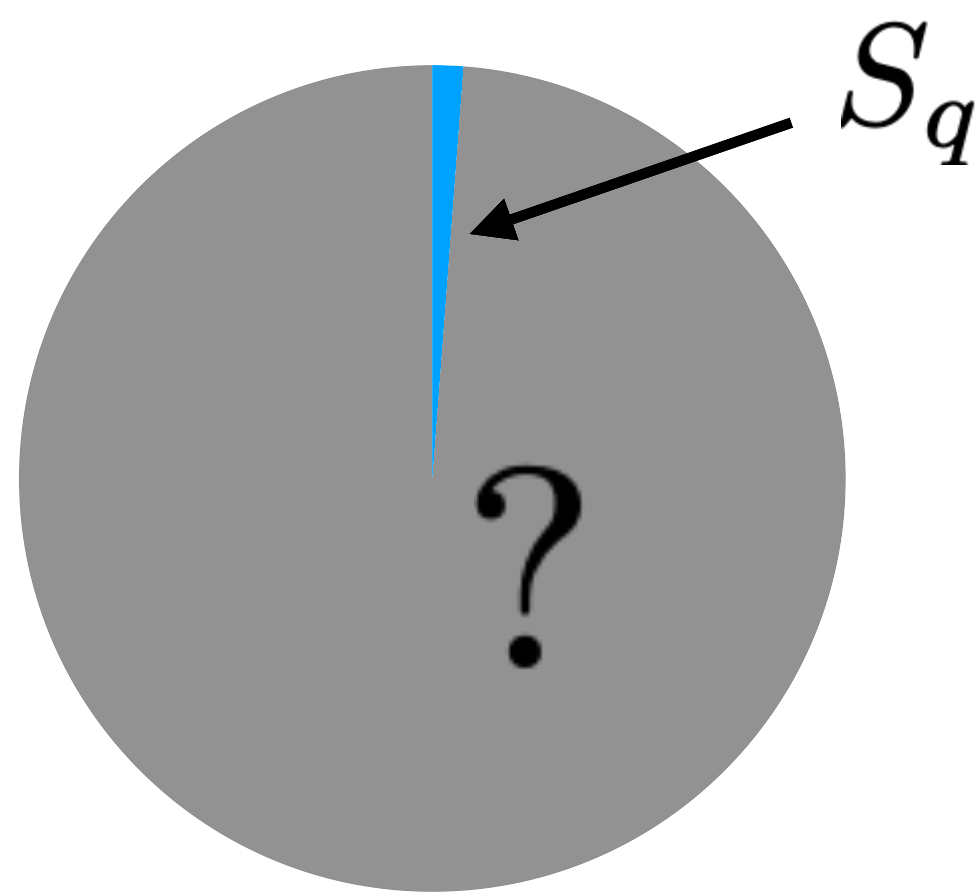
Overview

- Proton spin puzzle
- Review of polarized small-x toolkit
- Orbital angular momentum (OAM) at small x: moment amplitudes and evolution
- Elastic dijet production as a probe for OAM
- Feasibility of elastic dijets at EIC

Proton spin puzzle: Origins

- 1988 EMC experiment: quarks contribute very little to proton spin

$$S_q^{\mathrm{EMC}} = 0.006 \pm 0.058 \pm 0.117$$



European Muon Collaboration, J. Ashman et al., Phys. Lett. B206, 364 (1988)

Proton spin puzzle: Origins

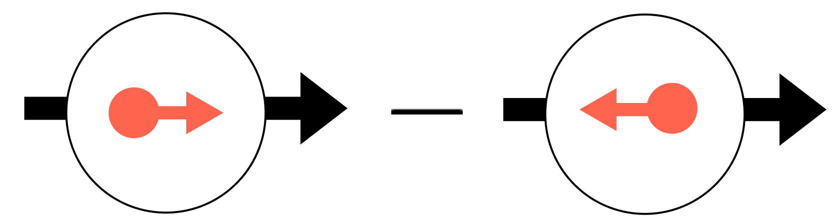
Jaffe-Manohar sum rule

- Theory response: spin sum rules

$$S_q + S_g + L_q + L_g = \frac{1}{2}$$

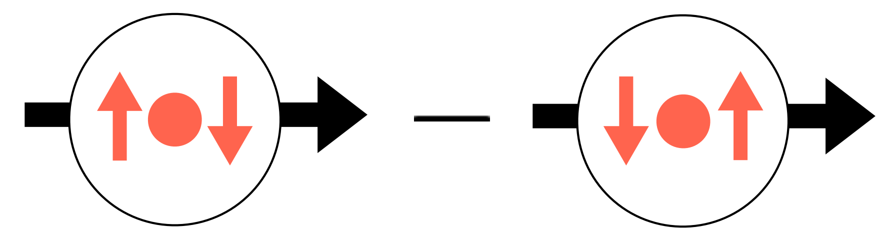
$$S_q = \int_0^1 dx\, \frac{1}{2}\Delta\Sigma(x, Q^2)$$

Quark helicity PDF



$$S_g = \int_0^1 dx\, \Delta g(x, Q^2)$$

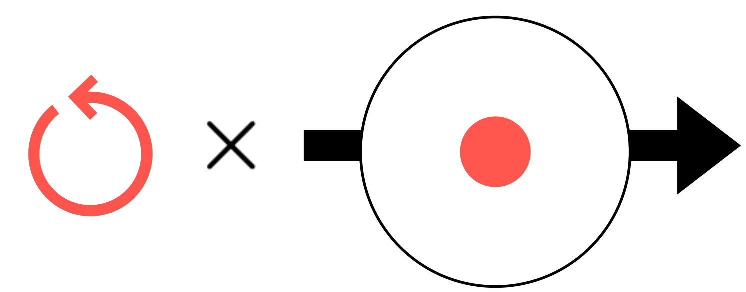
Gluon helicity PDF



$$L_q = \int_0^1 dx \sum_q L_{q+\bar{q}}(x, Q^2)$$

$$L_g = \int_0^1 dx\, L_g(x, Q^2)$$

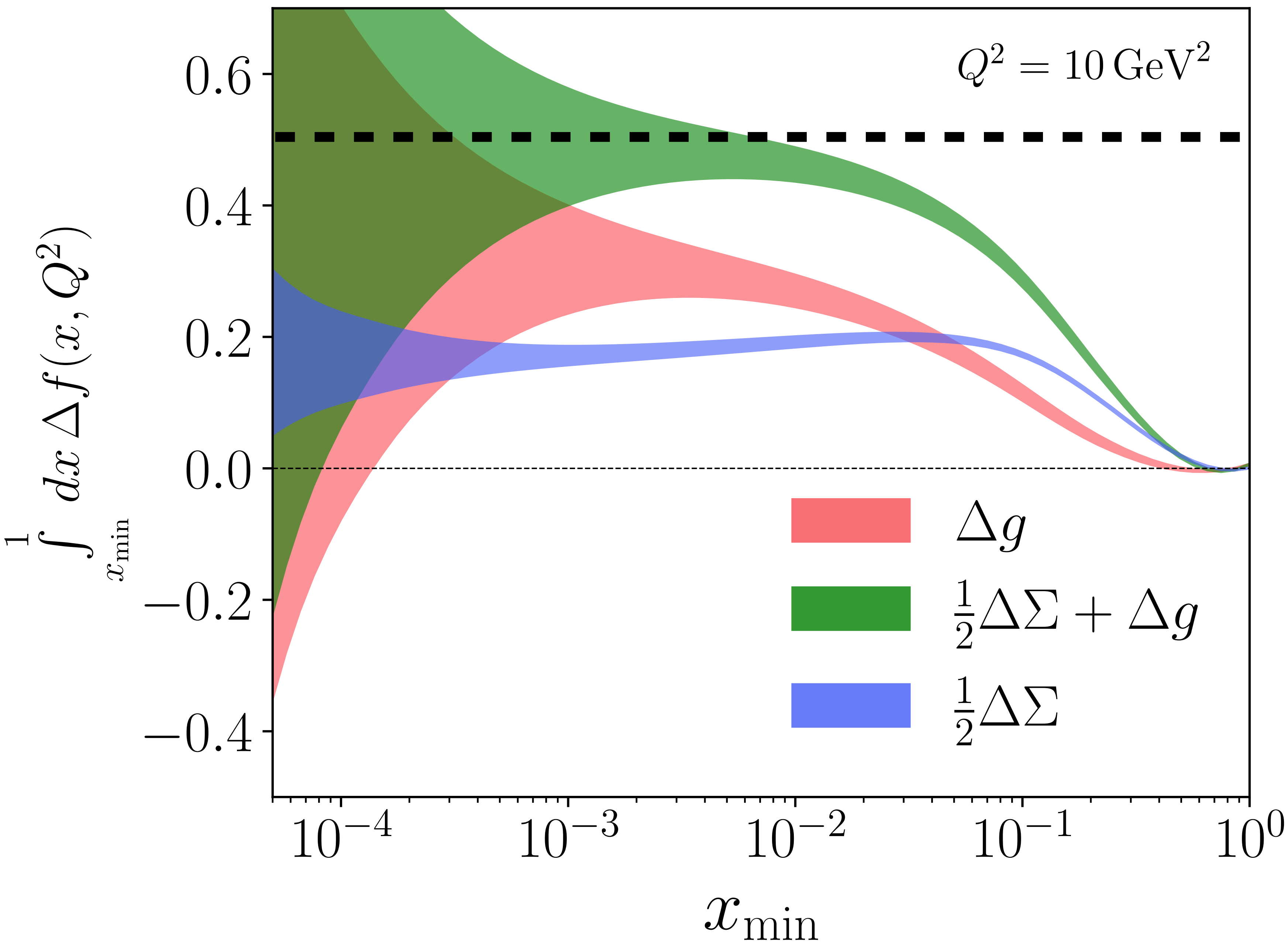
Quark and gluon orbital angular momentum (OAM) distributions



Jaffe and Manohar, Nucl. Phys. B337 509 (1990)

Proton spin puzzle: Current status

- Current status of the puzzle (from a recent collinear hPDF extraction)



- Uncertainties large in small x region (inaccessible to experiment since $x \sim 1/\text{energy}$)
- **No OAM extraction from experiments**

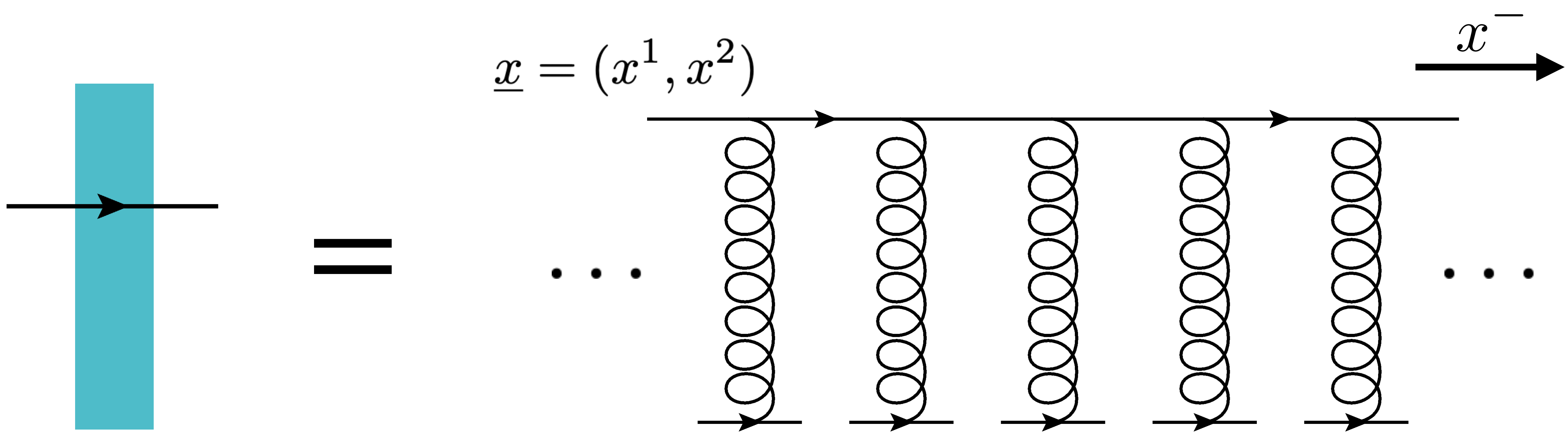
Borsa et al, Phys.Rev.Lett. 133 (2024) 15

Polarized Small x Tools

Wilson lines

- Scattering of quark on proton at small x: light-cone Wilson lines

$$V_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp \left[ig \int_{a^-}^{b^-} dx^- \, A^+(0^+, x^-, \underline{x}) \right]$$

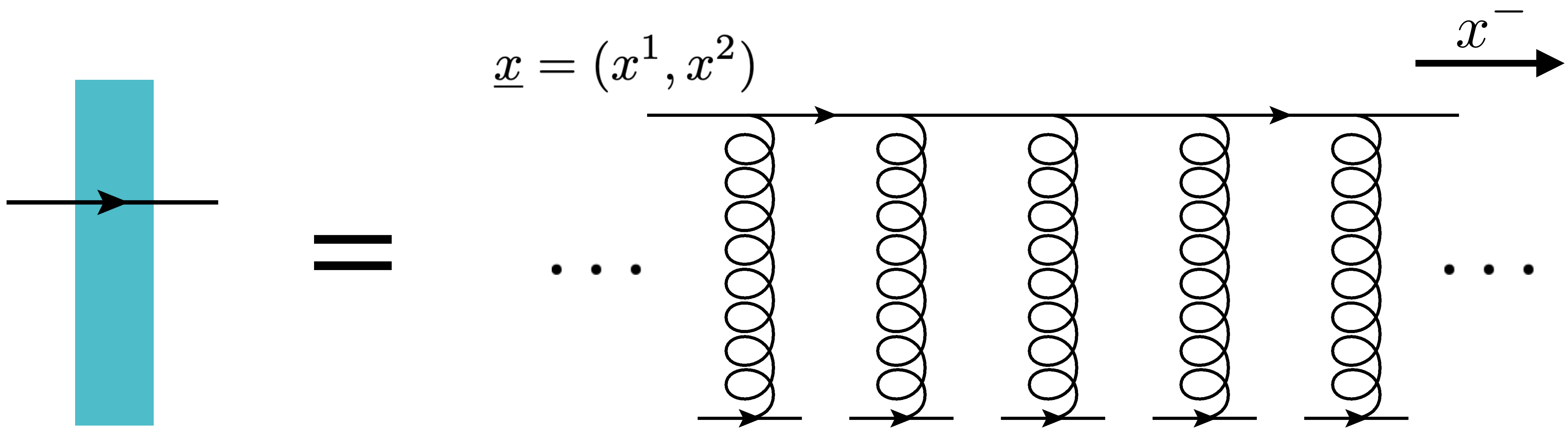


(fundamental) for adjoint, $A^+ \to \mathcal{A}^+ \quad V \to U$ $\qquad V_{\underline{x}} = V_{\underline{x}}[\infty, -\infty]$

Wilson lines

- Scattering of quark on proton at small x: light-cone Wilson lines

$$V_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp\left[ig \int\limits_{a^-}^{b^-} dx^- \, A^+(0^+, x^-, \underline{x}) \right]$$



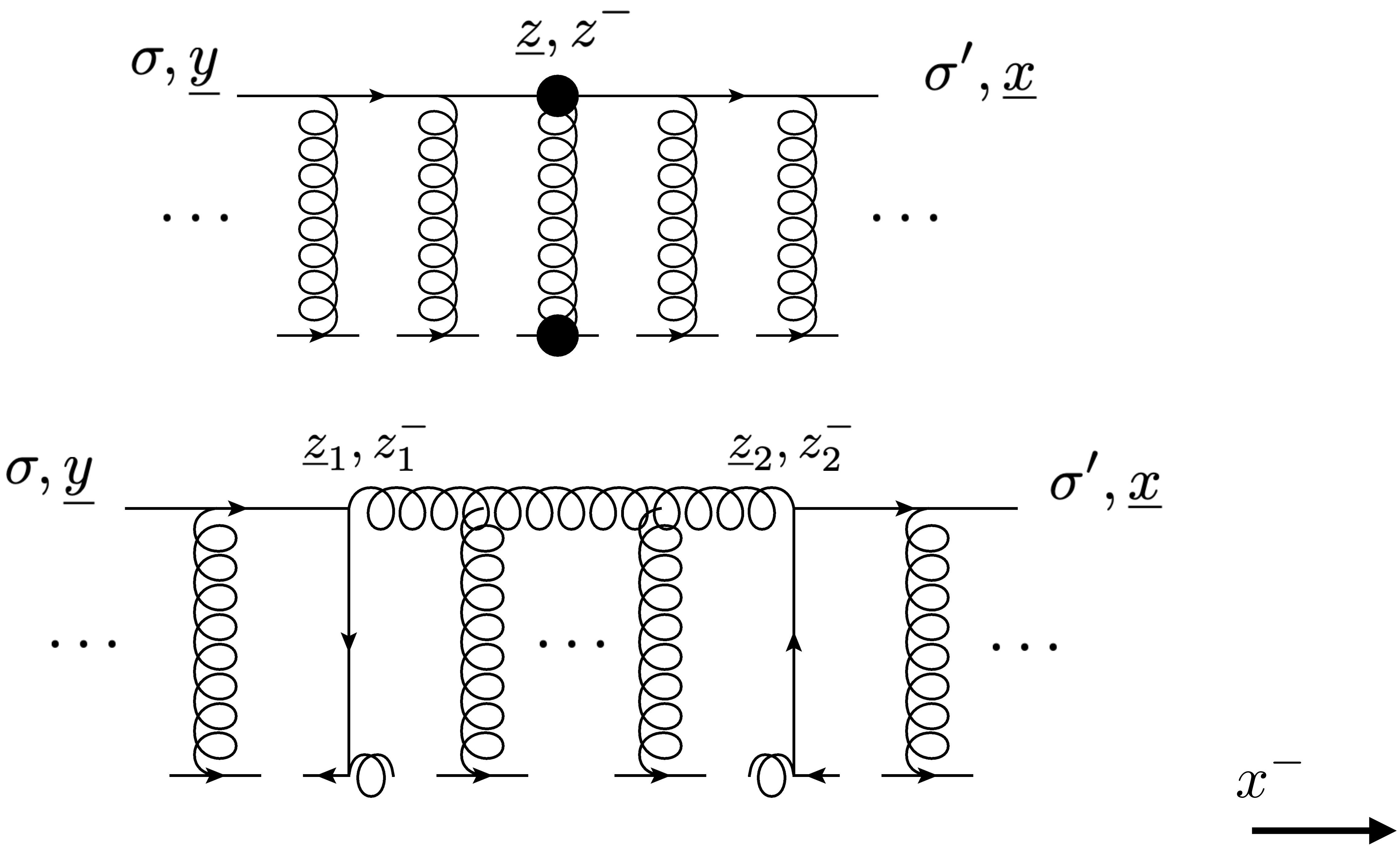
only color rotation, no spin dependence!

$V_{\underline{x}} = V_{\underline{x}}[\infty, -\infty]$

Altinoluk et al, *JHEP* 07 (2014) 068
Kovchegov, Pitonyak, Sievert, *JHEP* 10 (2017) 198
Kovchegov, Sievert, *Phys.Rev.D* 99 (2019) 5
Chirilli, *JHEP* 01 (2019) 118
Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095

Polarized Wilson Lines

- Semi-infinite Wilson lines sandwiching **spin-dependent** operators
  → **sub-eikonal:** suppressed by one power of x (or $1/s$)



Polarized Wilson lines

- Multiple operators exist at sub-eikonal order → type 1 and type 2

$$V_{\underline{x},\underline{y};\sigma',\sigma} = \delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\ V_{\underline{x}} + \sigma\,\delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\left[V_{\underline{x}}^{\mathrm{q}[1]} + V_{\underline{x}}^{\mathrm{G}[1]}\right] + \delta_{\sigma\sigma'}\left[\delta^2(\underline{x}-\underline{y})\,V_{\underline{x}}^{\mathrm{q}[2]} + V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}\right]$$

Kovchegov, Pitonyak, Sievert (1706.04236)
Kovchegov, Sievert (1808.09010)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

Polarized Wilson lines

- Multiple operators exist at sub-eikonal order → type 1 and type 2

$$V_{\underline{x},\underline{y};\sigma',\sigma} = \delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\; V_{\underline{x}} + \sigma\,\delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\left[V_{\underline{x}}^{\mathrm{q}[1]} + V_{\underline{x}}^{\mathrm{G}[1]}\right] + \delta_{\sigma\sigma'}\left[\delta^2(\underline{x}-\underline{y})\, V_{\underline{x}}^{\mathrm{q}[2]} + V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}\right]$$

$$V_{\underline{x}}^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\,s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^-\, V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\, \left[\gamma^+\gamma^5\right]_{\alpha\beta}\, \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty]$$

$$V_{\underline{x}}^{\mathrm{G}[1]} = \frac{i\,g\,P^+}{s}\int\limits_{-\infty}^{\infty} dx^-\, V_{\underline{x}}[\infty, x^-]\, F^{12}(x^-, \underline{x})\, V_{\underline{x}}[x^-, -\infty]$$

$$V_{\underline{x}}^{\mathrm{q}[2]} = -\frac{g^2 P^+}{2\,s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^-\, V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\, \left[\gamma^+\right]_{\alpha\beta}\, \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty]$$

$$V_{\underline{x},\underline{y}}^{\mathrm{G}[2]} = -\frac{i\,P^+}{s}\int\limits_{-\infty}^{\infty} dz^-\, d^2z\; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z})\, \overleftarrow{D}^i(z^-, \underline{z})\, D^i(z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z})$$

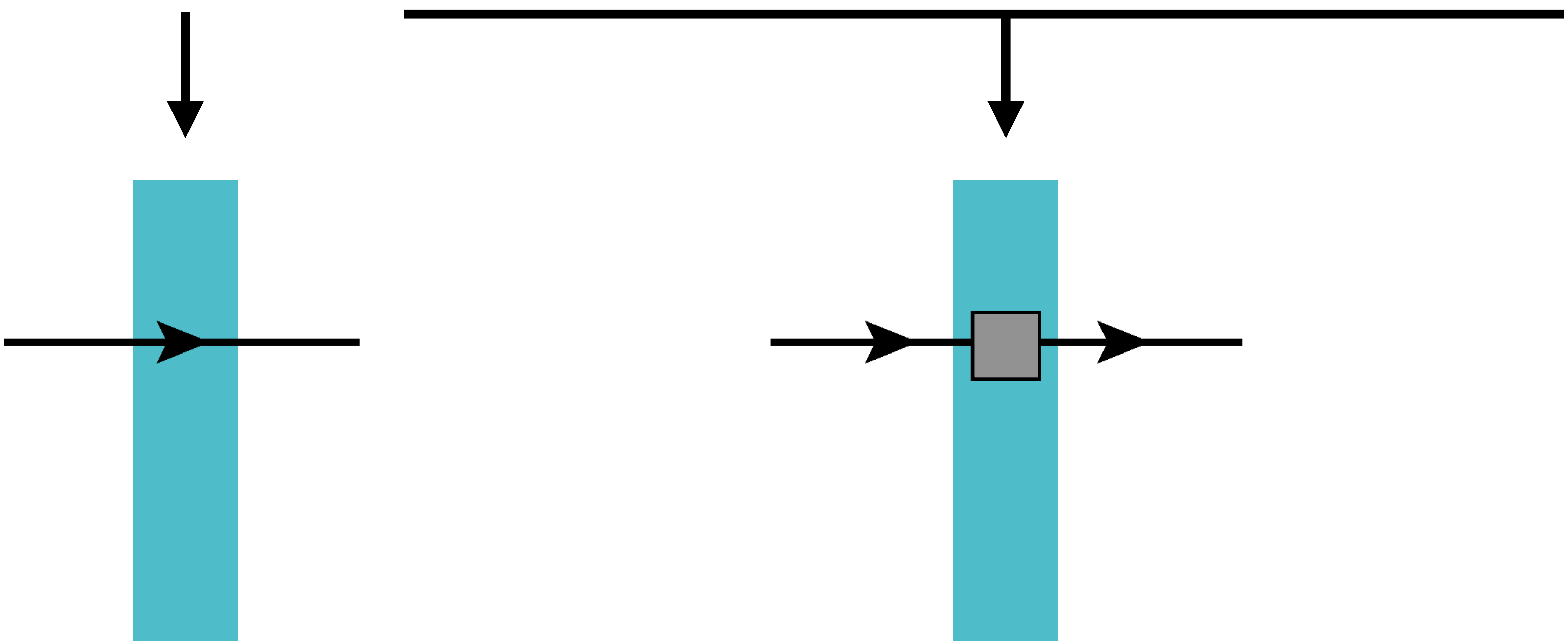
$$V_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{q}[1]} + V_{\underline{x}}^{\mathrm{G}[1]}$$

Kovchegov, Pitonyak, Sievert (1706.04236)
Kovchegov, Sievert (1808.09010)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

Polarized Wilson lines

- Multiple operators exist at sub-eikonal order $\rightarrow$ type 1 and type 2

$$V_{\underline{x},\underline{y};\sigma',\sigma} = \delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\ V_{\underline{x}} + \sigma\,\delta_{\sigma\sigma'}\delta^2(\underline{x}-\underline{y})\left[V_{\underline{x}}^{\mathrm{q}[1]} + V_{\underline{x}}^{\mathrm{G}[1]}\right] + \delta_{\sigma\sigma'}\left[\delta^2(\underline{x}-\underline{y})\,V_{\underline{x}}^{\mathrm{q}[2]} + V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}\right]$$



$$V_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{q}[1]} + V_{\underline{x}}^{\mathrm{G}[1]}$$

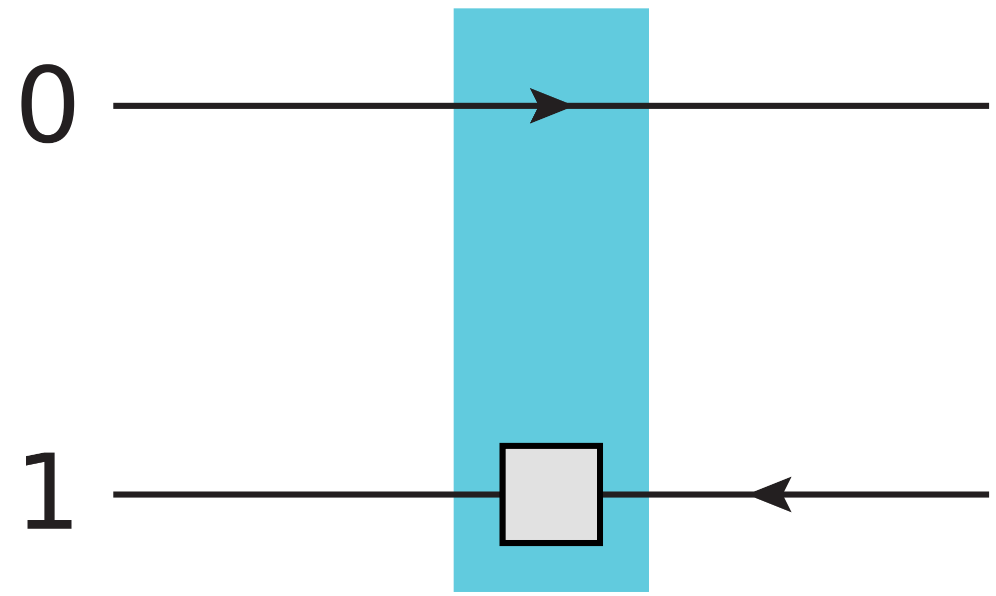
Kovchegov, Pitonyak, Sievert (1706.04236)
Kovchegov, Sievert (1808.09010)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

Polarized dipole amplitudes

- S-matrix for scattering of dipole with 1 regular and 1 polarized Wilson line

$$Q_{10}(s) = \frac{s}{2N_c} \mathrm{Re} \left\langle \mathrm{T}\,\mathrm{tr} \left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[1]\dagger} + V_{\underline{1}}^{\mathrm{pol}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle (s)$$

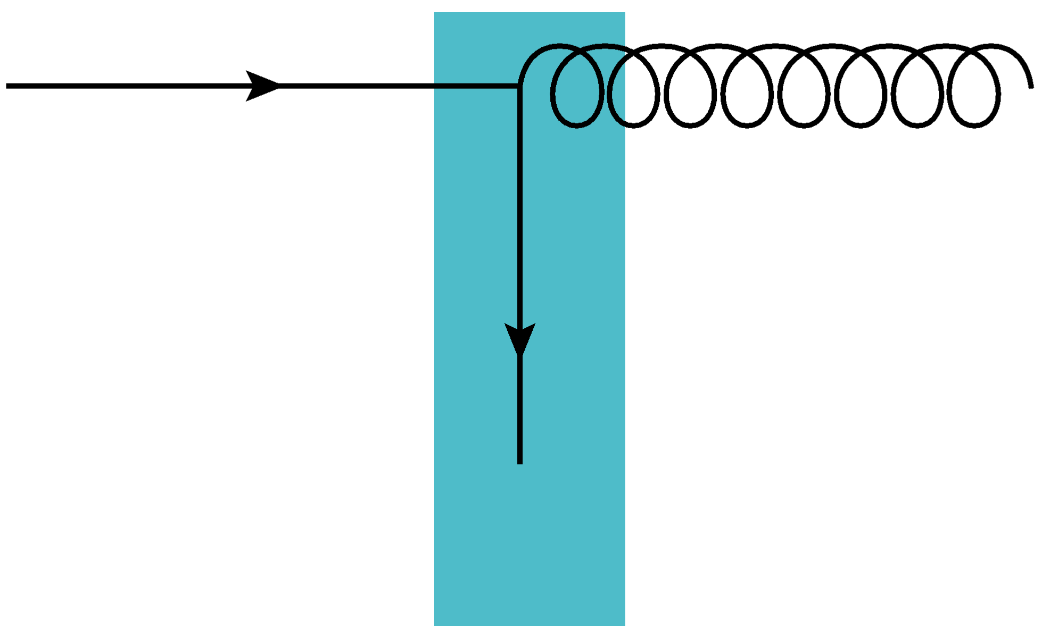
$$G^i_{10}(s) = \frac{s}{2N_c} \mathrm{Re} \left\langle \mathrm{T}\,\mathrm{tr} \left[V_{\underline{0}} V_{\underline{1}}^{i\mathrm{G}[2]\dagger} + V_{\underline{1}}^{i\mathrm{G}[2]} V_{\underline{0}}^{\dagger} \right] \right\rangle (s)$$



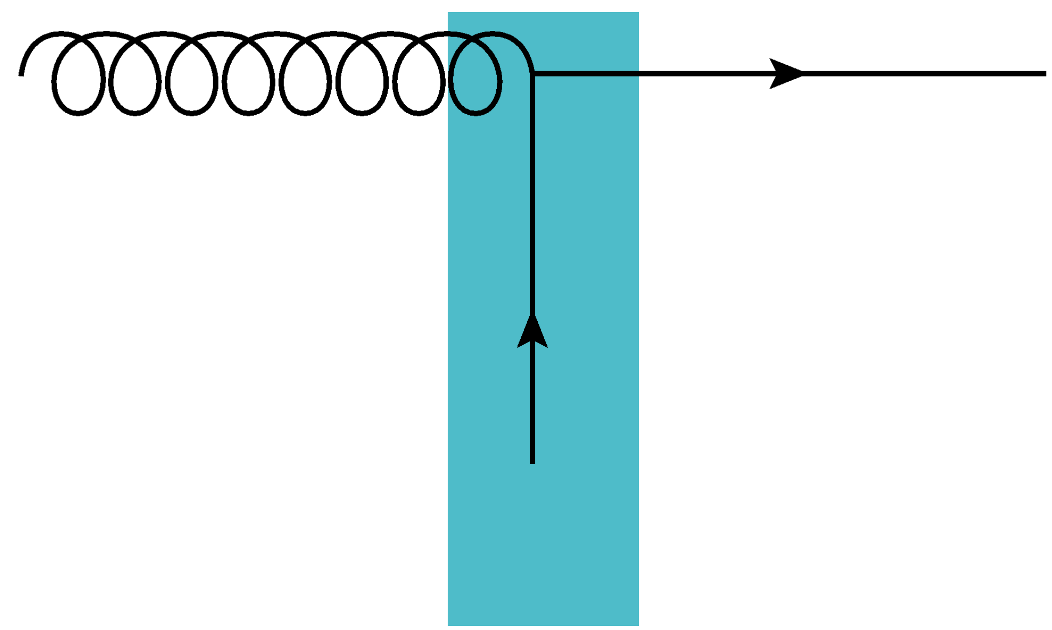
Kovchegov, Pitonyak, Sievert, *JHEP* 10 (2017) 198
Kovchegov, Sievert, *Phys.Rev.D* 99 (2019) 5
Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095

Polarized transition operators

- At sub-eikonal order, also have "transition" operators which do not preserve identity of projectile:



quark → gluon



gluon → quark

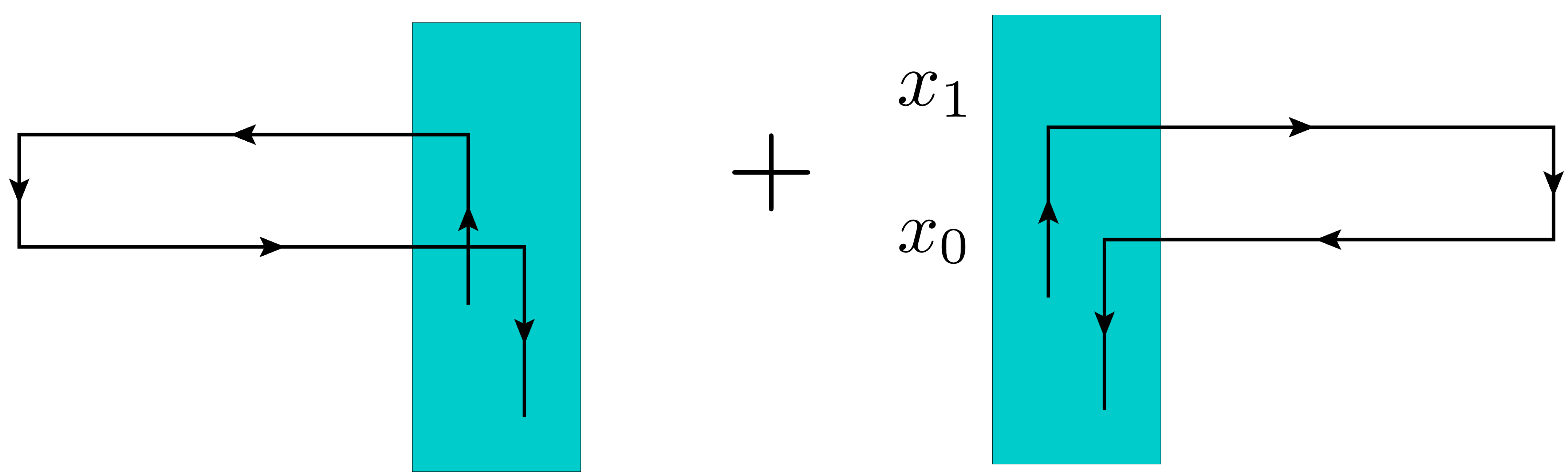
Chirilli, *JHEP* 06 (2021) 096
Borden, Kovchegov, Li, *JHEP* 09 (2024) 037

Polarized transition amplitude

- Polarized amplitude from transition operators:

$$\widetilde{Q}_{10}(s) \equiv \left\langle \frac{g^2}{8P^+} \int\limits_{-\infty}^{\infty} dx_0^- \int\limits_{-\infty}^{\infty} dx_1^- \Bigg[\bar{\psi}(x_0^-, \underline{x}_0) \left(\frac{1}{2}\gamma^+\gamma^5\right) V_{\underline{0}}[x_0^-, \infty] V_{\underline{1}}[\infty, x_1^-] \psi(x_1^-, \underline{x}_1) \right.$$

$$\left. + \bar{\psi}(x_0^-, \underline{x}_0) \left(\frac{1}{2}\gamma^+\gamma^5\right) V_{\underline{0}}[x_0^-, -\infty] V_{\underline{1}}[-\infty, x_1^-] \psi(x_1^-, \underline{x}_1) + \text{c.c.} \Bigg] \right\rangle(s)$$



Borden, Kovchegov, Li, *JHEP* 09 (2024) 037

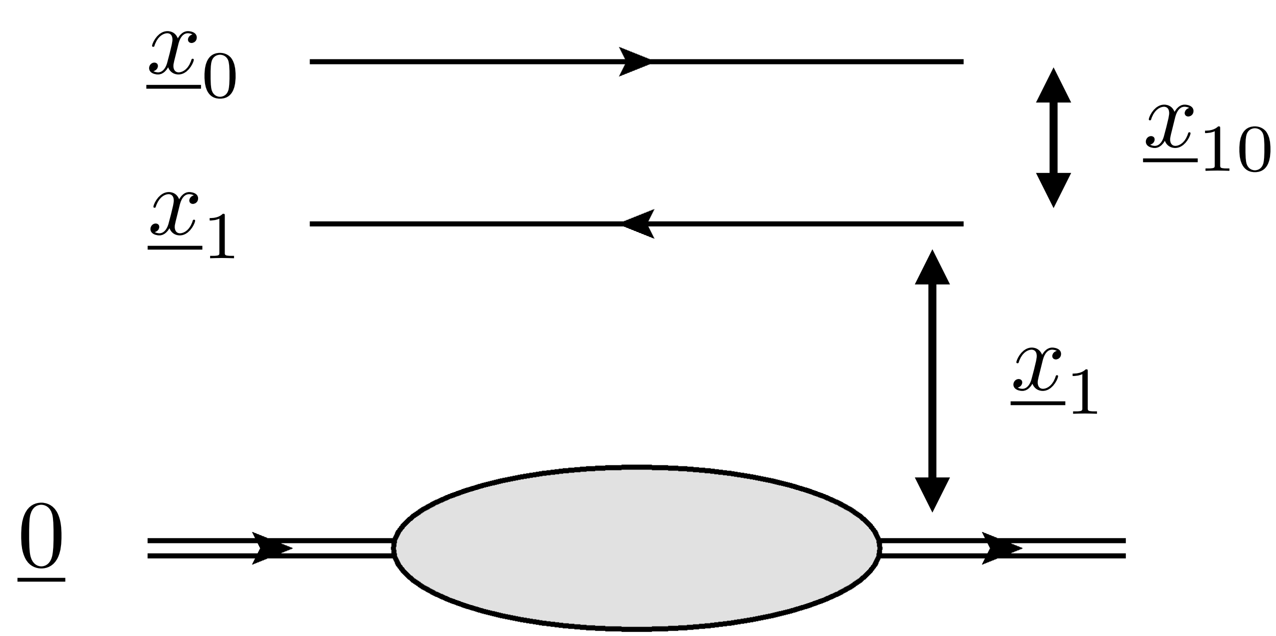
Integrated polarized amplitudes

- Helicity PDFs depend on **impact-parameter integrated** dipole amplitudes

$$\int d^2x_1\, Q_{10}(s) = Q(x_{10}^2, s)$$

$$\int d^2x_1\, G_{10}^i(s) = x_{10}^i\, G_1(x_{10}^2, s) + \epsilon^{ij}\, x_{10}^j\, G_2(x_{10}^2, s)$$

$$\int d^2x_1\, \widetilde{Q}_{10}(s) = \widetilde{Q}(x_{10}^2, s)$$



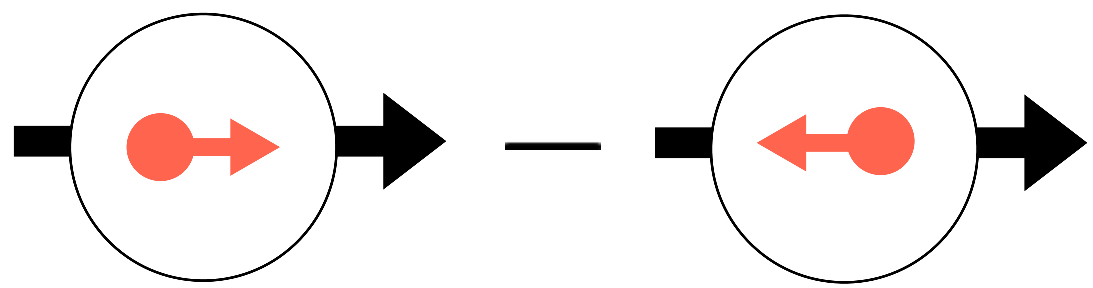
Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095
Borden, Kovchegov, Li, *JHEP* 09 (2024) 037

Helicity PDFs at small x

- Quark helicity PDF defined through Wigner function

$$\Delta\Sigma(x,Q^2) = \int \frac{d^2b_\perp\, db^-\, d^2k_\perp}{(2\pi)^3} \sum_q W^q_{LL}(k,b)\Big|_{k^+=xP^+}$$

$$=$$



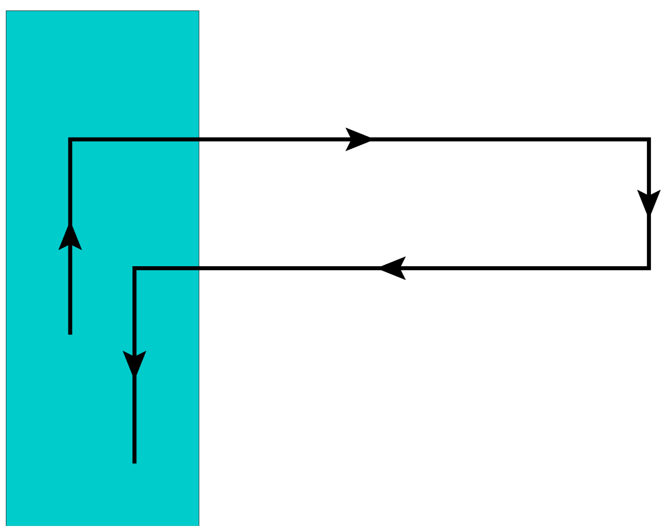
- Polarized quark/polarized nucleon SIDIS Wigner function

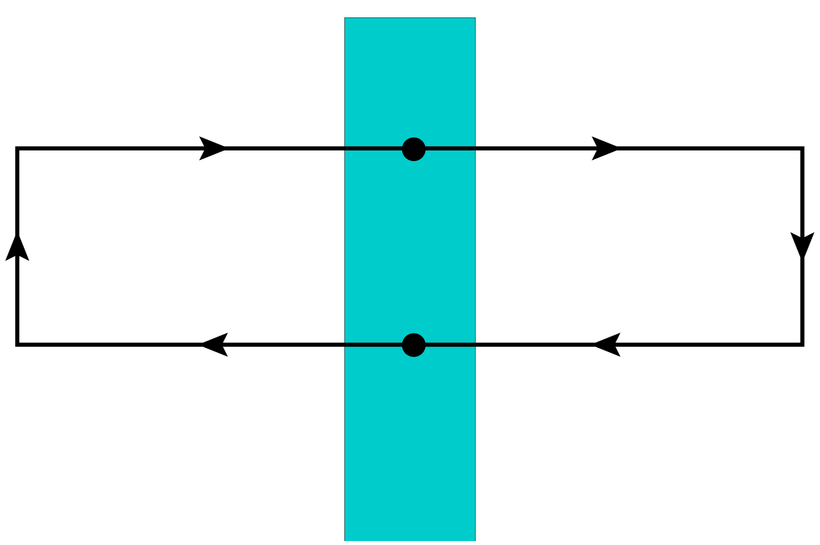
$$W^q_{LL}(k,b) = \int d^2r\, dr^-\, e^{ik\cdot r} \left\langle \bar{q}\left(b-\frac{1}{2}r\right)\mathcal{U}^{[+]}\left(\frac{1}{2}\gamma^+\gamma^5\right) q\left(b+\frac{1}{2}r\right)\right\rangle_{S_L}$$

Lorce, Pasquini (1106.0139)

Helicity PDFs at small x

- Start with operator definition, simplify for $x \ll 1$, result:

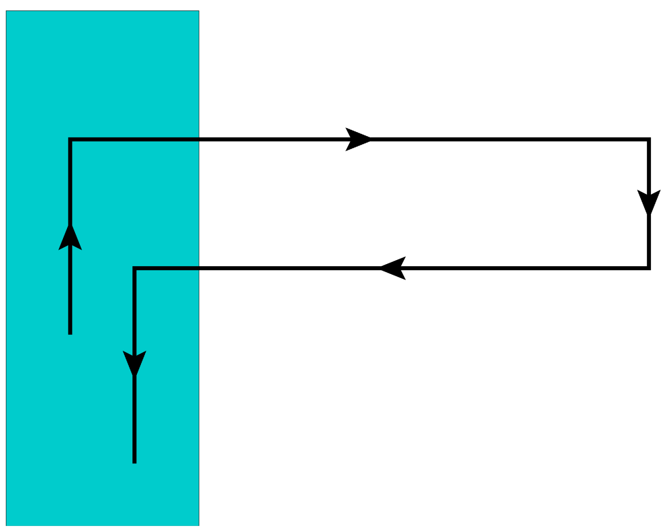


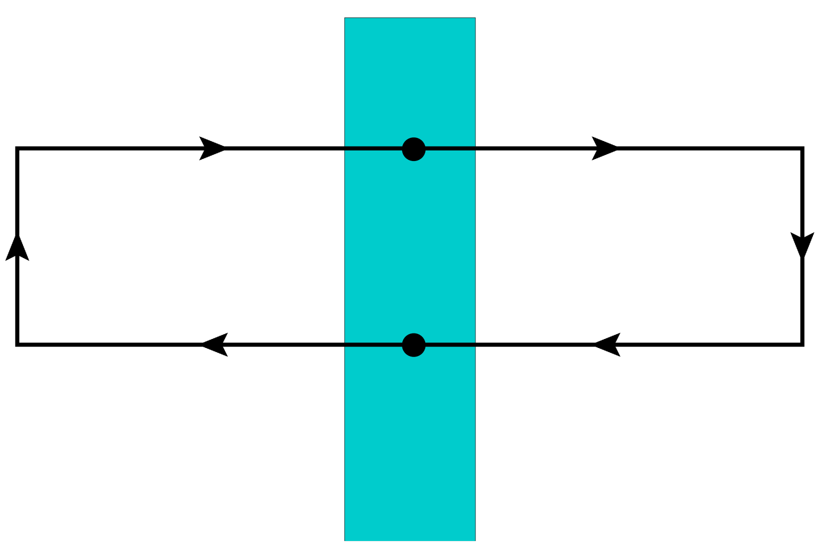


Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095
Borden, Kovchegov, Li, *JHEP* 09 (2024) 037

Helicity PDFs at small x

- Start with operator definition, simplify for $x \ll 1$, result:





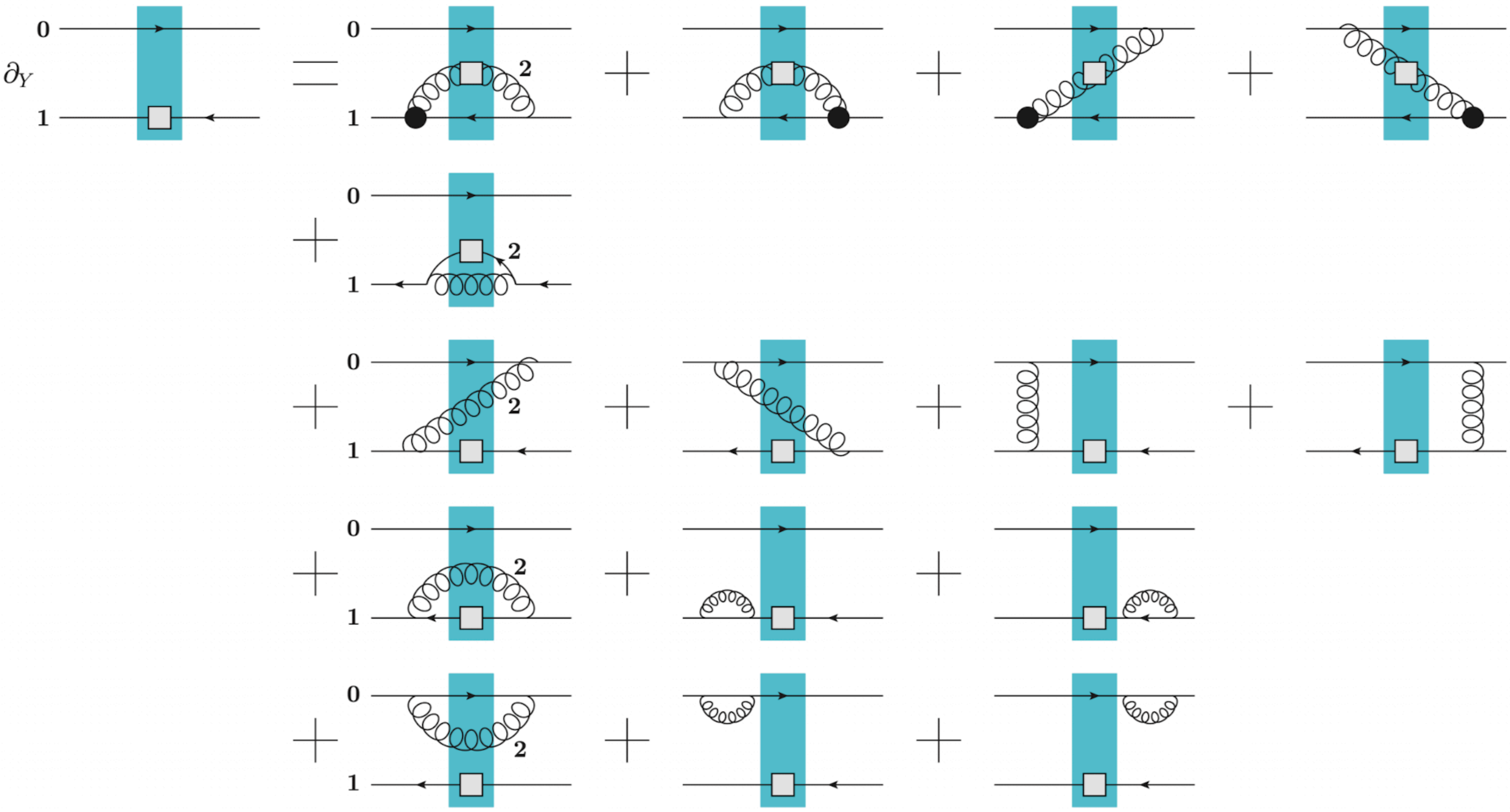
$$\Delta\Sigma(x, Q^2) = \frac{N_f}{\alpha_s \pi^2}\, \widetilde{Q}\left(x_{10}^2 = \frac{1}{Q^2}, s = \frac{Q^2}{x}\right)$$

$$\Delta g(x, Q^2) = \frac{2\, N_c}{\alpha_s \pi^2} G_2\left(x_{10}^2 = \frac{1}{Q^2}, s = \frac{Q^2}{x}\right)$$

Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095
Borden, Kovchegov, Li, *JHEP* 09 (2024) 037

Helicity evolution

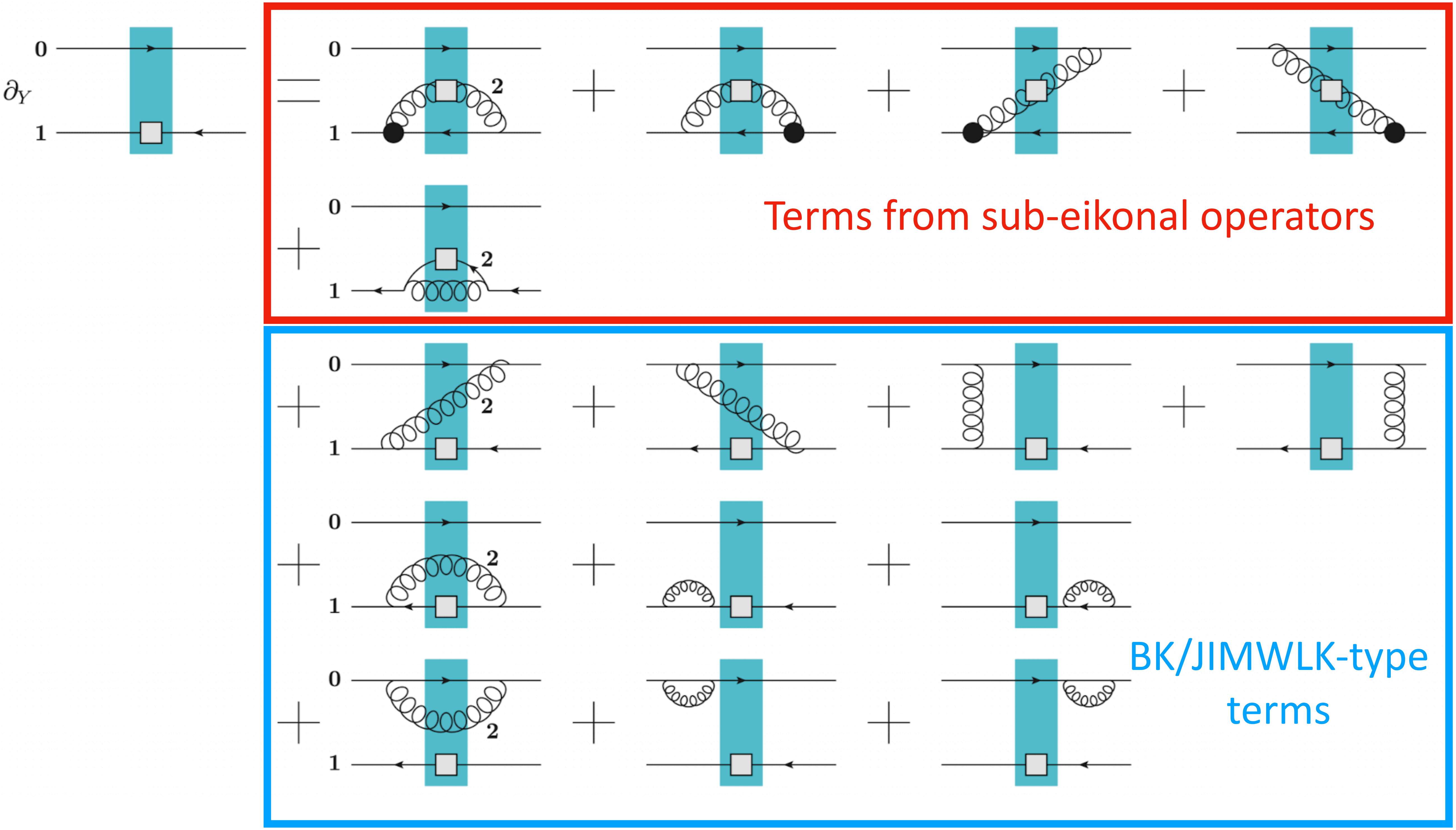
- Resums double logarithms of energy, $\alpha_s \ln^2 s$



Kovchegov, Pitonyak, Sievert (1511.06737)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

Helicity evolution

- Resums double logarithms of energy, $\alpha_s \ln^2 s$



Kovchegov, Pitonyak, Sievert (1511.06737)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

Helicity evolution at large N_c

- Not closed in general (cf. Balitsky hierarchy), but closed in large-N_c limit

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[3\, G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) + \Gamma(x_{10}^2, x_{21}^2, z' s) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z' s) \Big]$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z' s}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \Big[G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) \Big]$$

($G \equiv Q$ in large-N_c limit)

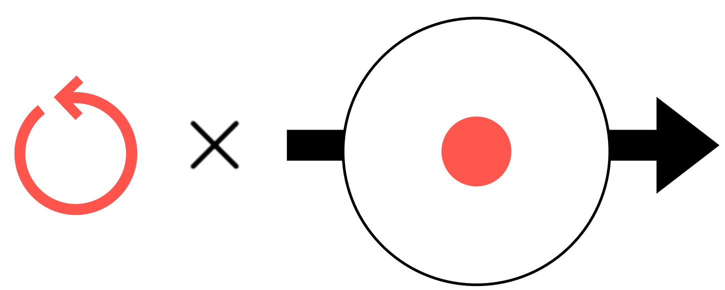
Kovchegov, Pitonyak, Sievert (1511.06737)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2204.11898)

OAM at Small x

OAM distributions at small x

- Quark OAM distribution defined through quark Wigner function

$$L_q(x,Q^2) = \int \frac{d^2b_\perp\, db^-\, d^2k_\perp}{(2\pi)^3}\,(\underline{b}\times\underline{k})\,W^q_{UL}(k,b)\Big|_{k^+=xP^+}$$

$=$ 

$$\underline{x} = (x^1, x^2)$$

$$\underline{b} = \frac{1}{2}(\underline{x}_1 + \underline{x}_0)$$

- *Unpolarized* quark/polarized nucleon SIDIS Wigner function

$$W^q_{UL}(k,b) = \int d^2r\, dr^-\, e^{ik\cdot r}\left\langle \bar{q}\left(b-\frac{1}{2}r\right)\mathcal{U}^{[+]}\left(\frac{1}{2}\gamma^+\right)q\left(b+\frac{1}{2}r\right)\right\rangle_{S_L}$$

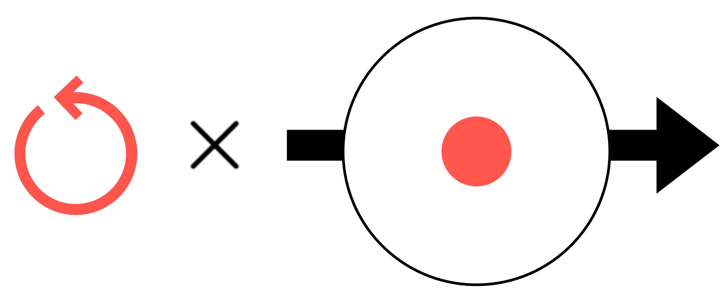
Lorce, Pasquini (1106.0139)
Hatta (1111.3547)
Ji, Xiong, Yuan (1207.5221)

OAM distributions at small x

- Quark OAM distribution defined through quark Wigner function

$$L_q(x, Q^2) = \int \frac{d^2 b_\perp \, db^- \, d^2 k_\perp}{(2\pi)^3} \, \boxed{(\underline{b}} \times \underline{k}) \, W^q_{UL}(k, b) \Big|_{k^+ = x P^+}$$

$=$



Need first impact-parameter moments of dipole amplitudes!

$$\underline{x} = (x^1, x^2)$$

$$\underline{b} = \frac{1}{2}(\underline{x}_1 + \underline{x}_0)$$

- *Unpolarized* quark/polarized nucleon SIDIS Wigner function

$$W^q_{UL}(k, b) = \int d^2 r \, dr^- \, e^{ik\cdot r} \left\langle \bar{q}\left(b - \frac{1}{2}r\right) \mathcal{U}^{[+]} \left(\frac{1}{2}\gamma^+\right) q\left(b + \frac{1}{2}r\right) \right\rangle_{S_L}$$

Lorce, Pasquini (1106.0139)
Hatta (1111.3547)
Ji, Xiong, Yuan (1207.5221)

Moment amplitudes

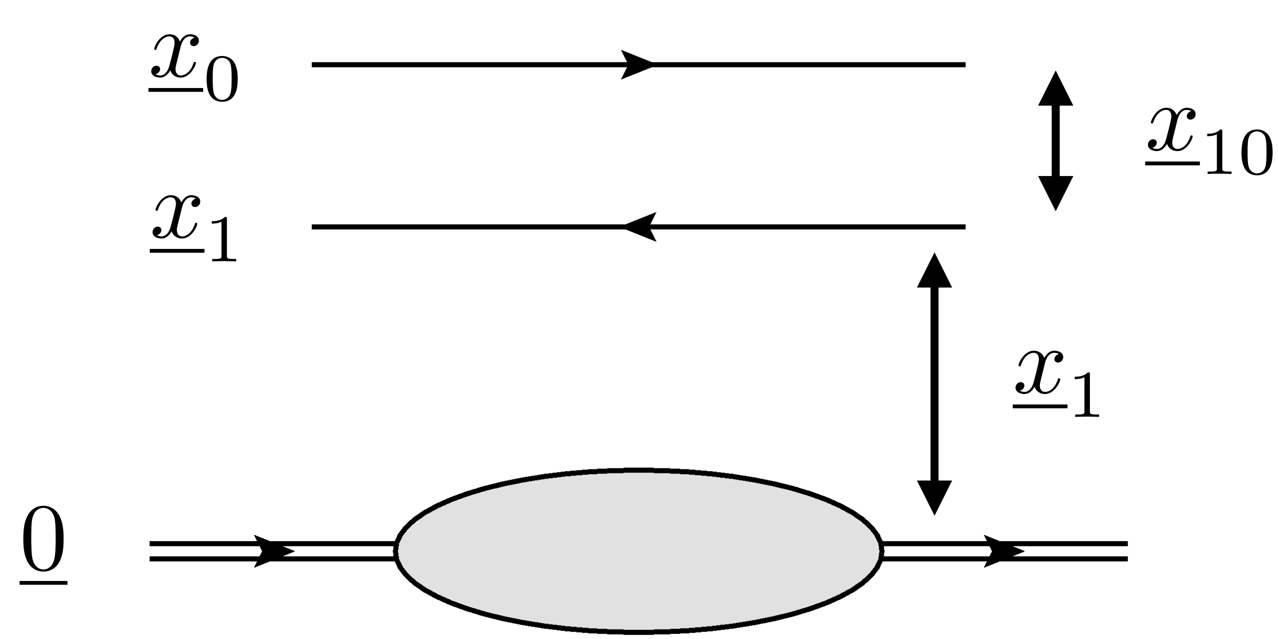
- First impact-parameter moments of polarized amplitudes
  → **moment amplitudes**

$$\int d^2x_1 \, x_1^m \, Q_{10}(s) = x_{10}^m \, I_3(x_{10}^2, s) + \dots$$

$$\int d^2x_1 \, x_1^m \, G_{10}^i(s) = \epsilon^{mi} x_{10}^2 \, I_4(x_{10}^2, s) + \epsilon^{mk} x_{10}^k x_{10}^i \, I_5(x_{10}^2, s) + \dots$$

$$\int d^2x_1 \, x_1^m \, \widetilde{L}_{10}(s) = i\epsilon^{mk} x_{10}^k \, \widetilde{I}(x_{10}^2, s) + \dots$$

$\widetilde{L}_{10} \approx \widetilde{Q}_{10}$ with $\gamma^+\gamma^5 \to \gamma^+$

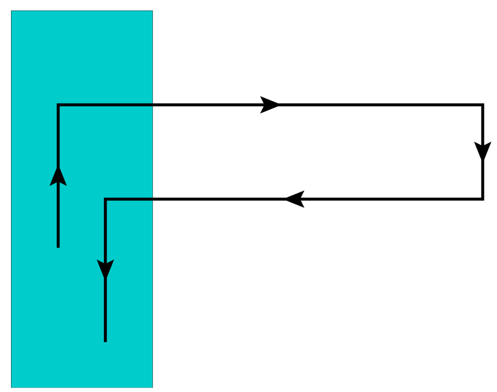


BM, Kovchegov (2310.18404)

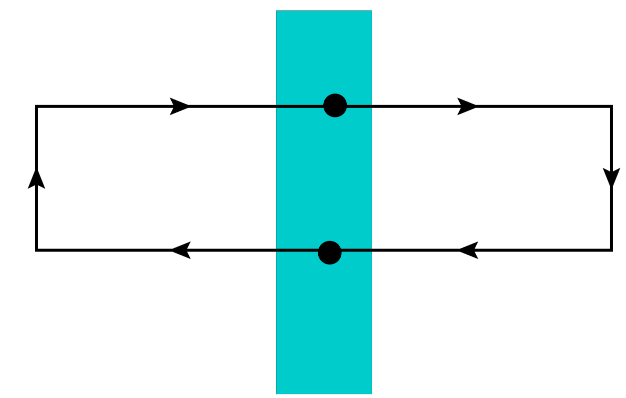
OAM distributions at small x

- Start with operator definition, simplify for $x \ll 1$, result:

$$\int d^2 b_\perp \, d^2 k_\perp \, (\underline{b} \times \underline{k})$$



$$\int d^2 b_\perp \, d^2 k_\perp \, (\underline{b} \times \underline{k})$$



$$L_{q+\bar{q}}(x, Q^2) = \frac{2\, N_f}{\alpha_s \pi^2} \, \widetilde{I} \left(x_{10}^2 = \frac{1}{Q^2}, s = \frac{Q^2}{x} \right)$$

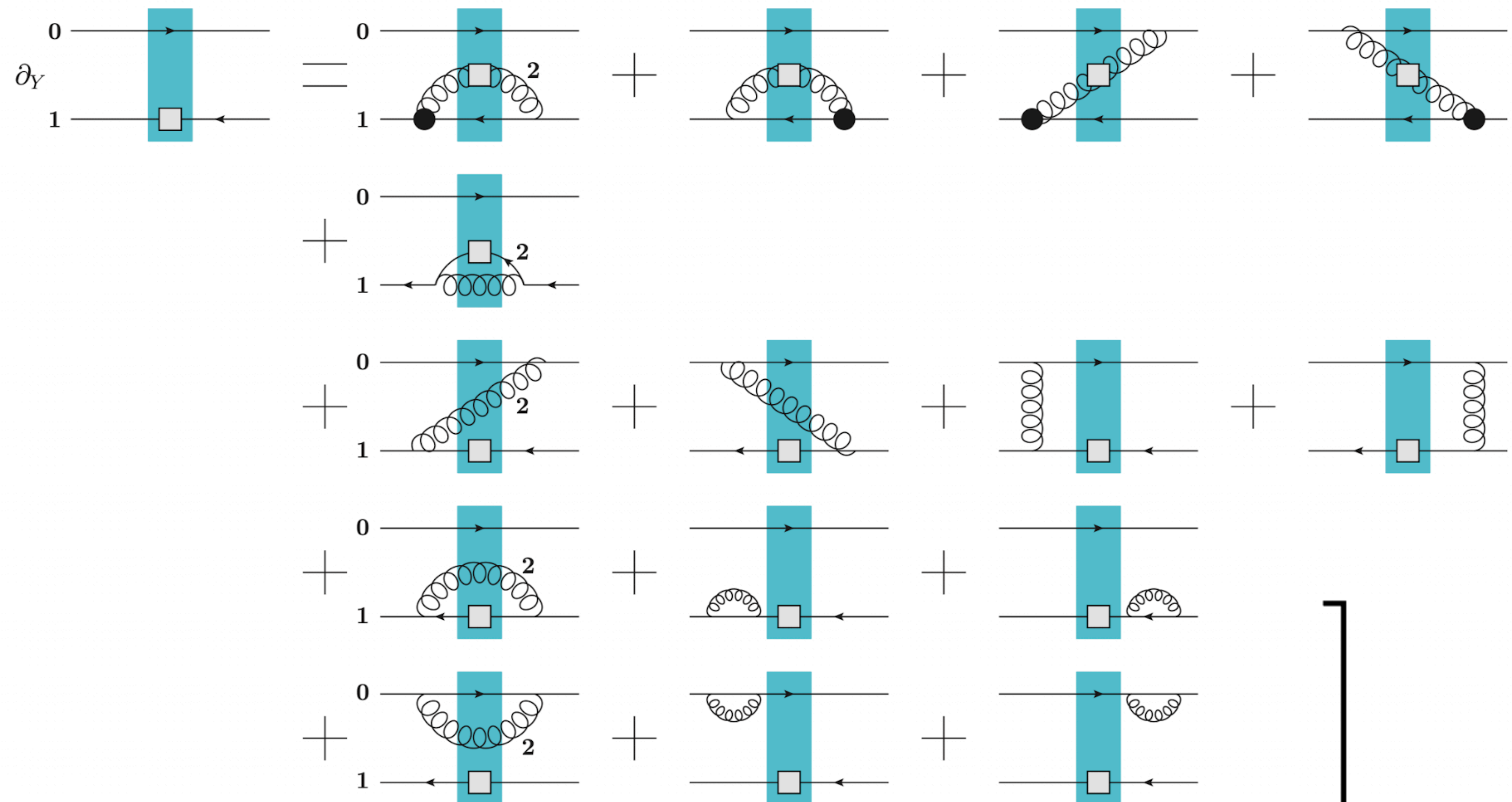
$$L_g(x, Q^2) = - \frac{2\, N_c}{\alpha_s \pi^2} \left[2\, I_4 + 3\, I_5 \right] \left(x_{10} = \frac{1}{Q^2}, \, zs = \frac{Q^2}{x} \right)$$

BM, Kovchegov (2310.18404)

OAM small-x evolution

- Moment evolution: take impact-parameter moments of helicity evolution

$$\int d^2x_1\, x_1^m \Bigg[$$



$$\Bigg]$$

Kovchegov (1901.07453)
Kovchegov, BM (2310.18404)

OAM small-x evolution

- Closed system of equations in large-N_c limit

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \end{pmatrix}(x_{10}^2, zs) = \begin{pmatrix} I_3^{(0)} \\ I_4^{(0)} \\ I_5^{(0)} \end{pmatrix}(x_{10}^2, zs) + \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 - 2\,\Gamma_2 \\ 0 \\ 0 \end{pmatrix}(x_{10}^2, x_{21}^2, z's)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 4 & -4 & 2 & -4 & -6 \\ 0 & 4 & 2 & -2 & -3 \\ -2 & 2 & -1 & 4 & 7 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ G \\ G_2 \end{pmatrix}(x_{21}^2, z's),$$

Kovchegov, BM (2310.18404)

OAM small-x evolution

- Closed system of equations in large-N_c limit

$$\begin{pmatrix} I_3\\ I_4\\ I_5\end{pmatrix}(x_{10}^2, zs) = \boxed{\begin{pmatrix} I_3^{(0)}\\ I_4^{(0)}\\ I_5^{(0)}\end{pmatrix}}(x_{10}^2,zs) + \frac{\alpha_s N_c}{4\pi}\int\limits_{\frac{1}{sx_{10}^2}}^{z}\frac{dz'}{z'}\int\limits_{\frac{1}{z's}}^{x_{10}^2}\frac{dx_{21}^2}{x_{21}^2}\begin{pmatrix}2\,\Gamma_3 - 4\,\Gamma_4+2\,\Gamma_5-2\,\Gamma_2\\0\\0\end{pmatrix}(x_{10}^2,x_{21}^2,z's)$$

$$+ \frac{\alpha_s N_c}{4\pi}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int\limits_{\max[x_{10}^2,\frac{1}{z's}]}^{\min[\frac{z}{z'}x_{10}^2,\frac{1}{\Lambda^2}]}\frac{dx_{21}^2}{x_{21}^2}\begin{pmatrix}4&-4&2&-4&-6\\0&4&2&-2&-3\\-2&2&-1&4&7\end{pmatrix}\begin{pmatrix}I_3\\I_4\\I_5\\G\\G_2\end{pmatrix}(x_{21}^2,z's),$$

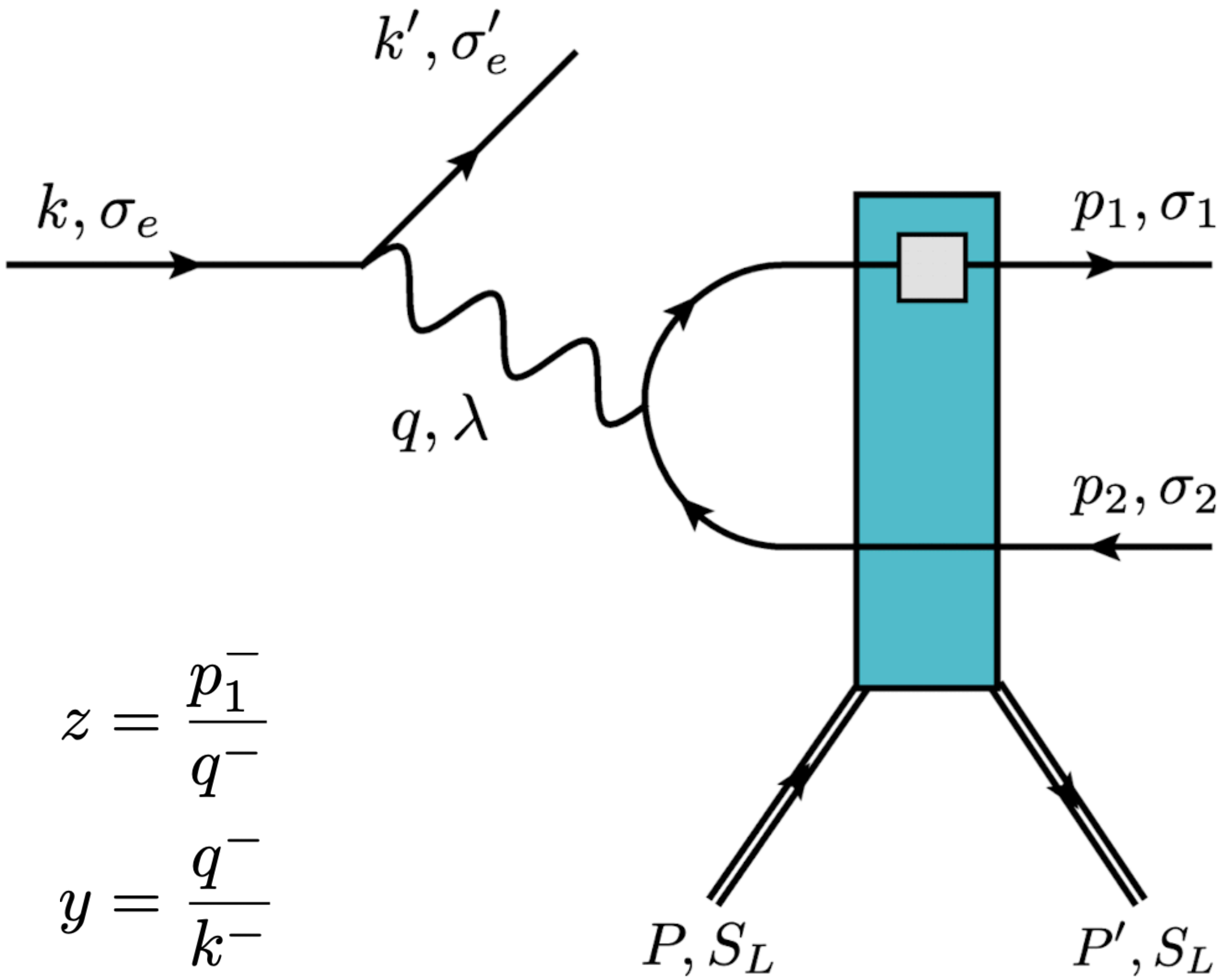
How do we obtain initial conditions from data?

Kovchegov, BM (2310.18404)

Elastic Dijet Production at the EIC

Elastic dijet production in ep collisions

- Dijet production in electron-proton collisions provides a two-transverse momentum process which could probe OAM



$$z = \frac{p_1^-}{q^-}$$

$$y = \frac{q^-}{k^-}$$

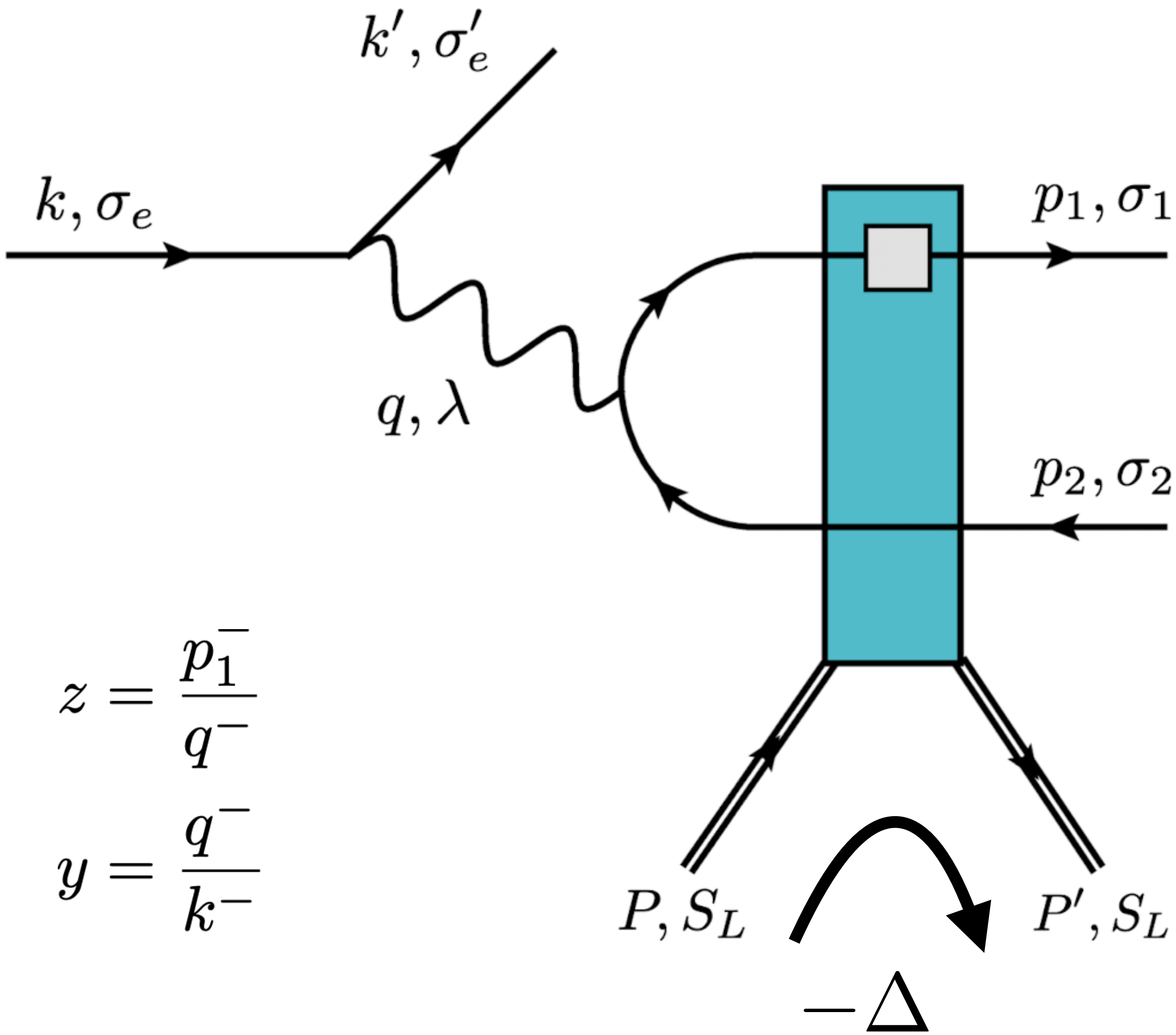
$$\underline{p} = (1-z)\,\underline{p}_1 - z\,\underline{p}_2,$$

$$\underline{\Delta} = \underline{p}_1 + \underline{p}_2 = -\underline{P}'.$$

Hatta et al, *Phys.Rev.D* 95 (2017) 11
Bhattacharya, Boussarie, Hatta, *Phys.Rev.Lett.* 128 (2022) 18
Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet production in ep collisions

- Dijet production in electron-proton collisions provides a two-transverse momentum process which could probe OAM



$$z = \frac{p_1^-}{q^-}$$

$$y = \frac{q^-}{k^-}$$

$$\underline{p} = (1-z)\,\underline{p}_1 - z\,\underline{p}_2,$$
$$\underline{\Delta} = \underline{p}_1 + \underline{p}_2 = -\underline{P}'.$$

Elastic: access $\underline{\Delta}$ conjugate to $\underline{b}$

Hatta et al, *Phys.Rev.D* 95 (2017) 11
Bhattacharya, Boussarie, Hatta, *Phys.Rev.Lett.* 128 (2022) 18
Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet double spin asymmetry

- Expand numerator of DSA to linear order in $\Delta_\perp / p_\perp \ll 1$ to access moment amplitudes:

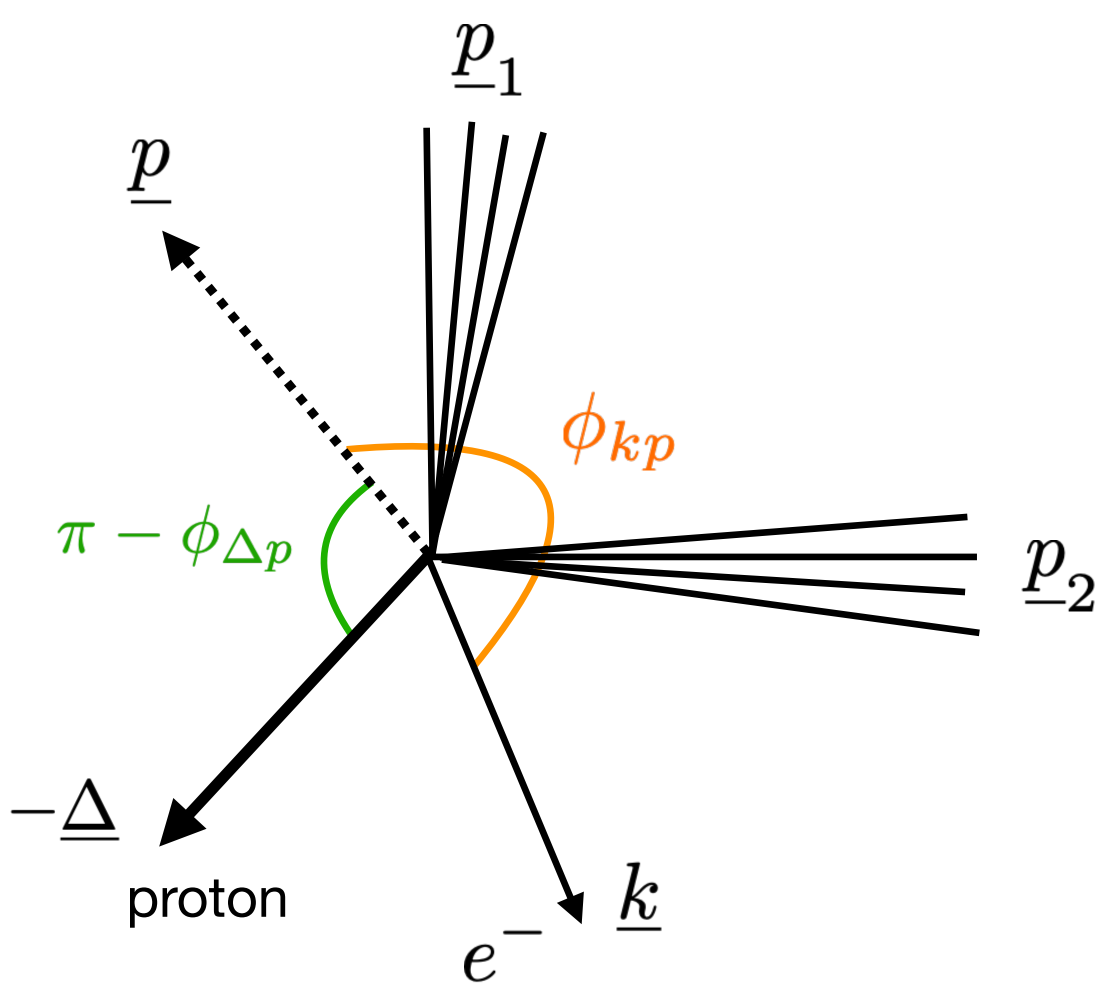
$$\frac{d\sigma^{\mathrm{DSA}}}{d^2p_\perp d^2\Delta_\perp d^2k'_\perp} = A + B\,\cos\phi_{kp} + \frac{\Delta_\perp}{p_\perp}\Big[C\,\cos\phi_{\Delta p} + D\,\cos\phi_{\Delta p}\cos\phi_{kp} + E\,\sin\phi_{\Delta p}\sin\phi_{kp}\Big] + \mathcal{O}\left(\frac{\Delta_\perp^2}{p_\perp^2}\right)$$

Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet double spin asymmetry

- Expand numerator of DSA to linear order in $\Delta_\perp / p_\perp \ll 1$ to access moment amplitudes:

$$\frac{d\sigma^{\mathrm{DSA}}}{d^2 p_\perp d^2 \Delta_\perp d^2 k'_\perp} = A + B\,\cos\phi_{kp} + \frac{\Delta_\perp}{p_\perp}\Big[C\,\cos\phi_{\Delta p} + D\,\cos\phi_{\Delta p}\cos\phi_{kp} + E\,\sin\phi_{\Delta p}\sin\phi_{kp}\Big] + \mathcal{O}\left(\frac{\Delta_\perp^2}{p_\perp^2}\right)$$

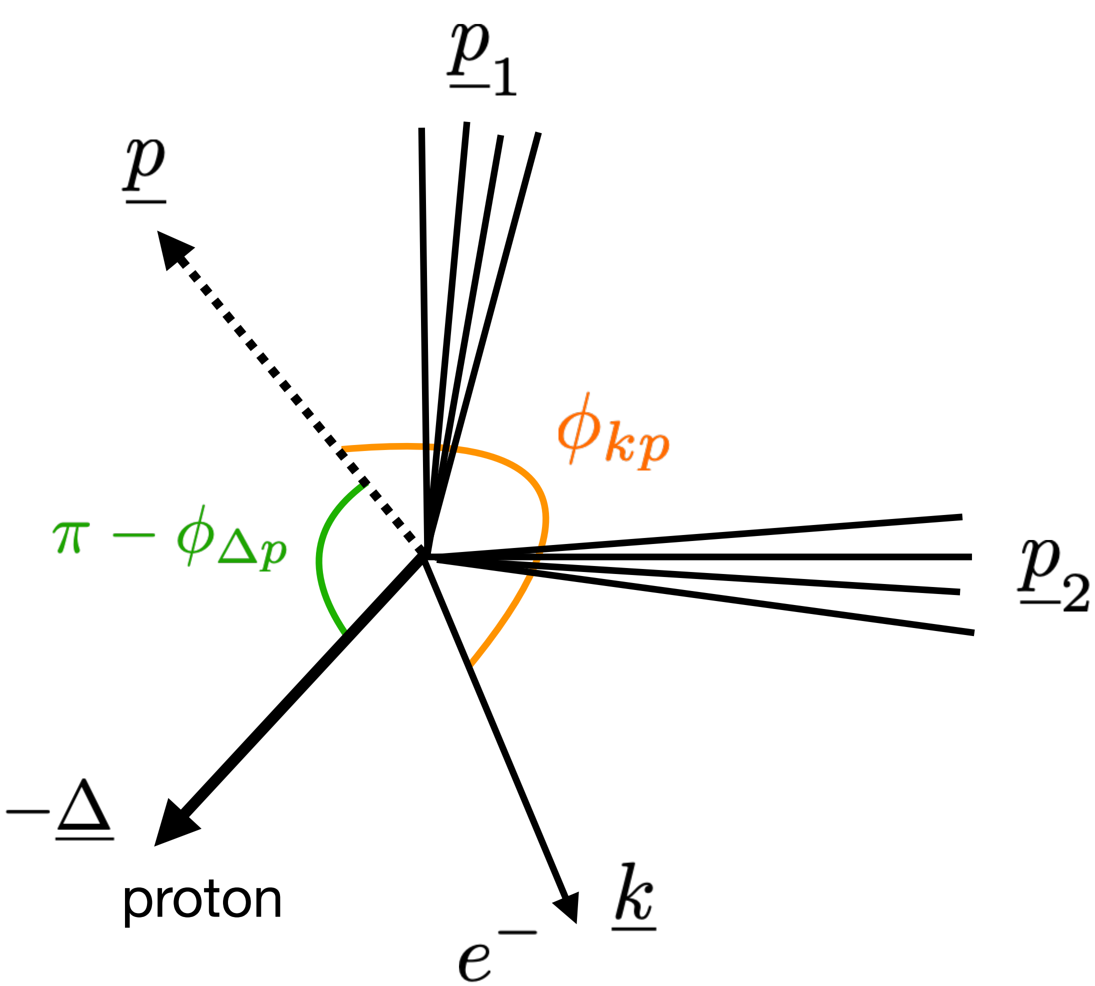


Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet double spin asymmetry

- Expand numerator of DSA to linear order in $\Delta_\perp / p_\perp \ll 1$ to access moment amplitudes:

$$\frac{d\sigma^{\mathrm{DSA}}}{d^2 p_\perp d^2 \Delta_\perp d^2 k'_\perp} = A + B\,\cos\phi_{kp} + \frac{\Delta_\perp}{p_\perp}\left[C\,\cos\phi_{\Delta p} + D\,\cos\phi_{\Delta p}\cos\phi_{kp} + E\,\sin\phi_{\Delta p}\sin\phi_{kp}\right] + \mathcal{O}\left(\frac{\Delta_\perp^2}{p_\perp^2}\right)$$



Depend on moment amplitudes!
(very messy expressions though)

Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet double spin asymmetry

- Expand numerator of DSA to linear order in $\Delta_\perp / p_\perp \ll 1$ to access moment amplitudes:

$$\frac{d\sigma^{\mathrm{DSA}}}{d^2p_\perp d^2\Delta_\perp d^2k'_\perp} = A + B\,\cos\phi_{kp} + \frac{\Delta_\perp}{p_\perp}\Big[C\,\cos\phi_{\Delta p} + D\,\cos\phi_{\Delta p}\cos\phi_{kp} + E\,\sin\phi_{\Delta p}\sin\phi_{kp}\Big] + \mathcal{O}\left(\frac{\Delta_\perp^2}{p_\perp^2}\right)$$

$$C = (2-y)\sum_q \frac{\alpha_{EM}^2 e_q^2 N_c^2\, p_\perp}{16\pi^5 W^2}(1-2z)$$
$$\times\left\{\frac{(1-2z)^2}{2} N^{(10)}_{(01)}\, Q_q{}^{(00)}_{(11)} + \left[z^2+(1-z)^2\right] N^{(10)}_{(01)}\, G_2{}^{(00)}_{(11)} + N^{(00)}_{(11)}\left[\frac{1}{2}Q_q{}^{(00)}_{(11)} + \underline{I_{3q}{}^{(00)}_{(11)} - I_{3q}{}^{(10)}_{(01)}}\right]\right.$$
$$\left. - \left[z^2+(1-z)^2\right] N^{(00)}_{(11)}\left[Q_q{}^{(10)}_{(21)} + 2\,G_2{}^{(10)}_{(21)} + \underline{I_4{}^{(10)}_{(21)} + I_4{}^{(01)}_{(10)} - I_5{}^{(00)}_{(11)} + I_5{}^{(01)}_{(10)} + I_5{}^{(10)}_{(01)}}\right]\right\}$$

Fourier-Bessel transform:

$$F^{(cd)}_{(ab)}(p_\perp, \overline{Q}, W^2) \equiv \int_0^\infty dx_{12}\, x_{12}\, (p_\perp x_{12})^c \left(\overline{Q} x_{12}\right)^d J_a(p_\perp x_{12}) K_b\left(\overline{Q}\, x_{12}\right)\, F(x_{12}^2, W^2)$$

Kovchegov, BM, *Phys.Rev.D* 111 (2025) 5

Elastic dijet DSA: Feasibility

- Is elastic dijet production a *feasible* probe for OAM at small x at future **Electron Ion Collider (EIC)?**

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- For pseudo-data, need value and error:
  → Value determined by initial conditions

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

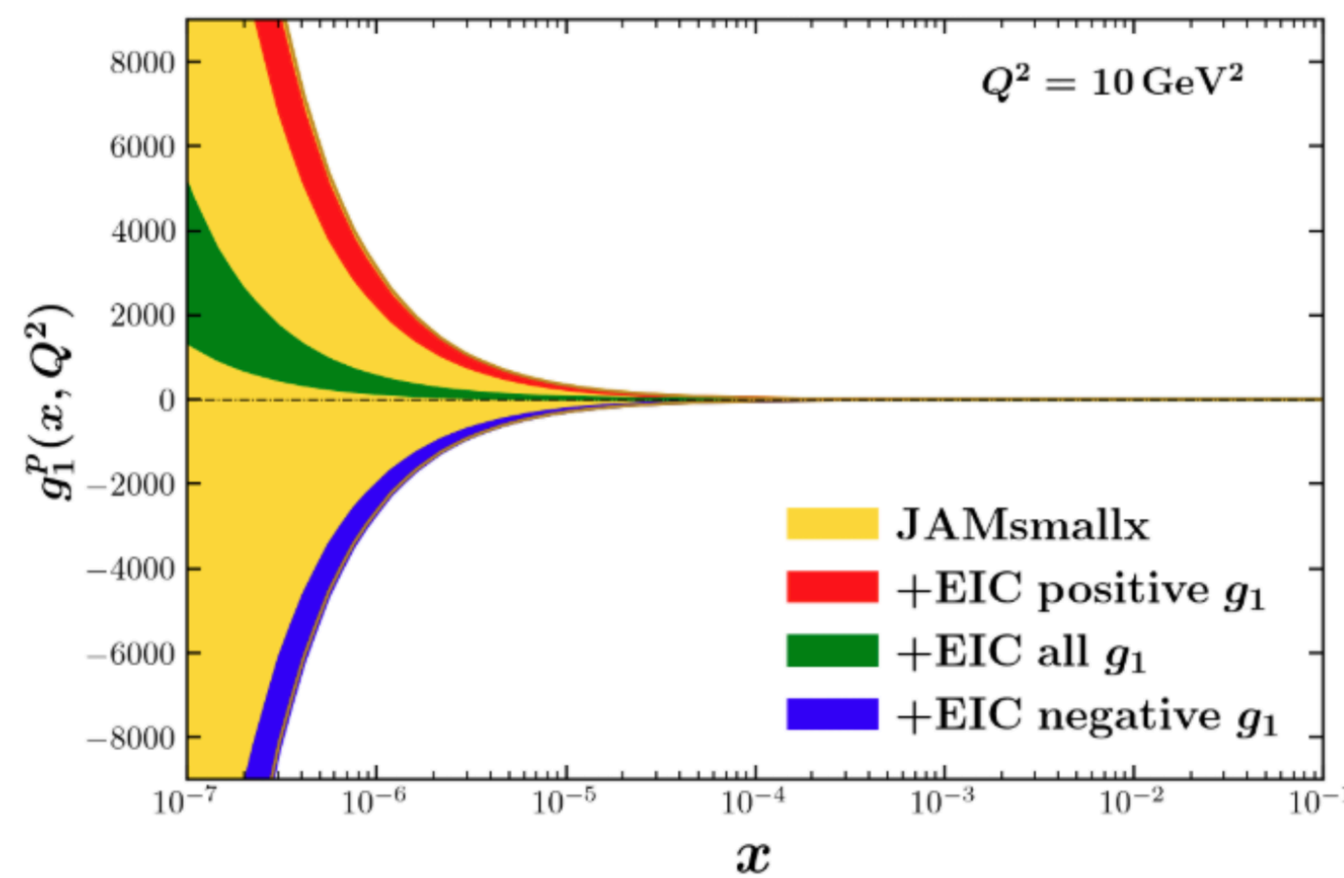
- For pseudo-data, need **value** and error:
  → Value determined by initial conditions

$$Q_u^{(0)}, Q_d^{(0)}, Q_s^{(0)},$$
$$\widetilde{G}^{(0)}, G_2^{(0)}$$
$$\widetilde{Q}_u^{(0)}, \widetilde{Q}_d^{(0)}, \widetilde{Q}_s^{(0)},$$

$$f^{(0)}(s_{10}, \eta) = a\,\eta + b\,s_{10} + c$$

Infer parameters from global analysis of DIS, SIDIS, and pp data

3 scenarios
(positive, all, and negative g_1^p replicas):



Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- For pseudo-data, need **value** and error:
  → Value determined by initial conditions

$$\begin{matrix} I_{3u}^{(0)} & I_{3d}^{(0)} & I_{3s}^{(0)} \\ \widetilde{I}_{3}^{(0)} & I_{4}^{(0)} & I_{5}^{(0)} \\ \widetilde{I}_{u}^{(0)} & \widetilde{I}_{d}^{(0)} & \widetilde{I}_{s}^{(0)} \end{matrix}$$

3 scenarios:

$$\int_{10^{-7}}^{0.1} dx\, L_{q+\bar{q}/g}(x, Q^2 = 10\,\mathrm{GeV}^2) = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

$$f^{(0)}(s_{10}, \eta) = a\,\eta + b\,s_{10} + c$$

Randomly chosen parameters in $[-20,20]$

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- For pseudo-data, need value and **error**:
  → Statistical error determined by unpolarized cross section

$$\delta\left(A_{LL}\right)_{\text{stat.}} \equiv \delta\left(\frac{N^{\uparrow\uparrow}-N^{\uparrow\downarrow}}{N^{\uparrow\uparrow}+N^{\uparrow\downarrow}}\right)_{\text{stat}} = \sqrt{\frac{1-A_{LL}^2}{N^{\uparrow\uparrow}+N^{\uparrow\downarrow}}} \approx \sqrt{\frac{1}{d\sigma^{\text{unpol}}\,\mathcal{L}_{\text{int}}}},$$

Unpolarized small-x theory

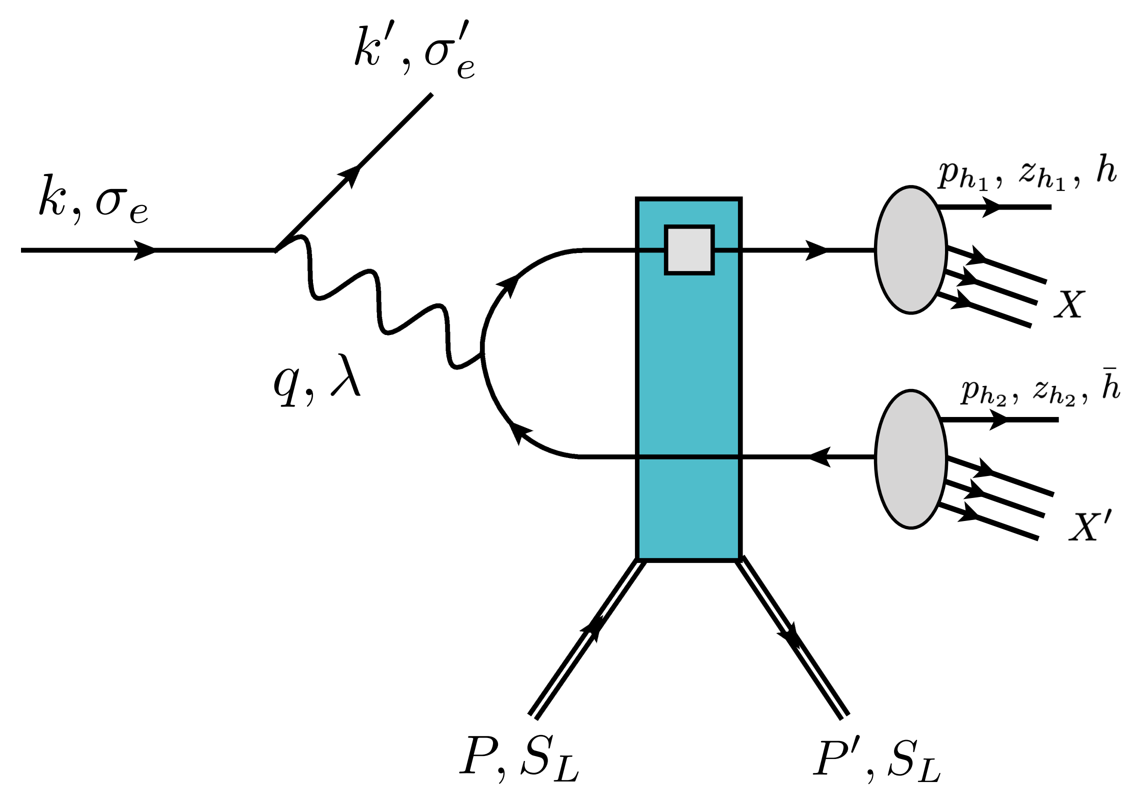
Choose $100\,\text{fb}^{-1}$ for baseline

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- To separate three flavors (u, d, s), also need elastic dihadron production

$$\frac{d\sigma^{ep\to e'p'h_1h_2}}{dz_{h_1}dz_{h_2}d^2p_{h_1}d^2p_{h_2}} = \sum_q \int_{z_{h_1}}^{1-z_{h_2}} \frac{dz}{z(1-z)}\, D_1^{h_1/q}\left(\frac{z_{h_1}}{z}\right) D_1^{h_2/\bar{q}}\left(\frac{z_{h_2}}{1-z}\right) \times \frac{d\sigma^{ep\to e'p'q\bar{q}}}{dz\, d^2p_1\, d^2p_2}\Bigg|_{\substack{\underline{p}_1=\frac{z}{z_{h_1}}\underline{p}_{h_1} \\ \underline{p}_2=\frac{1-z}{z_{h_2}}\underline{p}_{h_2}}} + (h_1 \leftrightarrow h_2)$$



Produce pseudo-data for $\pi^+\pi^-$ and K^+K^- production

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- Fit pseudo-data using Jefferson Lab Angular Momentum (JAM) collaboration Monte-Carlo framework
- Monte Carlo approximation of posterior distribution

$$\mathcal{P}(a|\,\text{data}) \propto \mathcal{L}(\text{data}\,|a)\,\pi(a) \propto \exp\left[-\frac{1}{2}\chi^2(a)\right]\pi(a)$$

$$\{\mathbf{a}_k; 1, \ldots, n\}$$

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- Fit pseudo-data using Jefferson Lab Angular Momentum (JAM) collaboration Monte-Carlo framework
- Monte Carlo approximation of posterior distribution

$$\mathcal{P}(a|\,\text{data}) \propto \mathcal{L}(\text{data}\,|a)\,\pi(a) \propto \exp\left[-\frac{1}{2}\chi^2(a)\right]\pi(a)$$

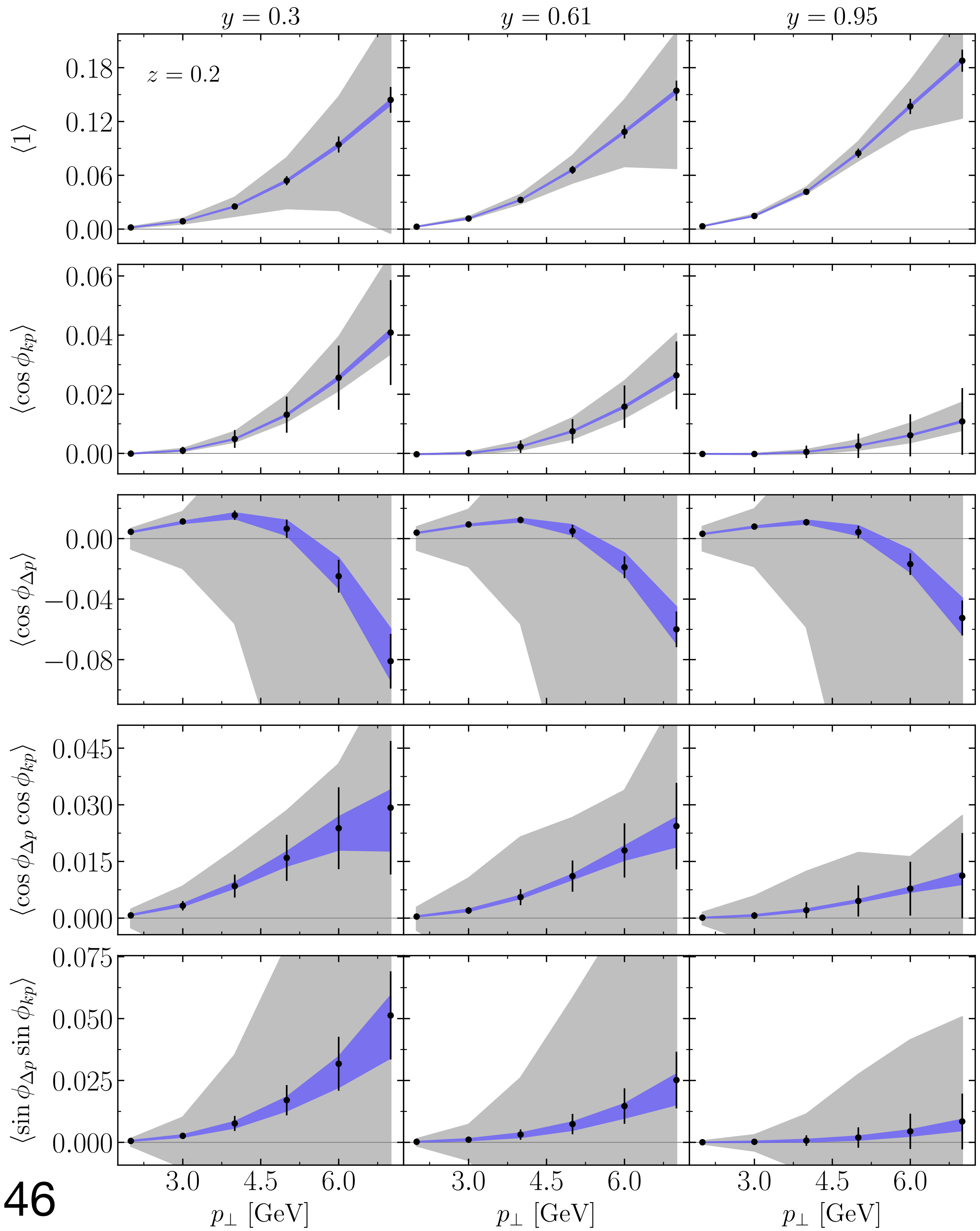
$$\{\mathbf{a}_k; 1, \ldots, n\}$$

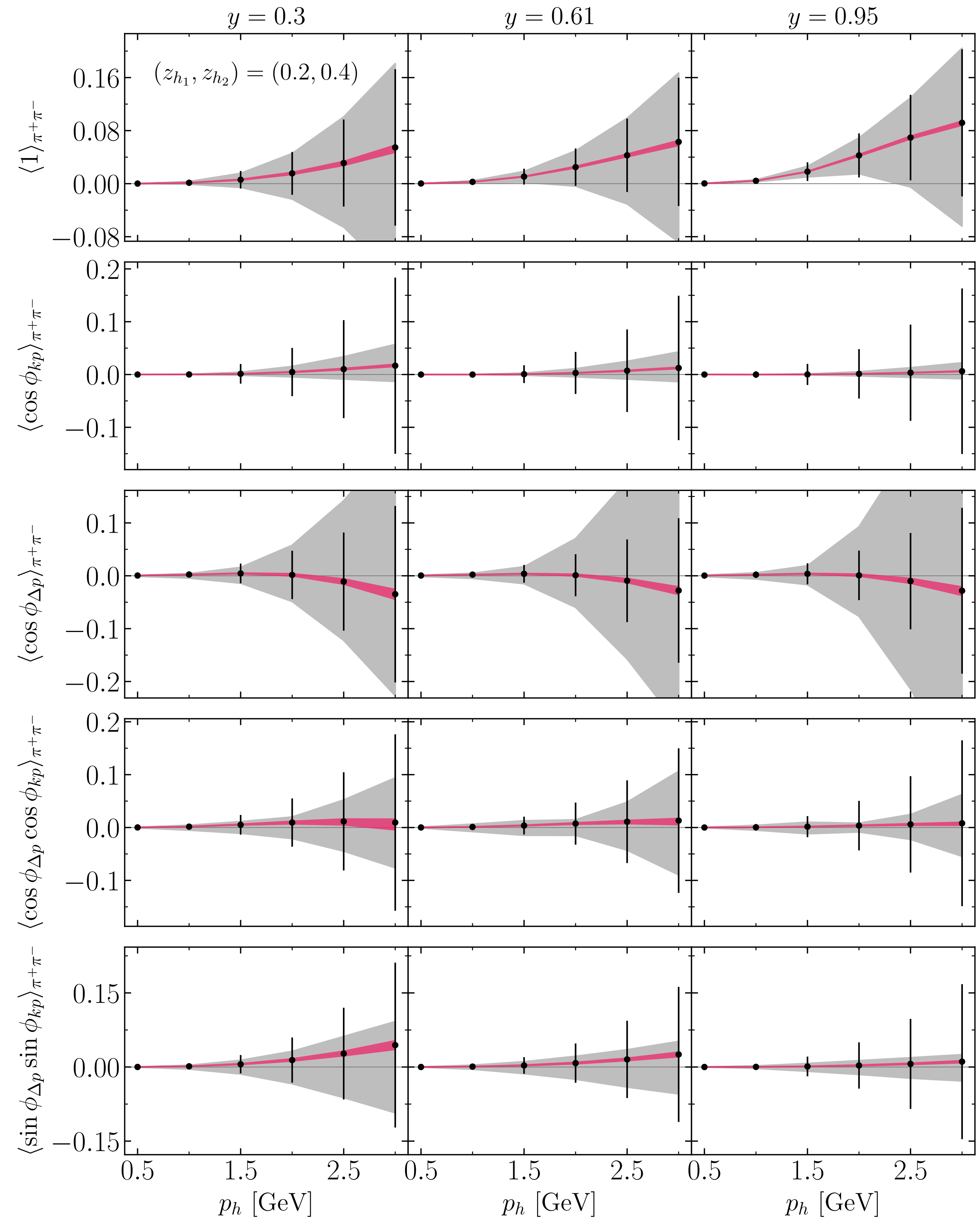
- Observables computed as sum over MC sample:

$$\mathrm{E}[\mathcal{O}] = \frac{1}{n}\sum_k \mathcal{O}(\mathbf{a}_k)$$

$$\mathrm{Var}[\mathcal{O}] = \frac{1}{n}\sum_k \left(\mathcal{O}(\mathbf{a}_k) - \mathrm{E}[\mathcal{O}]\right)^2$$

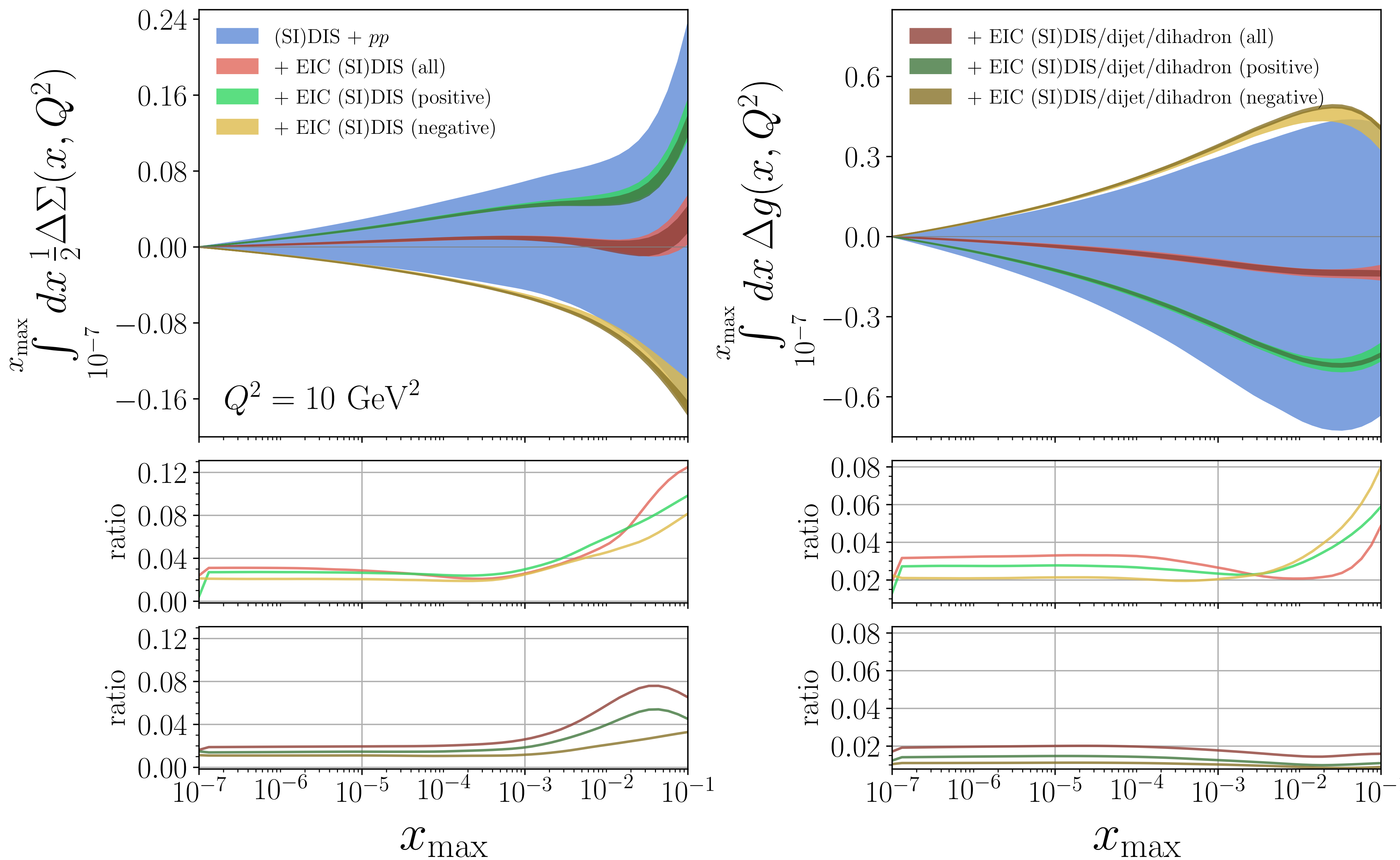
Produce elastic dijet/dihadron pseudo-data → Fit elastic dijet/dihadron pseudo-data → Extract OAM distributions (and helicity PDFs)





Elastic dijet DSA: Feasibility

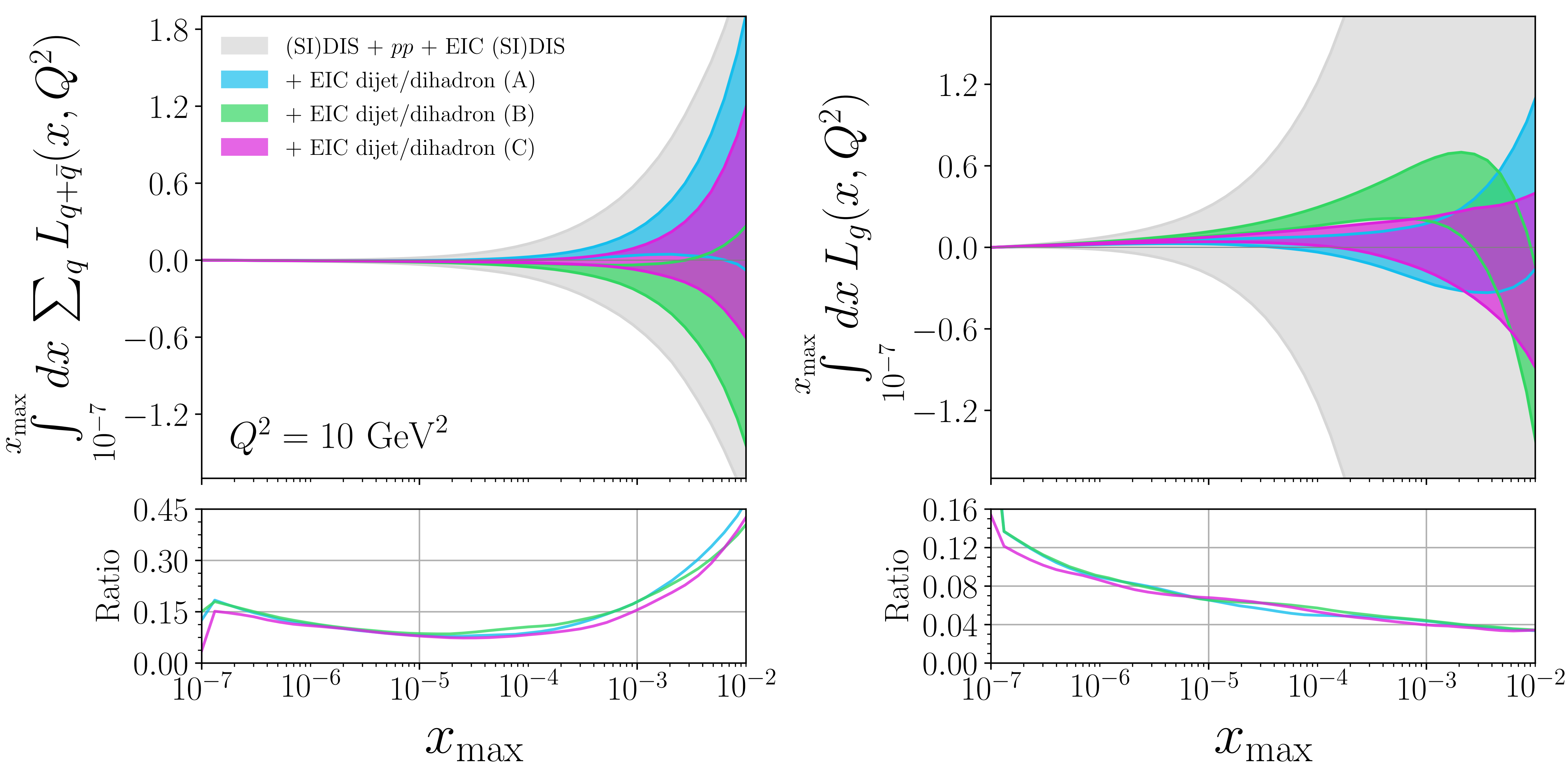
Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)



Elastic dijet DSA: Feasibility

JAM smallx preliminary

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

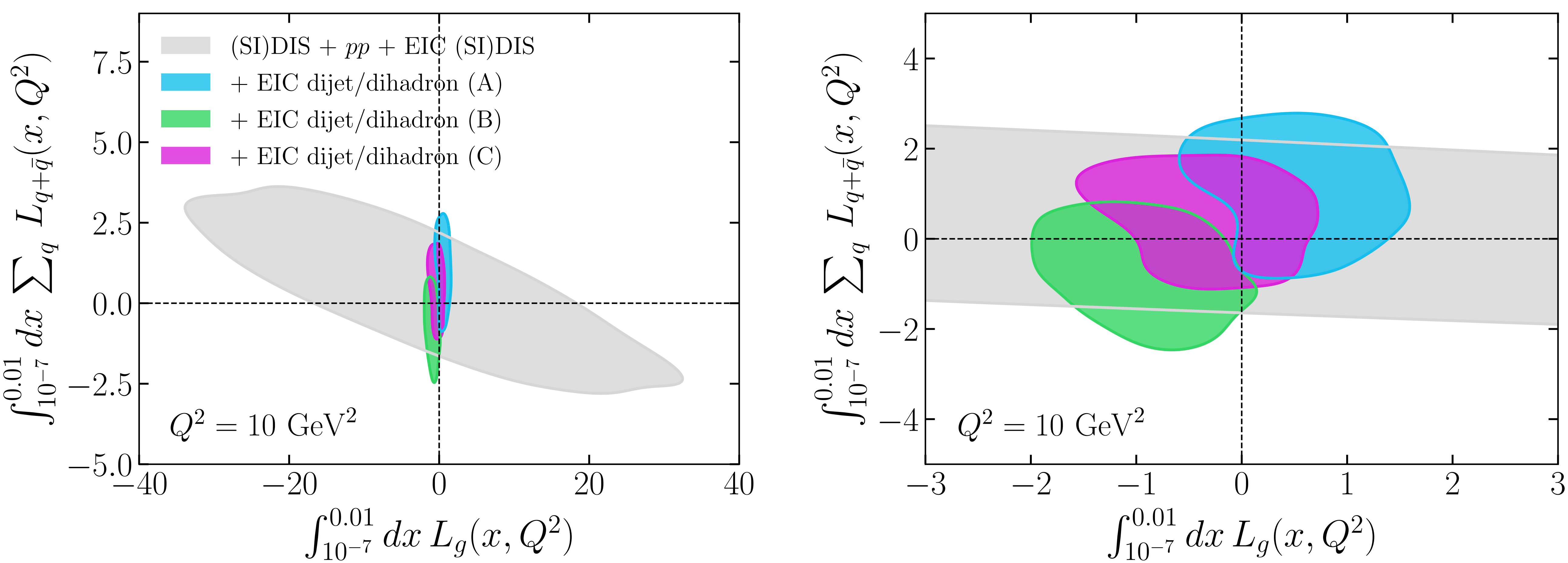


JAM smallx preliminary

Elastic dijet DSA: Feasibility

Produce elastic dijet pseudo-data → Fit elastic dijet pseudo-data → Extract OAM distributions (and helicity PDFs)

- Compared to prior, large improvement in knowledge of OAM



Take-home message

- **Novel description of OAM at small-x** using polarized small x tools
- Showed initial conditions for OAM evolution probed via **angular harmonics** in elastic dijet production
- **OAM can be extracted** from dijet/dihadron data
  $\Rightarrow$ *First* study — more iterations to come!

For first time ever,
EIC will measure orbital motion of partons &
access all contributions to proton spin!

Backup

Polarized Dipole Amplitudes

- Polarized dipole amplitudes in large-N_c or large-N_c & N_f limits:

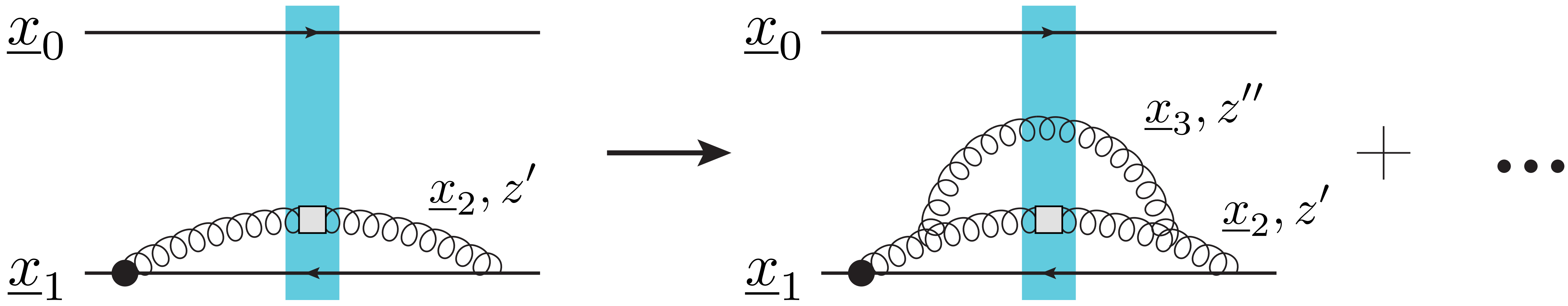
	Large-N_c limit	**Large-N_c & N_f limit**
$Q_{10} \to$	G_{10}	Q_{10}
$G^i_{10} \to$	G^i_{10}	G^i_{10}
$G^{\mathrm{adj}}_{10} \to$	$4\, G_{10}$	$4\, \widetilde{G}_{10}$
$G^{i\,\mathrm{adj}}_{10} \to$	$2\, G^i_{10}$	$2\, G^i_{10}$

- G_{10} = Q_{10} without contribution from $V_{\underline{x}}^{\mathrm{q}[1]}$
- $\widetilde{G}_{10}$ = Q_{10} with $V_{\underline{x}}^{\mathrm{pol}[1]} \to W_{\underline{x}}^{\mathrm{pol}[1]}$ where

$$W_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{G}[1]} + \frac{g^2 P^+}{4s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-] \psi_\alpha(x_2^-, \underline{x}) \left(\frac{1}{2} \gamma^+ \gamma^5 \right) \bar{\psi}_\beta(x_1^-, \underline{x}) V_{\underline{x}}[x_1^-, -\infty]$$

Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095

Neighbor Dipole Amplitudes



- Lifetime ordering: $x_{32}^2 z'' \ll x_{21}^2 z'$
- IR cutoff: $x_{32} \ll x_{20} \approx x_{10}$

Subsequent evolution of dipole 20 must know about dipole 21!

Polarized Small-x Evolution

- Closed, coupled, linear system in the large-N_c limit:

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \Big],$$

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \Big],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \Big[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \Big],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) \Big]$$

$$\widetilde{Q}(x_{10}^2, zs) = \widetilde{Q}^{(0)}(x_{10}^2, zs) - \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \Big[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \Big].$$

"Neighbor" dipole amplitudes: auxiliary functions needed to enforce lifetime ordering

Kovchegov, Pitonyak, Sievert, *JHEP* 10 (2016) 072
Cougoulic, Kovchegov, Tarasov, Tawabutr, *JHEP* 07 (2022) 095

Longitudinal Spin Asymmetries

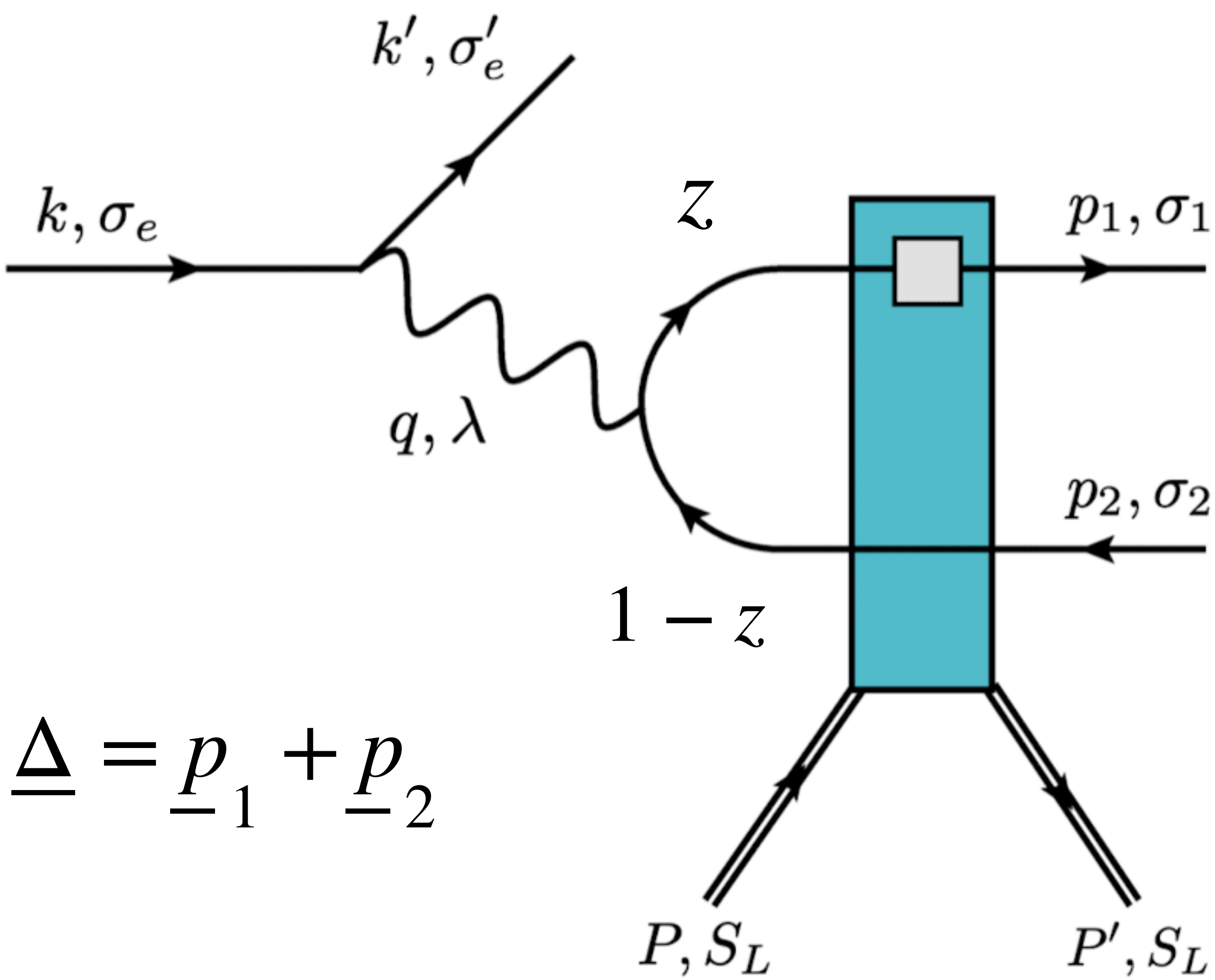
- Consider both double and single spin asymmetries (DSA & SSA)

$$d\sigma = d\sigma^{\text{unpol.}} + S_L\, d\sigma^{\text{SSA}} + S_L\, \sigma_e\, d\sigma^{\text{DSA}}$$

- Under a parity & time-reversal (PT) transformation,

$$d\sigma^{\text{SSA}} \to -d\sigma^{\text{SSA}}$$
$$d\sigma^{\text{DSA}} \to d\sigma^{\text{DSA}}$$

Elastic Dijet Production in ep Collisions



$$\underline{\Delta} = \underline{p}_1 + \underline{p}_2$$

- Depends on 6 kinematic variables + 2 azimuthal angles

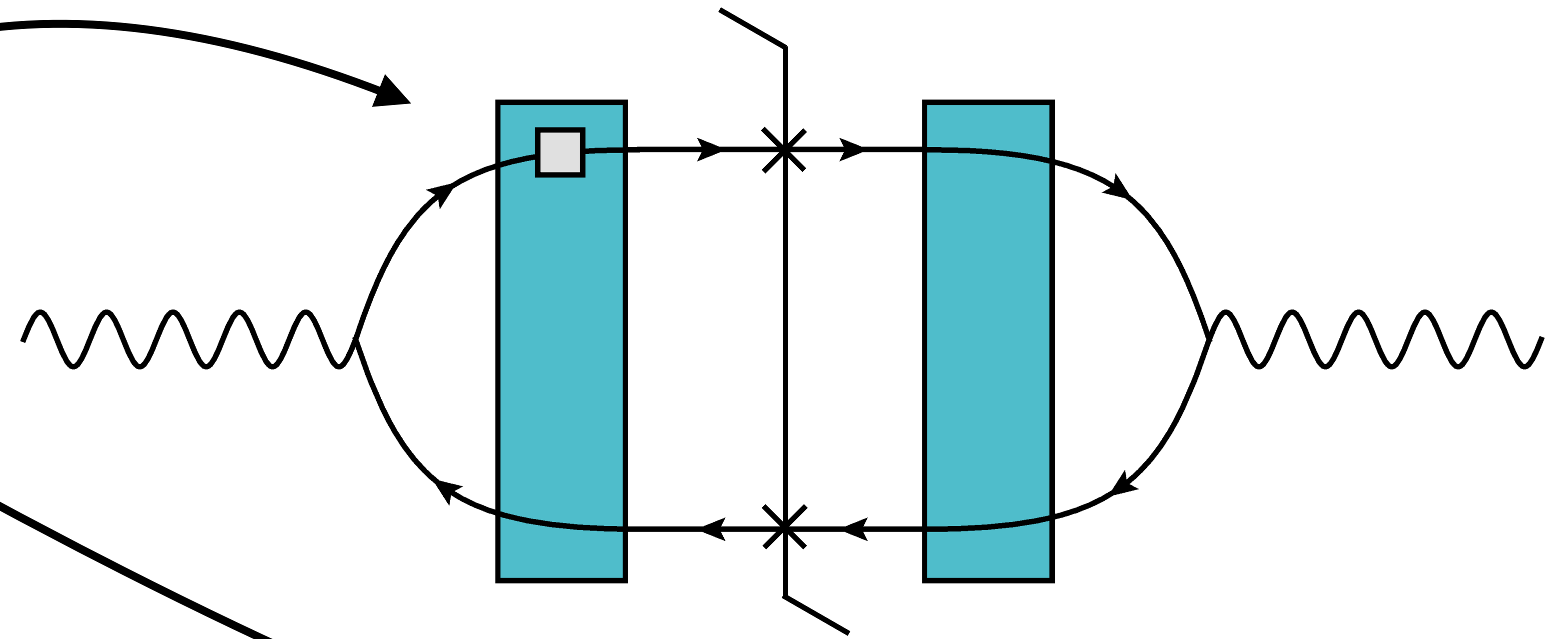
$$Q^2 = -q^2,$$
$$s = (P + k)^2,$$
$$y = \frac{P \cdot q}{P \cdot k}$$
$$t = (P' - P)^2 \approx -\Delta_\perp^2$$
$$z = \frac{P \cdot p_1}{P \cdot q}$$
$$p = (1 - z)\, p_1 - z\, p_2$$

$$\phi_{kp} = \phi_k - \phi_p$$
$$\phi_{\Delta p} = \phi_\Delta - \phi_p$$

Longitudinal Spin Asymmetries

- Consider elastic dijet spin asymmetries

$$d\sigma = |\Psi|^2 \otimes G \otimes S$$



- PT-*even* G *or* S $\Rightarrow$ **real** amplitudes like N_{10}, Q_{10}, G^i_{10}
- PT-*odd* G *or* S $\Rightarrow$ **imaginary** amplitudes like $\mathrm{Im}\, N_{10}, \mathrm{Im}\, Q_{10}, \mathrm{Im}\, G^i_{10}$

"Exotic" = Underdeveloped/non-existent phenomenology

Elastic Dijet Production in ep Collisions

$$\langle 1 \rangle = \frac{2\pi}{D} \int dX \, (2-y) \sum_{q=u,d,s} C_{\mathrm{TT}\,q},$$

$$\langle \cos\phi_{kp} \rangle = \frac{\pi}{D} \int dX \, \sqrt{2(1-y)} \sum_{q=u,d,s} C_{\mathrm{LT}\,q}^{\cos\phi_{kp}},$$

$$\langle \cos\phi_{\Delta p} \rangle = \frac{\pi}{D} \int dX \, (2-y) \frac{\sqrt{-t}}{p_\perp} \sum_{q=u,d,s} C_{\mathrm{TT}\,q}^{\cos\phi_{\Delta p}},$$

$$\langle \cos\phi_{kp} \cos\phi_{\Delta p} \rangle = \frac{\pi}{2\,D} \int dX \, \sqrt{2(1-y)} \frac{\sqrt{-t}}{p_\perp} \sum_{q=u,d,s} C_{\mathrm{LT}\,q}^{\cos\phi_{kp}\cos\phi_{\Delta p}},$$

$$\langle \sin\phi_{kp} \sin\phi_{\Delta p} \rangle = \frac{\pi}{2\,D} \int dX \, \sqrt{2(1-y)} \frac{\sqrt{-t}}{p_\perp} \sum_{q=u,d,s} C_{\mathrm{LT}\,q}^{\sin\phi_{kp}\sin\phi_{\Delta p}},$$

$$D = \int dX \sum_{q=u,d,s} \left\{ \left[1 + (1-y)^2\right] D_{\mathrm{TT}\,q} + 4(1-y) D_{\mathrm{LL}\,q} \right\}.$$

DSA Results: TT Coefficients

$$C_{\mathrm{TT}\,q} = -\frac{\alpha_{EM}^2 e_q^2 N_c^2\, p_\perp}{16\pi^5 W^2} N_{(11)}^{(00)} \left\{ (1-2z)^2 Q_q{}_{(11)}^{(00)} + 2\left[z^2 + (1-z)^2\right] G_2{}_{(11)}^{(00)} \right\}, \tag{47a}$$

$$\begin{aligned} C_{\mathrm{TT}\,q}^{\cos\phi_{\Delta p}} = {} & \frac{\alpha_{EM}^2 e_q^2 N_c^2\, p_\perp}{16\pi^5 W^2} (1-2z) \\ & \times \left\{ \frac{(1-2z)^2}{2} N_{(01)}^{(10)}\, Q_q{}_{(11)}^{(00)} + \left[z^2 + (1-z)^2\right] N_{(01)}^{(10)}\, G_2{}_{(11)}^{(00)} + N_{(11)}^{(00)} \left[\frac{1}{2} Q_q{}_{(11)}^{(00)} + I_{3q}{}_{(11)}^{(00)} - I_{3q}{}_{(01)}^{(10)} \right] \right. \\ & \left. - \left[z^2 + (1-z)^2\right] N_{(11)}^{(00)} \left[Q_q{}_{(21)}^{(10)} + 2\, G_2{}_{(21)}^{(10)} + I_4{}_{(21)}^{(10)} + I_4{}_{(10)}^{(01)} - I_5{}_{(11)}^{(00)} + I_5{}_{(10)}^{(01)} + I_5{}_{(01)}^{(10)} \right] \right\}, \end{aligned} \tag{47b}$$

Fourier-Bessel transform:

$$F_{(ab)}^{(cd)}(p_\perp, \overline{Q}, W^2) \equiv \int_0^\infty dx_{12}\, x_{12}\ (p_\perp x_{12})^c \left(\overline{Q} x_{12}\right)^d J_a(p_\perp x_{12}) K_b\left(\overline{Q}\, x_{12}\right)\ F(x_{12}^2, W^2)$$

DSA Results: LT Coefficients

$$C_{\mathrm{LT}\,q}^{\cos\phi_{kp}} = -\frac{\alpha_{EM}^2 e_q^2 N_c^2 \sqrt{z(1-z)}\,p_\perp}{8\sqrt{2}\,\pi^5 W^2}(1-2z)\left\{N_{(00)}^{(00)}\left[2\,G_2{}_{(11)}^{(00)} - Q_q{}_{(11)}^{(00)}\right] - N_{(11)}^{(00)} Q_q{}_{(00)}^{(00)}\right\}, \tag{49a}$$

$$C_{\mathrm{LT}\,q}^{\cos\phi_{kp}\cos\phi_{\Delta p}} = -\frac{\alpha_{EM}^2 e_q^2 N_c^2 \sqrt{z(1-z)}\,p_\perp}{8\sqrt{2}\,\pi^5 W^2}\left\{N_{(00)}^{(00)}\left[I_{3q}{}_{(11)}^{(00)} - I_{3q}{}_{(01)}^{(10)}\right] - \frac{1}{2}(1-2z)^2\left[N_{(11)}^{(00)} - N_{(01)}^{(10)}\right]Q_q{}_{(00)}^{(00)} \right. \tag{49b}$$

$$+ N_{(11)}^{(00)} I_{3q}{}_{(10)}^{(10)} + \left[z^2+(1-z)^2\right]N_{(00)}^{(00)}\left[Q_q{}_{(01)}^{(10)} - Q_q{}_{(11)}^{(00)}\right] - \left[z^2+(1-z)^2\right]N_{(11)}^{(00)}Q_q{}_{(10)}^{(10)} - \frac{1}{2}(1-2z)^2 N_{(10)}^{(10)} Q_q{}_{(11)}^{(00)}$$

$$\left. + (1-2z)^2 N_{(00)}^{(00)}\left[3\,G_2{}_{(11)}^{(00)} - 2\,G_2{}_{(01)}^{(10)} + I_4{}_{(21)}^{(10)} + I_4{}_{(10)}^{(01)} - I_5{}_{(11)}^{(00)} + I_5{}_{(01)}^{(10)} + I_5{}_{(10)}^{(01)}\right] + (1-2z)^2 N_{(10)}^{(10)} G_2{}_{(11)}^{(00)}\right\},$$

$$C_{\mathrm{LT}\,q}^{\sin\phi_{kp}\sin\phi_{\Delta p}} = -\frac{\alpha_{EM}^2 e_q^2 N_c^2 \sqrt{z(1-z)}\,p_\perp}{8\sqrt{2}\,\pi^5 W^2}\left\{\left[z^2+(1-z)^2\right]N_{(00)}^{(00)}Q_q{}_{(11)}^{(00)} - N_{(00)}^{(00)} I_{3q}{}_{(11)}^{(00)} \right. \tag{49c}$$

$$+ \frac{1}{2}(1-2z)^2 N_{(11)}^{(00)} Q_q{}_{(00)}^{(00)} + (1-2z)^2 N_{(11)}^{(00)}\left[G_2{}_{(10)}^{(10)} + I_4{}_{(11)}^{(11)} + I_4{}_{(00)}^{(20)} + I_5{}_{(11)}^{(11)} - I_5{}_{(10)}^{(10)} + I_5{}_{(00)}^{(20)}\right]$$

$$+ (1-2z)^2 N_{(00)}^{(00)}\left[G_2{}_{(01)}^{(10)} - 3\,G_2{}_{(11)}^{(00)} - 2\,I_4{}_{(11)}^{(00)} + I_4{}_{(01)}^{(10)} - I_4{}_{(11)}^{(20)} - I_4{}_{(10)}^{(01)} + I_4{}_{(00)}^{(11)} + I_5{}_{(11)}^{(00)}\right.$$

$$\left.\left. - I_5{}_{(11)}^{(20)} - I_5{}_{(10)}^{(01)} + I_5{}_{(00)}^{(11)}\right]\right\}.$$

Fourier-Bessel transform:

$$F_{(ab)}^{(cd)}(p_\perp, \overline{Q}, W^2) \equiv \int_0^\infty dx_{12}\, x_{12}\, \left(p_\perp x_{12}\right)^c \left(\overline{Q} x_{12}\right)^d J_a(p_\perp x_{12}) K_b\left(\overline{Q}\, x_{12}\right)\, F(x_{12}^2, W^2)$$

DSA Results: Unpolarized Coefficients

$$D = \int dX \sum_{q=u,d,s} \left\{ \left[1 + (1-y)^2\right] D_{\mathrm{TT}\,q} + 4(1-y) D_{\mathrm{LL}\,q} \right\}.$$

$$D_{\mathrm{TT}\,q} = \frac{\alpha_{EM}^2 e_q^2 N_c^2 z(1-z)\left[z^2 + (1-z)^2\right] p_\perp}{8\pi^3} \left[N_{(11)}^{(00)}\right]^2,$$

$$D_{\mathrm{LL}\,q} = \frac{\alpha_{EM}^2 e_q^2 N_c^2 z^2 (1-z)^2}{4\pi^3} \left[N_{(00)}^{(00)}\right]^2.$$

c-Problem

- Degeneracy in Fourier-Bessel transform

$$F^{(0)}(r^2, W^2) = a\,\ln\frac{W^2}{\Lambda^2} + b\,\ln\frac{1}{r^2\Lambda^2} + c$$

$$F^{(cd)}_{(ab)} = \int_0^{1/\Lambda} dr\, r\;(p_\perp r)^c\,\left(\overline{Q}r\right)^d J_a(p_\perp r) K_b(\overline{Q}r) F(r^2, W^2)$$

$$= \left(a\ln\frac{W^2}{\Lambda^2} + c\right)\# + b\,\#'$$

c-Problem: Solution

$$\sqrt{s} = 30,\ 40\,\mathrm{GeV}$$
$$y = 0.2,\ 0.3,\ 0.43,\ 0.61,\ 0.86,\ 0.95$$

- Binning in $W^2 = sy$ breaks the degeneracy