

Moving Beyond a Single Fit: Bayesian Likelihood Analysis in Nucleon Structure Studies

Saraswati Pandey

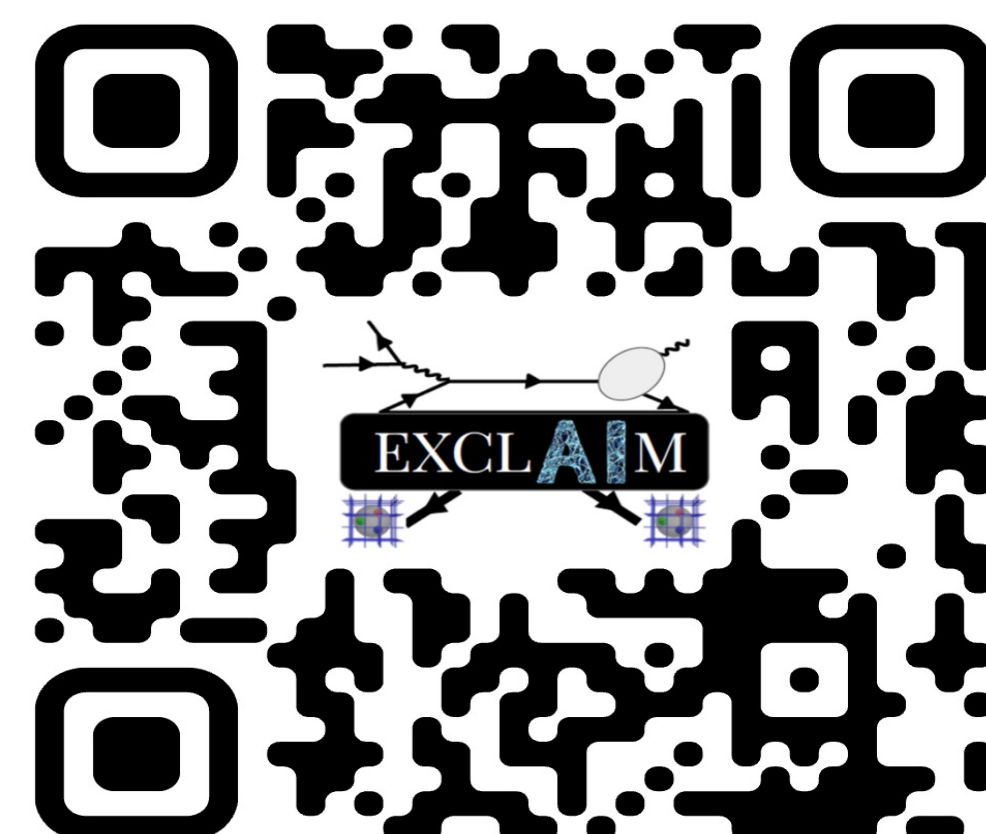
University of Virginia

Based on: [arXiv:2605.18589](https://arxiv.org/abs/2605.18589) and [arXiv:2410.23469](https://arxiv.org/abs/2410.23469)

Thanks to : Zaki, Douglas, Simonetta



Gluon Saturation, Diffraction and Multihadron Production
at the EIC: Theory, Simulations and Feasibility of
Measurements



*CFNS workshop, Stonybrook
University, July 14-17, 2026*



**Center for Frontiers
in Nuclear Science**

STONY BROOK UNIVERSITY +
BROOKHAVEN NATIONAL LABORATORY

Outline

□ Motivation

- Goal at EIC

- Inverse problems in QCD

□ Approach

- Maximum likelihood estimate

- MCMC Sampling Algorithms

□ Extraction CFFs from exclusive data

- Formalism

- Obtained Results (DVCS and $DV\pi^0$)

□ ΔG Extraction

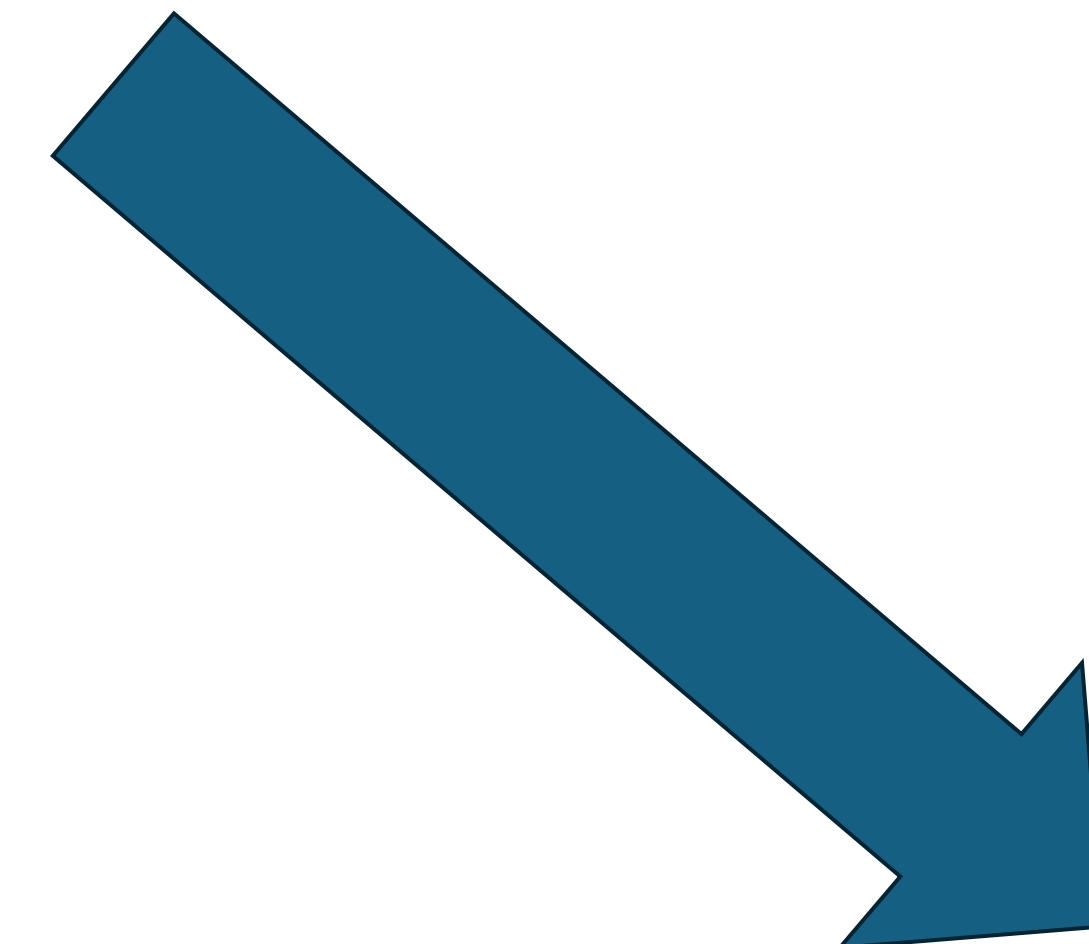
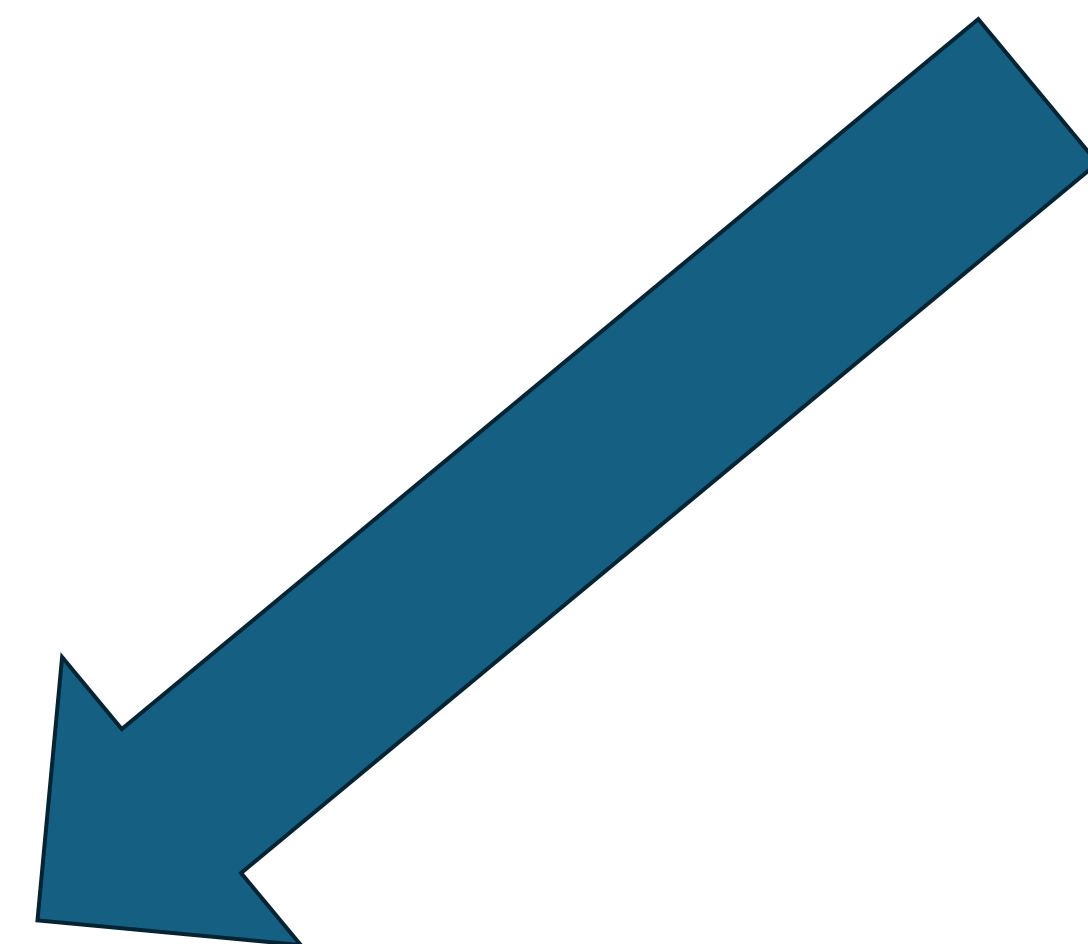
□ Conclusion and Next Steps

Motivation for EIC

Imaging transverse spatial distribution of
quarks and gluons

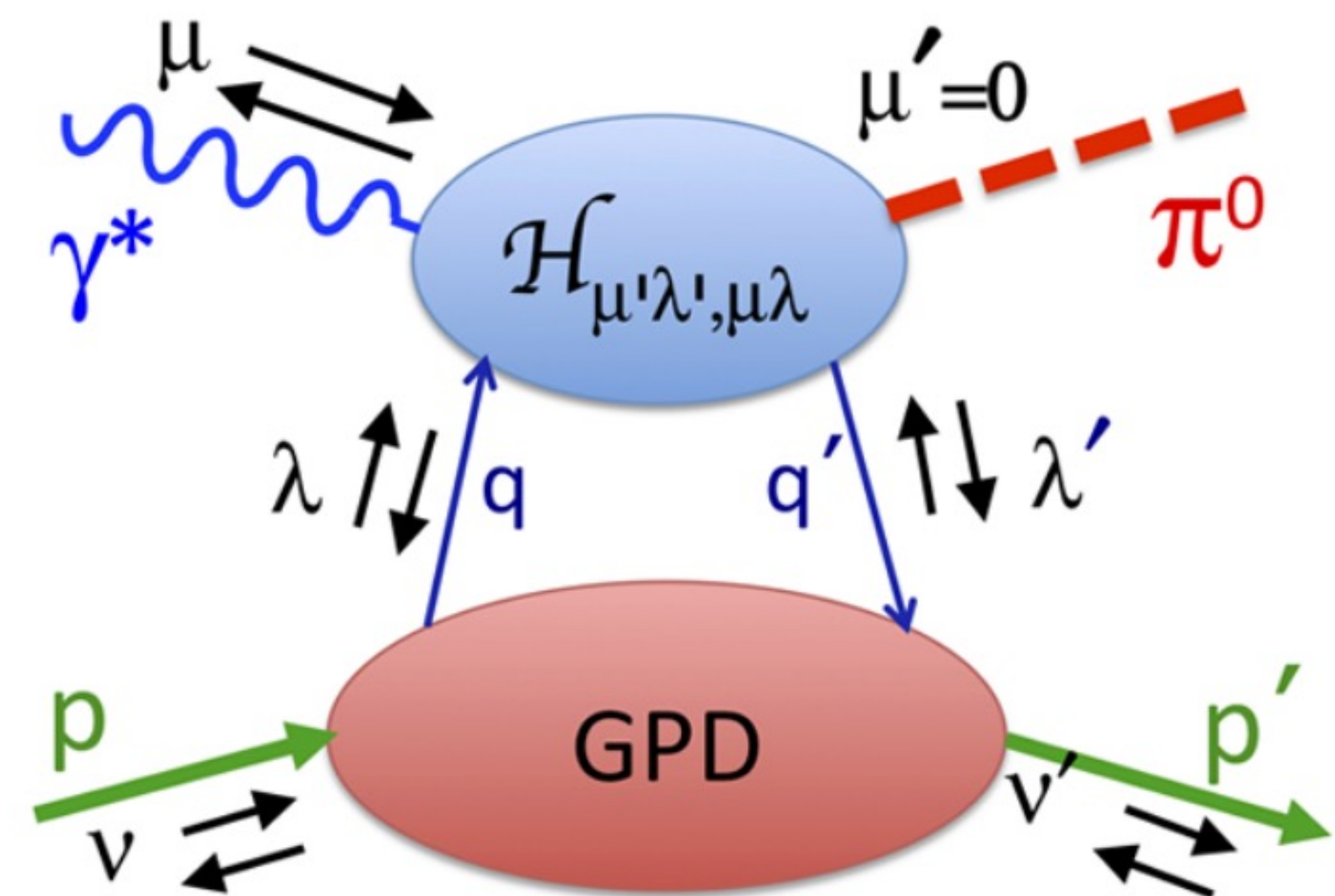
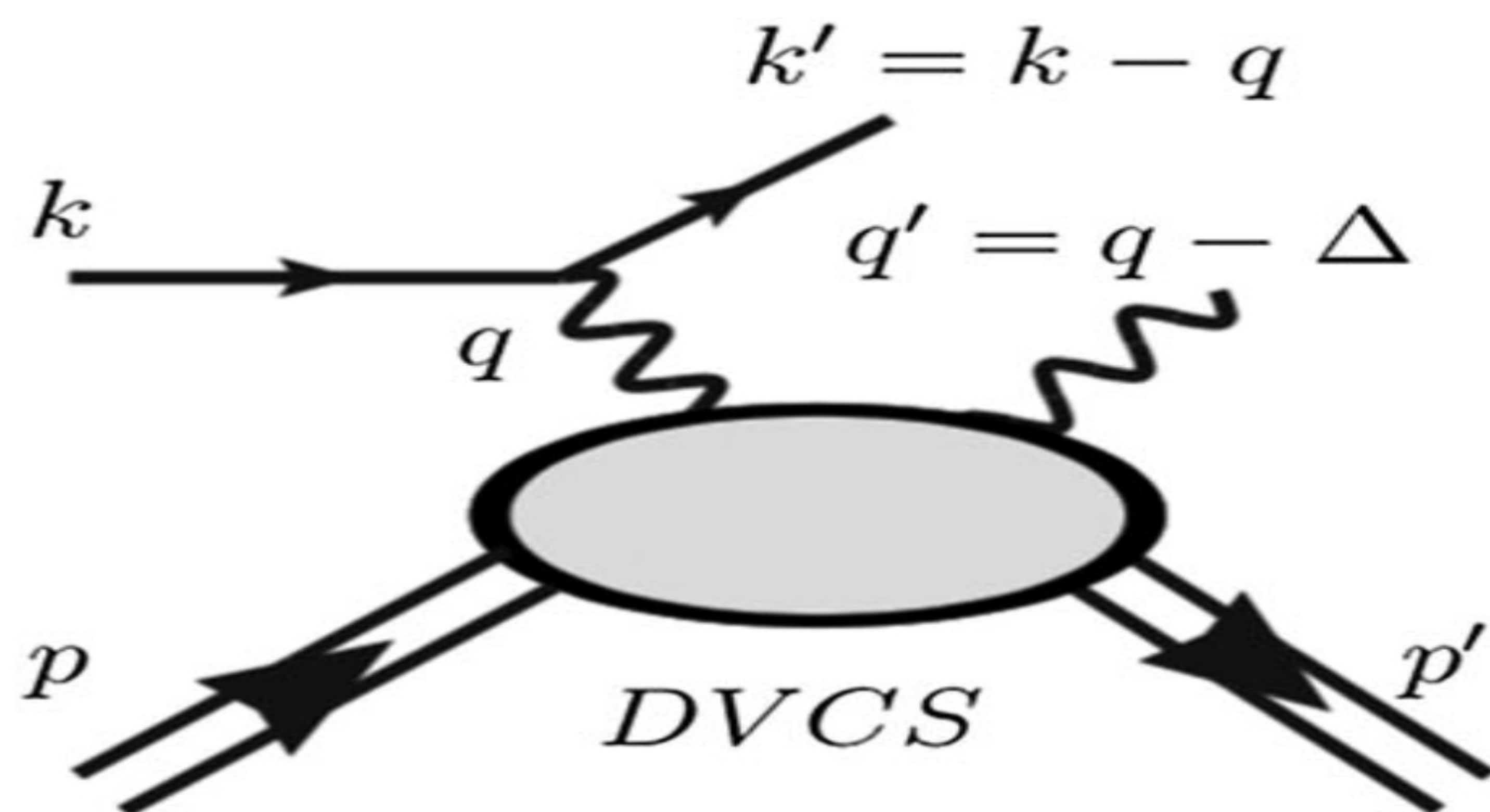


Exclusive reactions



DVCS

DVMP



Generalized parton distributions (GPDs)

□ QCD matrix element between the p and p'

□ Non-forward limit $\longrightarrow$ quark and gluon distributions

More information from GPDs

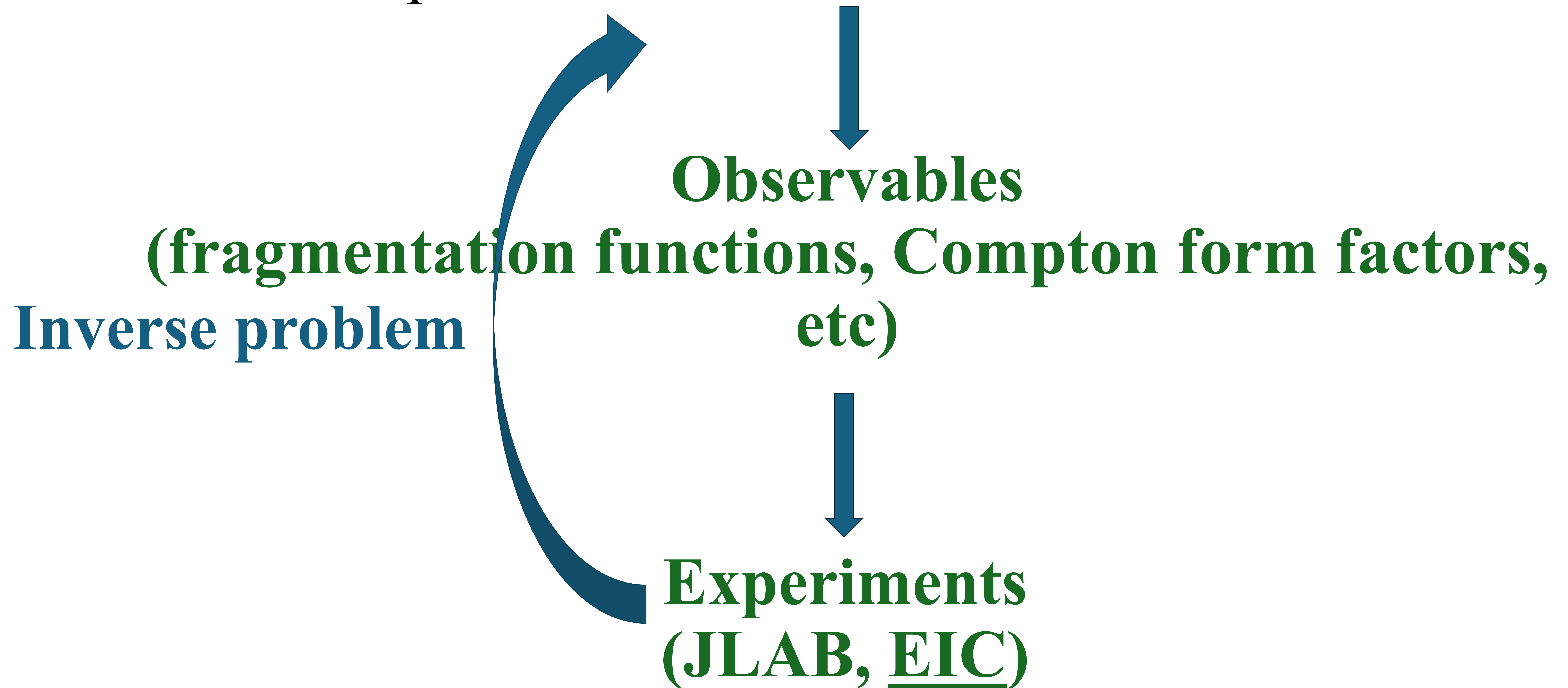
□ second moment of GPDs $\longrightarrow$ total angular momentum of quarks and gluons in the proton.

□ probe the energy-momentum tensor $\longrightarrow$ nucleon mass.

□ pressure and shear forces inside hadrons $\longrightarrow$ higher twist

Inverse problems in QCD

□ First inverse problems **QCD Theory**



□ Second inverse problem

$$\text{CFF} = \int (\text{QCD Kernel}) \times \text{GPD}$$

Approach

Maximum likelihood estimate

□ We use a “canonical” approach to perform a Bayesian likelihood analysis.

□ The joint likelihood of the parameters of interest is calculated as a simple product of Gaussians.

$$\mathcal{L} = \prod_{i=1}^N \text{Gaussian}(x, \mu, \sigma)$$

$$\text{Gaussian}(x, \mu, \sigma) \propto \exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right]$$

posterior

$$P(\theta | x) = \frac{P(x | \theta) P(\theta)}{P(x)}$$

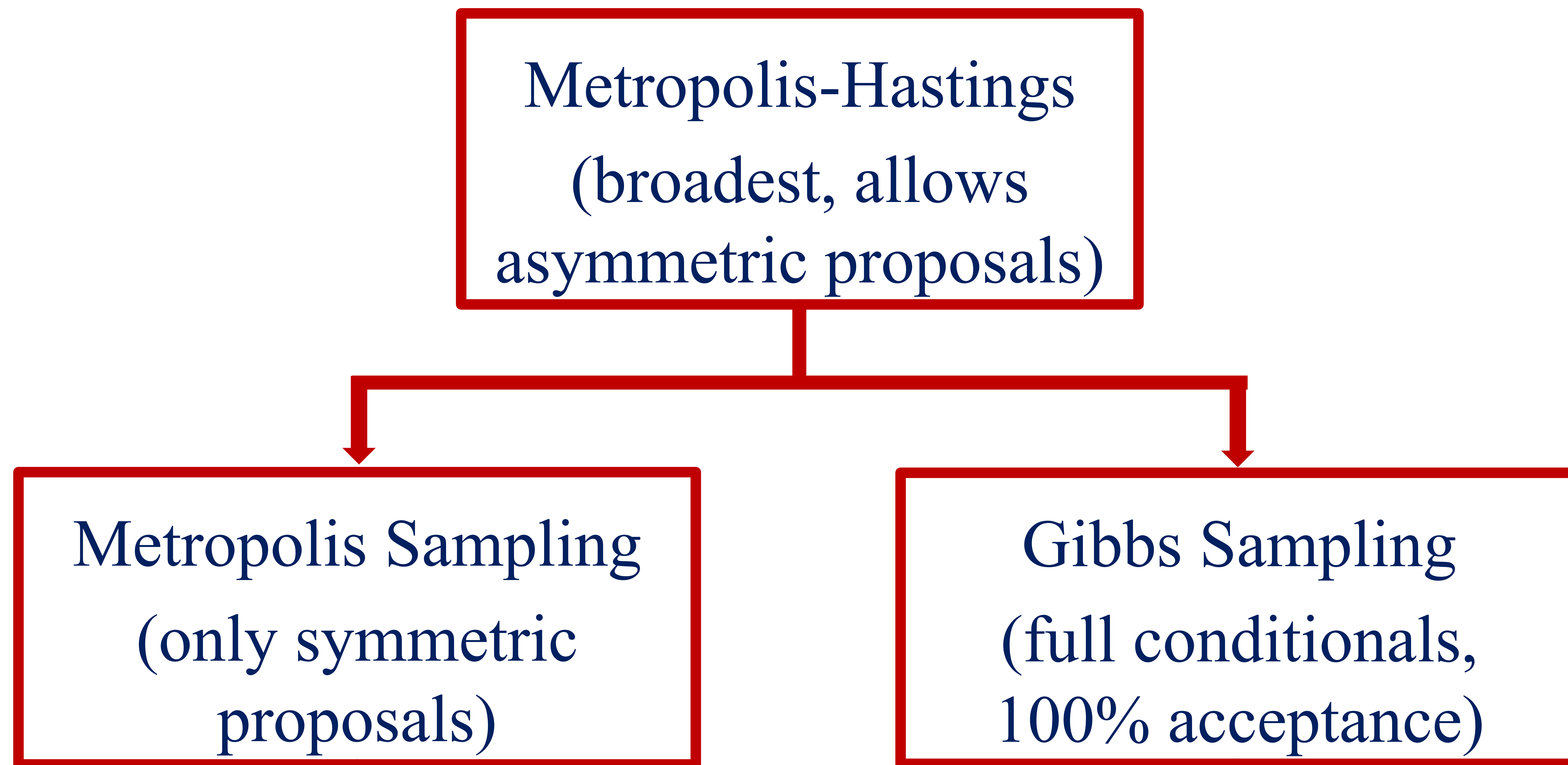
likelihood

prior

□ Markov Chain Monte Carlo (MCMC) algorithms are used to take multidimensional probability density functions and generate set of representative samples.

□ These samples are used to create easy visualizations of the samples in the form corner plots.

Markov Chain Monte Carlo Sampling



Main goal: sample from some kind of probability distribution $p(x)$

using some function $f(x)$. $\{ p(x) = f(x)/NC \}$

NC = normalization constant



[*arXiv:1202.3665*](#)

[*arXiv:1008.4686*](#)

Approach (cont.⁶)

Sampling process

□ Step 1:

- Take some distribution x_t
- Propose an independent candidate (x_t)
- Next candidate (x_{t+1}) centered at the mean of x_t
- Obtain a chain in this manner



Not definite

□ Step 2:

$$p(a)T(a \rightarrow b) = p(b)T(b \rightarrow a)$$

yet!



$$\frac{A(a \rightarrow b)}{A(b \rightarrow a)} = \underbrace{\frac{f(b)}{f(a)}}_{r_f} \times \underbrace{\frac{g(a|b)}{g(b|a)}}_{r_g}$$

**condition of
detailed balance**

$A(a \rightarrow b) = \text{acceptance probability}$

$$A(a \rightarrow b) = \text{Min}(1, r_f r_g)$$

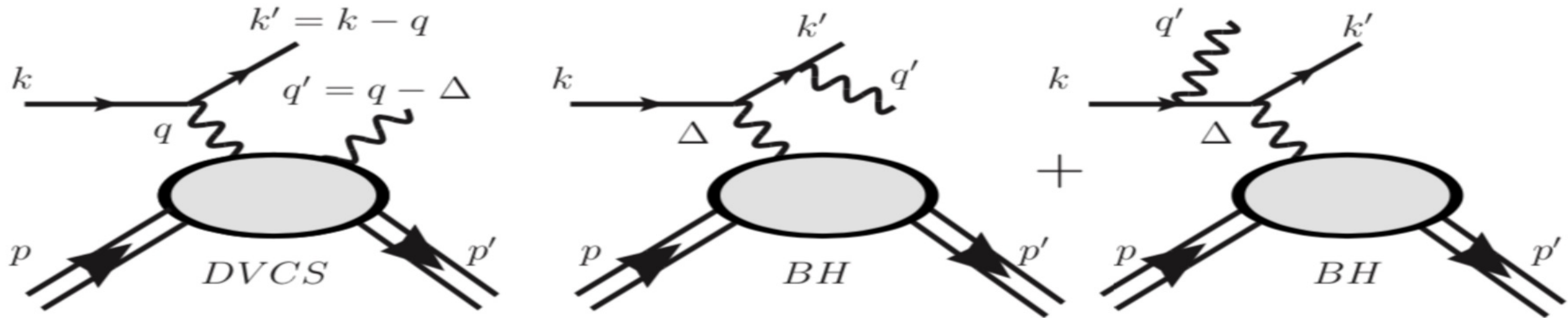
Approach (cont.⁷)

Markov Chain Monte Carlo Sampling

MONTE CARLO METHOD

Posterior Parameter Space Exploration

Chiral Even CFFs Extraction



$$e \ p \longrightarrow e' \ p' \ \gamma$$

$$\begin{aligned} \frac{d^5 \sigma_{\text{DVCS}}}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} &= \Gamma |T_{\text{DVCS}}|^2 \\ &= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + (2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right. \\ &\quad + (2\Lambda) \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + (2h) \left(\sqrt{1-\epsilon^2} F_{LL} + 2\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\ &\quad + (2\Lambda_T) \left[\sin(\phi - \phi_S) (F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ &\quad \left. + \sqrt{2\epsilon(1+\epsilon)} (\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)}) \right] \\ &\quad + (2h)(2\Lambda_T) \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\ &\quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \left. \right\}. \end{aligned} \quad (36)$$

$$\begin{aligned} \frac{d^5 \sigma_{\text{unpol}}^{\text{BH}}}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} &\equiv \frac{\Gamma}{t} F_{UU}^{\text{BH}} \\ &= \frac{\Gamma}{t} [A(y, x_{Bj}, t, Q^2, \phi) (F_1^2 + \tau F_2^2) \\ &\quad + B(y, x_{Bj}, t, Q^2, \phi) \tau G_M^2(t)] \quad (143) \end{aligned}$$

with,

$$\begin{aligned} A &= \frac{8M^2}{t(kq')(k'q')} [4\tau((kP^2) + (k'P^2)) \\ &\quad - (\tau + 1)((k\Delta^2) + (k'\Delta^2))] \quad (144) \end{aligned}$$

$$B = \frac{16M^2}{t(kq')(k'q')} [(k\Delta^2) + (k'\Delta^2)], \quad (145)$$

$$|T|^2 = |T_{\text{BH}} + T_{\text{DVCS}}|^2 = |T_{\text{BH}}|^2 + |T_{\text{DVCS}}|^2 + \mathcal{I},$$

$$\mathcal{I} = T_{\text{BH}}^* T_{\text{DVCS}} + T_{\text{DVCS}}^* T_{\text{BH}}.$$

*S. Liuti, G. R. Goldstein,
et al, Phys. Rev. D 101,
054021 (2020).* 9

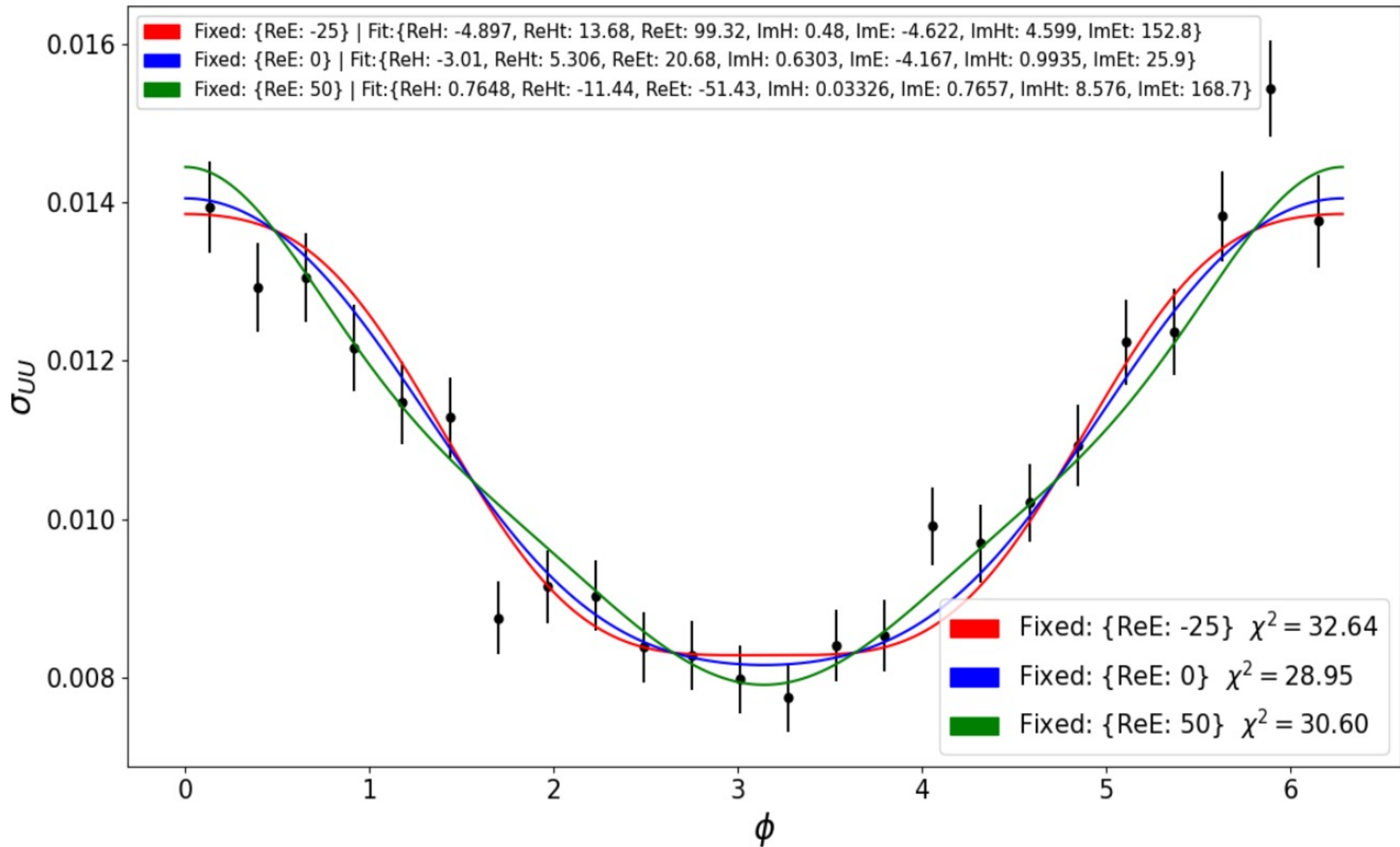
Experimental data

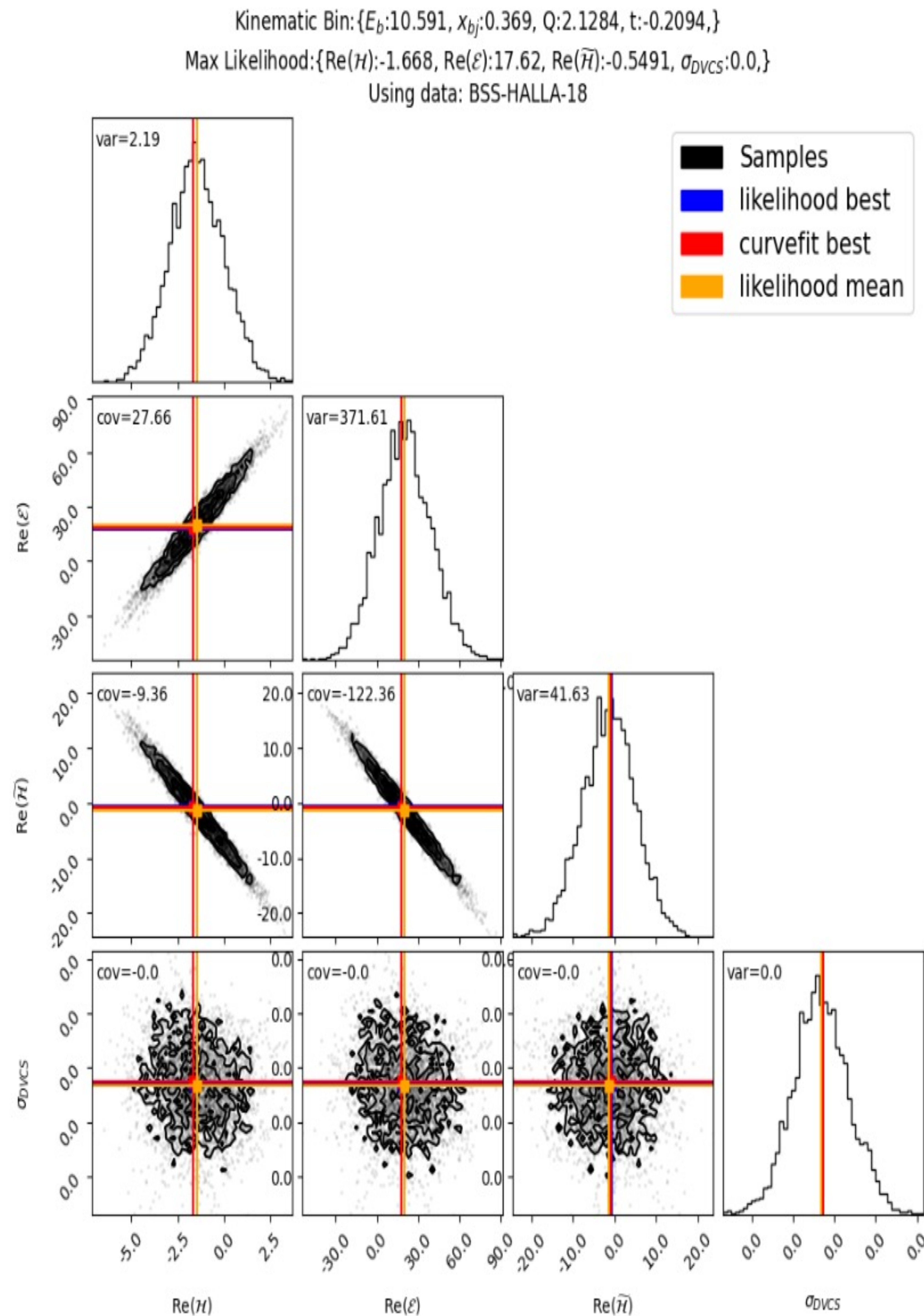
E_{beam} (GeV)	x_{Bj}	Q^2 (GeV ²)	t (GeV ²)	ϕ (deg)	σ_{total}	$\Delta\sigma$
10.591	0.369	4.53	-0.2094	7.5	0.01394	0.00058
10.591	0.369	4.53	-0.2094	22.5	0.01292	0.00056
10.591	0.369	4.53	-0.2094	37.5	0.01305	0.00056
10.591	0.369	4.53	-0.2094	52.5	0.01216	0.00054
10.591	0.369	4.53	-0.2094	67.5	0.01147	0.00052
10.591	0.369	4.53	-0.2094	82.5	0.01128	0.00051
10.591	0.369	4.53	-0.2094	97.5	0.00875	0.00046
10.591	0.369	4.53	-0.2094	112.5	0.00915	0.00046
10.591	0.369	4.53	-0.2094	127.5	0.00904	0.00045
10.591	0.369	4.53	-0.2094	142.5	0.00838	0.00044
10.591	0.369	4.53	-0.2094	157.5	0.00828	0.00044
10.591	0.369	4.53	-0.2094	172.5	0.00798	0.00043
10.591	0.369	4.53	-0.2094	187.5	0.00774	0.00043
10.591	0.369	4.53	-0.2094	202.5	0.00841	0.00045
10.591	0.369	4.53	-0.2094	217.5	0.00853	0.00045
10.591	0.369	4.53	-0.2094	232.5	0.00991	0.00049
10.591	0.369	4.53	-0.2094	247.5	0.00969	0.00049
10.591	0.369	4.53	-0.2094	262.5	0.01021	0.00049
10.591	0.369	4.53	-0.2094	277.5	0.01093	0.00051
10.591	0.369	4.53	-0.2094	292.5	0.01223	0.00054
10.591	0.369	4.53	-0.2094	307.5	0.01236	0.00054
10.591	0.369	4.53	-0.2094	322.5	0.01382	0.00057
10.591	0.369	4.53	-0.2094	337.5	0.01543	0.00061
10.591	0.369	4.53	-0.2094	352.5	0.01376	0.00058

**Jefferson Lab
Hall A**

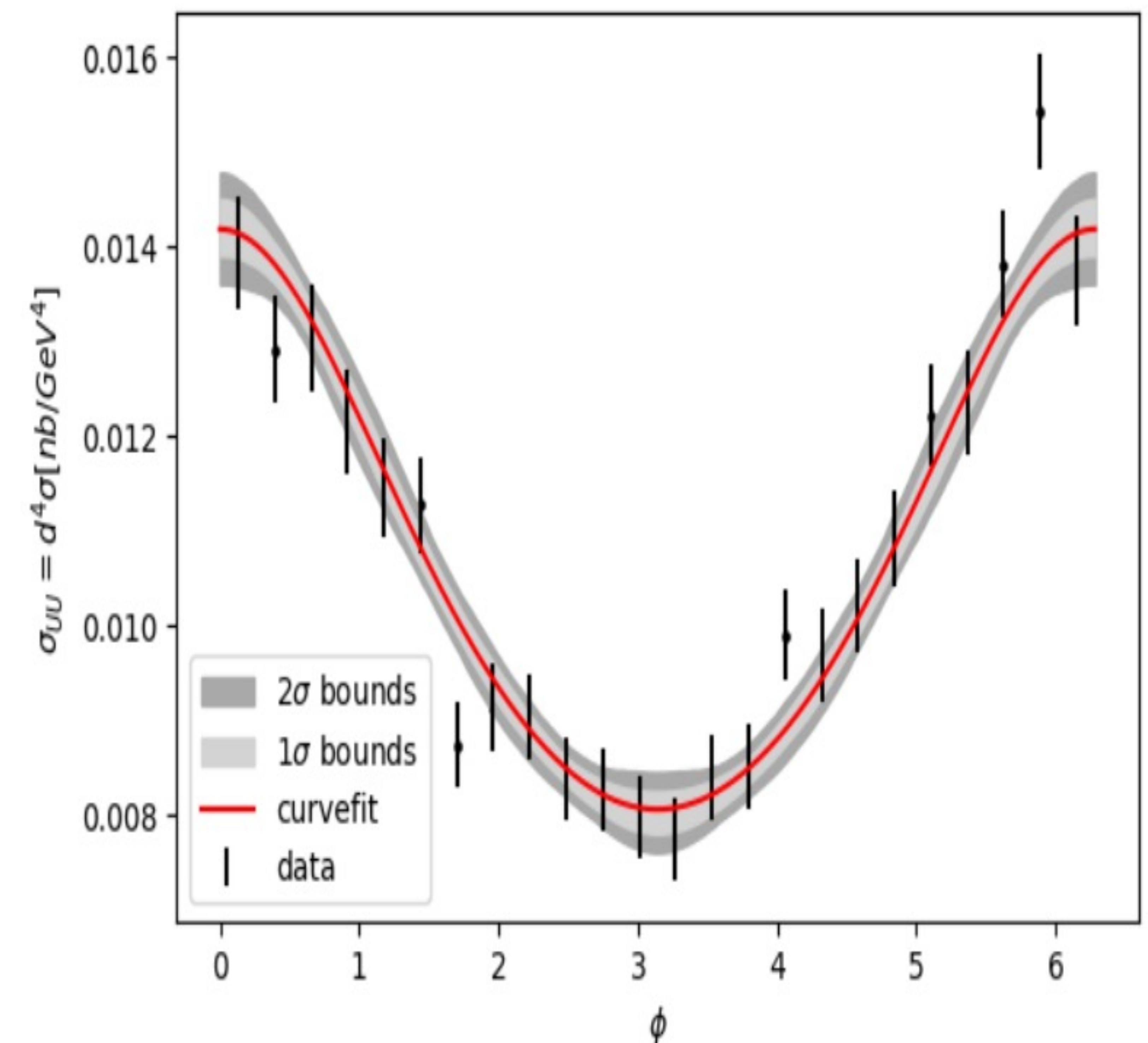
***Phys. Rev. Lett. 128,
252002 (2022),
2201.03714***

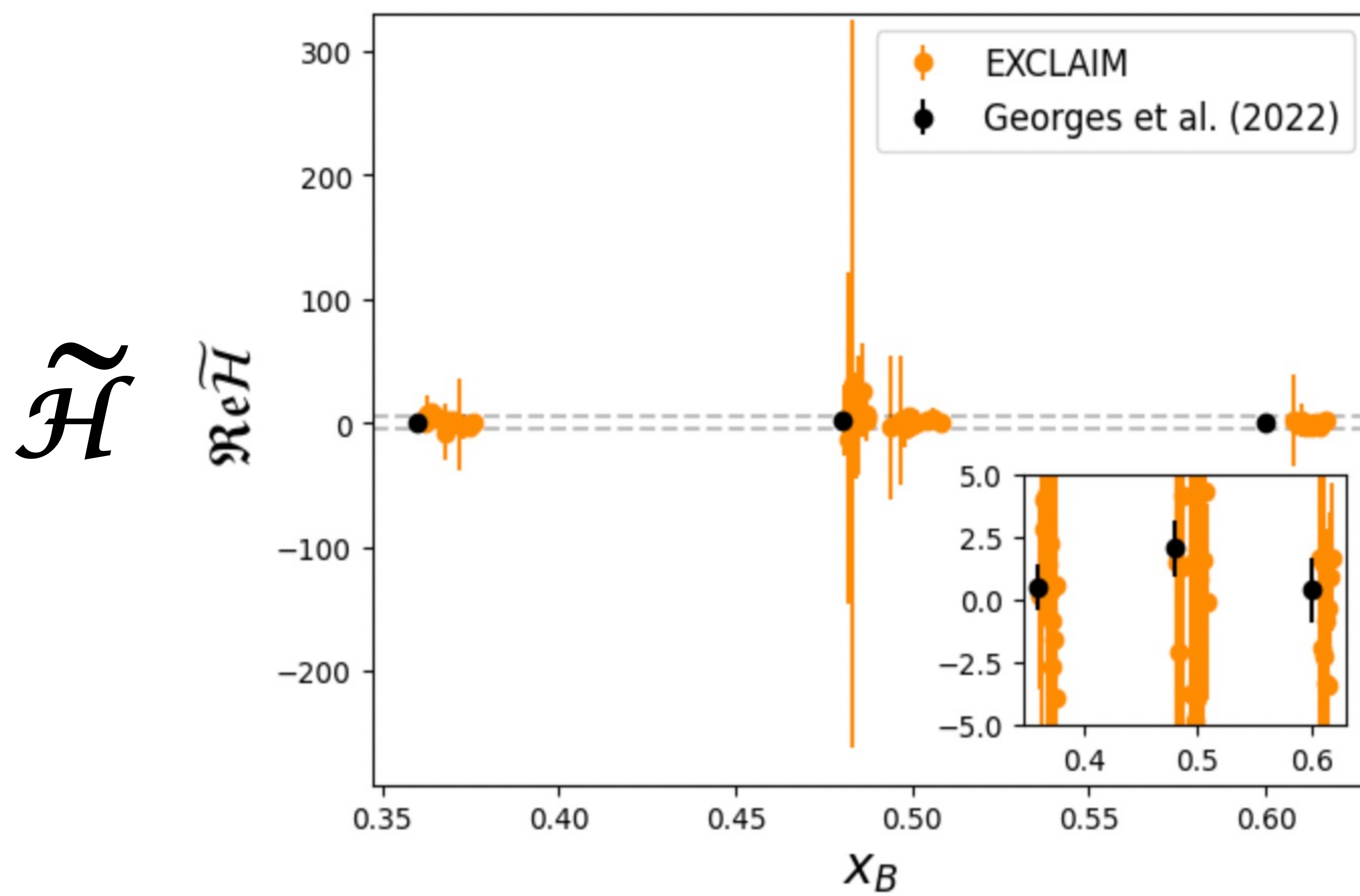
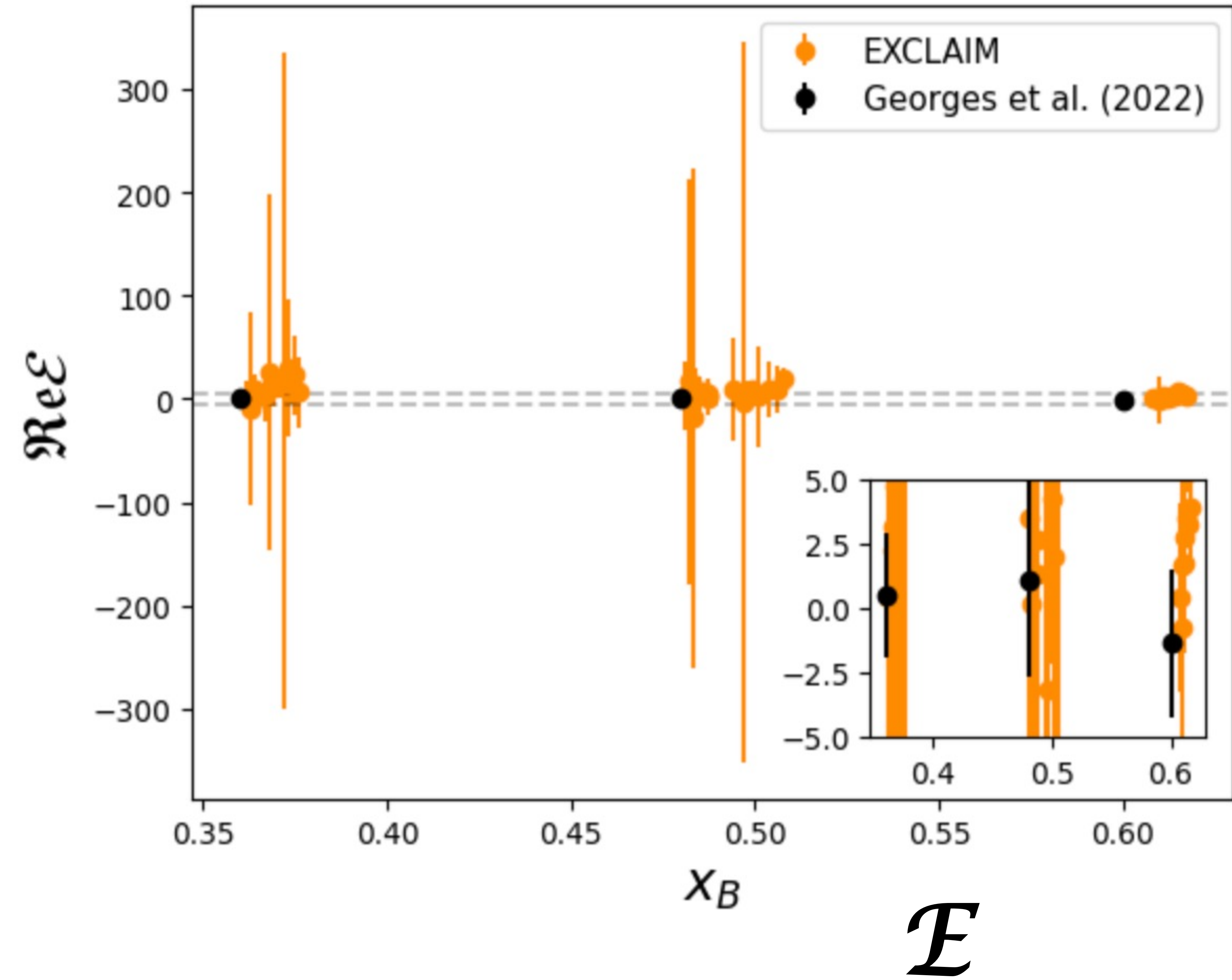
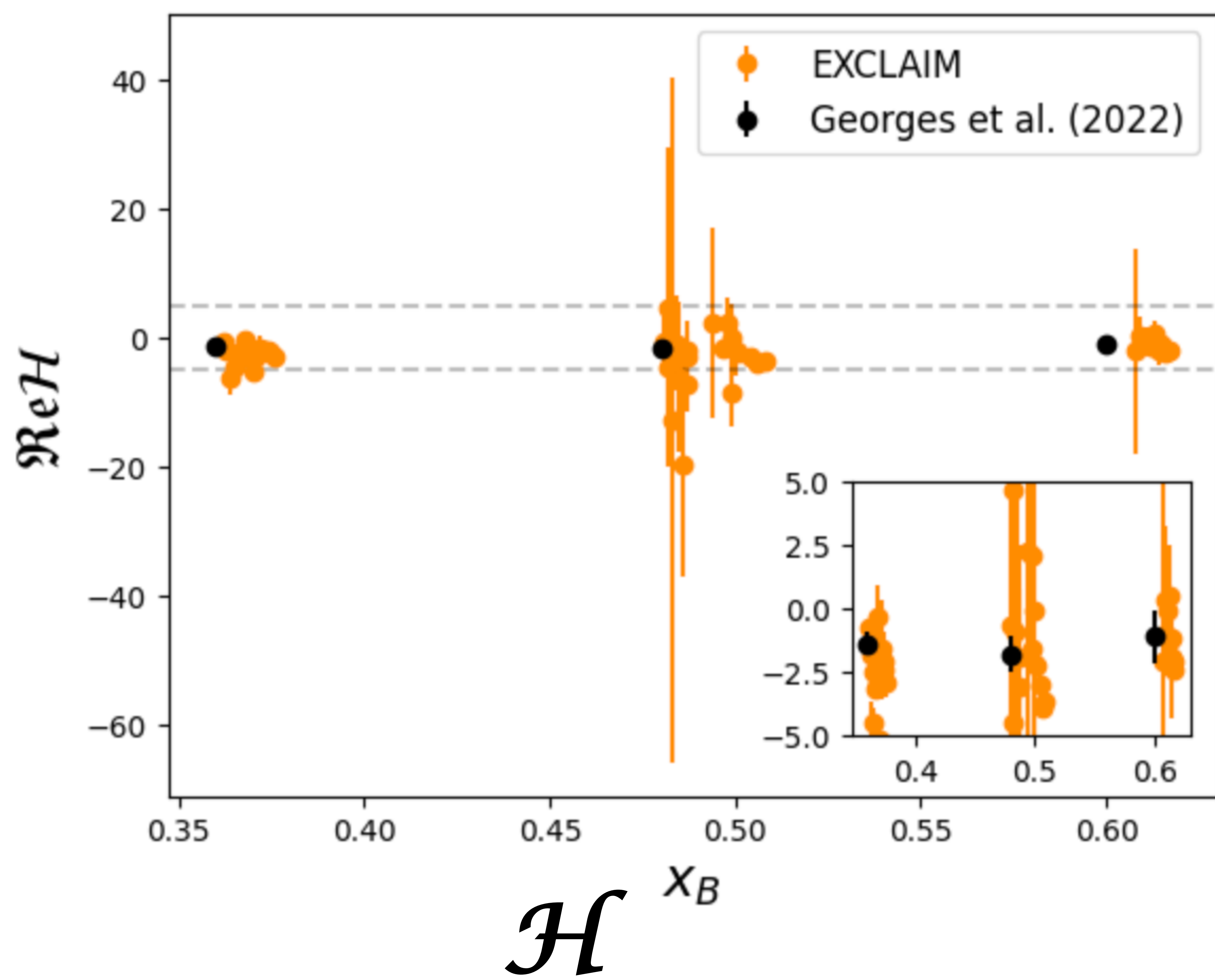
{Eb:10.591, x:0.369, Q:2.1284, t:-0.2094,}





Kinematic Bin: $\{E_b:10.591, x_{bj}:0.369, Q:2.1284, t:-0.2094,\}$
 Best Curve Fit: $\{\text{Re}(\mathcal{H}): -1.641, \text{Re}(\mathcal{E}): 18.13, \text{Re}(\overline{\mathcal{H}}): -0.7673, \sigma_{DVCS}: 0.0,\}$
 Using data: BSS-HALLA-18



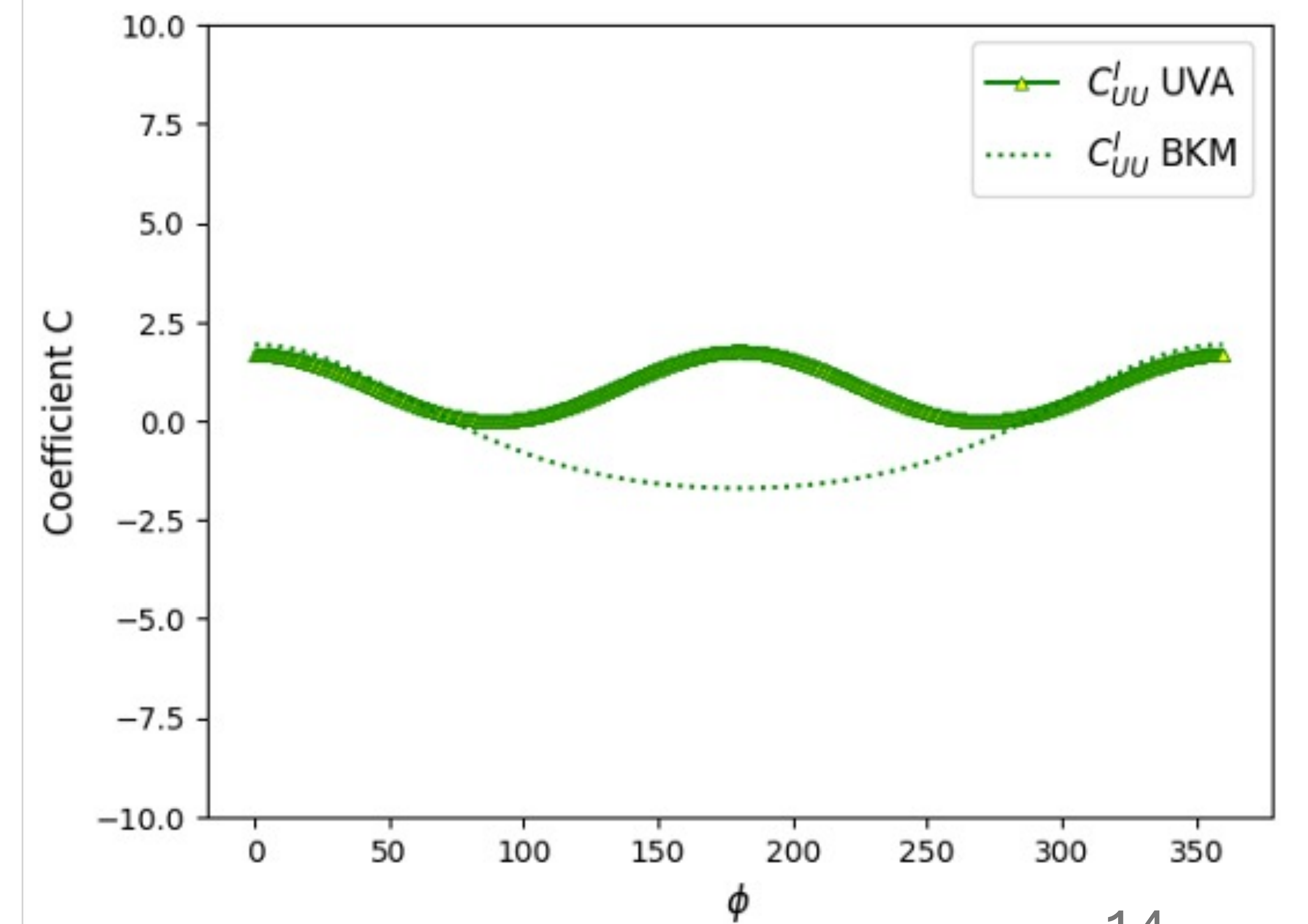
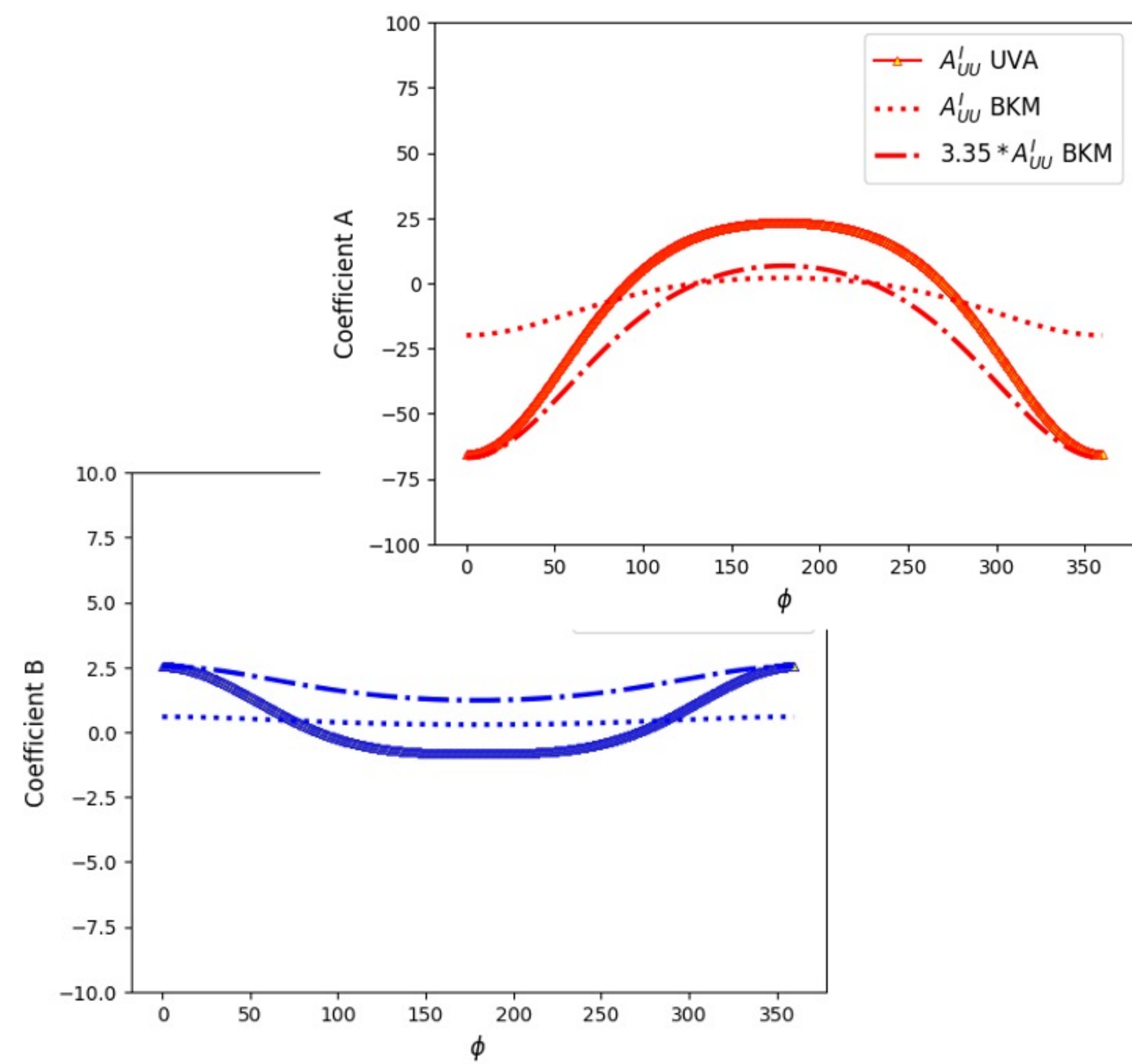
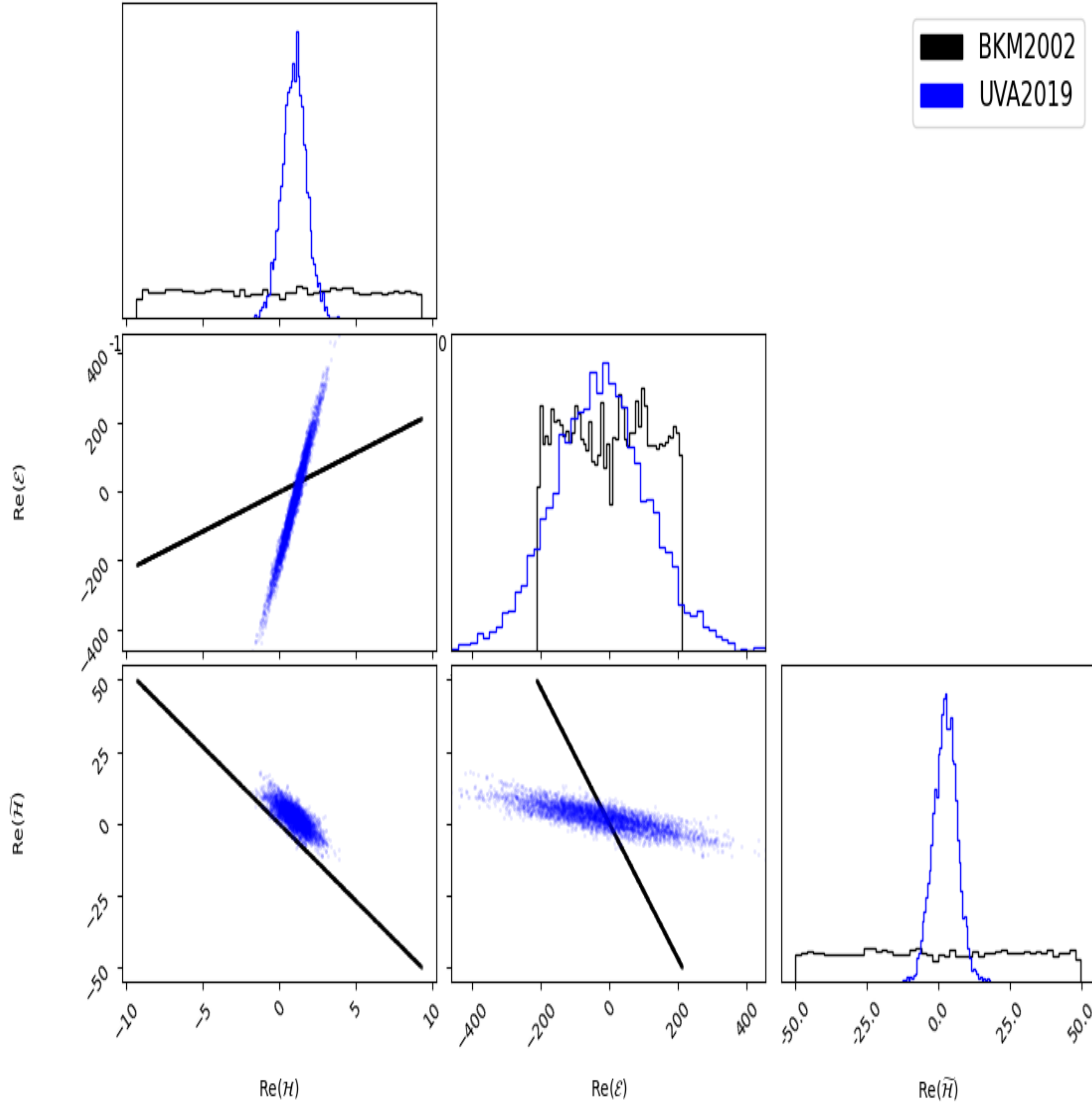


[arXiv:2410.23469](#)

Kinematic Bin: {Eb:10.591, x:0.369, Q:2.1284, t:-0.2094,}

Using data: BSS-HALLA-18

Using prior: (-50<Re($\mathcal{H}$)<50) & (-500<Re($\mathcal{E}$)<500) & (-50<Re($\tilde{\mathcal{H}}$)<50)

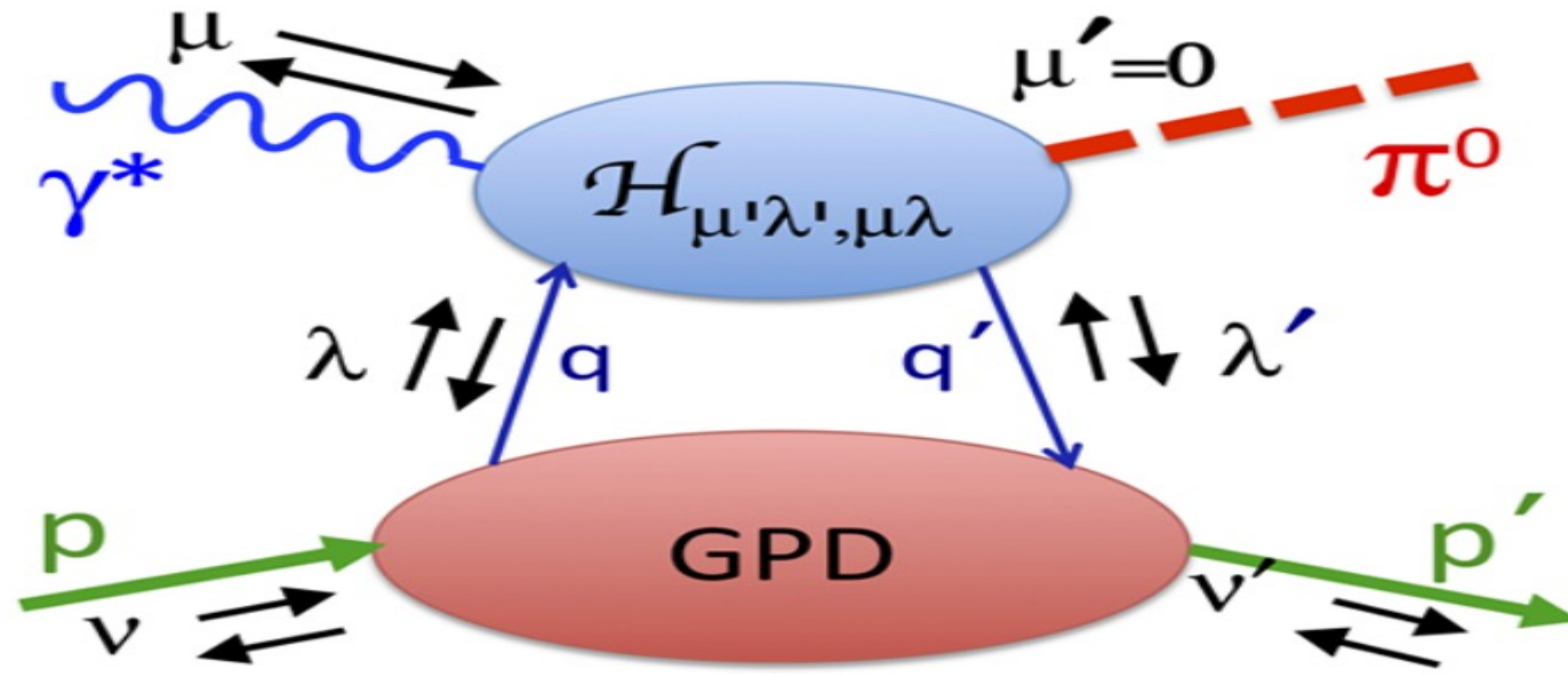


At twist2

$$\underline{\underline{F_{UU}^{\mathcal{I},tw2}}} = \underline{\underline{A_{UU}^{\mathcal{I}}}} \Re (F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + \underline{\underline{B_{UU}^{\mathcal{I}}}} G_M \Re (\mathcal{H} + \mathcal{E}) + \underline{\underline{C_{UU}^{\mathcal{I}}}} G_M \Re \tilde{\mathcal{H}}$$

ϕ = virtual photon polarization vector's phase

Chiral Odd CFFs Extraction



Liuti et. al, Phys. Rev. D 91, 114013 (2015)

Liuti et. al, Phys. Rev. D 79, 054014 (2009)

arxiv : 1401.0438

$e p \longrightarrow e' p' \pi^0$

$$\begin{aligned}
 \frac{d^4\sigma}{dx_{Bj} dy d\phi dt} = & \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\
 & + S_{||} \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\
 & - S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \frac{\epsilon}{2} \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \right. \\
 & + \left. \left. \sqrt{\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \right. \\
 & + \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \right\} \quad (74)
 \end{aligned}$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}} = \boxed{A_{UL}^{\sin \phi} \sin \phi} + \boxed{A_{UL}^{\sin 2\phi} \sin 2\phi}$$

$$A_{LL} = \frac{N_{s_z=+}^{\rightarrow} - N_{s_z=-}^{\rightarrow} + N_{s_z=+}^{\leftarrow} - N_{s_z=-}^{\leftarrow}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2} F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi}}{F_{UU,T} + \epsilon F_{UU,L}} = \boxed{A_{LL}} + \boxed{A_{LL}^{\cos \phi} \cos \phi}$$

Theoretical background

$$W_{\Lambda',\Lambda}^{[i\sigma^{i+}\gamma_5]}(x,\xi,t) = \frac{1}{2\bar{P}^+} \bar{U}(P',\Lambda') \left(i\sigma^{+i} H_T(x,\xi,t) + \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2M} E_T(x,\xi,t) \right. \\ \left. + \frac{P^+ \Delta^i - \Delta^+ P^i}{M^2} \tilde{H}_T(x,\xi,t) + \frac{\gamma^+ P^i - P^+ \gamma^i}{2M} \tilde{E}_T(x,\xi,t) \right) U(P,\Lambda)$$

$$f_{\Lambda_\gamma 0}^{\Lambda\Lambda'}(\zeta,t) = \sum_{\lambda,\lambda'} g_{\Lambda_\gamma 0}^{\lambda\lambda'}(X,\zeta,t,Q^2) \otimes A_{\Lambda'\lambda',\Lambda\lambda}(X,\zeta,t),$$

**hard
process
helicity
amplitude**

$$f_{10}^{++} = g_{10}^{+-} \otimes A_{+-,++}$$

$$f_{10}^{+-} = g_{10}^{+-} \otimes A_{--,++}$$

$$f_{10}^{-+} = g_{10}^{+-} \otimes A_{+-,-+}$$

$$f_{10}^{--} = g_{10}^{+-} \otimes A_{++,+-}$$

$$f_{00}^{+-} = g_{00}^{+-} \otimes (A_{--,++} - A_{+-,-+})$$

$$f_{00}^{++} = g_{00}^{+-} \otimes (A_{++,+-} - A_{+-,++}),$$

**chiral-odd
quark proton
helicity
amplitude**

Chiral-Odd quark proton helicity amplitudes



total helicity amplitudes



$$\begin{aligned}
 A_{++,-} &= \sqrt{1-\xi^2} \left[H_T + \frac{t_0-t}{4M^2} \tilde{H}_T - \frac{\xi^2}{1-\xi^2} E_T + \frac{\xi}{1-\xi^2} \tilde{E}_T \right] \\
 &= \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[H_T + \frac{t_0-t}{4M^2} \tilde{H}_T - \frac{\zeta^2/4}{1-\zeta} E_T + \frac{\zeta/2}{1-\zeta} \tilde{E}_T \right] \\
 A_{+-,-} &= -e^{-i2\phi} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{H}_T = -e^{-i2\phi} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0-t}{4M^2} \tilde{H}_T
 \end{aligned}$$

$$\begin{aligned}
 A_{++,+} &= e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{H}_T + (1-\xi) (E_T + \tilde{E}_T) \right] \\
 &= e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{H}_T + \frac{1-\zeta}{1-\zeta/2} (E_T + \tilde{E}_T) \right] , \\
 A_{-+,-} &= e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{H}_T + (1+\xi) (E_T - \tilde{E}_T) \right] \\
 &= e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{H}_T + \frac{1}{1-\zeta/2} (E_T - \tilde{E}_T) \right] .
 \end{aligned}$$

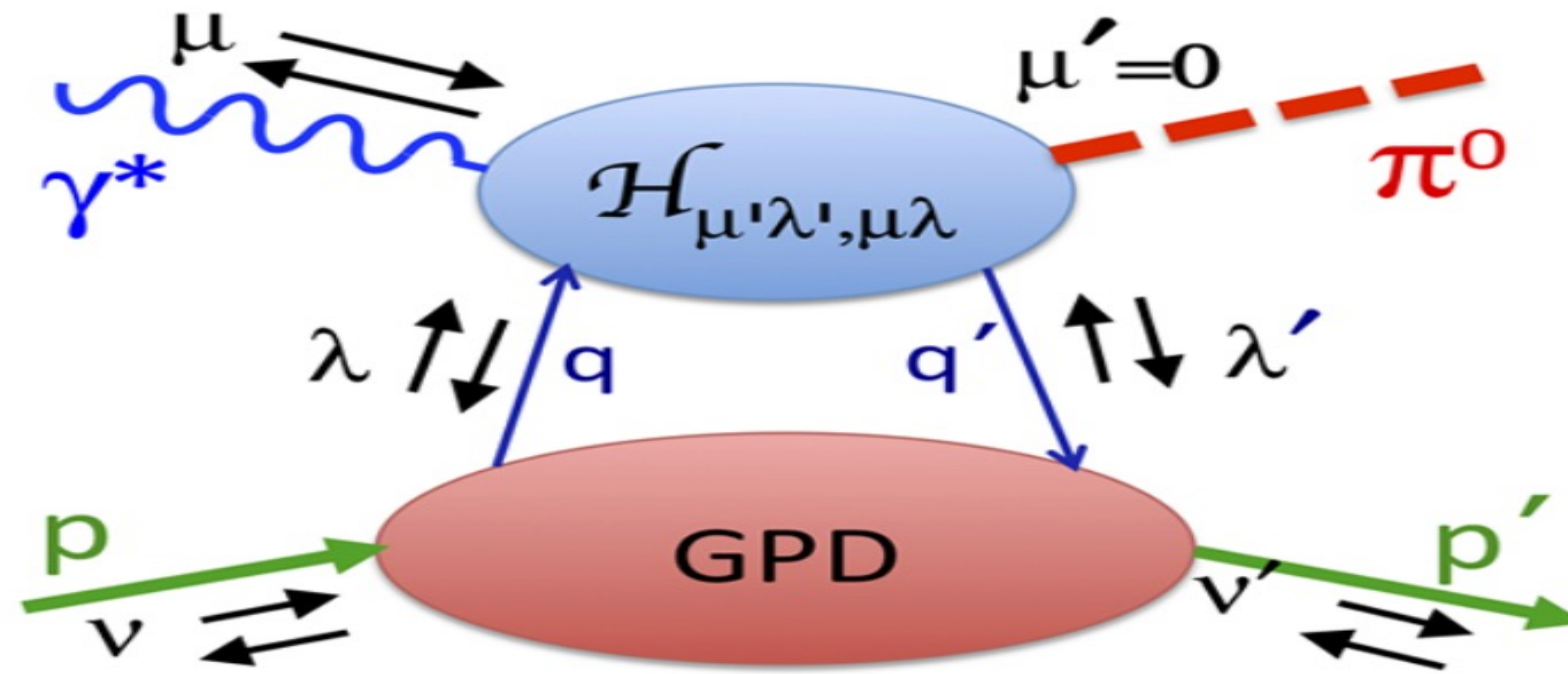
$$\begin{aligned}
 g_{\Lambda_{\gamma^*}, \lambda; 0, \lambda'}^V &= \int dx_1 dy_1 \int d^2b \hat{\psi}_V(y_1, b) \hat{\mathcal{F}}_{\Lambda_{\gamma^*}, \lambda; 0, \lambda'}(Q^2, x_1, x_2, b) \alpha_S(\mu_R) \exp[-S] \hat{\phi}_{\pi^0}(x_1, b) \\
 g_{\Lambda_{\gamma^*}, \lambda; 0, \lambda'}^A &= \int dx_1 dy_1 \int d^2b \hat{\psi}_A(y_1, b) \hat{\mathcal{F}}_{\Lambda_{\gamma^*}, \lambda; 0, \lambda'}(Q^2, x_1, x_2, b) \alpha_S(\mu_R) \exp[-S] \hat{\phi}_{\pi^0}(x_1, b)
 \end{aligned}$$



hard process helicity amplitudes

$$\begin{aligned}
 f_{10}^{++} &= g_{\pi}^{V, odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1+\xi) (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V, odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1}{1-\zeta/2} (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right] , \\
 f_{10}^{+-} &= \frac{g_{\pi}^{V, odd}(Q) + g_{\pi}^{A, odd}(Q)}{2} \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T - \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \\
 &= \frac{g_{\pi}^{V, odd}(Q) + g_{\pi}^{A, odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T - \frac{\zeta^2/4}{1-\zeta} \mathcal{E}_T + \frac{\zeta/2}{1-\zeta} \tilde{\mathcal{E}}_T \right] \\
 f_{10}^{-+} &= -\frac{g_{\pi}^{A, odd}(Q) - g_{\pi}^{V, odd}(Q)}{2} e^{-i2\phi} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 &= -\frac{g_{\pi}^{A, odd}(Q) - g_{\pi}^{V, odd}(Q)}{2} e^{-i2\phi} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 f_{10}^{--} &= g_{\pi}^{V, odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1-\xi) (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V, odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1-\zeta}{1-\zeta/2} (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right] \\
 f_{00}^{+-} &= g_{\pi}^{A, odd}(Q) e^{-i\phi} \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \\
 f_{00}^{++} &= -g_{\pi}^{A, odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\xi \mathcal{E}_T + \tilde{\mathcal{E}}_T \right] .
 \end{aligned}$$

Formalism



$$e p \longrightarrow e' p' \pi^0$$

Liuti et. al, Phys. Rev. D 91, 114013 (2015)

Liuti et. al, Phys. Rev. D 79, 054014 (2009)

arxiv : 1401.0438

$$\begin{aligned} \frac{d^4\sigma}{dx_B dy d\phi dt} = & \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\ & + S_{||} \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\ & - S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \frac{\epsilon}{2} \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \right. \\ & + \left. \left. \sqrt{\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \right. \\ & + \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \right\} \end{aligned} \quad (74)$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}} = \boxed{A_{UL}^{\sin \phi} \sin \phi} + \boxed{A_{UL}^{\sin 2\phi} \sin 2\phi}$$

$$A_{LL} = \frac{N_{s_z=+}^{\rightarrow} - N_{s_z=-}^{\rightarrow} + N_{s_z=+}^{\leftarrow} - N_{s_z=-}^{\leftarrow}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2} F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi}}{F_{UU,T} + \epsilon F_{UU,L}} = \boxed{A_{LL}} + \boxed{A_{LL}^{\cos \phi} \cos \phi}$$

$$F_{UU,T} = \frac{1}{2}(F_{11}^{++} + F_{11}^{--}) = \frac{1}{2} \sum_{\Lambda'} (f_{10}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{10}^{-\Lambda'} * f_{10}^{-\Lambda'})$$

$$= \frac{1}{2} (|f_{10}^{++}|^2 + |f_{10}^{+-}|^2 + |f_{10}^{-+}|^2 + |f_{10}^{--}|^2)$$

$$F_{UU,L} = F_{00}^{++} = \sum_{\Lambda'} f_{00}^{+\Lambda'} * f_{00}^{+\Lambda'} = |f_{00}^{++}|^2 + |f_{00}^{+-}|^2$$

$$F_{UU}^{\cos 2\phi} = -\Re F_{1-1}^{++} = -\Re \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{-10}^{+\Lambda'}$$

$$= -\Re [(f_{10}^{++})^* (f_{10}^{--}) - (f_{10}^{+-})^* (f_{10}^{-+})]$$

$$F_{UU}^{\cos \phi} = \Re (F_{10}^{++} + F_{10}^{--}) = \Re \sum_{\Lambda'} (f_{00}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'} * f_{10}^{-\Lambda'})$$

$$= \Re [(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})]$$

$$F_{LU}^{\sin \phi} = -\Im (F_{10}^{++} + F_{10}^{--}) = -\Im \sum_{\Lambda'} (f_{00}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'} * f_{10}^{-\Lambda'})$$

$$= -\Im [(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})]$$

Experimental data

Cross-section

Q2	W	xB	tmin-t	epsilon	sig0	dsig0 stat	dsig0 sys	sigLT	dsigLT stat	dsigLT sys	sigTT	dsigTT stat	dsigTT sys	sigLT'	dsigLT' stat	dsigLT' sys	
	3.11	2.53	0.36	0.03	0.61	195.46	3.66	11.93	7.05	3.19	0.25	-4.66	7.62	0.16	14.52	6.91	0.51
	3.11	2.53	0.36	0.1	0.61	217.29	4.22	13.26	-5.97	3.81	0.21	-67.45	9.03	2.36	18.91	8.05	0.66
	3.11	2.53	0.36	0.18	0.61	195.76	4.15	11.95	-11.55	4.01	0.4	-67.67	8.97	2.37	27.63	7.17	0.97
	3.11	2.53	0.36	0.29	0.61	183.18	4.54	11.18	-28.08	4.63	0.98	-87.12	10.32	3.05	9.05	6.38	0.32
	3.57	2.69	0.36	0.03	0.62	115.04	2.53	4.64	-2.37	2.18	0.08	-16.48	5.25	0.58	-5.08	4.97	0.18
	3.57	2.69	0.36	0.1	0.62	117.51	277	4.74	-9.7	2.54	0.34	-46.96	5.85	1.64	18.11	5.31	0.63
	3.57	2.69	0.36	0.17	0.62	119.61	3.35	4.82	-5.32	3.52	0.19	-35.39	6.99	1.24	14.58	5.31	0.51
	3.57	2.69	0.36	0.28	0.62	105.37	4.06	4.25	-5.32	3.52	0.19	-35.39	6.99	1.24	14.58	5.31	0.51
	4.44	2.96	0.36	0.03	0.63	57.04	1.88	2.08	-1.84	1.44	0.06	-2.42	3.43	0.08	4.92	3.24	0.17
	4.44	2.96	0.36	0.09	0.63	62.86	2.16	2.29	-0.23	1.83	0.01	-13.17	4.03	0.46	5.04	3.63	0.18
	4.44	2.96	0.36	0.17	0.63	64.53	2.47	2.35	0.62	2.35	0.02	-13.49	4.66	0.47	6.39	3.65	0.22
	4.44	2.96	0.36	0.28	0.63	51.63	2.56	1.88	-5.66	2.61	0.2	-28.8	4.76	1.01	5.79	3.13	0.2
	2.67	1.94	0.48	0.01	0.51	525.95	14.48	41.16	25.07	16.53	0.88	-29.14	43.88	1.02	7.6	30.21	0.27
	2.67	1.94	0.48	0.04	0.51	520.4	16.36	40.73	-38.25	19.21	1.34	-7.88	45.79	0.28	-5.32	31.83	0.19
	2.67	1.94	0.48	0.08	0.51	488.33	17.33	38.22	-31.6	21.72	1.11	-55.44	47.02	1.94	16.7	28.69	0.5
	2.67	1.94	0.48	0.14	0.51	480.77	23.45	37.63	-60.2	30.66	2.11	-116.12	57.67	4.06	14.05	27.05	0.49
	4.06	2.3	0.45	0.02	0.71	126.23	3.84	6.71	-3.93	3.36	0.14	-17.69	7.8	0.6	16.81	8.43	0.59
	4.06	2.3	0.45	0.07	0.71	128.7	4.65	6.84	-9.18	4.45	0.32	-13.9	8.66	0.49	26.38	8.65	0.92
	4.06	2.3	0.45	0.12	0.71	115.22	6.01	6.12	-16.42	6.24	0.57	-23.1	10.78	0.81	30.12	8.41	1.05

Jefferson Lab Hall A

Phys. Rev. Lett. 127, 152301

Phys. Rev. C 83, 025201

Phys. Lett. B 768 (2017)

168–173

Q2	W	xB	epsilon	-t	AUL^sinphi	dAUL^sinphi stat	dAUL^sinphi sys	AUL^sin2phi	dAUL^sin2phi stat	dAUL^sin2phi sys
1.94	2.59	0.25	0.51	0.15	0.272	0.037	0.03	-0.033	0.036	0.044
1.94	2.59	0.25	0.51	0.26	0.223	0.033	0.06	-0.047	0.032	0.03
1.94	2.59	0.25	0.51	0.49	0.292	0.036	0.021	-0.073	0.036	0.031
1.94	2.59	0.25	0.51	0.92	0.269	0.036	0.019	-0.12	0.036	0.024
1.94	2.59	0.25	0.51	1.46	0.188	0.047	0.032	-0.116	0.043	0.013
2.83	2.26	0.4	0.58	0.27	0.165	0.046	0.036	-0.006	0.042	0.029
2.83	2.26	0.4	0.58	0.52	0.208	0.03	0.022	-0.021	0.025	0.021
2.83	2.26	0.4	0.58	0.88	0.135	0.035	0.022	-0.043	0.026	0.012
2.83	2.26	0.4	0.58	1.24	0.164	0.049	0.051	-0.09	0.042	0.036
2.83	2.26	0.4	0.58	1.59	0.26	0.055	0.029	-0.155	0.047	0.029

Q2	W	xB	epsilon	-t	ALL^const	dALL^const stat	dALL^const sys	ALL^cosphi	dALL^cosphi stat	dALL^cosphi sys
1.94	2.59	0.25	0.51	0.15	0.594	0.029	0.019	0.163	0.041	0.064
1.94	2.59	0.25	0.51	0.26	0.593	0.028	0.03	0.162	0.042	0.081
1.94	2.59	0.25	0.51	0.49	0.564	0.036	0.031	0.2	0.064	0.078
1.94	2.59	0.25	0.51	0.92	0.395	0.03	0.025	0.158	0.049	0.05
1.94	2.59	0.25	0.51	1.46	0.214	0.036	0.037	-0.023	0.042	0.029
2.83	2.26	0.4	0.58	0.27	0.705	0.039	0.059	0.19	0.064	0.089
2.83	2.26	0.4	0.58	0.52	0.667	0.022	0.032	0.156	0.029	0.089
2.83	2.26	0.4	0.58	0.88	0.49	0.029	0.014	0.193	0.027	0.076
2.83	2.26	0.4	0.58	1.24	0.504	0.038	0.017	0.158	0.052	0.089
2.83	2.26	0.4	0.58	1.59	0.449	0.046	0.034	0.114	0.056	0.083

*Thanks to Prof. Garth
& Alicia*

Jefferson Lab Hall B

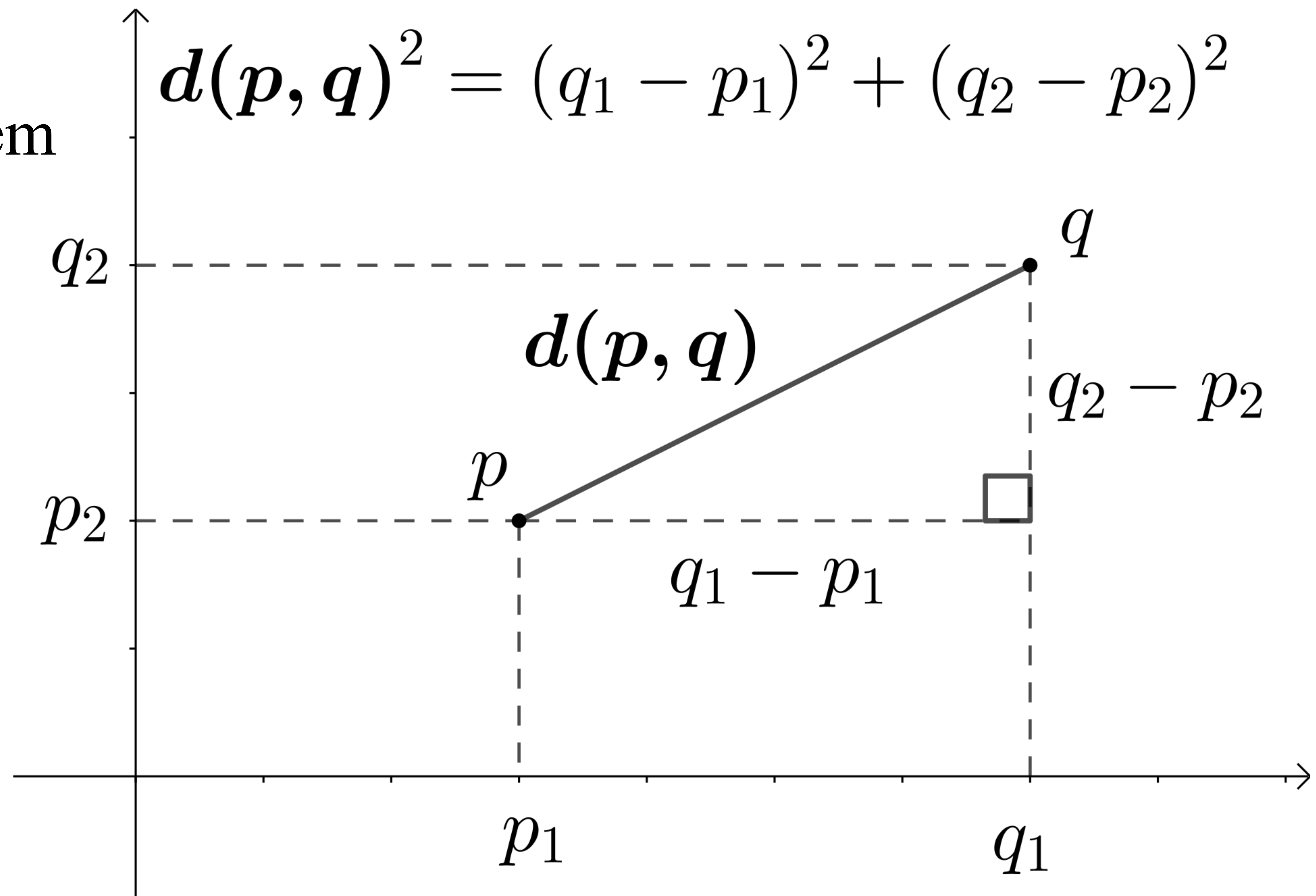
Asymmetry

Finding the best kinematic bin for analysis

□ Find the closest set of kinematic bins from all the data

□ Pythagorean theorem

□ Find the Euclidean distance



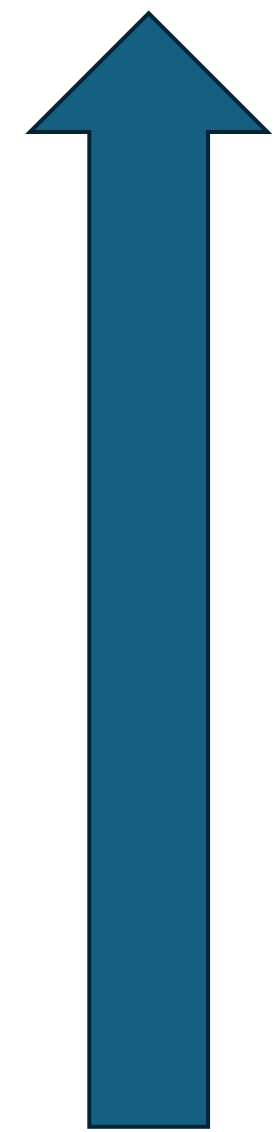
<https://commons.wikimedia.org/w/index.php?curid=67617313>

best overlapping kinematic bins

Datatype	Q^2	W	x_{Bj}	$ t $	ϵ
kinematic					
σ'_s	2.67	1.94	0.48	0.470	0.510
A	2.83	2.26	0.40	0.520	0.580
Joint (σ, A)	2.75	2.1	0.44	0.495	0.545

What is more important for CFFs fundamentally?

$$\begin{aligned}
 f_{10}^{++} &= g_{\pi}^{V,odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1+\xi) (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V,odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1}{1-\zeta/2} (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right], \\
 f_{10}^{+-} &= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T - \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \\
 &= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T - \frac{\zeta^2/4}{1-\zeta} \mathcal{E}_T + \frac{\zeta/2}{1-\zeta} \tilde{\mathcal{E}}_T \right] \\
 f_{10}^{-+} &= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} e^{-i2\phi} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 &= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} e^{-i2\phi} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 f_{10}^{--} &= g_{\pi}^{V,odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1-\xi) (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V,odd}(Q) e^{i\phi} \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1-\zeta}{1-\zeta/2} (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right]
 \end{aligned}$$



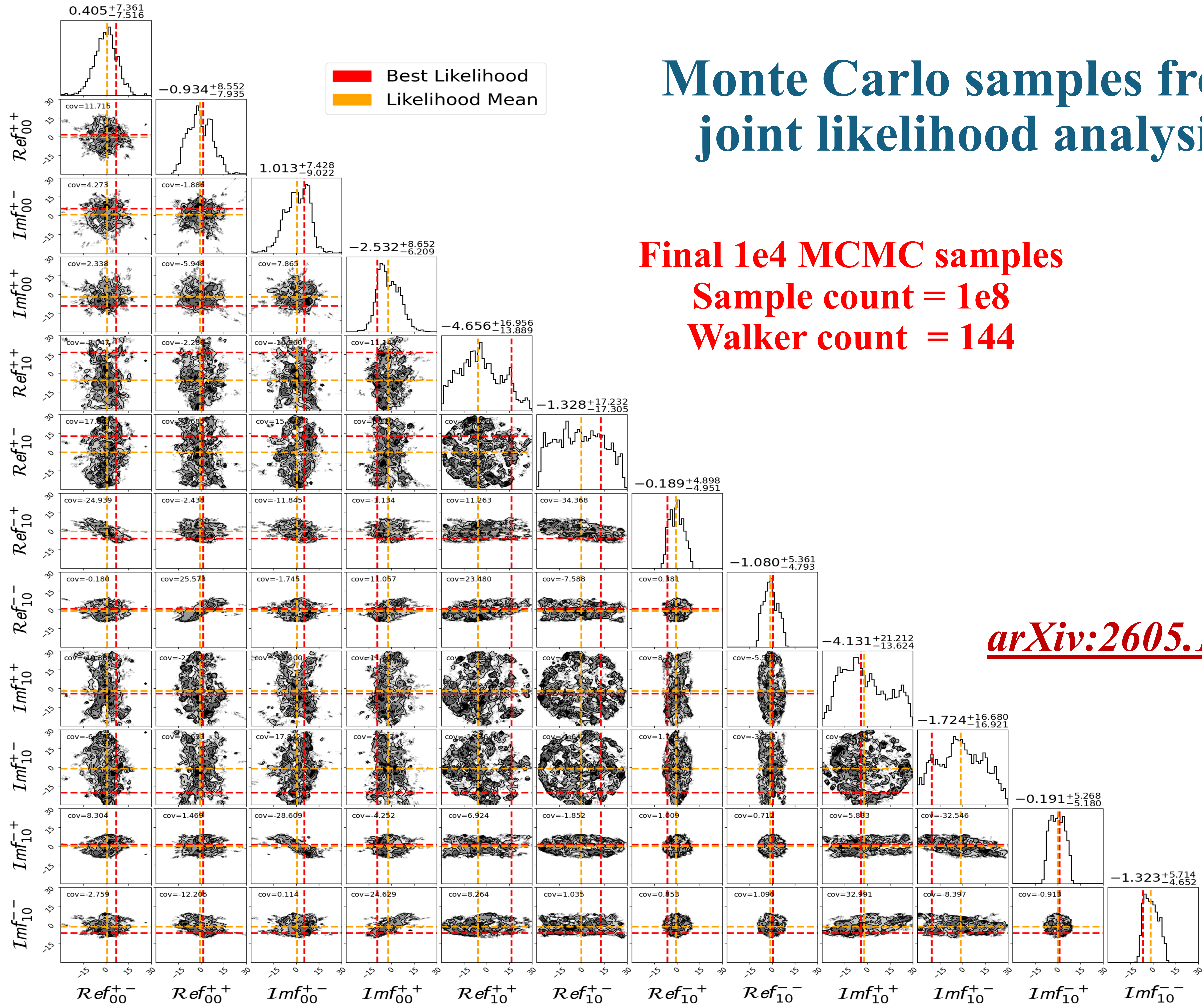
In a QCD factorized scenario

$$\begin{aligned}
 F_{UU,T} &= \frac{1}{2} (F_{11}^{++} + F_{11}^{--}) = \frac{1}{2} \sum_{\Lambda'} (f_{10}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{10}^{-\Lambda'} * f_{10}^{-\Lambda'}) \\
 &= \frac{1}{2} (|f_{10}^{++}|^2 + |f_{10}^{+-}|^2 + |f_{10}^{-+}|^2 + |f_{10}^{--}|^2) \\
 F_{UU,L} &= F_{00}^{++} = \sum_{\Lambda'} f_{00}^{+\Lambda'} * f_{00}^{+\Lambda'} = |f_{00}^{++}|^2 + |f_{00}^{+-}|^2 \\
 F_{UU}^{\cos 2\phi} &= -\Re F_{1-1}^{++} = -\Re \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{-10}^{+\Lambda'} \\
 &= -\Re [(f_{10}^{++})^* (f_{10}^{--}) - (f_{10}^{+-})^* (f_{10}^{-+})] \\
 F_{UU}^{\cos \phi} &= \Re (F_{10}^{++} + F_{10}^{--}) = \Re \sum_{\Lambda'} (f_{00}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'} * f_{10}^{-\Lambda'}) \\
 &= \Re [(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})] \\
 F_{LU}^{\sin \phi} &= -\Im (F_{10}^{++} + F_{10}^{--}) = -\Im \sum_{\Lambda'} (f_{00}^{+\Lambda'} * f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'} * f_{10}^{-\Lambda'}) \\
 &= -\Im [(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})]
 \end{aligned}$$

Monte Carlo samples from joint likelihood analysis

Final 1e4 MCMC samples
Sample count = 1e8
Walker count = 144

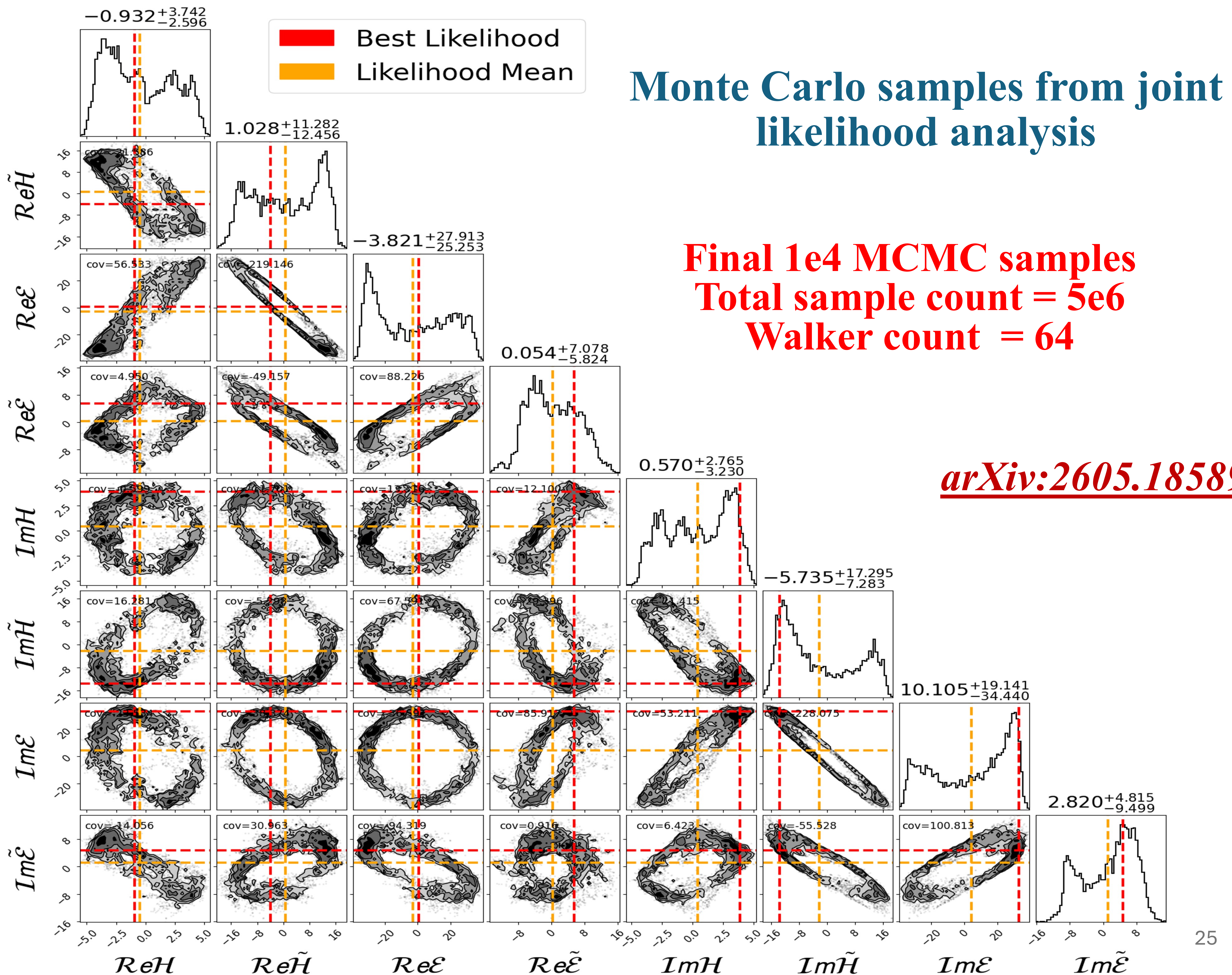
arXiv:2605.18589



Monte Carlo samples from joint likelihood analysis

Final 1e4 MCMC samples
Total sample count = 5e6
Walker count = 64

[arXiv:2605.18589](https://arxiv.org/abs/2605.18589)



481.8^{+26.56}_{-25.04}

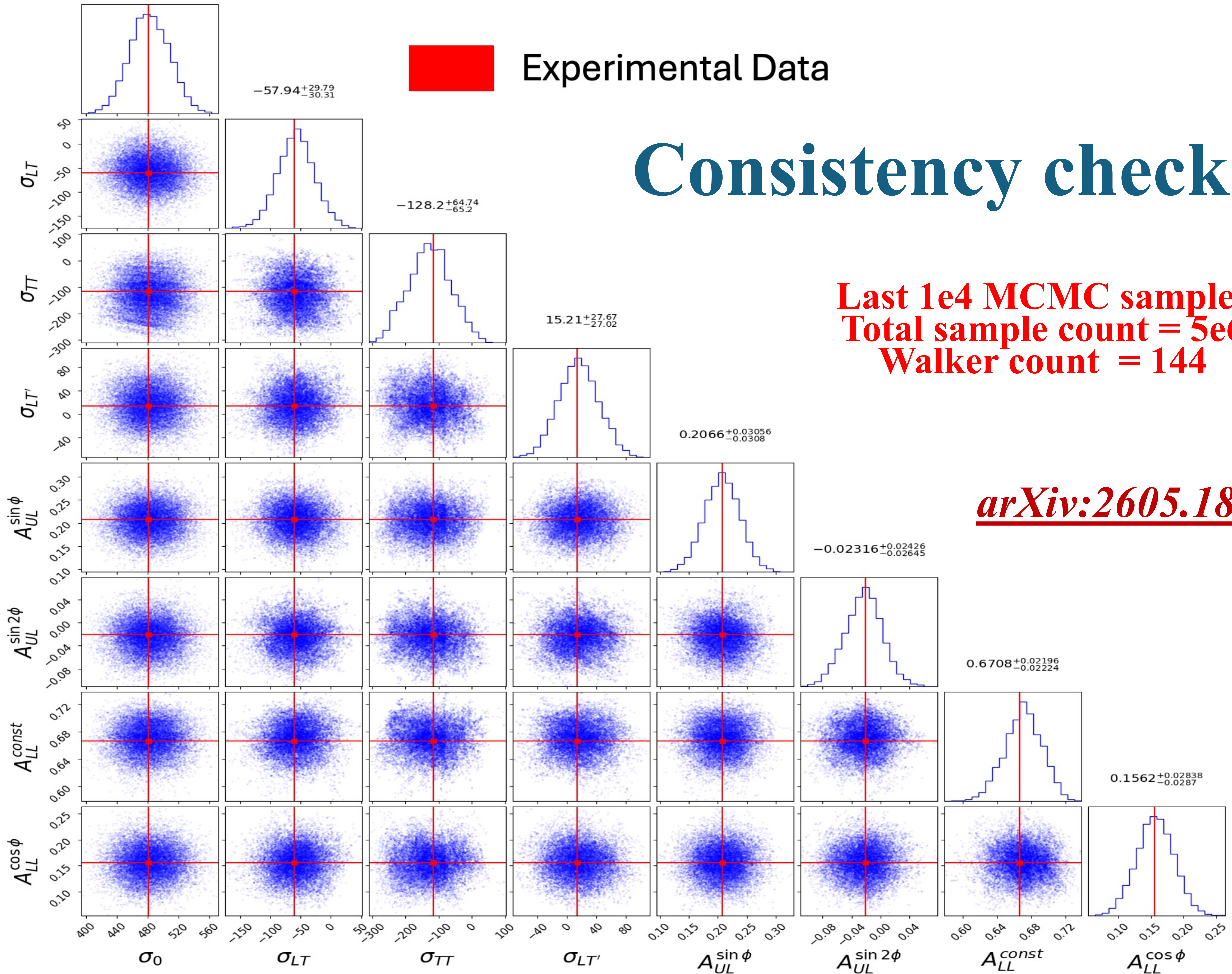


Experimental Data

Consistency check

Last 1e4 MCMC samples
Total sample count = 5e6
Walker count = 144

arXiv:2605.18589



$$F_{UU,T} = \frac{1}{2}(F_{11}^{++} + F_{11}^{--}) = \frac{1}{2} \sum_{\Lambda'} (f_{10}^{+\Lambda'}{}^* f_{10}^{+\Lambda'} + f_{10}^{-\Lambda'}{}^* f_{10}^{-\Lambda'})$$

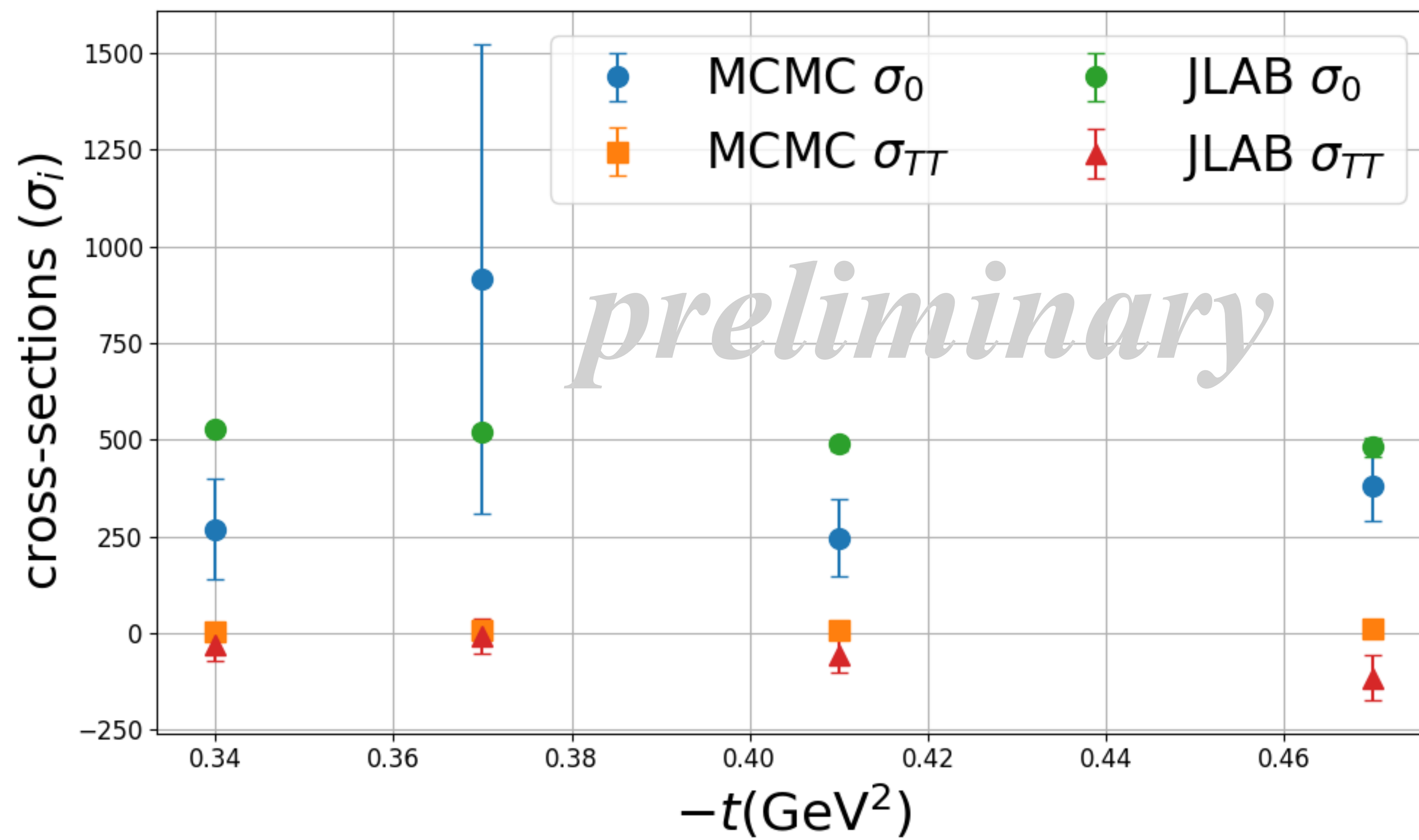
$$= \frac{1}{2} (|f_{10}^{++}|^2 + |f_{10}^{+-}|^2 + |f_{10}^{-+}|^2 + |f_{10}^{--}|^2)$$

$$F_{UU,L} = F_{00}^{++} = \sum_{\Lambda'} f_{00}^{+\Lambda'}{}^* f_{00}^{+\Lambda'} = |f_{00}^{++}|^2 + |f_{00}^{+-}|^2$$

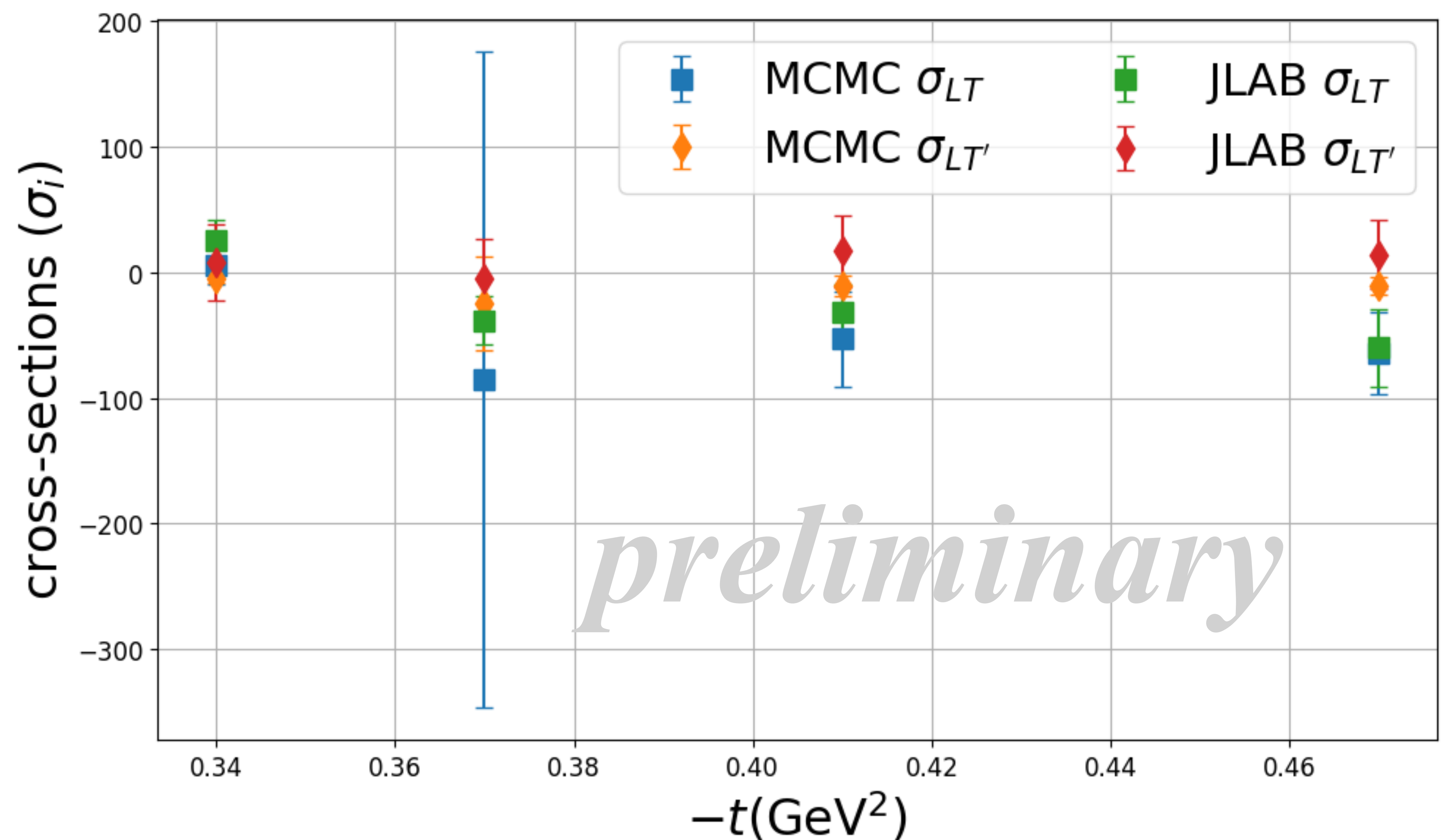
$$\begin{aligned} F_{UU}^{\cos 2\phi} &= -\Re F_{1-1}^{++} = -\Re \sum_{\Lambda'} f_{10}^{+\Lambda'}{}^* f_{-10}^{+\Lambda'} \\ &= -\Re [(f_{10}^{++})^* (f_{10}^{--}) - (f_{10}^{+-})^* (f_{10}^{-+})] \end{aligned}$$

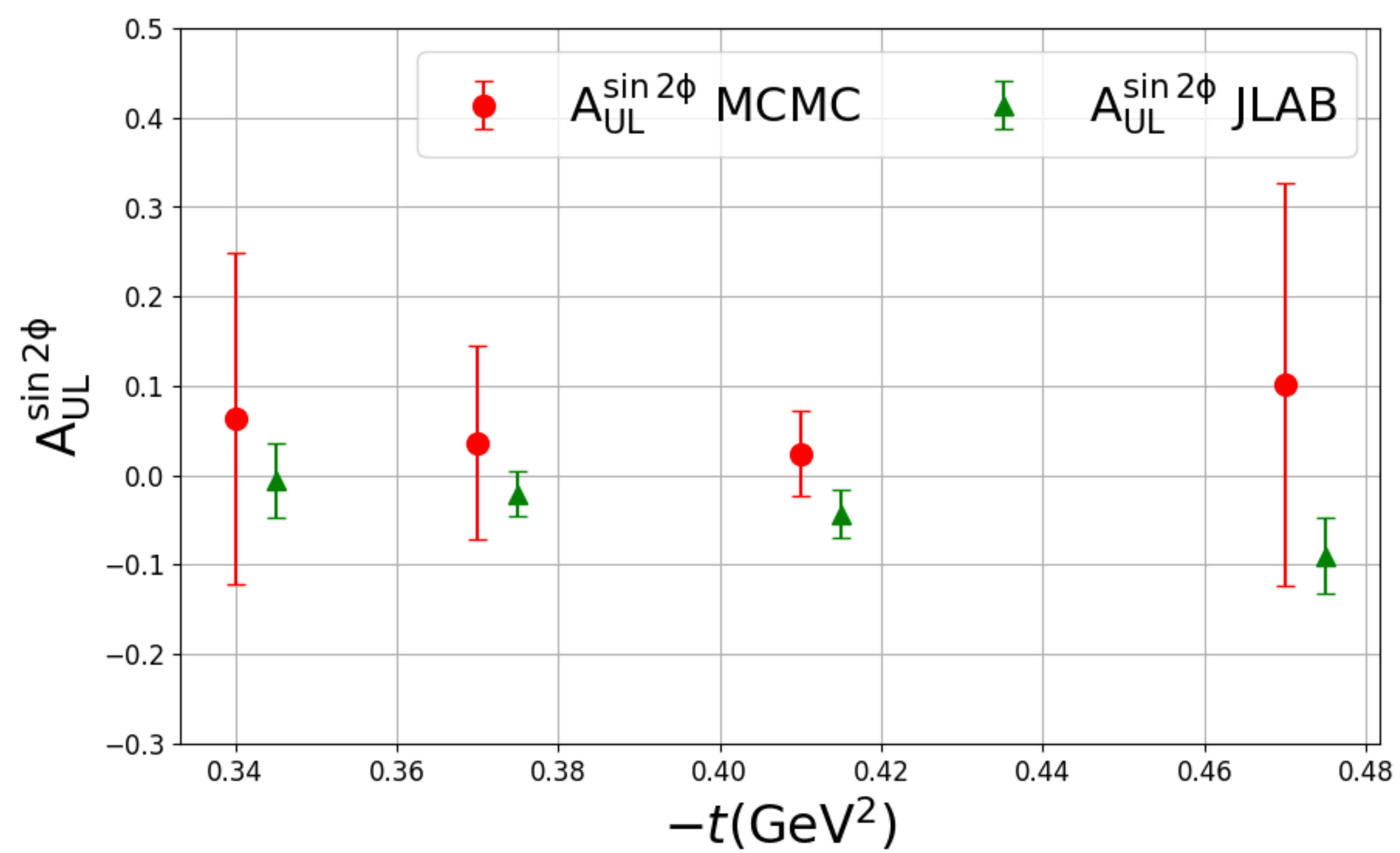
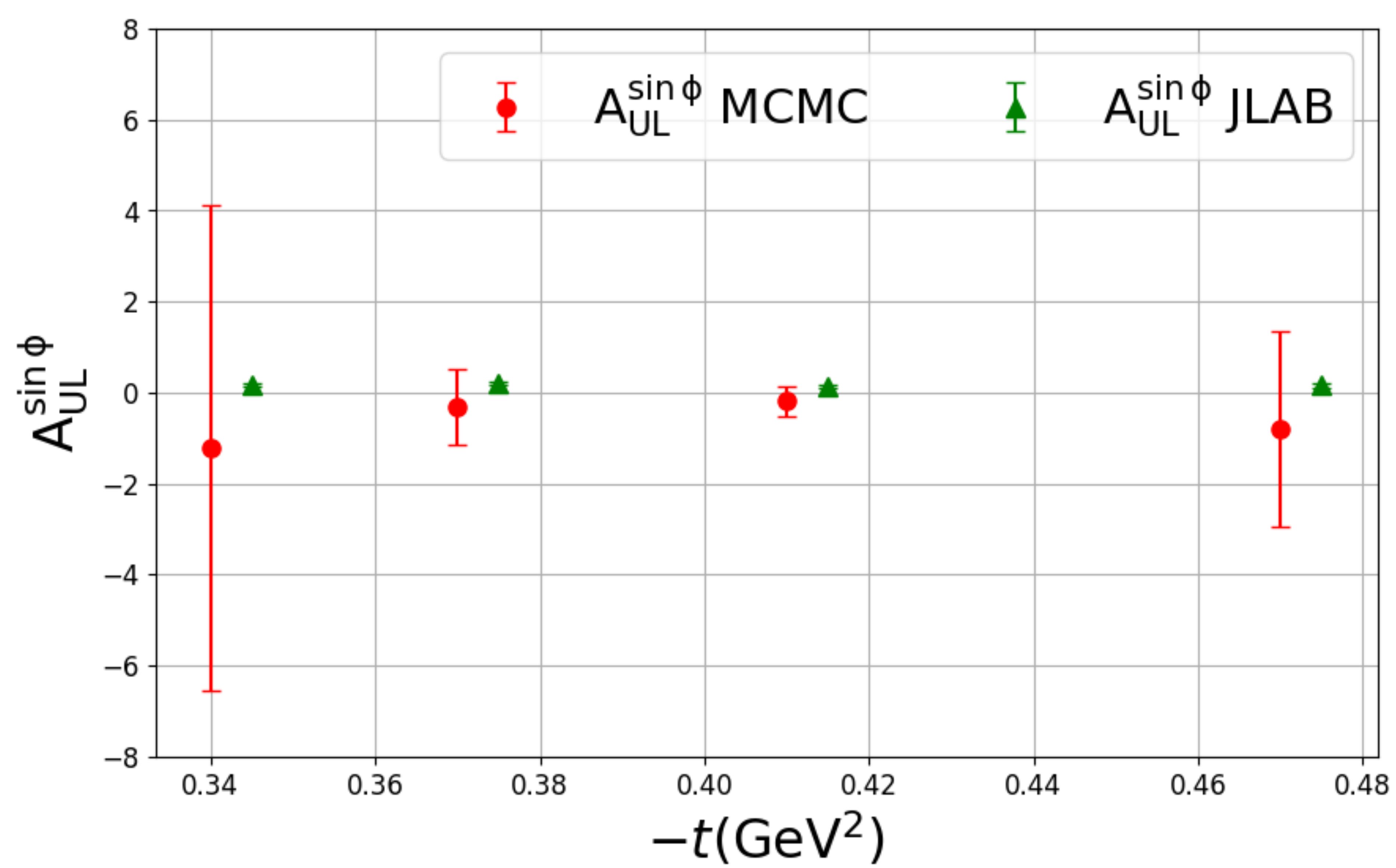
$$\begin{aligned} F_{UU}^{\cos \phi} &= \Re(F_{10}^{++} + F_{10}^{--}) = \Re \sum_{\Lambda'} (f_{00}^{+\Lambda'}{}^* f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'}{}^* f_{10}^{-\Lambda'}) \\ &= \Re[(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})] \end{aligned}$$

$$\begin{aligned} F_{LU}^{\sin \phi} &= -\Im(F_{10}^{++} + F_{10}^{--}) = -\Im \sum_{\Lambda'} (f_{00}^{+\Lambda'}{}^* f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'}{}^* f_{10}^{-\Lambda'}) \\ &= -\Im[(f_{00}^{+-})^* (f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})] \end{aligned}$$

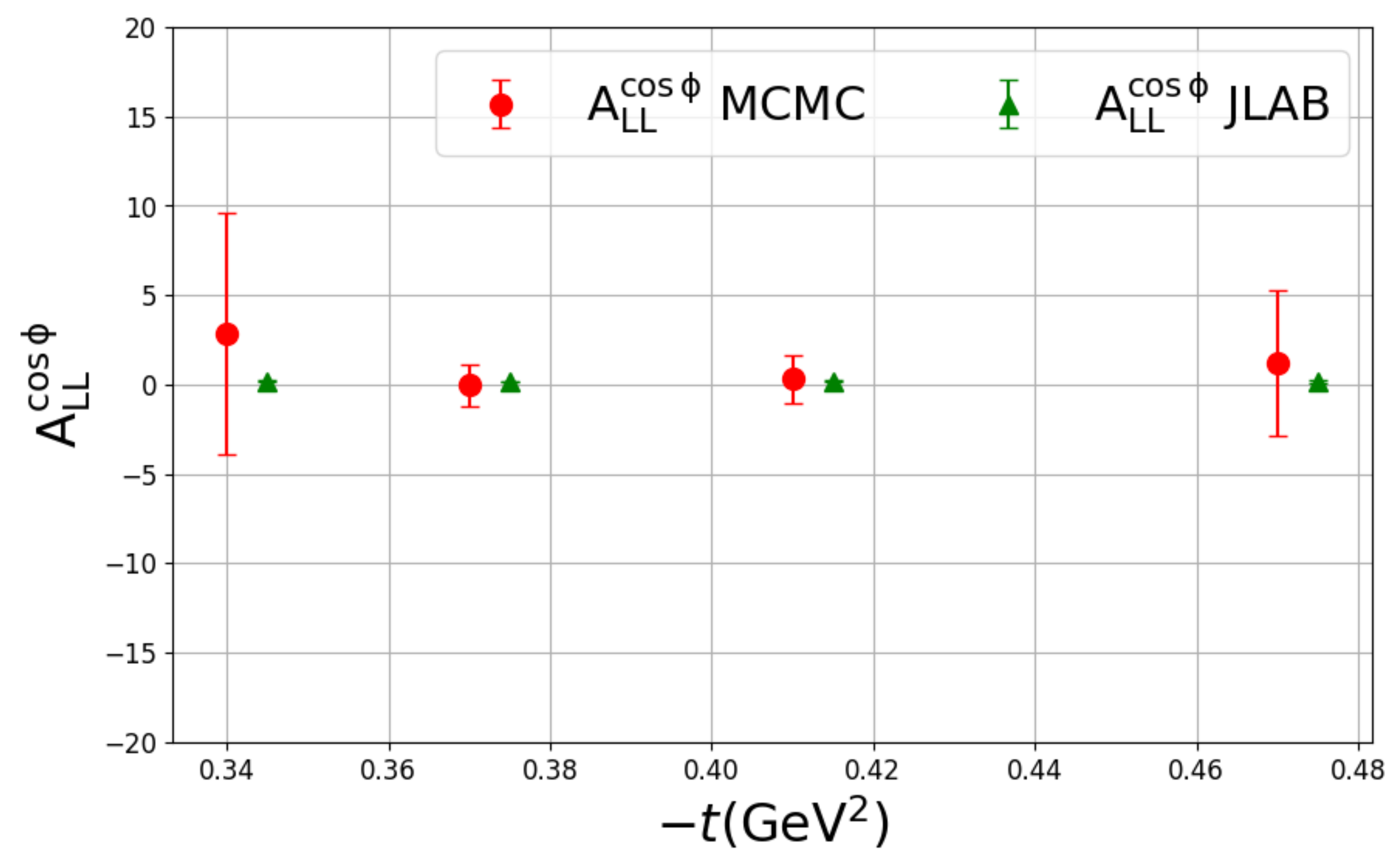
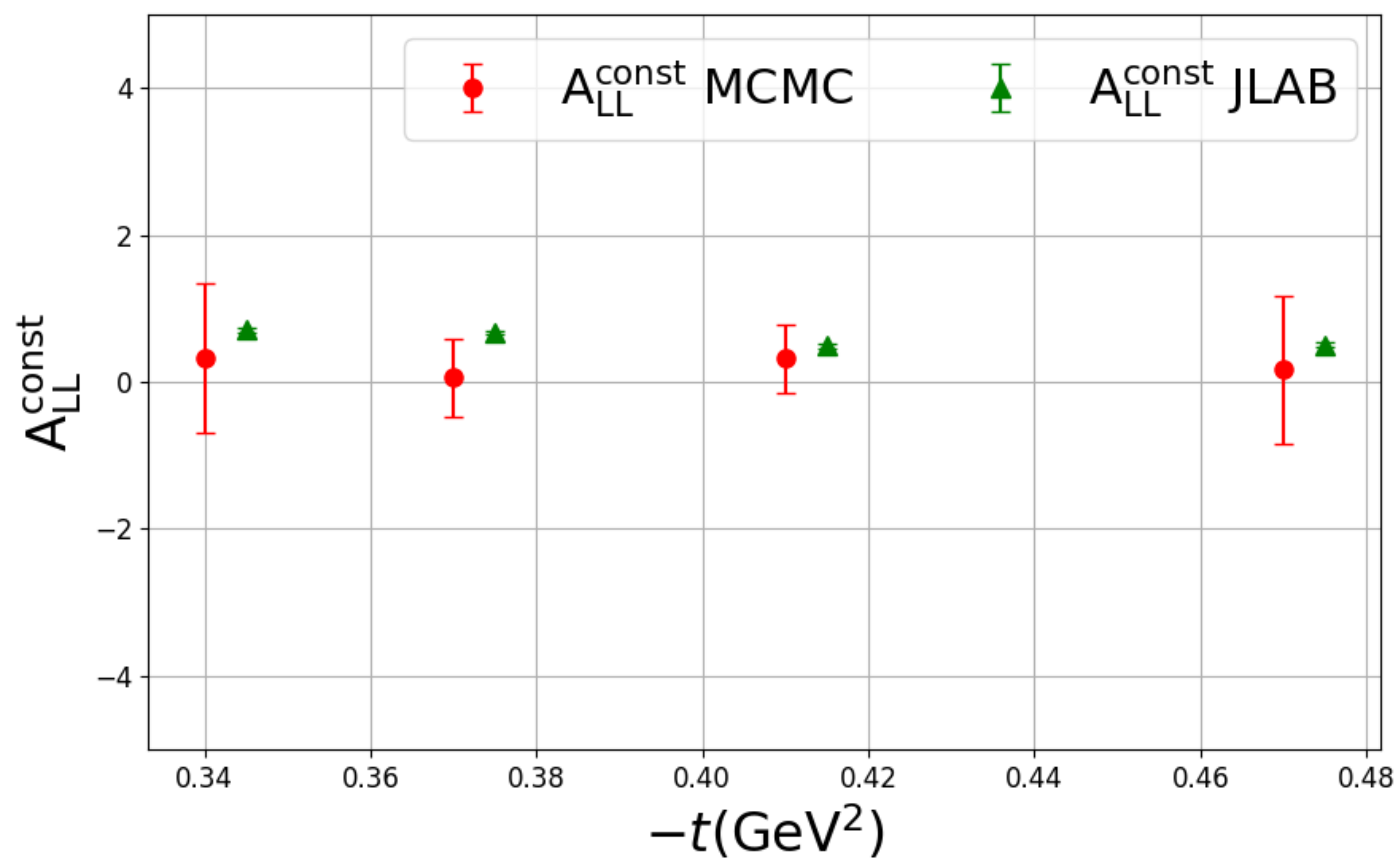


Phys. Rev. Lett. 127, 152301
Phys. Rev. C 83, 025201

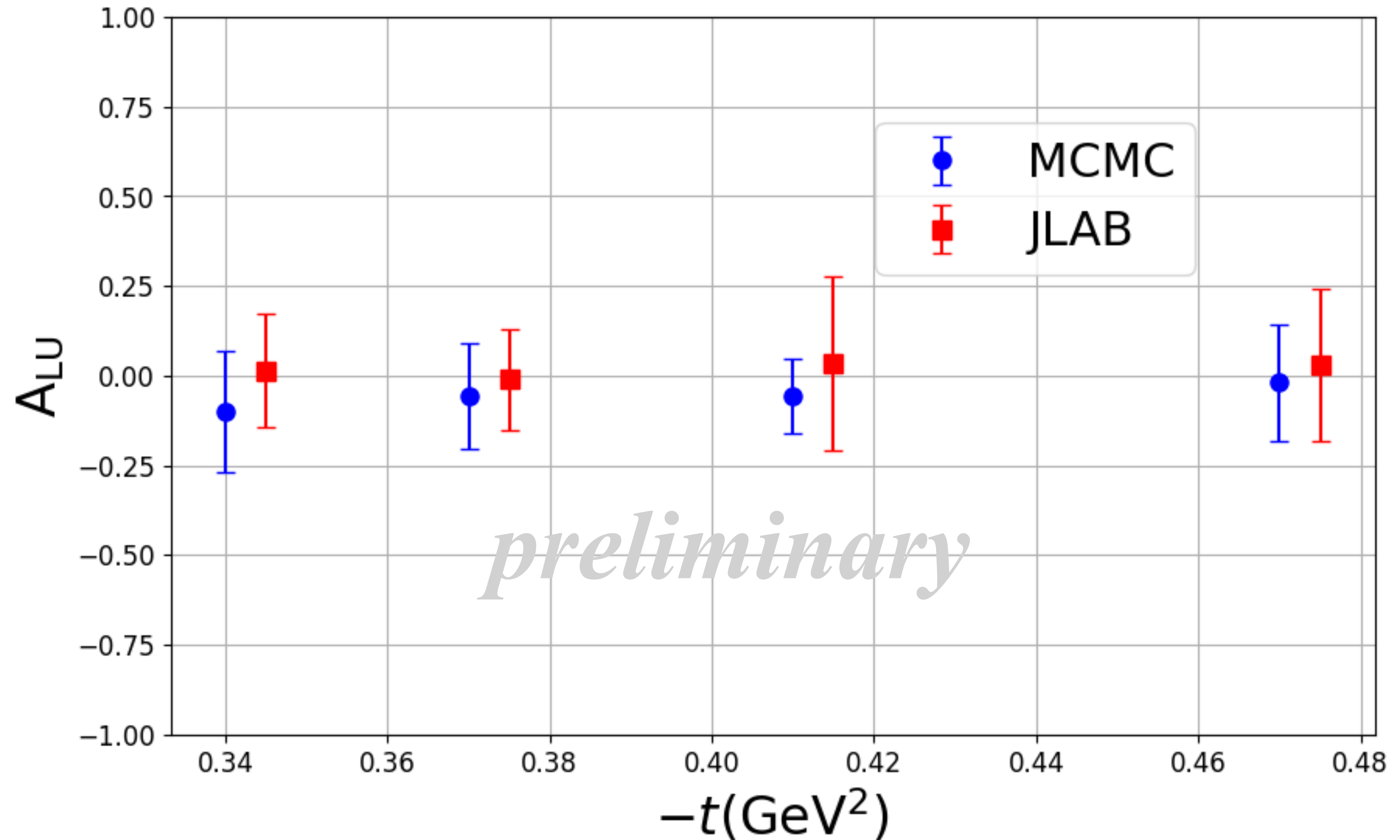




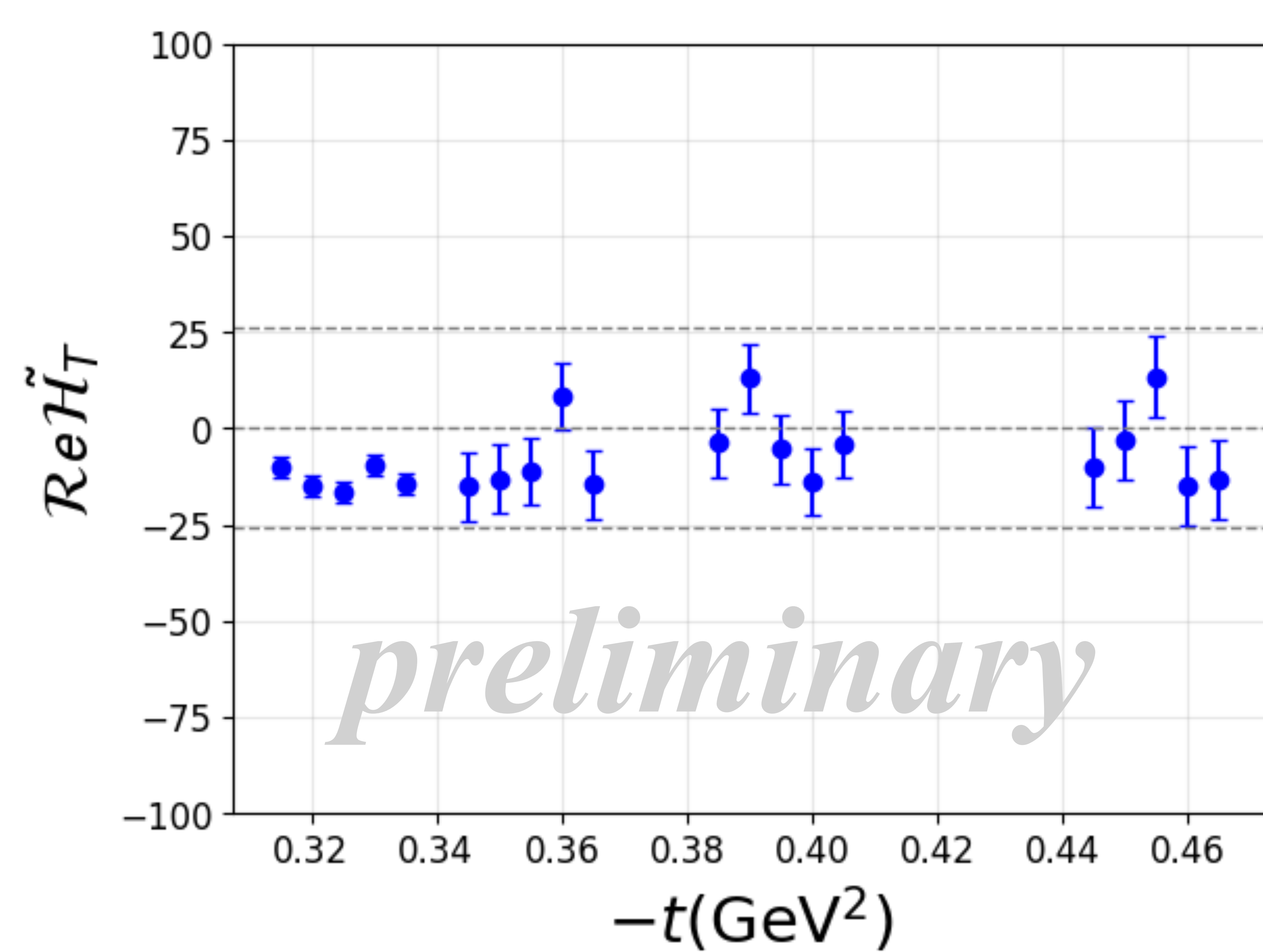
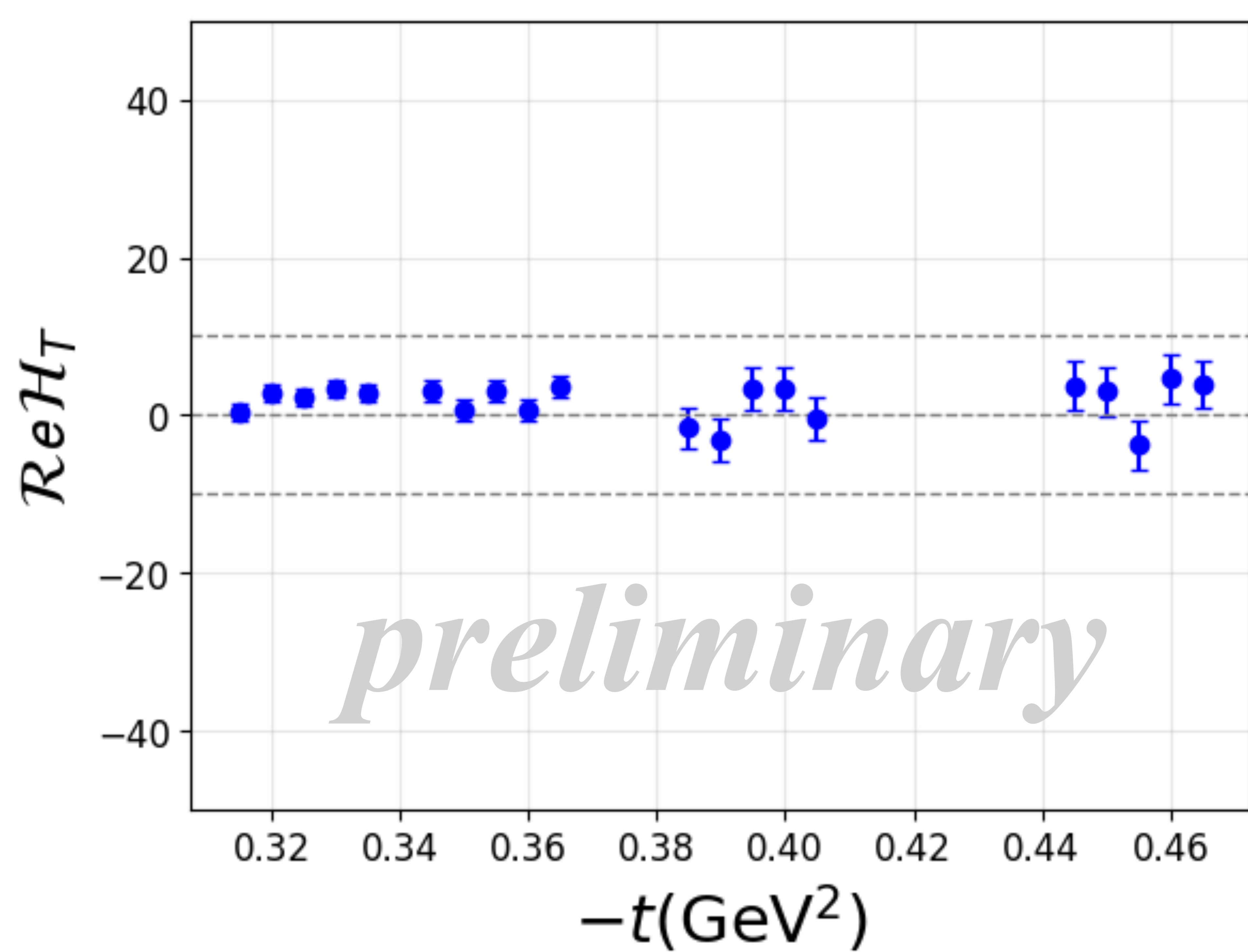
preliminary



Beam spin asymmetry

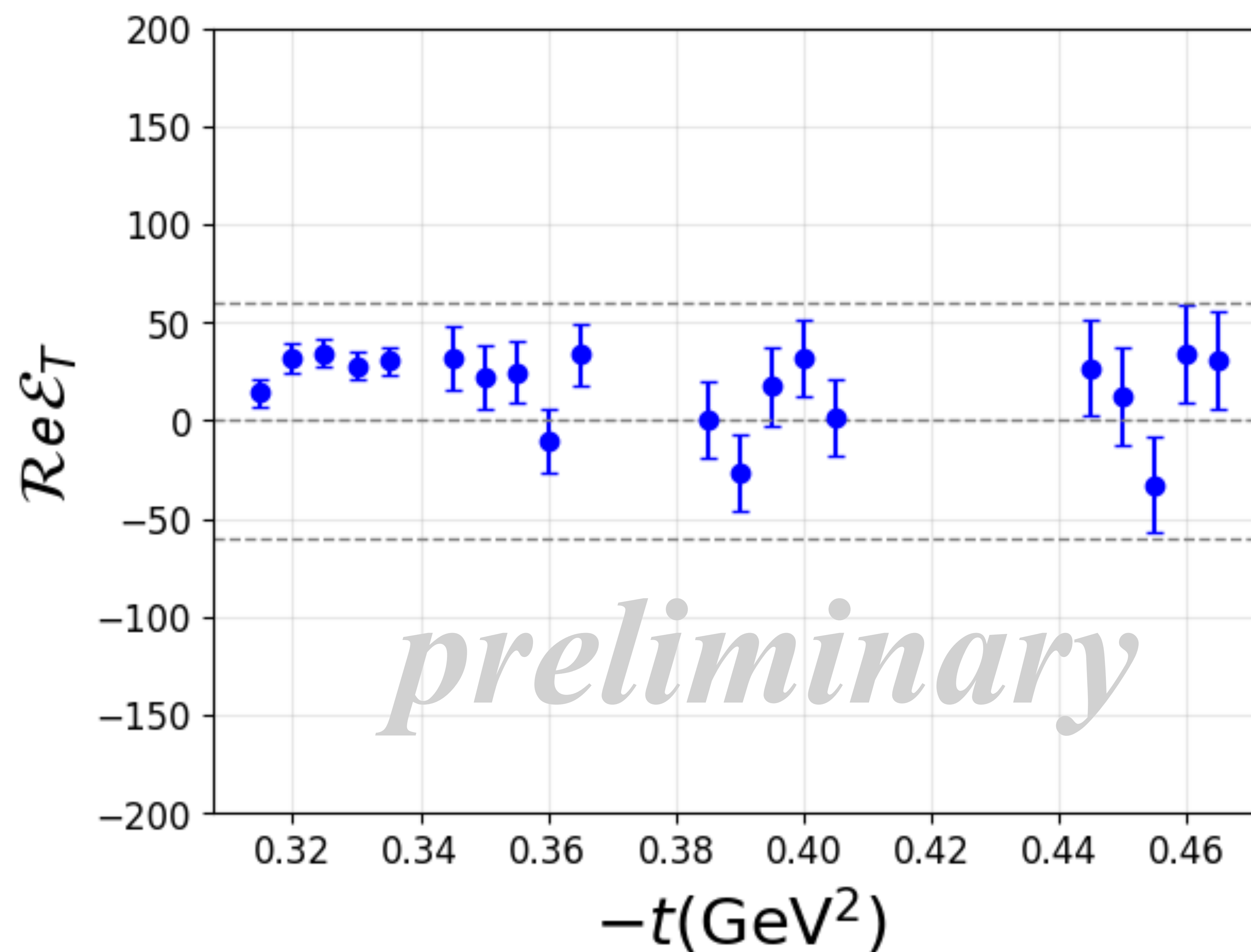


Phys. Rev. Lett. 127, 152301
Phys. Rev. C 83, 025201

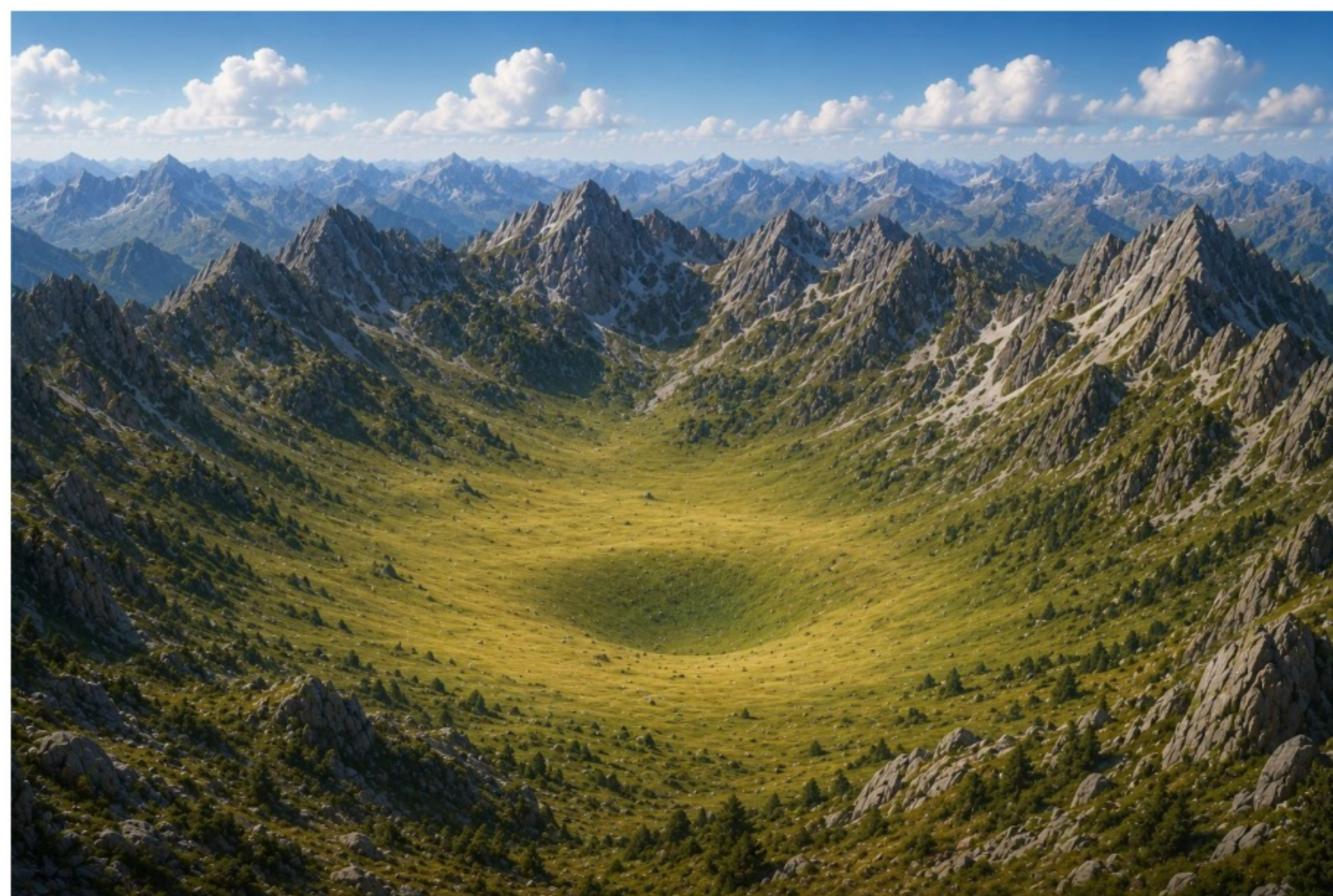


$\text{Re}\mathcal{H}_T$

$\text{Re}\mathcal{E}_T$



$\text{Re}\tilde{\mathcal{H}}_T$



Ideal χ^2 landscape



High-dimensional χ^2 landscape



2-dimensional χ^2 landscape



Presented by M. Boglione

4th Joint ECFA-NuPECC-APPEC Workshop “Synergies between the EIC and the LHC”

Jun 8 – 10, 2026, Cagliari, Italy

Likelihood analysis of parton helicity pdfs

□ Polarized Deep Inelastic scattering (DIS) spin structure functions, asymmetries, and cross-section data can be used to extract the parton helicity pdfs.

□ Parton model picture of proton, the structure function g_1^p :

$$g_1^p(x, Q^2) = \frac{1}{2} \Delta\Sigma(x, Q^2) - \frac{\alpha_s}{2\pi} \Delta G(x, Q^2),$$

$\Delta\Sigma$ = net quark, and ΔG = gluon helicity, contributions to the proton spin.

□ This spin structure function can also be used to understand the controversies associated with the sign problem of ΔG .

Preliminary Outcomes

□ Parametric form

$$\Delta\Sigma \approx Nq x^\alpha(1-x)^\beta(1+\gamma x^\kappa)$$

$$\Delta G \approx Ng x^{\alpha'}(1-x)^{\beta'}(1+\gamma' x^{\kappa'})$$

E143 experiment only

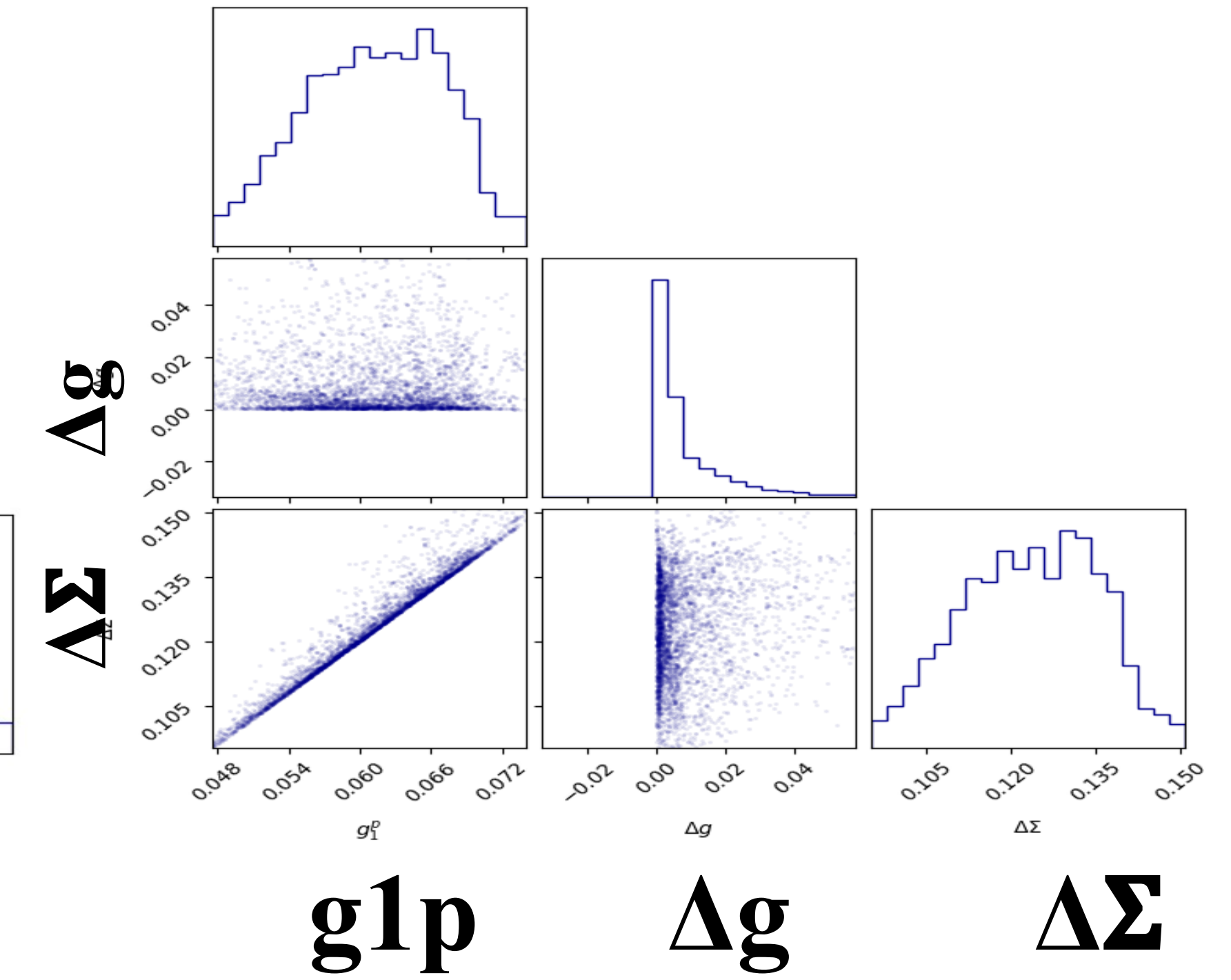
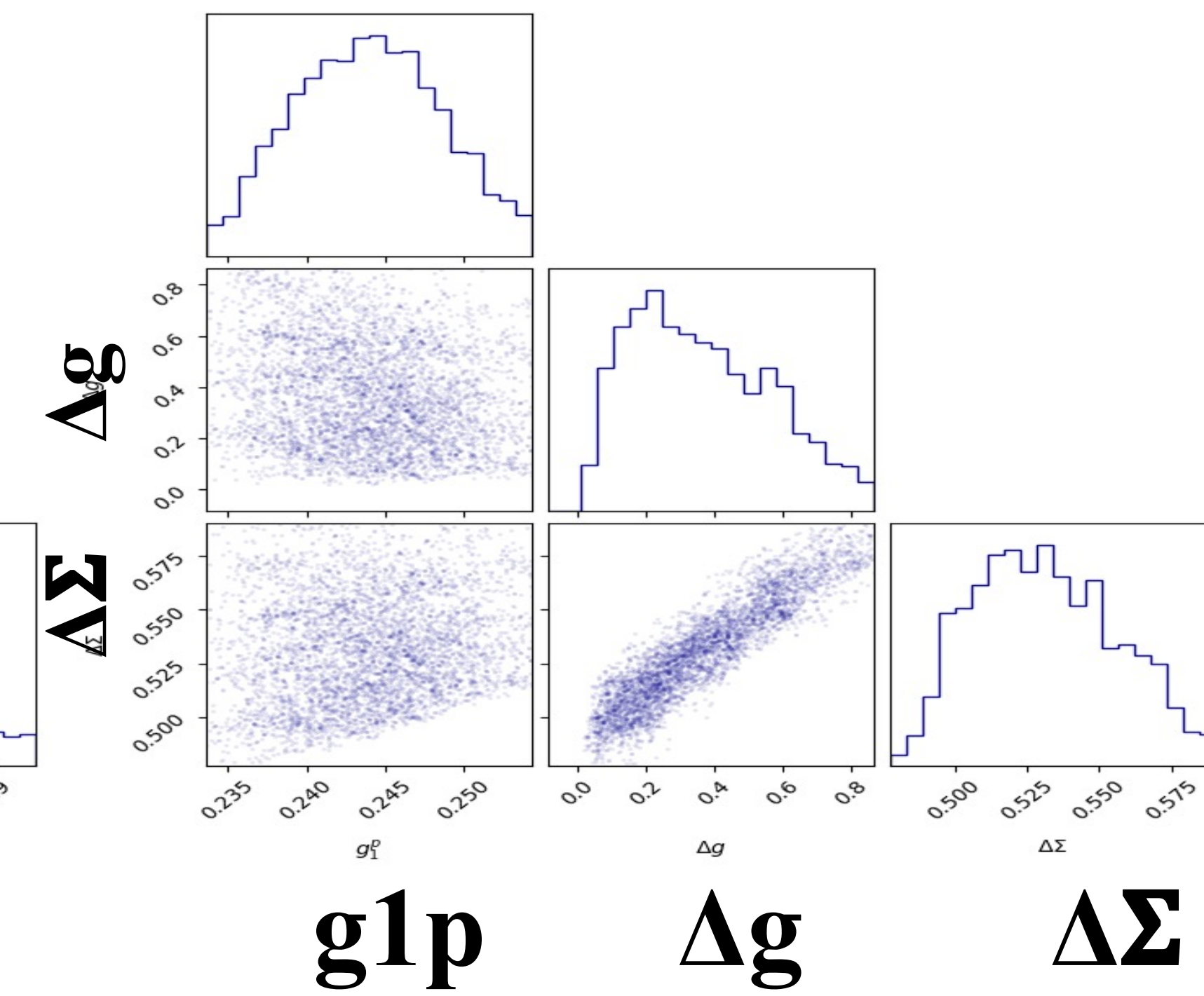
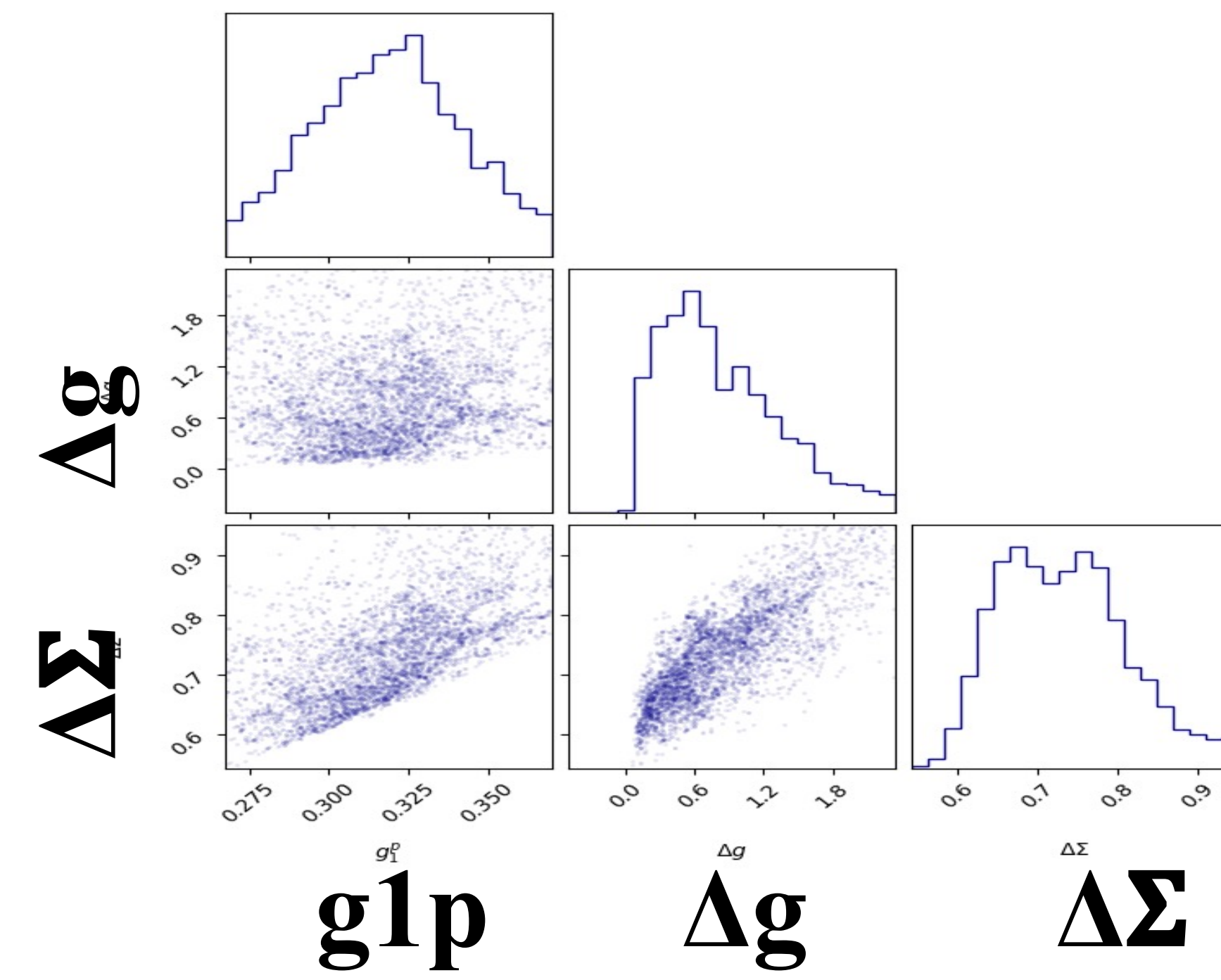
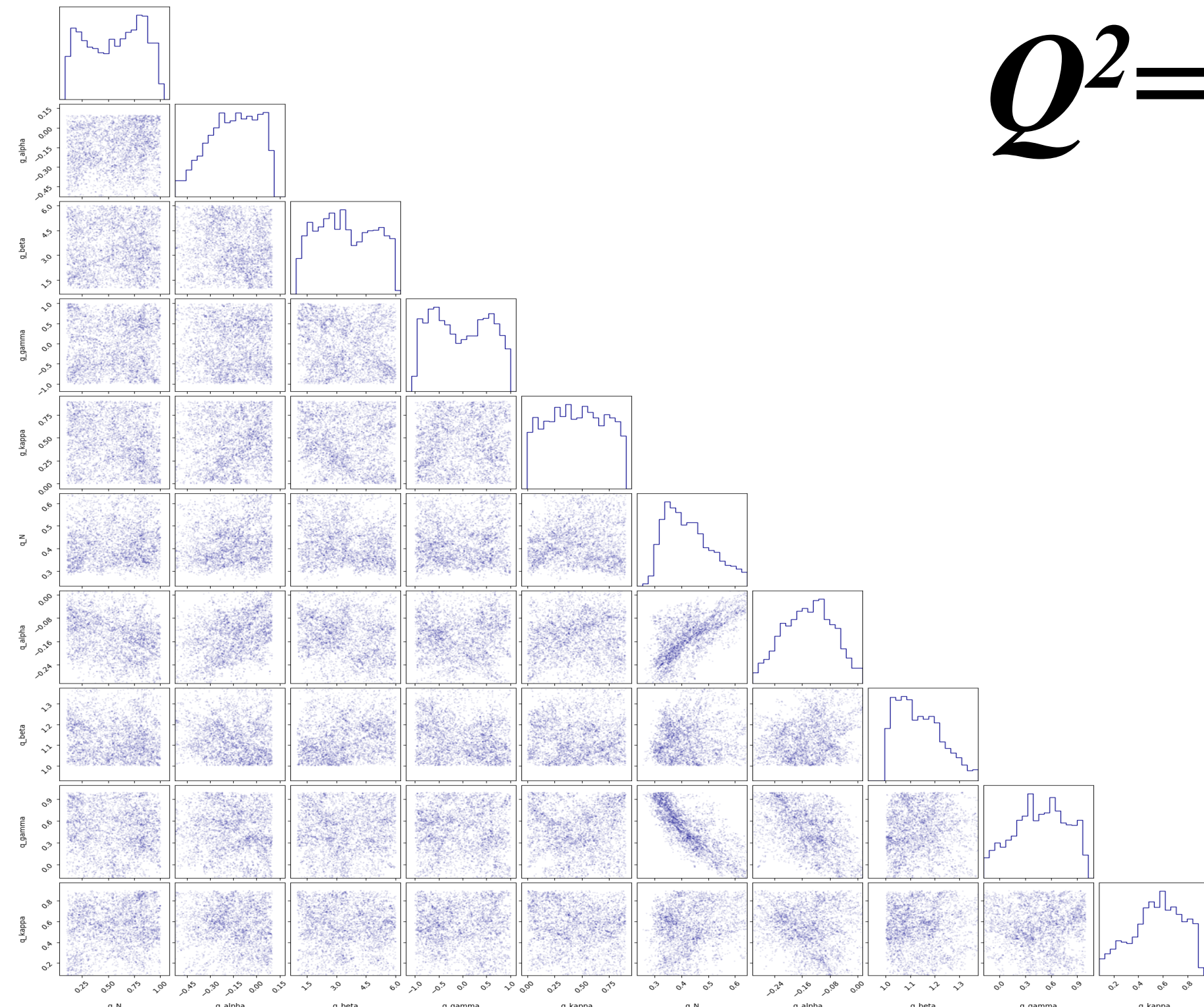
Uninformed prior

$Q^2 = 2 \text{ (GeV)}^2$

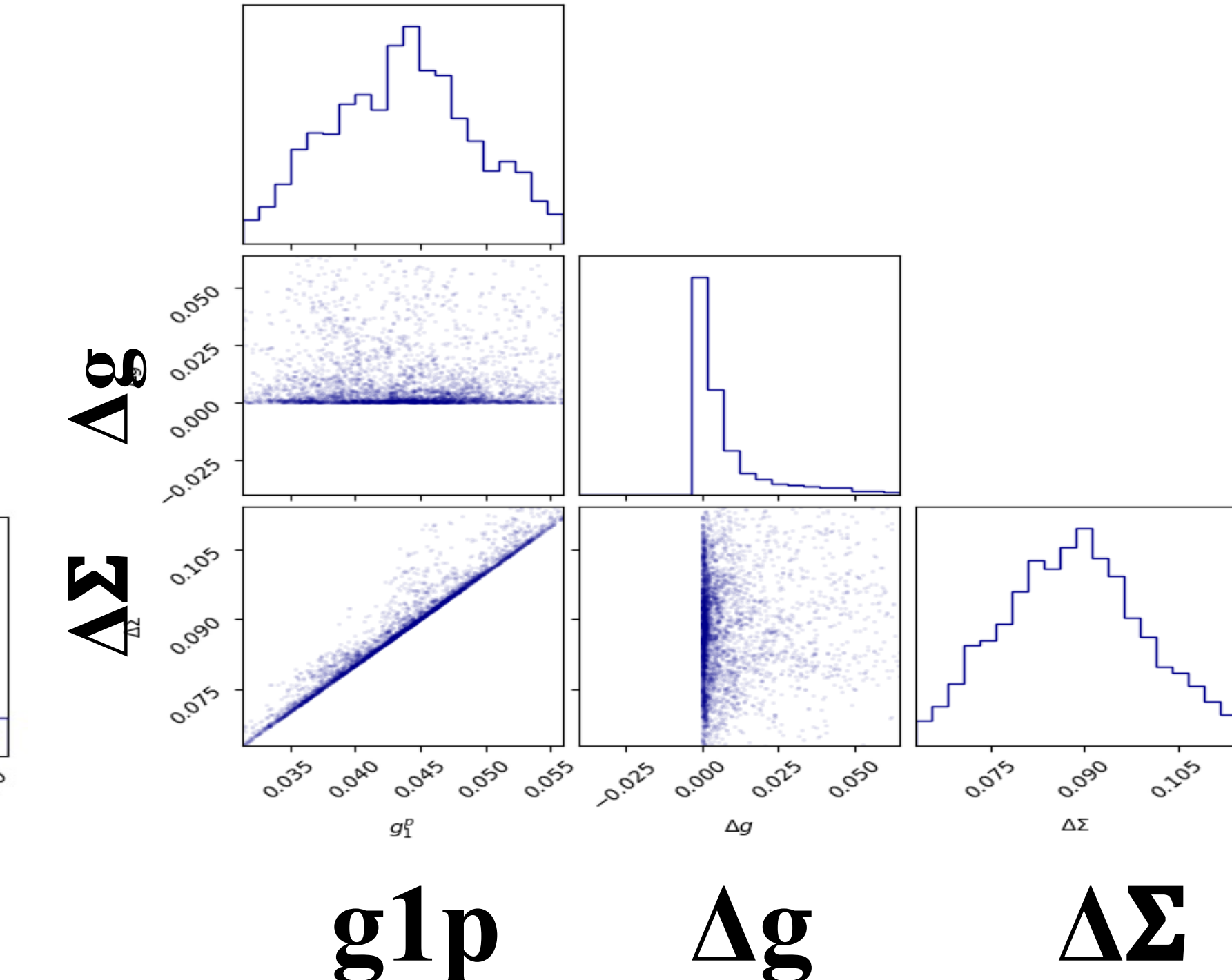
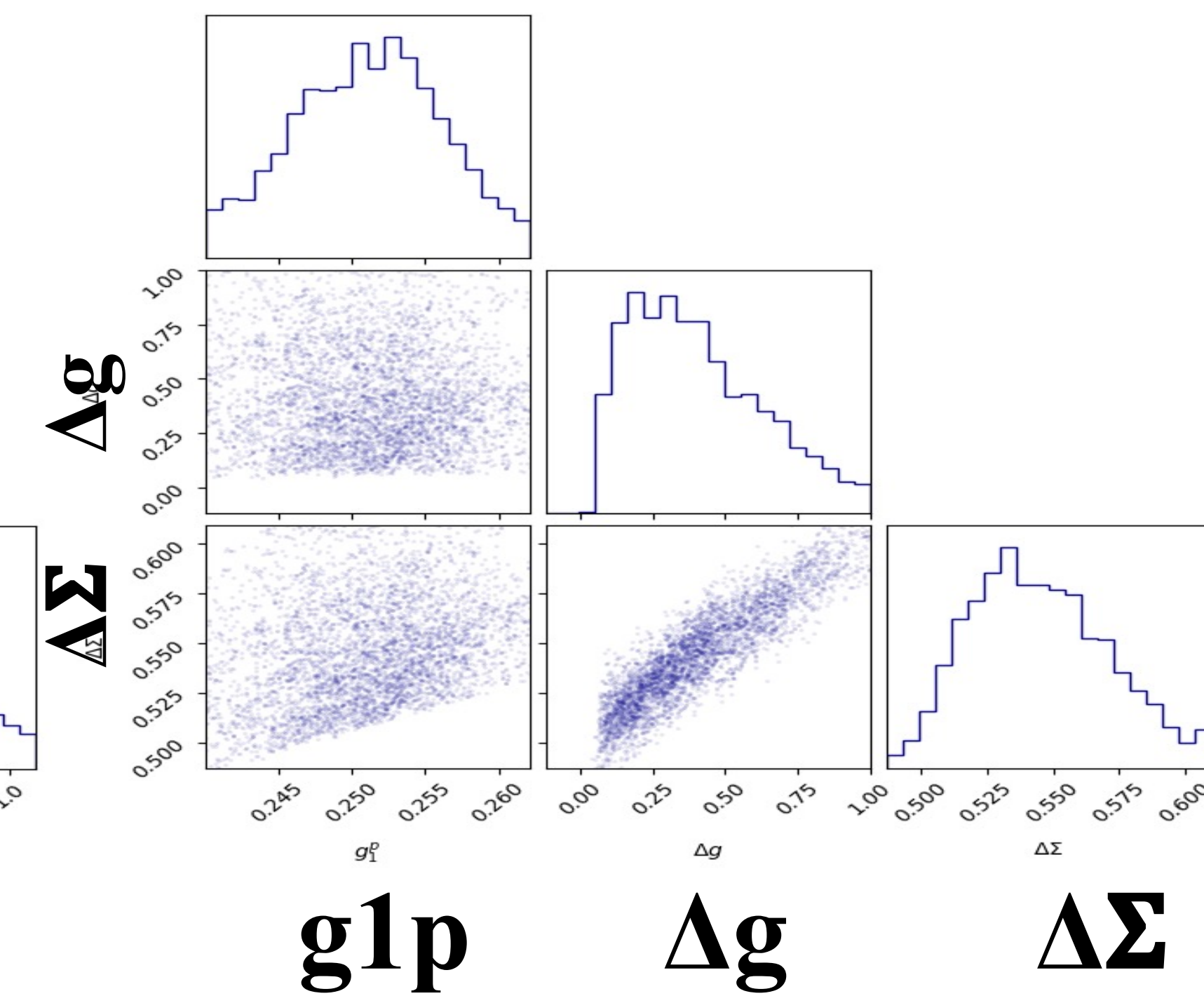
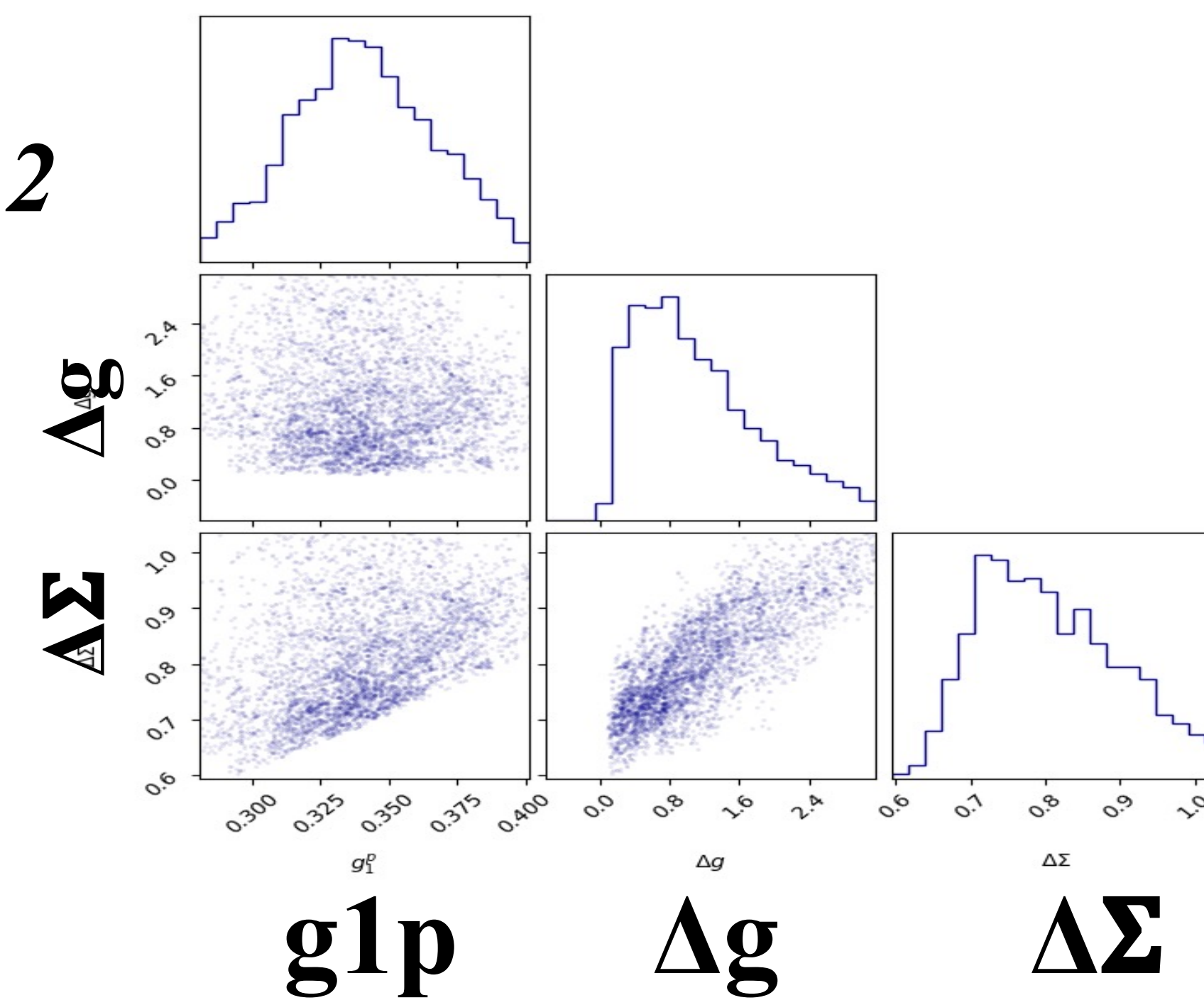
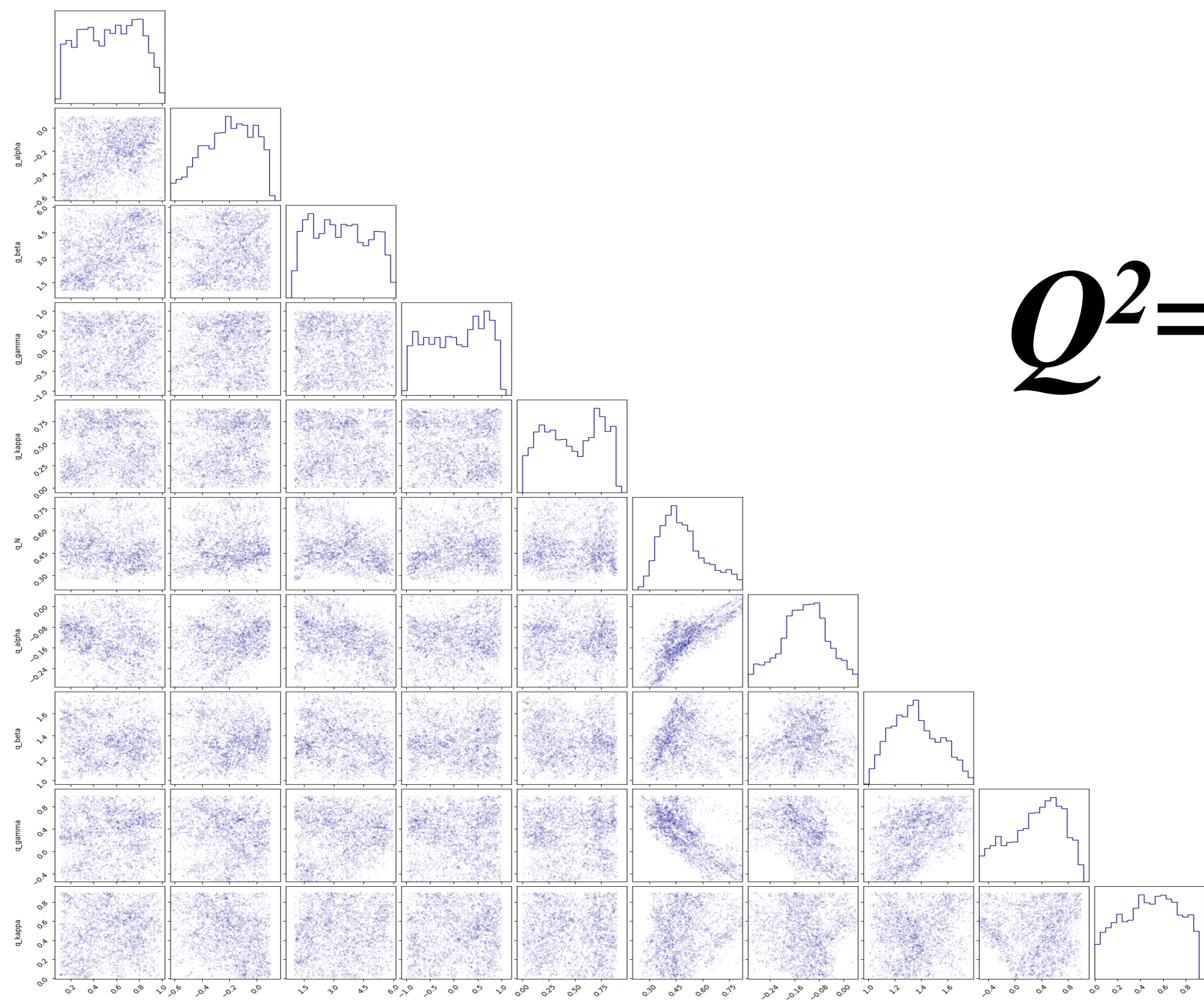
$x = 0.031$

$x = 0.162$

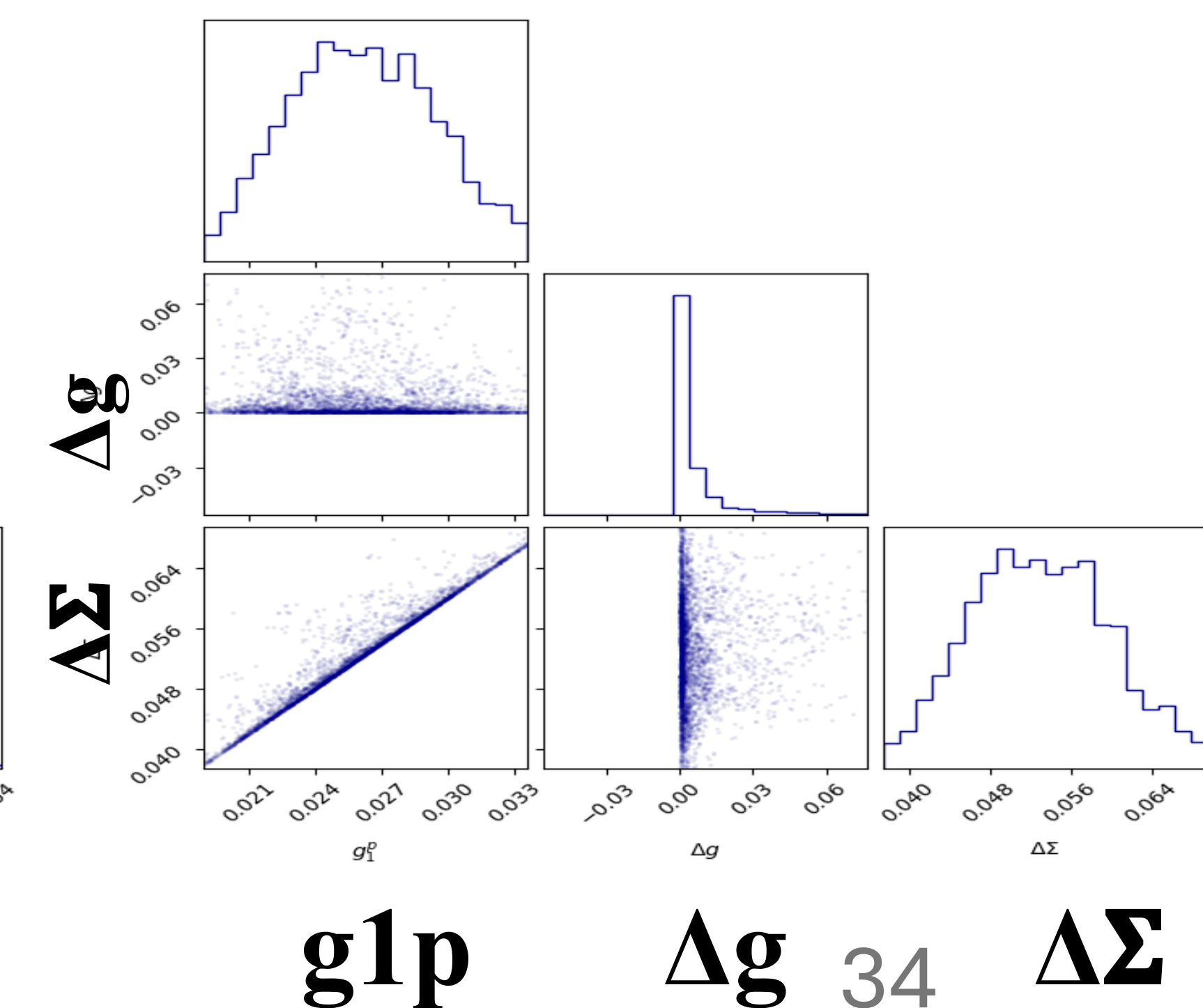
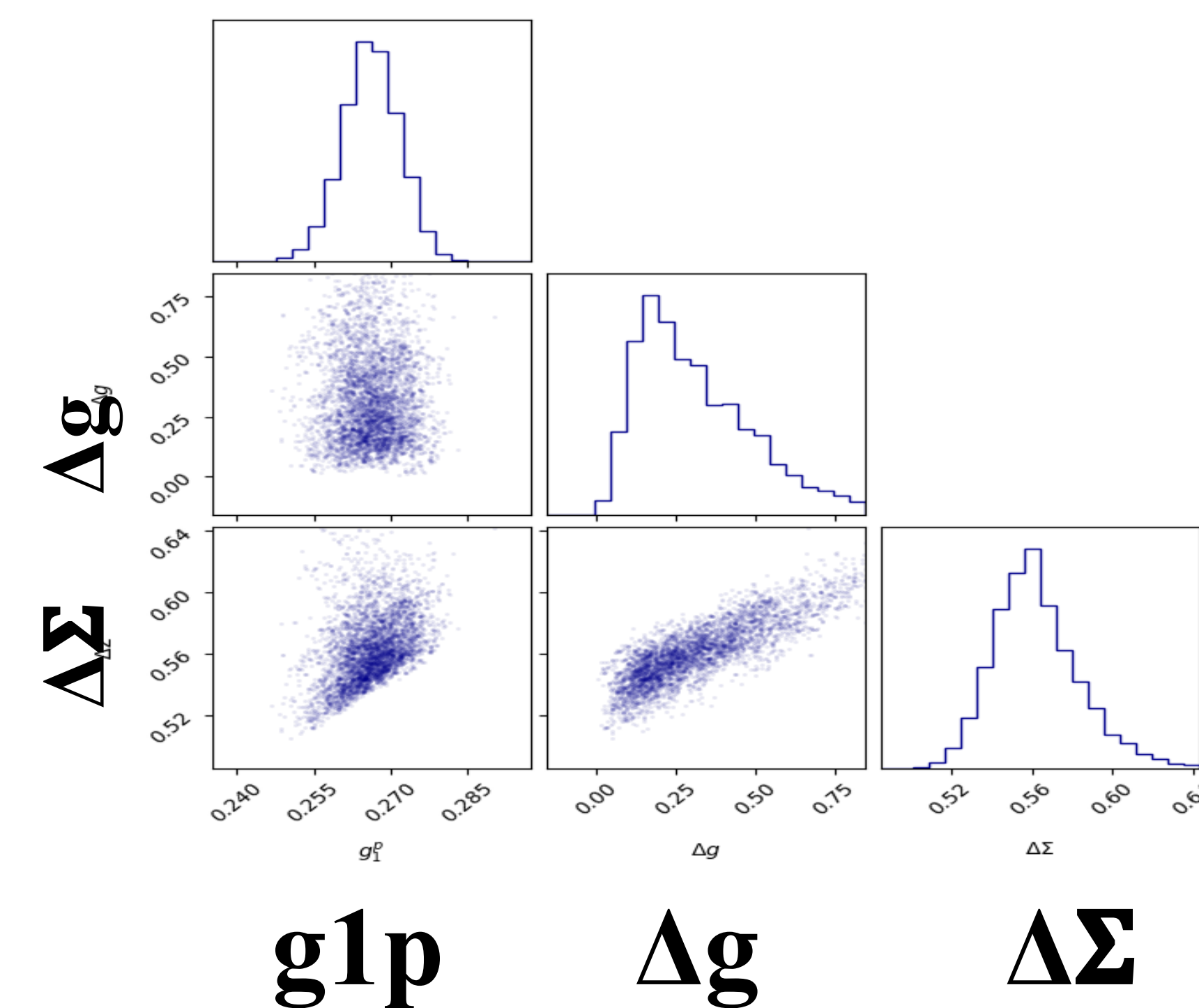
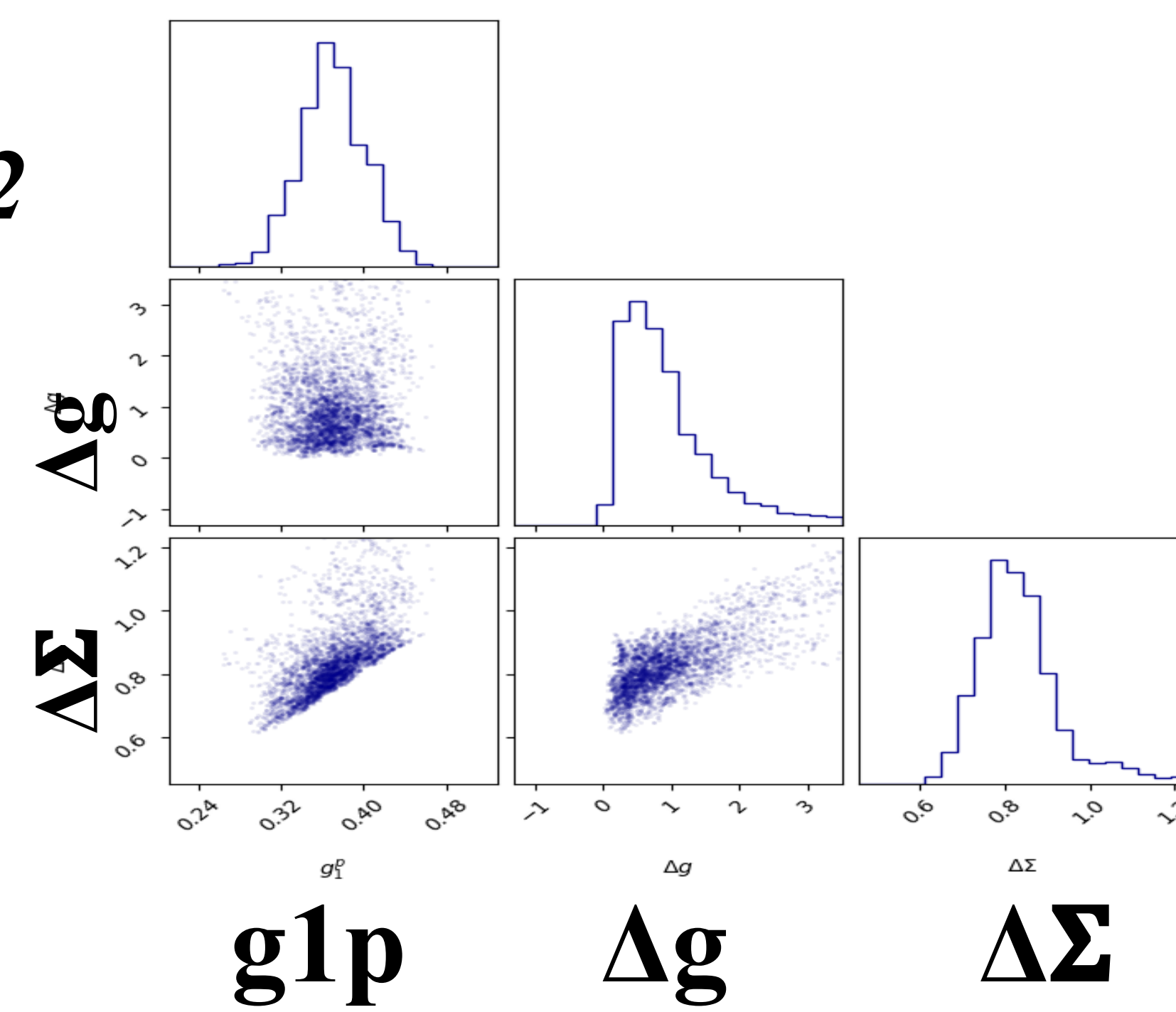
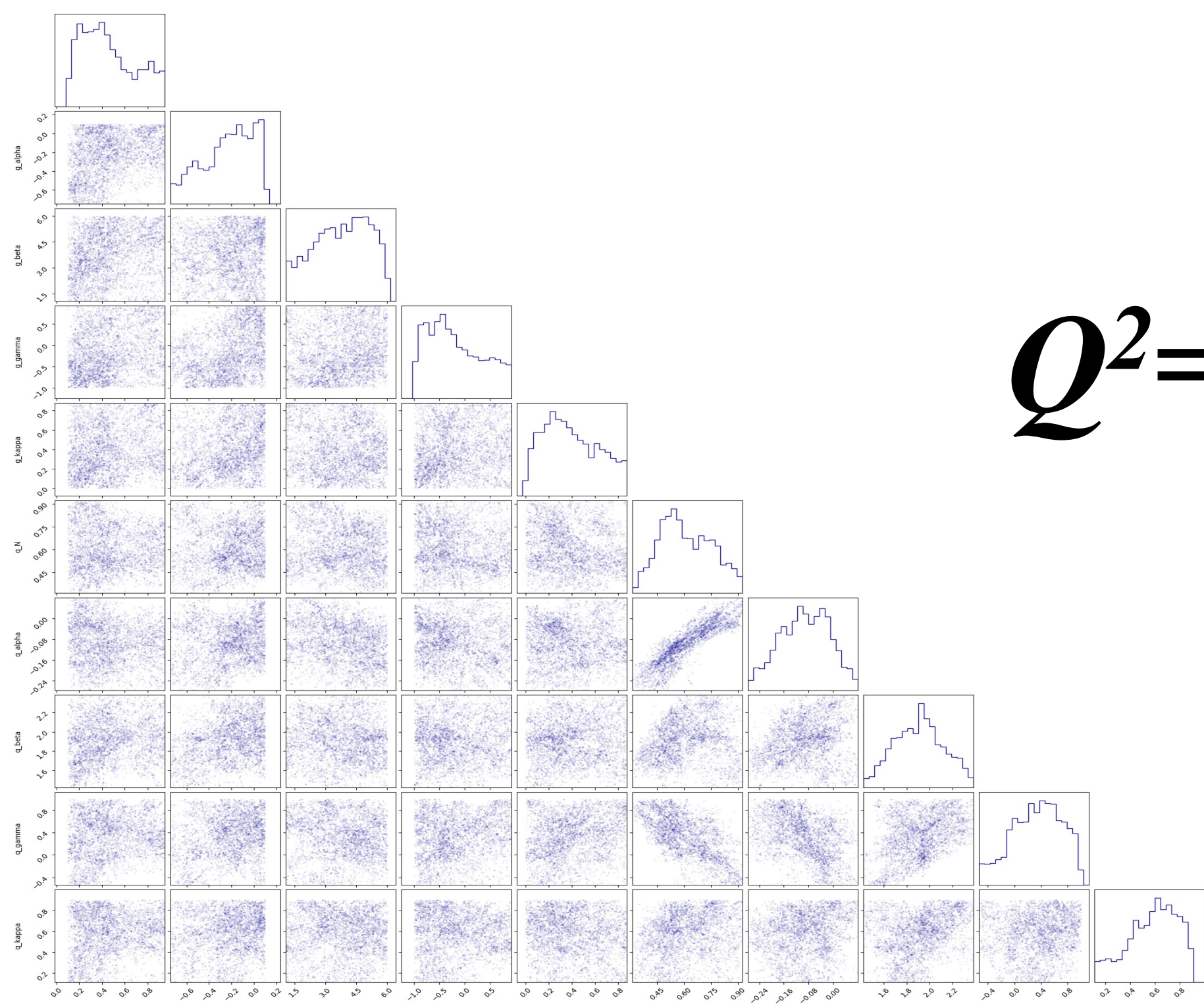
$x = 0.749$



$Q^2 = 3 \text{ (GeV)}^2$



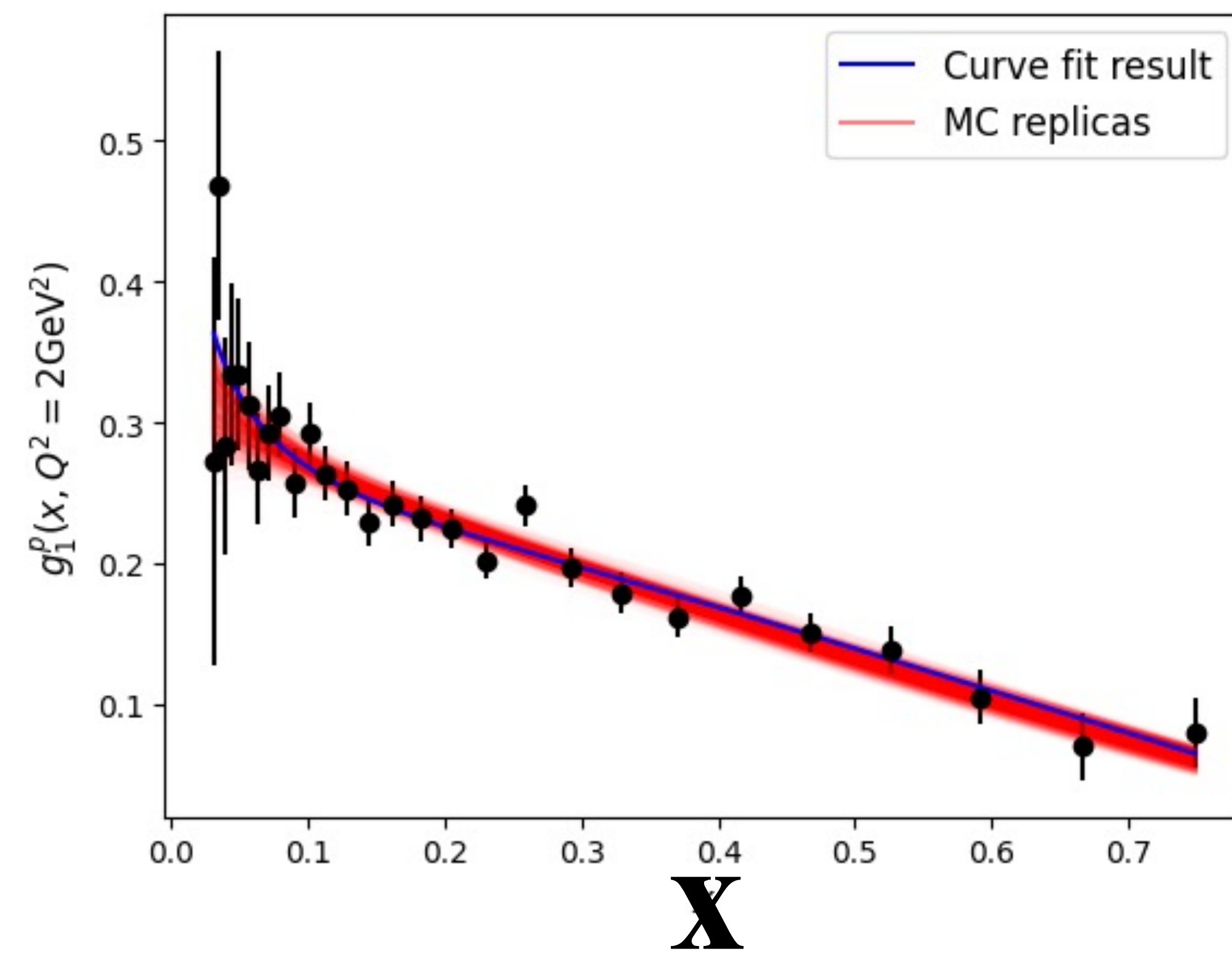
$Q^2 = 5 \text{ (GeV)}^2$



Preliminary Outcomes

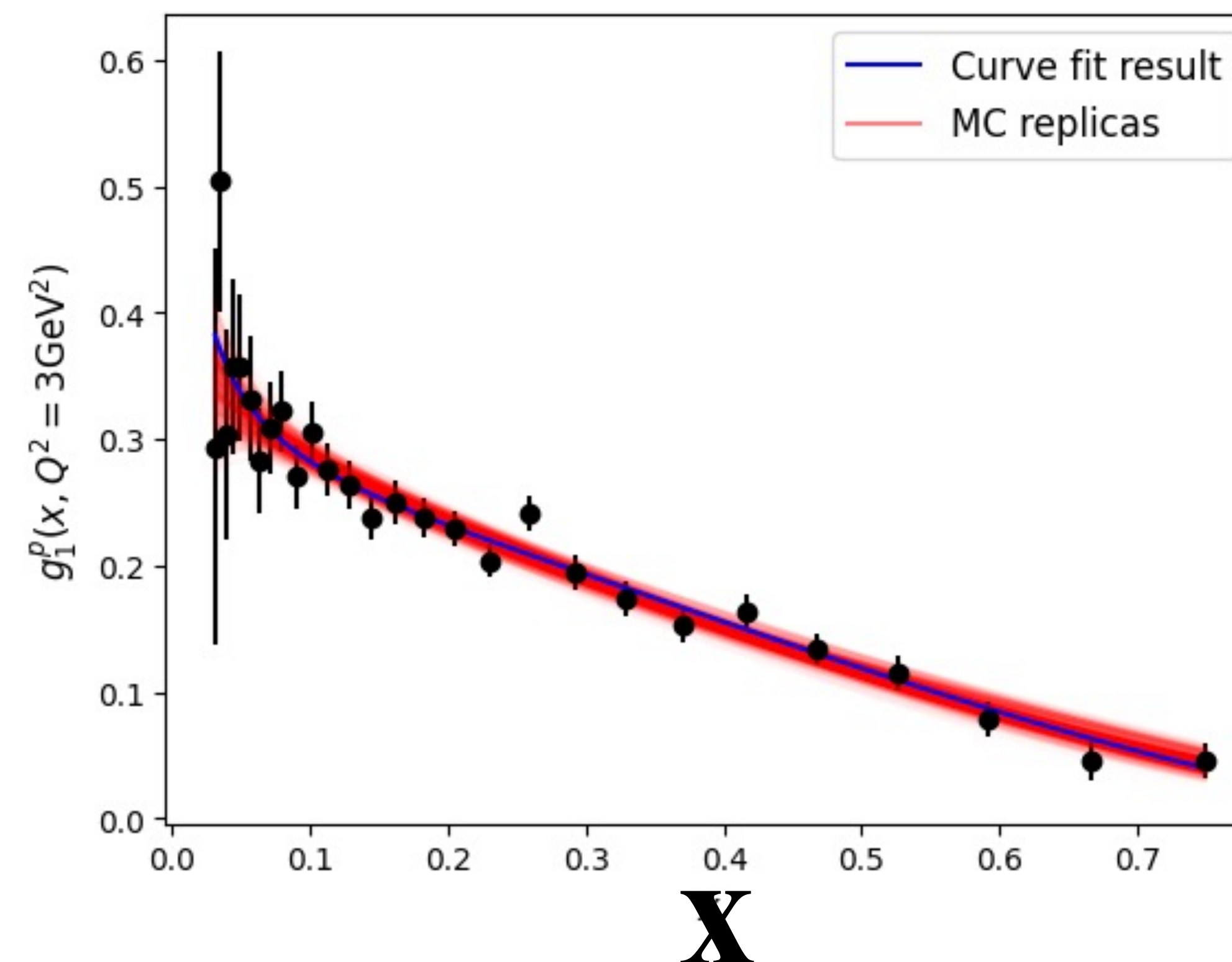
$$Q^2 = 2 \text{ (GeV)}^2$$

$g_1^p(x, Q^2)$



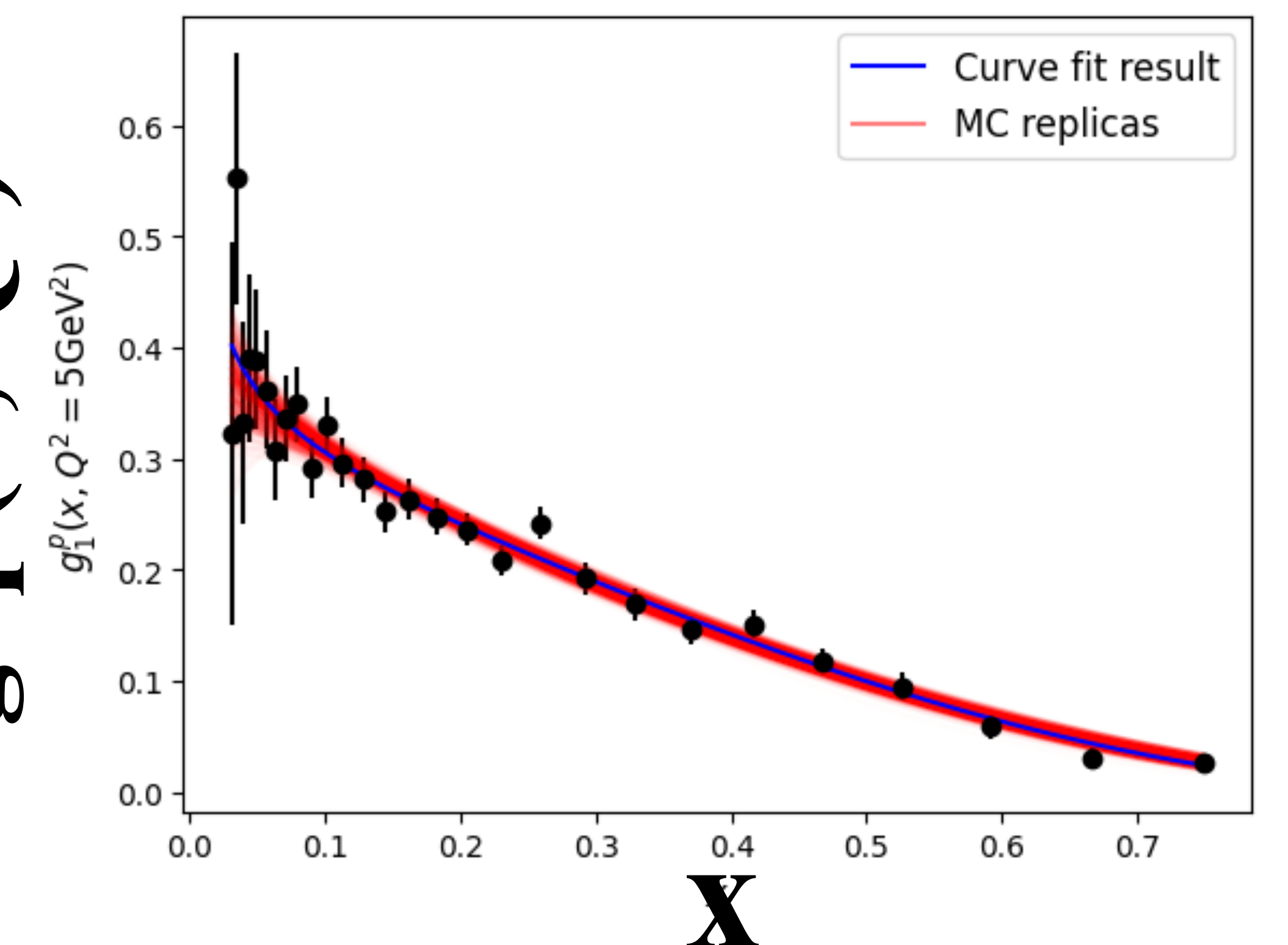
$$Q^2 = 3 \text{ (GeV)}^2$$

$g_1^p(x, Q^2)$

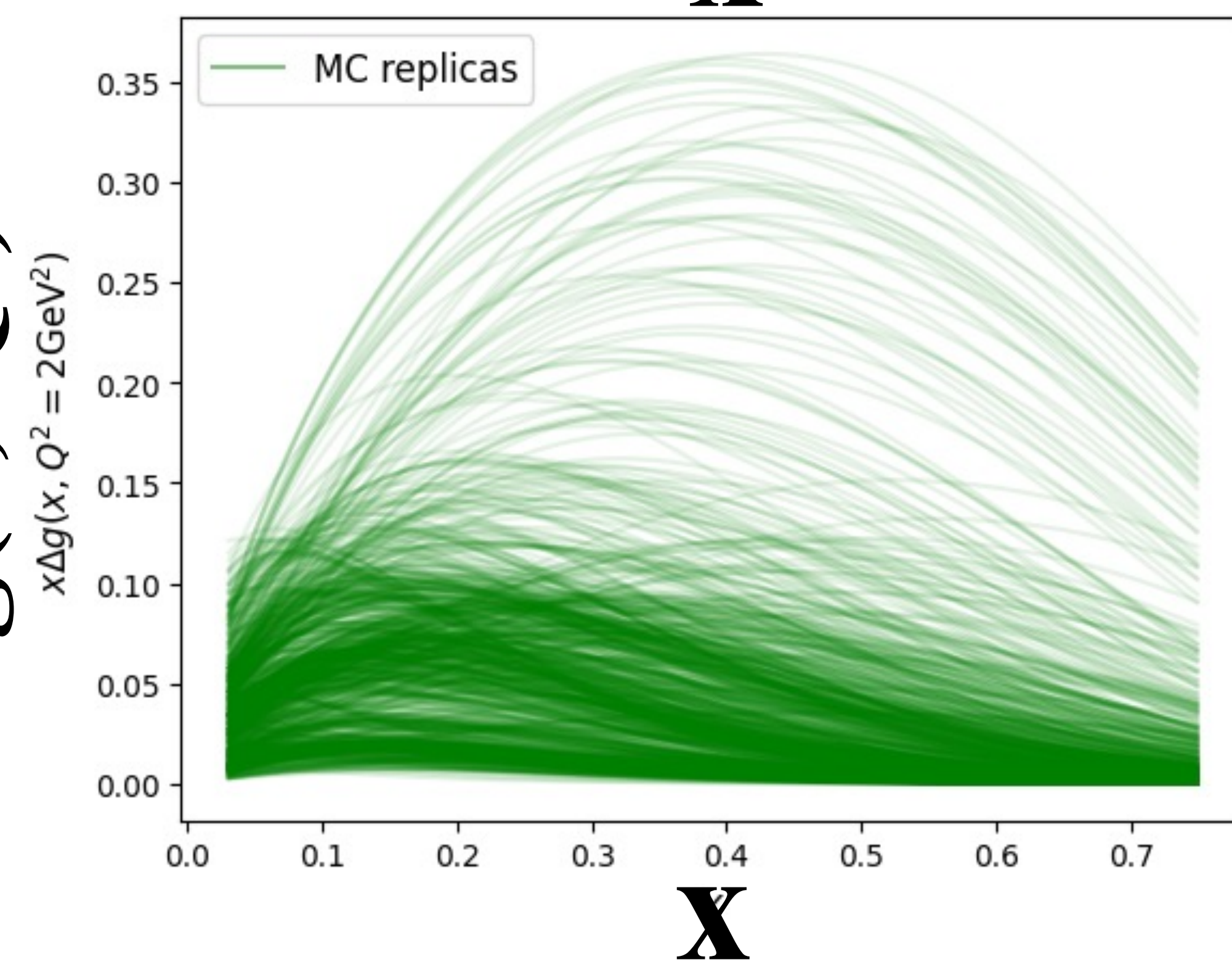


$$Q^2 = 5 \text{ (GeV)}^2$$

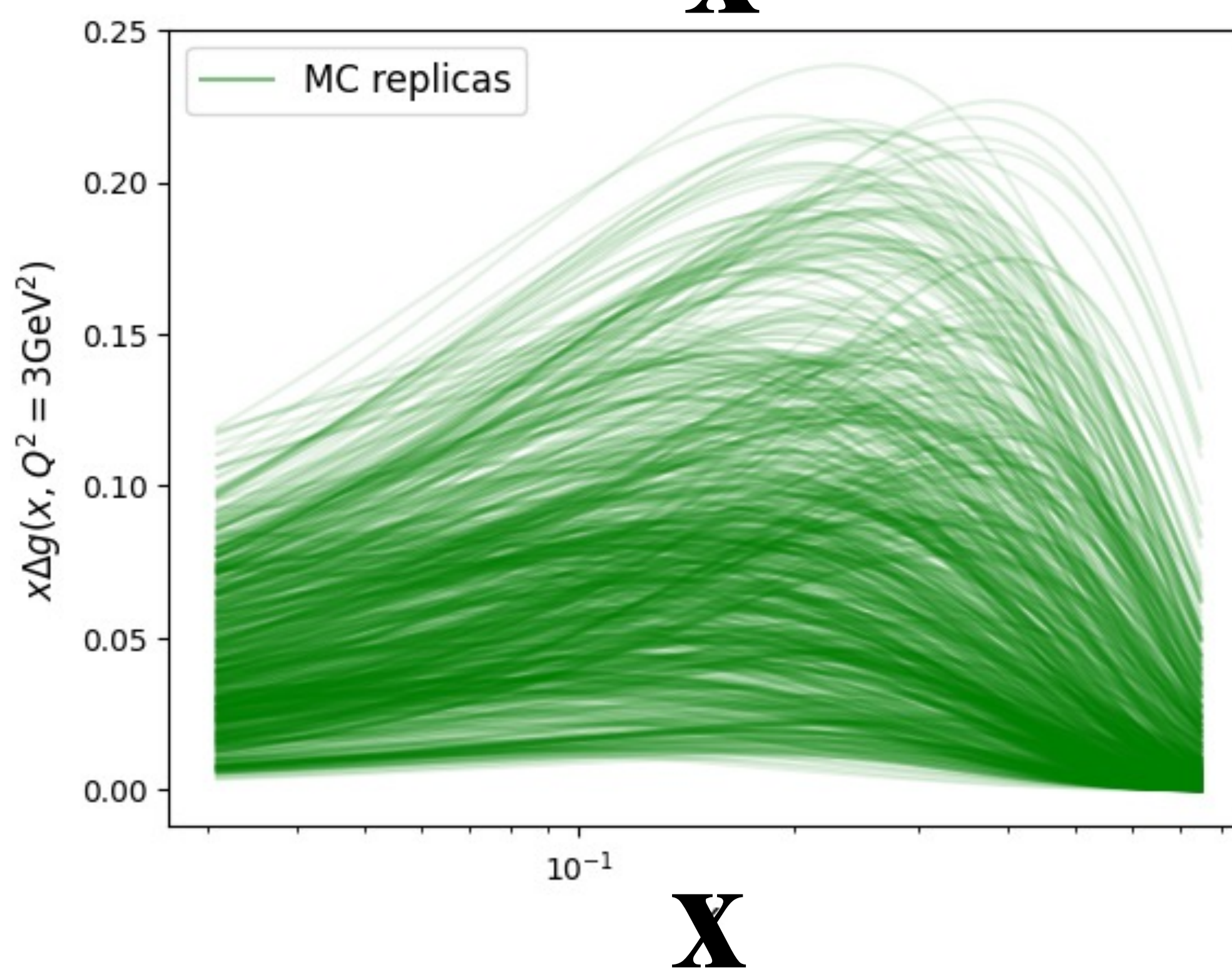
$g_1^p(x, Q^2)$



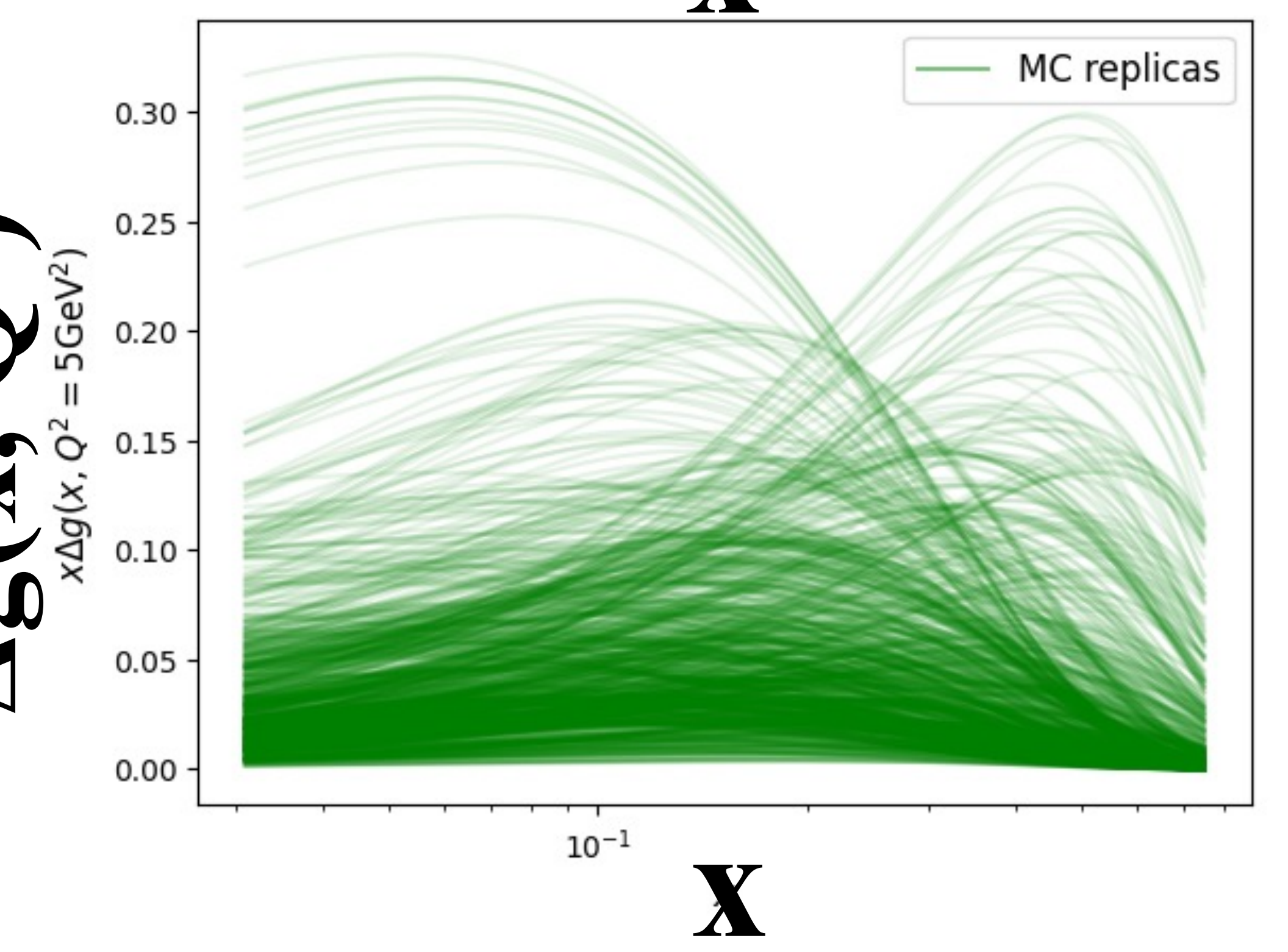
$\Delta g(x, Q^2)$



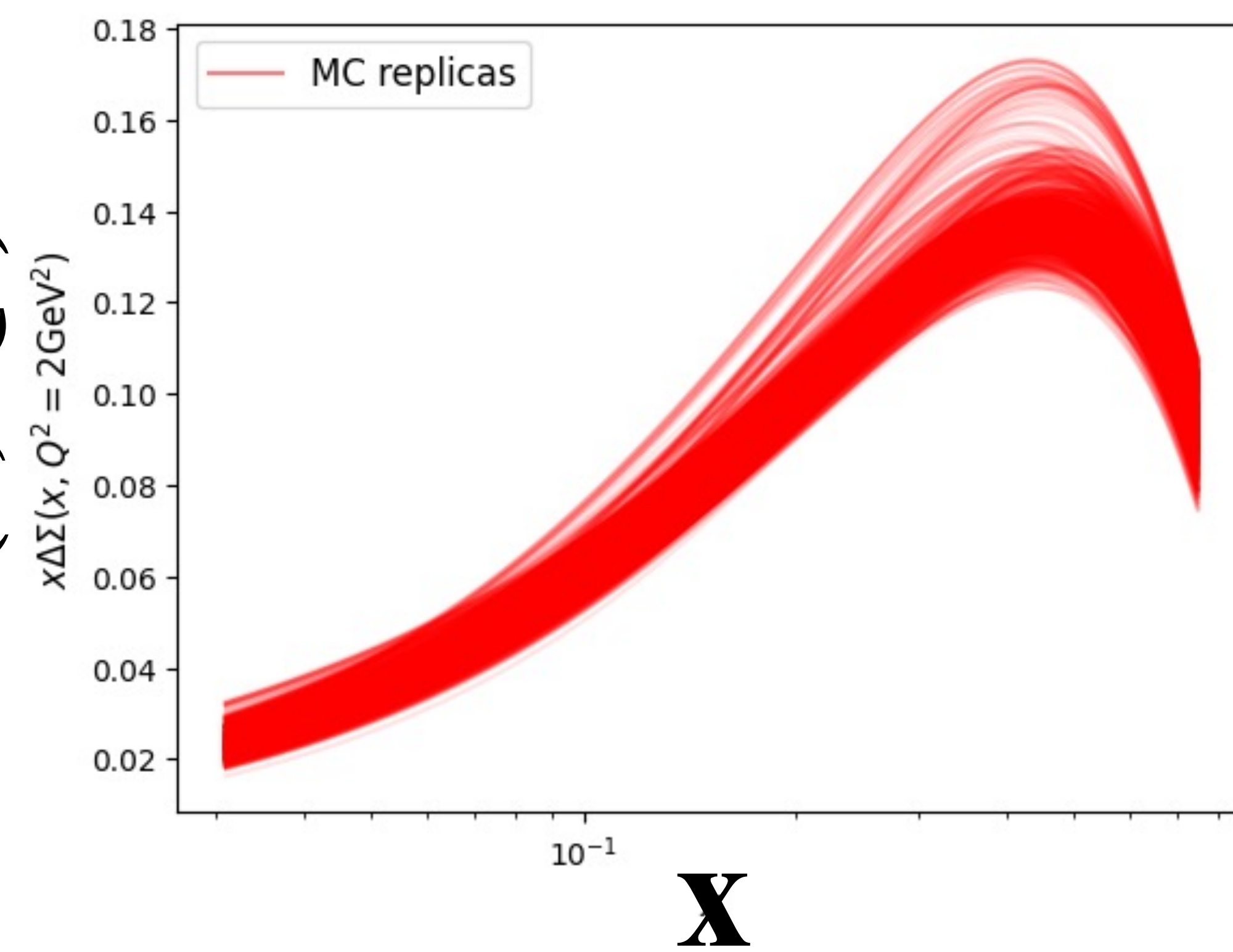
$\Delta g(x, Q^2)$



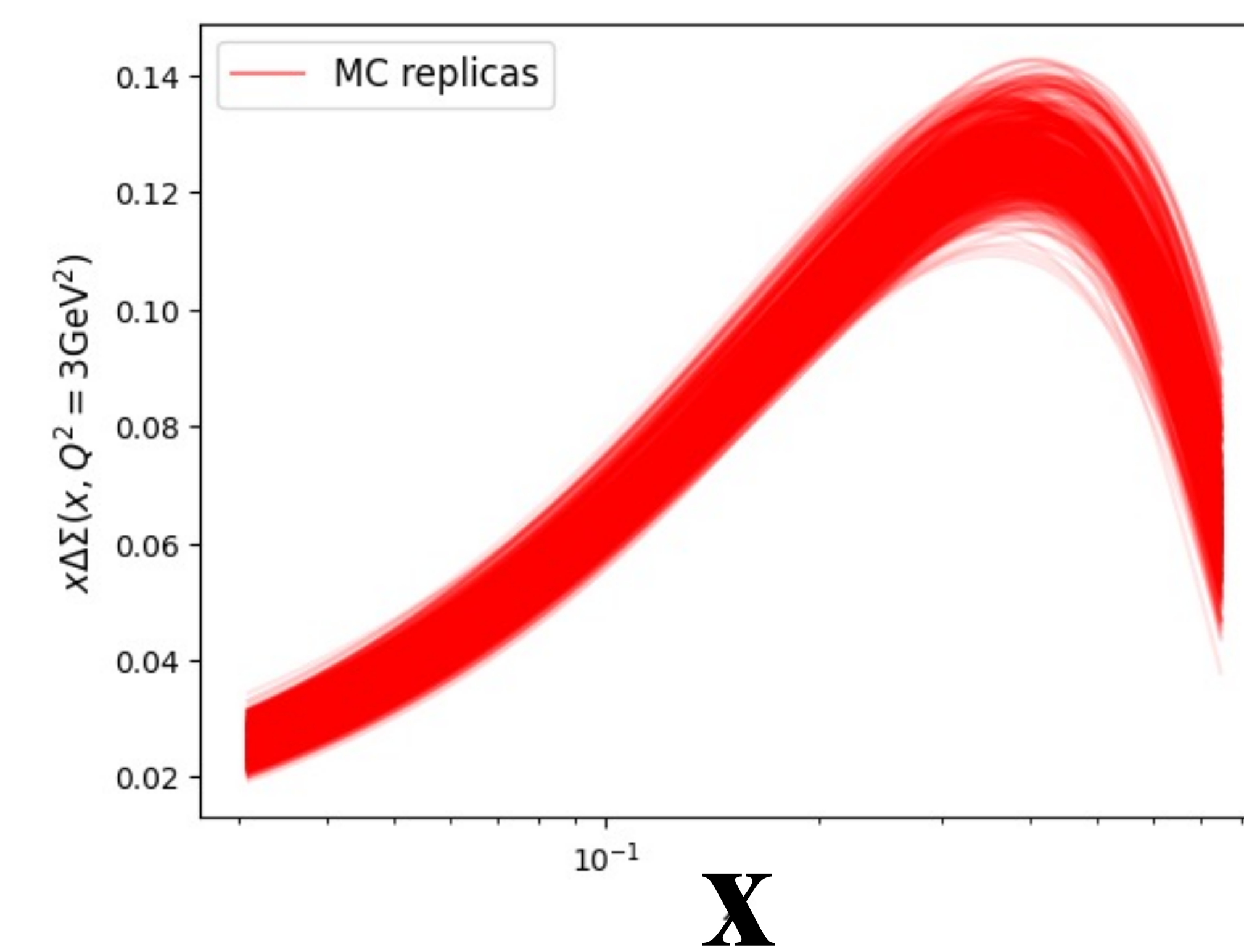
$\Delta g(x, Q^2)$



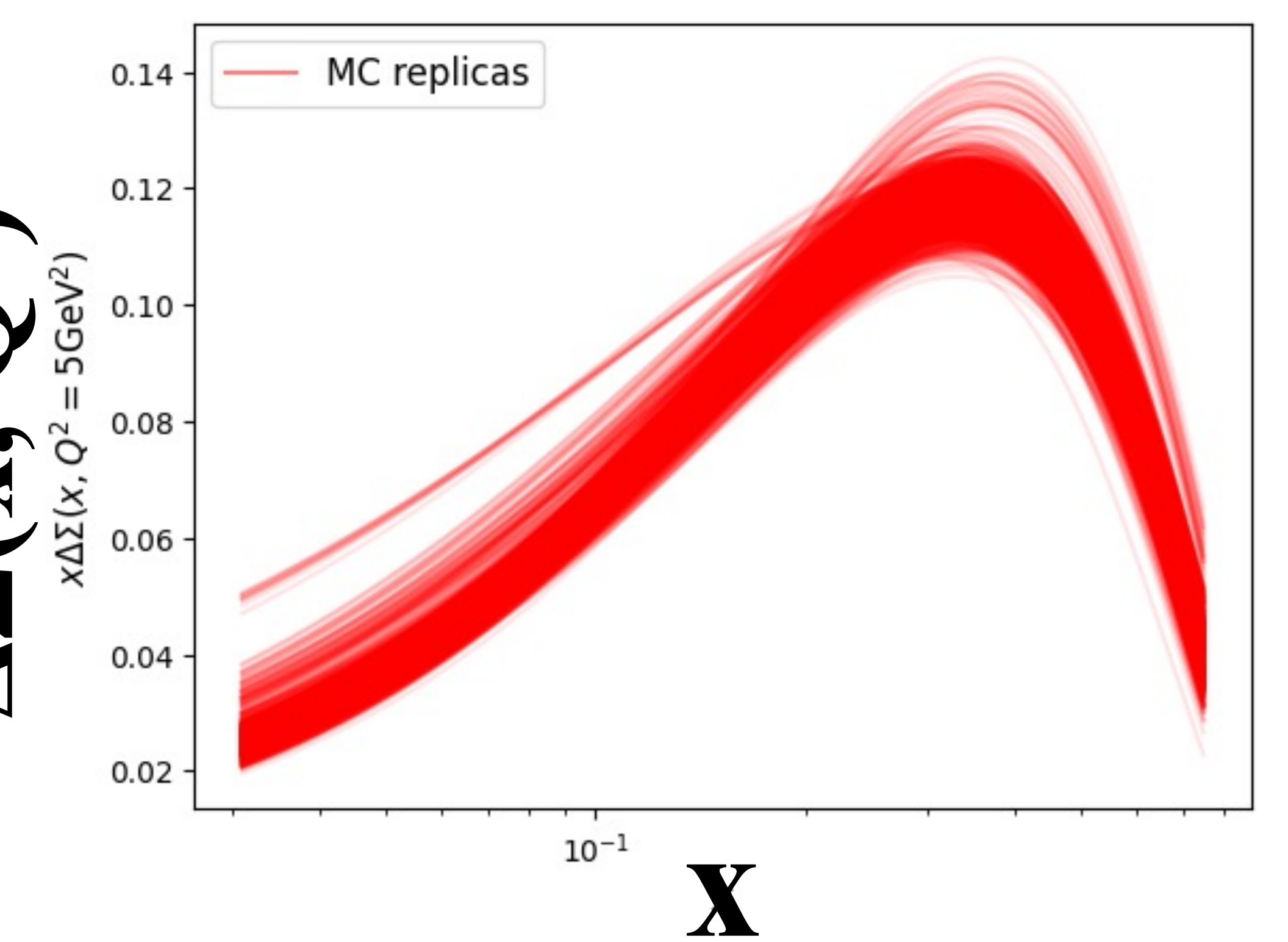
$\Delta \Sigma(x, Q^2)$



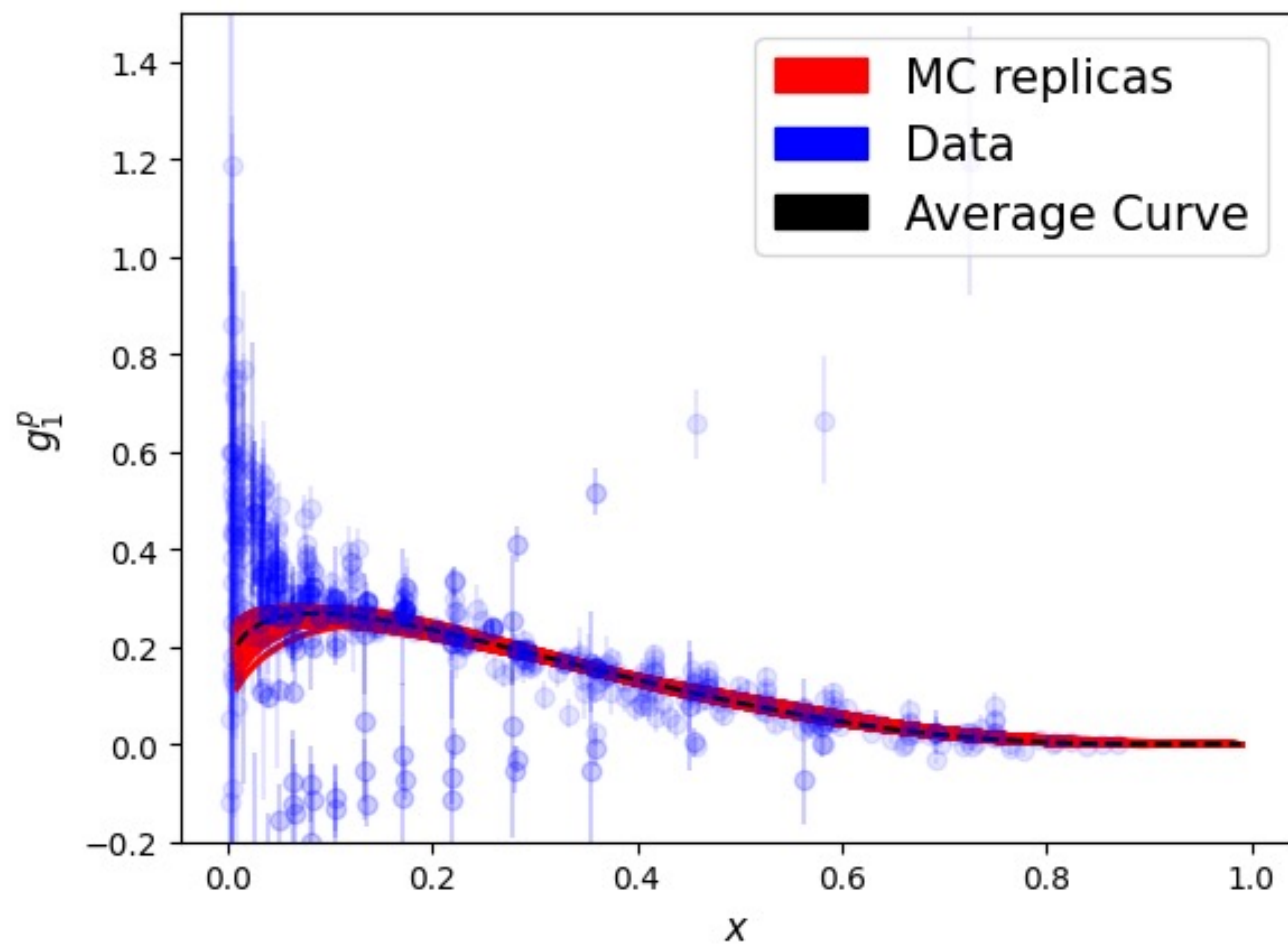
$\Delta \Sigma(x, Q^2)$



$\Delta \Sigma(x, Q^2)$



preliminary

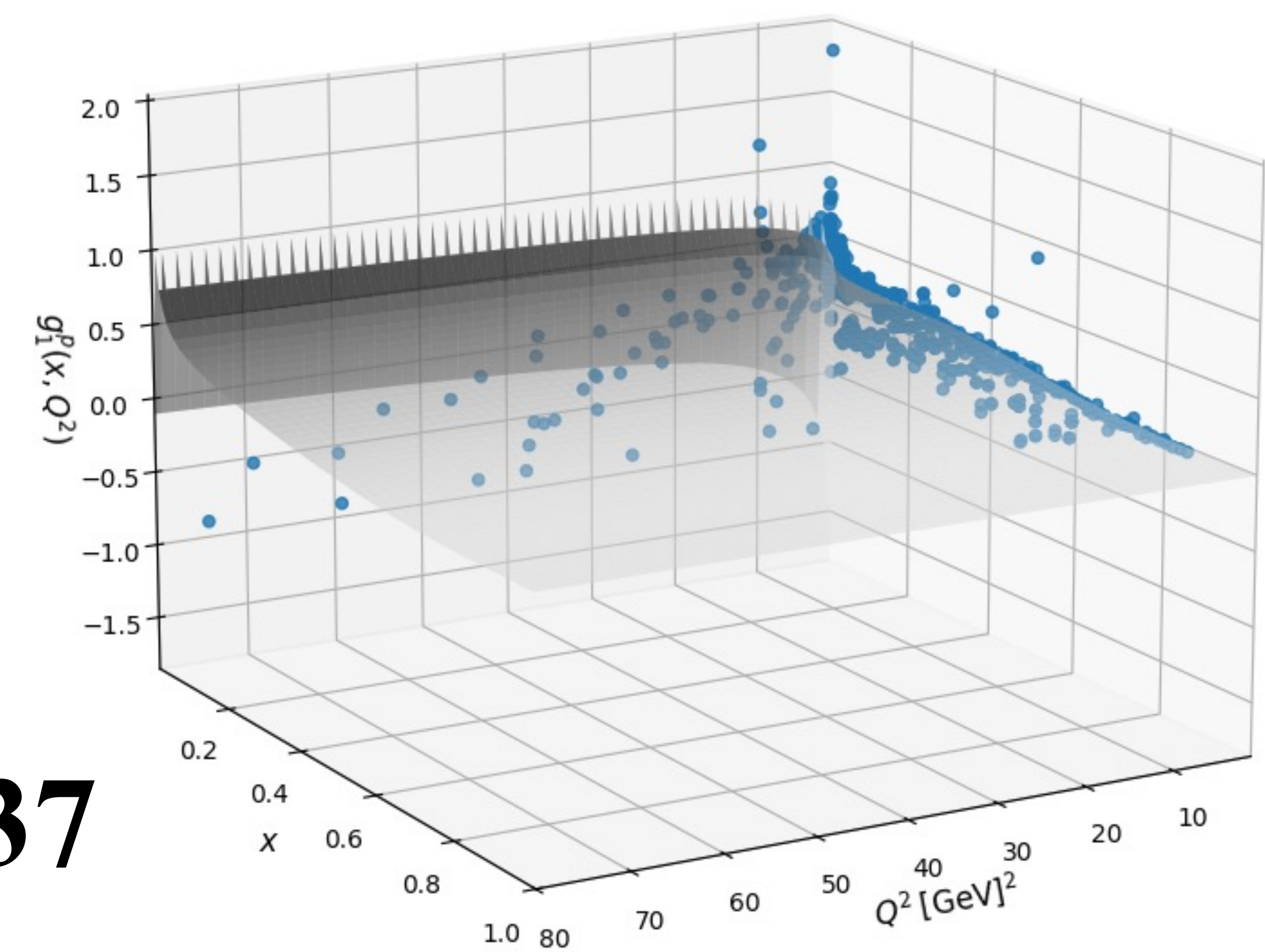


$$\Delta\Sigma \approx Nq x^\alpha (1-x)^\beta (1 + \delta\sqrt{x} + \gamma x^\kappa)$$

$$\Delta G \approx Ng x^{\alpha'} (1-x)^{\beta'} (1 + \delta'\sqrt{x} + \gamma' x^{\kappa'})$$

g_1^p with the best-fit parameters

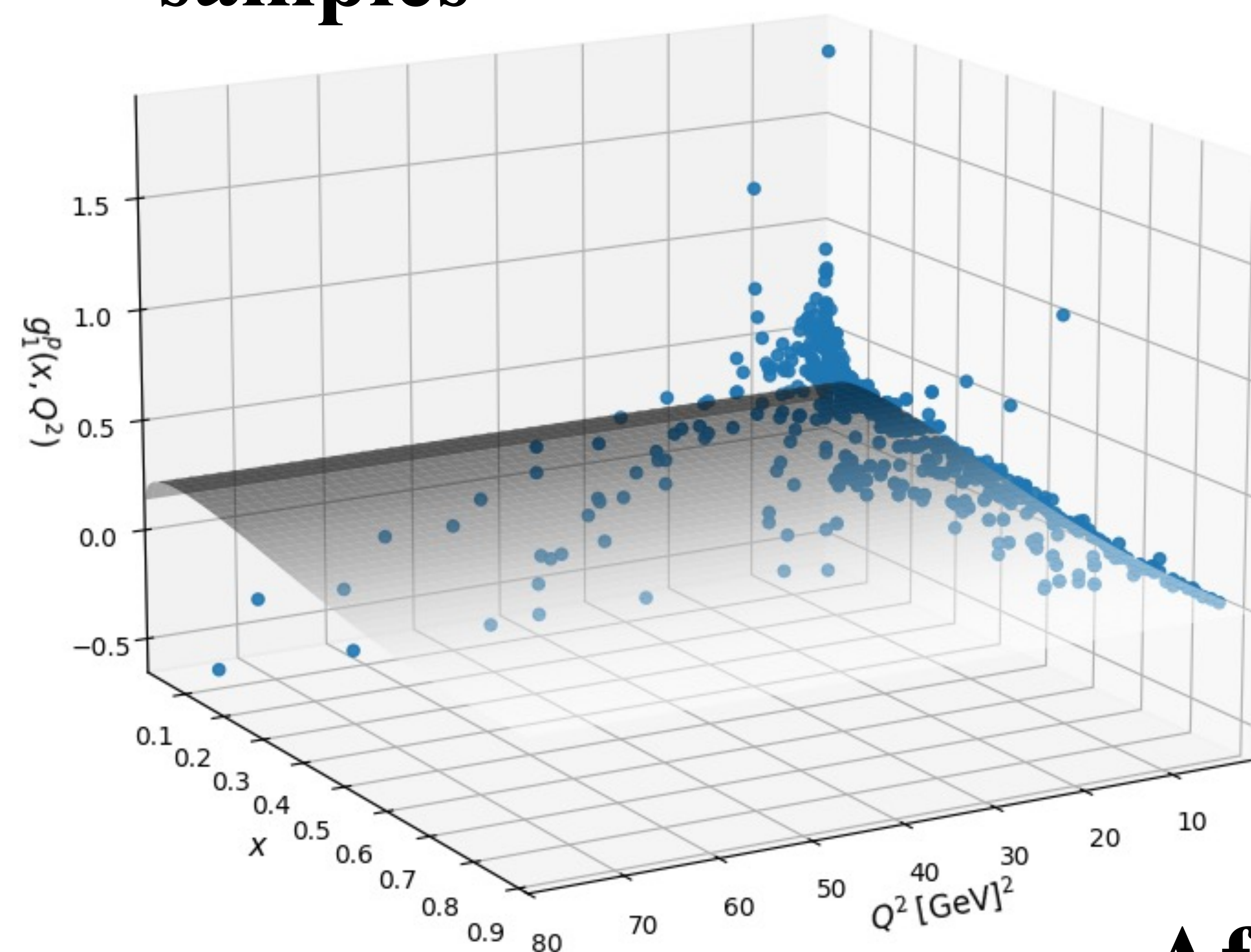
$$\chi^2/\text{ndf} = 4.37$$



g_1^p with the Monte Carlo samples

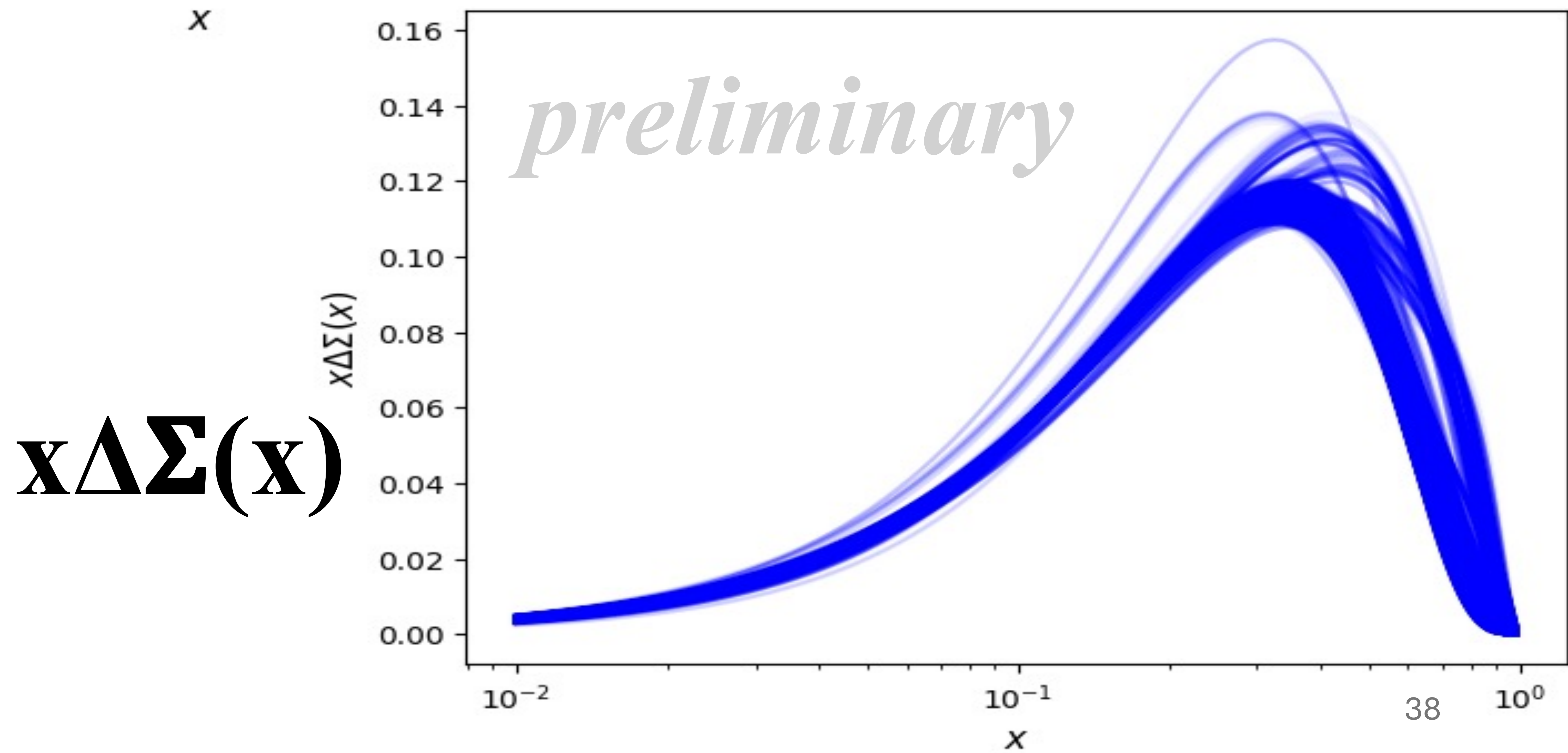
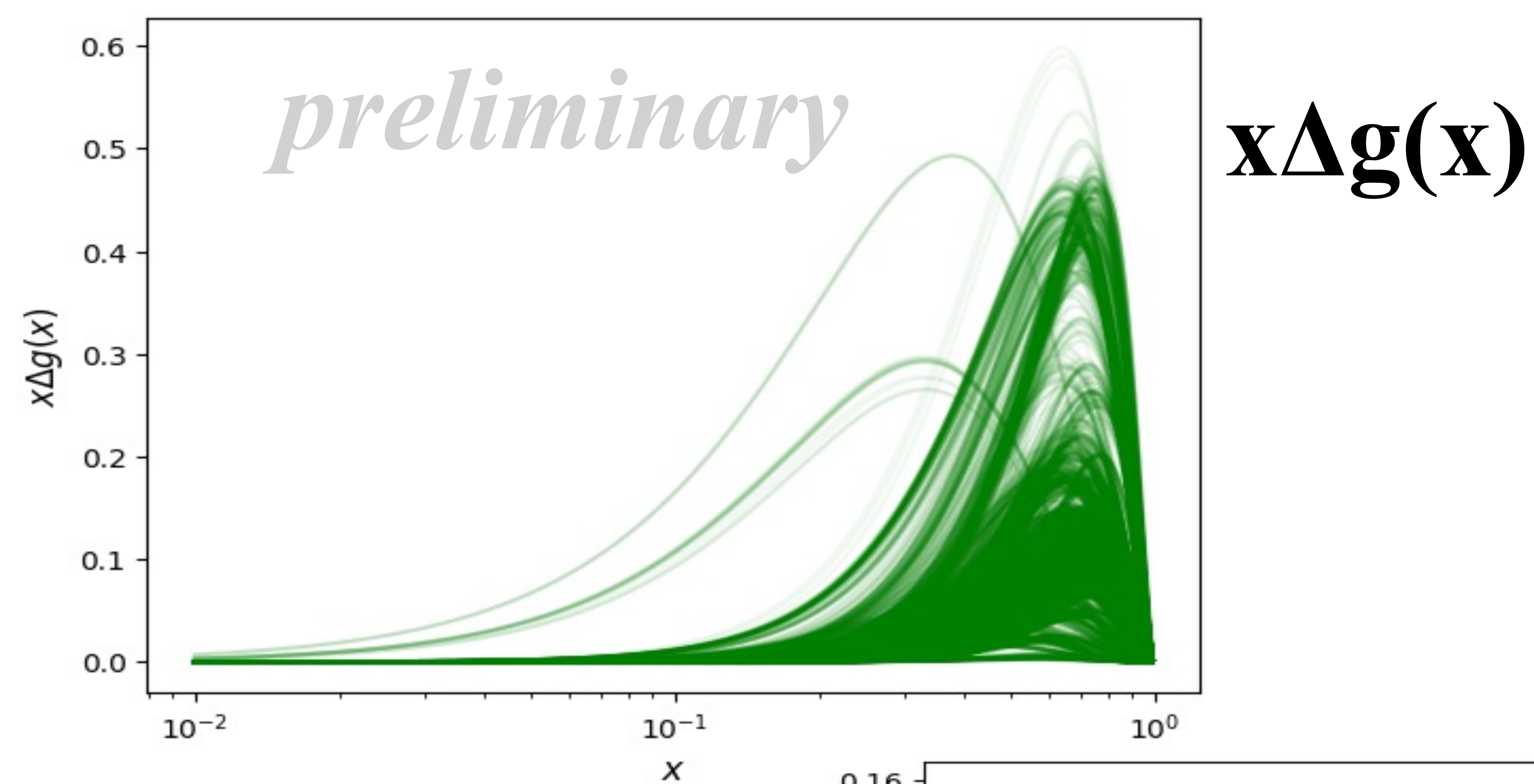
preliminary

Before MCMC sampling



Prior Domain of all parameters
[0.0001, 5]

After MCMC sampling



Lesson learned/ Conclusion

- ❑ Attempt to have a possible extraction of Compton form factors from π^0 cross-section and asymmetry data.
- ❑ Need of more different kinds data for any particular kinematic bin!
 - ❑ Like more Φ dependent observables
 - ❑ More kind of polarization observables.
- ❑ Broader range of x and Q^2



Lesson learned/ Conclusion

- ❑ Careful analysis of polarized DIS data to study helicity pdfs.
- ❑ Need of more robust data for wide range of kinematics (x , Q^2).
- ❑ We want to figure out what new experiment (set up) can help to solve our parametric degeneracy.

What next can be done!

□ Possibility to improve the Analysis **More Observables**

$$F_{LL} = 4[2(1 - \xi^2)(\Re \mathcal{H} \Re \tilde{\mathcal{H}} + \Im \mathcal{H} \Im \tilde{\mathcal{H}}) + 2 \frac{t_o - t}{2M^2} (\Re \mathcal{E}(\xi \Re \tilde{\mathcal{E}}) + \Im \mathcal{E}(\xi \Im \tilde{\mathcal{E}})) \\ + \frac{2\xi^2}{1 - \xi^2} (\Re \mathcal{H} \Re \tilde{\mathcal{E}} + \Im \mathcal{H} \Im \tilde{\mathcal{E}} + \Re \tilde{\mathcal{H}} \Re \mathcal{E} + \Im \tilde{\mathcal{H}} \Im \mathcal{E})],$$

$$F_{LL}^{\mathcal{I}, tw2} = A_{LL}^{\mathcal{I}} \Re(F_1(\tilde{\mathcal{H}} - \xi \tilde{\mathcal{E}}) + \tau F_2 \tilde{\mathcal{E}}) + B_{LL}^{\mathcal{I}} G_M \Re \tilde{\mathcal{H}} + C_{LL}^{\mathcal{I}} G_M \Re(\mathcal{H} + \mathcal{E})$$

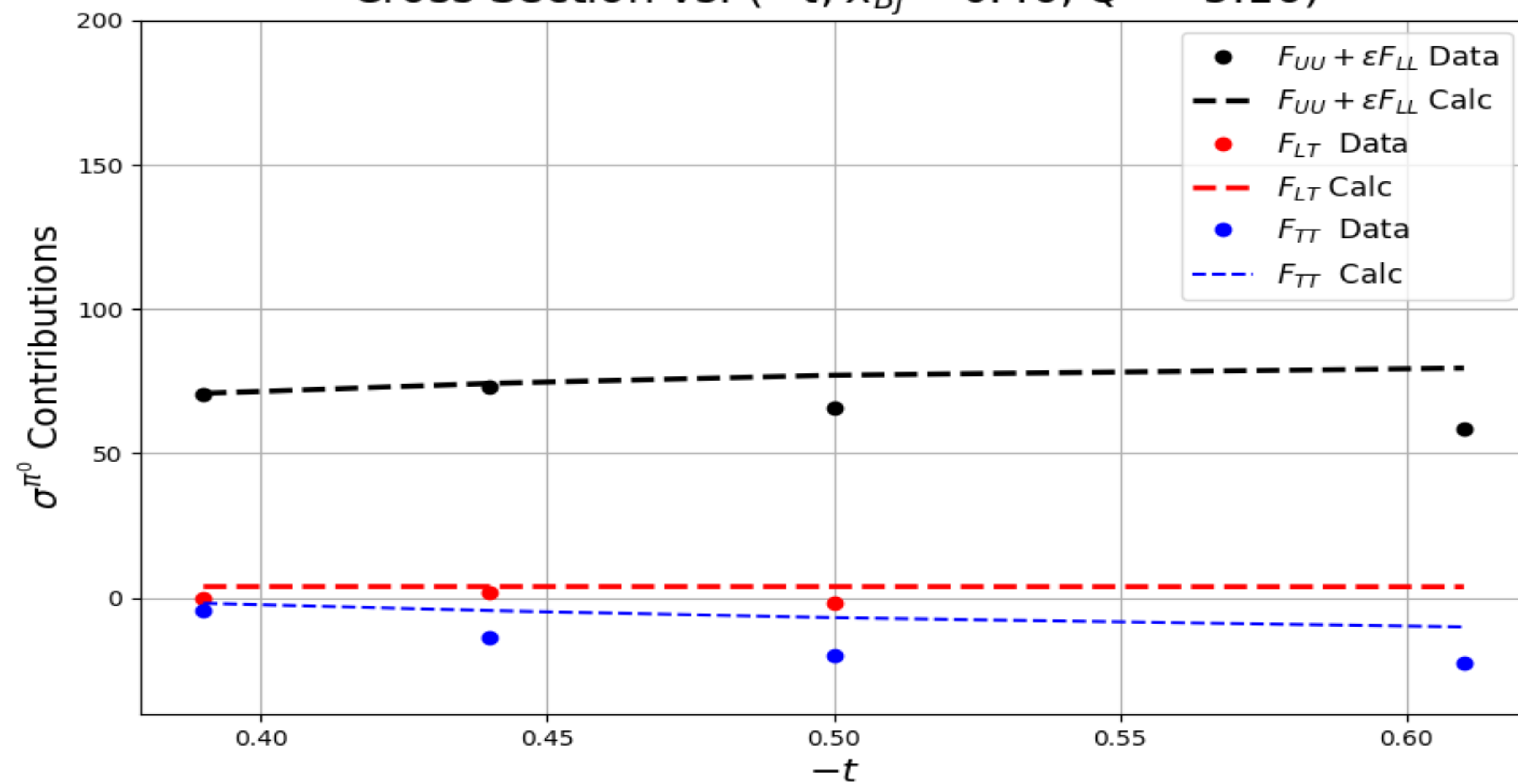
$$\frac{d^4\sigma}{dx_{Bj} dy d\phi dt} = \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\ + S_{||} \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\ - S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \frac{\epsilon}{2} \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \right. \\ + \left. \sqrt{\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \\ + \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \right\}$$

What next can be done!

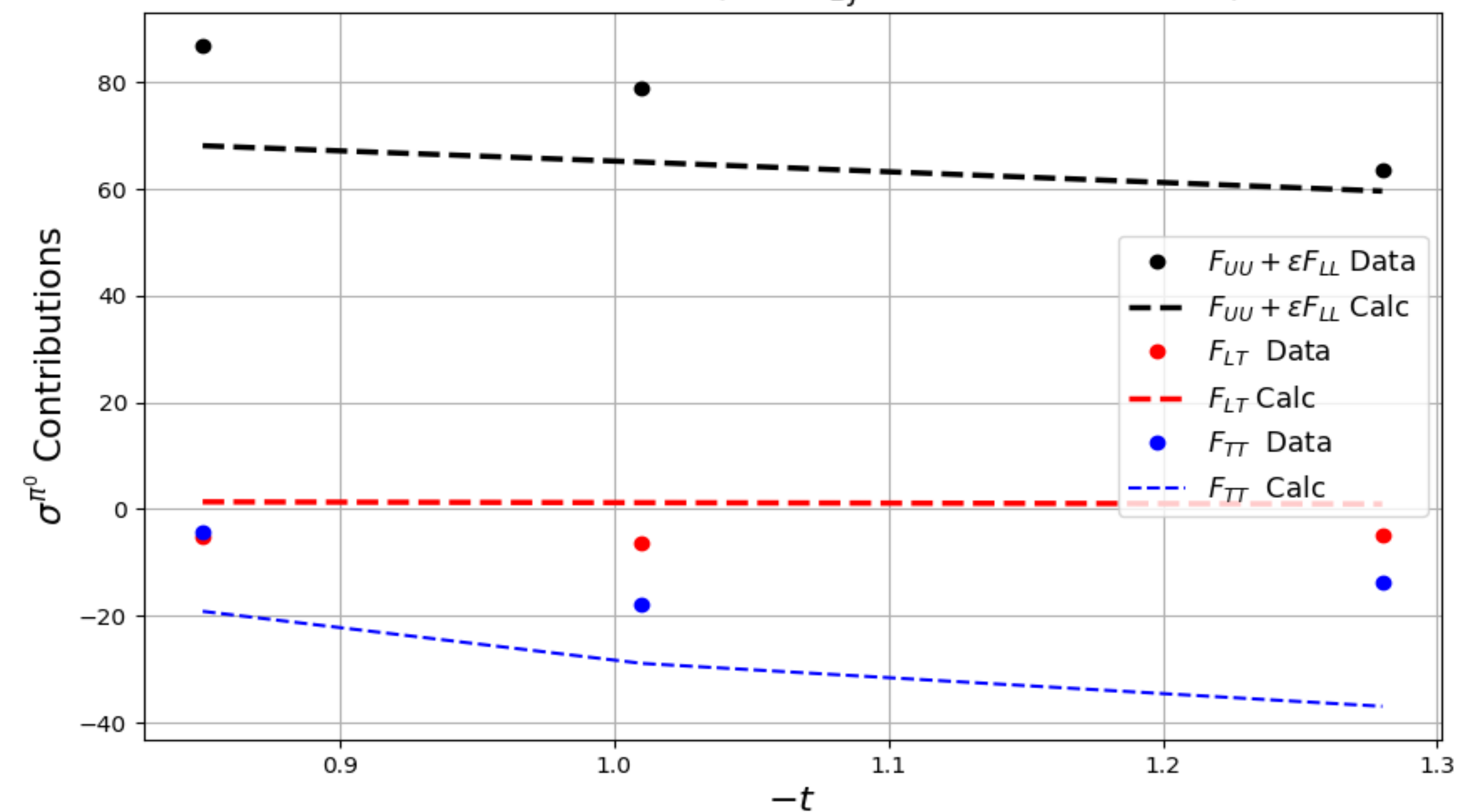
□ Analysis on more kinematic bins

Model comparison with data at HALL A

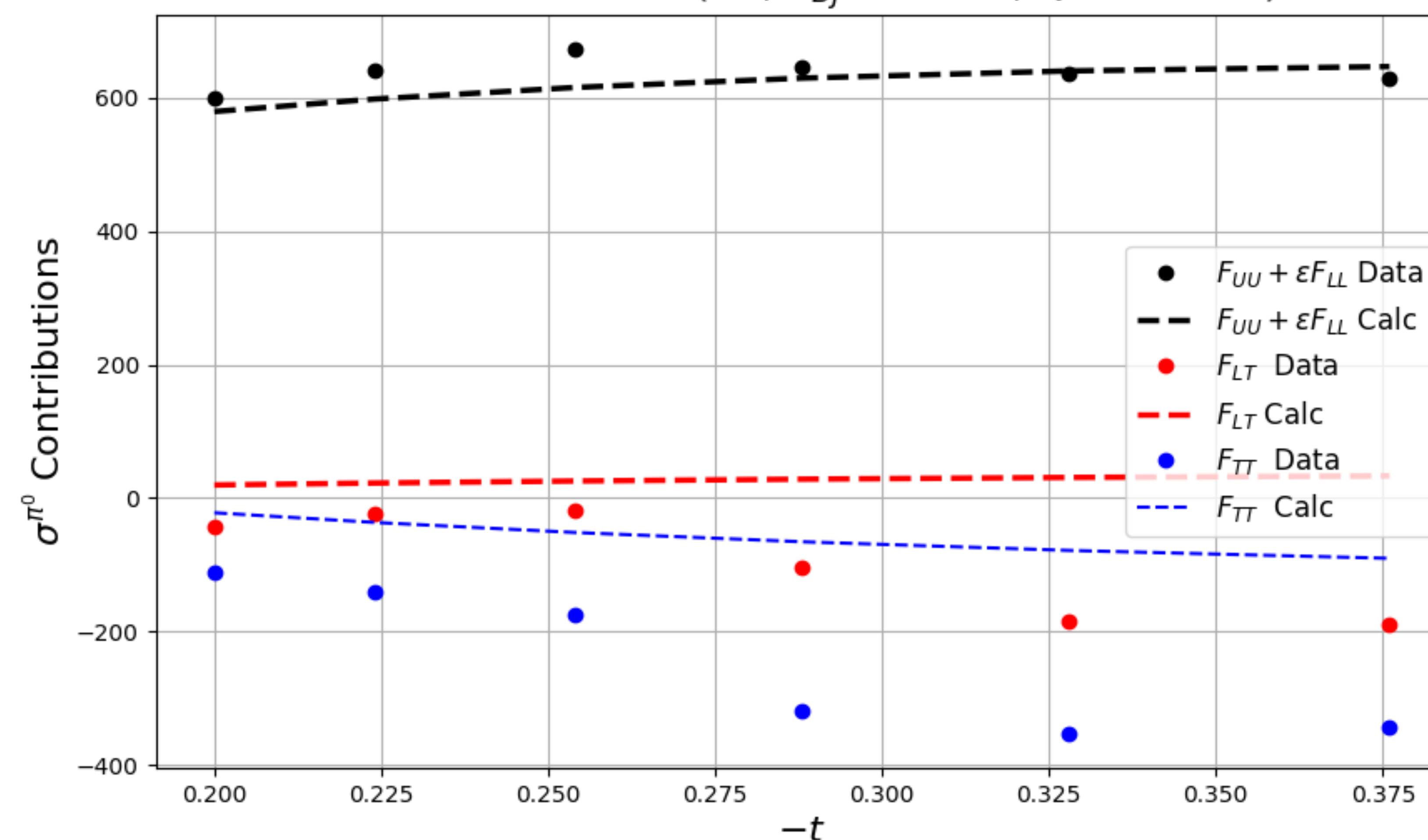
Cross Section vs. $(-t, x_{Bj} = 0.46, Q^2 = 5.16)$



Cross Section vs. $(-t, x_{Bj} = 0.59, Q^2 = 5.49)$

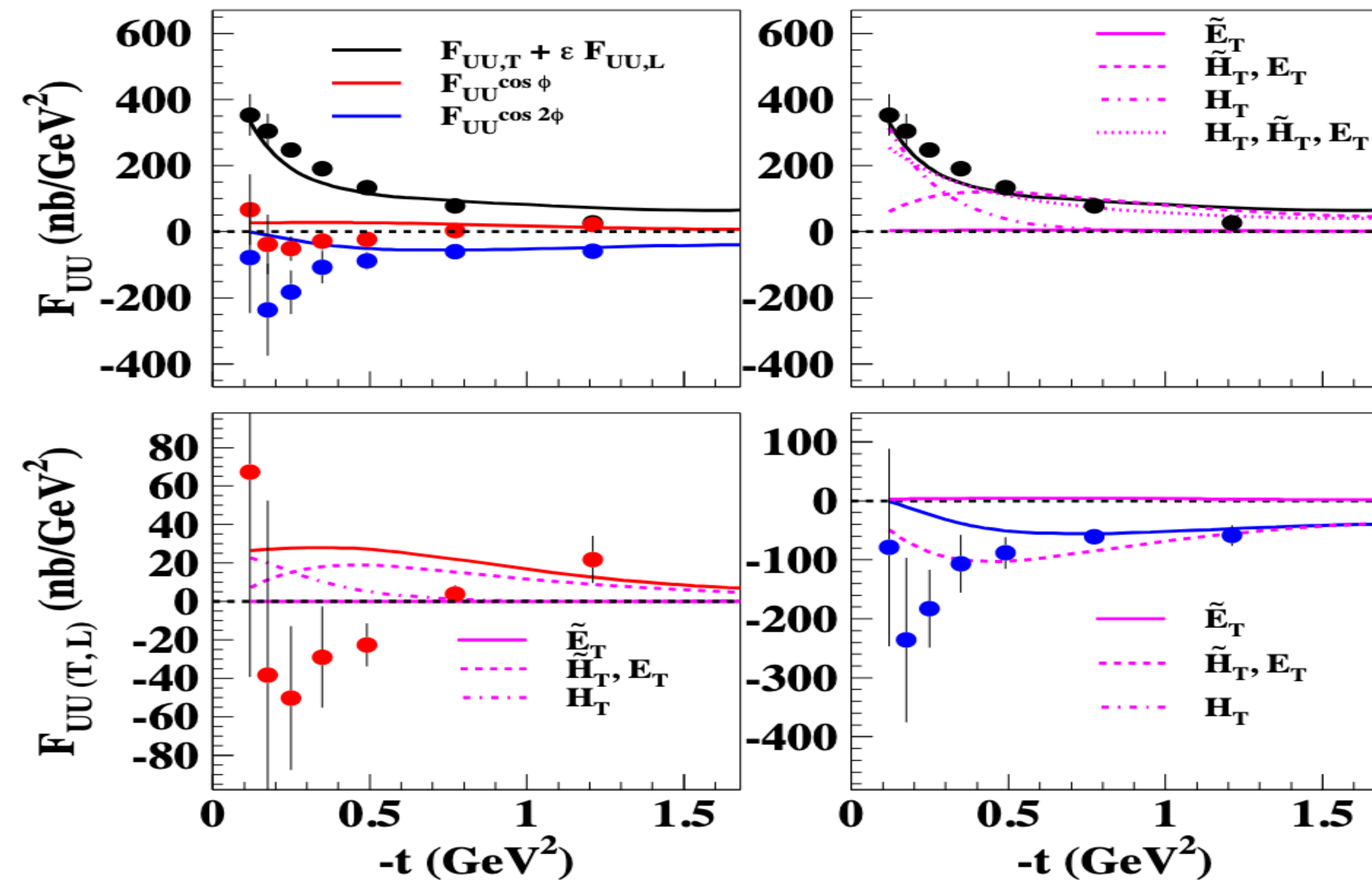


Cross Section vs. $(-t, x_{Bj} = 0.368, Q^2 = 1.941)$



Phys. Rev. Lett. 127,
152301
GGL, Phys. Rev. D 91,
114013 (2015)

$$F_{UU,T} + \epsilon F_{UU,L}$$



$$F_{UU}^{\cos \phi}$$

$$F_{UU}^{\cos 2\phi}$$

GPDs??

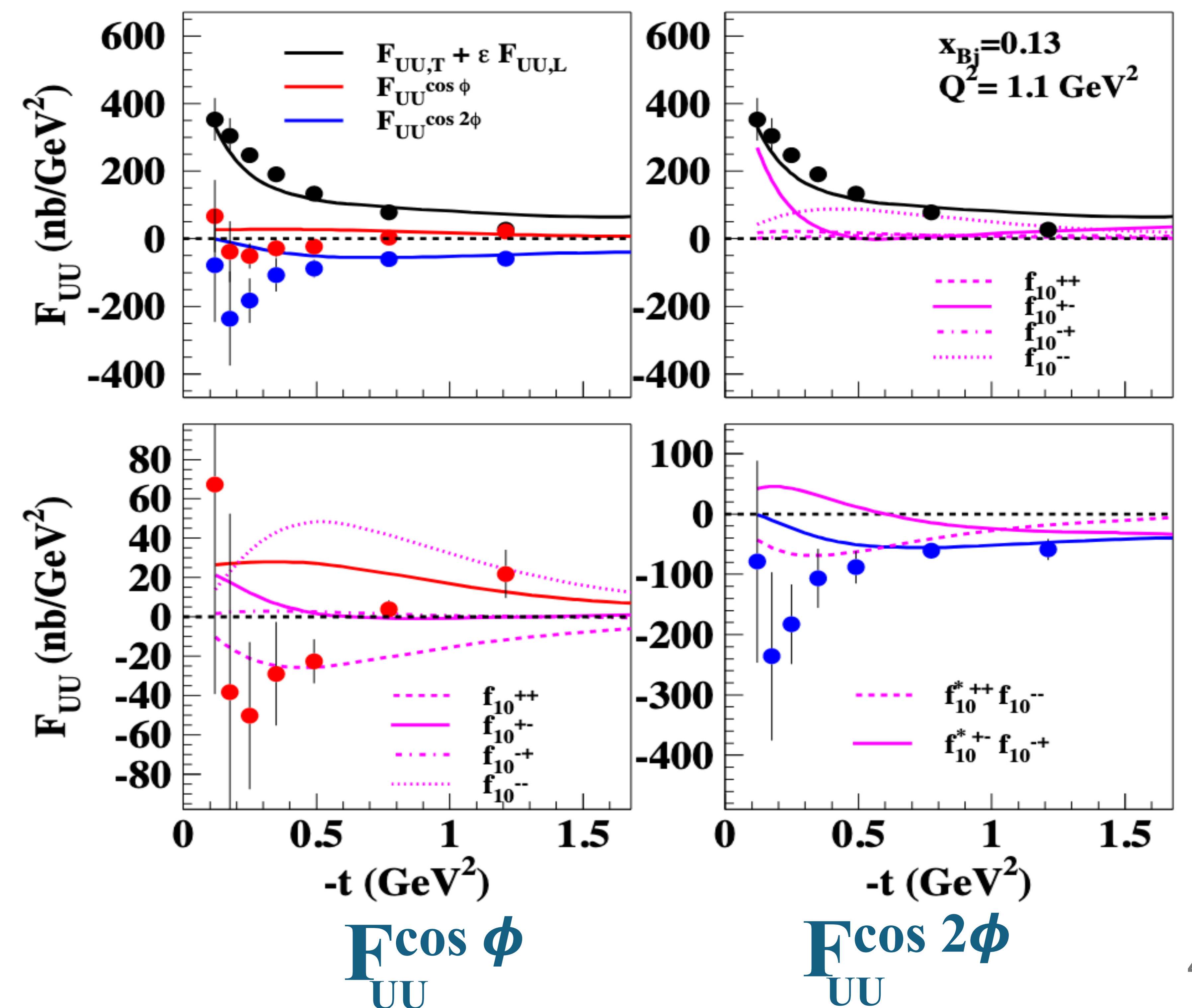
GGL, Phys. Rev. D 91, 114013 (2015)

GGL, Phys. Rev. D 79, 054014 (2009)

arxiv : 1401.0438

Second Inverse
problem

$$F_{UU,T} + \epsilon F_{UU,L}$$

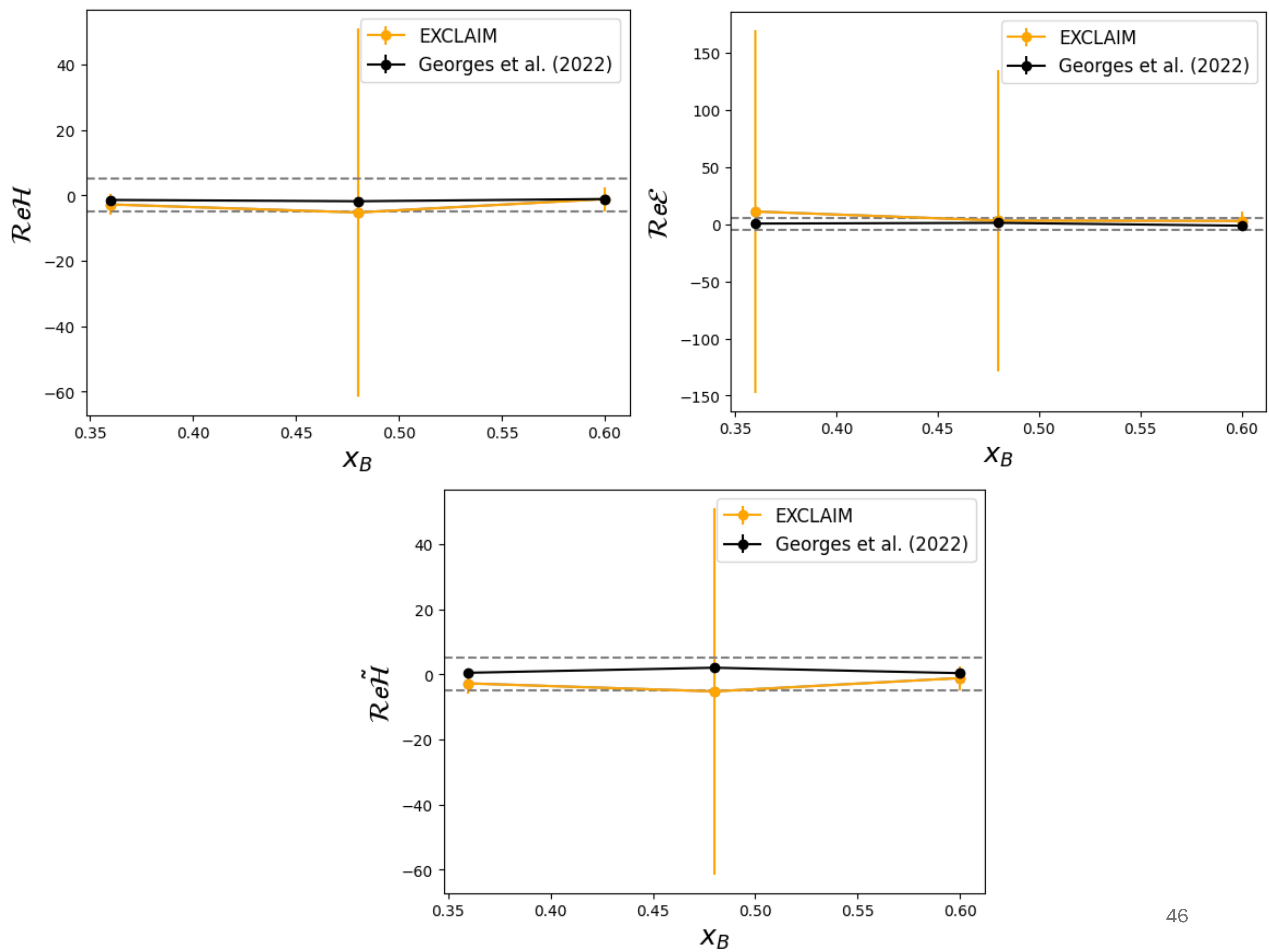


$$F_{UU}^{\cos \phi}$$

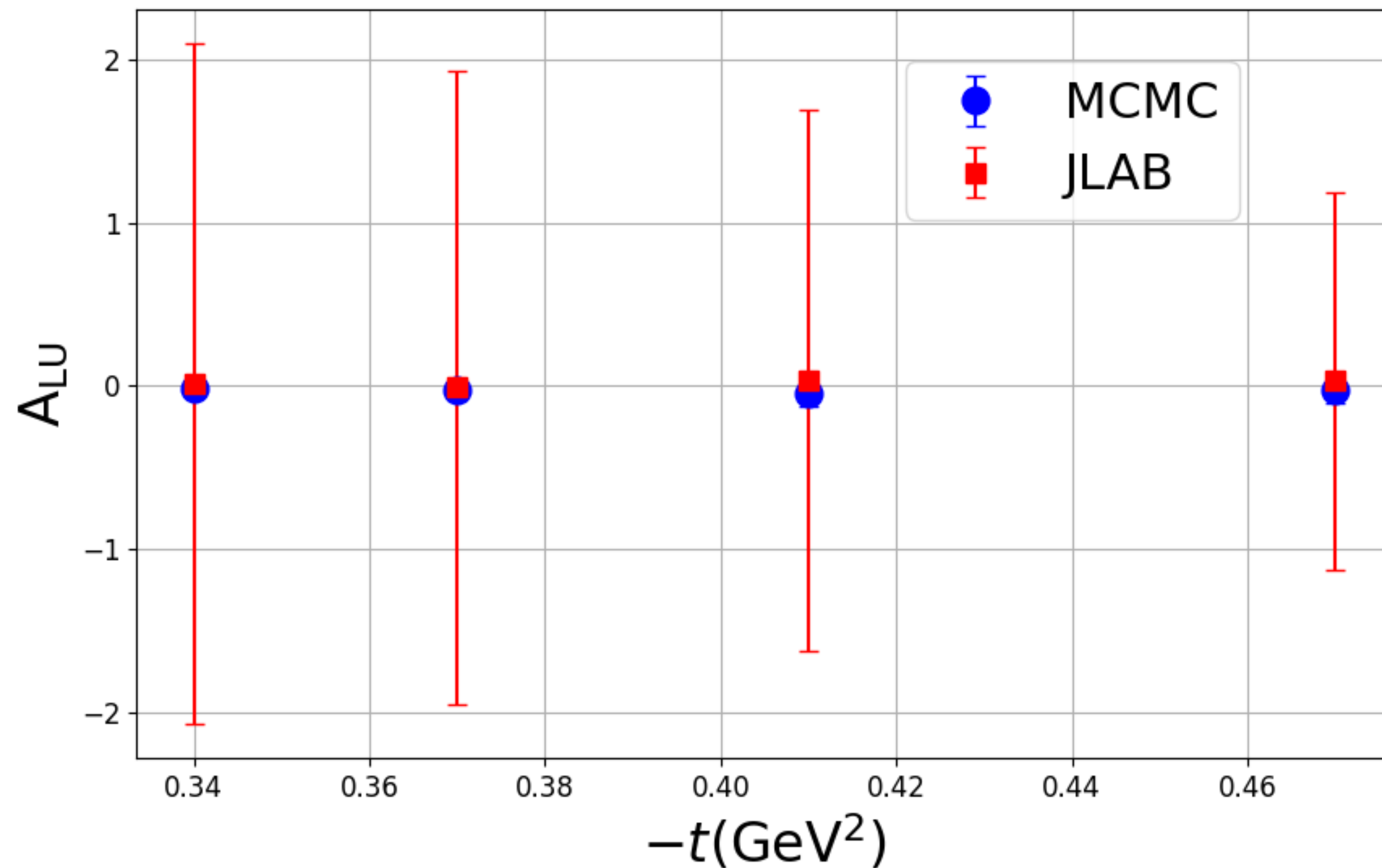
$$F_{UU}^{\cos 2\phi}$$

Thank you
Any Questions?

Backup slides



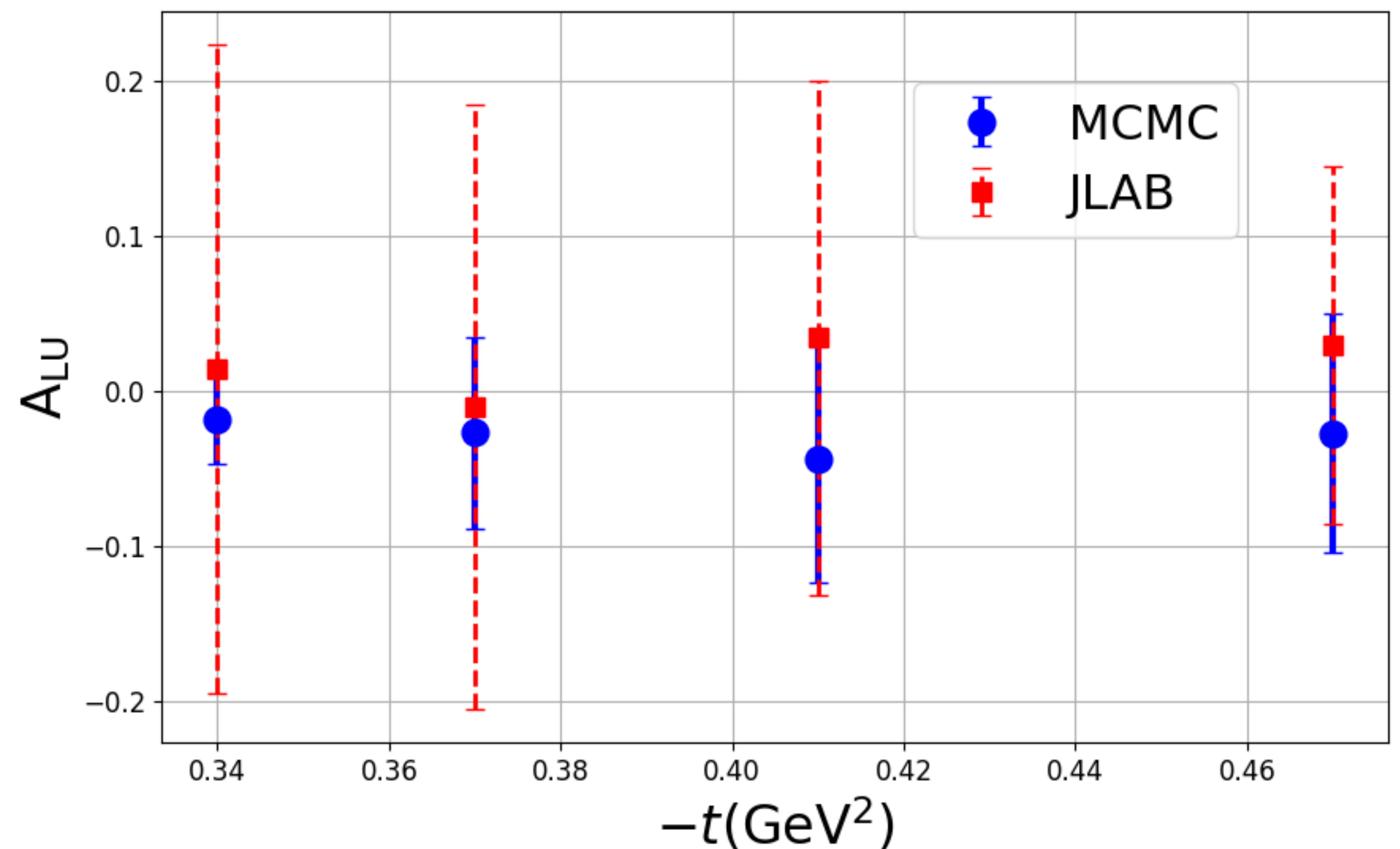
Beam spin asymmetry



A_{LU} from the mean of
the cross-sections
samples

data error scaled by 0.1

Phys. Rev. Lett. 127, 152301
Phys. Rev. C 83, 025201



$$\Delta\Sigma \approx Nq x^\alpha (1-x)^\beta (1 + \delta\sqrt{x} + \gamma x^\kappa)$$

$$\Delta G \approx Ng x^{\alpha'} (1-x)^{\beta'} (1 + \delta'\sqrt{x} + \gamma' x^{\kappa'})$$

Experiment	Datatype
E143, E154, E155	DIS
CLAS	DIS
COMPASS	DIS, SIDIS
EMC, SMC	DIS
SMC	SIDIS
HERMES	DIS, SIDIS

Next / Future Steps

- ❑ Incorporate more data and try to obtain more concrete results.

More Data to be used	Datatype
PHENIX	62.4 GeV (pp, π^0) 200 GeV (pp, π^0) 510 GeV (pp, π^0)
STAR	200 GeV (pp, jet)
LATTICE	HadStruc $\mathfrak{M}(p)$

- ❑ Look into much lower x region