Parton Distributions in the Shockwave Formalism

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Based on 2510.02254 and 2605.14092

“Gluon Saturation, Diffraction and Multihadron Production at the EIC: Theory, Simulations and Feasibility of Measurements”
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Matching Parton Distributions to small x

Lots of work has been done already

- Quark TMD : Marquet et al (2009), 0906.1454; Dong et al (2018), 1805.09479; Jalilian-Marian et al (2025), 2406.08277; Caucal et al (2025), 2503.16162 ...

- Gluon TMDs : Dominguez et al (2011), 1101.0715; Xiao et al (2017), 1703.06163 ...

- Quark GPD/PDF : Hatta et al (2017), 1703.02085; Kovchegov et al (2025), 2512.10086 ...

- Diffractive distributions : Hatta et al (2022), 2205.08060; Hauksson et al (2024), 2402.14748, Caucal et al (2026), 2606.04169 ...

- Checking consistency with TMD factorization at NLO : Taels et al (2022), 2204.11650; Caucal et al (2022), 2208.13872; Caucal et al (2023), 2304.03304; Caucal et al (2023), 2308.00022; Caucal et al (2024), 2405.19404 ...

- Subeikonal distributions, higher-twist distributions...

Matching Parton Distributions to small x

In this workshop:

- Diffractive distributions: See talk by Edmond Iancu.
- BFKL/Sudakov resummation: See talk by Feng Yuan.
- Subeikonal distributions/Spin effects : See talks by Gabriel Santiago and by Brandon Manley.
- Including higher-twist effects: See talk by Andrey Tarasov and Shaswat Tiwari.

Computing Parton Distributions in Shockwave Limit

General parton distribution definition:

$$F^{ij}\left(\mathbf{k}, \boldsymbol{\Delta}, x, \xi\right) = \left(P^{+}\right)^{n} \int \frac{dr^{-}\, d^{2}\mathbf{r}}{(2\pi)^{3}}\, e^{ixP^{+}r^{-} - i\mathbf{k}\cdot\mathbf{r}} \langle P' |\, T\, \Phi^{i}\left(-\frac{r}{2}\right) \mathscr{W}_{C}^{R}\left[-\frac{r}{2}, \frac{r}{2}\right] \Phi^{j}\left(\frac{r}{2}\right) | P \rangle$$

Here Φ^{i} can be a quark field or a gluon strength tensor.

$$\downarrow$$

Compute in the background of a strong gluon field in the strict eikonal limit:

$$A^{+}_{cl}(x) \approx \delta\left(x^{-}\right) \rho\left(\mathbf{x}\right) \sim \frac{1}{g_s}$$

Computing Parton Distributions in Shockwave Limit

We will only be concerned with the leading twist (twist 2) operators.

$\downarrow$

Assume we take the kinematic limit where small-x effects are important, but collinear factorization holds.

Computing Parton Distributions in Shockwave Limit

Example: Inclusive DIS

Valid for All twist

$$F_2 = Q^4 \frac{N_c}{2\pi^3} \sum_f e_f^2 \int d^2\mathbf{x} d^2\mathbf{y} \int_0^1 \frac{dz}{4\pi z\bar{z}} N(\mathbf{x},\mathbf{y}) \left[(z\bar{z})^2 \left(z^2+\bar{z}^2\right) K_1\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right) + 4z^3\bar{z}^3 K_0\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right)^2 \right].$$

Take $Q^2 \gg Q_s^2$

$$F_2 = \frac{8N_c}{3\pi} \sum_f e_f^2 \int d^2\mathbf{q}\ \tilde{N}(-\mathbf{q},\mathbf{q}) \left[\log\left(\frac{Q^2}{\mathbf{q}^2}\right) + \frac{1}{6} \right] + \mathcal{O}\left(\frac{Q_s^2}{Q^2}\right) \longrightarrow$$

Only sensitive to leading twist parton distributions

Sharply peaked at $|\mathbf{q}| \sim Q_s$

Should match with twits-2 parton distributions.

Computing Parton Distributions in Shockwave Limit

Example: Inclusive DIS

Valid for All twist

$$F_2 = Q^4 \frac{N_c}{2\pi^3} \sum_f e_f^2 \int d^2\mathbf{x} d^2\mathbf{y} \int_0^1 \frac{dz}{4\pi z\bar{z}} N(\mathbf{x},\mathbf{y}) \left[(z\bar{z})^2 \left(z^2+\bar{z}^2\right) K_1\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right) + 4z^3\bar{z}^3 K_0\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right)^2 \right].$$

Take $Q^2 \gg Q_s^2$

$$F_2 = \frac{8N_c}{3\pi} \sum_f e_f^2 \int d^2\mathbf{q}\ \tilde{N}(-\mathbf{q},\mathbf{q}) \left[\log\left(\frac{Q^2}{\mathbf{q}^2}\right) + \frac{1}{6} \right] + \mathcal{O}\left(\frac{Q_s^2}{Q^2}\right)$$

$\longrightarrow$ Only sensitive to leading Twist

Collinear-log dominates for $Q^2 \gg Q_s^2$
Track this collinear log to a quark PDF.

Should match with twits-2 parton distributions.

Computing Parton Distributions in Shockwave Limit

Conventions: Target has large P^+ momentum. $A^-_{cl}(x) = 0$ (Lightcone gauge) .

$A^i_{cl}(x) = 0$

(Solution of EOM in eikonal approximation)

Projectile has large q^- momentum and small $q^+ \ll P^+$, sees target as shockwave.

Gluon PDF: $$f_g(x) = \frac{1}{xP^+}\int \frac{dr^-}{2\pi} e^{ixP^+r^-} \langle p \,|\, G_a^{+i}(0)\, \mathscr{W}^{ab}(0,r^-)\, G_b^{+i}(r^-) \,|\, p \rangle\big|_{r^+=\mathbf{r}=0}$$

Eikonal approximation of correlator + semiclassical treatment of the Nucleus

$$\langle p(\mathbf{b}', b'^-) \,|\, \mathcal{O}\left(y\right) |\, p(\mathbf{b}, b^-)\rangle \approx \delta^{(2)}(\mathbf{b}' - \mathbf{b})\, \delta\left(b'^- - b^-\right) \langle \mathcal{O}\left(y - b\right)\rangle$$

$$A^\mu(x) = A^\mu_{cl}(x) + a^\mu(x)$$

Average over target fluctuations

$$f_g(x) = \frac{1}{x\pi}\int dx^- dy^- d^2\mathbf{x}\, e^{-ixP^+\left(x^- - y^-\right)} \langle G_a^{+i}(x)\, \mathscr{W}^{ab}\left[x, y\right] G_b^{+i}\rangle\big|_{x^+=y^+=0 \;\; \mathbf{x}=\mathbf{y}} .$$

Starting point for our calculations.

Gluon PDF in the Shockwave Formalism

$$\mathscr{W}^{ab}_{\mathbf{x}}\left[x^-, y^-\right] = \mathscr{P} \exp\left[-ig_s \int_{y^-}^{x^-} dz^- \, A^+_{cl}(z^-, \mathbf{x})\right]$$

$$f_g(x) = \frac{1}{x\pi} \int_{-L^-/2}^{L^-/2} dx^- \, dy^- \int d^2\mathbf{x} \, e^{-ixP^+(x^- - y^-)} \langle \partial^i \left(A^+_{cl,a}(x^-, \mathbf{x})\right) \mathscr{W}^{ab}_{\mathbf{x}}\left[x^-, y^-\right] \partial^i \left(A^+_{cl,b}(y^-, \mathbf{x})\right) \rangle \, .$$

Since $xL^-P^+ \ll 1$, then in the integrand $e^{-ixP^+(x^- - y^-)} \approx 1$.

And using the identity:

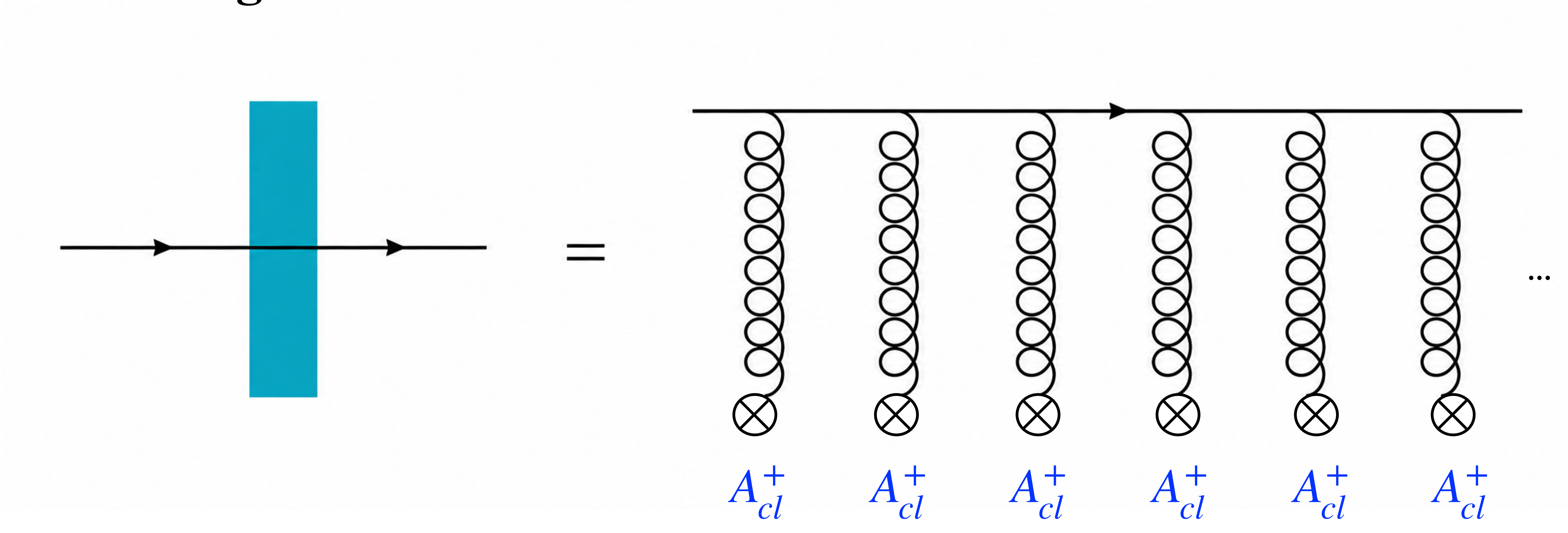
$$\partial^i \mathscr{W}_{\mathbf{x}}[u^-, v^-] = -\, ig_s \int_{v^-}^{u^-} dz^- \, \mathscr{W}_{\mathbf{x}}[u^-, z^-] \, \partial^i A^+_{cl}(z^-, \mathbf{x}) \mathscr{W}_{\mathbf{x}}[z^-, v^-] \, .$$

We arrive at:

$$f_g(x) = \frac{2}{x\pi g_s^2} \int d^2\mathbf{x} \, \mathrm{Tr}\left[\partial^i \left(\mathscr{W}_{\mathbf{x}}\left[L^-/2, -L^-/2\right]\right) \partial^i \left(\mathscr{W}_{\mathbf{x}}\left[-L^-/2, L^-/2\right]\right)\right] = \frac{2}{x\pi g_s^2} \int d^2\mathbf{x} \, \mathrm{Tr}\left[\partial^i \left(V(\mathbf{x})\right) \partial^i \left(V^\dagger(\mathbf{x})\right)\right]$$

Feynman Rules

Building Block: Resummed coherent interaction with the Shockwave

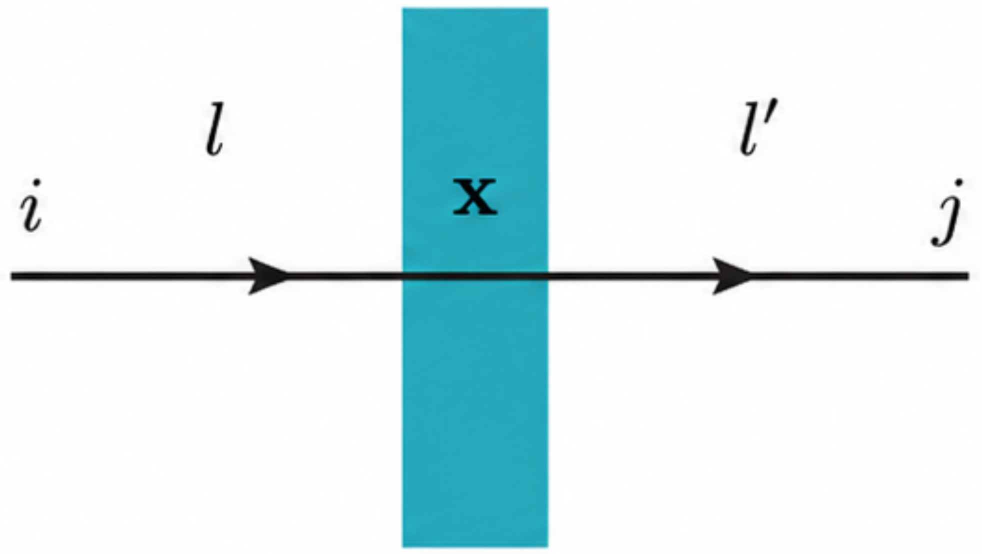


Interaction exponentiates into Wilson lines in the eikonal limit

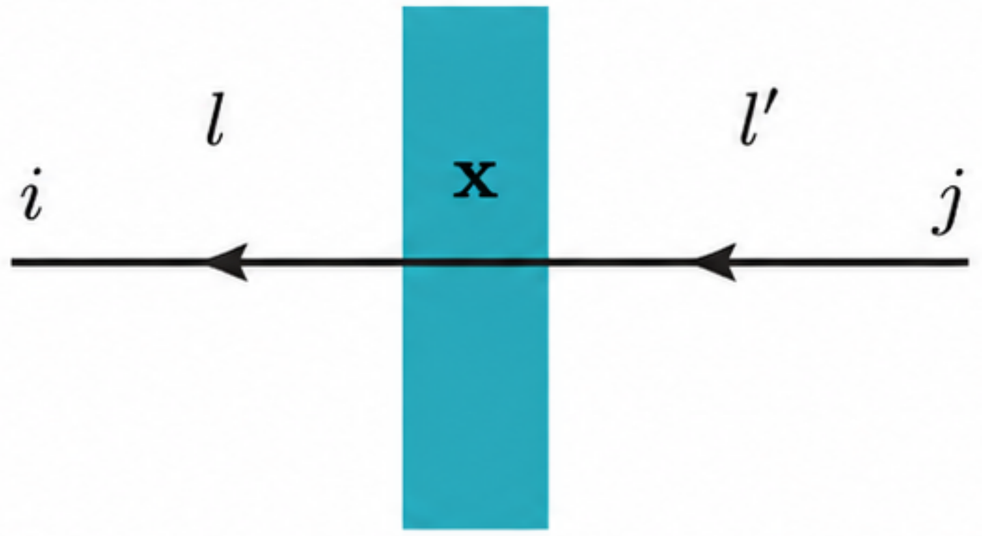
$$V(\mathbf{x}) = \mathscr{P} \exp\left(-ig\int_{-\infty}^{\infty} dy^- \, A^{a,+}_{\rm cl}(y^-, \mathbf{x})t^a\right)$$

Feynman Rules

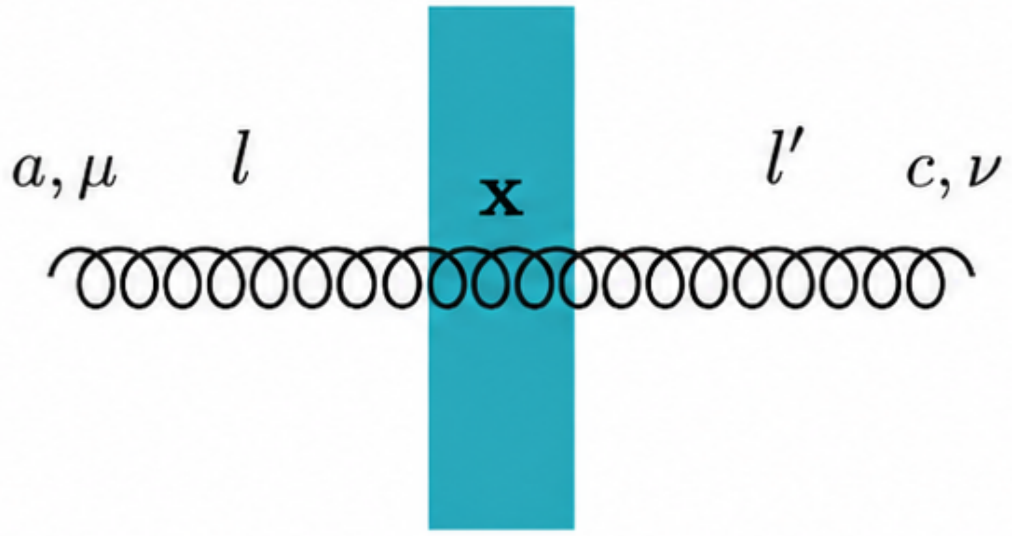
Shockwave interaction with quarks/gluons



$$= \gamma^{-} 2\pi\delta(l'^{-} - l^{-}) \int \mathrm{d}^2\mathbf{x}\, e^{-i\mathbf{x}\cdot(l'-l)}\, V(\mathbf{x})_{ji}$$

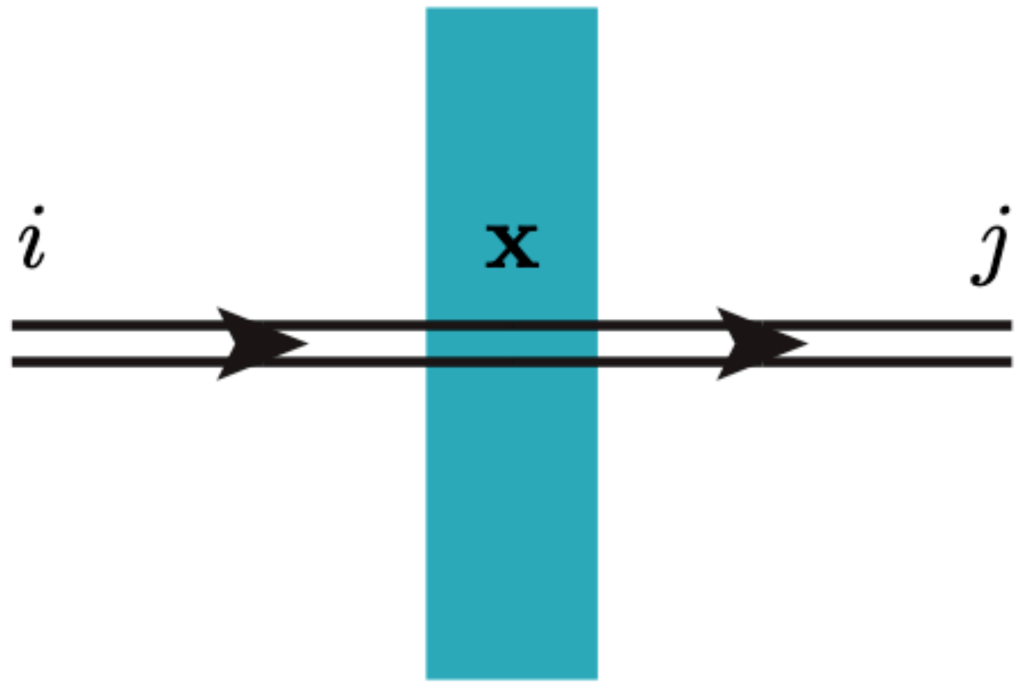


$$= -\gamma^{-} 2\pi\delta(l'^{-} - l^{-}) \int \mathrm{d}^2\mathbf{x}\, e^{-i\mathbf{x}\cdot(l'-l)}\, V^{\dagger}(\mathbf{x})_{ij}$$



$$= -2l^{-} g^{\mu\nu} 2\pi\delta(l'^{-} - l^{-}) \int \mathrm{d}^2\mathbf{x}\, e^{-i\mathbf{x}\cdot(l'-l)}\, U(\mathbf{x})_{ca}$$

Shockwave interaction with a gauge link

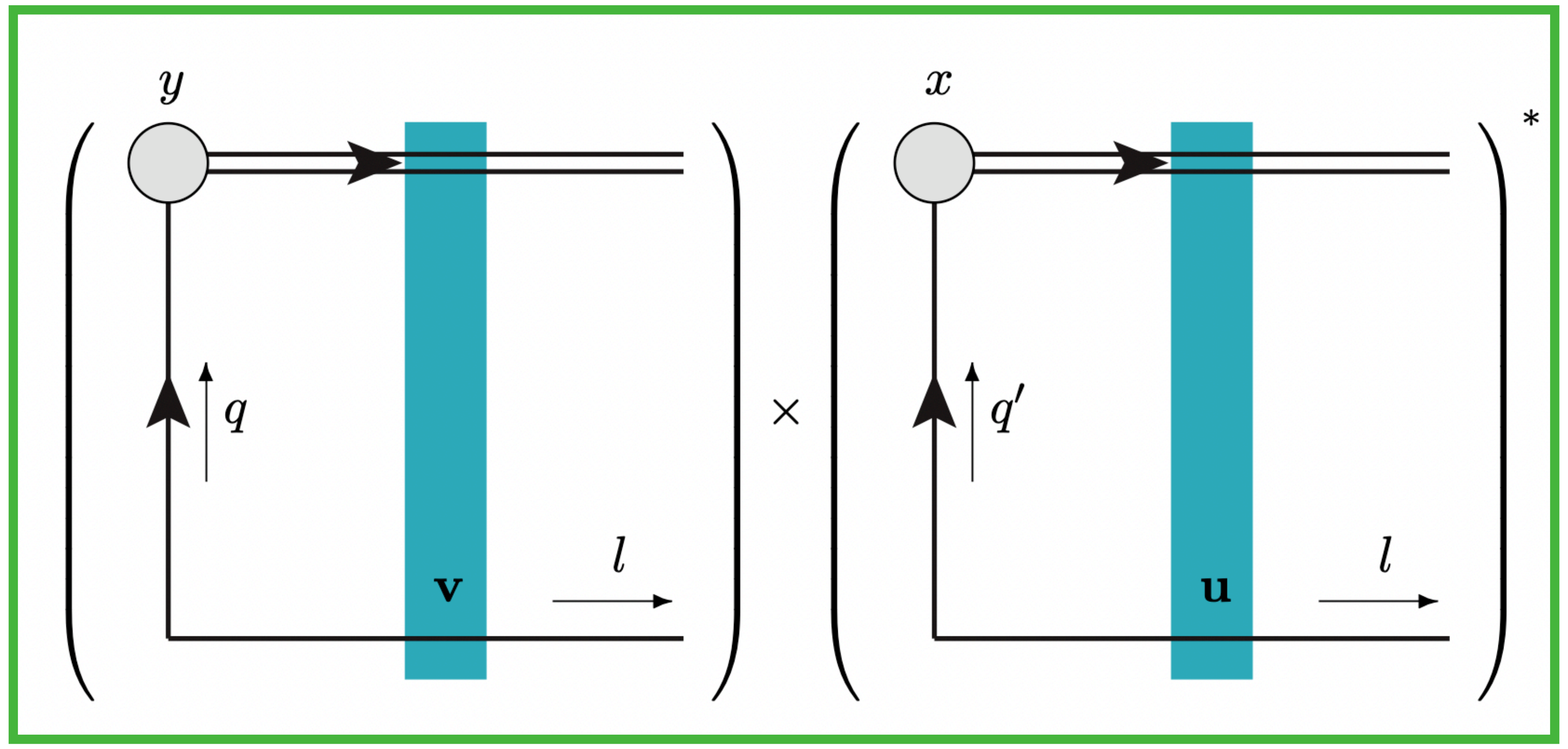


$$= V(\mathbf{x})_{ji}$$

Quark PDF in the Shockwave Formalism

$$f_q^{\text{bare}}(x) = \int \frac{dr^-}{2\pi} e^{ixP^+r^-} \langle p \,|\, \bar{\psi}(0)\mathscr{W}(0,r^-)\frac{\gamma^+}{2}\psi(r^-)\,|\, p\rangle\Big|_{r^+=\mathbf{r}=0}$$

$$= \frac{P^+}{2\pi}\int dx^-\,dy^-\,d^{D-2}\mathbf{x}\; e^{-ixP^+(x^- - y^-)} \boxed{\langle \bar{\psi}(x^-,\mathbf{x})\mathscr{W}_{\mathbf{x}}\left(x^-,y^-\right)\frac{\gamma^+}{2}\psi(y^-,\mathbf{x})\rangle}$$



Compute correlation function using shockwave Feynman rules

Quark PDF in the Shockwave Formalism

$$f_q^{\text{bare}}(x) = \frac{2^D N_c}{x\,(2\pi)^D}\frac{\Gamma\,(D/2)^2}{D-1}\int d^{D-2}\mathbf{x}\, d^{D-2}\mathbf{u}\,\frac{N\,(\mathbf{x},\mathbf{u})}{|\,\mathbf{x}-\mathbf{u}\,|^{2(D-2)}}\,.$$

Where $N\,(\mathbf{x},\mathbf{u}) = 1 - \frac{1}{N_c}\mathrm{Tr}\langle V\,(\mathbf{x})\,V^\dagger\,(\mathbf{u})\rangle$.

Introduce the weighted Fourier Transform: $$\tilde{N}(\mathbf{p},\mathbf{q}) = \int \frac{d^{D-2}\,\mathbf{x}\,d^{D-2}\mathbf{y}}{(2\pi)^{2(D-2)}}e^{i\mathbf{p}\cdot\mathbf{x}+i\mathbf{q}\cdot\mathbf{y}}\frac{N(\mathbf{x},\mathbf{y})}{|\,\mathbf{x}-\mathbf{y}\,|^2}$$

$$f_q^{\text{bare}}(x) = \frac{4N_c}{3\pi x}\int d^2\mathbf{q}\;\tilde{N}(-\mathbf{q},\mathbf{q})\left[\frac{1}{\epsilon}-\log\left(\pi\mathbf{q}^2\right)-\gamma_E-\frac{4}{3}\right],$$

Why is the quark PDF divergent at this order?

Gluon PDF:

$$f_g\,(x) = \frac{4N_c\,(D-2)}{\alpha_s}\frac{1}{x}\left(\frac{e^{-\gamma_E}}{\pi\mu^2}\right)^{\epsilon}\int d^{D-2}\mathbf{q}\,\tilde{N}\left(-\mathbf{q},\mathbf{q}\right)$$

$f_q^{\text{bare}}(x) = \mathcal{O}\left(\alpha_s^0\right)$ and $f_g\,(x) = \mathcal{O}\left(\alpha_s^{-1}\right)$

Renormalization of the Quark PDF

Standard quark pdf conunterterm:

$$f_q^{\text{Renorm}}(x,\mu_R) = f_q^{\text{bare}}(x) - \frac{1}{\epsilon}\left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon} \frac{\alpha_s}{2\pi}\int_x^{1} \frac{dy}{y}\left[P_{qg}\left(\frac{x}{y}\right) f_g(y,\mu_R) + P_{qq}\left(\frac{x}{y}\right) f_q\left(y,\mu_R\right)\right].$$

In shockwave limit:

$$f_q^{\text{Renorm}}(x,\mu_R) = f_q^{\text{bare}}(x) - \frac{1}{\epsilon}\left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon} \frac{\alpha_s}{2\pi}\int_x^{\infty} \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) f_g(y,\mu_R).$$

My interpretation: $\longrightarrow$ **Recoilless** background with narrow support in x^-, p^+ momentum conservation not enforced.

Renormalization of the Quark PDF

$$f_q^{\text{Renorm}}(x,\mu_R) = f_q^{\text{bare}}(x) - \frac{1}{\epsilon}\left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon}\frac{\alpha_s}{2\pi}\int_x^{\infty}\frac{dy}{y}P_{qg}\left(\frac{x}{y}\right)f_g(y,\mu_R)\,. \qquad P_{qg}(z) = T_F\left[z^2+(1-z)^2\right].$$

$$f_g(x) = \frac{8N_c}{\alpha_s}\frac{1}{x}\left(\frac{e^{-\gamma_E}}{\pi\mu^2}\right)^{\epsilon}\int d^{D-2}\mathbf{q}\,\tilde{N}(-\mathbf{q},\mathbf{q})\,.$$

At this order $f_g(x) \propto \frac{1}{x}$, no BK yet!

$$\boxed{f_q^{\text{Renorm}}(x,\mu_R) = \frac{4N_c}{3\pi}\frac{1}{x}\int d^2\mathbf{q}\,\tilde{N}(-\mathbf{q},\mathbf{q})\left[\log\frac{\mu_R^2}{\mathbf{q}^2}-\frac{1}{3}\right]}$$

Matching: Inclusive DIS

We look at the F2 structure function computed within both factorization formalisms.

$$F_\lambda(x_B, Q^2) \equiv \frac{Q^2}{4\pi^2\alpha_{em}}\sigma(\gamma^*_\lambda + p)$$

$$F_2(x_B, Q^2) \equiv F_L(x_B, Q^2) + F_T(x_B, Q^2),$$

In Collinear factorization:

$$F_2 = \sum_f e_f^2 x \left[f_q(x,\mu_F) + f_{\bar{q}}(x,\mu_F)\right] +$$
$$+ \, x\frac{\alpha_s}{2\pi}\sum_f e_f^2 \int_x^{\infty} \frac{dy}{y} 2\, f_g(y,\mu_F) \times \left[P_{qg}\left(\frac{x}{y}\right)\log\frac{Q^2}{\mu_F^2} + C_g\left(\frac{x}{y}\right)\right].$$

$$C_g(z) = \frac{1}{2}\left[8z(1-z) - 1 - \left(z^2 + (1-z)^2\right)\log\frac{z}{1-z}\right].$$

Bardeen et al Phys.Rev.D 18 (1978) 3998.
Furmanski et al Z.Phys.C 11 (1982) 293.
Ellis et al “QCD and collider physics” (1996).

In CGC:

$$F_2 = Q^4\frac{N_c}{2\pi^3}\sum_f e_f^2 \int d^2\mathbf{x} d^2\mathbf{y} \int_0^1 \frac{dz}{4\pi z\bar{z}} N(\mathbf{x},\mathbf{y}) \times$$
$$\times \left[(z\bar{z})^2\left(z^2 + \bar{z}^2\right)K_1\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right) + 4z^3\bar{z}^3 K_0\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right)^2\right].$$

Golec-Biernat et al, 9807513, 1998.

Matching: Inclusive DIS

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$$+\, x\frac{\alpha_s}{2\pi}\sum_f e_f^2 \int_x^{\infty} \frac{dy}{y} 2\, f_g(y,\mu_F) \times \left[P_{qg}\left(\frac{x}{y}\right)\log\frac{Q^2}{\mu_F^2} + C_g\left(\frac{x}{y}\right)\right].$$

In CGC:

$$F_2 = Q^4\frac{N_c}{2\pi^3}\sum_f e_f^2 \int d^2\mathbf{x} d^2\mathbf{y} \int_0^1 \frac{dz}{4\pi z\bar{z}} N(\mathbf{x},\mathbf{y}) \times$$
$$\times \left[(z\bar{z})^2\left(z^2+\bar{z}^2\right) K_1\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right) + 4z^3\bar{z}^3 K_0\left(\sqrt{z\bar{z}}Q\left|\mathbf{x}-\mathbf{y}\right|\right)^2\right].$$

Take small-x limit
by using PDF calculations
and integrate

$$F_2 = \frac{8N_c}{3\pi}\sum_f e_f^2 \int d^2\mathbf{q}\; \tilde{N}(-\mathbf{q},\mathbf{q}) \left[\log\left(\frac{Q^2}{\mathbf{q}^2}\right) + \frac{1}{6}\right]$$

Matching: Inclusive DIS

We look at the F2 structure function computed within both factorization formalisms.

$$F_\lambda(x_B, Q^2) \equiv \frac{Q^2}{4\pi^2\alpha_{em}}\sigma(\gamma^*_\lambda + p)$$

$$F_2(x_B, Q^2) \equiv F_L(x_B, Q^2) + F_T(x_B, Q^2),$$

In Collinear factorization:

$$F_2 = \sum_f e_f^2 x \left[f_q(x, \mu_F) + f_{\bar{q}}(x, \mu_F)\right] +$$
$$+ x\frac{\alpha_s}{2\pi}\sum_f e_f^2 \int_x^\infty \frac{dy}{y} 2\, f_g(y, \mu_F) \times \left[P_{qg}\left(\frac{x}{y}\right)\log\frac{Q^2}{\mu_F^2} + C_g\left(\frac{x}{y}\right)\right].$$

In CGC:

$$F_2 = Q^4\frac{N_c}{2\pi^3}\sum_f e_f^2 \int d^2\mathbf{x} d^2\mathbf{y} \int_0^1 \frac{dz}{4\pi z\bar{z}} N(\mathbf{x}, \mathbf{y}) \times$$
$$\times \left[(z\bar{z})^2\left(z^2 + \bar{z}^2\right) K_1\left(\sqrt{z\bar{z}}Q\,|\mathbf{x} - \mathbf{y}|\right) + 4z^3\bar{z}^3 K_0\left(\sqrt{z\bar{z}}Q\,|\mathbf{x} - \mathbf{y}|\right)^2\right].$$

Take small-x limit by using PDF calculations and integrate

$$F_2 = \frac{8N_c}{3\pi}\sum_f e_f^2 \int d^2\mathbf{q}\ \tilde{N}(-\mathbf{q}, \mathbf{q})\left[\log\left(\frac{Q^2}{\mathbf{q}^2}\right) + \frac{1}{6}\right]$$

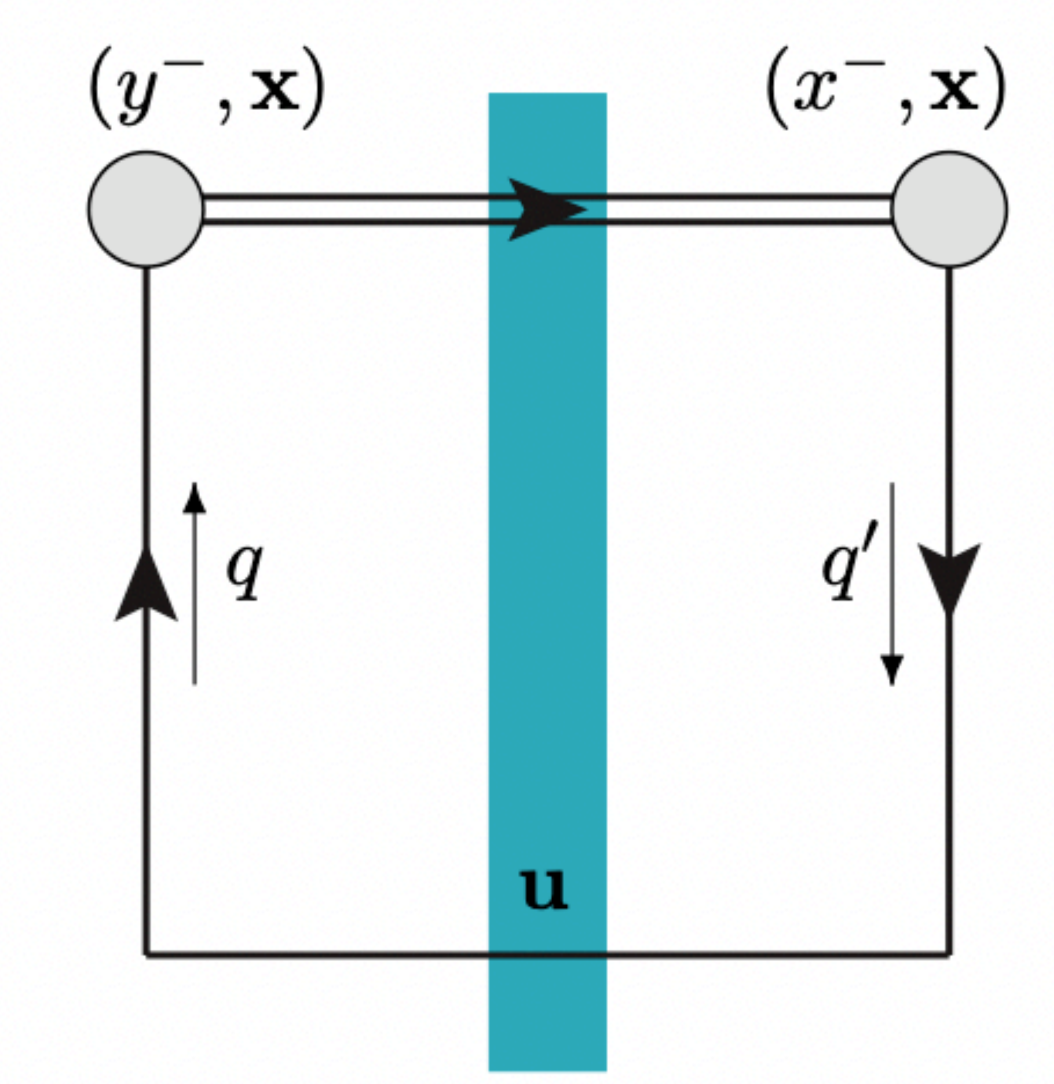
Take $Q^2 \gg Q_s^2$ and integrate

Quark GPD in the Shockwave Formalism

$$f_q\left(x,\xi,\boldsymbol{\Delta}\right) = \frac{P^+}{2\pi}\sqrt{1-\xi^2}\int dx^-\, dy^-\, d^2\mathbf{x}\, e^{i\boldsymbol{\Delta}\cdot\mathbf{x}+i\xi P^+\left(x^-+y^-\right)-ixP^+\left(x^--y^-\right)}$$

$$\times \langle \mathrm{T}\bar{\psi}(x^-,\mathbf{x})\mathscr{W}_{\mathbf{x}}\left(x^-,y^-\right)\frac{\gamma^+}{2}\psi(y^-,\mathbf{x})\rangle|_{\mathbf{x}=\mathbf{y}}$$

$$\xi = \frac{p^+-p'^+}{p^++p'^+}, \qquad \boldsymbol{\Delta} = \mathbf{p}'-\mathbf{p}\,.$$



$$f_q^{\mathrm{bare}}\left(x,\xi,\boldsymbol{\Delta}\right) = \frac{N_c}{2\pi}\frac{1}{\epsilon}\times\frac{2\sqrt{1-\xi^2}}{\xi^3}\left[2x\xi+\left(\xi^2-x^2\right)\log\left|\frac{x+\xi}{x-\xi}\right|\right]\int d^2\mathbf{q}\,\tilde{N}\left(\boldsymbol{\Delta}+\mathbf{q},-\mathbf{q}\right)+\text{finite}$$

Why is the quark GPD divergent at this order?

$$f_g\left(x,\xi,\boldsymbol{\Delta},\mu\right) = \frac{4N_c(D-2)}{\alpha_s}\left(\frac{e^{-\gamma_E}}{\pi\mu^2}\right)^{\epsilon}\sqrt{1-\xi^2}\int d^{D-2}\mathbf{q}\,\tilde{N}\left(\mathbf{q}+\frac{1}{2}\boldsymbol{\Delta},-\mathbf{q}+\frac{1}{2}\boldsymbol{\Delta}\right)$$

Renormalization of the Quark GPD

$$f_q^{\text{Renorm}}\left(x,\xi,\boldsymbol{\Delta},\mu_R\right)=f_q^{\text{bare}}\left(x,\xi,\boldsymbol{\Delta}\right)-\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu}{\mu_R}\right)^{\epsilon}\int_{-\infty}^{\infty}\frac{dy}{y^2}K^{qg}\left(x,y;\xi\right)\times f_g\left(y,\xi,\boldsymbol{\Delta}\right).$$

$$K^{qg}\left(x,y;\xi\right)=P_{qg}\left(\frac{x}{y},\frac{\xi}{y}\right)\left[\theta\left(\xi<x<y\right)-\theta\left(y<x<-\xi\right)\right]$$
$$+\frac{1}{2}\theta\left(-\xi<x<\xi\right)\left[P'_{qg}\left(\frac{x}{y},\frac{\xi}{y}\right)\theta\left(x<y\right)-P'_{qg}\left(\frac{x}{y},-\frac{\xi}{y}\right)\theta\left(y<x\right)\right]$$

$$P_{qg}\left(\hat{x},\hat{\xi}\right)=T_F\frac{\hat{x}^2+\left(1-\hat{x}\right)^2-\hat{\xi}^2}{\left(1-\hat{\xi}^2\right)^2}\qquad P'_{qg}\left(\hat{x},\hat{\xi}\right)=T_F\frac{\left(\hat{x}+\hat{\xi}\right)\left(1-2\hat{x}+\hat{\xi}\right)}{\hat{\xi}\left(1+\hat{\xi}\right)\left(1-\hat{\xi}^2\right)}$$

Renormalization of the Quark GPD

$$f_q^{\text{Renorm}}\left(x,\xi,\boldsymbol{\Delta},\mu_R\right)=\frac{\sqrt{1-\xi^2}}{\xi}\frac{N_c}{2\pi}\int d^2\mathbf{q}\left[\tilde{N}\left(\boldsymbol{\Delta}+\mathbf{q},-\mathbf{q}\right)\hat{F}_q\left(\hat{\xi}_-,\frac{\mu_R^2}{\mathbf{q}^2}\right)+\tilde{N}\left(\mathbf{q},\boldsymbol{\Delta}-\mathbf{q}\right)\hat{F}_q\left(\hat{\xi}_+,\frac{\mu_R^2}{\mathbf{q}^2}\right)\right]$$

$$\hat{F}_q\left(\hat{\xi},\frac{\mu_R^2}{\mathbf{q}^2}\right)=\frac{1}{\hat{\xi}^2}\left\{\left[\log\frac{\mu_R^2}{\mathbf{q}^2}-\frac{1}{3}\right]\left[2\hat{\xi}-\left(1-\hat{\xi}^2\right)\log\frac{1+\hat{\xi}}{1-\hat{\xi}}+h\left(\hat{\xi}\right)\right]\right\}. \qquad \hat{\xi}_\pm=\frac{\xi}{x\pm i\epsilon}.$$

$$h\left(\hat{\xi}\right)=\frac{2}{3}\hat{\xi}-\left(1-\hat{\xi}^2\right)\left[2\left(\mathrm{Li}_2\left(\hat{\xi}\right)-\mathrm{Li}_2\left(-\hat{\xi}\right)\right)-\frac{5}{3}\log\frac{1+\hat{\xi}}{1-\hat{\xi}}\right.$$

$$\left.+\mathrm{Li}_2\left(\frac{1-\hat{\xi}}{2}\right)-\mathrm{Li}_2\left(\frac{1+\hat{\xi}}{2}\right)+\frac{1}{2}\log^2\frac{1-\hat{\xi}}{2}-\frac{1}{2}\log^2\frac{1+\hat{\xi}}{2}\right]$$

Matching: DVCS

We now look at the scattering amplitude $\mathscr{M}^{\gamma^*_{\pm1}+p\to\gamma_{\pm1}+p'}$ computed with GPDs first:

$$-i\mathscr{M}^{\gamma^*_{\pm1}+p\to\gamma_{\pm1}+p'} = -ie^2\int_{-\infty}^{\infty}\frac{dx}{\xi}\left[\sum_q f_q^{\text{Renorm}}\left(x,\xi,\boldsymbol{\Delta},\mu_R\right)C_0^{S,q}\left(z,\mu_F\right)+\frac{1}{\xi}f_g\left(x,\xi,\boldsymbol{\Delta},\mu\right)C_1^{S,g}\left(z,\mu_F\right)\right]$$

Ji et al, 9707254, 1998.
Mankiewicz et al, 9712251, 1998.
Schönleber, PhD thesis, 2023.

$$C_1^{S,g}\left(z,\mu_F\right) = \sum_f e_f^2\frac{\alpha_s}{4\pi}\frac{1}{4z\bar{z}}\left[\log\left(\frac{\mu_F^2}{Q^2}\right)\frac{z}{\bar{z}}\log z-\frac{z}{2\bar{z}}\log^2 z+\frac{1+z}{\bar{z}}+(z\leftrightarrow\bar{z})\right].$$

Take small-x limit
by using GPD calculations
and integrate

$$C_0^{S,q}\left(z,\mu_F\right) = \frac{e_q^2}{2}\left[\frac{1}{z}-\frac{1}{\bar{z}}\right]. \qquad z=\frac{1}{2}\left(1-\frac{x}{\xi-i\epsilon}\right), \qquad \bar{z}=1-z.$$

$$-i\mathscr{M}^{\gamma^*_{\pm1}+p\to\gamma_{\pm1}+p'} = 2N_c\sum_f\left(ee_f\right)^2\frac{\sqrt{1-\xi^2}}{\xi}\int d^2\mathbf{q}\left[\log\frac{Q^2}{\mathbf{q}^2}-2\right]\left[\tilde{N}\left(\boldsymbol{\Delta}+\mathbf{q},-\mathbf{q}\right)+\tilde{N}\left(\mathbf{q},\boldsymbol{\Delta}-\mathbf{q}\right)\right]$$

Matching: DVCS

Now we look at the scattering amplitude $\mathscr{M}^{\gamma^*_{\pm 1}+p\to\gamma_{\pm 1}+p'}$ computed with the CGC:

$$-i\mathscr{M}^{\gamma^*_{\pm 1}+p\to\gamma_{\pm 1}+p'} = Q^2\frac{\sqrt{1-\xi^2}}{\xi}\frac{N_c}{2}\sum_f\left(\frac{ee_f}{\pi}\right)^2\int d^2\mathbf{x}\,d^2\mathbf{y}\int_0^1\frac{dz}{4\pi z\bar{z}}\left[e^{i\boldsymbol{\Delta}\cdot\mathbf{b}}N\left(\mathbf{x},\mathbf{y}\right)(z\bar{z})^{3/2}\left(z^2+\bar{z}^2\right)\frac{Q}{|\mathbf{x}-\mathbf{y}|}\mathrm{K}_1\left(\sqrt{z\bar{z}}Q|\mathbf{x}-\mathbf{y}|\right)\right]$$

Bartels et al ,0301192, 2003.
Kowalski et al, 0606272, 2006.
Hatta et al, 1703.02085, 2017.

Take $Q^2 \gg Q_s^2$ and integrate

$$-i\mathscr{M}^{\gamma^*_{\pm 1}+p\to\gamma_{\pm 1}+p'} = 2N_c\sum_f\left(ee_f\right)^2\frac{\sqrt{1-\xi^2}}{\xi}\int d^2\mathbf{q}\left[\log\frac{Q^2}{\mathbf{q}^2}-2\right]\left[\tilde{N}\left(\boldsymbol{\Delta}+\mathbf{q},-\mathbf{q}\right)+\tilde{N}\left(\mathbf{q},\boldsymbol{\Delta}-\mathbf{q}\right)\right]$$

Agreement again!

Summary/Future directions

- Performed a comprehensive calculation of (twist-2) parton distributions in the shockwave limit (at LO and at the eikonal level). (See also Benić et al, 2603.06092).

- Results of calculation allow a consistency check of high energy and collinear factorization in the region of overlap.

- Even at lowest order, collinear evolution equations for quark PDF and GPD were found.

- Going to higher orders: We expect to encounter both collinear/CSS evolution and small-x evolution. Interplay between high energy and collinear/Sudakov logarithms? (See for instance Caucal et al, 3065768 and Mukherjee et al, 2311.16402).

- In principle could also extend to subeikonal and higher twist distributions.

- Work in progress: Gluon TMD in the Shockwave limit at NLO.

Thanks for your attention!

Any questions?

Background image: CERN PhotoLab