Recent results on entropy production in DIS

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Presented results are obtained in collaboration with:

P. Caputa; M. Hentschinski; D. Kharzeev; H. Jung; M. Praszałowicz; W. Płaczek;
M. Rohrmoser, R. Straka; K. Tu

Entropy, entanglement and my motivation

Properties of entanglement entropy may provide new insight into understanding the behavior of parton density functions.

Links to other areas: thermodynamics, gravity, quantum information, and conformal field theory.

Interesting in the context of parton's growth and parton's saturation.

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy or entropy corresponding to parton density.

K. Kutak '11, Peschanski '12

I. Zahed, A. Stoffers '13

A. Kovner, M. Lublinsky '15,

Kharzeev, Levin '17,

Berges, Florchinger, Venugopalan '17

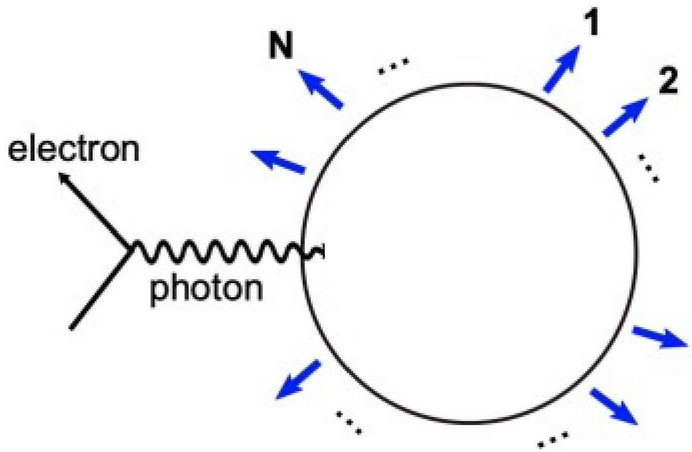
Rough analogy

black hole + coffee → Hawking radiation + back hole

electron + proton → electron +radiation of hadrons

J. A. Wheller thought experiment.

Dall-E



Kharzeev, Levin ’17
Kharzeev’21

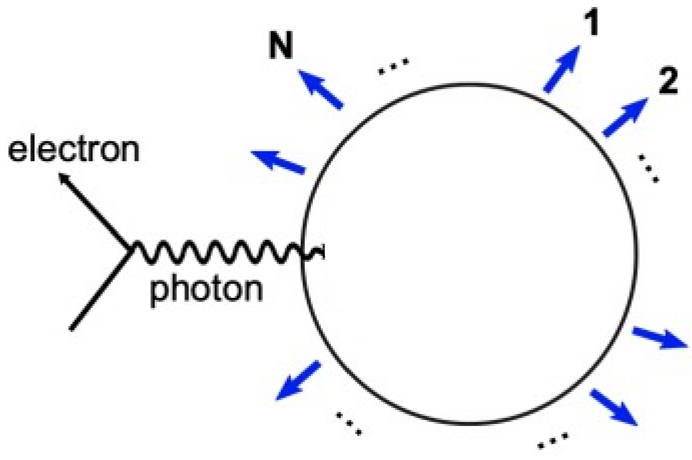
Bekenstein: BH has to have entropy because coffee has entropy and overall entropy would decrease if we toss coffee into black hole without increasing entropy

Comment:
better analogy would be proton – ion collision

Entanglement and entropy intro

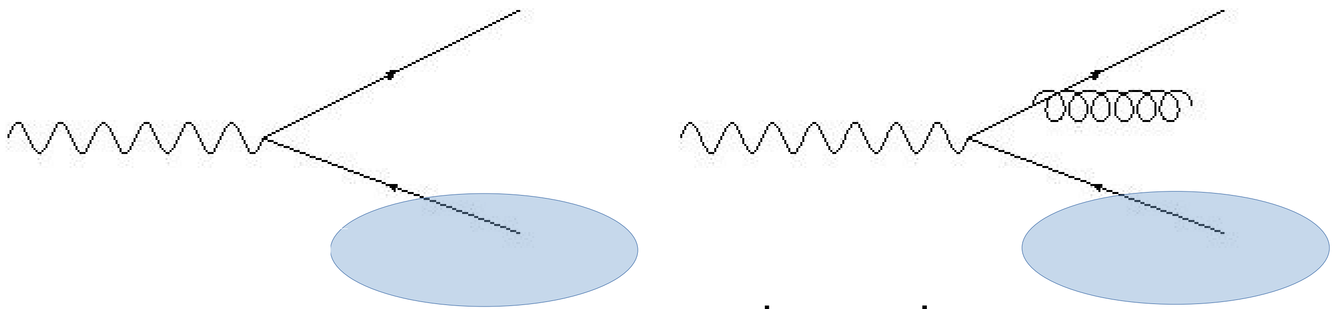
electron + proton → electron +radiation of hadrons

- collision of color singlets
- final state is color singlet
- color can not be isolated
- quark causally disconnected over period 1/Q from rest of proton. Hadronisation over period $1/\Lambda_{QCD}$ not enough time to exchange information information about color neutralization
- quark and remaining quarks and gluons form a color singlet. Strongly correlated by causaly disconnected
- DIS at large photon virtuality Q^2: a partial observation of the entangled quantum state
- on observable level i.e. hadrons you do not have complete information. Color i.e. phases are traced out, observation happened at certain resolution.



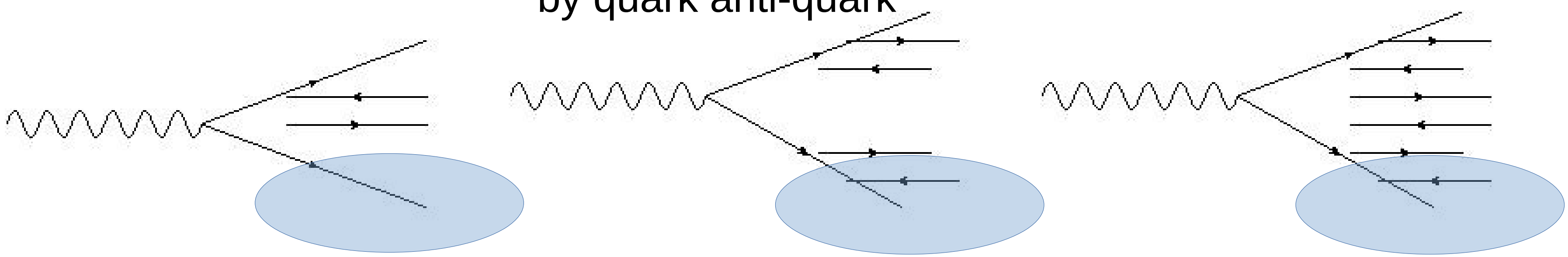
Kharzeev, Tu, Ulrich'19

Cascade of dipoles and entanglement entropy



Mueller 95,
Lublinsky, Levin ‘03, ...

approximate gluon
by quark anti-quark



BFKL dipole cascade

set of partons is described by set of dipoles with fixed sizes , Y is rapidity and is related to energy

$$|\psi\rangle = |q\bar{q}; color; spin; g - coordinates\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\hat{\rho}_r(Y, d_{q\bar{q}}, Q) = Tr(\hat{\rho}(quarksources; color; spin; glu_{\perp}))$$

$$S = -Tr(\hat{\rho}_r \ln \hat{\rho}_r) = -\sum_n p_n \ln\, p_n$$

Microstates p_n – probabilities for given number of dipoles

Explicit construction in DLL
Liu Nowak, Zahed ‘22
KNO scalling in DLL
Liu, Nowak, Zahed ’23

Cascade of dipoles – fixed dipole size

$$\frac{dp_n(y)}{dy} = -\lambda\, n\, p_n(y) + (n-1)\,\lambda\, p_{n-1}(y)$$

Initial conditions

$p_1(0) = 1$ at initial rapidity there is only 1 dipole

$p_{n>1}(0) = 0$

Lublinsky, Levin ‘03

rate at which number of dipoles grow. The phenomenological value is $\lambda = 0.3$.
It is an observable.

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

the growth due to the splitting of (n − 1) dipoles into n dipoles.

See for density matrix in
Liu, Nowak, Zahed ’22 PRD
KNO scalling in DLL
Liu, Nowak, Zahed ’23

Exact analytical solution is:

$$p_n(y) = e^{-\lambda y}(1 - e^{-\lambda y})^{n-1} \qquad y = \ln\left(\frac{1}{x}\right)$$

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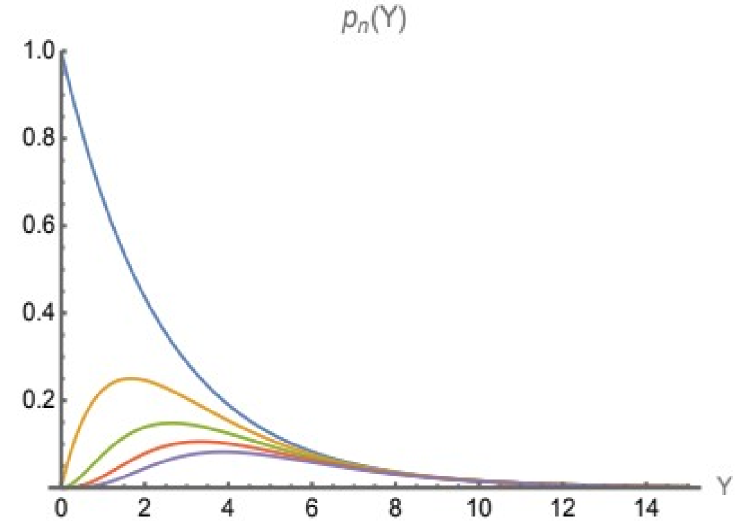

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KL entropy formula - interpretation

$$p_n(y) = e^{-\lambda y}(1 - e^{-\lambda y})^{n-1}$$

At low x partonic microstates have equal probabilities.

In this equipartitioned state the entropy is maximal – the partonic state at small *x* is maximally entangled.

In terms of information theory as Shanon entropy:

- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely

- it is impossible to predict how many partons will be detected in a give event.

- structure function at small x should become universal for all hadrons.

.

Cascade of dipoles – multiplicity and entropy

$$\langle n\rangle = \sum_{n=1} n\, p_n(y) = e^{\lambda y} = \left(\frac{1}{x}\right)^{\lambda} = xg(x)$$

$$S = -\sum_n p_n \ln p_n = \ln\langle n\rangle + (1-\langle n\rangle)\ln\frac{\langle n\rangle - 1}{\langle n\rangle}$$

In the large y limit we have $\langle n\rangle >> 1$

Entropy expressed in terms of gluon momentum density

$$S = \ln\langle n\rangle \qquad p_n = 1/\langle n\rangle$$

$$S = \lambda\, y = \ln\left(\frac{1}{x}\right)^{\lambda}$$

$$\boxed{S = \ln xg(x)}$$

$$S(x,Q) = \ln(xg(x,Q))$$

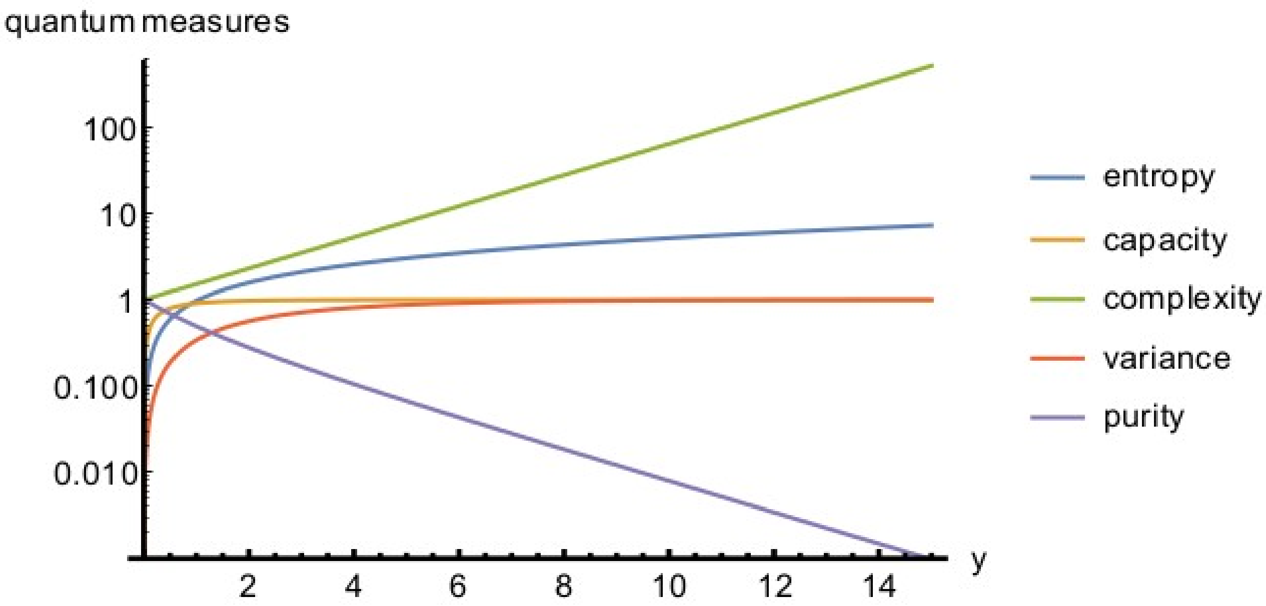
In agreement with calculations based on Unruh effect and saturation applied to p-p collisions
K. Kutak
1103.3654v1

see also
K. Kutak 2310.18510

Conjectured in Kharzeev, Levin 21

QI measures and dipole equations

P. Caputa, K. Kutak ’24



$$S_K(t) = -\sum_n p_n(t) \log p_n(t)$$

$$C_E = \lim_{m\to 1} m^2 \partial_m^2 [(1-m)\, S_K^{(m)}].$$

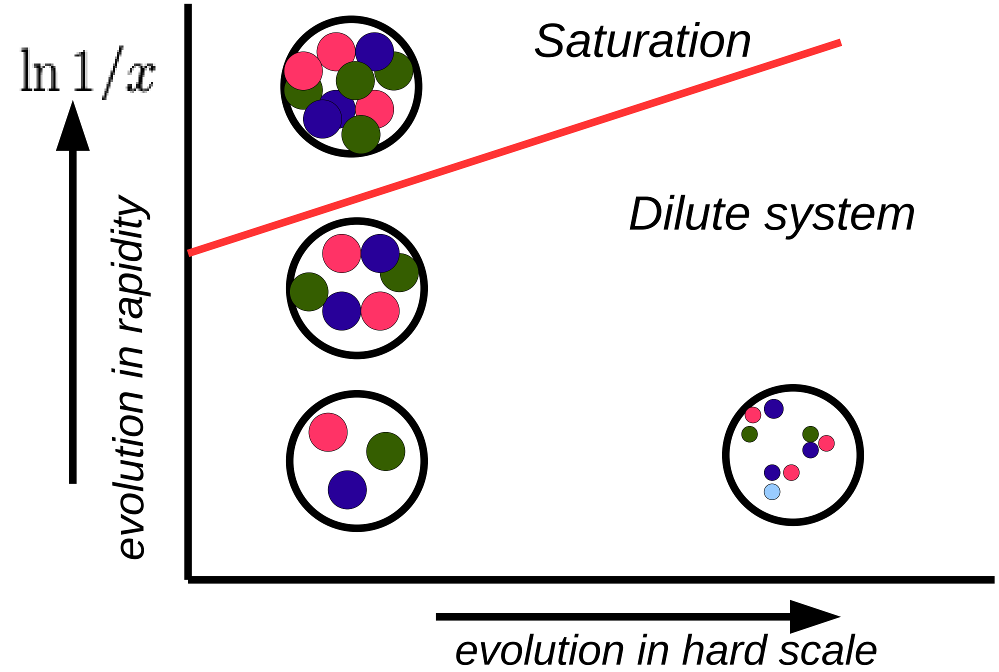
$$\mathcal{C}_K(t) = \langle n \rangle = \sum_n n\, p_n(t)$$

$$\delta_K^2 = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2}$$

$$\gamma_K = \sum_n p_n^2(t).$$

$$\partial_Y p_n(Y) = -\lambda n p_n(Y) + \lambda(n-1) p_{n-1}(Y)$$

Gluons at high energies



Saturation – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

L.V. Gribov, E.M. Levin, M.G. Ryskin
Phys.Rept. 100 (1983) 1-150

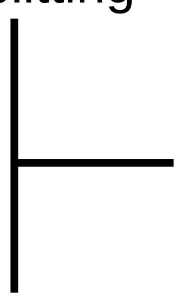
Larry D. McLerran, Raju Venugopalan
Phys.Rev. D49 (1994) 3352-3355

Phenomenological model:
Golec-Biernat, Wusthoff '99

On microscopic level it means that
gluon apart splitting recombine

splitting

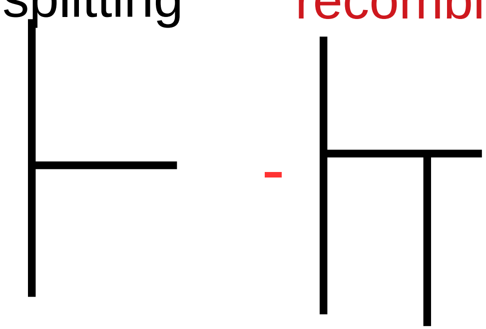
Linear evolution
Equation
BFKL



splitting recombination

Nonlinear evolution equations
BK, JIMWLK
Balitcky-Kovchegov,

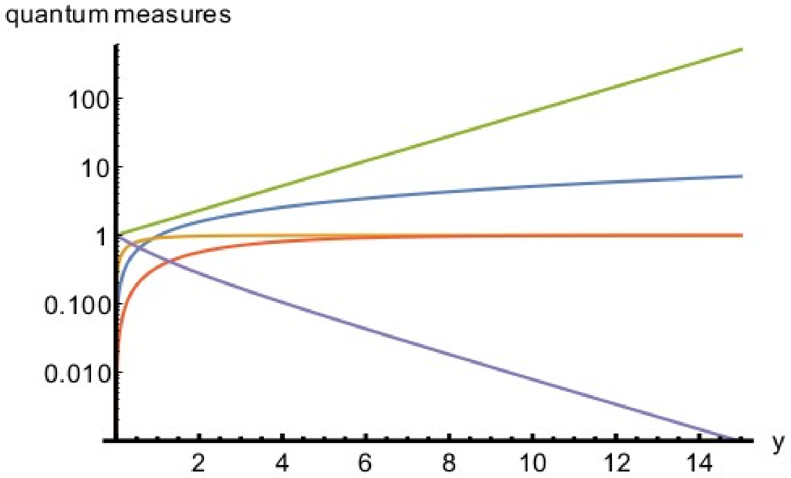
Jailian-Marian,Iancu
McLerran,Weigert,Leonidov,Kovner

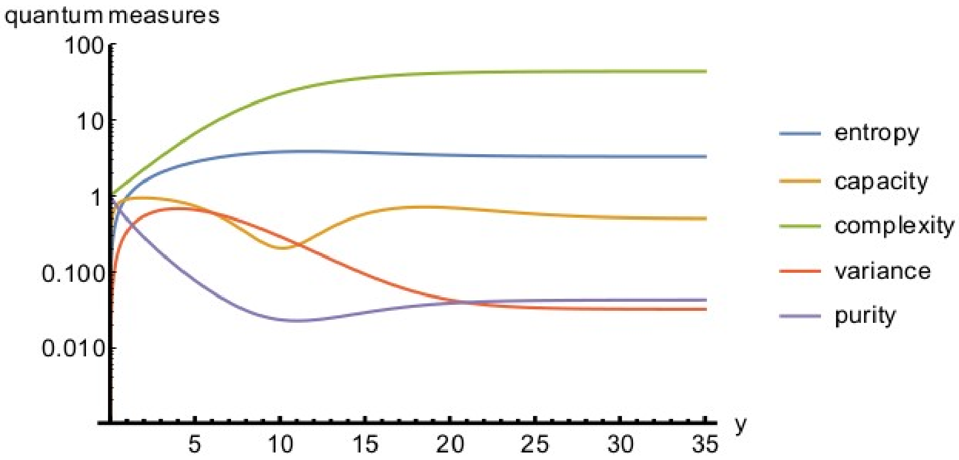


Bartels,
Wusthoff ‘93

QI measures and dipole equations

P. Caputa, K. Kutak, 2404.07657





$$\partial_Y p_n(Y) = -\lambda n p_n(Y) + \lambda(n-1)p_{n-1}(Y)$$

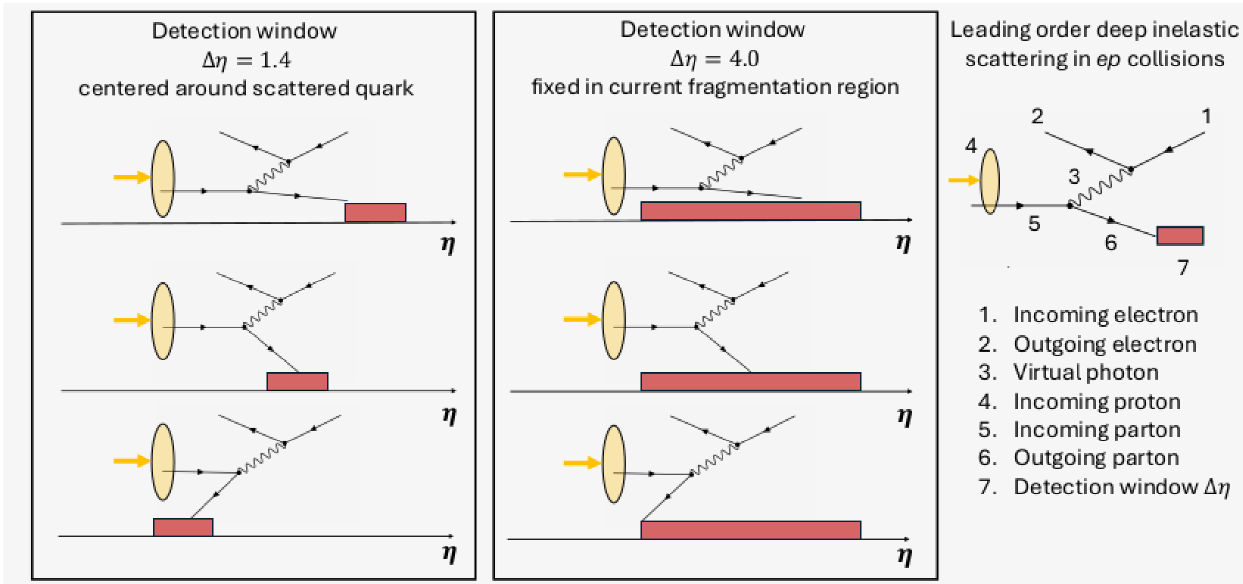
$$\partial_Y p_n(Y) = -\lambda n p_n(Y) + \lambda(n-1)p_{n-1}(Y) + \beta n(n+1)p_{n+1}(Y) - \beta n(n-1)p_n(Y)$$

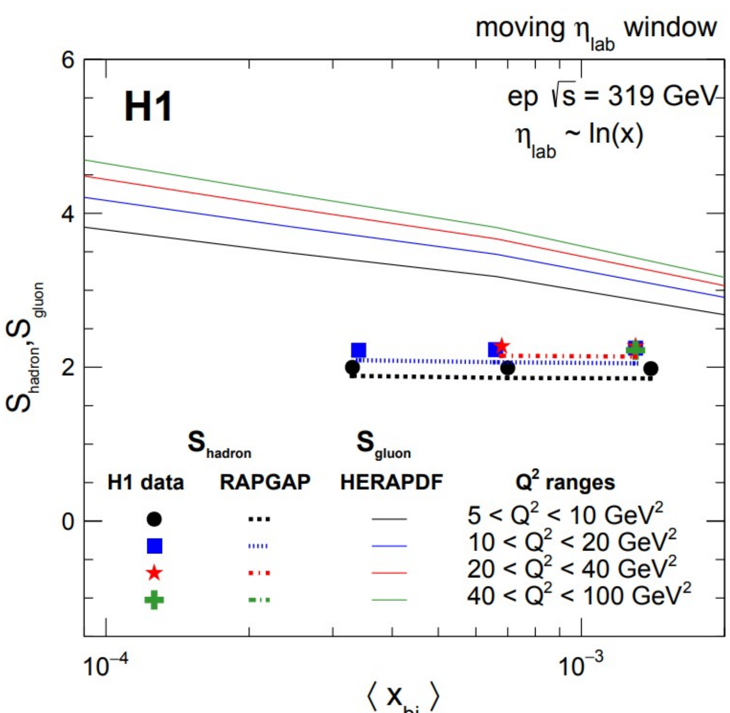
E. Iancu, D.N. Triantafyllopoulos ’05

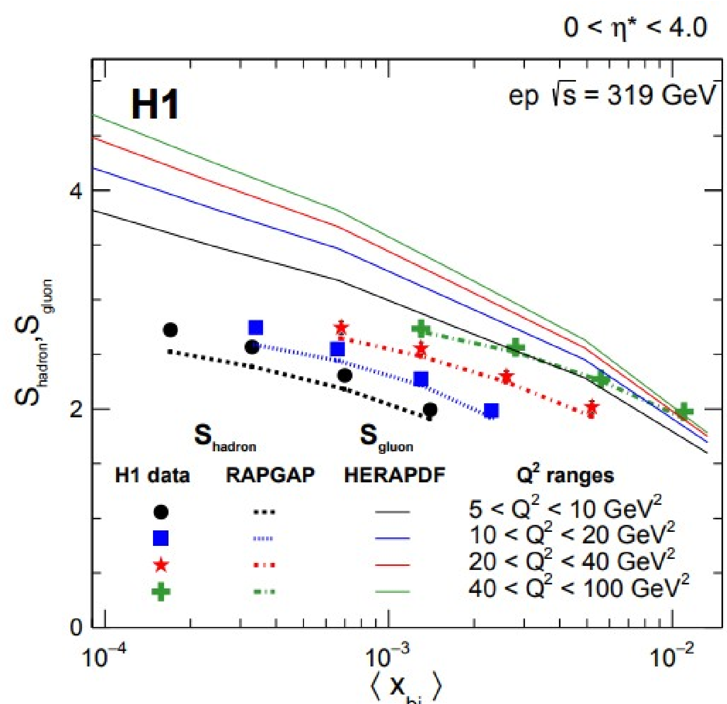
Bondarenko, Motyka, Mueller, Shoshi, Xiao ‘07

Hagiwara,Hatta,Xiao,Yuan ‘18

Entropy measurements

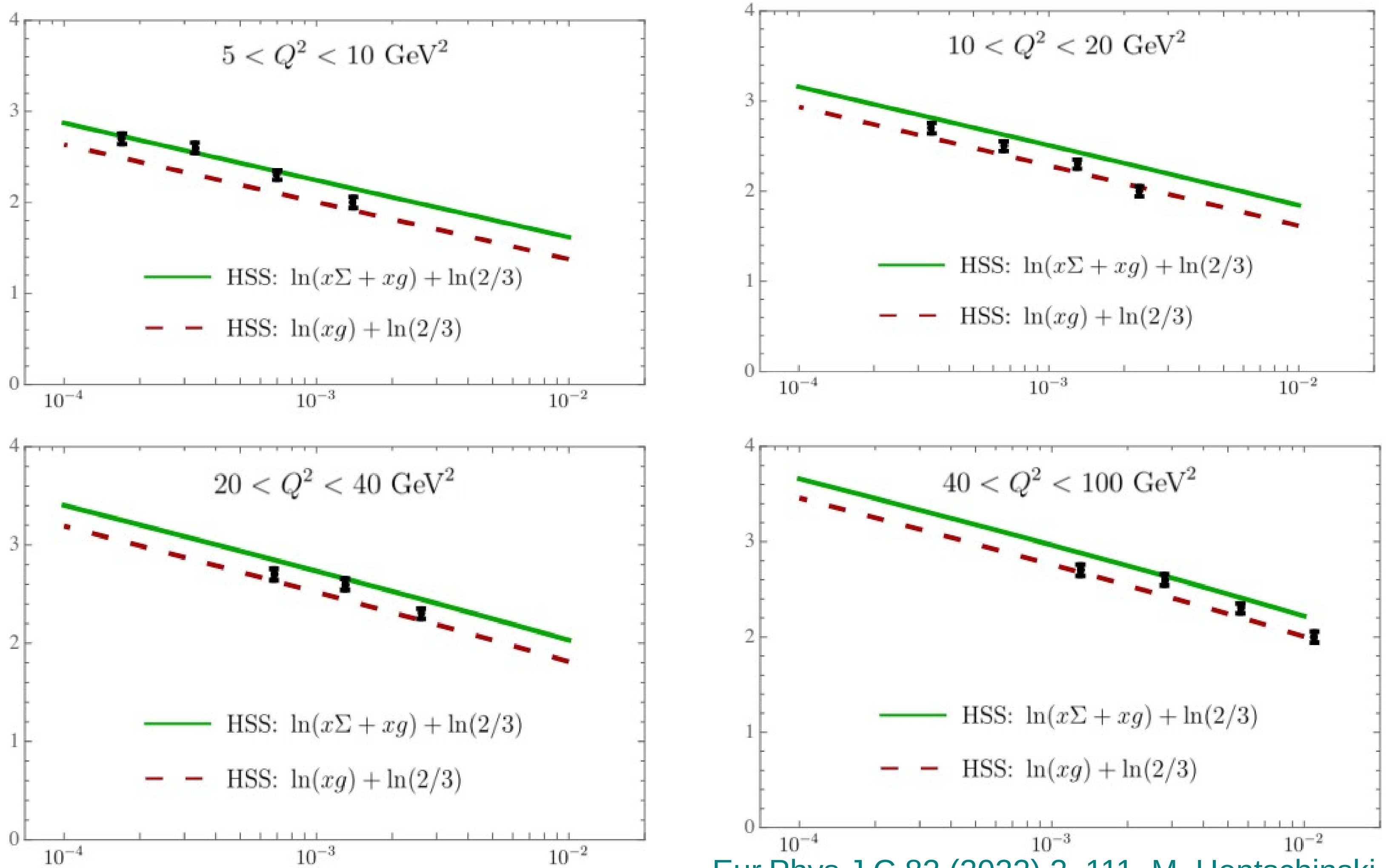






Problem with H1 escription of data...

Results – fixed rapidity window

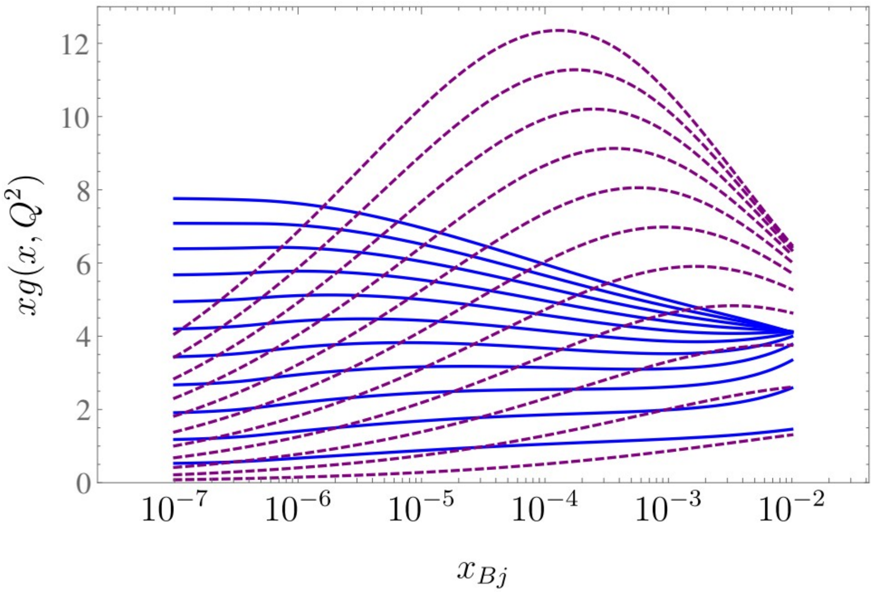


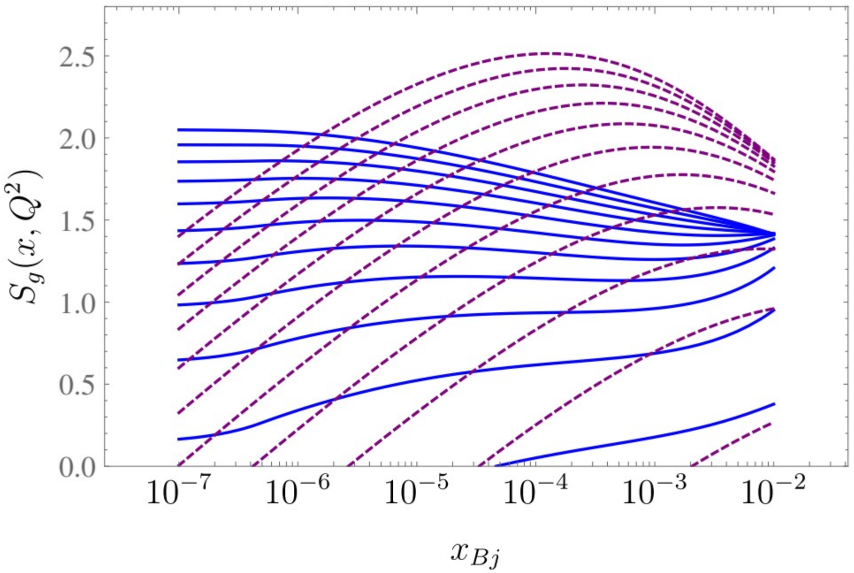
Eur.Phys.J.C 82 (2022) 2, 111 M. Hentschinski, K. Kutak

Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

Integrated gluon and entropy

Hentschinski, Kutak,Straka’23





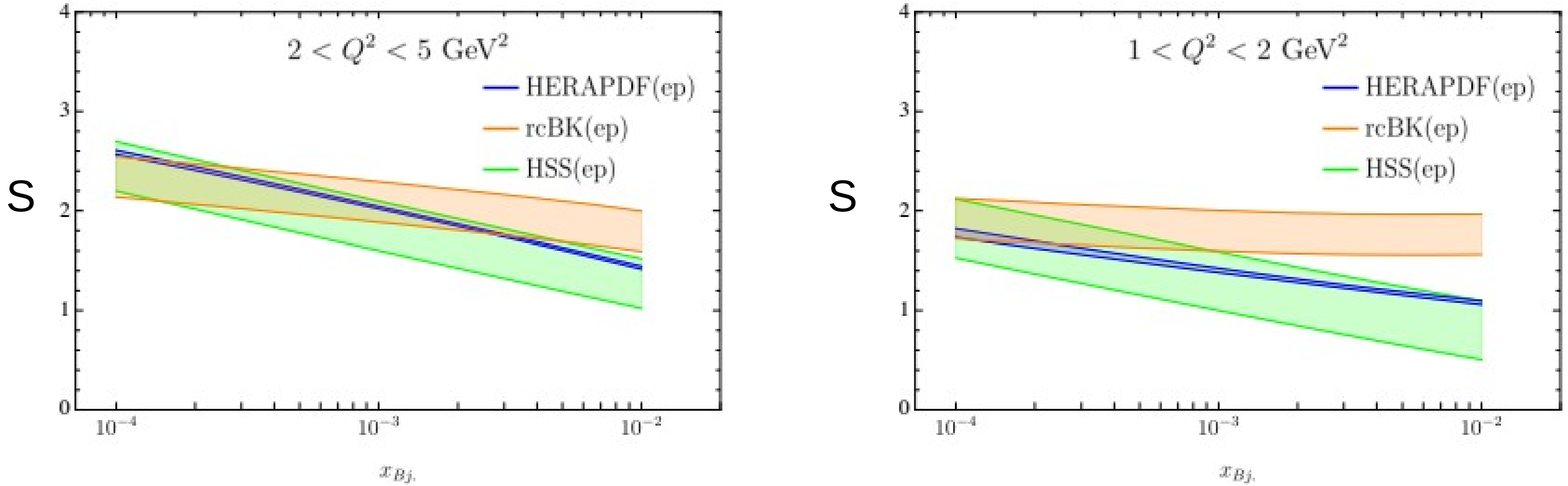
Gluon density from BK drops as x gets lower
It has consequences on entropy.

$$S(x) = \ln(xg(x))$$

No factor 2/3 because we
do not compare to data here

Entropy from BK

Hentschinski, Kutak,Straka'23



Entropy saturates as a consequence of gluon saturation

$$S(x) = \ln(xg(x))$$

Generalization of KL to moving rapidity window

Hentschinski, Kharzeev, Kutak, Tu ‘24

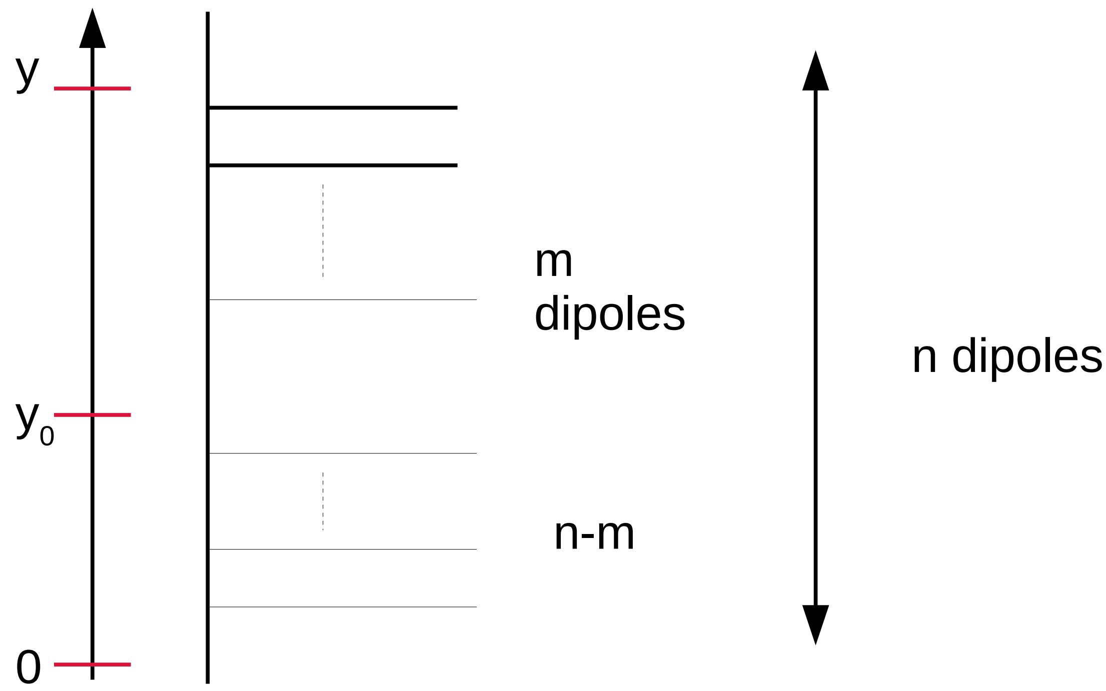
Generalize the KL formula to address
measurement in rapidity window

$$p_n(y) = \sum_{m=0}^{n} p_{n-m}(y_0)\tilde{p}_m(y, y_0)$$

$$\tilde{p}_0(y, y_0) = e^{-\lambda(y-y_0)}$$

$$\tilde{p}_{n\geq 1}(y, y_0) = p_n(y) \cdot (1 - \tilde{p}_0(y, y_0))$$

$$\sum_{m=0} \tilde{p}_m(y, y_0) = 1$$



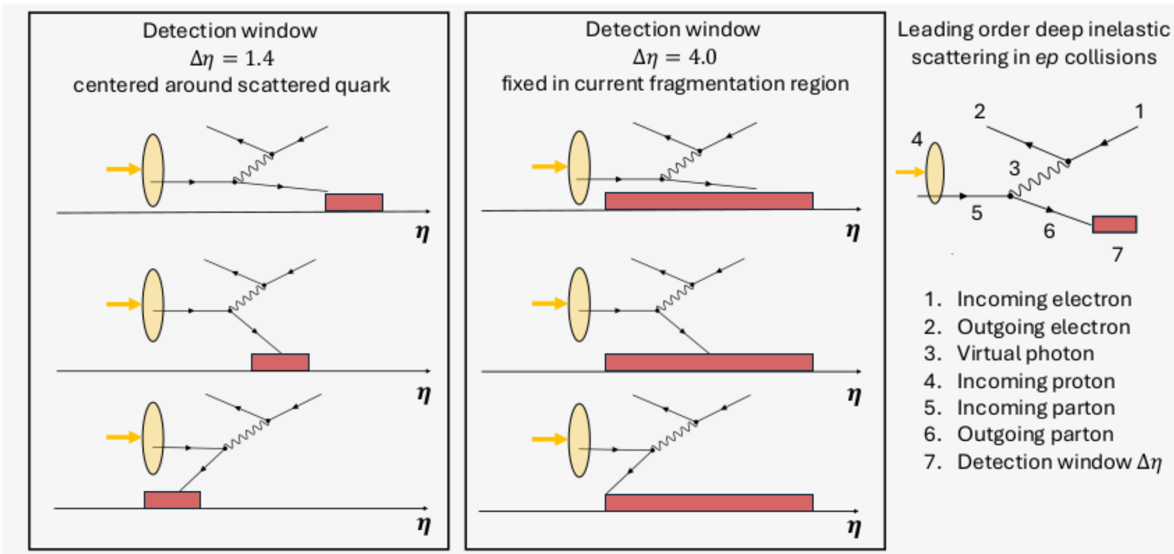
probability to have m dipoles in $[y_0,y]$

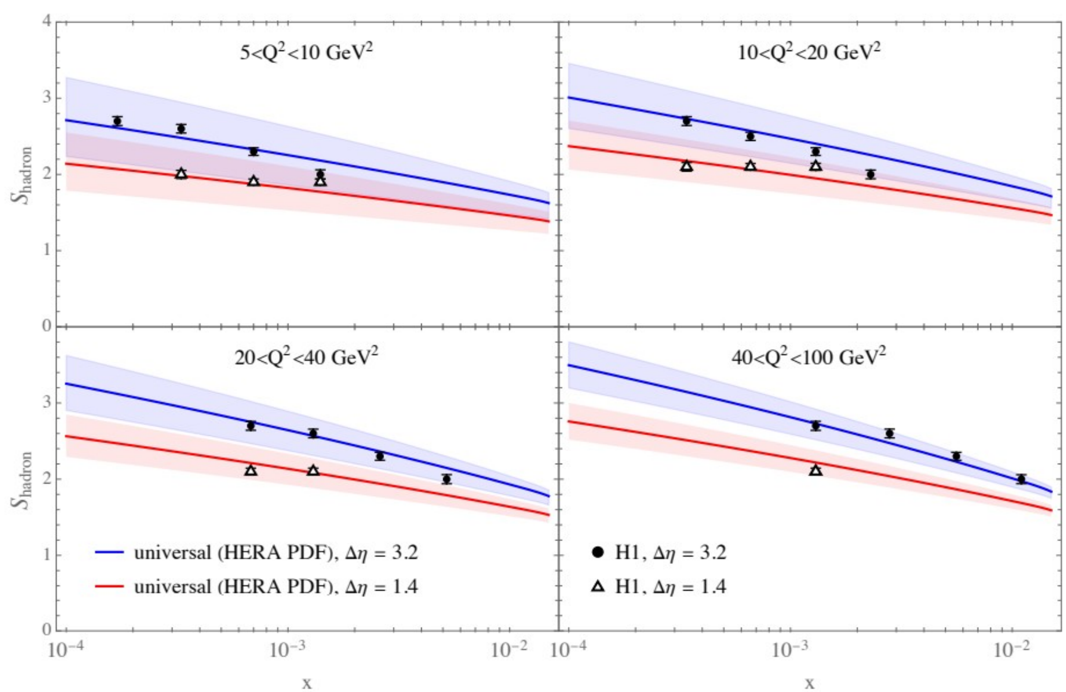
n-m dipoles in the range $[0,y_0]$

probability to have n dipoles in $[0,y]$

Entropy measurements

Hentschinski, Kharzeev, Kutak, Tu ‘24

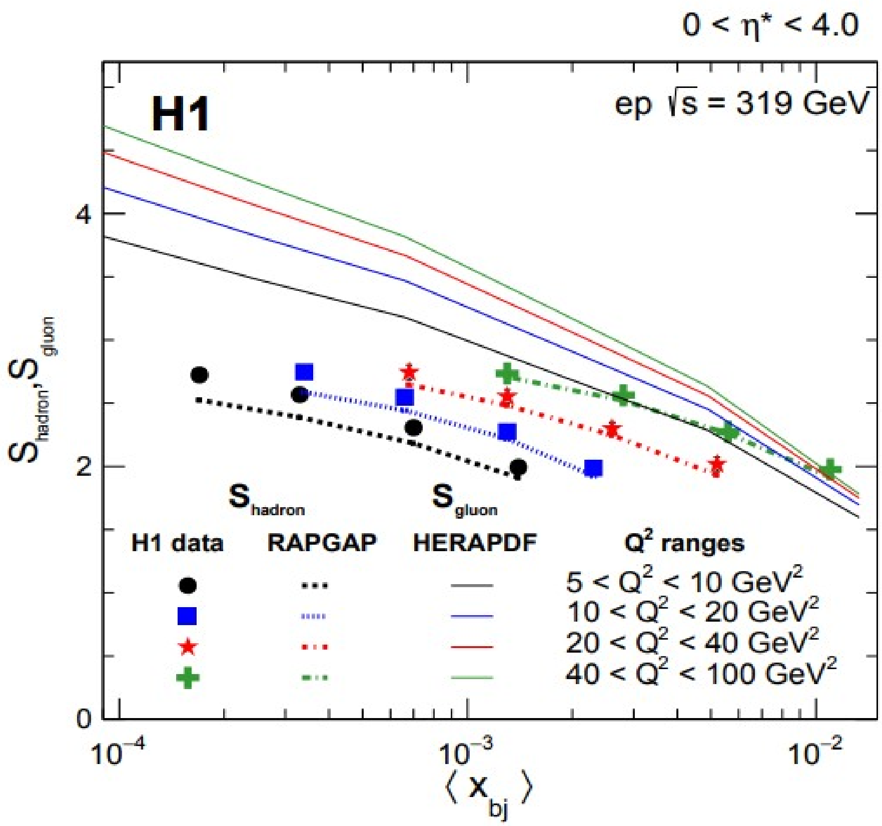




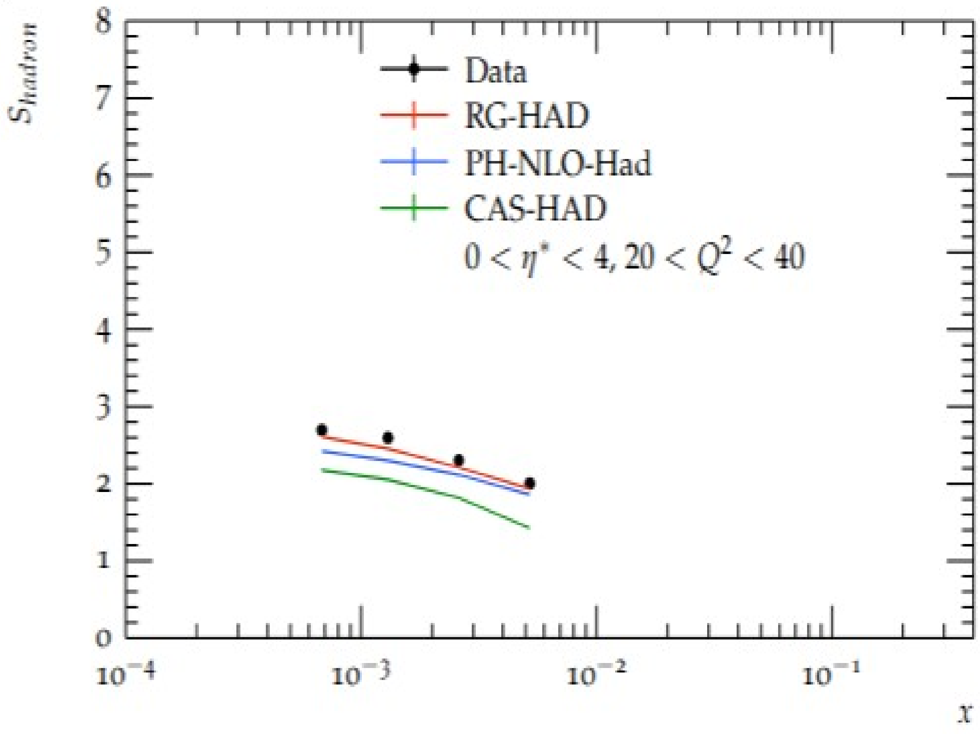
blue – fixed rapidity window

red - moving rapidity window

Entropy from H1 and CASCADE, RapGap, PowHeg



Hadronisation accounted for



M. Hentschinski, H. Jung, K. Kutak ’25

Z. Tu, D. E. Kharzeev, and T. Ullrich, Phys. Rev. Lett. 124 (2020) 062001 – statement that Monte Carlo event generators at parton level can not describe the entropy.

Entropy from measurement and Monte Carlo at parton level

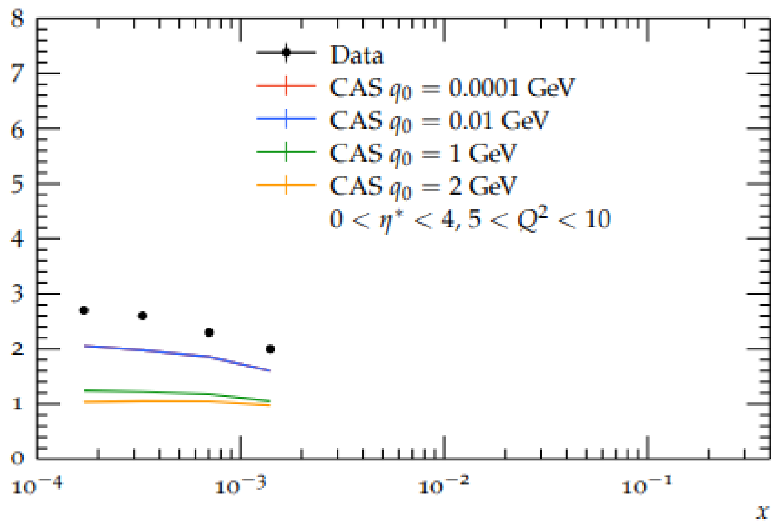
$$xf_a(x,\mu^2) = \Delta_a^S(\mu^2)\; xf_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq^2}{q^2} \frac{\Delta_a^S(\mu^2)}{\Delta_a^S(q^2)} \int_x^{z_{\rm M}} dz\, \frac{\alpha_s}{2\pi} \hat{P}_{ab}(z) \frac{x}{z}\; f_b\left(\frac{x}{z}, q^2\right)$$

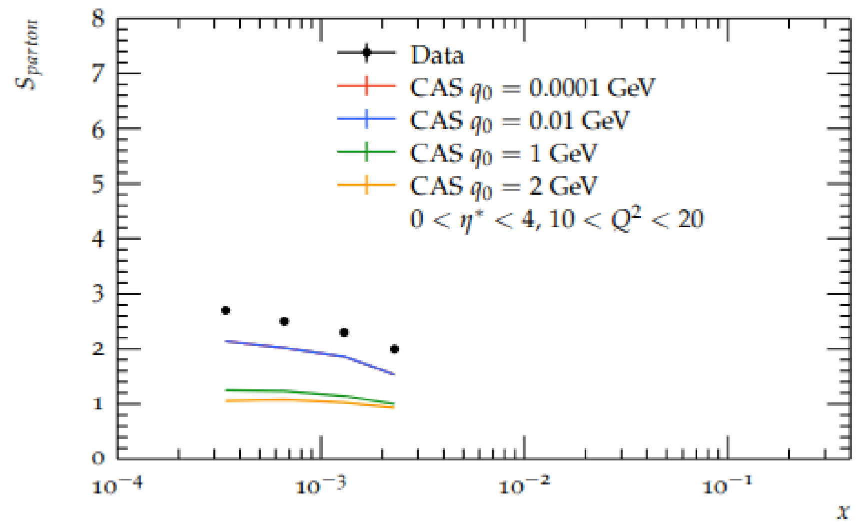
Hentschinski, Jung, Kutak ’25

$$z_M = 1 - \frac{q_0}{q}$$

$$q = \frac{q_t}{1-z}$$

minimal scale

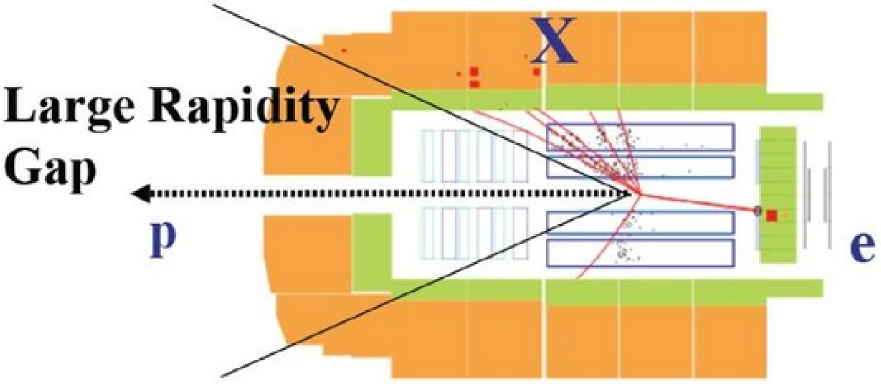


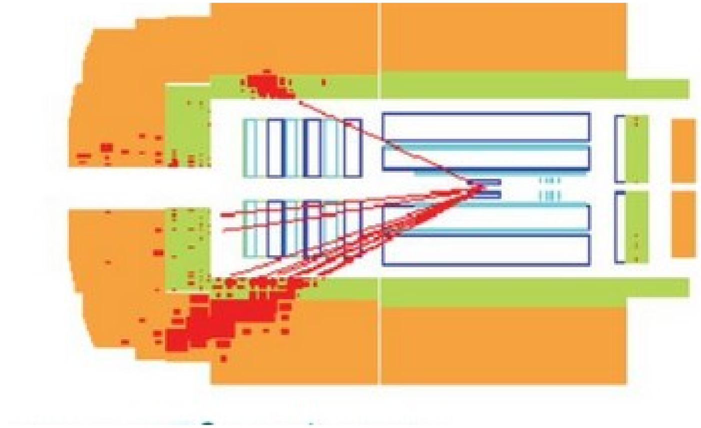


Motivated by Kharzeev, Tu Ullrich’19

Typically in Monte Carlo generators q_0=1 or 2 GeV. Now if pdf (obtained with no cut) are used in backward shower and the cut is 2 GeV one looses some multiplicity, do not obey the shower equation. This has consequences for entropy….

Diffraction vs. nondiffraction



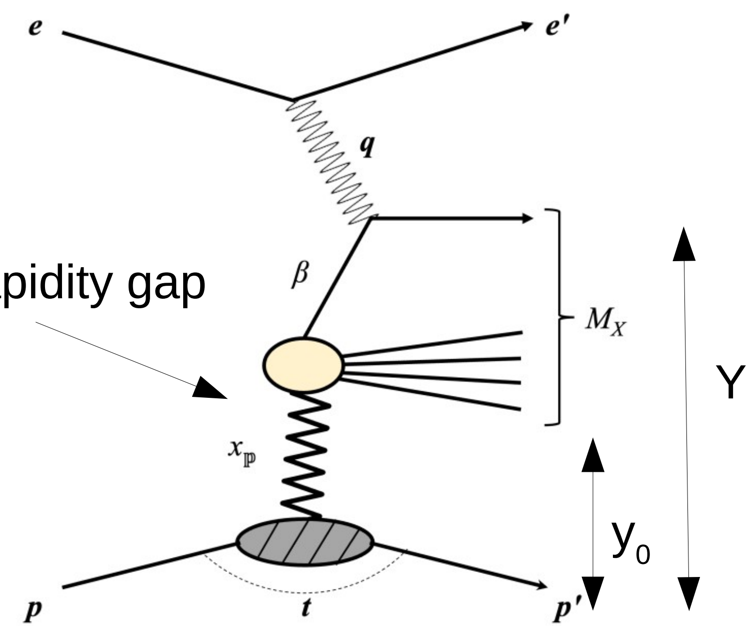


H1 detector

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in Diffractive Deep Inelastic Scattering

rapidity gap



colorless
diffractive exchange

$x_{\mathbb{P}}$ proton’s momentum fraction carried by the Pomeron

β denotes the Pomeron’s momentum fraction carried by the quark interacting with the virtual photon

$$x = \beta \cdot x_{\mathbb{P}}$$ Bjorken x

$$y_0 \simeq \ln 1/x_{\mathbb{P}}$$ size of rpidity gap

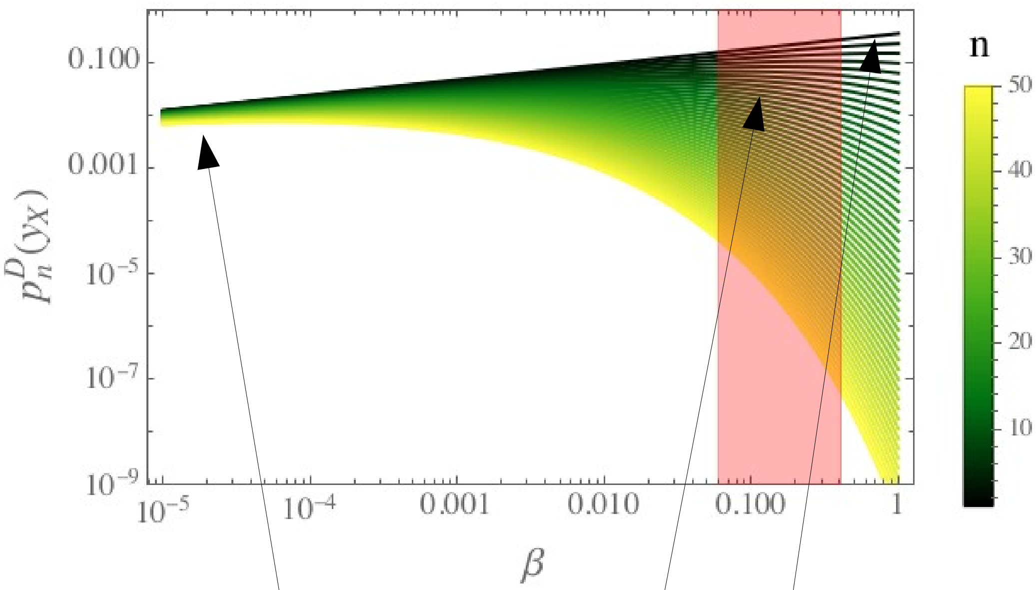
$$Y = \ln 1/x$$

$$y_X = Y - y_0 \simeq \ln 1/\beta$$

Analogous evolution equation as for non-diffractive case but Initial conditions are different and there is delay because of rapidity gap.

Munier, Mueller Phys. Rev. D 98, 034021 (2018)

EE in DDIS



$$\left\langle \frac{dn(\beta)}{d\beta} \right\rangle_{\text{charged}} \simeq \frac{2}{3} \left\langle \frac{dn(\beta)}{d\beta} \right\rangle$$

$$p_n^D(y_X) = \frac{1}{C} e^{-\Delta y_X} \left(1 - \frac{1}{C} e^{-\Delta y_X}\right)^{n-1}$$

$$p_n \equiv 1/Z$$

H1 data region

$$S(Z) = -\sum_n p_n \ln p_n = (1-Z) \ln \frac{Z-1}{Z} + \ln Z$$

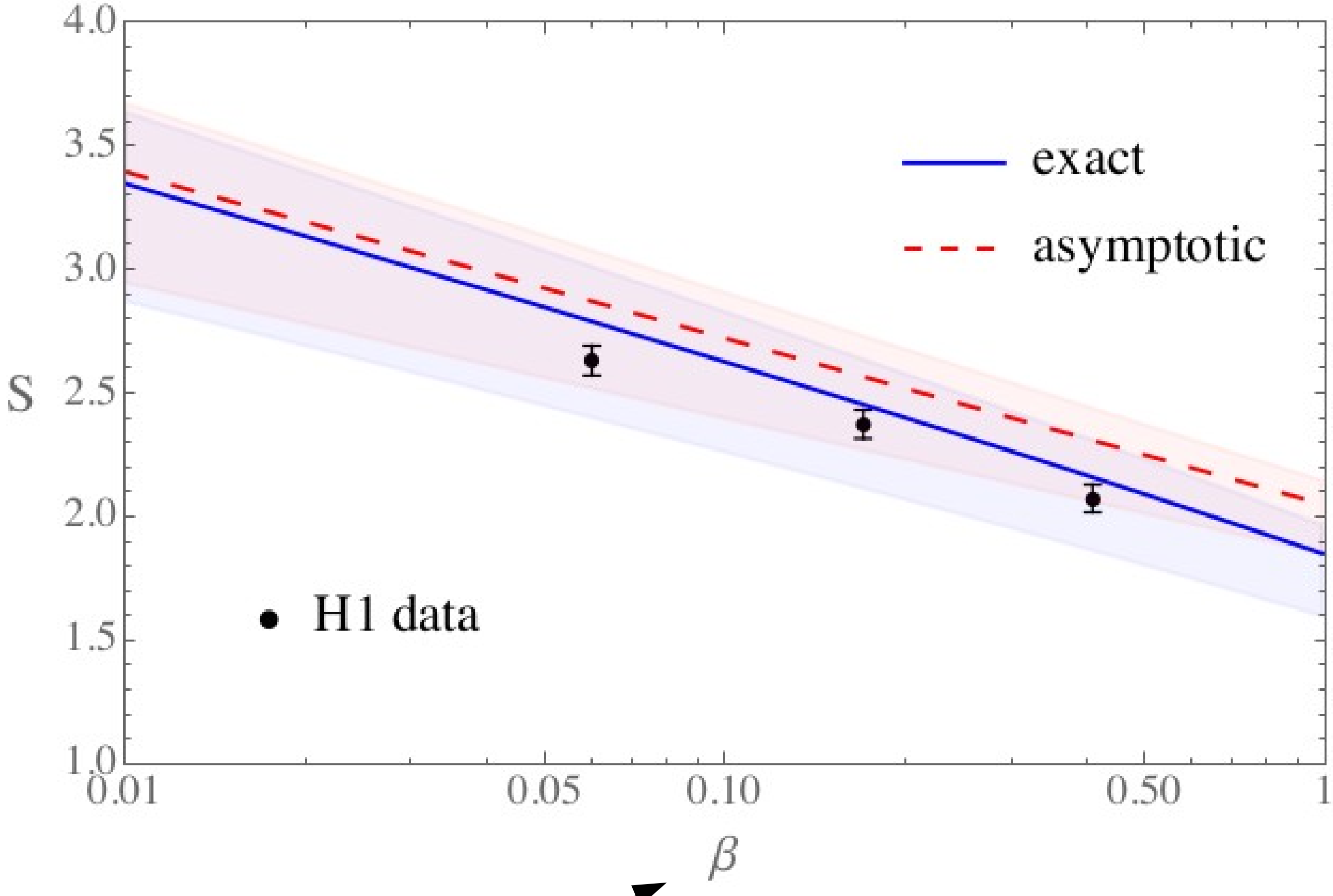
Asymptotic or maximal entangled region. All configurations have the same probability

$$S_{\text{asym.}}(Z) = \ln Z + 1 + \mathcal{O}(1/Z)$$

High probability of configurations with few partons

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu '23

EE in DDIS



momentum fraction carried by the
quark interacting with the virtual photon

LHCb and charged hadron multiplicity

y	$\bar{n}_{\rm ch}(y)$	$S(y)$
2.25	2.01 ± 0.12	1.91 ± 0.11
2.75	2.42 ± 0.10	2.06 ± 0.11
3.25	2.41 ± 0.10	2.05 ± 0.06
3.75	2.12 ± 0.09	1.95 ± 0.06
4.25	1.85 ± 0.07	1.84 ± 0.06

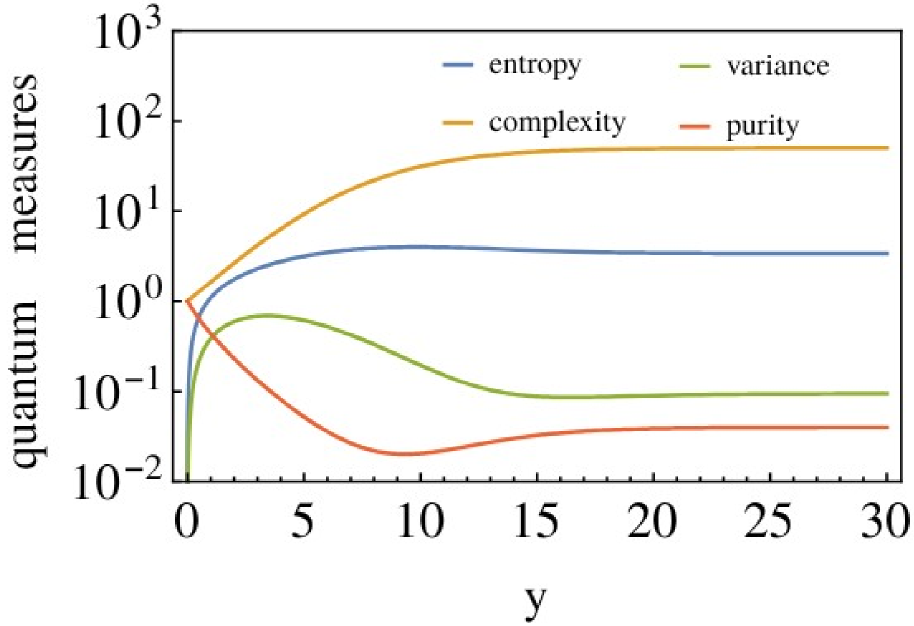
decreasing entropy
in forward region

$$S(y) = -\sum_n p_n(y) \ln(p_n(y))$$

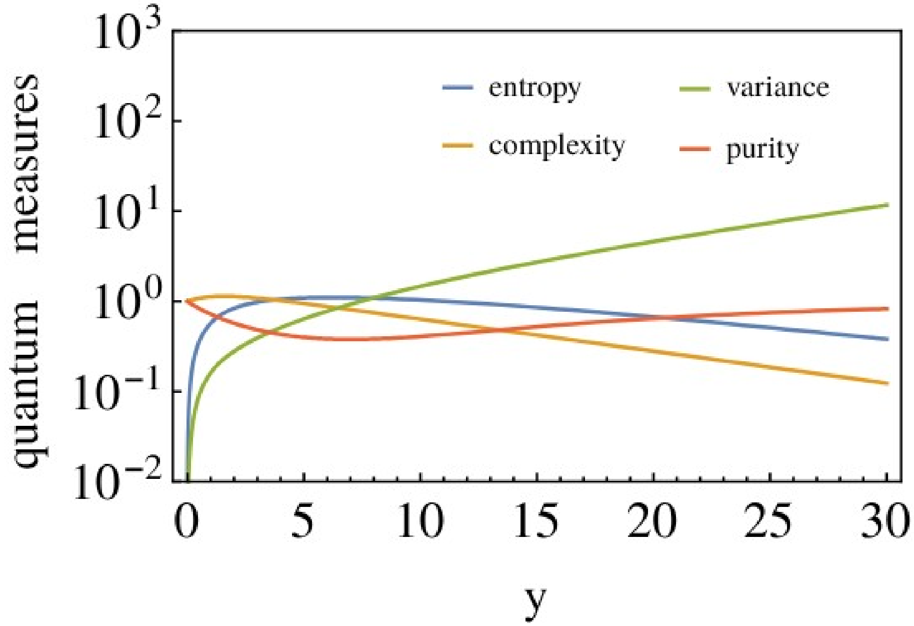
Kutak, Praszałowicz'25

$$\langle n \rangle = \sum_n n p_n$$

Production, recombination and transitions to vacuum – QI measures



β - cascade

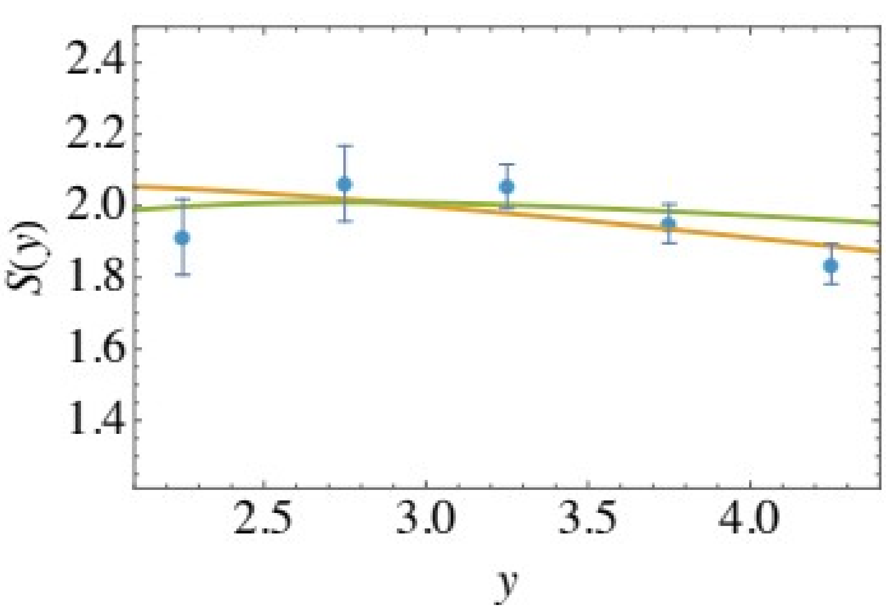


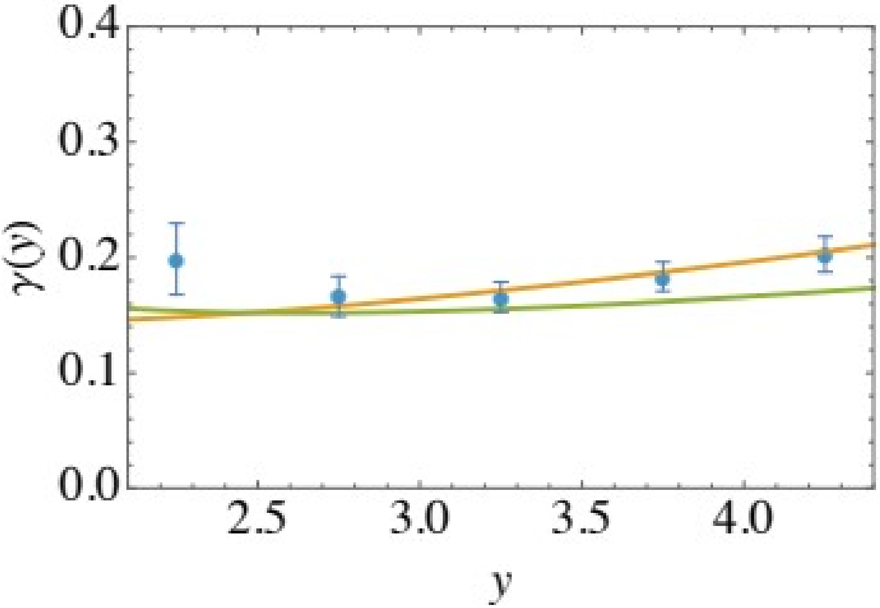
γ - cascade

Kutak, Praszałowicz’25

LHCb entropy, purity in forward direction

Kutak, Praszałowicz’25





Entropy

Purity

$$S(y) = -\sum_n p_n(y) \ln(p_n(y))$$

$$\gamma_K = \sum_n p_n^2(t).$$

Entropy from complete BFKL cascade

work in progress:
Hentschinski, Kutak,Płaczek, Rohrmoser

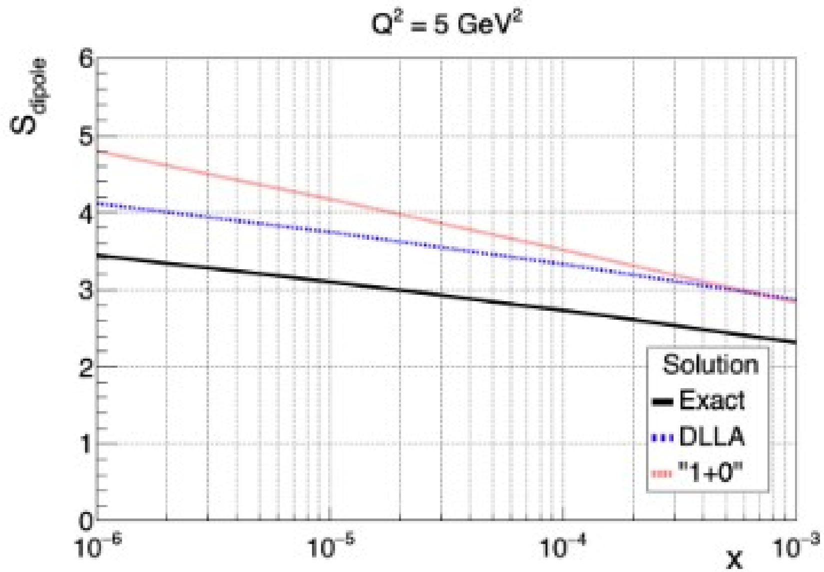
$$\frac{dp_n(y)}{dy} = -\lambda\, n\, p_n(y) + (n-1)\,\lambda\, p_{n-1}(y)$$

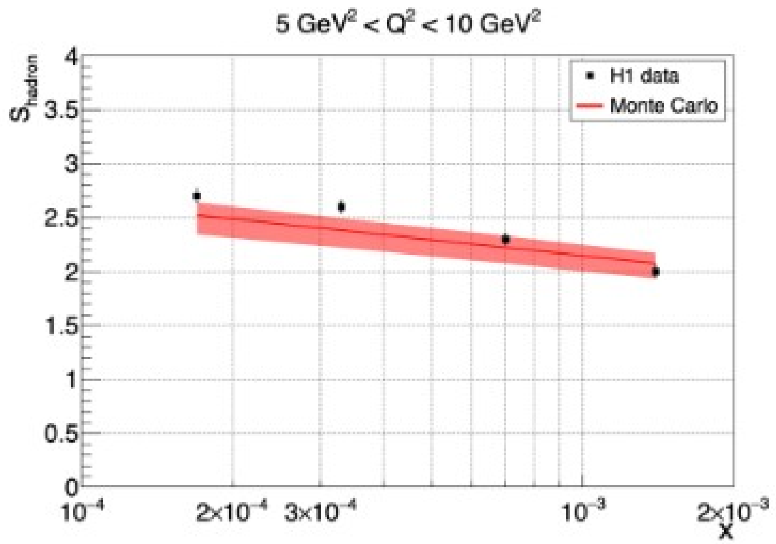
the approximated equation

Two new Monte Carlo programs:
by W. Placzek, M. Rohrmoser

$$\frac{dP_n(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n)}{dY} = -\sum_{i=1}^{n}\sigma(\boldsymbol{r}_i)\,P_n(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n)$$
$$+\sum_{i=1}^{n-1}K(\boldsymbol{r}_i,\boldsymbol{r}_n)\,P_{n-1}(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_i+\boldsymbol{r}_n,\ldots,\boldsymbol{r}_{n-1})$$

the the complete equation





Krylov subspace, complexity – motivation

Simple reference quantum state spreads and becomes complex in Hilbert space

Visvanath, Muller ‘63
Altman, Avdoshkin, Cao, Parker, Scaffidi ‘19
Balasubramanian, Caputa, Magan, Wu ‘22,...

$$i\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle \qquad |\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$$

Krylov basis

$$|\Psi(t)\rangle = e^{-iHt} |\Psi_0\rangle = \sum_n \phi_n(t) |K_n\rangle \qquad p_n(t) = |\phi_n(t)|^2 \qquad \sum_n |\phi_n(t)|^2 = 1$$

Comes from studies of efficient diagonalization of matrices and computation of characteristic polynomial coefficients

The complexity has simple form in Krylov basis.
It can be to used to quantify chaotic behavior of quantum systems.

$$\mathcal{C}_K(t) = \langle n \rangle = \sum_n n\, p_n(t)$$

Krylov basis and dipole evolution and complexity

P. Caputa, K. Kutak, 2404.07657

$$|\Psi(t)\rangle = e^{-iHt}\,|\Psi_0\rangle = \sum_n \phi_n(t)\,|K_n\rangle$$

build from consecutive application of hamiltonian

$$i\partial_t\phi_n(t) = a_n\phi_n(t) + b_n\phi_{n-1}(t) + b_{n+1}\phi_{n+1}(t)$$

Krylov basis minimalizes cost function

The equation above can be solved for

$$a_n = 0$$

$$b_n = \alpha n$$

boost operator

$$|\psi(t)\rangle = e^{-i\alpha(L_1+L_{-1})t}\,|0\rangle\otimes|0\rangle$$

vacuum of un-observed part of proton

vacuum of observed part of proton

$$p_n(t) = |\phi_n(t)|^2$$

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One can calculate reduced density matrix

$$\rho(t) = \sum_{n=0}^{\infty} p_n(t)\,|n\rangle\,\langle n|$$

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After expressing it in terms rapidity and probabilities rapidity variable one gets

$$p_n(Y) = \frac{\Gamma(2h+n)}{n!\Gamma(2h)}(e^{-\alpha Y})^{2h}\left(1-e^{-\alpha Y}\right)^n$$

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$$\mathcal{C}_K(Y) = e^{\alpha Y} - 1 \qquad Y = \ln(1/x)$$

$$\boxed{\mathcal{C}_K = xg(x)}$$

Conclusions and outlook and comments

- We described results of published by H1 on entropy both in sliding and fixed rapidity window

- We applied various quantum measures to the multiplicity densities of the dipole model. New handle to look for saturation

- The Monte Carlo simulation at parton level demonstrated to give bulk contribution to entropy. This further provides evidence that most of the entropy is generated on parton level i.e. entropy is associated with pdfs. Soft gluons are crucial element

- This result might be of relevance for Monte Carlo simulations in heavy ion physics and studies of quark gluon plasma and thermalisation. Also in studies of TMD in Monte Carlo settings.

- We described entropy data using complete Mueller cascade

Gluon production and entropy – another assumptions

Kutak '11

Bialas; Janik; Fialkowski, Wit; Iancu, Blaizot, Peschanski,...

$$M_G(x) = Q_s(x)$$ energy dependent gluon's mass

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$$M(x) = N_G(x) M_G(x)$$ mass of system of gluons

Many-body interactions

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Similarly in QED. Cut on photon`s kt
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In presented approach mass is not fixed it is x dependent

$$[L_0, \phi(z)] = \left(z\frac{d}{dz} + h \right) \phi(z)$$ scaling

$$[L_{-1}, \phi(z)] = \frac{d}{dz}\phi(z)$$ translations

$$[L_1, \phi(z)] = \left(z^2\frac{d}{dz} + 2hz \right) \phi(z)$$ special conformal

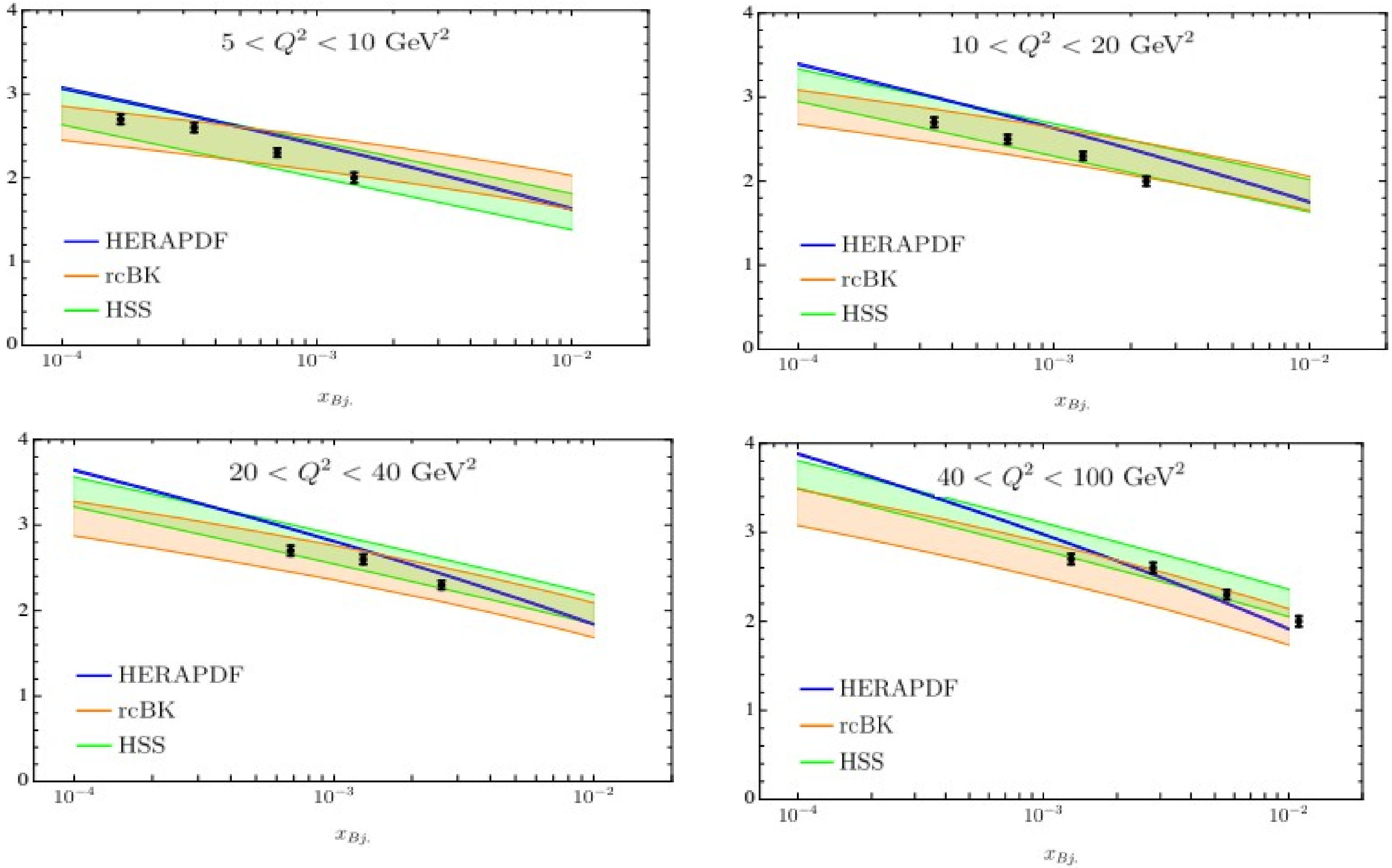
$$L_0 = \frac{1}{2}(a^\dagger a + b^\dagger b + 1), \qquad L_{-1} = a^\dagger b^\dagger, \qquad L_1 = ab$$

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}, \qquad [L_1, L_{-1}] = 2L_0$$

$$[a, a^\dagger] = [b, b^\dagger] = 1$$

Large scales - description

Martin Hentschinski, K. Kutak, Robert Straka '23



Krylov subspace, complexity – motivation

Simple reference quantum state spreads and becomes complex in Hilbert space

Visvanath, Muller ‘63
Altman, Avdoshkin, Cao, Parker, Scaffidi ‘19
Balasubramanian, Caputa, Magan, Wu ‘22,...

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P. Caputa, K. Kutak, 2404.07657

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$$\boxed{\mathcal{C}_K = xg(x)}$$

Krylov subspace – construction

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}|\Psi_n\rangle \qquad |\Psi_n\rangle \equiv \{|\Psi_0\rangle, H|\Psi_0\rangle, ..., H^n|\Psi_0\rangle, ...\}$$

n consequtive application of Hamiltonian

$$|K_0\rangle = |\psi(0)\rangle = |\psi_0\rangle$$

Gram-Schmidt orthogonalization procedure. Construct with K_2 by subtracting the previous two vectors, K_3 by subtracting the previous 3 vectors, and so forth

$$|z_1\rangle = \hat{H}|K_0\rangle - a_0|K_0\rangle \qquad |K_1\rangle = \frac{|z_1\rangle}{\langle z_1|z_1\rangle}$$

$$|z_{n+1}\rangle = (\hat{H} - a_n)|K_n\rangle - b_n|K_{n-1}\rangle$$

Strength of the Lanczos algorithm. n + 1 is determined by n and n – 1. Low memory requirements Visvanath, Muller '63

$$|K_n\rangle = b_n^{-1}|z_n\rangle \qquad b_n = \langle z_n|z_n\rangle^{\frac{1}{2}} \qquad a_n = \langle K_n|\hat{H}|K_n\rangle$$

Project a high-dimensional problem onto a lower-dimensional Krylov subspace

$$H|K_n\rangle = a_n|K_n\rangle + b_{n+1}|K_{n+1}\rangle + b_n|K_{n-1}\rangle$$

$$\langle K_n|K_m\rangle = \delta_{nm}$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = \sum_n \phi_n(t)|K_n\rangle$$

$$H_{nm} := \langle K_n|\hat{H}|K_m\rangle = \begin{pmatrix} a_1 & b_1 & 0 & 0 & \cdots \\ b_1 & a_2 & b_2 & 0 & \cdots \\ 0 & b_2 & a_3 & b_3 & \cdots \\ 0 & 0 & b_3 & a_4 & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix}$$

probability amplitudes for each vector

$$i\partial_t\phi_n(t) = a_n\phi_n(t) + b_n\phi_{n-1}(t) + b_{n+1}\phi_{n+1}(t)$$

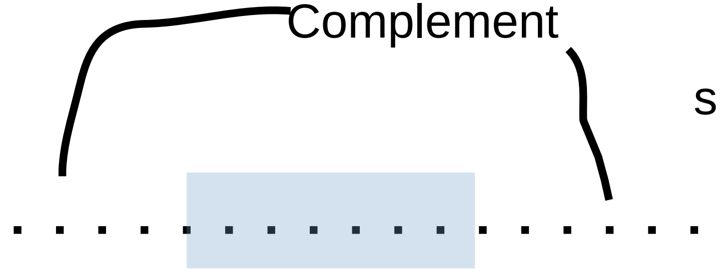
Comments

CFT result for EE

central charge

$$S = \frac{c}{3} \ln \frac{L}{\epsilon}$$

UV cutoff



spnis in 1 D

Region A of length L

Entanglement entropy obtained from CFT
calculations as well as from gravity using
Ryu-Takayanagi formula

See also
Callan, Wilczek '94
Calabrese, Cardy '04

and lectures by
Headrick

Relation to Kharzeev-Levin formula

$$L = (mx)^{-1} \qquad \epsilon \equiv 1/m$$

Length of tube probed in DIS

Proton's Compton wave length

$$S = \ln \left(\frac{1}{x} \right)^{1/3}$$

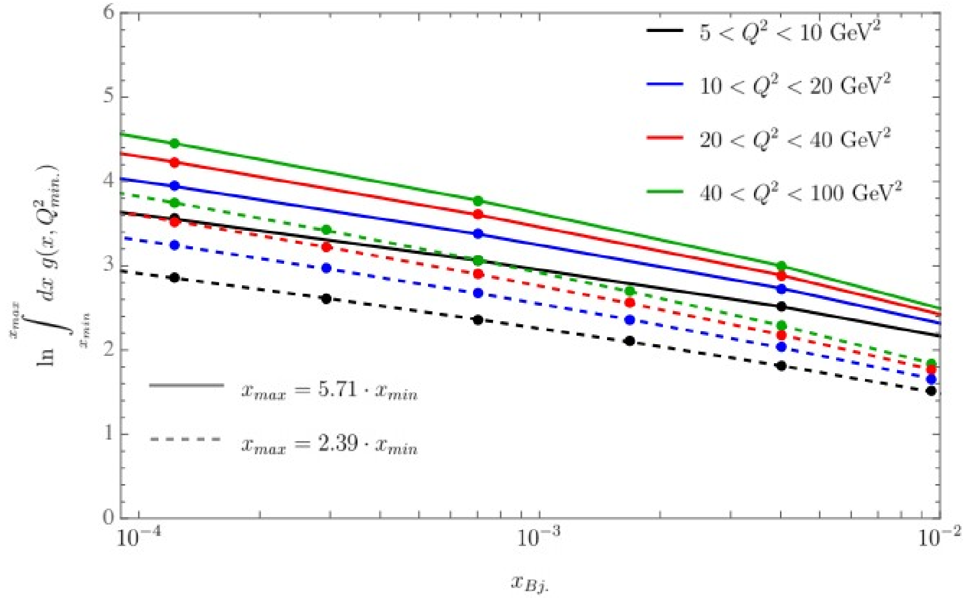
$$S(x) = \ln(xg(x))$$

Studied also in the context of 2 D QCD

Liu, Nowak, Zahed, '22

Casini, Huerta, Hosco '05

Bining and KL formula



plot showing dependence of the result on the sie of bins if binig is naive

$$n_g(Q^2) = \int_0^1 dx g(x, Q^2)$$

Formal definition of number of gluons

$$n_g(\bar{x}) = \int_{x_{\min}}^{x_{\max}} dx g(x, Q^2)$$

$$\bar{x} \in [x_{\min}, x_{\max}]$$

$$\bar{x} = \frac{\int_{x_{\min}}^{x_{\max}} dx\, x g(x, Q^2)}{\int_{x_{\min}}^{x_{\max}} dx g(x, Q^2)}$$

average *x*

Data binning takes place in rapidity

$$\bar{n}_g(\bar{x}) = \frac{1}{y_{max} - y_{min}} \int_{y_{min}}^{y_{max}} dy \frac{dn_g}{dy} = \frac{n_g(y_{max}) - n_g(y_{min})}{y_{max} - y_{min}}$$

$$y_{max,min} = \ln 1/x_{min,max}$$

for small bins

$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\ln(1/x)} = xg(x, Q^2)$$

$$\langle \bar{n}(x, Q^2) \rangle_{Q^2} = \frac{1}{Q^2_{\max} - Q^2_{\min}} \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \left[xg(x, Q^2) + x\Sigma(x, Q^2) \right]$$

$$\langle S(x, Q^2) \rangle_{Q^2} = \ln \langle \bar{n}(x, Q^2) \rangle_{Q^2}$$

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In presented approach
mass is not \fixed it is x dependent

$$S = 3\pi \left[N_G(x) + N_{G\,0}\right]$$