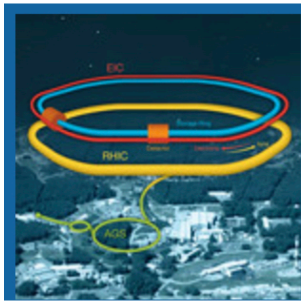

The 2026 CFNS Summer School on the Physics of the Electron-Ion Collider

The Multidimensional Parton Structure of Hadrons

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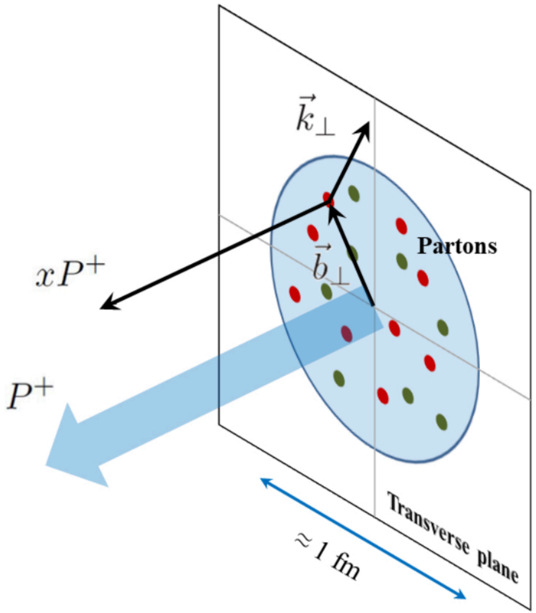
supported by the NSF

Outline



Light-cone variables for generic 4-vector $a^\mu = (a^0, a^1, a^2, a^3)$

$$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3) \qquad \vec{a}_\perp = (a^1, a^2)$$

1. Parton Distribution Functions (PDFs)
2. Transverse Momentum Dependent Parton Distributions (TMDs)
3. Generalized Parton Distributions (GPDs)
4. Generalized TMDs (GTMDs)

$\longrightarrow$ overlap with lectures by Olness, Surrow, Tu, Nadolsky, Siritsyn, ...

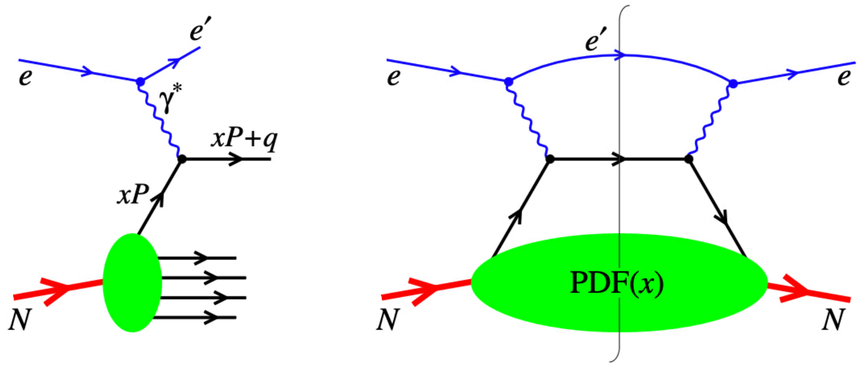
Disclaimer: we can only discuss bits and pieces

Some Literature

- M. Diehl: *Introduction to GPDs and TMDs*, 1512.01328
- Various articles on 3D parton structure of hadrons, Eur. Phys. J **A52** (2016)
- C. Lorcé, A. Metz, B. Pasquini, P. Schweitzer, *Parton distribution functions and their generalizations*, 2507.12664
- A. Signori, *Introduction to transverse momentum imaging*, 2604.19997
- R. Boussarie et al, *TMD Handbook*, 2304.03302
- C. Mezrag, *An introductory lecture on generalized parton distributions*, 2207.13584
- A. Accardi et al, *Electron Ion Collider: The next QCD frontier – understanding the glue that binds us all*, 1212.1701
- R. Abdul Khalek et al, *Science requirements and detector concepts for the Electron Ion Collider: EIC Yellow Report*, 2103.05419
- (and many more ...)

Inclusive Deep-Inelastic Scattering (DIS) and 1D Imaging

- Process: $e + N \to e + X$



- Independent variables (unpolarized cross section)

$$Q^2 = -q^2 > 0 \qquad x_{\rm B} = \frac{Q^2}{2P\cdot q} \qquad y \approx \frac{Q^2}{x_{\rm B} S} \ \text{(not independent)}$$

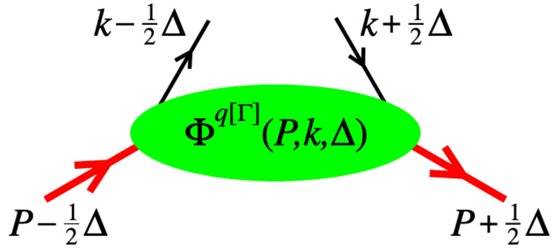
- Unpolarized DIS cross section in parton model

$$d\sigma_{\rm unp} = \sum_{q,\bar{q}} \int_0^1 dx\, f_1^q(x)\, d\hat{\sigma}_{eq\to eq}$$

$$\frac{d^2\hat{\sigma}_{eq\to eq}}{dx_{\rm B}dQ^2} = e_q^2 \frac{2\pi\alpha_{\rm em}}{Q^4}\left[1 + (1-y)^2\right]\delta(x - x_{\rm B})$$

PDFs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p + p') \qquad \Delta = p' - p$$

$$\Phi^q_{ij}(P,k,\Delta) = \int \frac{d^4z}{(2\pi)^4}\, \mathrm{e}^{ik\cdot z}\, \langle p'|\, \bar{\psi}^q_j(-\tfrac{z}{2})\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})\, |p\rangle$$

$$\Phi^{q[\Gamma]}(P,k,\Delta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, \mathrm{e}^{ik\cdot z}\, \langle p'|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})\, |p\rangle$$

- PDFs defined through

$$\Phi^{q[\Gamma]}_{\mathrm{PDF}}(P,x) = \int d^4k\; \Phi^{q[\Gamma]}(P,k,0)\; \delta(k^+ - xP^+)$$

$$= \int \frac{dz^-}{4\pi}\, \mathrm{e}^{ik\cdot z}\, \langle P|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})\, |P\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

PDFs of Quarks

- Including polarization for spin-$\frac{1}{2}$: $|P\rangle \to |P,S\rangle$

$$S = \left(\frac{\Lambda P^+}{M}, -\frac{\Lambda P^-}{M}, \vec{S}_\perp\right) \qquad S^2 = -\Lambda^2 - \vec{S}_\perp^{\,2} = -1 \qquad P \cdot S = 0$$

- Definition of leading-twist (twist-2) quark PDFs

$$f_1^q(x) = \Phi_{\mathrm{PDF}}^{q[\gamma^+]}(P,S,x)$$

$$= \int \frac{dz^-}{4\pi}\, \mathrm{e}^{ik\cdot z}\, \langle P,S|\, \bar{\psi}^q(-\tfrac{z}{2})\, \gamma^+\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})\, |P,S\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

$$\Lambda\, g_1^q(x) = \Phi_{\mathrm{PDF}}^{q[\gamma^+\gamma_5]}(P,S,x)$$

$$S_\perp^i\, h_1^q(x) = \Phi_{\mathrm{PDF}}^{q[i\sigma^{i+}\gamma_5]}(P,S,x) \qquad \left(\sigma^{\mu\nu} = \tfrac{i}{2}[\gamma^\mu,\gamma^\nu]\right)$$

  – general structure of qq-correlator follows from parity, hermiticity, time-reversal

  – PDFs depend on renormalization scale μ, which has been suppressed

- More on polarization dependence
  - quarks with definite helicity/chirality

$$P_\lambda = \frac{1}{2}(1 + \lambda\gamma_5) \quad (\lambda = \pm 1) \qquad P_\lambda^2 = P_\lambda \qquad P_+ P_- = P_- P_+ = 0$$

$$\psi_\lambda = P_\lambda \psi \qquad \bar{\psi}_\lambda = \bar{\psi} P_{-\lambda}$$

  - rewriting of operator for f_1^q

$$\bar{\psi}\gamma^+\psi = \bar{\psi}_+\gamma^+\psi_+ + \bar{\psi}_-\gamma^+\psi_-$$

$\rightarrow f_1^q$ describes sum of two densities

  - rewriting of operator for g_1^q

$$\bar{\psi}\gamma^+\gamma_5\psi = \bar{\psi}_+\gamma^+\psi_+ - \bar{\psi}_-\gamma^+\psi_-$$

$\rightarrow g_1^q$ describes difference of two densities (can become negative)

  - h_1 describes difference of two densities for transverse quark polarization

$$\bar{\psi}\, i\sigma^{i+}\gamma_5\psi = \bar{\psi}_+ i\sigma^{i+}\gamma_5\psi_- + \bar{\psi}_- i\sigma^{i+}\gamma_5\psi_+$$

$\rightarrow h_1^q$ is chiral odd (decouples from DIS)

- Interpretation as number densities using light-front quantization
- Density interpretation spoiled by QCD effects (radiative corrections)
- g_1^q and h_1^q can also be interpreted as strength of spin-spin correlations

$$\lambda\, \Phi_{\mathrm{PDF}}^{q[\gamma^+\gamma_5]}(P,S,x) = \lambda\, \Lambda\, g_1^q(x)$$

$$s_\perp^i\, \Phi_{\mathrm{PDF}}^{q[i\sigma^{i+}\gamma_5]}(P,S,x) = \vec{s}_\perp \cdot \vec{S}_\perp\, h_1^q(x)$$

- Decomposing the qq-correlator in terms of PDFs

$$\Phi^q = \frac{1}{2}\,\Phi^{q[\gamma^+]}\,\gamma^- - \frac{1}{2}\,\Phi^{q[\gamma^+\gamma_5]}\,\gamma^-\gamma_5 + \frac{1}{2}\,\Phi^{q[i\sigma^{i+}\gamma_5]}\,i\sigma^{i-}\gamma_5 + \ldots$$

  - all other terms suppressed for large P^+

- Leading-twist PDFs of gluons $\big(\langle P|F^{+i}\,F^{+j}|P\rangle\big)$

$$f_1^g(x) = g(x) \quad \to \text{unpolarized gluon PDF}$$

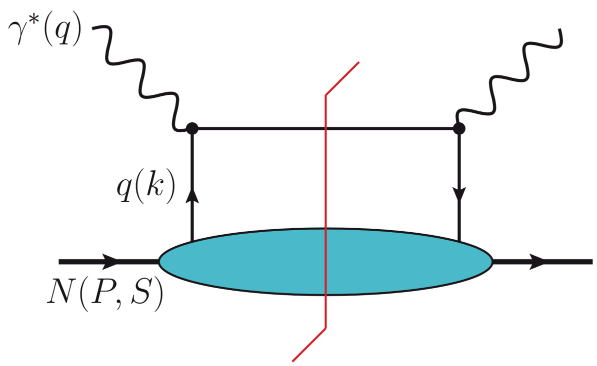
$$g_1^g(x) = \Delta g(x) \to \text{gluon helicity PDF}$$

Inclusive DIS with qq-Correlator

- Process

$$e(l,\lambda_\ell) + N(P,S) \to e(l',\lambda'_\ell) + X$$

- Handbag diagram



- Cross section

$$d\sigma \sim L_{\mu\nu}\, W^{\mu\nu}$$

- Leptonic tensor

$$L^{\mu\nu} = \Big[\bar{u}(l',\lambda'_\ell)\,\gamma^\nu\, u(l,\lambda_\ell)\Big]\Big[\bar{u}(l',\lambda'_\ell)\,\gamma^\mu\, u(l,\lambda_\ell)\Big]^*$$

$$= 2\Big(l^\mu l'^\nu + l'^\mu l^\nu - l\cdot l' g^{\mu\nu}\Big) + 2\,i\,\lambda_\ell\,\varepsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma$$

- Hadronic tensor (neglect m_q)

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k \, \mathrm{Tr}\left[\Phi^q(k)\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)$$

$$= \sum_q e_q^2 \int d^4k \, \Phi^{q[\gamma^+]}(k)\,\frac{1}{2}\,\mathrm{Tr}\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big) + \ldots$$

$$= \sum_q \frac{e_q^2}{2} \int d^4k \, \Phi^{q[\gamma^+]}(k)\,\mathrm{Tr}\left(\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)\right)\Big|_{k^-=\vec{k}_\perp=0} + \ldots$$

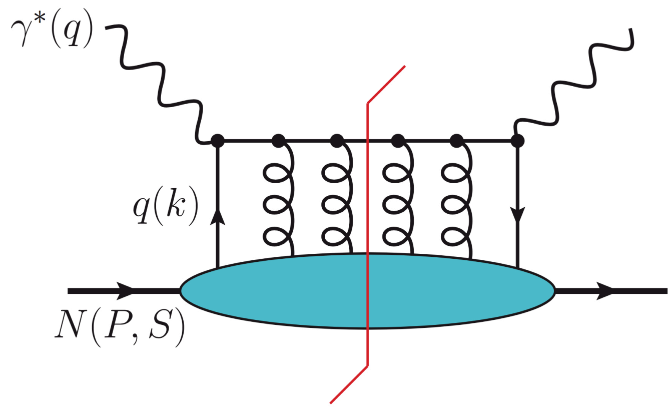
$$= \sum_q \frac{e_q^2}{2} \int dk^+ \, \Phi^{q[\gamma^+]}(x)\,\mathrm{Tr}\left(\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)\right)\Big|_{k^-=\vec{k}_\perp=0}$$

$$= \sum_q \frac{e_q^2}{2} \int \frac{dk^+}{k^+} \, \underbrace{\Phi^{q[\gamma^+]}(x)}_{f_1^q(x)}\,\mathrm{Tr}\left(\left[\not{k}\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\underbrace{\delta\big((k+q)^2\big)}_{\frac{x_B}{Q^2}\,\delta(x-x_B)}\right)\Big|_{k^-=\vec{k}_\perp=0} + \ldots$$

$$= \frac{1}{Q^2}\sum_q \frac{e_q^2}{2}\, f_1^q(x_B)\,\mathrm{Tr}\left(\left[\not{k}\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\right)\Big|_{k^+=x_B P^+,\; k^-=\vec{k}_\perp=0}$$

– result agrees with the one obtained in parton model

- Handbag diagram including re-scattering of quark



  - including re-scattering graphs (to all orders) renders gauge-invariant correlator

$$\Phi^q_{ij}(P,S,x) = \int \frac{dz^-}{2\pi}\, \mathrm{e}^{ik\cdot z}\, \langle P,S|\, \bar\psi^q_j(-\tfrac{z}{2})\, \mathcal{W}_{\mathrm{PDF}}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})\, |P,S\rangle\Big|_{z^+=\vec z_\perp=0}$$

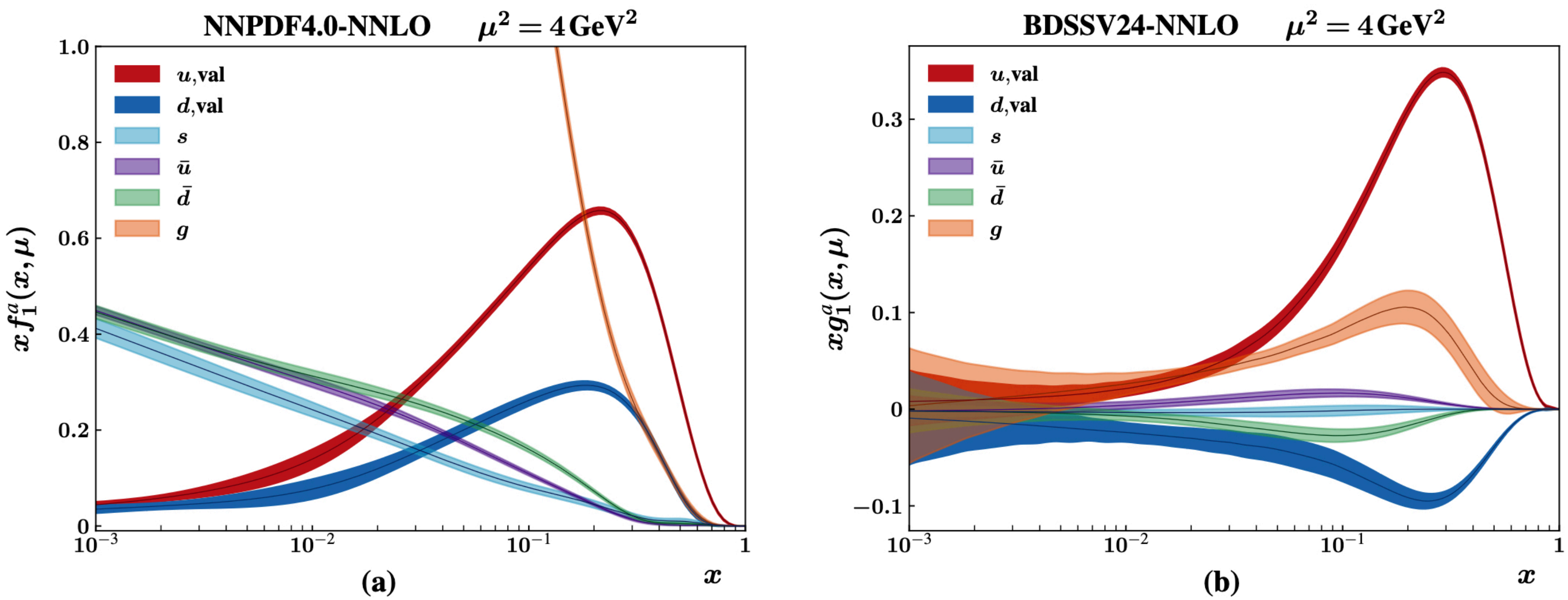
$$\mathcal{W}[a,b] = \mathcal{P}\exp\Big(ig\int_a^b dy^\mu A_\mu(y)\Big)$$

  - in other words: $\mathcal{W}_{\mathrm{PDF}}$ generated by final-state interaction (FSI) of active quark
  - path of $\mathcal{W}_{\mathrm{PDF}}$ is straight line $\rightarrow$ $\mathcal{W}_{\mathrm{PDF}} = \mathbb{1}$ for $A^+ = 0$ (light-cone gauge)

PDFs: Results from Experimental Data

- Unpolarized and helicity PDFs of the proton

(figures from 2507.12664)

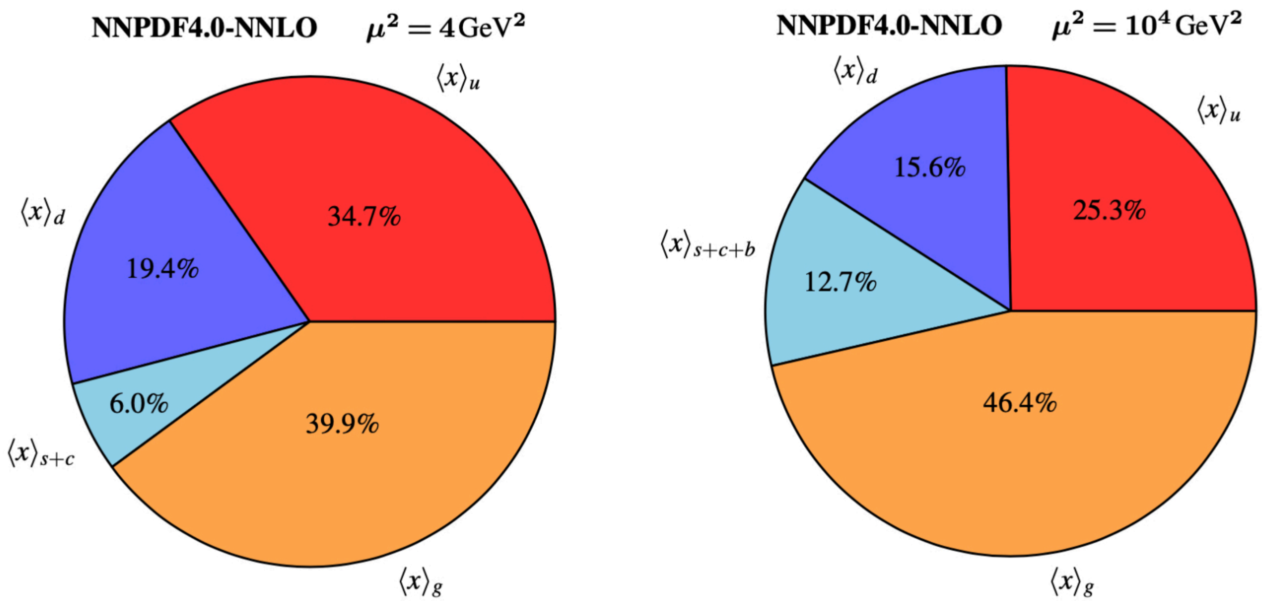


  - valence distributions: $f_1^{u,\mathrm{val}} = f_1^u - f_1^{\bar{u}} \qquad f_1^{d,\mathrm{val}} = f_1^d - f_1^{\bar{d}}$
  - PDFs depend on renormalization scheme ($\overline{\mathrm{MS}}$) and renormalization scale μ
  - unpolarized PDFs rather well known; helicity PDFs have larger uncertainties

- Parton momentum fractions

$$\int_{-1}^{1} dx\, x\, f_1^q(x,\mu) = \langle x\rangle_q(\mu) \qquad \int_{0}^{1} dx\, x\, f_1^g(x,\mu) = \langle x\rangle_g(\mu) \qquad \sum_a \langle x\rangle_a(\mu) = 1$$

(figures from 2507.12664)

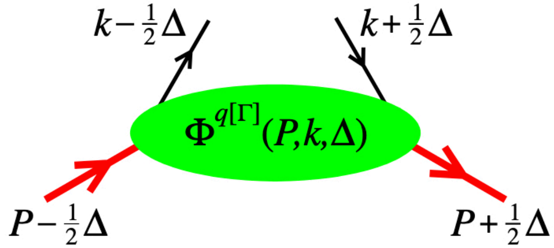


– asymptotic values

$$\lim_{\mu\to\infty} \frac{\langle x\rangle_q(\mu)}{\langle x\rangle_g(\mu)} = \frac{3}{16} \quad \to \quad \langle x\rangle_q(\infty) \approx 8.8\% \qquad \langle x\rangle_g(\infty) \approx 47.1\%$$

TMDs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p+p') \qquad \Delta = p' - p$$

$$\Phi^{q[\Gamma]}(P,k,\Delta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p'|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})\, |p\rangle$$

- TMDs defined through

$$\Phi^{q[\Gamma]}_{\mathrm{TMD}}(P,x,\vec{k}_\perp) = \int dk^+\, dk^-\; \Phi^{q[\Gamma]}(P,k,0)\; \delta(k^+ - xP^+)$$

$$= \int \frac{dz^-\, d^2\vec{z}_\perp}{2(2\pi)^3}\, e^{ik\cdot z}\, \langle P|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})\, |P\rangle\Big|_{z^+=0}$$

TMDs of Quarks

- Definition of leading-power (leading-twist) quark TMDs
  (Mulders, Tangerman, 1995 / Boer, Mulders, 1997 / Bacchetta et al, 2006)

$$\Phi^{q[\gamma^+]}_{\mathrm{TMD}}(x,\vec{k}_\perp) = f_1^q - \frac{\varepsilon_\perp^{ij}\,k_\perp^i\,S_\perp^j}{M}\,f_{1T}^{\perp q}$$

$$\lambda\,\Phi^{q[\gamma^+\gamma_5]}_{\mathrm{TMD}}(x,\vec{k}_\perp) = \lambda\,\Lambda\,g_1^q + \frac{\lambda\,\vec{k}_\perp\cdot\vec{S}_\perp}{M}\,g_{1T}^{\perp q}$$

$$s_\perp^i\,\Phi^{q[i\sigma^{i+}\gamma_5]}_{\mathrm{TMD}}(x,\vec{k}_\perp) = \vec{s}_\perp\cdot\vec{S}_\perp\,h_1^q + \frac{\Lambda\,\vec{k}_\perp\cdot\vec{s}_\perp}{M}\,h_{1L}^{\perp q} - \frac{\varepsilon_\perp^{ij}\,k_\perp^i\,s_\perp^j}{M}\,h_1^{\perp q} + \frac{1}{2M^2}\Big(2\,\vec{k}_\perp\cdot\vec{s}_\perp\;\vec{k}_\perp\cdot\vec{S}_\perp - \vec{k}_\perp^{\,2}\,\vec{s}_\perp\cdot\vec{S}_\perp\Big)\,h_{1T}^{\perp q}$$

  - TMDs depend on x and $\vec{k}_\perp^{\,2}$ $\rightarrow$ forward limit is readily recovered
  - general structure of qq-correlator follows from parity, hermiticity, time-reversal
  - TMDs depend on two auxiliary scales (μ,ζ)
  - 8 leading-power gluon TMDs

- Quark TMDs have names

  $f_{1T}^{\perp q}$: Sivers function (Sivers, 1989)

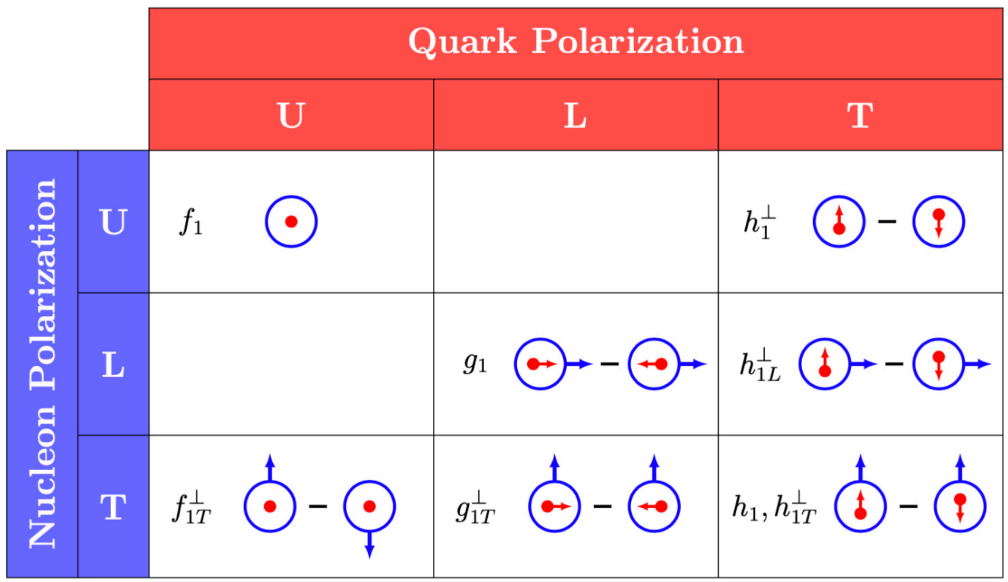
  $h_1^{\perp q}$: Boer-Mulders function (Boer, Mulders, 1997)

  $g_{1T}^{\perp q}$ $h_{1L}^{\perp q}$: worm-gear functions (polarization of hadron and quark perpendicular)

  $h_{1T}^{\perp q}$: pretzelosity (quadrupole pattern of pre-factor)

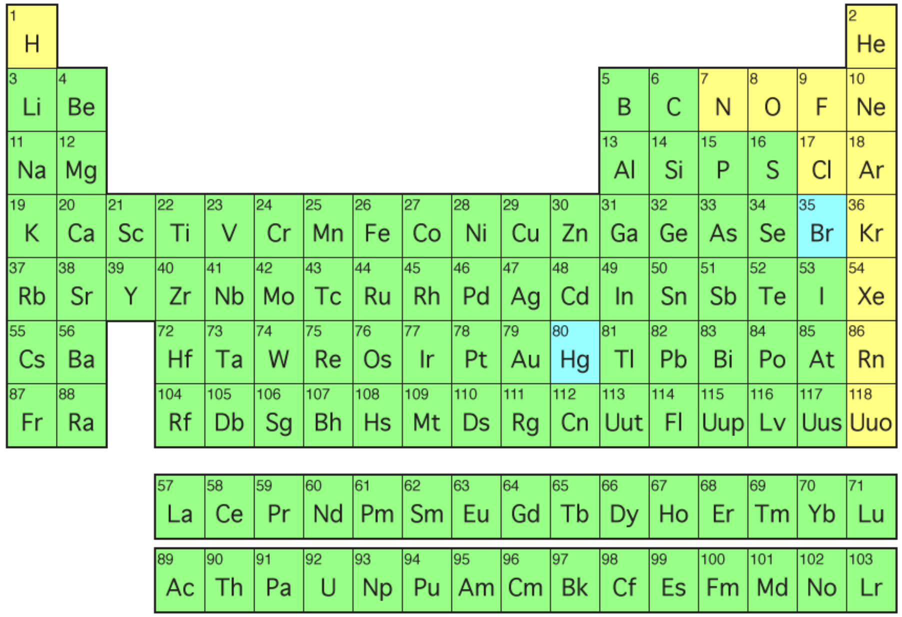
- Overview

(figure from 2507.12664)



  – no TMD for U/L and L/U polarizations due to parity invariance

- “Stamp collecting”? ... maybe ... but we are in good company
  - periodic table of elements



  don’t forget the isotopes ...

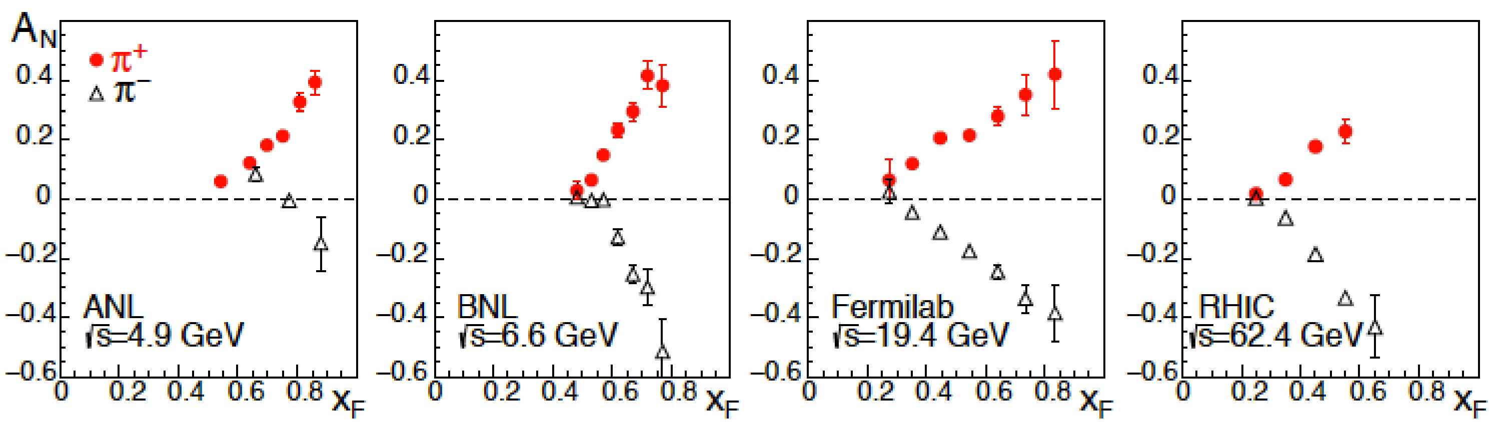
  - (supersymmetric) extensions of the Standard Model
  - materials science
  - etc.

- Sivers function: a closer look
  - density of unpolarized quarks in (transversely) polarized nucleon

$$\Phi^{q[\gamma^+]}_{\mathrm{TMD}}(P,S,x,\vec{k}_\perp) = f_1^q(x,\vec{k}_\perp^{\,2}) - \frac{(\vec{k}_\perp \times \vec{S}_\perp)\cdot \hat{P}}{M}\, f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2})$$

  - $f_{1T}^{\perp q}$ describes difference of two densities for transverse nucleon polarization
  - $f_{1T}^{\perp q}$ can generate transverse single-spin asymmetries (SSAs) in scattering processes
  - observed large transverse SSAs were motivation for Sivers to explore this effect

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \sim \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$



(figure from Aidala et al, 1209.2803)

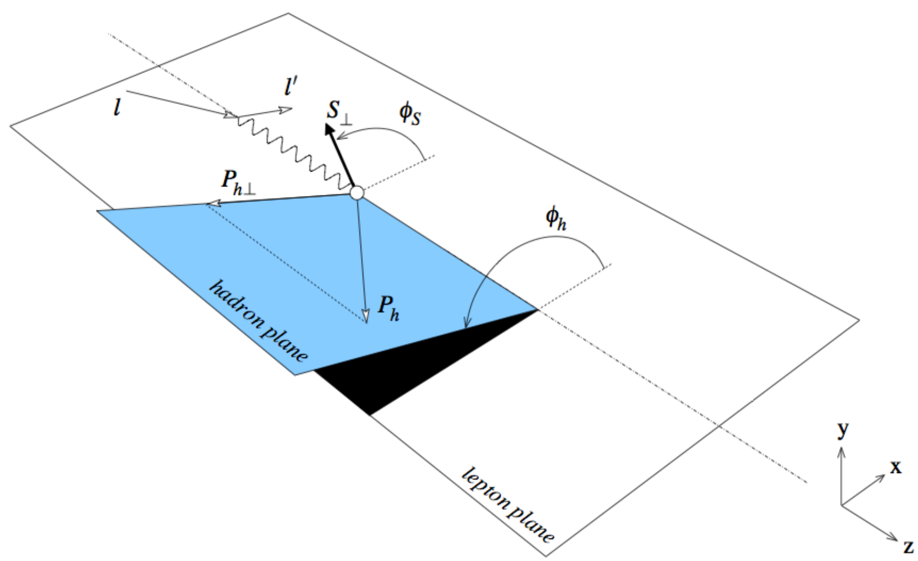
TMDs in Semi-Inclusive DIS (SIDIS)

- Process

$$\ell(l, \lambda_\ell) + N(P, S) \to \ell(l', \lambda'_\ell) + h(P_h) + X$$

- 6 independent kinematic variables

$$Q^2 \qquad x_B = \frac{Q^2}{2P \cdot q} \qquad \phi_S \qquad z_h = \frac{P \cdot P_h}{P \cdot q} \qquad P_{h\perp} = |\vec{P}_{h\perp}| \qquad \phi_h$$



(figure from Bacchetta et al, hep-ph/0611265)

- Model-independent form of cross section (in notation of hep-ph/0611265)

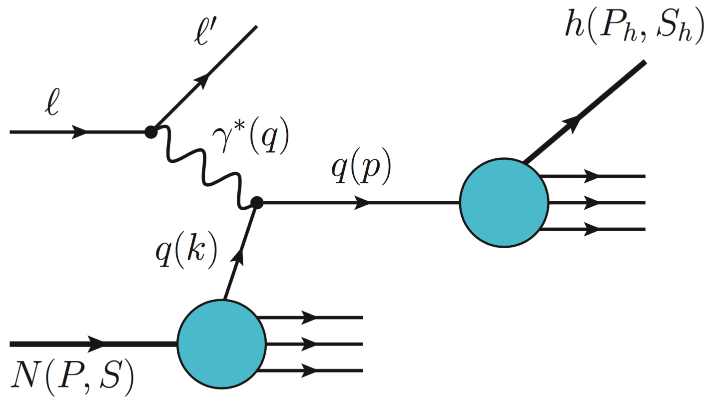
$$
\begin{aligned}
\frac{d\sigma}{dx_{\mathrm{B}}\,dy\,d\phi_S\,dz_h\,d\phi_h\,dP_{h\perp}^2} \sim \Big\{ & \left(1-y+\tfrac{1}{2}y^2\right) F_{UU,T} + (1-y)\,\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
& + \Lambda\,(1-y)\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h} + \lambda_\ell\,\Lambda\,y\left(1-\tfrac{1}{2}y\right) F_{LL} \\
& + |\vec{S}_\perp|\left(1-y+\tfrac{1}{2}y^2\right)\,\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} \\
& + |\vec{S}_\perp|\left(1-y\right)\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
& + |\vec{S}_\perp|\left(1-y\right)\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& + \lambda_\ell\,|\vec{S}_\perp|\,y\left(1-\tfrac{1}{2}y\right)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + 10 \text{ additional terms} \Big\}
\end{aligned}
$$

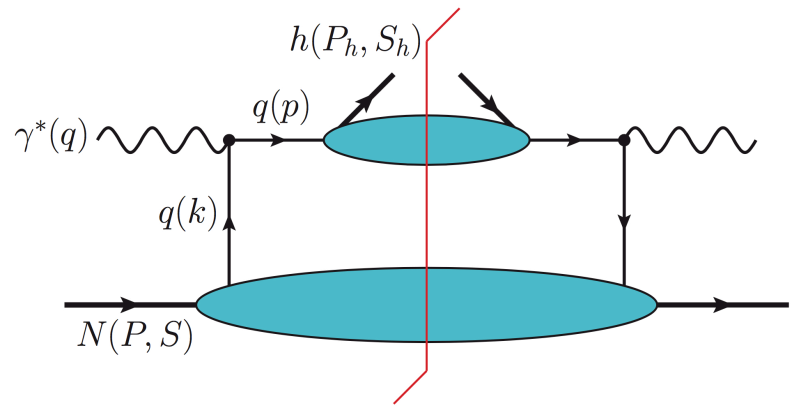
  - structure functions depend on 4 variables:

$$F_i = F_i(x_{\mathrm{B}}, z_h, P_{h\perp}^2, Q^2)$$

  - at low $P_{h\perp}$, leading contribution to $d\sigma$ given by 8 listed structure functions

- Feynman diagram at tree level





$$d\sigma \sim d\hat{\sigma}_{\rm pert} \otimes \Phi \otimes \Delta$$

- factorization formula depends on kinematics:
  1. cross section integrated upon $P_{h\perp}$
  2. cross section differential in $P_{h\perp}$, and $P_{h\perp} \sim Q$
  3. cross section differential in $P_{h\perp}$, and $P_{h\perp} \ll Q \rightarrow$ realm of TMDs

- full description of differential cross section from small to large $P_{h\perp}$ still area of active research (see, e.g., Collins et al, 1605.00671)

- Hadronic tensor at tree level

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k\, d^4p\, \delta^{(4)}(k+q-p)\, \mathrm{Tr}\left[\Phi^q(k)\,\gamma^\mu\,\Delta^q(p)\,\gamma^\nu\right]$$

  - consider P^+ and P_h^- large, as well as $k^+ = xP^+$ and $P_h^- = zp^-$
  - consider frame with $\vec{P}_{h\perp} = 0$, and small $\vec{q}_\perp \neq 0$
  - neglect small light-cone components of parton momenta k^- and p^+ in delta-function (approximation for TMD parton model)

$$W^{\mu\nu} \sim \frac{2\,x_\mathrm{B}\,z_h}{Q^2} \sum_q e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)$$
$$\times\, \mathrm{Tr}\left[\Phi^q(x_\mathrm{B}, \vec{k}_\perp)\,\gamma^\mu\,\Delta^q(z_h, \vec{p}_\perp)\,\gamma^\nu\right]$$

  - express Φ^q and Δ^q in terms of TMD-PDFs and TMD-FFs, respectively
  - transverse parton momenta of TMD-PDFs and TMD-FFs are convoluted
  - contract with leptonic tensor $L^{\mu\nu}$
  - compare with model-independent form of cross section to find tree-level results for structure functions F_i

- Structure functions at tree level (e.g., hep-ph/0611265)

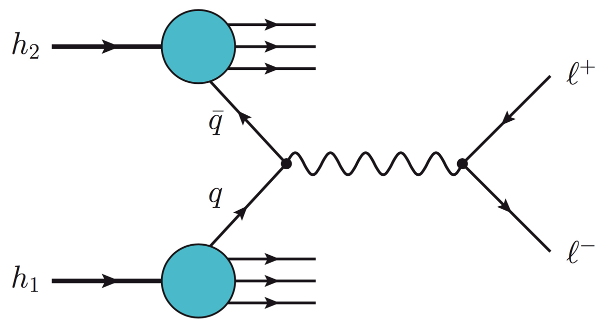
$$
\begin{aligned}
F_{UU,T} &= x_{\mathrm{B}} \sum_q e_q^2 \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1^q(x_{\mathrm{B}}, \vec{k}_\perp^{\,2}) \, D_1^q(z_h, \vec{p}_\perp^{\,2}) \\
F_{UU}^{\cos 2\phi_h} &\sim h_1^\perp \otimes H_1^\perp \\
F_{UL}^{\sin 2\phi_h} &\sim h_{1L}^\perp \otimes H_1^\perp \\
F_{LL} &\sim g_1 \otimes D_1 \\
F_{UT,T}^{\sin(\phi_h - \phi_S)} &\sim f_{1T}^\perp \otimes D_1 \qquad \text{[Sivers effect]} \\
F_{UT}^{\sin(\phi_h + \phi_S)} &\sim h_1 \otimes H_1^\perp \qquad \text{[Collins effect]} \\
F_{UT}^{\sin(3\phi_h - \phi_S)} &\sim h_{1T}^\perp \otimes H_1^\perp \\
F_{LT}^{\cos(\phi_h - \phi_S)} &\sim g_{1T}^\perp \otimes D_1
\end{aligned}
$$

  - except for $F_{UU,T}$ expressions are symbolic; in most cases convolutions contain additional powers of transverse parton momenta
  - all 8 TMD-PDFs can be studied
  - all 8 structure functions have been measured

TMDs in Other Processes

1. Drell-Yan process: $h_1 + h_2 \to \ell^+ + \ell^- + X$

- Feynman diagram at tree level



- Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

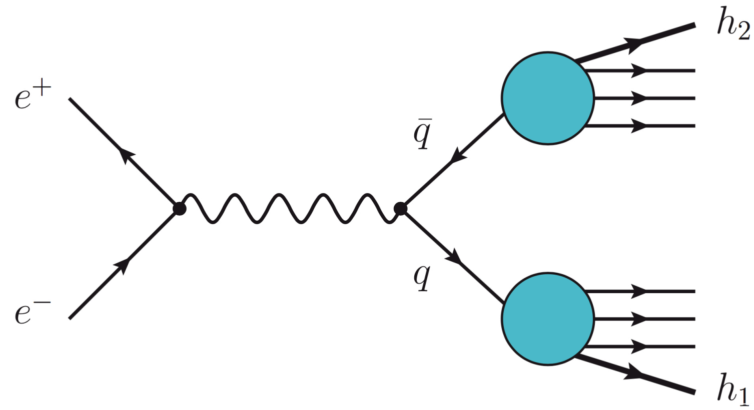
$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{k}_{a\perp}\, d^2\vec{k}_{b\perp}\, \delta^{(2)}(\vec{k}_{a\perp} + \vec{k}_{b\perp} - \vec{q}_\perp)$$
$$\times \mathrm{Tr}\left[\Phi^q(x_a, \vec{k}_{a\perp})\, \gamma^\mu\, \Phi^{\bar{q}}(x_b, \vec{k}_{b\perp})\, \gamma^\nu\right]$$

  - sensitivity to "product" of two TMD-PDFs
  - longitudinal momentum fractions x_a and x_b fixed by kinematics of reaction

- 48 structure functions; cross checks possible for various TMDs (Arnold, A.M., Schlegel, 2008)

2. Electron-positron annihilation: $e^+ + e^- \to h_1 + h_2 + X$

- Feynman diagram at tree level



- Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{p}_{a\perp}\, d^2\vec{p}_{b\perp}\, \delta^{(2)}(\vec{p}_{a\perp} + \vec{p}_{b\perp} - \vec{q}_\perp)$$

$$\times \operatorname{Tr}\left[\Delta^q(z_a, \vec{p}_{a\perp})\, \gamma^\mu\, \Delta^{\bar{q}}(z_b, \vec{p}_{b\perp})\, \gamma^\nu\right]$$

– sensitivity to "product" of two TMD-FFs

– longitudinal momentum fractions z_a and z_b fixed by kinematics of reaction

3. Some additional processes

(a) $\ell\, N \to \ell\, \text{jet}\, \text{jet}\, X$ $\qquad$ $\ell\, N \to \ell\, J/\psi\, X$

(b) $p\, p \to \gamma\, \gamma\, X$ $\qquad$ $p\, p \to \gamma\, \text{jet}\, X$ $\qquad$ $p\, p \to \text{jet}\, \text{jet}\, X$

(c) $p\, p \to (h\, \text{jet})\, X$

(d) $p\, p \to J/\psi\, X$ $\qquad$ $p\, p \to \eta_c\, X$ $\qquad$ $p\, p \to \text{Higgs}\, X$

(e) $p\, A$-collisions

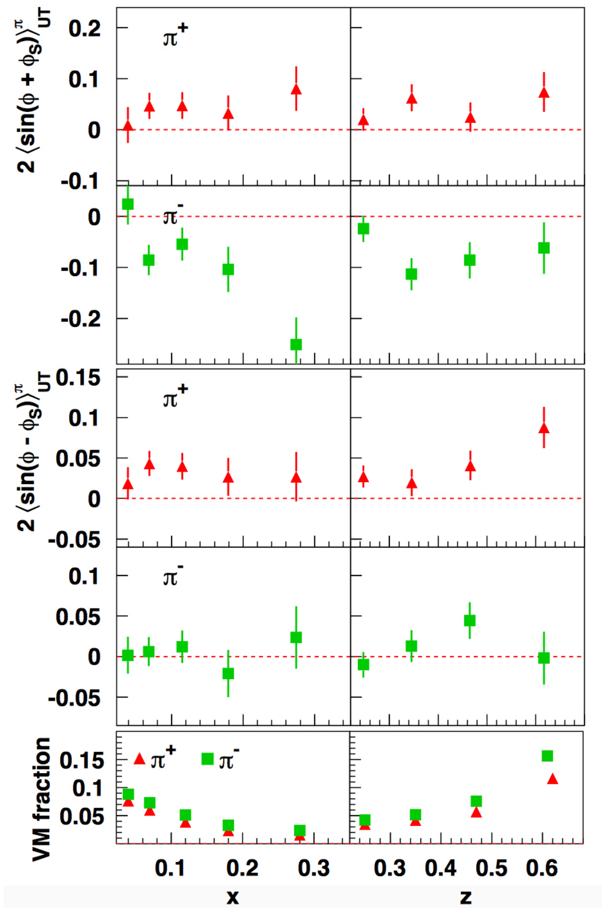
(f) etc.

- Very rich phenomenology

- Status of TMD factorization for additional processes:
  - holds in some cases (according to current knowledge)
  - breaks down in some cases
  - unclear in some cases (further studies needed) $\to$ active research area

Measurements of TMD Observables

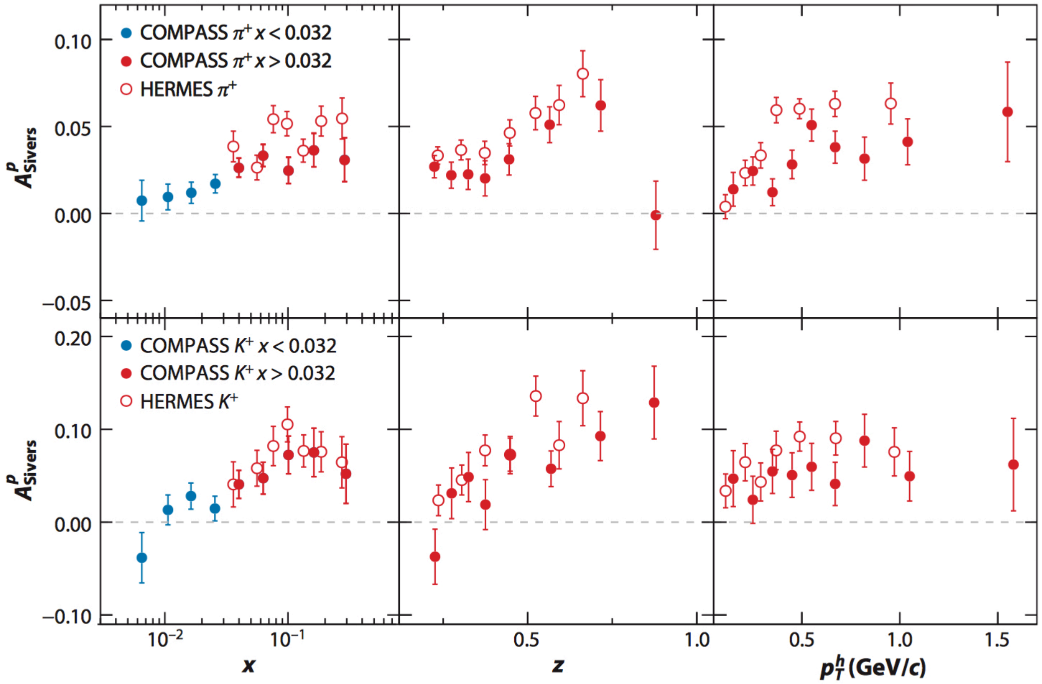
1. Pioneering steps for Sivers and Collins effects (HERMES, hep-ph/0408013)



- hydrogen target
- first measurement of Sivers asymmetry, $F_{UT,T}^{\sin(\phi_h-\phi_S)}/F_{UU,T}$
- first measurement of Collins asymmetry, $F_{UT}^{\sin(\phi_h+\phi_S)}/F_{UU,T}$
- nonzero Sivers effect for π^+
- nonzero Collins effect for π^+ and π^-
- in the meantime, many more data from COMPASS, HERMES, JLab, with hydrogen, deuterium, and ^{3}He targets

2. More SIDIS data on Sivers effect

Comparison of data from COMPASS and HERMES



(compilation from Grosse Perdekamp, Yuan, 1510.06783)

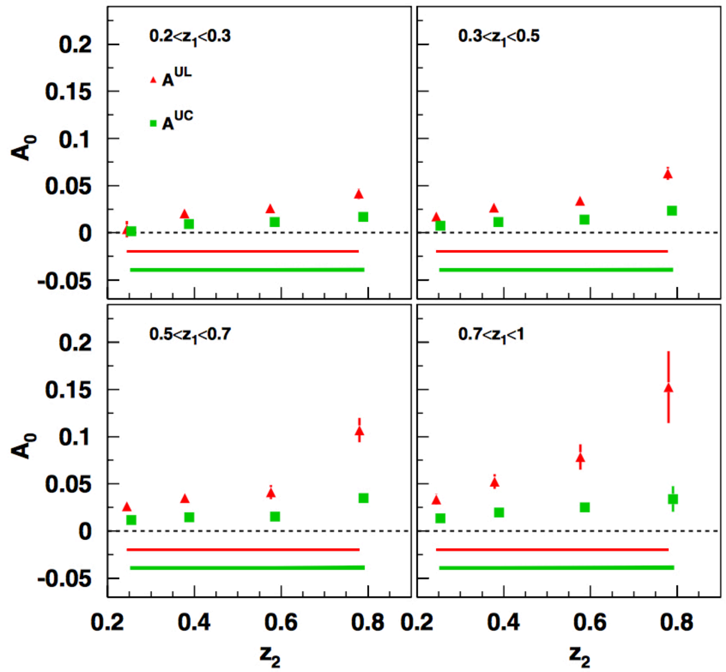
- measurements with hydrogen target, for π^+ and K^+ production
- results from COMPASS and HERMES largely agree
- robust nonzero effects
- asymmetry for K^+ at least as large as for π^+

3. (Double) Collins effect in electron-positron annihilation: $e^+e^- \to h_1 h_2 X$

- Azimuthal modulation due to Collins effect (Boer, Jakob, Mulders, hep-ph/9702281)

$$d\sigma \sim \sum_q e_q^2 \left[D_1^{h_1/q} \otimes D_1^{h_2/\bar{q}} + \cos(2\phi)\, H_1^{\perp\, h_1/q} \otimes H_1^{\perp h_2/\bar{q}} \right]$$

- Sample data (Belle, 0805.2975)

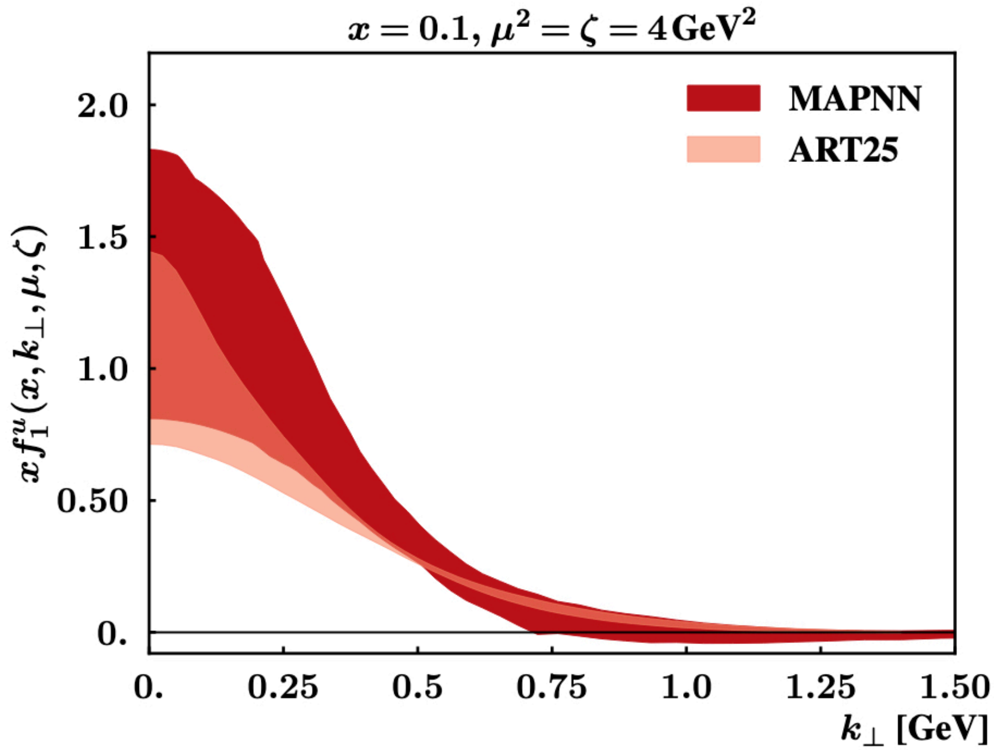


- More data available, also from BaBar and BES

TMD Extractions from Data

1. Unpolarized TMD f_1^q

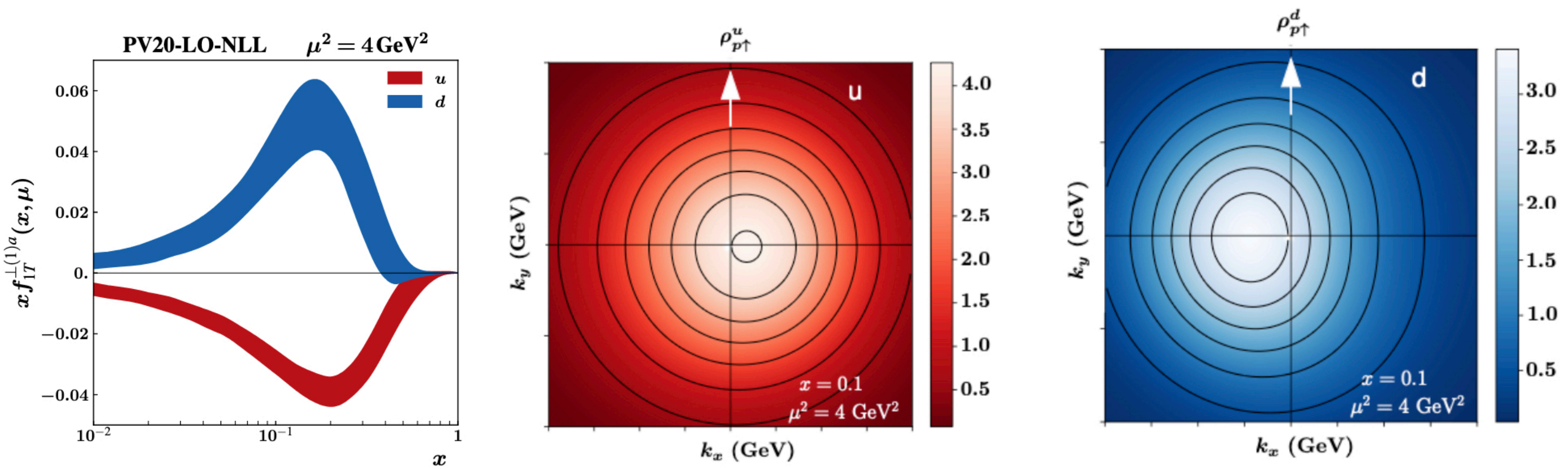
(figure from 2507.12664)



- MAPNN (Bacchetta et al, 2502.04166): DY data only, and neural network approach
- ART25 (Moos et al, 2503.11201): DY and SIDIS data

2. Sivers function $f_{1T}^{\perp q}$

(figure from 2507.12664)



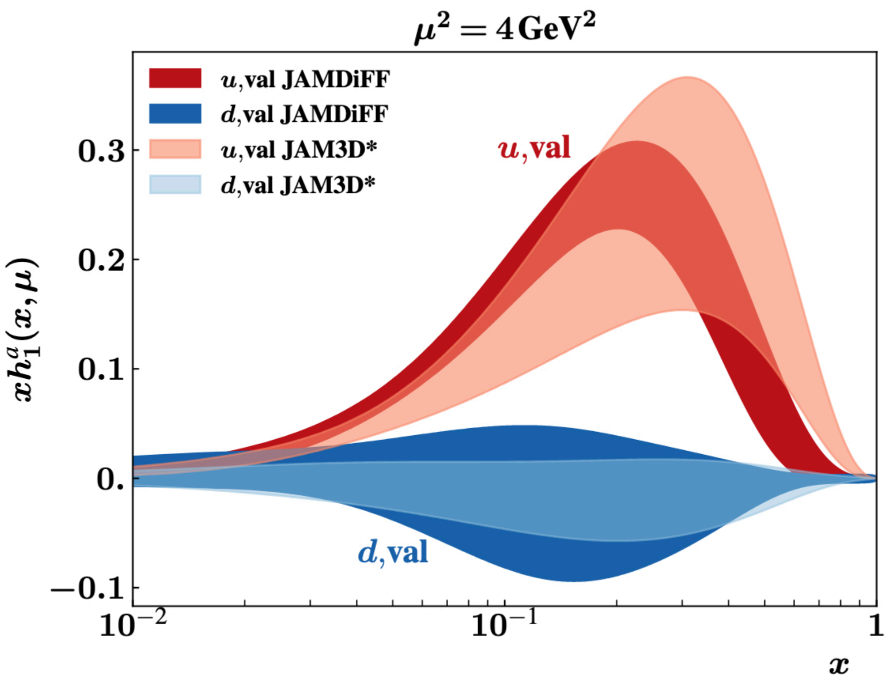
$$f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^{\,2}}{2M^2} \, f_{1T}^{\perp q}(x, \vec{k}_\perp^{\,2})$$

- PV20 (Bacchetta et al, 2004.14278): SIDIS and DY data for Sivers effect
- Nice agreement with large-N_c prediction: $f_{1T}^{\perp u} = - f_{1T}^{\perp d} + \mathcal{O}(1/N_c)$
  (Pobylitsa, hep-ph/0301236)
- Density of unpolarized quarks in (transversely) polarized nucleon

$$\Phi^{q[\gamma^+]}_{\text{TMD}}(P,S,x,\vec{k}_\perp) = f_1^q(x,\vec{k}_\perp^{\,2}) - \frac{(\vec{k}_\perp \times \vec{S}_\perp)\cdot\hat{P}}{M} f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2})$$

3. Transversity distribution h_1^q

(figure from 2507.12664)



- Transversity distribution in SIDIS via single-hadron and dihadron production

$$A_{UT}^{h} \sim h_1(x, \vec{k}_\perp^{\,2}) \otimes H_1^\perp(z, \vec{p}_\perp^{\,2}) \qquad A_{UT}^{hh} \sim h_1(x) \otimes H_1^\sphericalangle(z, M_h)$$

- JAM3D* (Gamberg et al, 2205.00999): Data on single-hadron production
- JAMDiFF (Cocuzza et al, 2306.12998): Data on dihadron production

Some Remarks on the Evolution of TMDs

- Further reading, for instance:
  - Collins, *Foundations of perturbative QCD*, 2011
  - Rogers, 1509.04766
  - TMD Handbook, 2304.03302

- In QCD, TMDs depend on two (auxiliary) scales

$$f_1^q(x, \vec{k}_\perp^{\,2}; \mu, \zeta)$$

  - μ: due to UV divergence
  - ζ: due to rapidity divergence (different regulators in use)

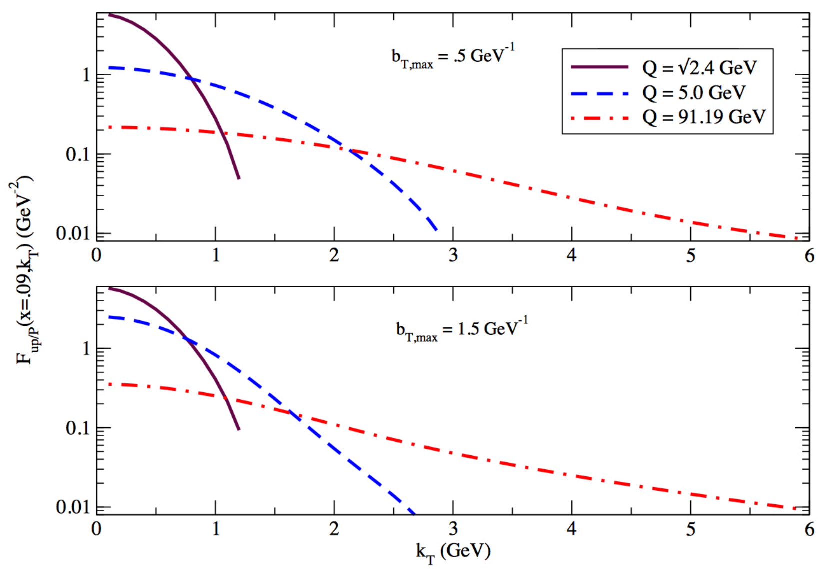
- Rapidity divergence complicates the relation between TMDs and PDFs
  - after regulating rapidity divergence one has

$$f_1^q(x, \mu) \neq \int d^2\vec{k}_\perp \, f_1^q(x, \vec{k}_\perp^{\,2}; \mu, \zeta)$$

  - rapidity divergence cancels for PDF $f_1(x, \mu)$ (between real and virtual diagrams) (e.g., Collins, hep-ph/0304122)

- Evolution of TMDs
  - μ-dependence: differs from DGLAP evolution
  - ζ-dependence: governed by Collins-Soper evolution equation (Collins, Soper, 1981)
  - TMD evolution has also dependence on non-perturbative input
    $\rightarrow$ progress in phenomenology and lattice QCD (e.g., Avkhadiev et al, 2307,12359)

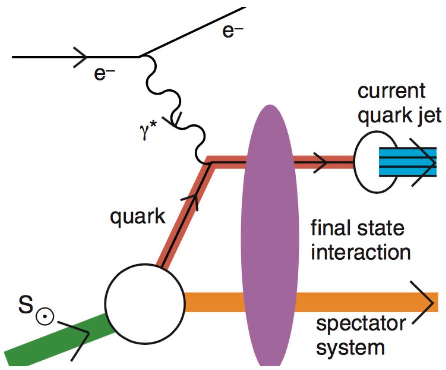
- Numerical study (Aybat, Rogers, 1101.5057)



  - evolution broadens TMDs
  - evolution typically dilutes effects like Sivers asymmetry

Eventful History of the Sivers Function

- In 1989, Dennis Sivers suggested the function (correlation)
- In 1992, John Collins argued that $f_{1T}^{\perp q} = 0$ due to T-reversal invariance
- Model calculation of transverse SSA in DIS (Brodsky, Hwang, Schmidt, hep-ph/0201296)



  - spectator system modeled by scalar diquark
  - FSI modeled by single photon exchange
  - nonzero transverse target SSA A_{UT}
  - A_{UT} given by interference of lowest-order graph and (imaginary part of) box graph

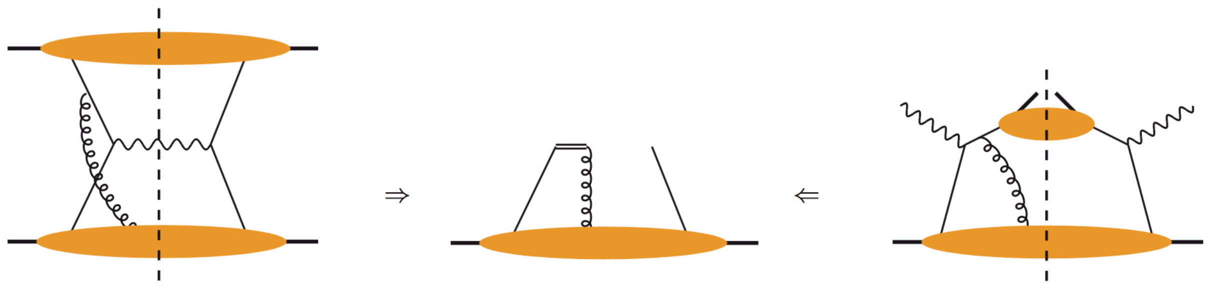
- Interpretation of BHS calculation, correction, prediction (Collins, hep-ph/0204004)
  - nonzero A_{UT} of BHS can be described in TMD factorization using $f_{1T}^{\perp q}$
  - if $\mathcal{W}_{\text{TMD}}$ taken into account, T-reversal does not forbid existence of $f_{1T}^{\perp q}$
  - T-reversal rather predicts process dependence:

$$f_{1T}^{\perp q}\big|_{\text{DY}} = -\, f_{1T}^{\perp q}\big|_{\text{SIDIS}} \qquad\qquad h_{1}^{\perp q}\big|_{\text{DY}} = -\, h_{1}^{\perp q}\big|_{\text{SIDIS}}$$

- These developments were of utmost importance for entire TMD field

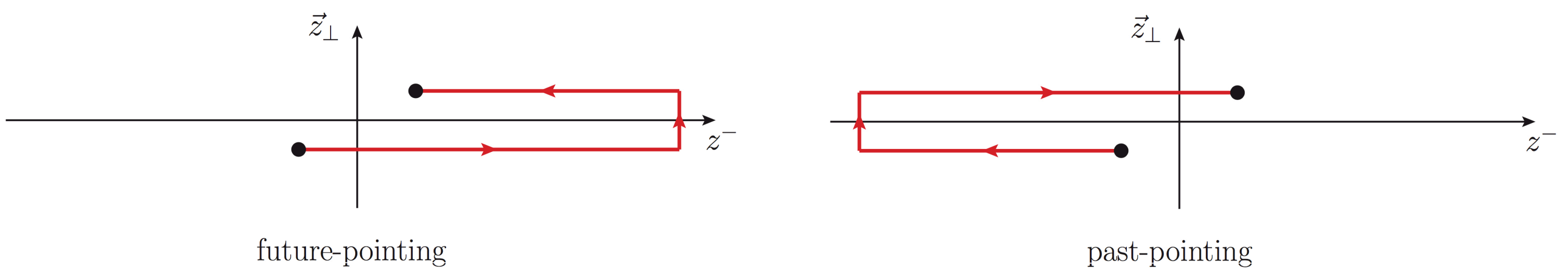
Process Dependence of the Sivers Function

1. Comparing SIDIS and DY



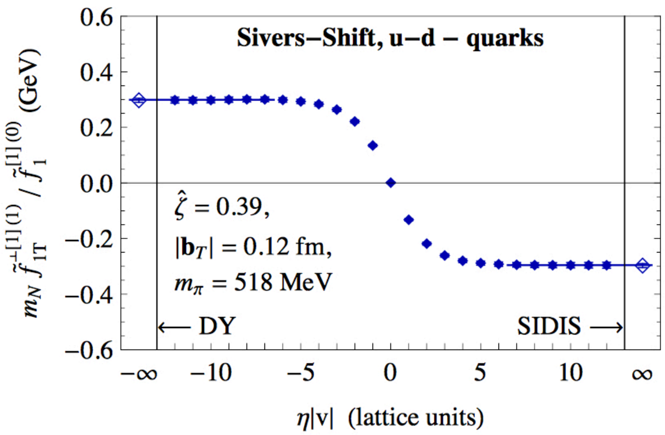
(figure from Diehl, 1512.01328)

- Gauge link structure in SIDIS and DY (FSI vs ISI)



- T-reversal allows one to relate definitions in two processes (Collins, hep-ph/0204004)
  - Six T-even TMDs are universal
  - Two T-odd TMDs change sign between SIDIS and DY
  - breakdown of universality, but in well-defined way
  - strictly speaking, T-odd TMDs not exclusively property of nucleon

- Sign reversal in lattice QCD (Musch et al, 1111.4249)



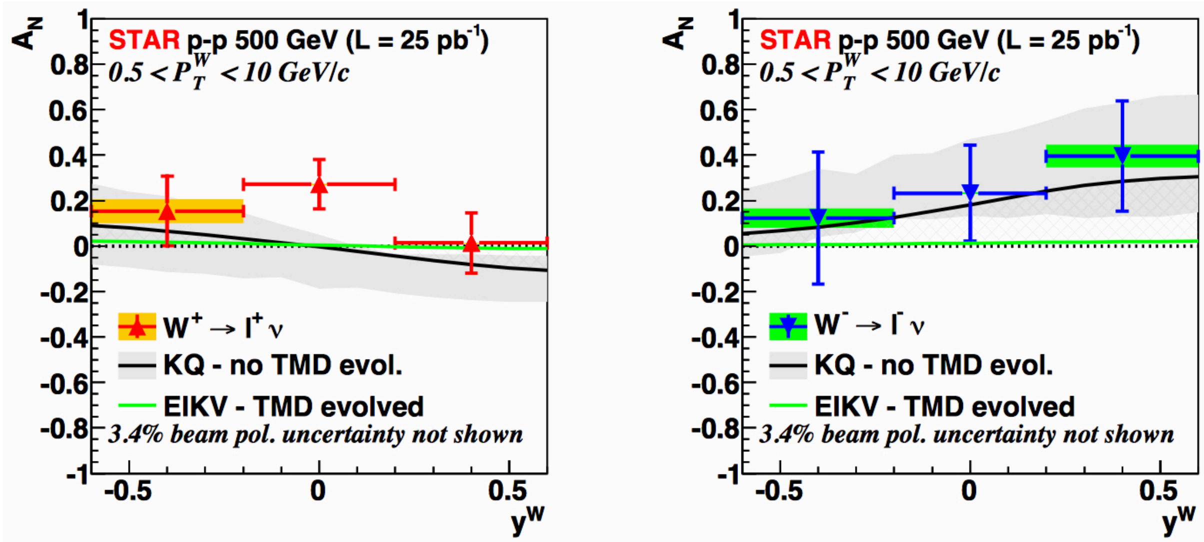
  - calculation for staple-like gauge links with finite length
  - results saturate for distances of about 0.4 fm

- What if sign reversal of $f_{1T}^{\perp q}$ not confirmed by experiment ?
  - would not imply that QCD is wrong
  - would imply that transverse SSAs not understood in QCD
  - problem with TMD factorization
  - implication on resummation of large transverse momentum logarithms, many observables (at LHC), collinear twist-3 factorization, ...

- Experimental check of process dependence of $f_{1T}^{\perp}$ was crucial
  (DOE Hadron Physics Performance Milestone, HP13: *Test unique QCD predictions for relations between single transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering*)

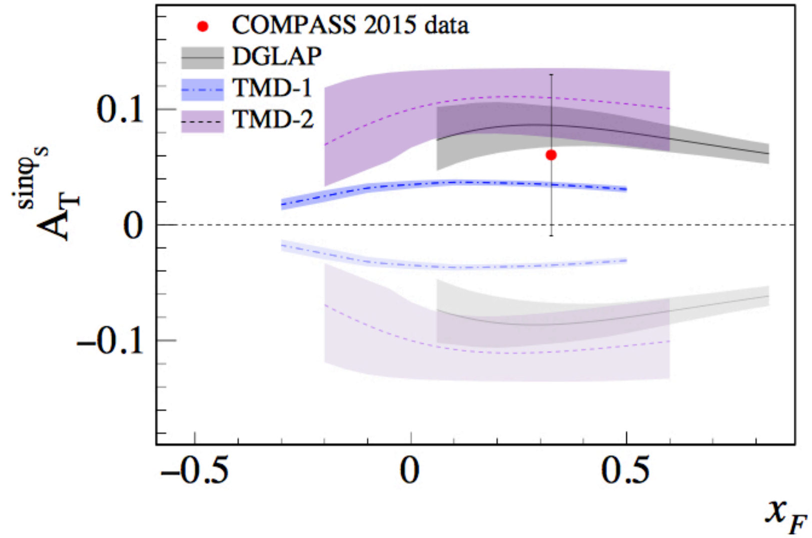
2. Phenomenology of process dependence of the Sivers function

- Measurement of Sivers asymmetry in $p^{\uparrow}p \to W^{\pm}/Z^{0}\,X$ at RHIC (STAR, 1511.06003)



  - relevant scale is mass of heavy gauge bosons
  - long evolution from measurements of Sivers effect in SIDIS
  - calculations with and without TMD evolution lead to very different results for asymmetry
  - such measurements can help constrain the (TMD) evolution
  - some indication of sign reversal, but not conclusive

- Measurement of Sivers asymmetry in $\pi^- p^\uparrow \to \mu^+ \mu^- X$ at COMPASS
  (COMPASS, 1704.00488)



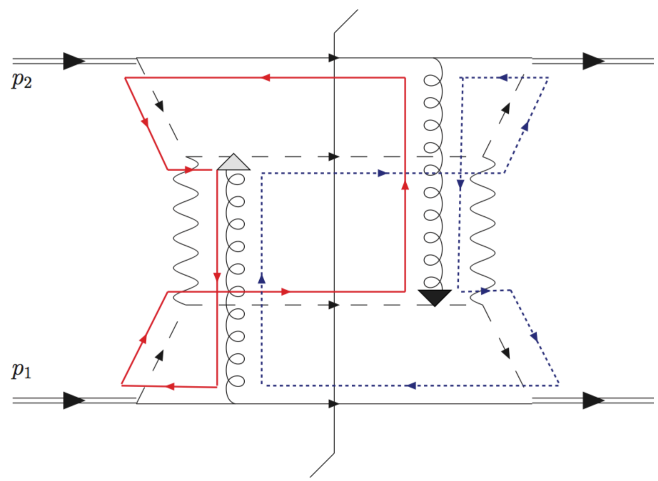
  - scale of measurement: $4.3\,\mathrm{GeV}^2 \leq m_{\mu\mu}^2 \leq 8.5\,\mathrm{GeV}^2$
  - data point favors sign reversal of Sivers fucntion

- Other work on process dependence of Sivers effect
  - simultaneous study of transverse SSAs in DIS and in SIDIS
    (A.M. et al, 1209.3138)
  - study of transverse SSA in $p^\uparrow p \to \mathrm{jet} X$
    (Gamberg, Kang, Prokudin, 1302.3213)

- Overall, strong indication from phenomenology that Sivers effect depends on process

Breakdown of TMD factorization

- Sample process: $p\,p \rightarrow \text{jet jet } X$
- Originally thought to obey generalized TMD factorization
  $\rightarrow$ definition of TMDs depends on partonic subprocess
  (Bomhof, Mulders, Pijlman, 2004 / ... / Collins, Qiu, 2007 / Collins, 2007)
- But, even generalized TMD factorization breaks down (Rogers, Mulders, 1001.2977)

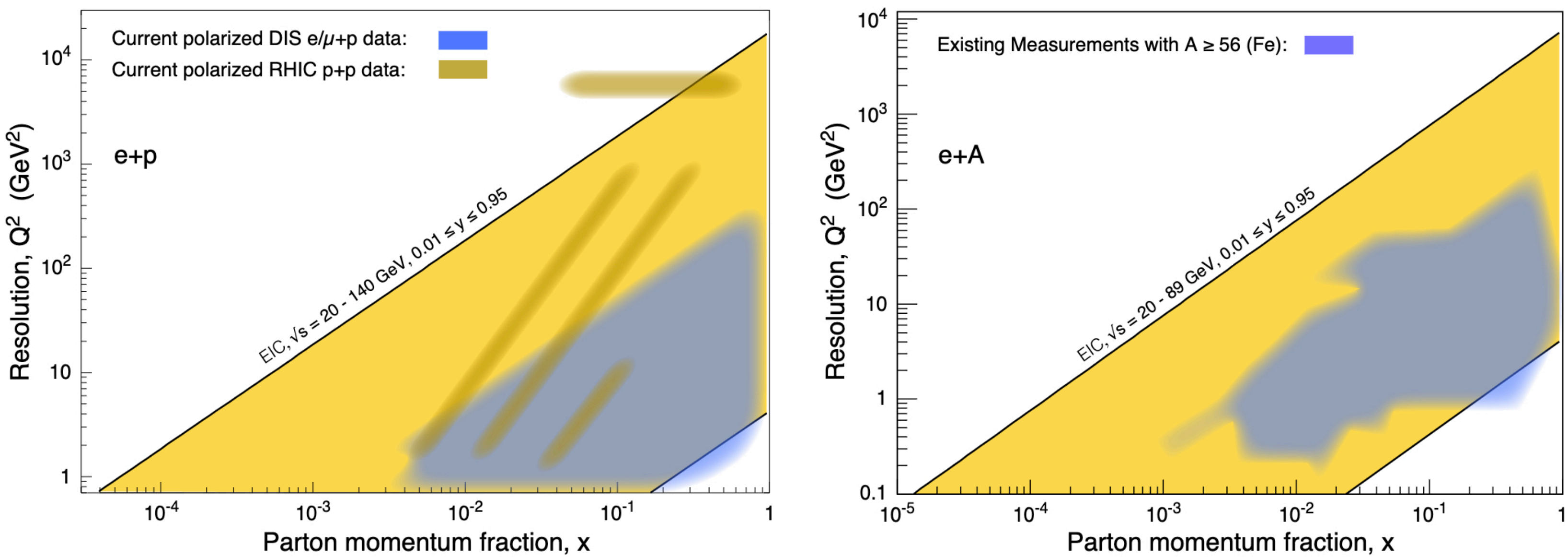


  - complicated color flow does not allow one to define two individual TMDs (color-entanglement)
  - specific to non-Abelian gauge theory

TMDs: Opportunities at the EIC

- Kinematics

(figure from EIC Yellow Report, 2103.05419)

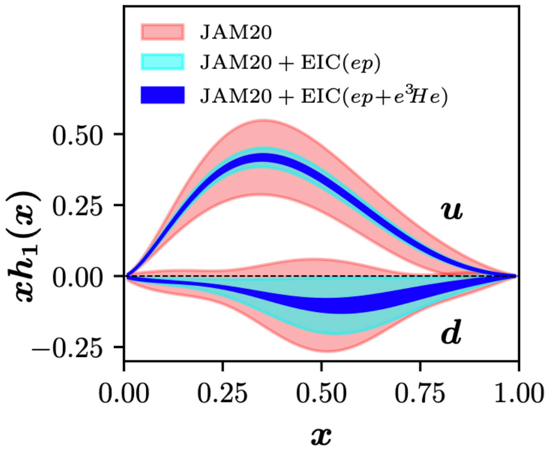


  - Compared to fixed-target experiments: much larger kinematic coverage
  - Compared to HERA collider: higher luminosity, polarization, nuclear targets

- Example: transversity distribution h_1^q

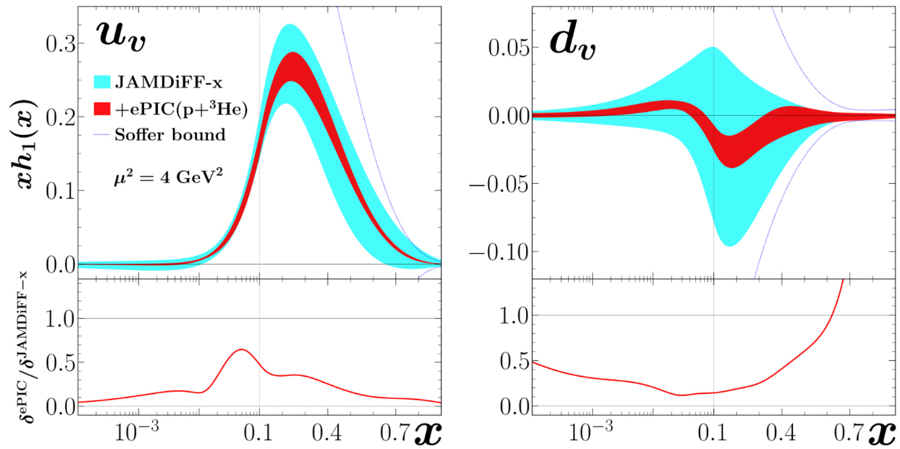
single-hadron production

(EIC Yellow Report, 2103.05419)



dihadron production

(Sawaya et al, 2606.00362)



  - significant reduction of uncertainties
  - proton and ^{3}He target needed for constraints on both h_1^u and h_1^d
  - reduced uncertainties provide strong constraints on tensor charges

$$\delta q(\mu) = \int_{-1}^{1} dx\, h_1^q(x, \mu)$$