


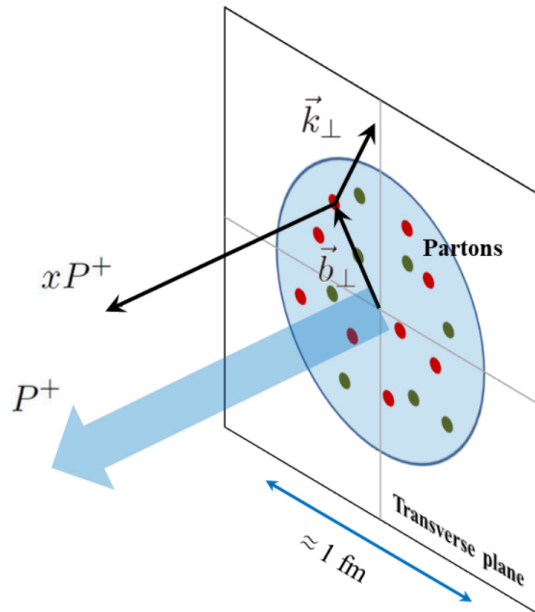
The 2026 CFNS Summer School on the Physics of the  
Electron-Ion Collider

# The Multidimensional Parton Structure of Hadrons

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# Outline



Light-cone variables for generic  
4-vector  $a^\mu = (a^0, a^1, a^2, a^3)$

$$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3) \quad \vec{a}_\perp = (a^1, a^2)$$

1. Parton Distribution Functions (PDFs)
2. Transverse Momentum Dependent Parton Distributions (TMDs)
3. Generalized Parton Distributions (GPDs)
4. Generalized TMDs (GTMDs)

→ overlap with lectures by Olness, Surrow, Tu, Nadolsky, Siritsyn, ...

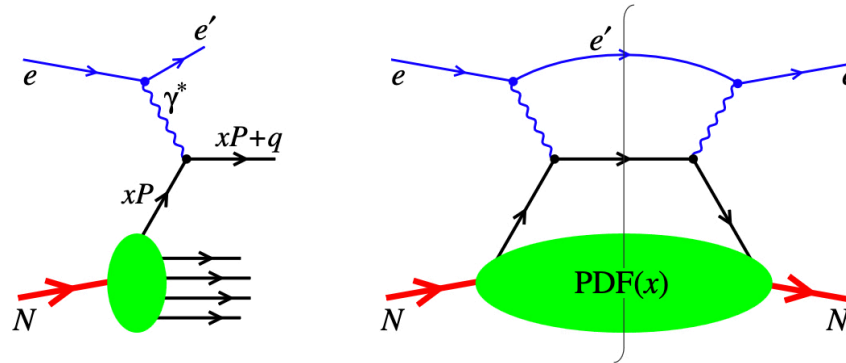
**Disclaimer:** we can only discuss bits and pieces

## Some Literature

- M. Diehl: *Introduction to GPDs and TMDs*, 1512.01328
- Various articles on 3D parton structure of hadrons, Eur. Phys. J **A52** (2016)
- C. Lorcé, A. Metz, B. Pasquini, P. Schweitzer, *Parton distribution functions and their generalizations*, 2507.12664
- A. Signori, *Introduction to transverse momentum imaging*, 2604.19997
- R. Boussarie et al, *TMD Handbook*, 2304.03302
- C. Mezrag, *An introductory lecture on generalized parton distributions*, 2207.13584
- A. Accardi et al, *Electron Ion Collider: The next QCD frontier – understanding the glue that binds us all*, 1212.1701
- R. Abdul Khalek et al, *Science requirements and detector concepts for the Electron Ion Collider: EIC Yellow Report*, 2103.05419
- (and many more ...)

# Inclusive Deep-Inelastic Scattering (DIS) and 1D Imaging

- Process:  $e + N \rightarrow e + X$



- Independent variables (unpolarized cross section)

$$Q^2 = -q^2 > 0 \quad x_B = \frac{Q^2}{2P \cdot q} \quad y \approx \frac{Q^2}{x_B S} \quad (\text{not independent})$$

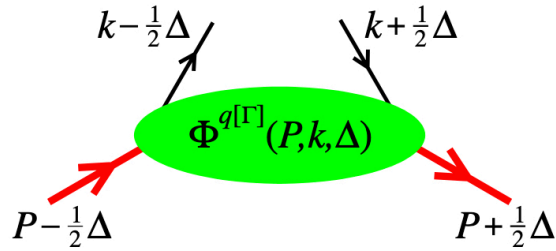
- Unpolarized DIS cross section in parton model

$$d\sigma_{\text{unp}} = \sum_{q, \bar{q}} \int_0^1 dx f_1^q(x) d\hat{\sigma}_{eq \rightarrow eq}$$

$$\frac{d^2 \hat{\sigma}_{eq \rightarrow eq}}{dx_B dQ^2} = e_q^2 \frac{2\pi\alpha_{\text{em}}}{Q^4} [1 + (1 - y)^2] \delta(x - x_B)$$

# PDFs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p + p') \quad \Delta = p' - p$$

$$\Phi_{ij}^q(P, k, \Delta) = \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p' | \bar{\psi}_j^q(-\frac{z}{2}) \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi_i^q(\frac{z}{2}) | p \rangle$$

$$\Phi^{q[\Gamma]}(P, k, \Delta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle$$

- PDFs defined through

$$\begin{aligned} \Phi_{\text{PDF}}^{q[\Gamma]}(P, x) &= \int d^4 k \Phi^{q[\Gamma]}(P, k, 0) \delta(k^+ - xP^+) \\ &= \int \frac{dz^-}{4\pi} e^{ik \cdot z} \langle P | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | P \rangle \Big|_{z^+ = z_\perp = 0} \end{aligned}$$

# PDFs of Quarks

- Including polarization for spin- $\frac{1}{2}$ :  $|P\rangle \rightarrow |P, S\rangle$

$$S = \left( \frac{\Lambda P^+}{M}, -\frac{\Lambda P^-}{M}, \vec{S}_\perp \right) \quad S^2 = -\Lambda^2 - \vec{S}_\perp^2 = -1 \quad P \cdot S = 0$$

- Definition of leading-twist (twist-2) quark PDFs

$$\begin{aligned} f_1^q(x) &= \Phi_{\text{PDF}}^{q[\gamma^+]}(P, S, x) \\ &= \int \frac{dz^-}{4\pi} e^{ik \cdot z} \langle P, S | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | P, S \rangle \Big|_{z^+ = z_\perp = 0} \end{aligned}$$

$$\Lambda g_1^q(x) = \Phi_{\text{PDF}}^{q[\gamma^+ \gamma_5]}(P, S, x)$$

$$S_\perp^i h_1^q(x) = \Phi_{\text{PDF}}^{q[i\sigma^{i+} \gamma_5]}(P, S, x) \quad (\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu])$$

- general structure of qq-correlator follows from parity, hermiticity, time-reversal
- PDFs depend on renormalization scale  $\mu$ , which has been suppressed

- More on polarization dependence
  - quarks with definite helicity/chirality

$$P_\lambda = \frac{1}{2} (1 + \lambda \gamma_5) \quad (\lambda = \pm 1) \quad P_\lambda^2 = P_\lambda \quad P_+ P_- = P_- P_+ = 0$$

$$\psi_\lambda = P_\lambda \psi \quad \bar{\psi}_\lambda = \bar{\psi} P_{-\lambda}$$

- rewriting of operator for  $f_1^q$

$$\bar{\psi} \gamma^+ \psi = \bar{\psi}_+ \gamma^+ \psi_+ + \bar{\psi}_- \gamma^+ \psi_-$$

→  $f_1^q$  describes **sum** of two densities

- rewriting of operator for  $g_1^q$

$$\bar{\psi} \gamma^+ \gamma_5 \psi = \bar{\psi}_+ \gamma^+ \psi_+ - \bar{\psi}_- \gamma^+ \psi_-$$

→  $g_1^q$  describes **difference** of two densities (can become negative)

- $h_1$  describes **difference** of two densities for **transverse** quark polarization

$$\bar{\psi} i\sigma^{i+} \gamma_5 \psi = \bar{\psi}_+ i\sigma^{i+} \gamma_5 \psi_- + \bar{\psi}_- i\sigma^{i+} \gamma_5 \psi_+$$

→  $h_1^q$  is **chiral odd** (decouples from DIS)

- Interpretation as number densities using light-front quantization
- Density interpretation spoiled by QCD effects (radiative corrections)
- $g_1^q$  and  $h_1^q$  can also be interpreted as strength of spin-spin correlations

$$\lambda \Phi_{\text{PDF}}^{q[\gamma^+ \gamma_5]}(P, S, x) = \lambda \Lambda g_1^q(x)$$

$$s_{\perp}^i \Phi_{\text{PDF}}^{q[i\sigma^{i+} \gamma_5]}(P, S, x) = \vec{s}_{\perp} \cdot \vec{S}_{\perp} h_1^q(x)$$

- Decomposing the qq-correlator in terms of PDFs

$$\Phi^q = \frac{1}{2} \Phi^{q[\gamma^+]} \gamma^- - \frac{1}{2} \Phi^{q[\gamma^+ \gamma_5]} \gamma^- \gamma_5 + \frac{1}{2} \Phi^{q[i\sigma^{i+} \gamma_5]} i\sigma^{i-} \gamma_5 + \dots$$

– all other terms suppressed for large  $P^+$

- Leading-twist PDFs of gluons ( $\langle P | F^{+i} F^{+j} | P \rangle$ )

$$f_1^g(x) = g(x) \quad \rightarrow \text{unpolarized gluon PDF}$$

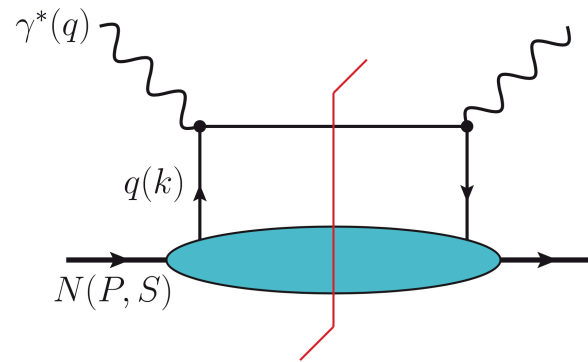
$$g_1^g(x) = \Delta g(x) \quad \rightarrow \text{gluon helicity PDF}$$

# Inclusive DIS with qq-Correlator

- Process

$$e(l, \lambda_\ell) + N(P, S) \rightarrow e(l', \lambda'_\ell) + X$$

- Handbag diagram



- Cross section

$$d\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

- Leptonic tensor

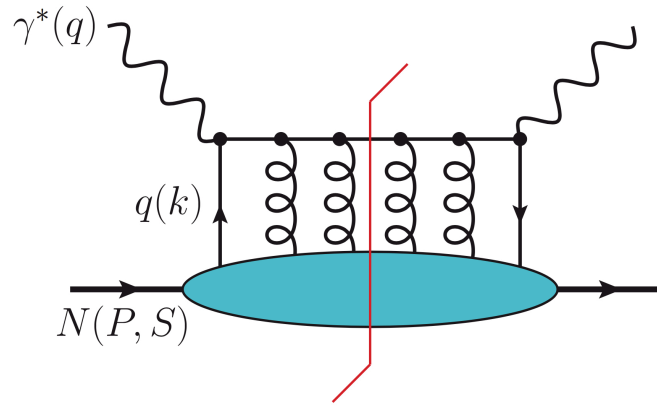
$$\begin{aligned} L^{\mu\nu} &= \left[ \bar{u}(l', \lambda'_\ell) \gamma^\nu u(l, \lambda_\ell) \right] \left[ \bar{u}(l', \lambda'_\ell) \gamma^\mu u(l, \lambda_\ell) \right]^* \\ &= 2 \left( l^\mu l'^\nu + l'^\mu l^\nu - l \cdot l' g^{\mu\nu} \right) + 2i \lambda_\ell \varepsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma \end{aligned}$$

- Hadronic tensor (neglect  $m_q$ )

$$\begin{aligned}
W^{\mu\nu} &\sim \sum_q e_q^2 \int d^4k \text{Tr} \left[ \Phi^q(k) \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \delta((k+q)^2) \\
&= \sum_q e_q^2 \int d^4k \Phi^{q[\gamma^+]}(k) \frac{1}{2} \text{Tr} \left[ \gamma^- \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \delta((k+q)^2) + \dots \\
&= \sum_q \frac{e_q^2}{2} \int d^4k \Phi^{q[\gamma^+]}(k) \text{Tr} \left( \left[ \gamma^- \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \delta((k+q)^2) \right) \Big|_{k^- = \vec{k}_\perp = 0} + \dots \\
&= \sum_q \frac{e_q^2}{2} \int dk^+ \Phi^{q[\gamma^+]}(x) \text{Tr} \left( \left[ \gamma^- \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \delta((k+q)^2) \right) \Big|_{k^- = \vec{k}_\perp = 0} \\
&= \sum_q \frac{e_q^2}{2} \int \frac{dk^+}{k^+} \underbrace{\Phi^{q[\gamma^+]}(x)}_{f_1^q(x)} \text{Tr} \left( \left[ \not{k} \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \underbrace{\delta((k+q)^2)}_{\frac{x_B}{Q^2} \delta(x-x_B)} \right) \Big|_{k^- = \vec{k}_\perp = 0} + \dots \\
&= \frac{1}{Q^2} \sum_q \frac{e_q^2}{2} f_1^q(x_B) \text{Tr} \left( \left[ \not{k} \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \right] \right) \Big|_{k^+ = x_B P^+, k^- = \vec{k}_\perp = 0}
\end{aligned}$$

- result agrees with the one obtained in parton model

- Handbag diagram including re-scattering of quark



- including re-scattering graphs (to all orders) renders gauge-invariant correlator

$$\Phi_{ij}^q(P, S, x) = \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle P, S | \bar{\psi}_j^q(-\frac{z}{2}) \mathcal{W}_{\text{PDF}}[-\frac{z}{2}, \frac{z}{2}] \psi_i^q(\frac{z}{2}) | P, S \rangle \Big|_{z^+ = z_{\perp} = 0}$$

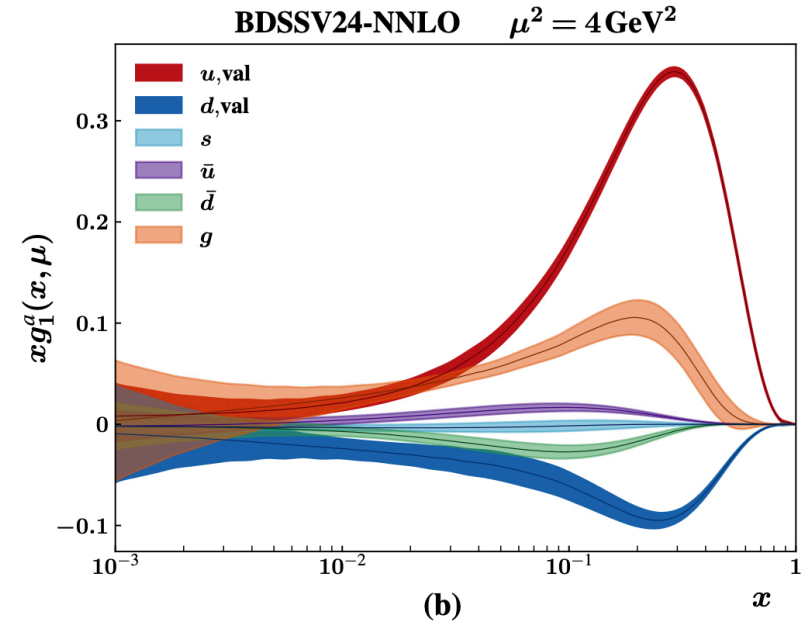
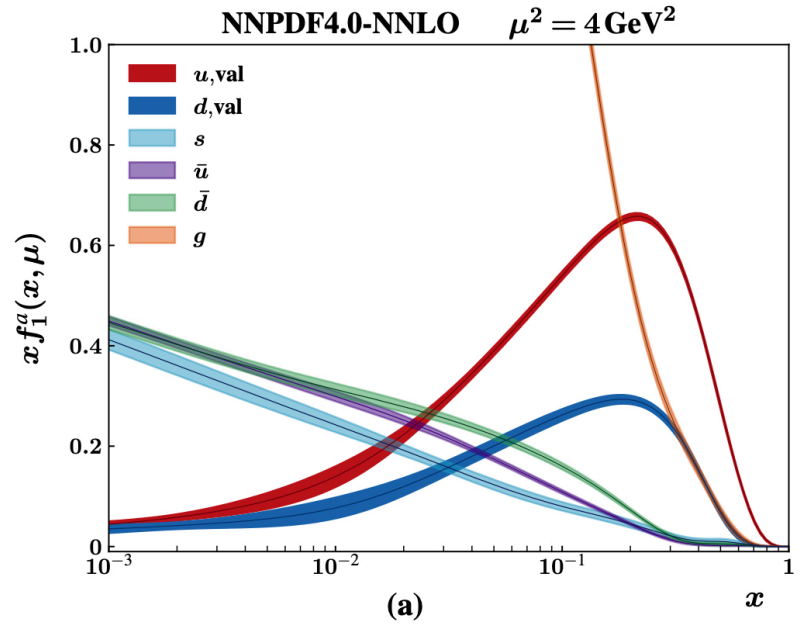
$$\mathcal{W}[a, b] = \mathcal{P} \exp \left( ig \int_a^b dy^\mu A_\mu(y) \right)$$

- in other words:  $\mathcal{W}_{\text{PDF}}$  generated by final-state interaction (FSI) of active quark
- path of  $\mathcal{W}_{\text{PDF}}$  is straight line  $\rightarrow \mathcal{W}_{\text{PDF}} = \mathbb{1}$  for  $A^+ = 0$  (light-cone gauge)

# PDFs: Results from Experimental Data

- Unpolarized and helicity PDFs of the proton

(figures from 2507.12664)

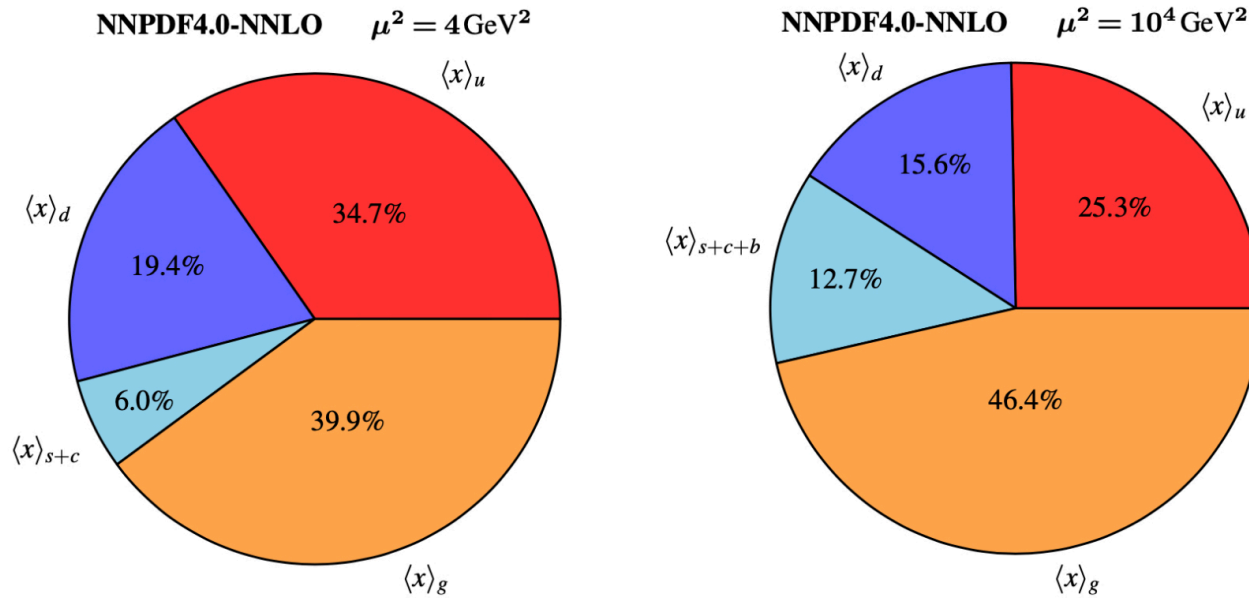


- valence distributions:  $f_1^{u, \text{val}} = f_1^u - f_1^{\bar{u}}$        $f_1^{d, \text{val}} = f_1^d - f_1^{\bar{d}}$
- PDFs depend on renormalization scheme ( $\overline{\text{MS}}$ ) and renormalization scale  $\mu$
- unpolarized PDFs rather well known; helicity PDFs have larger uncertainties

- Parton momentum fractions

$$\int_{-1}^1 dx x f_1^q(x, \mu) = \langle x \rangle_q(\mu) \quad \int_0^1 dx x f_1^g(x, \mu) = \langle x \rangle_g(\mu) \quad \sum_a \langle x \rangle_a(\mu) = 1$$

(figures from 2507.12664)

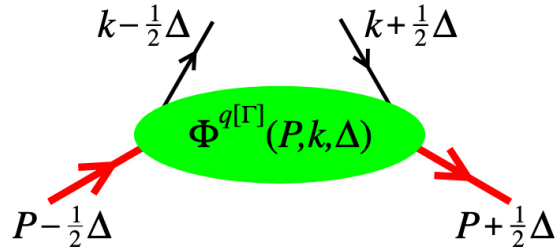


- asymptotic values

$$\lim_{\mu \rightarrow \infty} \frac{\langle x \rangle_q(\mu)}{\langle x \rangle_g(\mu)} = \frac{3}{16} \rightarrow \langle x \rangle_q(\infty) \approx 8.8\% \quad \langle x \rangle_g(\infty) \approx 47.1\%$$

# TMDs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p + p') \quad \Delta = p' - p$$

$$\Phi^{q[\Gamma]}(P, k, \Delta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle$$

- TMDs defined through

$$\begin{aligned} \Phi_{\text{TMD}}^{q[\Gamma]}(P, x, \vec{k}_\perp) &= \int dk^+ dk^- \Phi^{q[\Gamma]}(P, k, \mathbf{0}) \delta(k^+ - xP^+) \\ &= \int \frac{dz^- d^2 \vec{z}_\perp}{2(2\pi)^3} e^{ik \cdot z} \langle P | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | P \rangle \Big|_{z^+ = 0} \end{aligned}$$

# TMDs of Quarks

- Definition of leading-power (leading-twist) quark TMDs

(Mulders, Tangerman, 1995 / Boer, Mulders, 1997 / Bacchetta et al, 2006)

$$\Phi_{\text{TMD}}^{q[\gamma^+]}(x, \vec{k}_\perp) = f_1^q - \frac{\varepsilon_\perp^{ij} k_\perp^i S_\perp^j}{M} f_{1T}^{\perp q}$$

$$\lambda \Phi_{\text{TMD}}^{q[\gamma^+ \gamma_5]}(x, \vec{k}_\perp) = \lambda \Lambda g_1^q + \frac{\lambda \vec{k}_\perp \cdot \vec{S}_\perp}{M} g_{1T}^{\perp q}$$

$$\begin{aligned} s_\perp^i \Phi_{\text{TMD}}^{q[i\sigma^{i+} \gamma_5]}(x, \vec{k}_\perp) &= \vec{s}_\perp \cdot \vec{S}_\perp h_1^q + \frac{\Lambda \vec{k}_\perp \cdot \vec{s}_\perp}{M} h_{1L}^{\perp q} - \frac{\varepsilon_\perp^{ij} k_\perp^i s_\perp^j}{M} h_1^{\perp q} \\ &+ \frac{1}{2M^2} \left( 2 \vec{k}_\perp \cdot \vec{s}_\perp \vec{k}_\perp \cdot \vec{S}_\perp - \vec{k}_\perp^2 \vec{s}_\perp \cdot \vec{S}_\perp \right) h_{1T}^{\perp q} \end{aligned}$$

- TMDs depend on  $x$  and  $\vec{k}_\perp^2 \rightarrow$  forward limit is readily recovered
- general structure of qq-correlator follows from parity, hermiticity, time-reversal
- TMDs depend on two auxiliary scales ( $\mu, \zeta$ )
- 8 leading-power gluon TMDs

- Quark TMDs have names

$f_{1T}^{\perp q}$  : Sivers function (Sivers, 1989)

$h_1^{\perp q}$  : Boer-Mulders function (Boer, Mulders, 1997)

$g_{1T}^{\perp q}$   $h_{1L}^{\perp q}$  : worm-gear functions (polarization of hadron and quark perpendicular)

$h_{1T}^{\perp q}$  : pretzelosity (quadrupole pattern of pre-factor)

- Overview

(figure from 2507.12664)

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1$		$h_1^{\perp}$
	L		$g_1$	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	$h_1, h_{1T}^{\perp}$

– no TMD for U/L and L/U polarizations due to parity invariance

- “Stamp collecting”? ... maybe ... but we are in good company
  - periodic table of elements

1 H																	2 He															
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne															
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar															
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr															
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe															
55 Cs	56 Ba											72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn						
87 Fr	88 Ra											104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo						
																		57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
																		89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

don't forget the isotopes ...

- (supersymmetric) extensions of the Standard Model
- materials science
- etc.

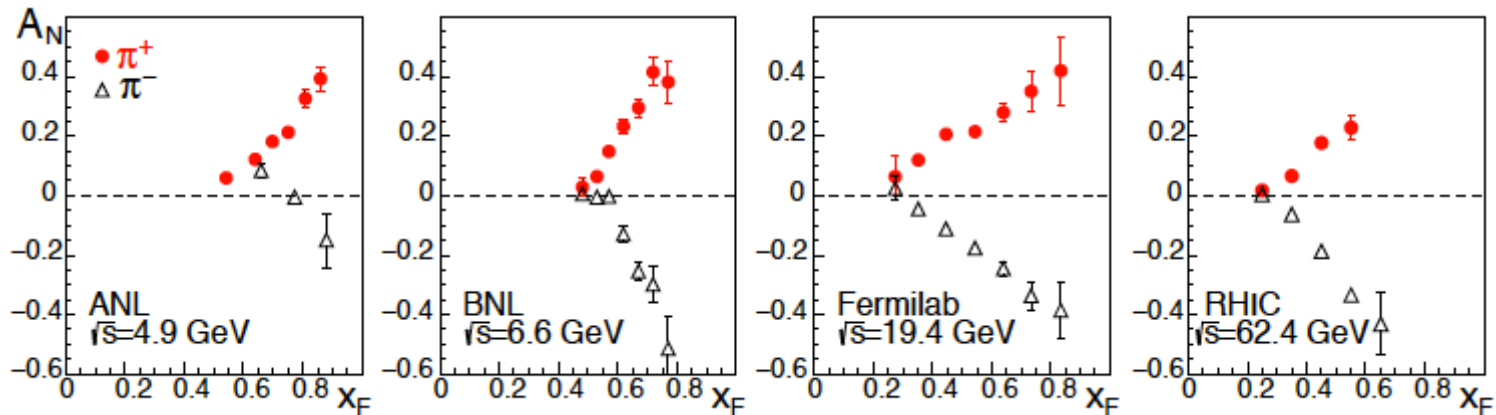
- Siverts function: a closer look

- density of unpolarized quarks in (transversely) polarized nucleon

$$\Phi_{\text{TMD}}^{q[\gamma^+]}(P, S, x, \vec{k}_\perp) = f_1^q(x, \vec{k}_\perp^2) - \frac{(\vec{k}_\perp \times \vec{S}_\perp) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_\perp^2)$$

- $f_{1T}^{\perp q}$  describes **difference** of two densities for transverse nucleon polarization
- $f_{1T}^{\perp q}$  can generate transverse single-spin asymmetries (SSAs) in scattering processes
- observed large transverse SSAs were motivation for Siverts to explore this effect

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \sim \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$



(figure from Aidala et al, 1209.2803)

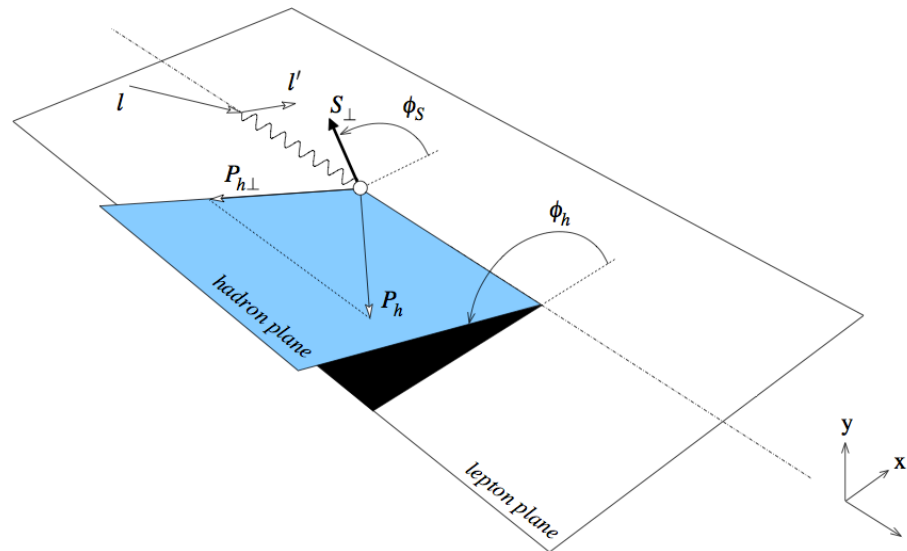
# TMDs in Semi-Inclusive DIS (SIDIS)

- Process

$$\ell(l, \lambda_\ell) + N(P, S) \rightarrow \ell(l', \lambda'_\ell) + h(P_h) + X$$

- 6 independent kinematic variables

$$Q^2 \quad x_B = \frac{Q^2}{2P \cdot q} \quad \phi_S \quad z_h = \frac{P \cdot P_h}{P \cdot q} \quad P_{h\perp} = |\vec{P}_{h\perp}| \quad \phi_h$$



(figure from Bacchetta et al, hep-ph/0611265)

- Model-independent form of cross section (in notation of hep-ph/0611265)

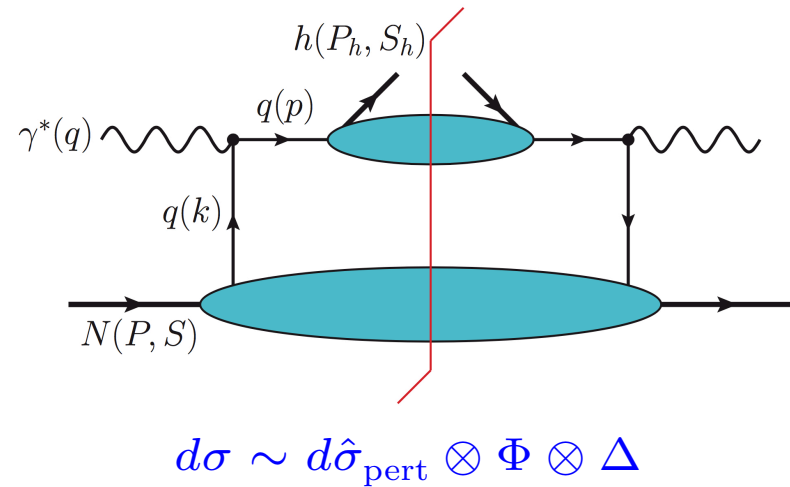
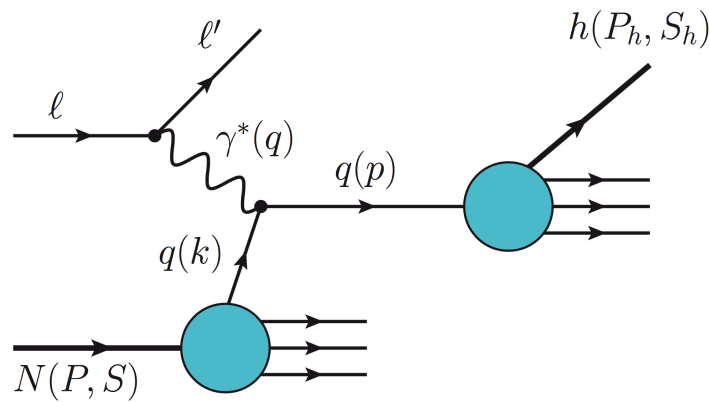
$$\begin{aligned}
\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{h\perp}^2} \sim & \left\{ (1 - y + \frac{1}{2}y^2) F_{UU,T} + (1 - y) \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
& + \Lambda (1 - y) \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} + \lambda_\ell \Lambda y (1 - \frac{1}{2}y) F_{LL} \\
& + |\vec{S}_\perp| (1 - y + \frac{1}{2}y^2) \sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h - \phi_S)} \\
& + |\vec{S}_\perp| (1 - y) \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \\
& + |\vec{S}_\perp| (1 - y) \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& \left. + \lambda_\ell |\vec{S}_\perp| y (1 - \frac{1}{2}y) \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + 10 \text{ additional terms} \right\}
\end{aligned}$$

- structure functions depend on 4 variables:

$$F_i = F_i(x_B, z_h, P_{h\perp}^2, Q^2)$$

- at low  $P_{h\perp}$ , leading contribution to  $d\sigma$  given by 8 listed structure functions

- Feynman diagram at tree level



– factorization formula depends on kinematics:

1. cross section integrated upon  $P_{h\perp}$
2. cross section differential in  $P_{h\perp}$ , and  $P_{h\perp} \sim Q$
3. cross section differential in  $P_{h\perp}$ , and  $P_{h\perp} \ll Q \rightarrow$  realm of TMDs

– full description of differential cross section from small to large  $P_{h\perp}$  still area of active research (see, e.g., Collins et al, 1605.00671)

- Hadronic tensor at tree level

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k d^4p \delta^{(4)}(k + q - p) \text{Tr} \left[ \Phi^q(k) \gamma^\mu \Delta^q(p) \gamma^\nu \right]$$

- consider  $P^+$  and  $P_h^-$  large, as well as  $k^+ = xP^+$  and  $P_h^- = zp^-$
- consider frame with  $\vec{P}_{h\perp} = 0$ , and small  $\vec{q}_\perp \neq 0$
- neglect small light-cone components of parton momenta  $k^-$  and  $p^+$  in delta-function (approximation for TMD parton model)

$$W^{\mu\nu} \sim \frac{2x_B z_h}{Q^2} \sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \\ \times \text{Tr} \left[ \Phi^q(x_B, \vec{k}_\perp) \gamma^\mu \Delta^q(z_h, \vec{p}_\perp) \gamma^\nu \right]$$

- express  $\Phi^q$  and  $\Delta^q$  in terms of TMD-PDFs and TMD-FFs, respectively
- transverse parton momenta of TMD-PDFs and TMD-FFs are convoluted
- contract with leptonic tensor  $L^{\mu\nu}$
- compare with model-independent form of cross section to find tree-level results for structure functions  $F_i$

- Structure functions at tree level (e.g., hep-ph/0611265)

$$F_{UU,T} = x_B \sum_q e_q^2 \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) f_1^q(x_B, \vec{k}_\perp^2) D_1^q(z_h, \vec{p}_\perp^2)$$

$$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$

$$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

$$F_{LL} \sim g_1 \otimes D_1$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^\perp \otimes D_1 \quad [\text{Sivers effect}]$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} \sim h_1 \otimes H_1^\perp \quad [\text{Collins effect}]$$

$$F_{UT}^{\sin(3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$$

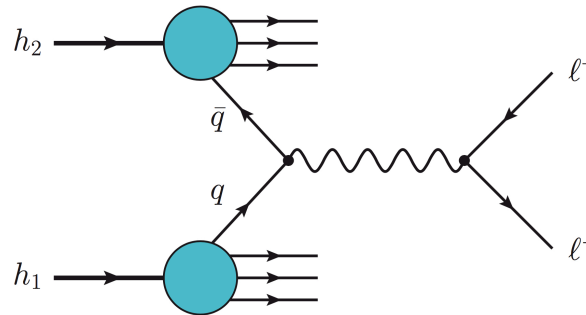
$$F_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T}^\perp \otimes D_1$$

- except for  $F_{UU,T}$  expressions are symbolic; in most cases convolutions contain additional powers of transverse parton momenta
- all 8 TMD-PDFs can be studied
- all 8 structure functions have been measured

# TMDs in Other Processes

## 1. Drell-Yan process: $h_1 + h_2 \rightarrow \ell^+ + \ell^- + X$

- Feynman diagram at tree level



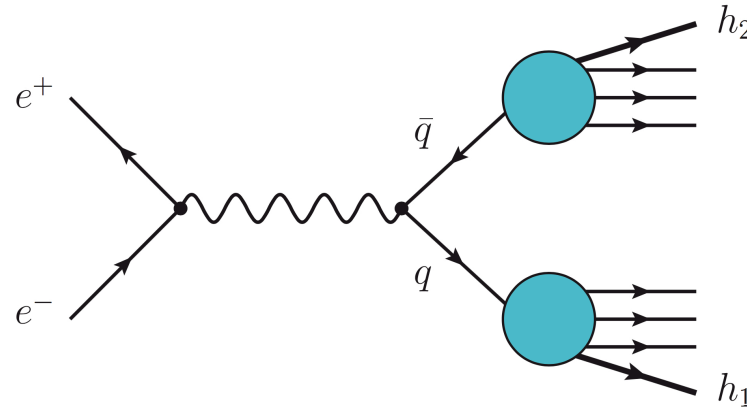
- Hadronic tensor at tree level, for low  $\vec{q}_\perp$  of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{k}_{a\perp} d^2\vec{k}_{b\perp} \delta^{(2)}(\vec{k}_{a\perp} + \vec{k}_{b\perp} - \vec{q}_\perp) \\ \times \text{Tr} \left[ \Phi^q(x_a, \vec{k}_{a\perp}) \gamma^\mu \Phi^{\bar{q}}(x_b, \vec{k}_{b\perp}) \gamma^\nu \right]$$

- sensitivity to “product” of two TMD-PDFs
- longitudinal momentum fractions  $x_a$  and  $x_b$  fixed by kinematics of reaction
- 48 structure functions; cross checks possible for various TMDs  
(Arnold, A.M., Schlegel, 2008)

2. Electron-positron annihilation:  $e^+ + e^- \rightarrow h_1 + h_2 + X$

- Feynman diagram at tree level



- Hadronic tensor at tree level, for low  $\vec{q}_\perp$  of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{p}_{a\perp} d^2\vec{p}_{b\perp} \delta^{(2)}(\vec{p}_{a\perp} + \vec{p}_{b\perp} - \vec{q}_\perp) \\ \times \text{Tr} \left[ \Delta^q(z_a, \vec{p}_{a\perp}) \gamma^\mu \Delta^{\bar{q}}(z_b, \vec{p}_{b\perp}) \gamma^\nu \right]$$

- sensitivity to “product” of two TMD-FFs
- longitudinal momentum fractions  $z_a$  and  $z_b$  fixed by kinematics of reaction

### 3. Some additional processes

(a)  $\ell N \rightarrow \ell \text{jet jet } X$        $\ell N \rightarrow \ell J/\psi X$

(b)  $pp \rightarrow \gamma \gamma X$        $pp \rightarrow \gamma \text{jet } X$        $pp \rightarrow \text{jet jet } X$

(c)  $pp \rightarrow (h \text{jet}) X$

(d)  $pp \rightarrow J/\psi X$        $pp \rightarrow \eta_c X$        $pp \rightarrow \text{Higgs } X$

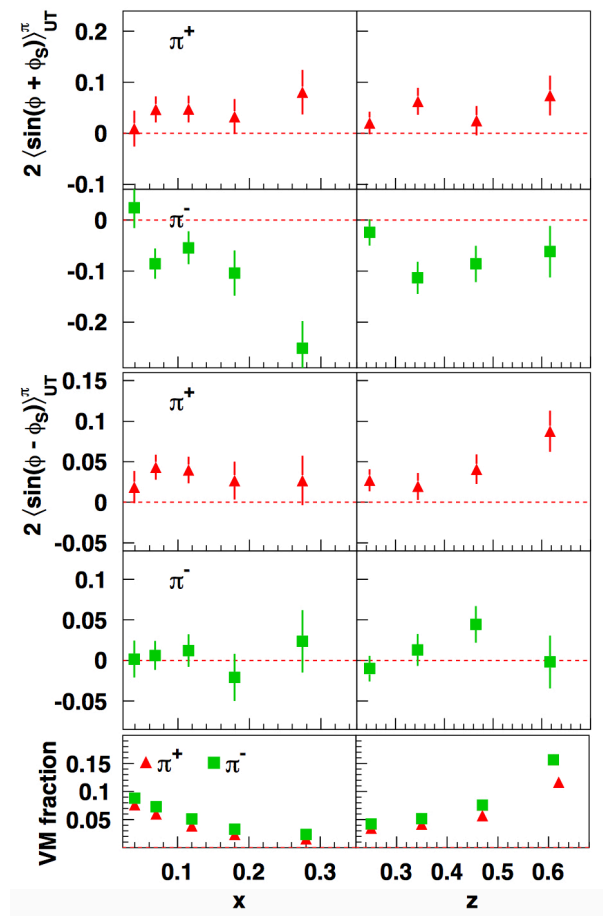
(e)  $p A$ -collisions

(f) etc.

- Very rich phenomenology
- Status of TMD factorization for additional processes:
  - holds in some cases (according to current knowledge)
  - breaks down in some cases
  - unclear in some cases (further studies needed) → active research area

# Measurements of TMD Observables

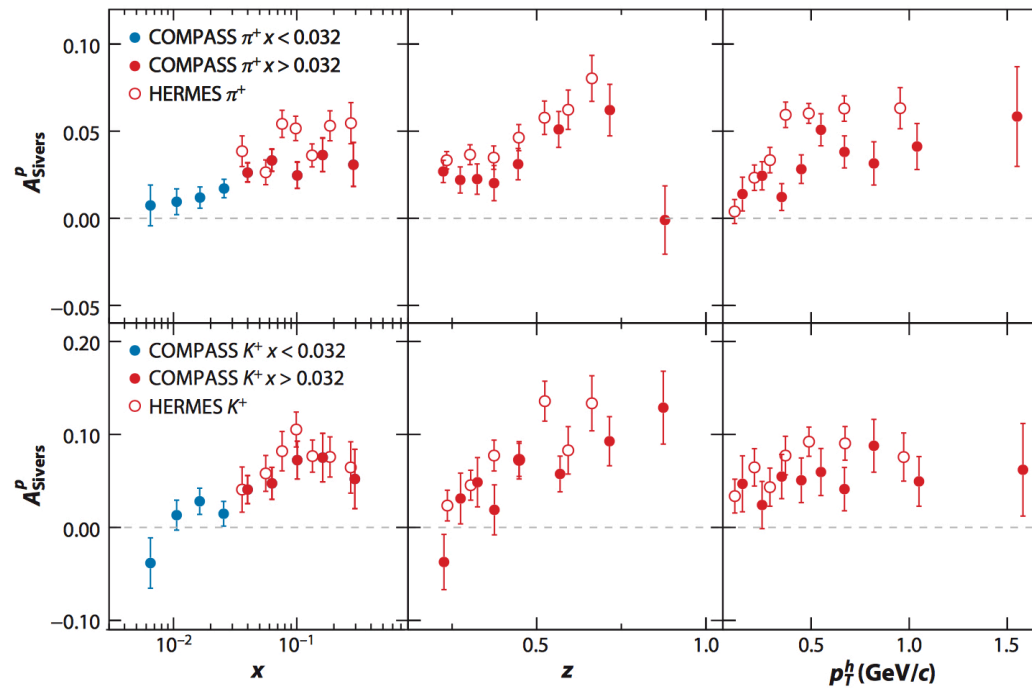
## 1. Pioneering steps for Sivers and Collins effects (HERMES, hep-ph/0408013)



- hydrogen target
- first measurement of Sivers asymmetry,  $F_{UT,T}^{\sin(\phi_h - \phi_S)} / F_{UU,T}$
- first measurement of Collins asymmetry,  $F_{UT}^{\sin(\phi_h + \phi_S)} / F_{UU,T}$
- nonzero Sivers effect for  $\pi^+$
- nonzero Collins effect for  $\pi^+$  and  $\pi^-$
- in the meantime, many more data from COMPASS, HERMES, JLab, with hydrogen, deuterium, and  $^3\text{He}$  targets

## 2. More SIDIS data on Sivers effect

### Comparison of data from COMPASS and HERMES



(compilation from Grosse Perdekamp, Yuan, 1510.06783)

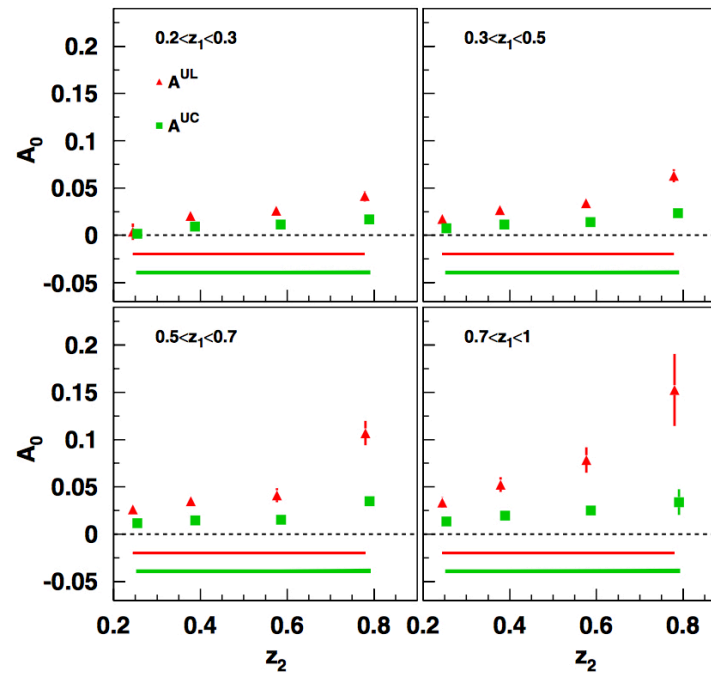
- measurements with hydrogen target, for  $\pi^+$  and  $K^+$  production
- results from COMPASS and HERMES largely agree
- robust nonzero effects
- asymmetry for  $K^+$  at least as large as for  $\pi^+$

### 3. (Double) Collins effect in electron-positron annihilation: $e^+e^- \rightarrow h_1h_2X$

- Azimuthal modulation due to Collins effect (Boer, Jakob, Mulders, hep-ph/9702281)

$$d\sigma \sim \sum_q e_q^2 \left[ D_1^{h_1/q} \otimes D_1^{h_2/\bar{q}} + \cos(2\phi) H_1^\perp{}^{h_1/q} \otimes H_1^\perp{}^{h_2/\bar{q}} \right]$$

- Sample data (Belle, 0805.2975)

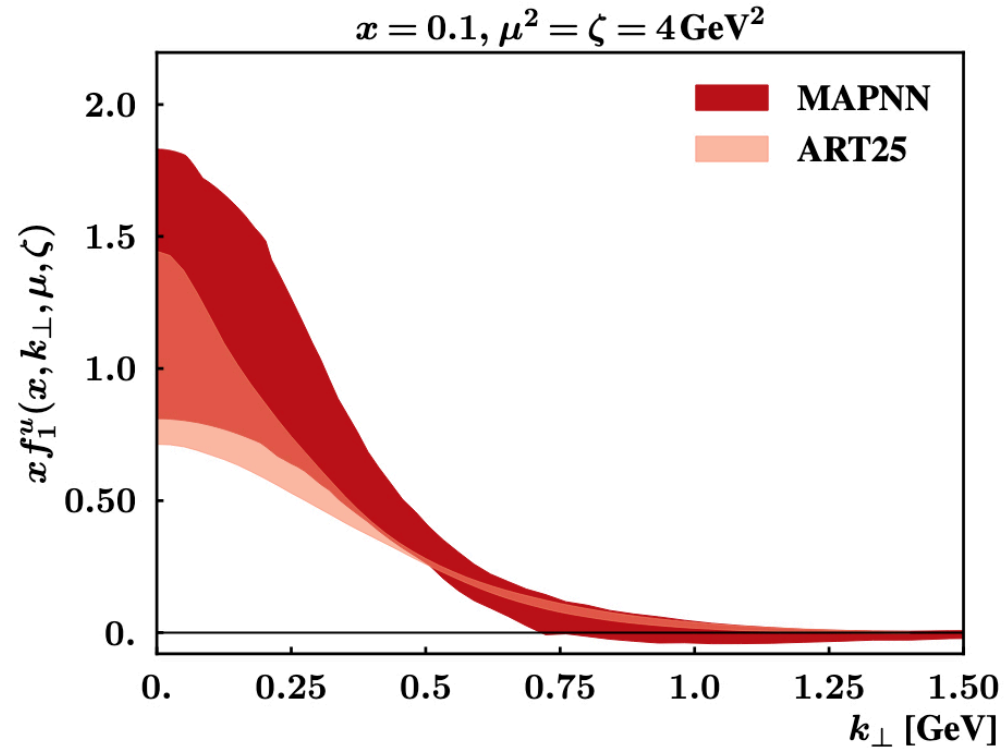


- More data available, also from BaBar and BES

# TMD Extractions from Data

## 1. Unpolarized TMD $f_1^q$

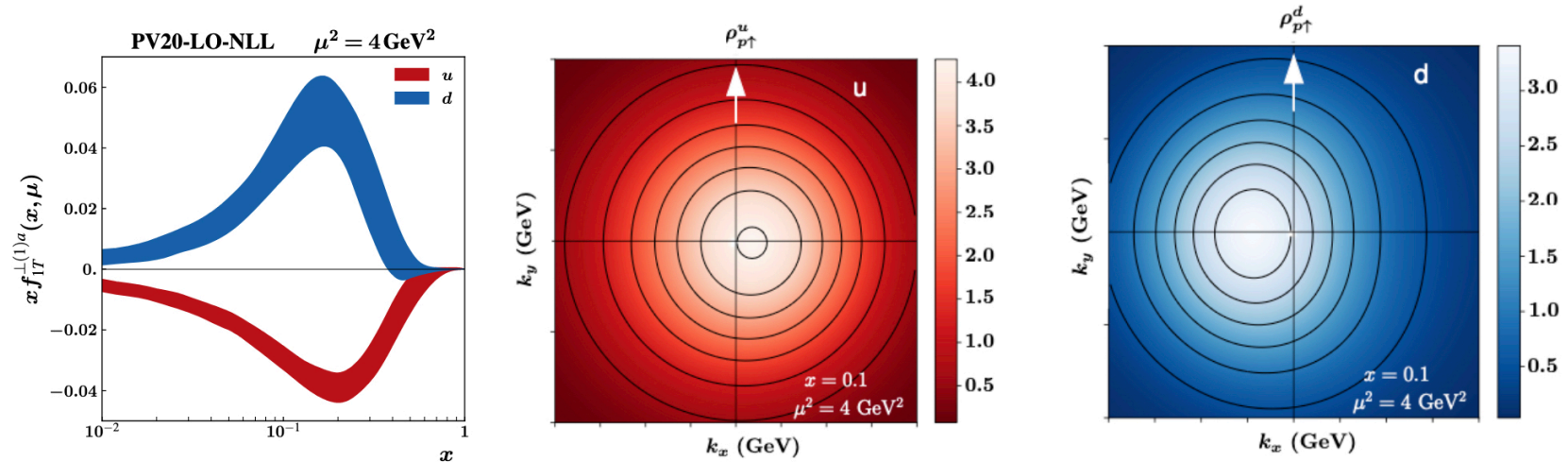
(figure from 2507.12664)



- MAPNN (Bacchetta et al, 2502.04166): DY data only, and neural network approach
- ART25 (Moos et al, 2503.11201): DY and SIDIS data

## 2. Sivers function $f_{1T}^{\perp q}$

(figure from 2507.12664)



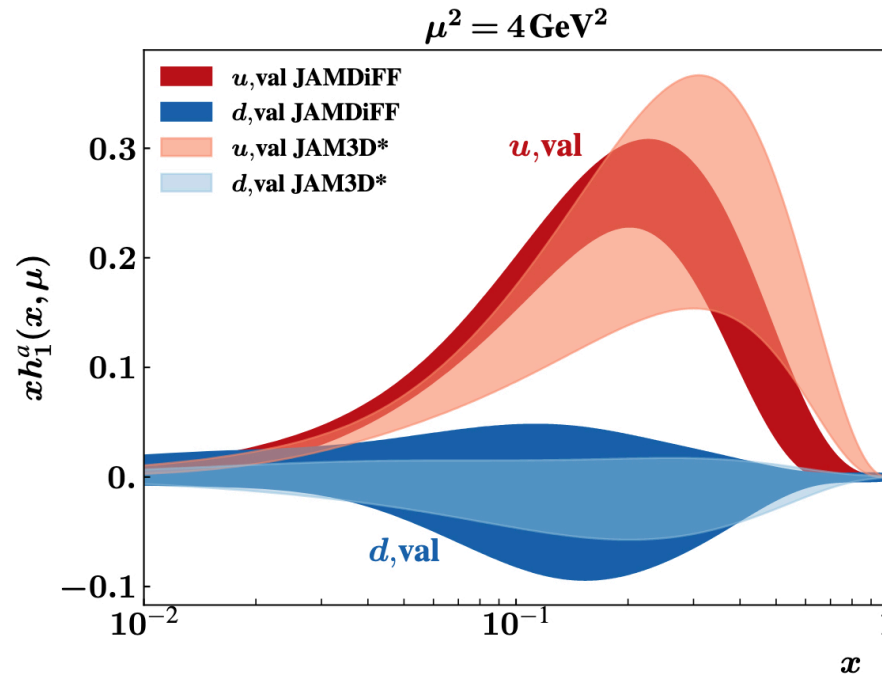
$$f_{1T}^{\perp(1)q}(x) = \int d^2 \vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{2M^2} f_{1T}^{\perp q}(x, \vec{k}_{\perp}^2)$$

- PV20 (Bacchetta et al, 2004.14278): SIDIS and DY data for Sivers effect
- Nice agreement with large- $N_c$  prediction:  $f_{1T}^{\perp u} = -f_{1T}^{\perp d} + \mathcal{O}(1/N_c)$  (Pobylitsa, hep-ph/0301236)
- Density of unpolarized quarks in (transversely) polarized nucleon

$$\Phi_{\text{TMD}}^{q[\gamma^+]}(P, S, x, \vec{k}_{\perp}) = f_1^q(x, \vec{k}_{\perp}^2) - \frac{(\vec{k}_{\perp} \times \vec{S}_{\perp}) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_{\perp}^2)$$

### 3. Transversity distribution $h_1^q$

(figure from 2507.12664)



- Transversity distribution in SIDIS via single-hadron and dihadron production

$$A_{UT}^h \sim h_1(x, \vec{k}_\perp^2) \otimes H_1^\perp(z, \vec{p}_\perp^2) \quad A_{UT}^{hh} \sim h_1(x) \otimes H_1^\triangleleft(z, M_h)$$

- JAM3D\* (Gamberg et al, 2205.00999): [Data on single-hadron production](#)
- JAMDiFF (Cocuzza et al, 2306.12998 ): [Data on dihadron production](#)

## Some Remarks on the Evolution of TMDs

- Further reading, for instance:
  - Collins, *Foundations of perturbative QCD*, 2011
  - Rogers, 1509.04766
  - TMD Handbook, 2304.03302

- In QCD, TMDs depend on two (auxiliary) scales

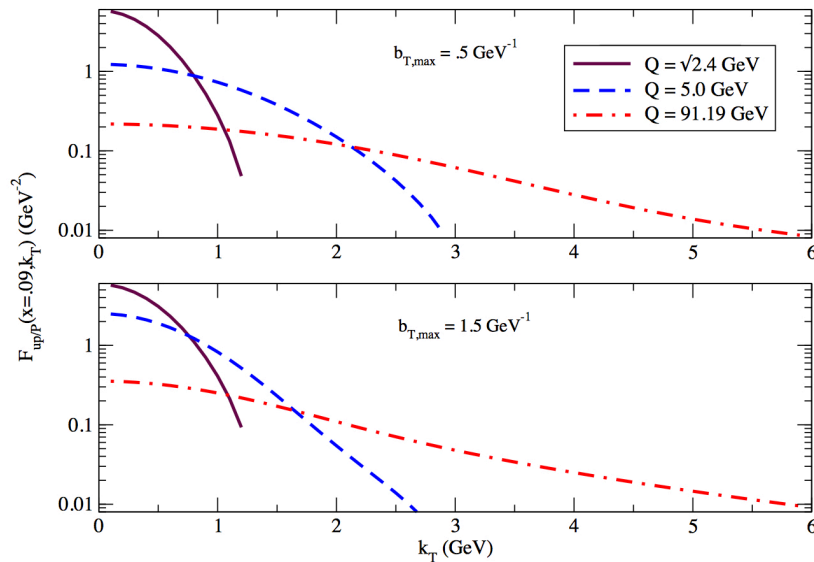
$$f_1^q(x, \vec{k}_\perp^2; \mu, \zeta)$$

- $\mu$ : due to UV divergence
  - $\zeta$ : due to rapidity divergence (different regulators in use)
- Rapidity divergence complicates the relation between TMDs and PDFs
    - after regulating rapidity divergence one has

$$f_1^q(x, \mu) \neq \int d^2\vec{k}_\perp f_1^q(x, \vec{k}_\perp^2; \mu, \zeta)$$

- rapidity divergence cancels for PDF  $f_1(x, \mu)$  (between real and virtual diagrams)  
(e.g., Collins, hep-ph/0304122)

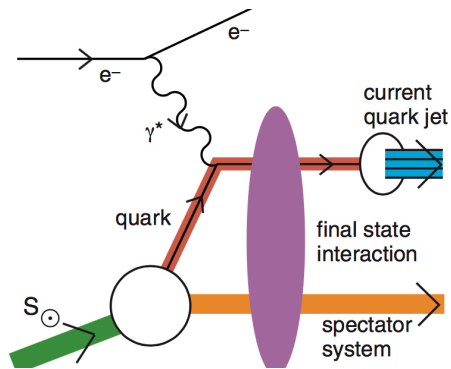
- Evolution of TMDs
  - $\mu$ -dependence: differs from DGLAP evolution
  - $\zeta$ -dependence: governed by Collins-Soper evolution equation (Collins, Soper, 1981)
  - TMD evolution has also dependence on non-perturbative input
    - progress in phenomenology and lattice QCD (e.g., Avkhadiev et al, 2307.12359)
- Numerical study (Aybat, Rogers, 1101.5057)



- evolution broadens TMDs
- evolution typically dilutes effects like Siverts asymmetry

# Eventful History of the Siverson Function

- In 1989, Dennis Siverson suggested the function (correlation)
- In 1992, John Collins argued that  $f_{1T}^{\perp q} = 0$  due to T-reversal invariance
- Model calculation of transverse SSA in DIS (Brodsky, Hwang, Schmidt, hep-ph/0201296)



- spectator system modeled by scalar diquark
- FSI modeled by single photon exchange
- nonzero transverse target SSA  $A_{UT}$
- $A_{UT}$  given by interference of lowest-order graph and (imaginary part of) box graph

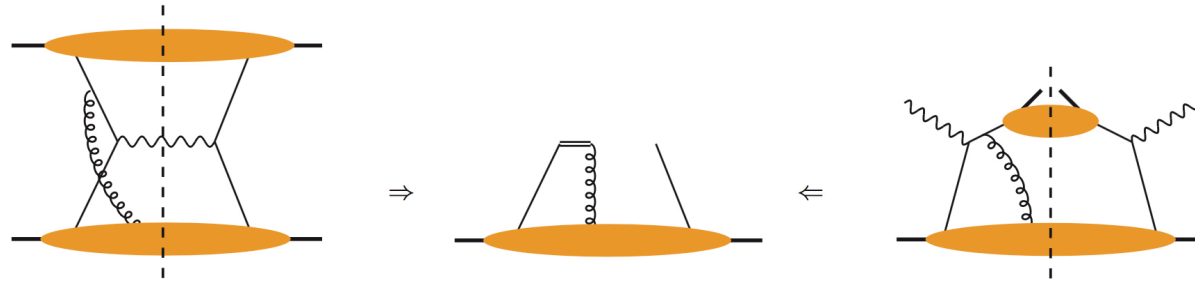
- Interpretation of BHS calculation, correction, prediction (Collins, hep-ph/0204004)
  - nonzero  $A_{UT}$  of BHS can be described in TMD factorization using  $f_{1T}^{\perp q}$
  - if  $\mathcal{W}_{\text{TMD}}$  taken into account, T-reversal does not forbid existence of  $f_{1T}^{\perp q}$
  - T-reversal rather predicts process dependence:

$$f_{1T}^{\perp q} \Big|_{\text{DY}} = - f_{1T}^{\perp q} \Big|_{\text{SIDIS}} \quad h_1^{\perp q} \Big|_{\text{DY}} = - h_1^{\perp q} \Big|_{\text{SIDIS}}$$

- These developments were of utmost importance for entire TMD field

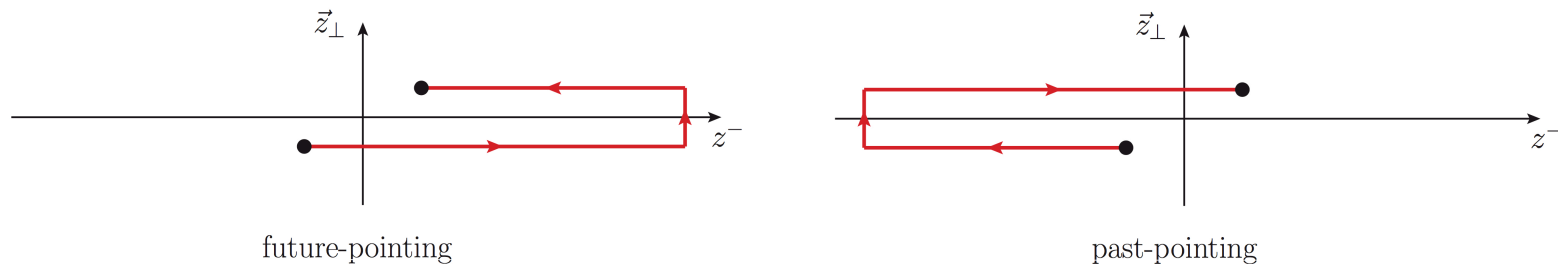
# Process Dependence of the Sivers Function

## 1. Comparing SIDIS and DY



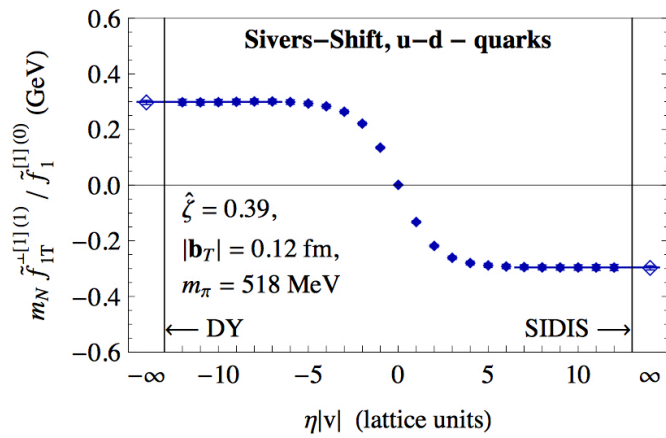
(figure from Diehl, 1512.01328)

- Gauge link structure in SIDIS and DY (FSI vs ISI)



- T-reversal allows one to relate definitions in two processes (Collins, hep-ph/0204004)
  - Six T-even TMDs are universal
  - Two T-odd TMDs change sign between SIDIS and DY
  - **breakdown of universality**, but in well-defined way
  - **strictly speaking, T-odd TMDs not exclusively property of nucleon**

- Sign reversal in lattice QCD (Musch et al, 1111.4249)

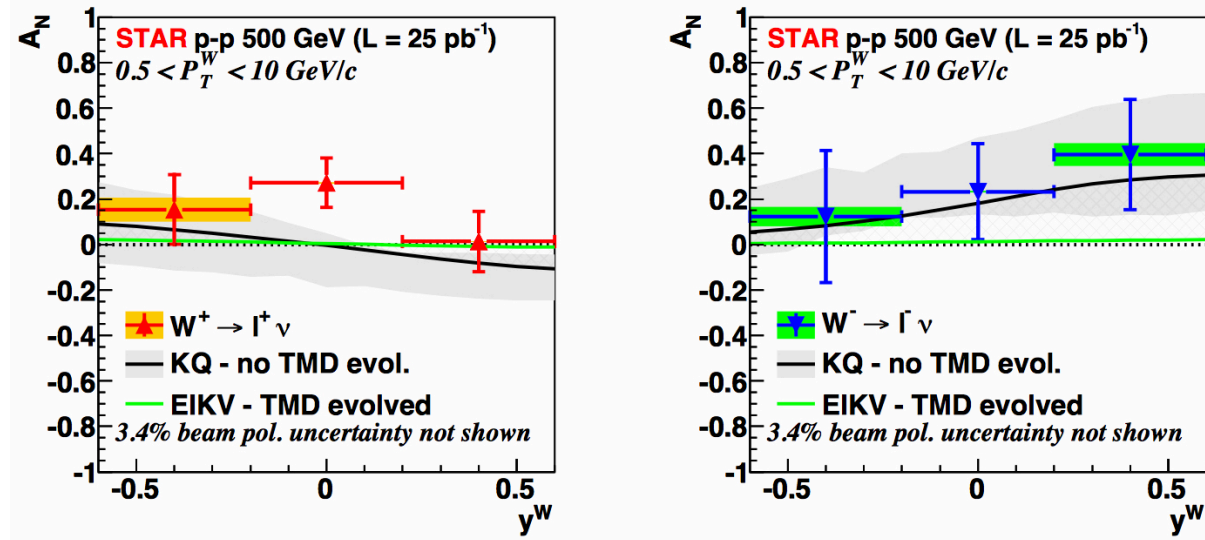


- calculation for staple-like gauge links with finite length
- results saturate for distances of about 0.4 fm

- What if sign reversal of  $f_{1T}^{\perp q}$  not confirmed by experiment?
  - would not imply that QCD is wrong
  - would imply that transverse SSAs not understood in QCD
  - problem with TMD factorization
  - implication on resummation of large transverse momentum logarithms, many observables (at LHC), collinear twist-3 factorization, ...
- Experimental check of process dependence of  $f_{1T}^{\perp}$  was crucial (DOE Hadron Physics Performance Milestone, HP13: *Test unique QCD predictions for relations between single transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering*)

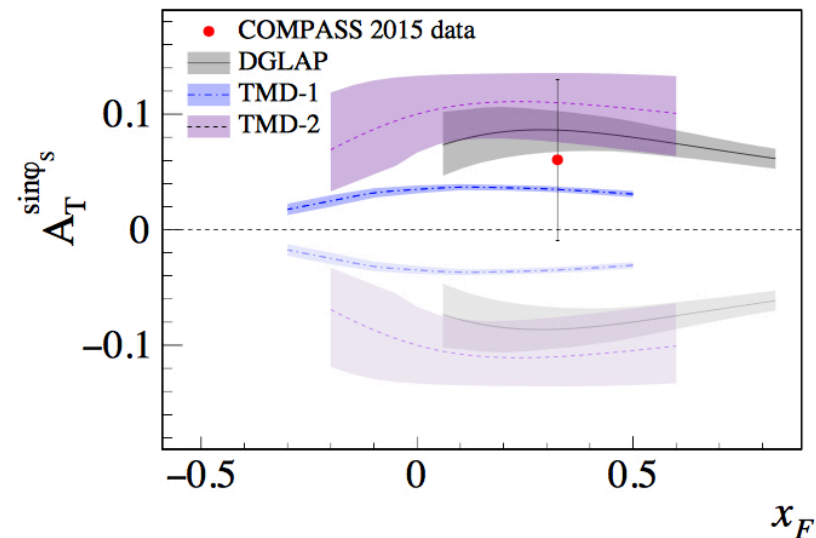
## 2. Phenomenology of process dependence of the Sivers function

- Measurement of Sivers asymmetry in  $p^\uparrow p \rightarrow W^\pm/Z^0 X$  at RHIC (STAR, 1511.06003)



- relevant scale is mass of heavy gauge bosons
- long evolution from measurements of Sivers effect in SIDIS
- calculations with and without TMD evolution lead to very different results for asymmetry
- such measurements can help constrain the (TMD) evolution
- some indication of sign reversal, but not conclusive

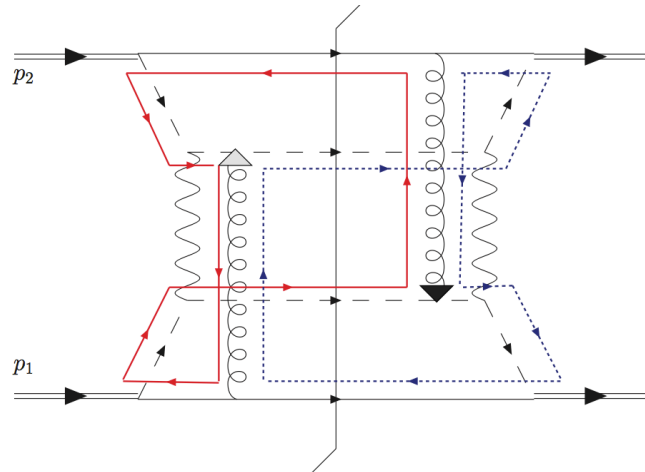
- Measurement of Sivers asymmetry in  $\pi^- p^\uparrow \rightarrow \mu^+ \mu^- X$  at COMPASS  
(COMPASS, 1704.00488)



- scale of measurement:  $4.3 \text{ GeV}^2 \leq m_{\mu\mu}^2 \leq 8.5 \text{ GeV}^2$
- data point favors sign reversal of Sivers function
- Other work on process dependence of Sivers effect
  - simultaneous study of transverse SSAs in DIS and in SIDIS  
(A.M. et al, 1209.3138)
  - study of transverse SSA in  $p^\uparrow p \rightarrow \text{jet} X$   
(Gamberg, Kang, Prokudin, 1302.3213)
- Overall, strong indication from phenomenology that Sivers effect depends on process

# Breakdown of TMD factorization

- Sample process:  $pp \rightarrow \text{jet jet } X$
- Originally thought to obey generalized TMD factorization  
→ definition of TMDs depends on partonic subprocess  
(Bomhof, Mulders, Pijlman, 2004 / ... / Collins, Qiu, 2007 / Collins, 2007)
- But, even generalized TMD factorization breaks down (Rogers, Mulders, 1001.2977)

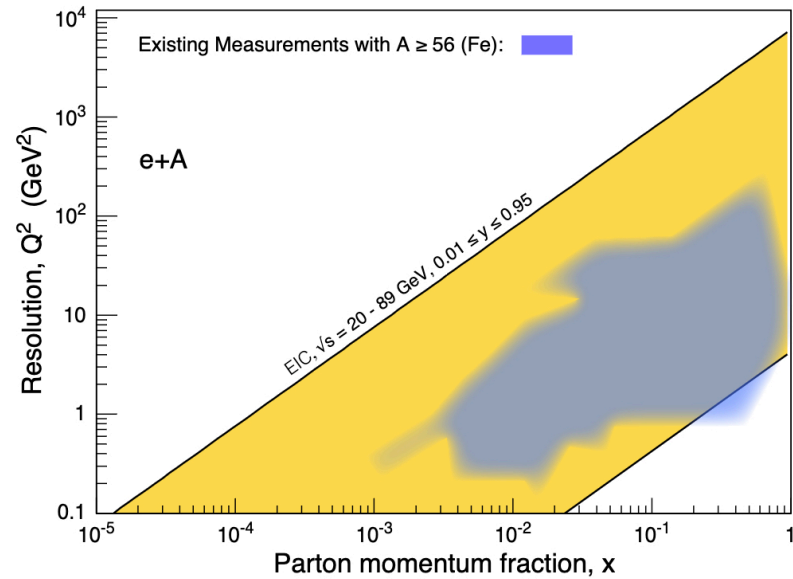
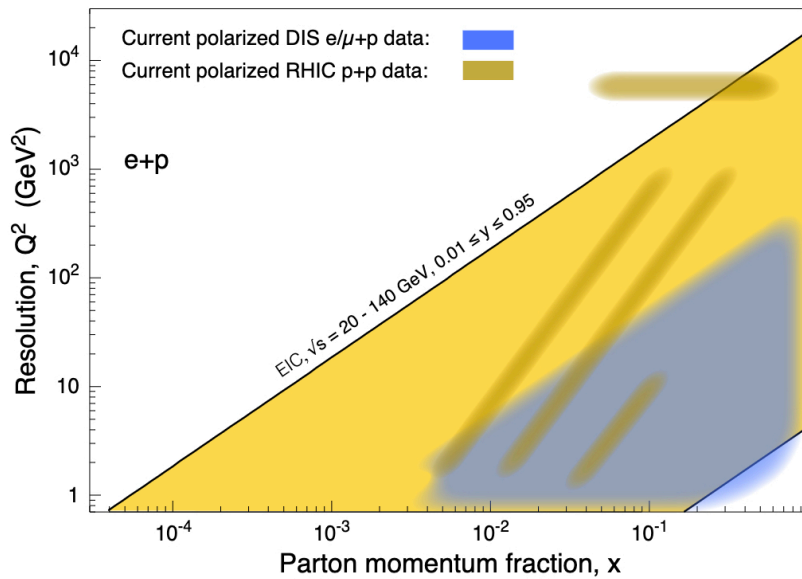


- complicated color flow does not allow one to define two individual TMDs (color-entanglement)
- specific to non-Abelian gauge theory

# TMDs: Opportunities at the EIC

- Kinematics

(figure from EIC Yellow Report, 2103.05419)

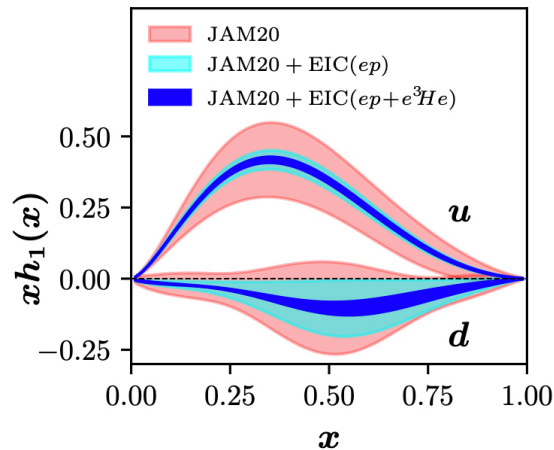


- Compared to fixed-target experiments: much larger kinematic coverage
- Compared to HERA collider: higher luminosity, polarization, nuclear targets

- Example: transversity distribution  $h_1^q$

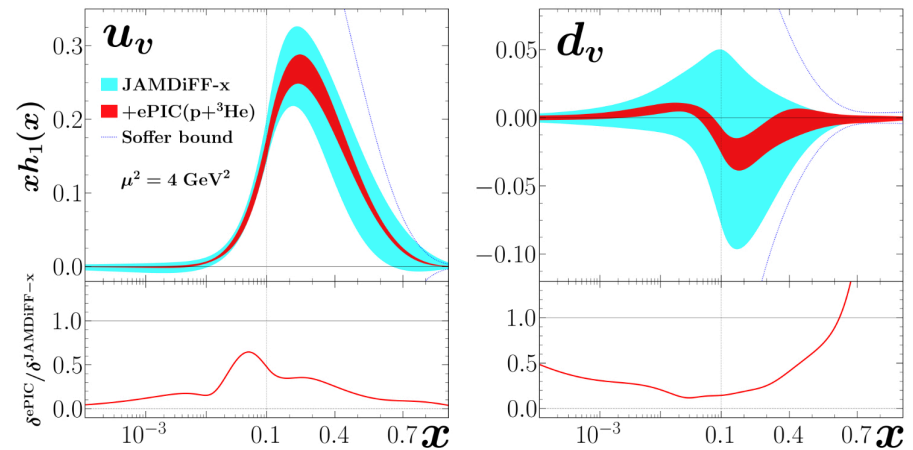
### single-hadron production

(EIC Yellow Report, 2103.05419)



### dihadron production

(Sawaya et al, 2606.00362)

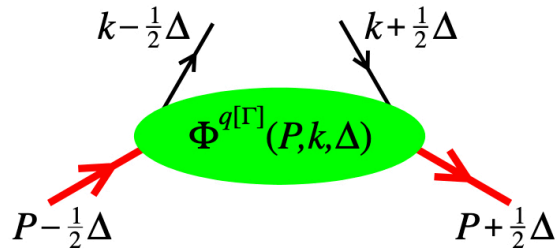


- significant reduction of uncertainties
- proton and  $^3\text{He}$  target needed for constraints on both  $h_1^u$  and  $h_1^d$
- reduced uncertainties provide strong constraints on tensor charges

$$\delta q(\mu) = \int_{-1}^1 dx h_1^q(x, \mu)$$

# GPDs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p + p') \quad \Delta = p' - p$$

$$\Phi^{q[\Gamma]}(P, k, \Delta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle$$

$$\Delta = \left( -2\xi P^+, \frac{\xi \vec{\Delta}_\perp^2 + 4\xi M^2}{4(1 - \xi^2)P^+}, \vec{\Delta}_\perp \right)$$

- GPDs defined through

$$\begin{aligned} \Phi_{\text{GPD}}^{q[\Gamma]}(P, x, \xi, \vec{\Delta}_\perp) &= \int d^4 k \Phi^{q[\Gamma]}(P, k, \Delta) \delta(k^+ - xP^+) \\ &= \int \frac{dz^-}{4\pi} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle \Big|_{z^+ = \vec{z}_\perp = 0} \end{aligned}$$

# GPDs of Quarks

- Definition of chiral-even leading-twist quark GPDs (Diehl, hep-ph/0101335)

$$\Phi_{\text{GPD}}^{q[\gamma^+]} = \frac{1}{2P^+} \bar{u}(p') \left[ \gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu} \Delta_\mu}{2M} E^q(x, \xi, t) \right] u(p)$$

$$\Phi_{\text{GPD}}^{q[\gamma^+ \gamma_5]} = \frac{1}{2P^+} \bar{u}(p') \left[ \gamma^+ \gamma_5 \tilde{H}^q(x, \xi, t) + \frac{\Delta^+ \gamma_5}{2M} \tilde{E}^q(x, \xi, t) \right] u(p)$$

$$t = -\frac{1}{1 - \xi^2} \left( 4 \xi^2 M^2 - \vec{\Delta}_T^2 \right)$$

- 8 leading-twist quark GPDs, and 8 leading-twist gluon GPDs
- chiral-odd GPDs hard to measure (Collins, Diehl, hep-ph/9907498)
- relation to PDFs in forward limit  $\Delta = 0$

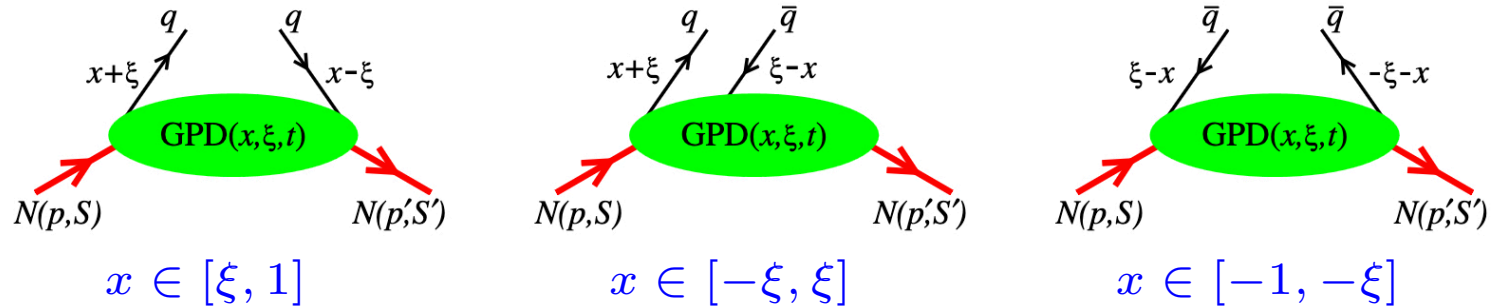
$$H^q(x, 0, 0) = f_1^q(x) \quad \tilde{H}^q(x, 0, 0) = g_1^q(x) \quad H_T^q(x, 0, 0) = h_1^q(x)$$

- relation to form factors (special case of polynomiality)

$$\int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t) \quad \int_{-1}^1 dx E^q(x, \xi, t) = F_2^q(t)$$

- Interpretation of GPDs (for  $\xi > 0$ )

(figure from 2507.12664)



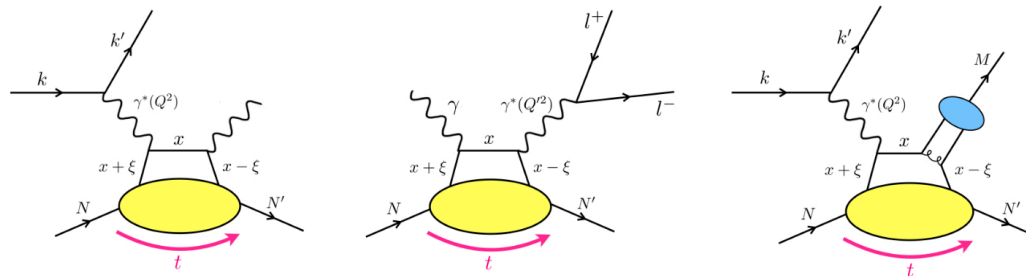
- GPDs themselves have no density interpretation
- For  $x \in [\xi, 1]$ , GPDs can be considered generalizations of quark PDFs
- For  $x \in [-1, -\xi]$ , GPDs can be considered generalizations of antiquark PDFs

$$H^q(x, 0, 0) = f_1^q(x) = -f_1^{\bar{q}}(-x) \quad \text{etc}$$

- ERBL (Efremov, Radyushkin, Brodsky, Lepage) region has no counterpart in PDFs

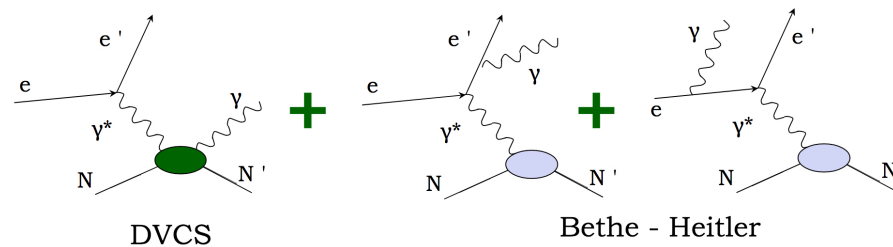
# Key Processes for Measuring GPDs

- Deep virtual Compton scattering (DVCS), Time-like Compton scattering (TCS), Deep virtual meson production (DVMP)



- GPDs appear at the amplitude level in hard exclusive reactions
- data available for all those processes

- More details on important DVCS process

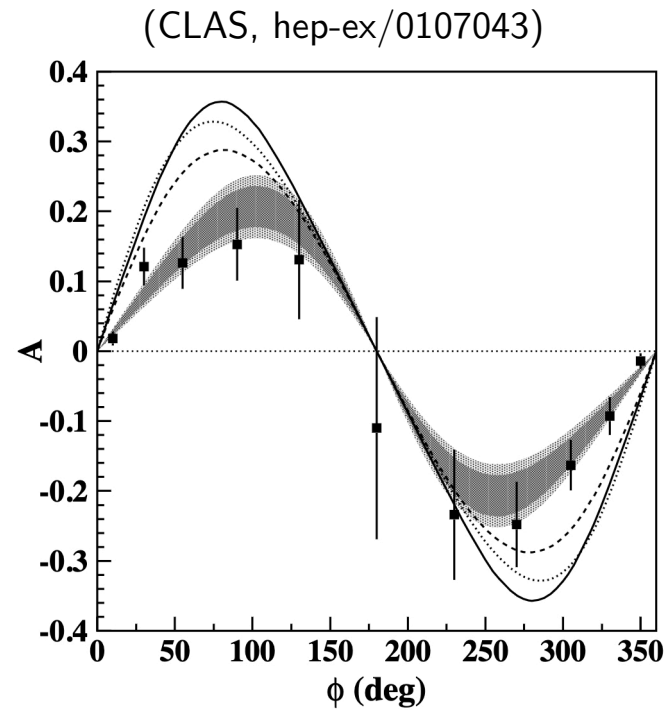
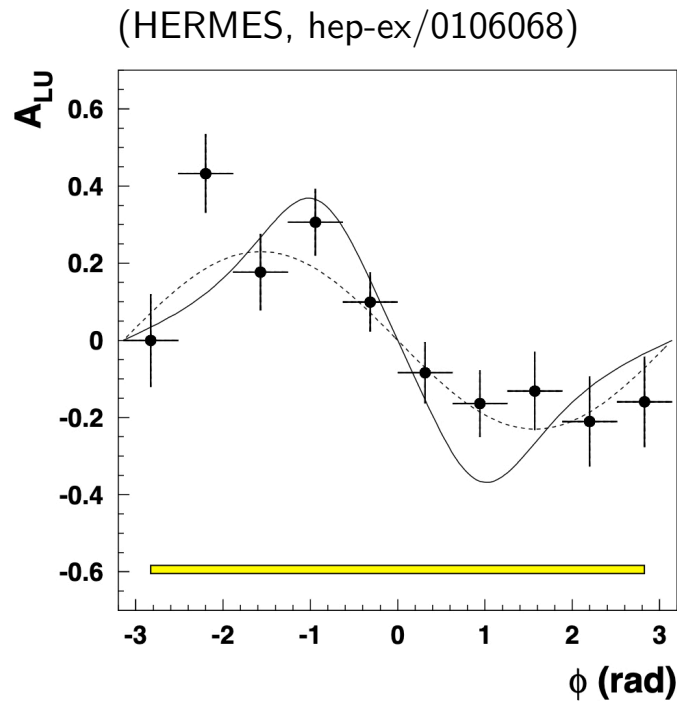


(figure from Sokhan, HUGS Summer School, 2018)

- interference between DVCS and Bethe-Heitler amplitude plays key role
- $\sigma_{\text{int}} \sim \text{Compton form factor} \times \text{electromagnetic form factor}$

# Measurements of GPD Observables

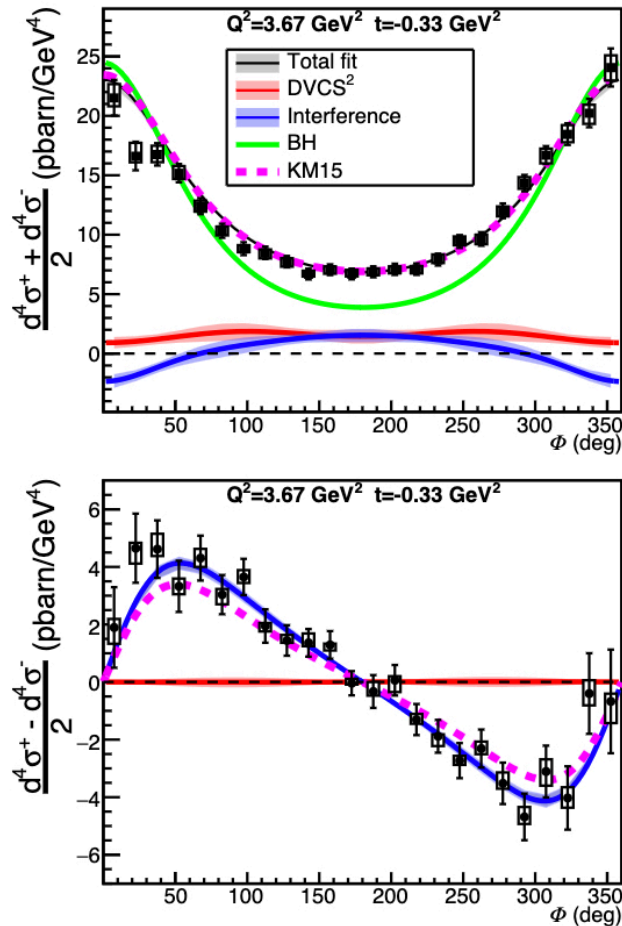
## 1. Pioneering steps: beam-spin asymmetry $A_{LU}$ for DVCS



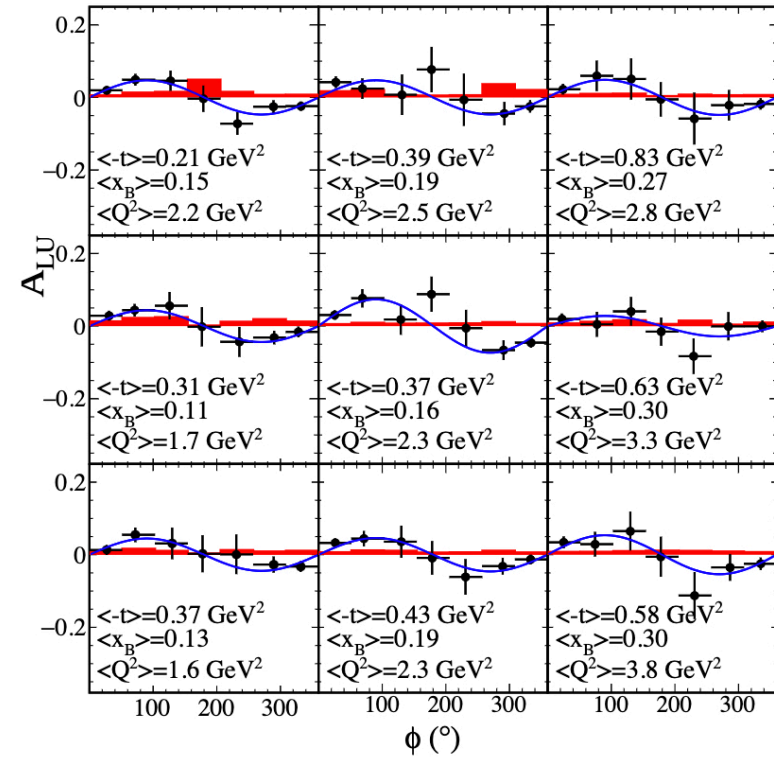
- hydrogen target for both experiments
- pure Bethe-Heitler contributions drops out in  $A_{LU}$
- $A_{LU}$  dominated by interference of DVCS and Bethe-Heitler amplitudes

## 2. Recent DVCS measurements from Jefferson Lab

(Hall A, 2201.03714)



(CLAS, 2406.15539)



- hydrogen target
- data for  $\sigma_{UU}$  and  $\sigma_{LU}$

- DVCS off neutron,  $ed \rightarrow e'n\gamma(p)$
- important for flavor separation

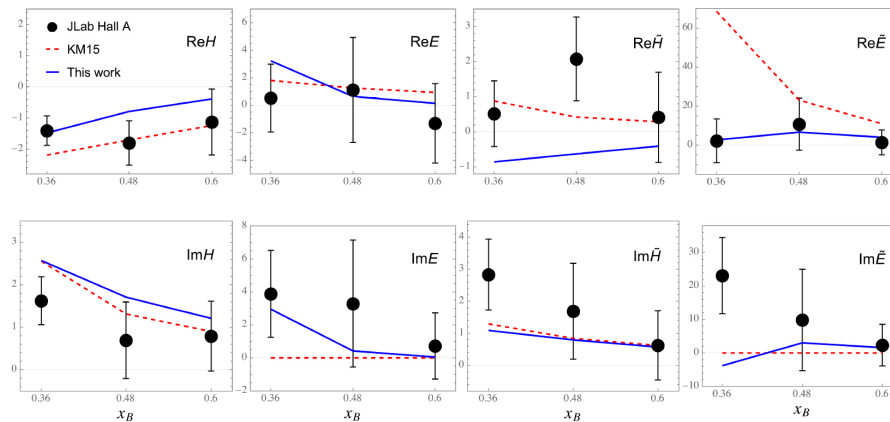
# Compton Form Factors

- Example of Compton form factor (CFF)

$$\mathcal{H}(\xi, t; \mu) = \sum_q e_q^2 \int_{-1}^1 dx H^q(x, \xi, t; \mu) \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) + \mathcal{O}(\alpha_s)$$

- $\mathcal{H}$  has real and imaginary part  $\rightarrow$  total of 8 CFFs

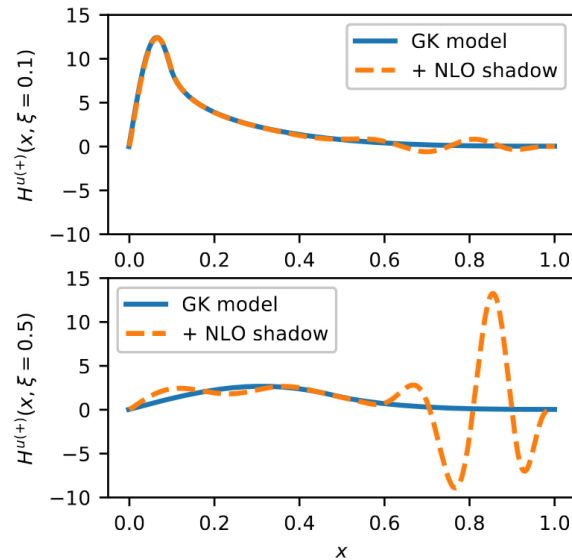
- Extracted CFFs (plot from Guo et al, 2302.07279; data points from JLab Hall A, 2201.03714)



- extraction of CFFs is difficult (multi-variable problem, complicated structure of cross section, power corrections, ...)
- very little known about CFFs at small  $\xi$  (Moutarde, Sznajder, Wagner, 1905.02098)
- how to get from CFFs to GPDs?  $\rightarrow$  deconvolution problem

# Deconvolution Problem

- Systematic study (Bertone et al, 2104.03836)



- by construction, “shadow GPD” does not contribute to CFF at given scale  $\mu_0$
- evolution hardly changes this picture
- conceptual problem for model-independent extraction of GPDs

- Related recent work (Moffat et al, 2303.12006)

- confirms qualitative finding of Bertone et al
- but, with sufficient leverage in  $\xi$  and  $Q^2$  situation may be more optimistic

- Potential way out: other processes, with direct sensitivity to  $x$ -dependence of GPDs

- double DVCS (Guidal, Vanderhaeghen, hep-ph/0208275 / Deja et al, 2303.13668 / ...)
- $\gamma\gamma, \gamma M$  production (Boussarie et al, 1609.03830 / Qiu, Yu, 2205.07846 / ...)

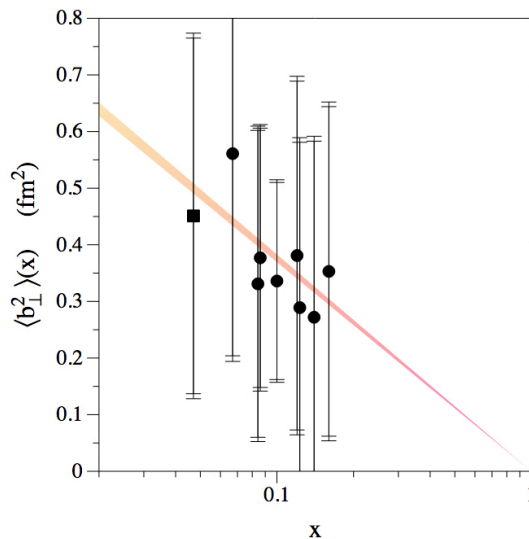
# GPDs and Imaging

- Impact parameter distributions ( $\xi = 0$ ) (Burkardt, hep-ph/0005108)

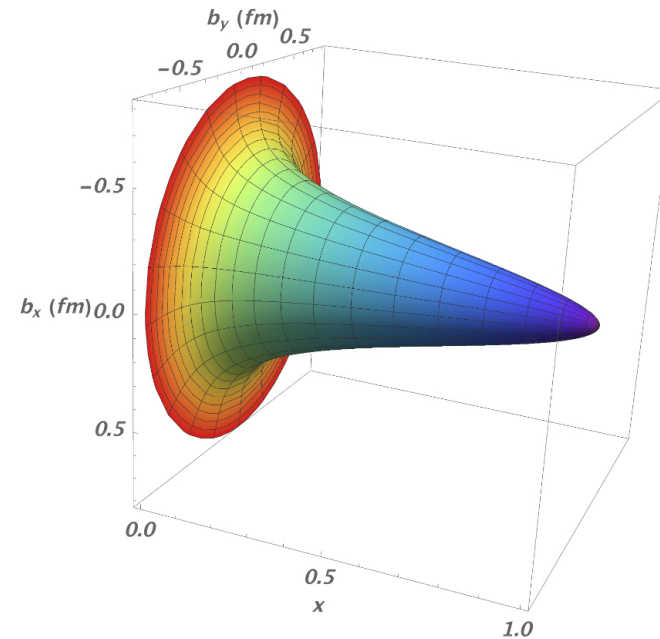
$$\rho^{q[\Gamma]}(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} \Phi_{\text{GPD}}^{q[\Gamma]}(x, \vec{\Delta}_\perp) \Big|_{\xi=0}$$

- Sample results (Dupré, Guidal, Vanderhaeghen, 1704.07330)

$$\langle b_\perp^2 \rangle(x) = \frac{\int d^2 \vec{b}_\perp \vec{b}_\perp^2 H(x, \vec{b}_\perp^2)}{\int d^2 \vec{b}_\perp H(x, \vec{b}_\perp^2)}$$



impact parameter distribution



- extraction of  $H(x, \vec{b}_\perp^2)$ , using data from Jefferson Lab and HERMES

# GPDs and Spin Sum Rule

- Ji spin sum rule (Ji, hep-ph/9603249)

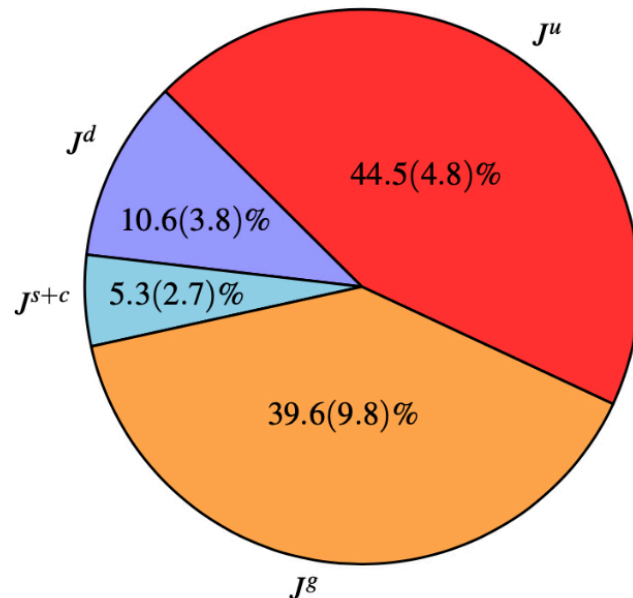
$$\frac{1}{2} = \sum_q J_{Ji}^q + J_{Ji}^g = \frac{1}{2} \Delta\Sigma + \sum_q L_{Ji}^q + J_{Ji}^g$$

$$J_{Ji}^q = \left( A^q(t) + B^q(t) \right) \Big|_{t=0} = \int_{-1}^1 dx x (H^q + E^q) \Big|_{t=0} \quad \text{similarly for } J_{Ji}^g$$

- Lattice results for parton angular momenta

(figure from 2507.12664)

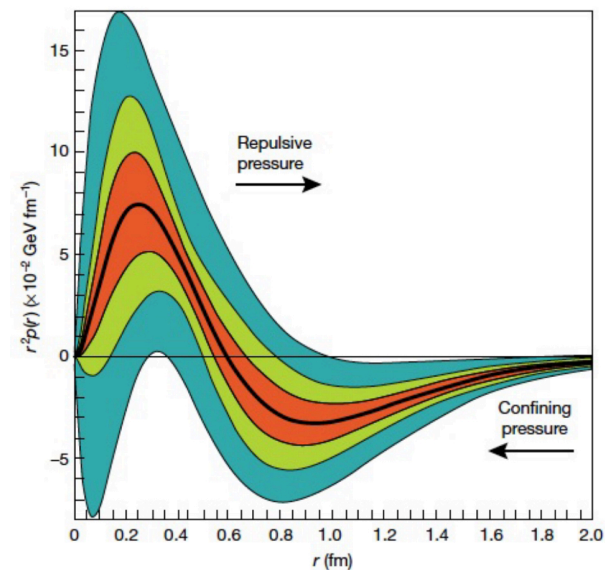
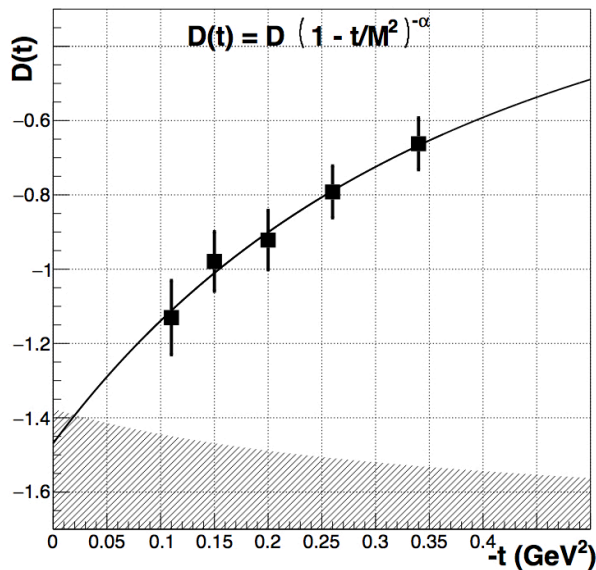
**ETMC20**  $\mu^2 = 4 \text{ GeV}^2$



- based on results from ETMC, 2003.08486
- sum rule not imposed in lattice calculation
- results from experiment exist, but errors are large

# GPDs and Pressure Distribution

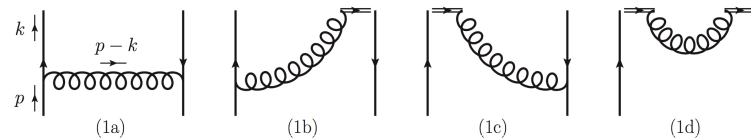
- Form factor  $D(t)$  of energy momentum tensor contains information about pressure distribution of partons (Polyakov, hep-ph/0210165 / Polyakov, Schweitzer, 1805.06596)
  - $D(t)$  appears in real part of CFF  $\mathcal{H}(\xi, t)$
- Results based on data from JLab (Burkert, Elouadrhiri, Girod, Nature 2018, 2104.02031)



- results for pressure distribution for 3 scenarios:
  - (i) prior to JLab data, (ii) with JLab data, (iii) with future JLab data
- exciting developments
- caveats: Kumerički, Nature 2019 / Ji, Yang, 2508.16727 / ...

# GPDs and Lattice QCD

- Access to  $x$ -dependence of PDFs in LQCD via Euclidean parton correlators
  - quasi-PDFs (Ji, 1305.1539)
  - pseudo-PDFs (Radyushkin, 1705.01488)
  - closely related previous works (Braun et al, hep-ph/9410318 / ...)
- Light-cone PDFs and quasi-PDFs have the same non-perturbative physics

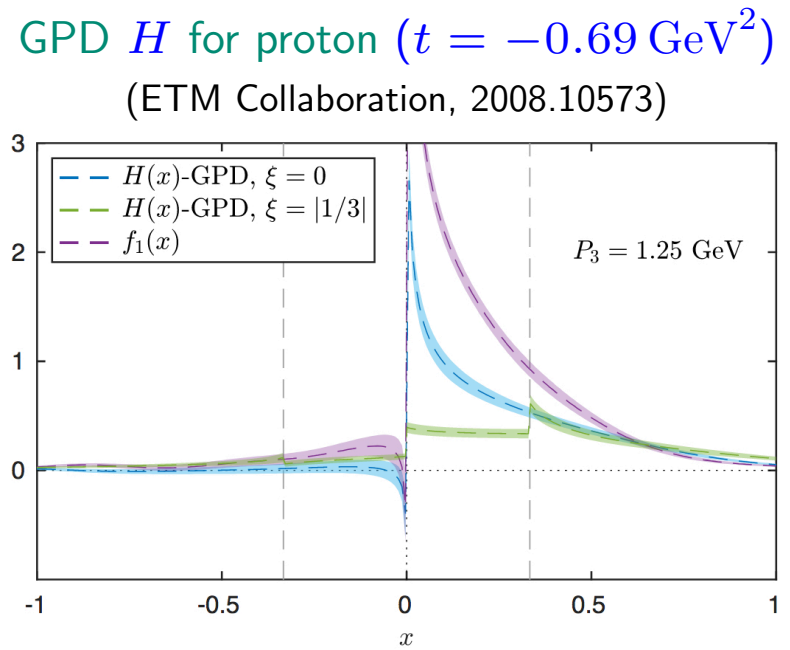
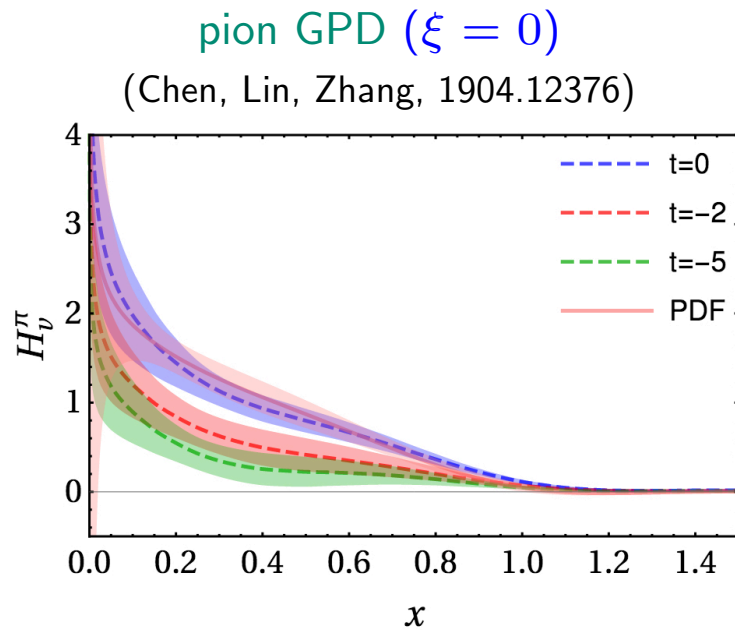


sample result, with nonzero gluon mass  $m_g$  as IR regulator

$$f_1^{(1a)} = \frac{\alpha_s C_F}{2\pi} (1-x) \left( \ln \frac{\mu^2}{x m_g^2} - 2 \right) \quad 0 < x < 1$$

$$f_{1,Q}^{(1a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x) \ln \frac{x-1}{x} - 1 & x < 0 \\ (1-x) \ln \frac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x) \ln \frac{x}{x-1} + 1 & x > 1 \end{cases}$$

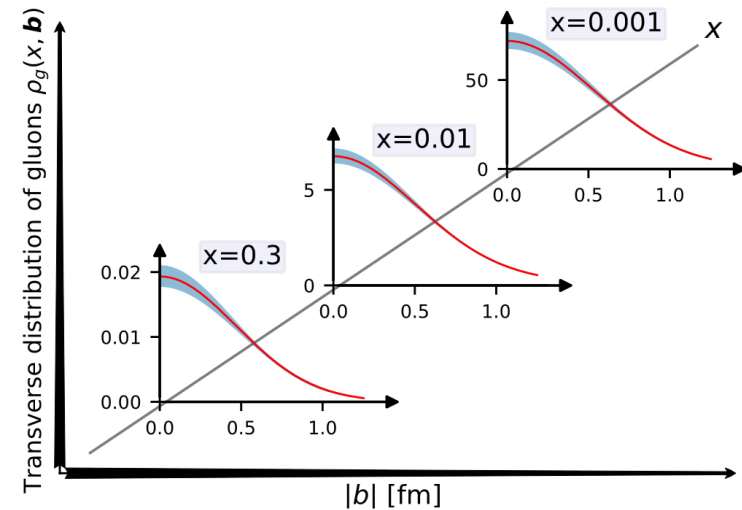
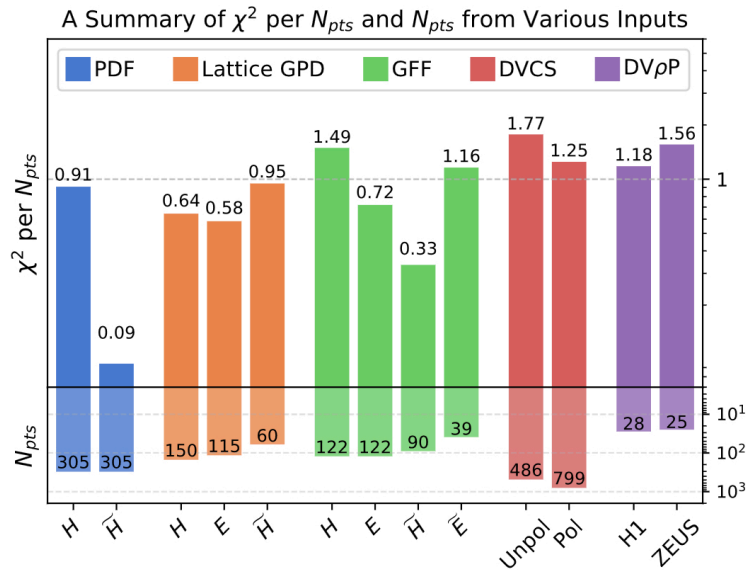
- Pioneering results for GPDs



- pioneering LQCD results of GPDs already encouraging
- significant progress in recent years

# GPDs from Experimental and LQCD Data

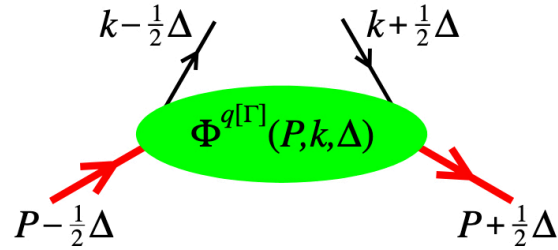
- Experimental and LQCD data can play complementary roles
- Recent results from GUMP analysis (Guo et al, 2509.08037)



- experimental data for DVCS and exclusive  $\rho$  production
- LQCD data for moments of GPDs and  $x$ -dependent GPDs
- In the future, more such combined analyses of this kind are expected to emerge

# GTMDs: Correlator and Kinematics

- Object of interest (neglecting spin labels)



$$P = \frac{1}{2}(p + p') \quad \Delta = p' - p$$

$$\Phi^{q[\Gamma]}(P, k, \Delta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle$$

$$\Delta = \left( -2\xi P^+, \frac{\xi \vec{\Delta}_\perp^2 + 4\xi M^2}{4(1 - \xi^2)P^+}, \vec{\Delta}_\perp \right)$$

- GTMDs defined through

$$\begin{aligned} \Phi_{\text{GTMD}}^{q[\Gamma]}(P, x, \vec{k}_\perp, \xi, \vec{\Delta}_\perp) &= \int dk^+ dk^- \Phi^{q[\Gamma]}(P, k, \Delta) \delta(k^+ - xP^+) \\ &= \int \frac{dz^- d^2 \vec{z}_\perp}{2(2\pi)^3} e^{ik \cdot z} \langle p' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi^q(\frac{z}{2}) | p \rangle \Big|_{z^+ = 0} \end{aligned}$$

# GTMDs of Quarks

- Definition of chiral-even leading-power quark GTMDs (Meissner, A.M., Schlegel, 0906.5323)

$$\Phi_{\text{GTMD}}^{q[\gamma^+]} = \frac{1}{2M} \bar{u}(p') \left[ F_{1,1}^q + \frac{i\sigma^{i+} k_{\perp}^i}{P^+} F_{1,2}^q + \frac{i\sigma^{i+} \Delta_{\perp}^i}{P^+} F_{1,3}^q + \frac{i\sigma^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} F_{1,4}^q \right] u(p)$$

$$\Phi_{\text{GTMD}}^{q[\gamma^+ \gamma_5]} = \frac{1}{2M} \bar{u}(p') \left[ -\frac{i\varepsilon_{\perp}^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} G_{1,1}^q + \frac{i\sigma^{i+} \gamma_5 k_{\perp}^i}{P^+} G_{1,2}^q + \frac{i\sigma^{i+} \gamma_5 \Delta_{\perp}^i}{P^+} G_{1,3}^q + i\sigma^{+-} \gamma_5 G_{1,4}^q \right] u(p)$$

- General results

- GTMDs depend on 5 variables:  $x, \xi, \vec{k}_{\perp}^2, \vec{k}_{\perp} \cdot \vec{\Delta}_{\perp}, \vec{\Delta}_{\perp}^2$
- 16 leading-power GTMDs for quarks (Meissner, A.M., Schlegel, 0906.5323)
- 16 leading-power GTMDs for gluons (Lorcé, Pasquini, 1307.4497)
- GTMDs have real and imaginary part

# Wigner Functions

- Wigner functions in non-relativistic QM (Wigner, 1932)
  - counterpart of classical phase space distribution
  - calculable from wave function

$$\begin{aligned}\mathcal{W}(x, k) &= \int \frac{dx'}{2\pi} e^{-i k x'} \psi^* \left( x - \frac{x'}{2} \right) \psi \left( x + \frac{x'}{2} \right) \\ &= \int \frac{dk'}{2\pi} e^{+i k' x} \tilde{\psi}^* \left( k - \frac{k'}{2} \right) \tilde{\psi} \left( k + \frac{k'}{2} \right)\end{aligned}$$

- relation to densities and observables

$$|\psi(x)|^2 = \int dk \mathcal{W}(x, k)$$

$$|\tilde{\psi}(k)|^2 = \int dx \mathcal{W}(x, k)$$

$$\langle O(x, k) \rangle = \int dx dk O(x, k) \mathcal{W}(x, k)$$

- $\mathcal{W}(x, k)$  can become negative (quantum effect)  $\rightarrow$  quasi-probability distribution
- $\mathcal{W}(x, k)$  may help to study transition btw quantum and classical mechanics

- Partonic Wigner functions

(Ji, hep-ph/0304037 / Belitsky, Ji, Yuan, hep-ph/0307383 / Lorcé, Pasquini, 1106.0139)

- correlator for Wigner functions ( $\xi = 0$ ) as Fourier transform of GTMD correlator

$$\mathcal{W}^{q[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} \Phi_{\text{GTMD}}^{q[\Gamma]}(x, \vec{k}_\perp, \vec{\Delta}_\perp) \Big|_{\xi=0}$$

- relation to densities and observables

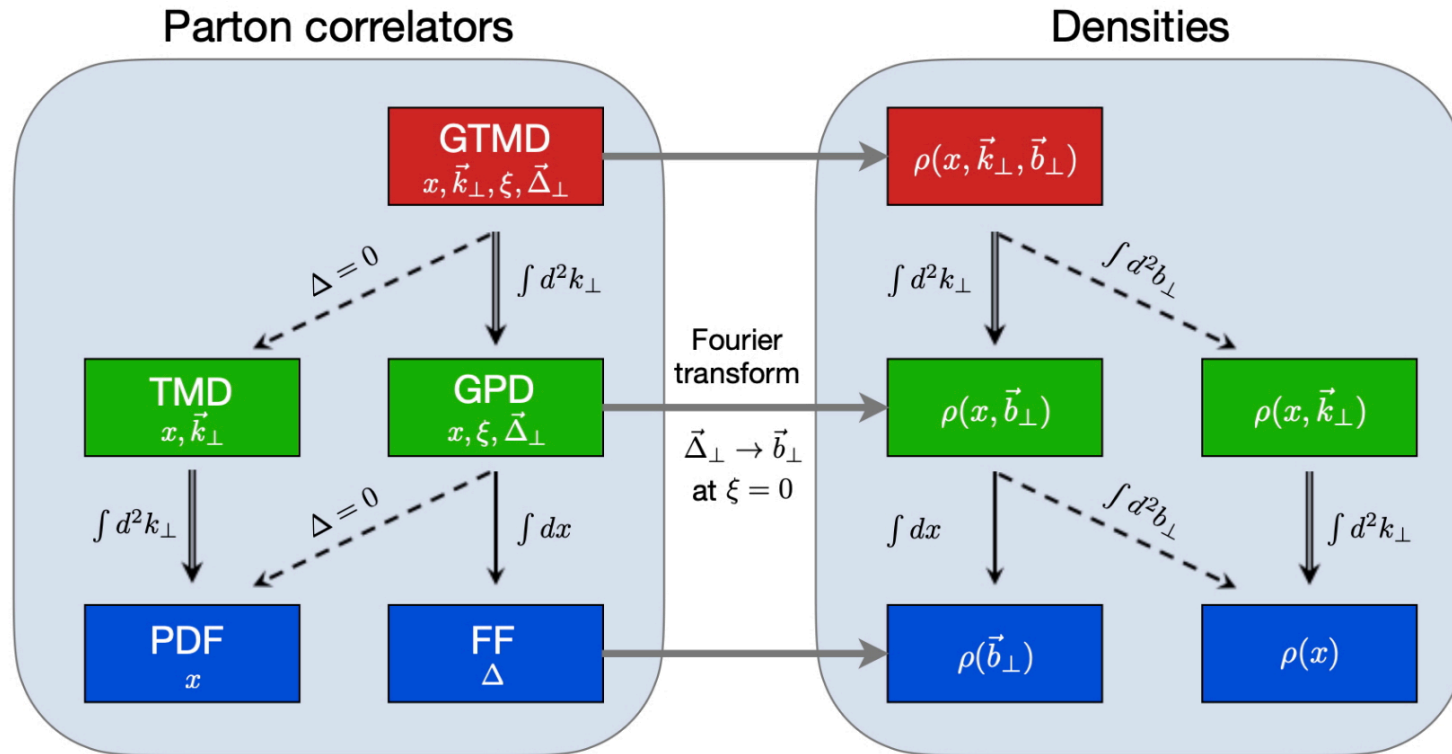
$$\rho^{q[\Gamma]}(x, \vec{b}_\perp) = \int d^2 \vec{k}_\perp \mathcal{W}^{q[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

$$\Phi_{\text{TMD}}^{q[\Gamma]}(x, \vec{k}_\perp) = \int d^2 \vec{b}_\perp \mathcal{W}^{q[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

$$\langle O(x, \vec{k}_\perp, \vec{b}_\perp) \rangle = \int dx d^2 \vec{k}_\perp d^2 \vec{b}_\perp O(x, \vec{k}_\perp, \vec{b}_\perp) \mathcal{W}^{q[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

# Parton Structure: Overview

(figure from 2507.12664)



- GTMDs/Wigner functions describe most general parton structure of hadrons
- GTMDs/Wigner functions contain physics beyond GPDs and TMDs
- Calculating/measuring GTMDs is an important goal of parton structure studies

# Orbital Angular Momentum of Partons

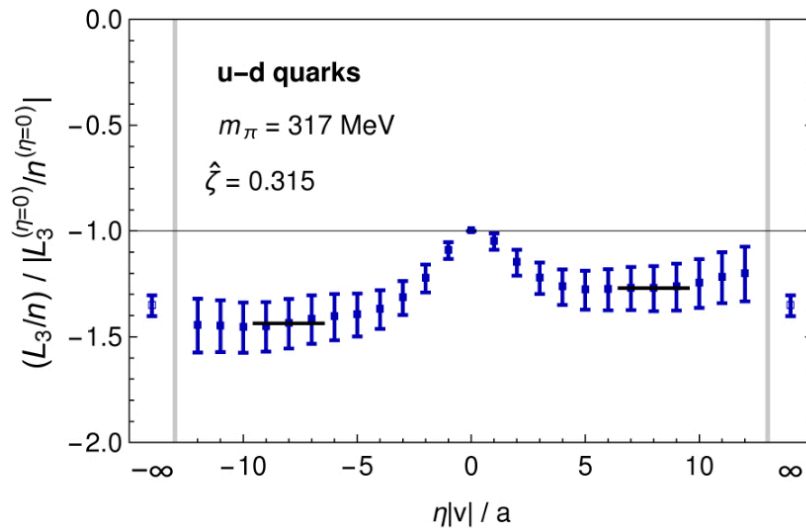
- Jaffe-Manohar spin sum rule (Jaffe, Manohar, 1989)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + \sum_q L_{\text{JM}}^q + L_{\text{JM}}^g$$

- Parton OAM in longitudinally polarized nucleon (Lorcé, Pasquini, 1106.0139 / Hatta, 1111.3547)

$$L_z^q = \int dx d^2\vec{k}_\perp d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho_L^{q[\text{unp}]}(x, \vec{k}_\perp, \vec{b}_\perp) = - \int dx d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q |_{\Delta=0}$$

- Exploratory calculation of  $L_{\text{JM}}^{u-d}$  in lattice QCD (Engelhardt et al, 1701.01536, 1901.00843)

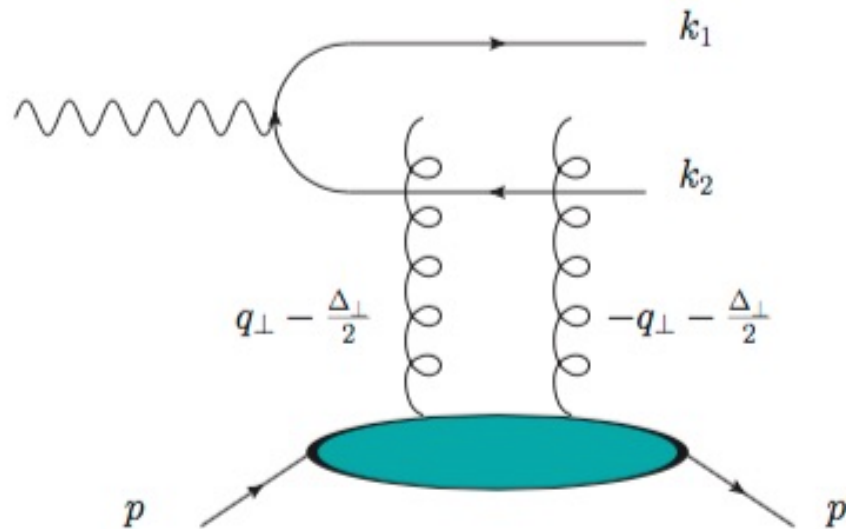


- figure shows  $L_{\text{JM}}^{u-d} / |L_{\text{Ji}}^{u-d}|$  (for large  $\eta|v|/a$ )
- considerable numerical difference btw  $L_{\text{JM}}^{u-d}$  and  $L_{\text{Ji}}^{u-d}$

- Developments in this area are milestone in spin physics

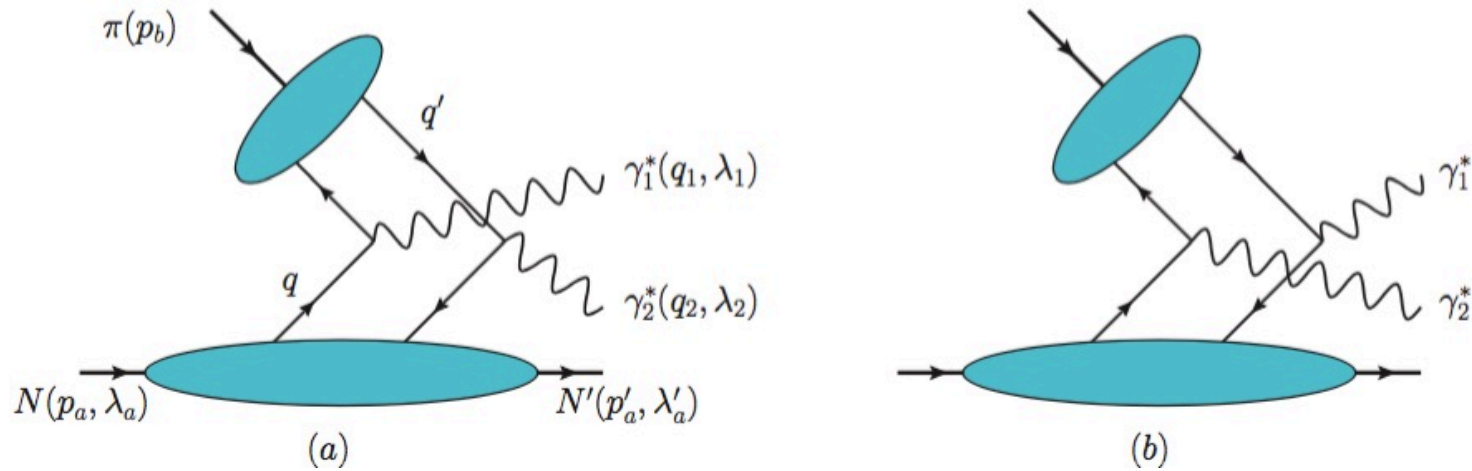
# Observables for GTMDs

- **Example 1: Diffractive Exclusive Back-To-Back Dijet Production in  $\ell N/\ell A$  Collisions**  
(Hatta, Xiao, Yuan, 1601.01585 / Altinoluk et al, 1511.07452)



- direct sensitivity to transverse parton momenta  $\rightarrow$  (gluon) GTMDs
- pioneering proposal for studying gluon GTMDs
- challenge: precise measurement of jet (transverse) momenta
- with target polarization one may address GTMD  $F_{1,4}^g \rightarrow$  gluon OAM  $L_{JM}^g$   
(Ji, Yuan, Zhao, 1612.02438 / Hatta et al, 1612.02445 / Bhattacharya, Boussarie, Hatta, 2201.08709)

- **Example 2: Exclusive Double Drell-Yan Process:  $\pi N \rightarrow (\ell_1^- \ell_1^+) (\ell_2^- \ell_2^+) N'$**   
(Bhattacharya, A.M., Zhou, 1702.04387)



- direct sensitivity to transverse parton momenta → **quark GTMDs**
- **pioneering proposal for studying quark GTMDs**
- at leading-order one is **sensitive to ERBL region only**
- **discussion of all-order factorization** (Echevarria, Gutierrez Garcia, Scimemi, 2208.00021)
- low count rate (amplitude  $T \sim \alpha_{em}^2$ , like double-DVCS)

# Conclusions

- Nucleon structure studies have a long and successful history
- Multi-dimensional structure of hadronic systems (nucleons, nuclei, pions, ...), expressed through TMDs, GPDs and GTMDs, is an extremely rich and dynamic field, which has seen tremendous progress
- Getting reliable information on the 3D parton structure from experiment remains complicated
- Models/approximations can help to further elucidate the underlying physics and guide experiments
- Future experimental data, especially from the EIC, will move the field to the next level
- Impact of LQCD results in this area will continue to increase
- Combining information from experiment and LQCD, wherever appropriate, will further the field