Lattice QCD: From Quarks and Gluons to Hadrons, Nucleons and Nuclei

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- Day 2 : Lattice Applications to Hadron Structure and Beyond

  Form Factors

  Nucleon Spin, Mass, Momentum

  Parton Disributions: Mellin moments and x-dependence

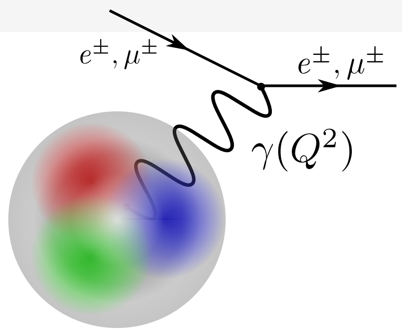
  Nuclear Sensitivity to Symmetry Violations

  (if time allows) Confinement, Entanglement and Quantum Information

Nucleon Vector Form Factors and Radii

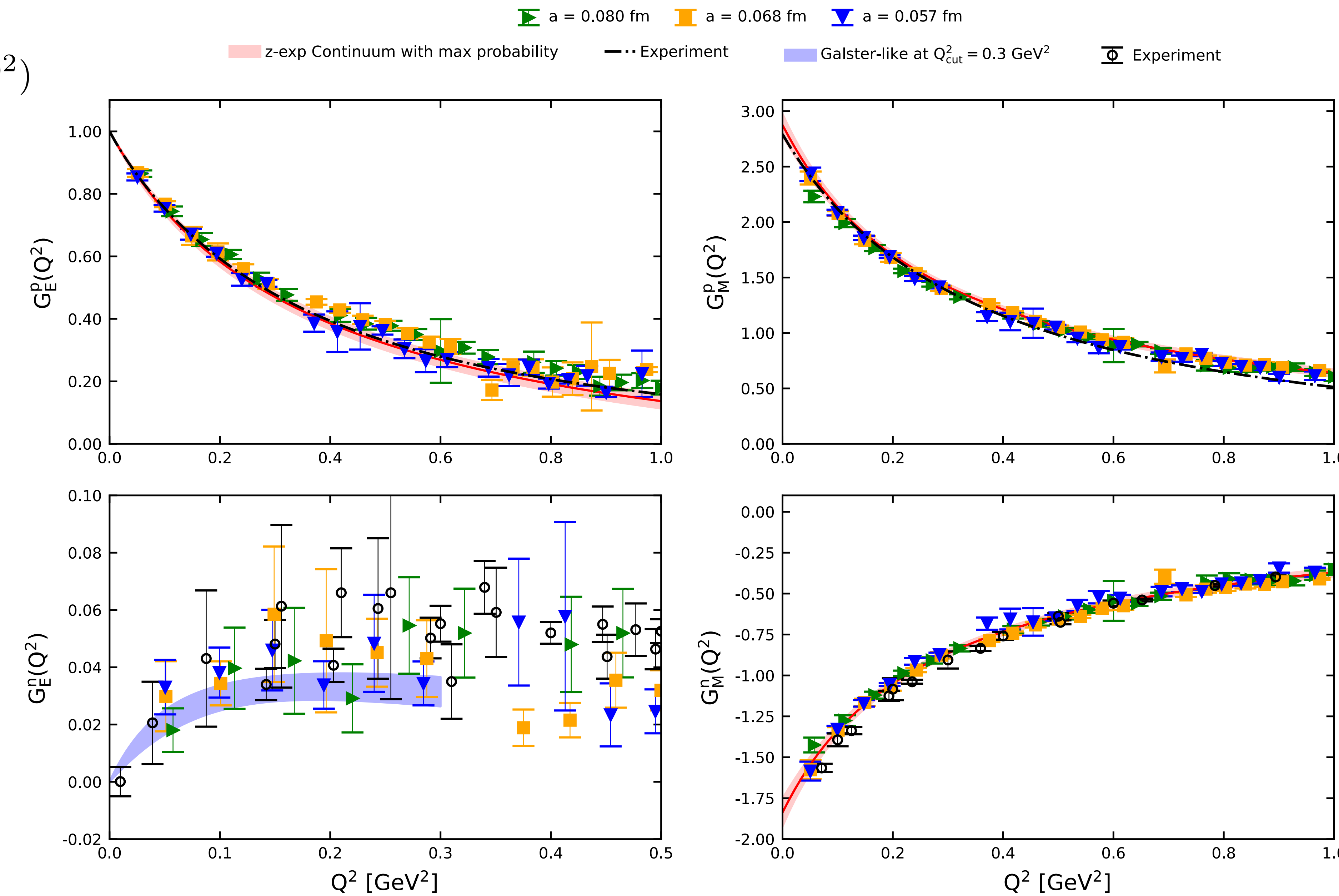
- Vector form factors: charge & magnetization structure

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$



$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

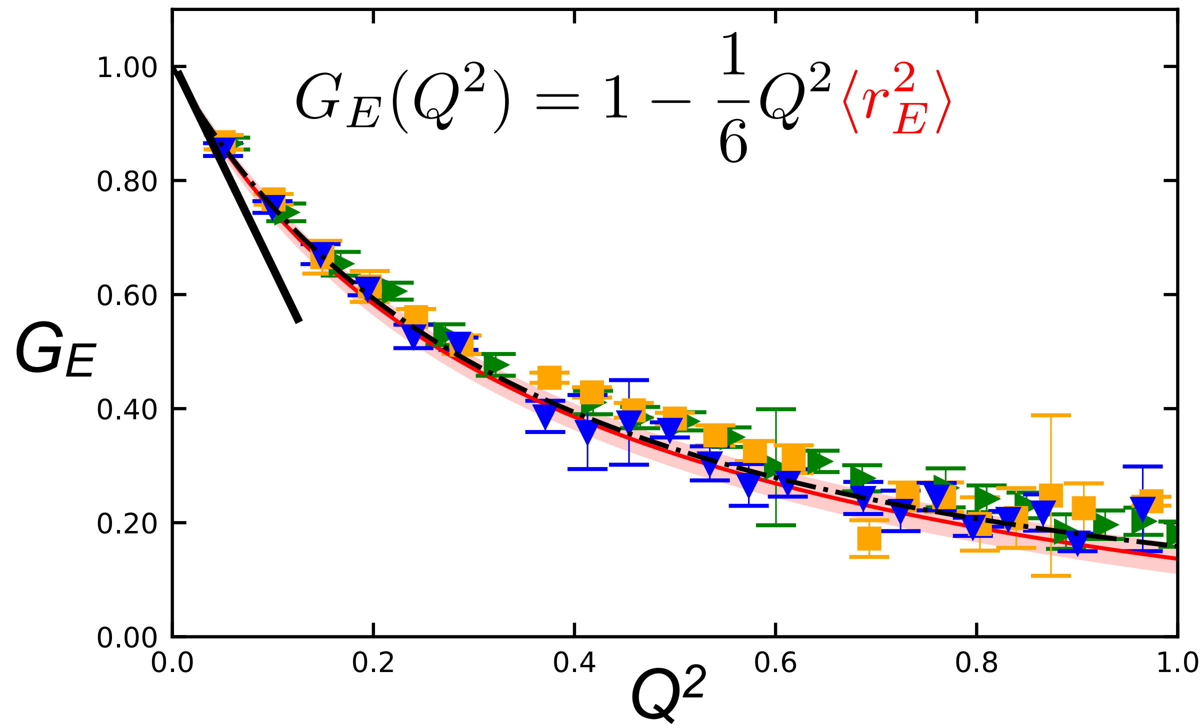


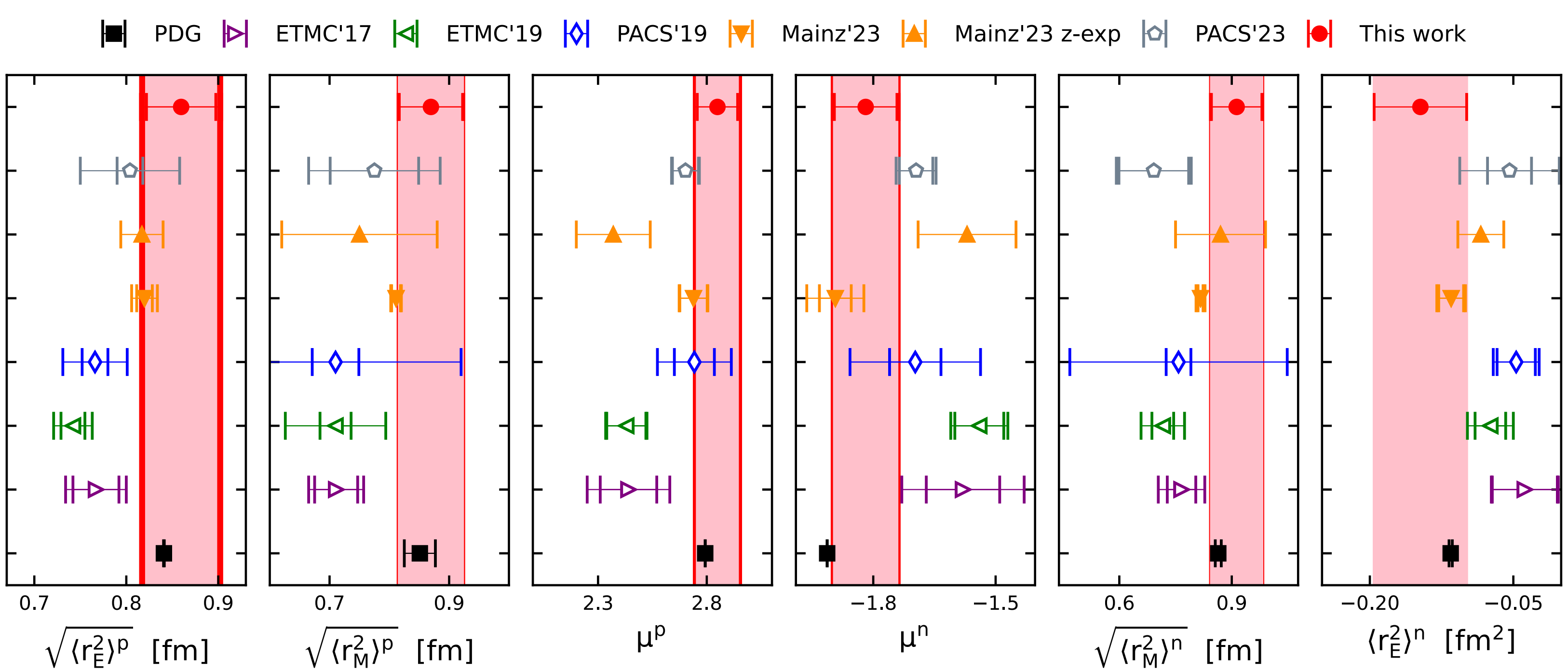
[Alexandrou et al (ETM collaboration), arXiv:2507.20910]

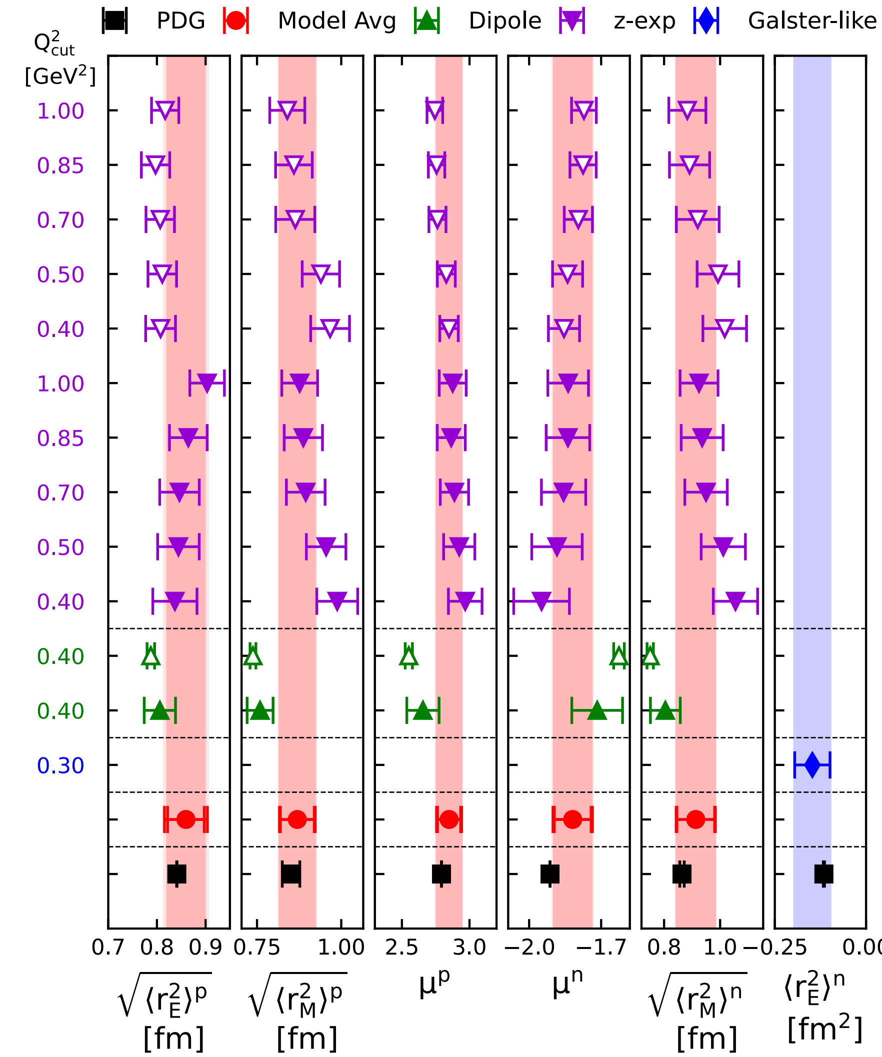
Nucleon Vector Form Factors and Radii (2)

- Electric and Magnetic Radii

$$G_{E,M}(Q^2) = G_{E,M}(0)\left[1 - \frac{1}{6}Q^2\langle r_E^2\rangle + O(Q^4)\right]$$



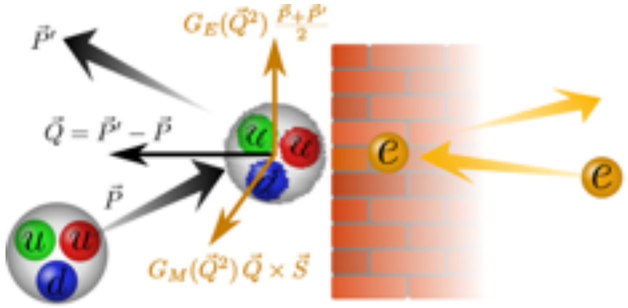


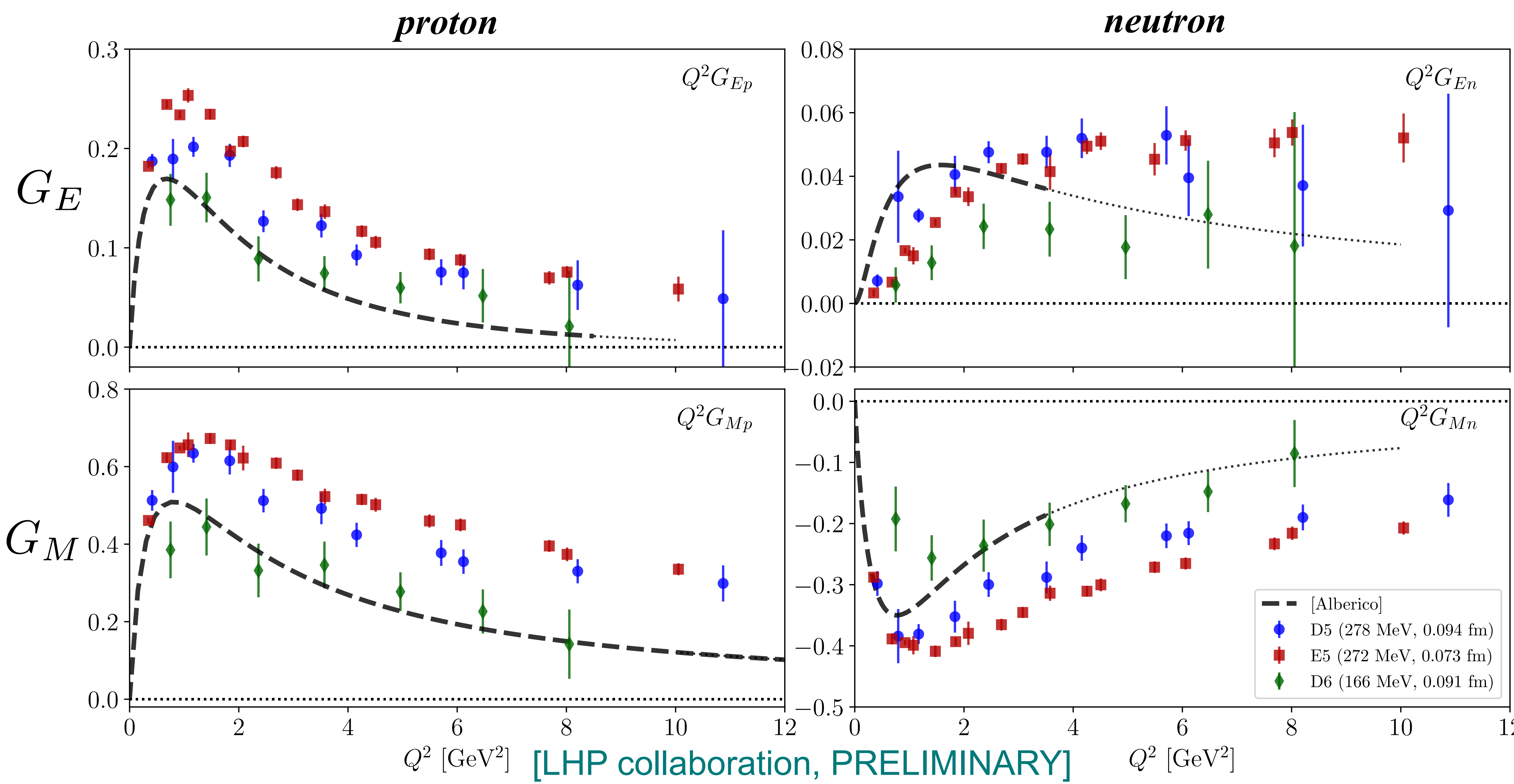


[Alexandrou et al (ETM collaboration), arXiv:2507.20910]

Nucleon Vector Form Factors : High Momentum

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$

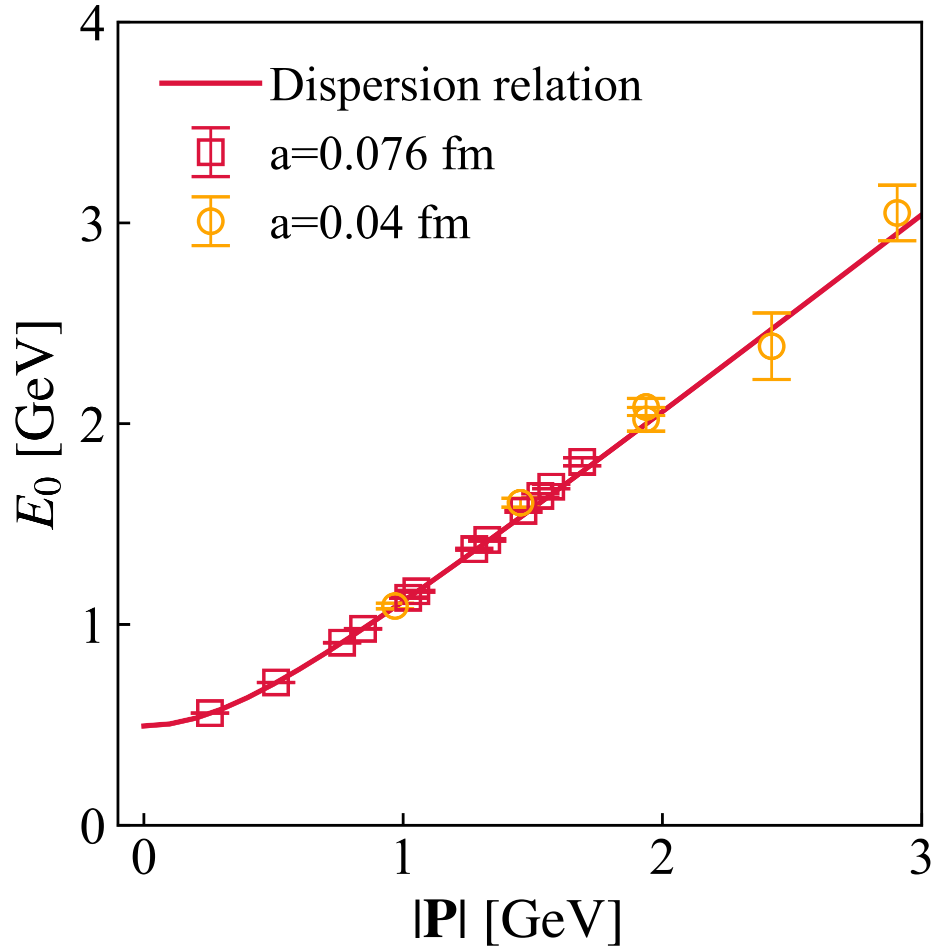




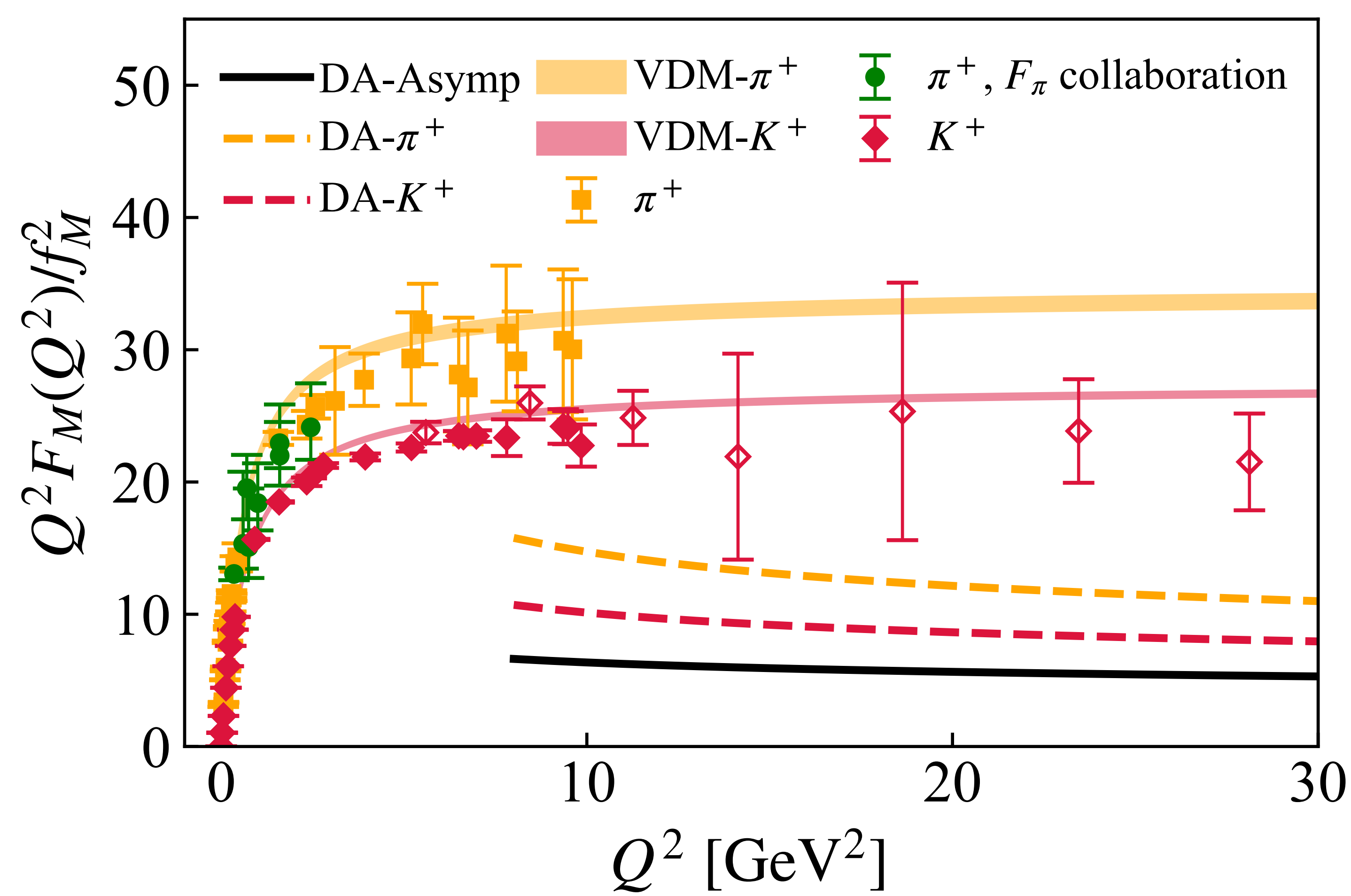
[LHP collaboration, PRELIMINARY]

Pion&Kaon Vector Form Factors : High Momentum

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$



Kaon dispersion relation from lattice

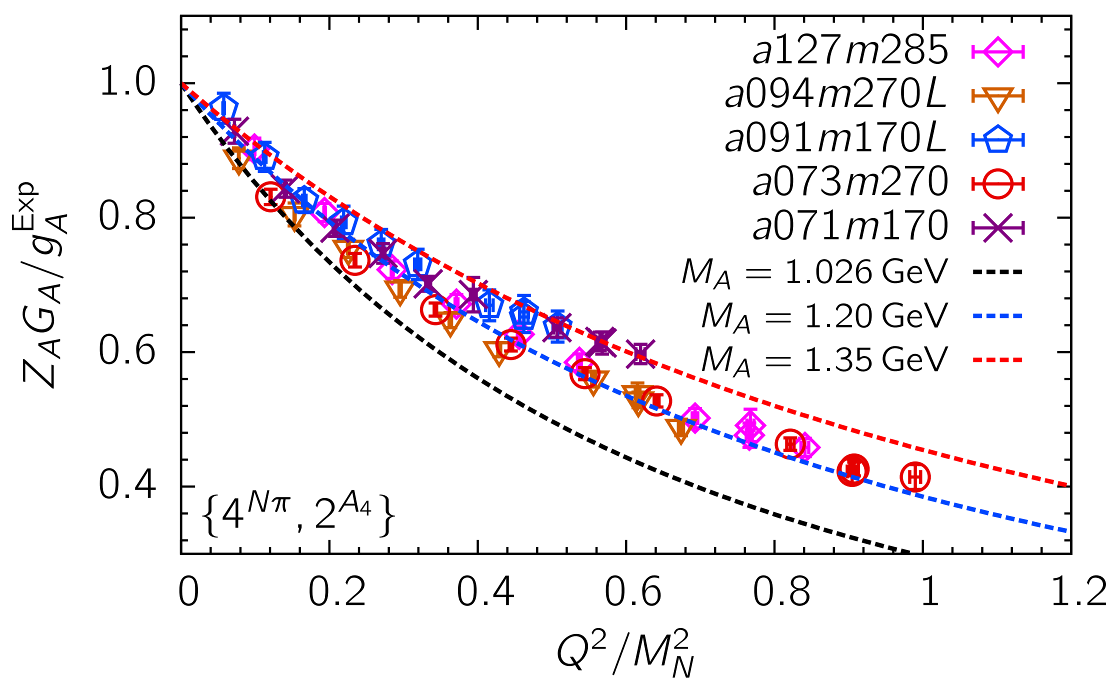


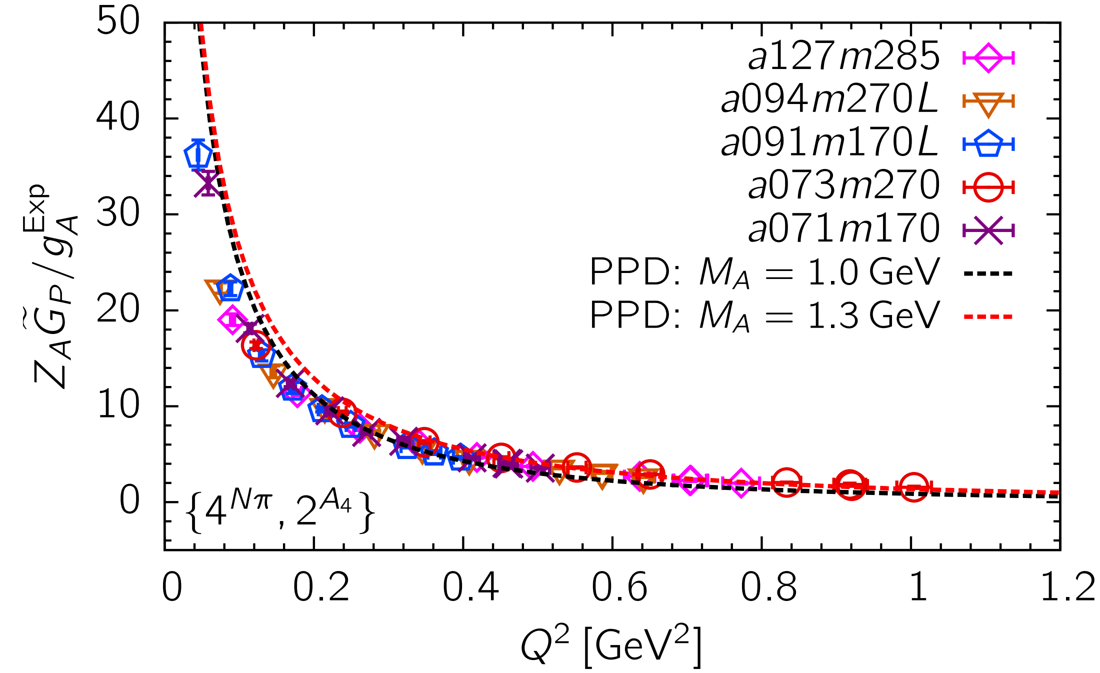
Pion and Kaon form factors

Nucleon Axial Form Factors

- Axial form factors: neutrino scattering , muon capture

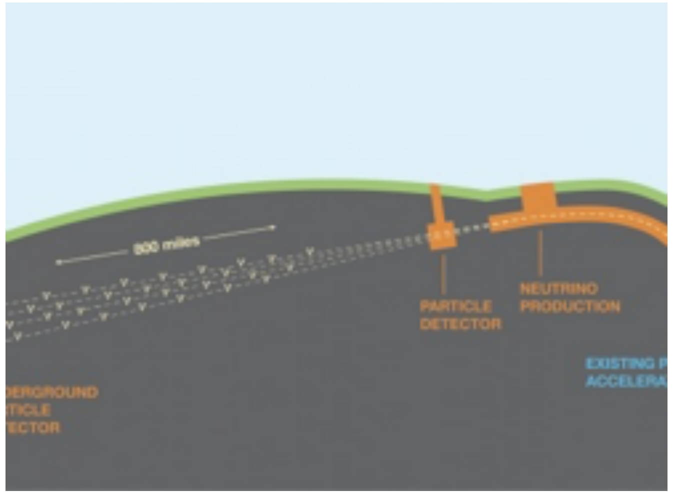
$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

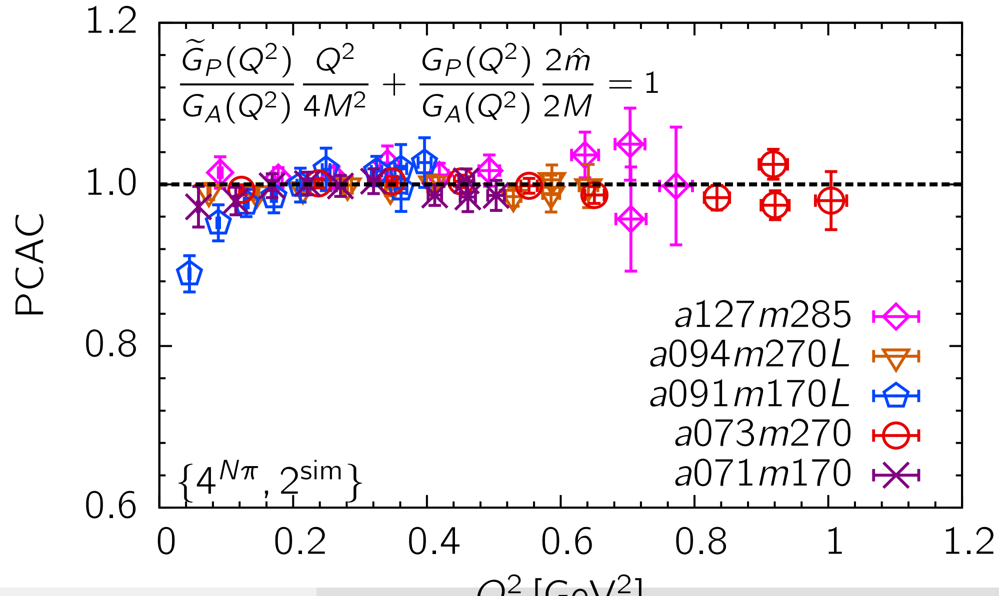




G_A : Neutrino Flux measurement at DUNE

G_P : Muon capture $p+\mu \;\to n+\nu_\mu$
Partial axial current conservation (PCAC)





Proton Spin Decomposition and Sum Rule

Deeply-Virtual Compton Scattering(DVCS)

DGLAP ;
J/ψ - production
at Electron-Ion Collider

polarized p-p collisions at RHIC
scaling violations in g_1

Spin puzzle:
quarks contribute ~30%
of the nucleon spin [EMC'89]

$$\frac{1}{2} = J^{quarks} + J^{Glue}$$

$$(S^q + L^q)$$

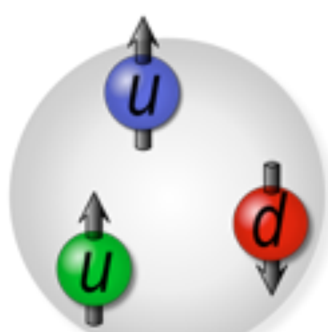
$$(S^G + L^G + L^{qG\text{-int}})$$

No local QFT definitions for S^G, L^G, L^{qG};
interpretation depends on experiment

polarized parton
distributions $\Delta q(x)$
from DIS (EMC exp)

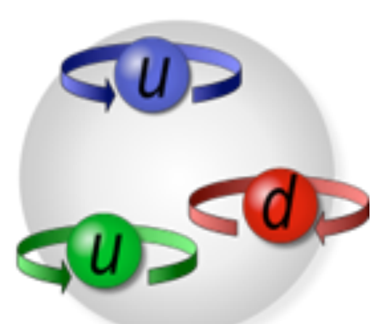
Wigner-like parton
distributions (GTMDs)?
Twist-3 corrections to DVCS?

[K.F.Liu and C.Lorce, contribution to EPJA
"3D Structure of the Nucleon", 1508.00911]



$$\frac{1}{2}\bar{q}\Sigma^{0i}q$$

+



$$\vec{L}^q = \vec{r} \times \vec{p}$$

=

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x\,\left[x^j\,T^{0k} - x^k\,T^{0j}\right]$$

Proton Spin Decomposition

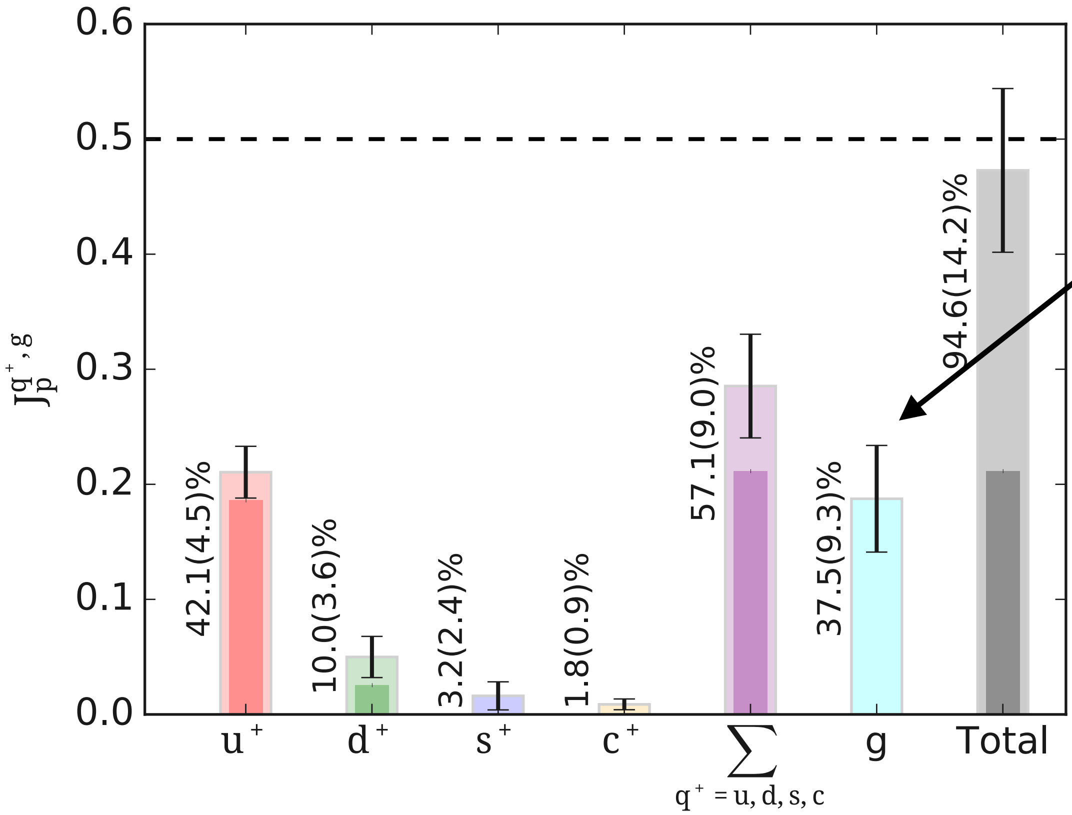
- Energy-Momentum distribution in nucleon: Generalized Form Factors A_{20}, B_{20}, D_{20}

$$\langle N(p+q)|T^{q,g}_{\mu\nu}|N(p)\rangle \;\to \{A_{20}, B_{20}, D_{20}\}^{q,g}(q^2)$$

$$\begin{cases} T^q_{\mu\nu} = \bar q\,\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q \\ T^{\rm glue}_{\mu\nu} = G^a_{\mu\lambda}G^a_{\nu\lambda} - \frac{1}{4}\delta_{\mu\nu}(G_{\mu\nu})^2 \end{cases}$$

- Ji's angular momentum decomposition [X.Ji, PRL78:610(1997)]

$$J_{q,g} = \frac{1}{2}\left[A^{q,g}_{20}(0) + B^{q,g}_{20}(0)\right]$$

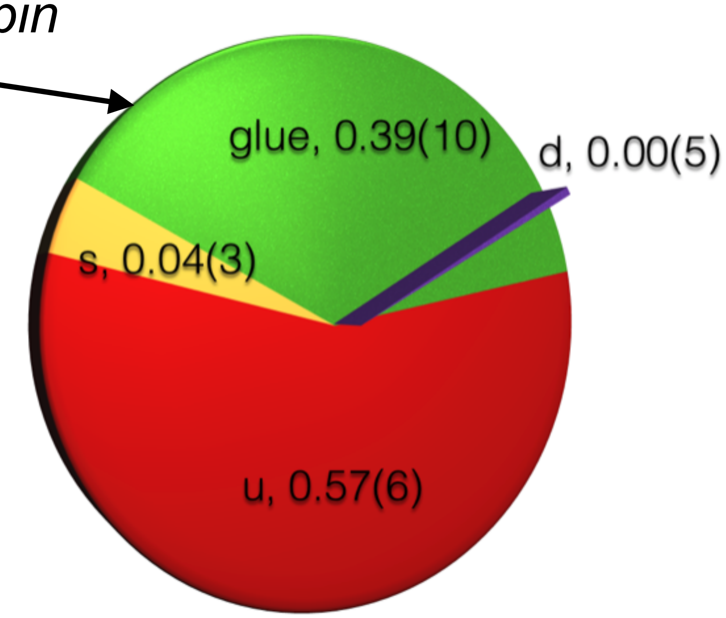


Nf=2+1+1 physical light quarks
[C.Alexandrou et al (ETMC) PRD101:094513 (2020)]

J_g = 38% of nucleon spin

Spin sum rule

$$\frac{1}{2} = J_q + J_g$$

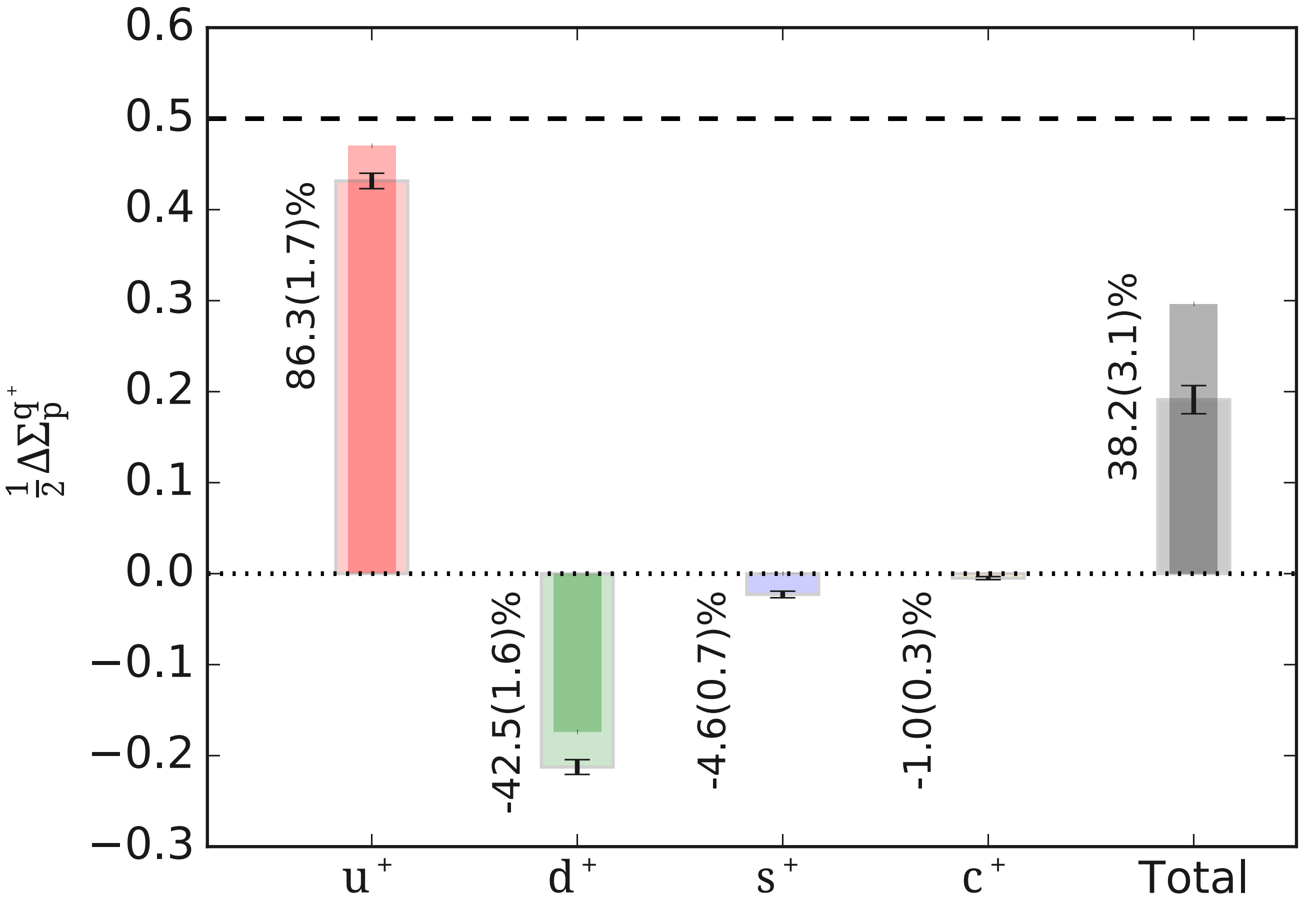


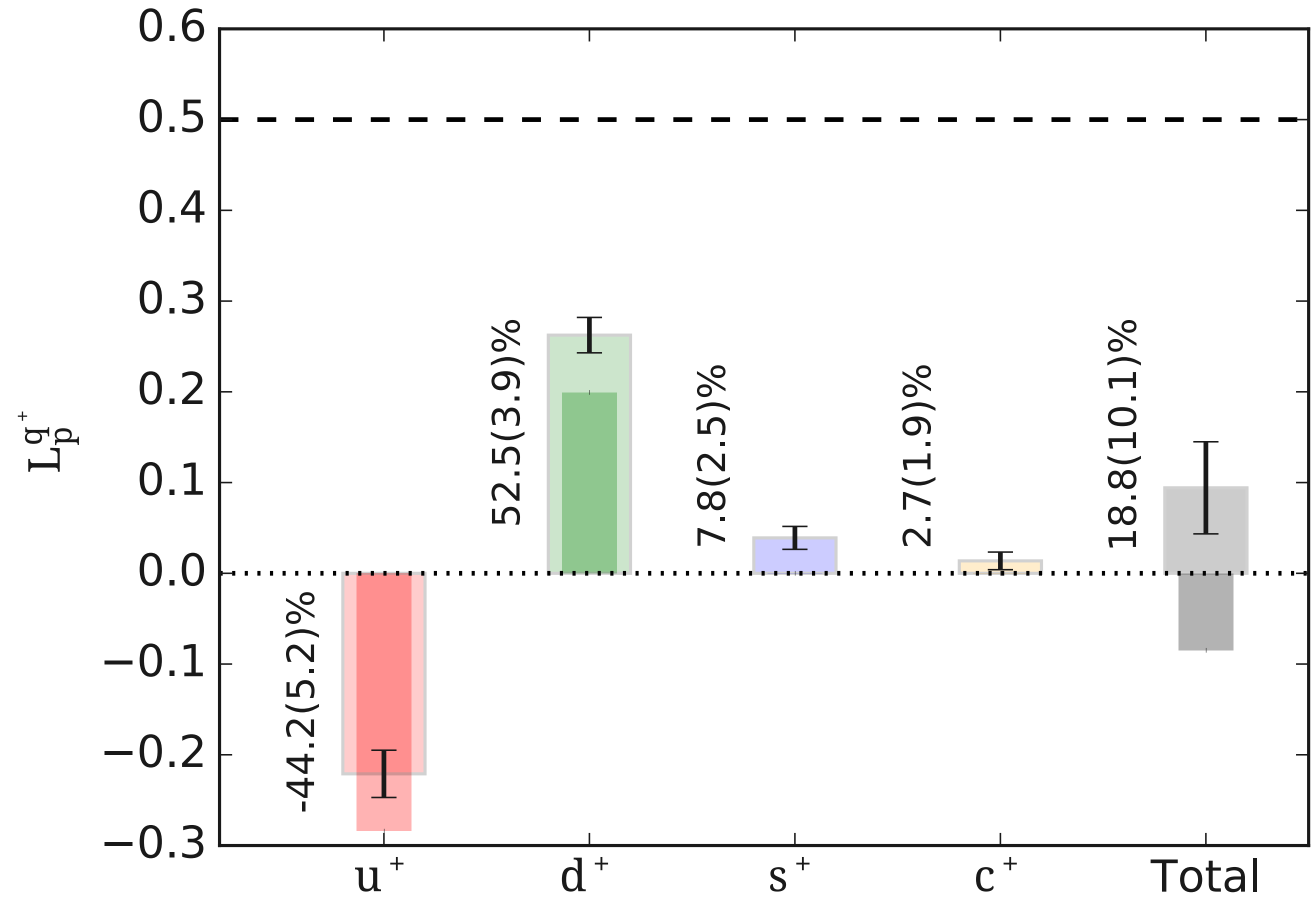
Nf=2+1 physical light+strange quarks
[Y.B.Yang, LATTICE 2018; arXiv:1904.04138]

Proton Spin Decomposition

- Quark Orbital angular momentum and Spin

$$L^{u,d,s} = J^{u,d,s} - \frac{1}{2}\Sigma^{u,d,s}$$



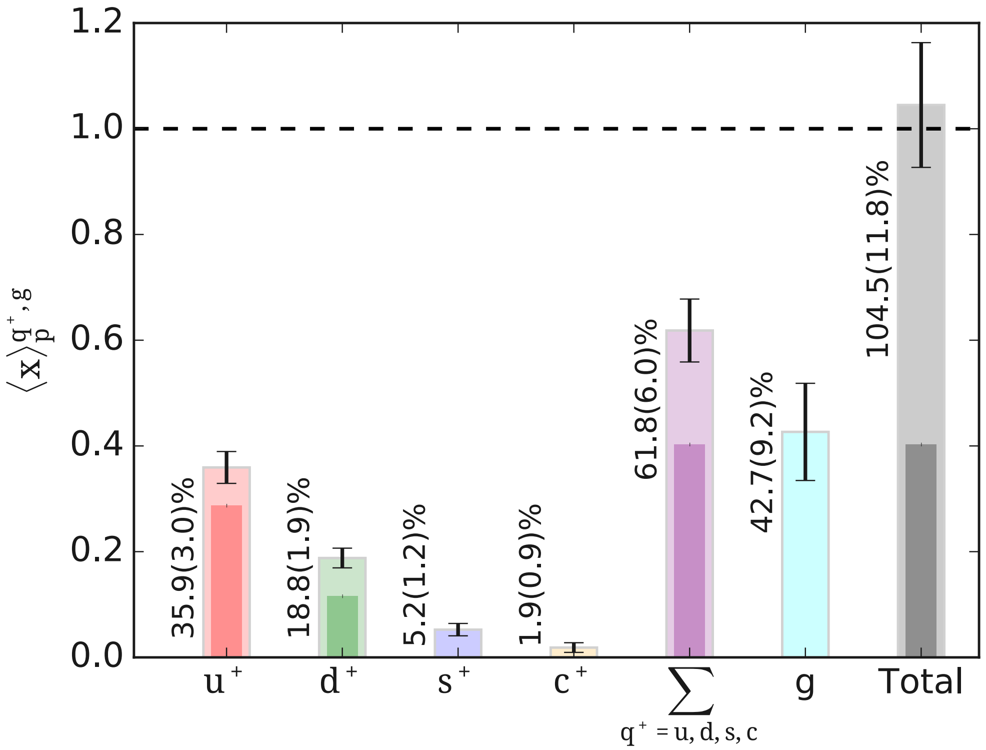


Nf=2+1+1 physical light quarks
[C.Alexandrou et al (ETMC) PRD101:094513 (2020)]

Nucleon Momentum and Mass Decomposition

- Momentum fraction
  (2nd Mellin moment of parton distribution)

$$\langle x \rangle^{q,g} = \int dx\, f_{q,g}(x) = A_{20}^{q,g}(0)$$



[C.Alexandrou et al (ETMC) PRD101:094513 (2020)]

- Momentum sum rule

$$1 = \sum_{quarks} \langle x \rangle^q + \langle x \rangle^{\text{gluon}}$$

- Proton mass decomposition:

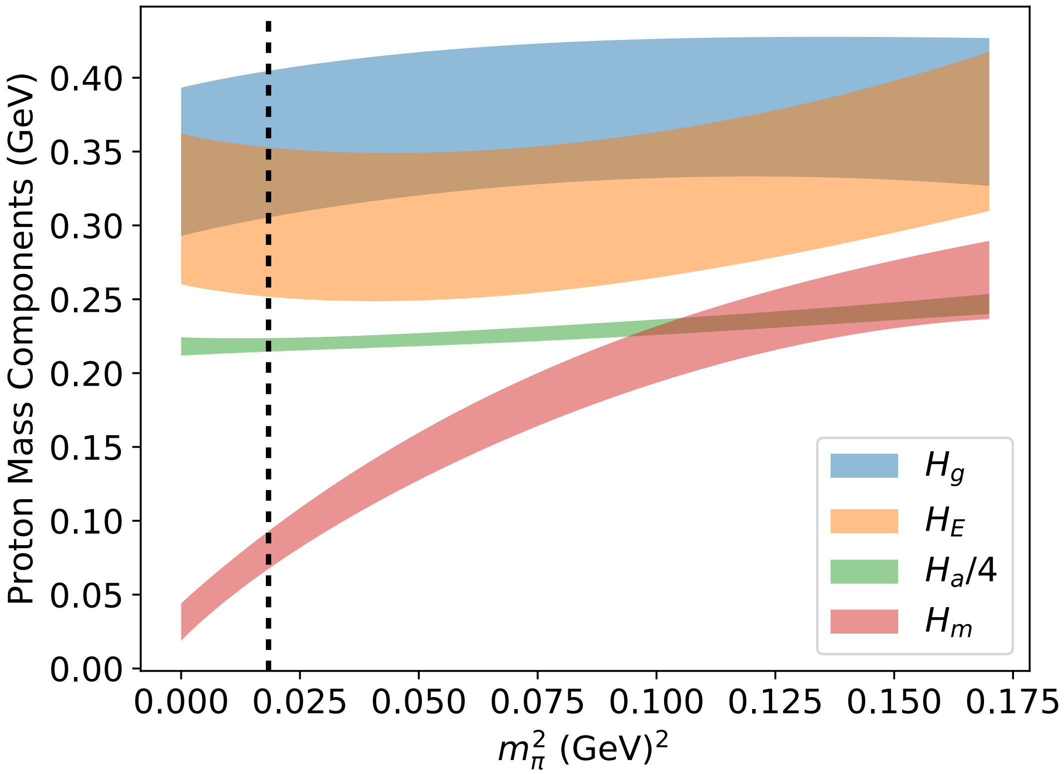
$$M = -\langle T_{44} \rangle = \langle H_m \rangle + \langle H_E \rangle(\mu) + \langle H_g \rangle(\mu) + \frac{1}{4}\langle H_a \rangle$$

	quark condensate	quark kin.energy	glue field energy	EM trace anomaly
At the physical point:	9(2)(1)%	32(4)(4)%	36(5)(4)%	23(1)(1)%

Quark and gluon energy contriubtions:

$$\langle H_E^R \rangle = \frac{3}{4}\langle x \rangle_q^R M - \frac{3}{4}\langle H_m \rangle$$

$$\langle H_g^R \rangle = \frac{3}{4}\langle x \rangle_g^R M$$



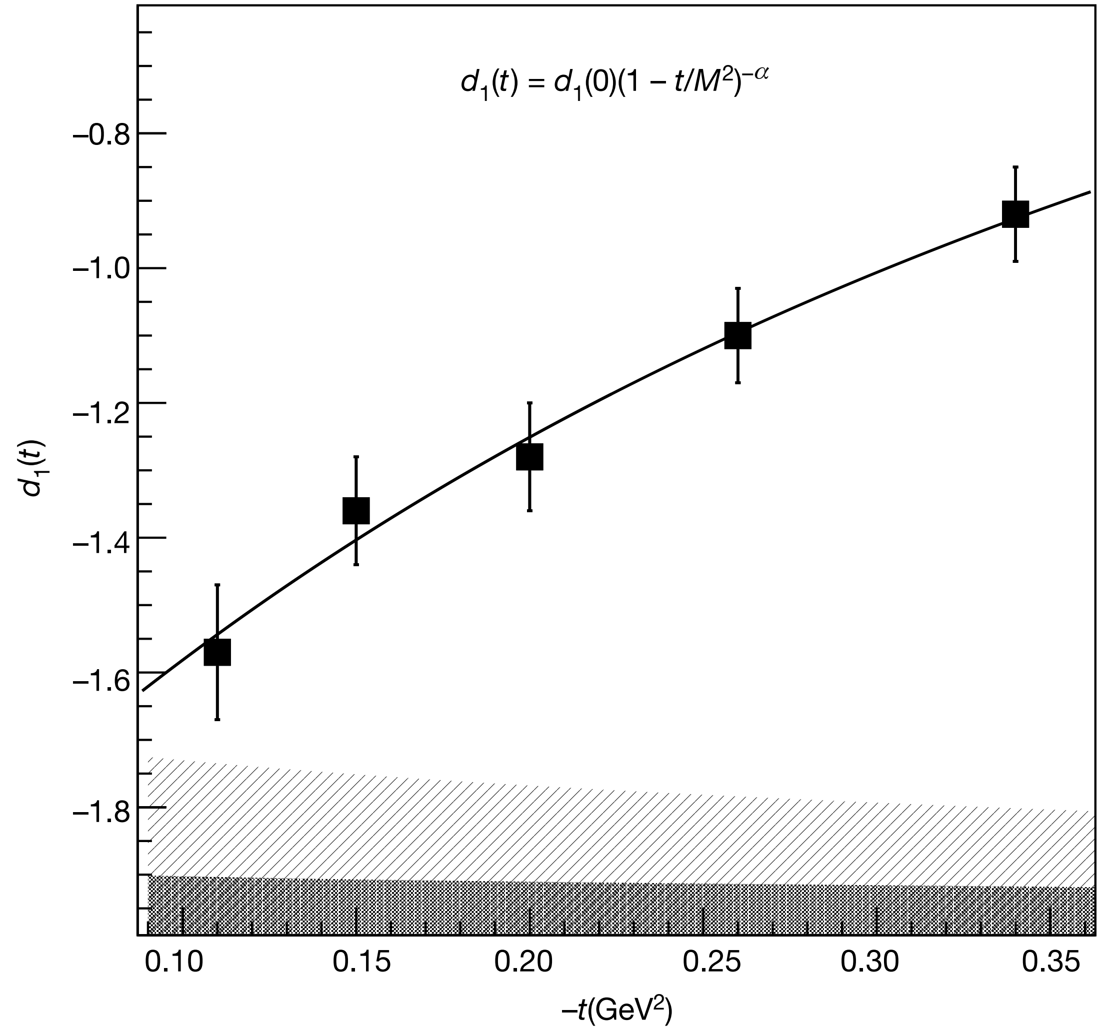
[Yang et al (χQCD), PRL121:212001(2018)]

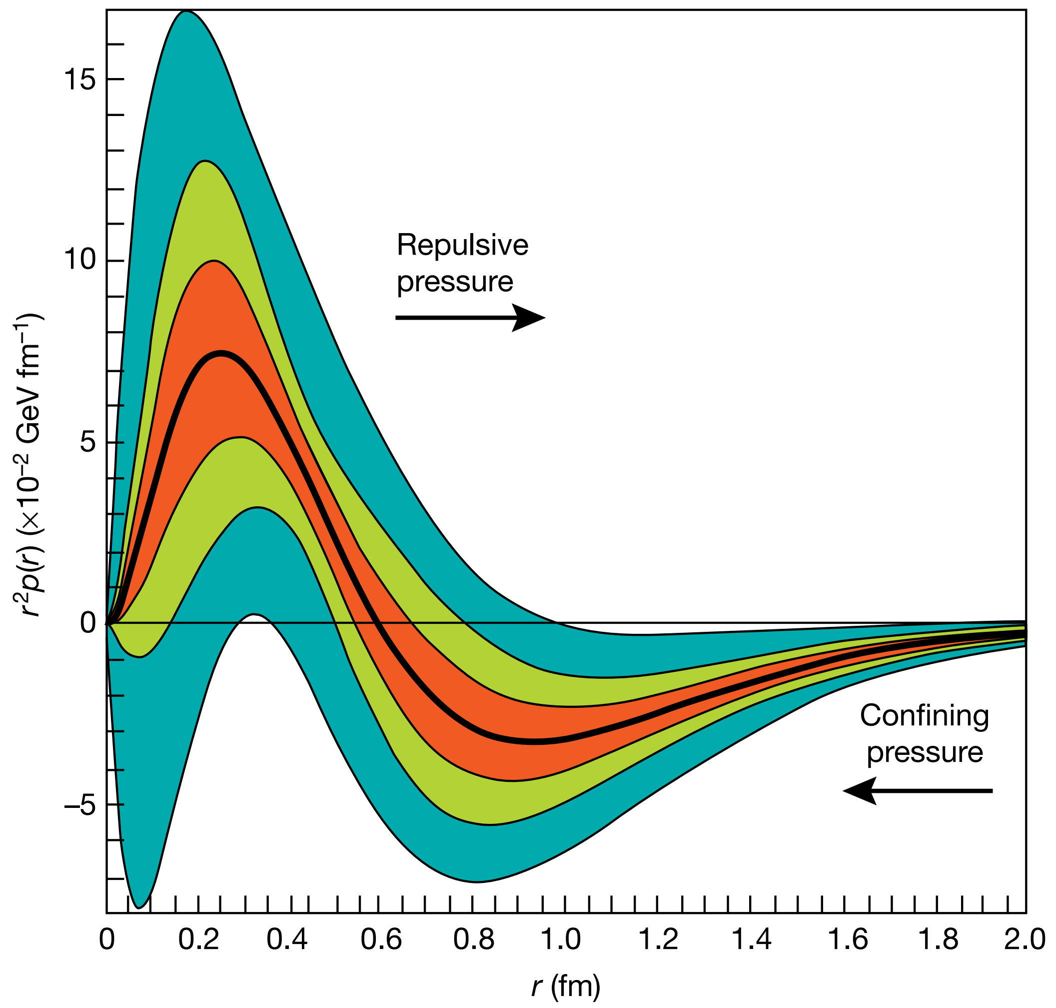
Gluon D-term: Pressure and Shear Stress

Energy-momentum-stress tensor: Generalized Form Factors A, B, D

pressure and shear stress

$$\langle p', \sigma' | T^{q,g}_{\mu\nu} | p, \sigma \rangle = \bar{U}(p', s') \Big[A^{q,g}(Q^2) \gamma_{\{\mu} P_{\nu\}} + B^{q,g}(Q^2) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} q^\rho}{2M} + D^{q,g}(Q^2) \frac{q_{\{\mu} q_{\nu\}}}{4M} \Big]$$

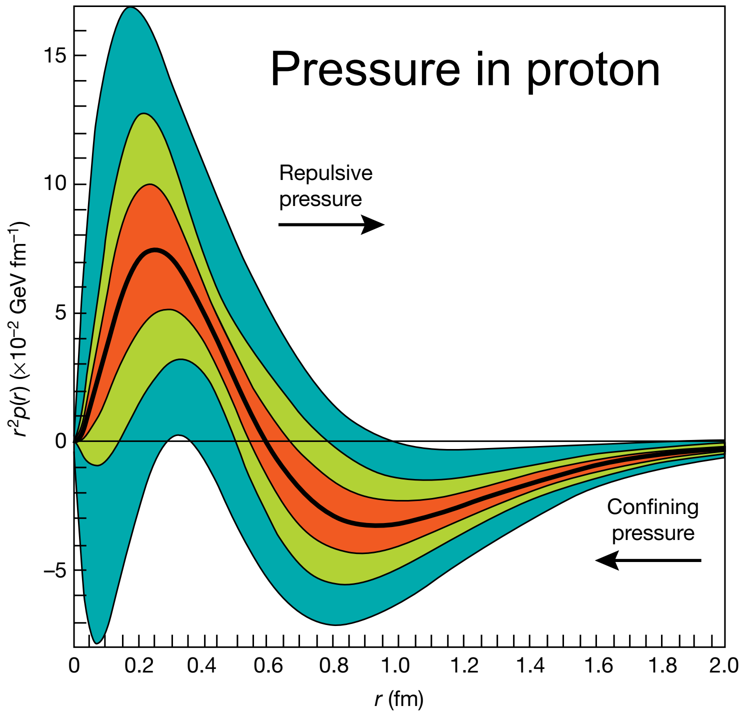




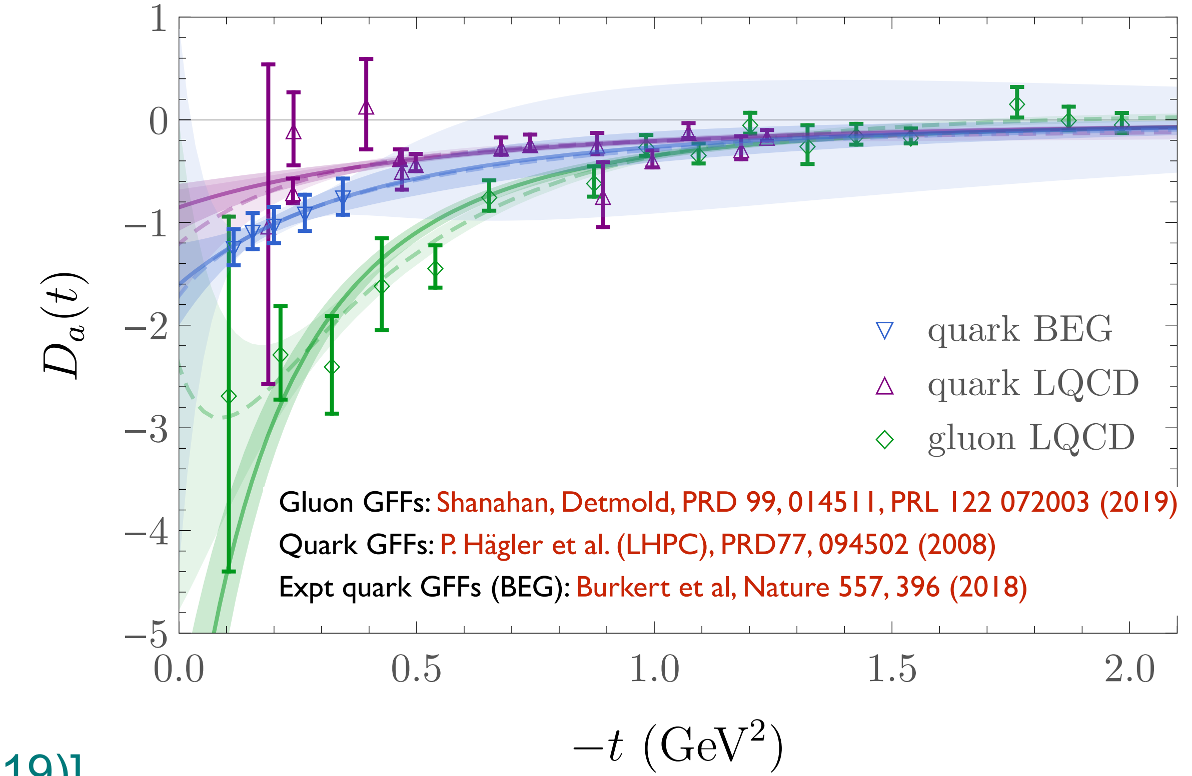
Quark D-term (DVCS measurements at CEBAF)
[Burkert, Elouadrhiri, Girod, Nature 557:396 (2018)]

D-term: Pressure and Shear Stress

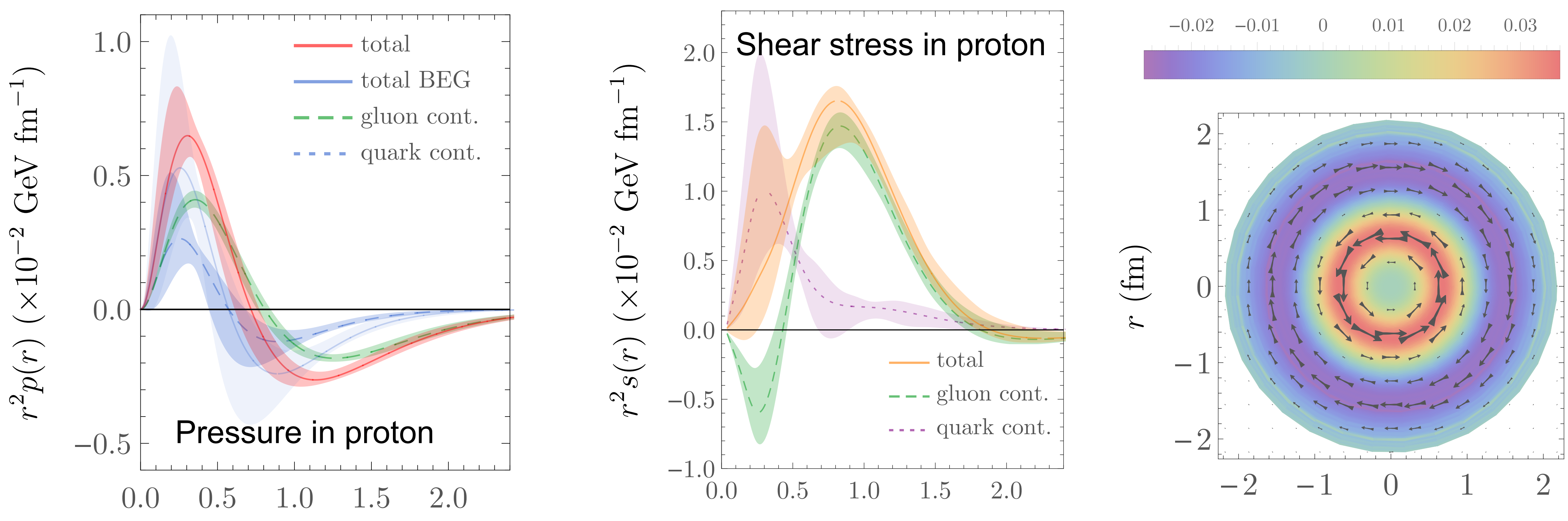
CEBAF [Burkert, Elouadrhiri, Girod, Nature 557:396 (2018)]



Quark and glue D-terms from lattice
[P.Shanahan, CFNS workshop 2019]



Lattice data [Shanahan, Detmold; PRL122:072003 (2019)]



Parton Distributions : Moments

$$q(x) \propto \left\langle P \right| \int \frac{d\lambda}{2\pi}\, e^{i2x\lambda(P\cdot n)}\, [\bar{q}(-\lambda n)\Gamma W q(\lambda n)] \left| P \right\rangle$$

W = Wilson line
in light-cone direction ***n***

$$\langle x^{n-1}\rangle = \int dx\, x^{n-1} q(x) \longrightarrow \langle P|\bar{q}\Gamma(iD\cdot n)^{n-1}q|P\rangle = n_{\mu_1}\cdots n_{\mu_n}\langle P|\mathcal{O}_{\mu_1\ldots\mu_n}|P\rangle$$

Lorenz/rotational symmetry broken on a lattice:
only the lowest (n ≤ 4) moments possible

$$n=1 \qquad \bar{q}\gamma_\mu q \rightarrow \mathbf{4}_1^-$$

$$n=2 \qquad \bar{q}[\gamma_{\{\mu} i\overleftrightarrow{D}_{\nu\}} - \langle\mathrm{Tr}\rangle]q \rightarrow \mathbf{3}_1^+ \oplus \mathbf{6}_3^+$$

$$n=3 \qquad \bar{q}[\gamma_{\{\mu} i\overleftrightarrow{D}_\nu i\overleftrightarrow{D}_{\rho\}} - \langle\mathrm{Tr}\rangle]q \rightarrow \mathbf{8}_1^- \oplus \mathbf{4}_1^- \oplus \mathbf{4}_2^-$$

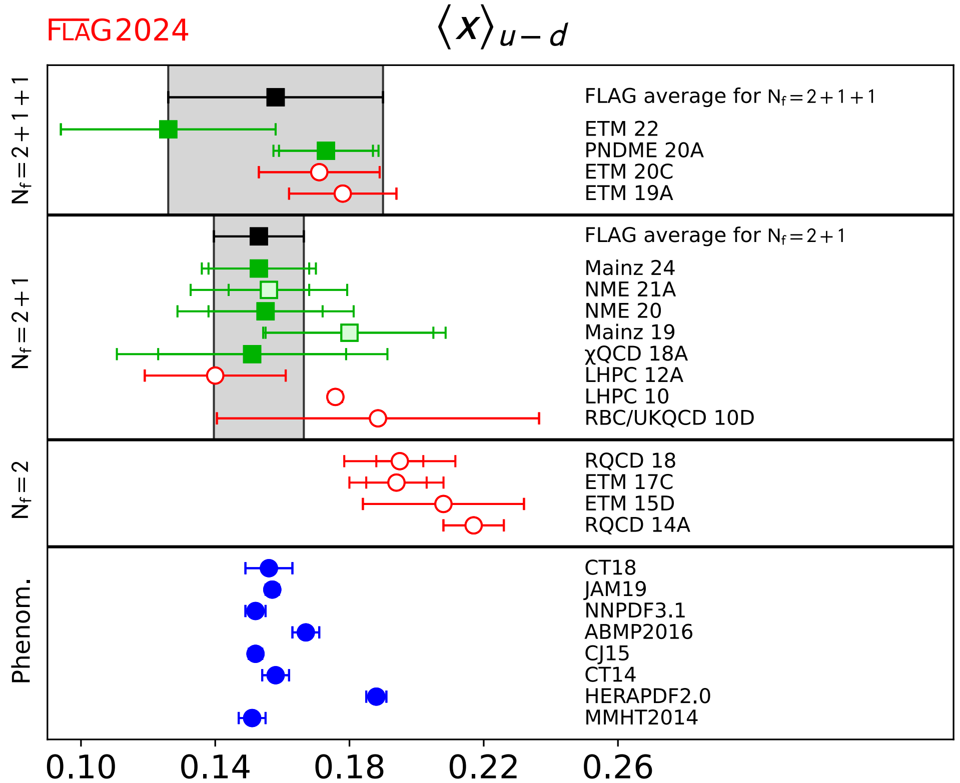
$$n=4 \qquad \bar{q}[\gamma_{\{\mu} i\overleftrightarrow{D}_\nu i\overleftrightarrow{D}_\rho i\overleftrightarrow{D}_{\sigma\}} - \langle\mathrm{Tr}\rangle]q \rightarrow \mathbf{1}_1^+ \oplus \mathbf{3}_1^+ \oplus \mathbf{6}_3^+ \oplus \mathbf{2}_1^+ \oplus \mathbf{1}_2^+ \oplus \mathbf{6}_1^+ \oplus \mathbf{6}_2^+$$

lower moments mix
into higher moments

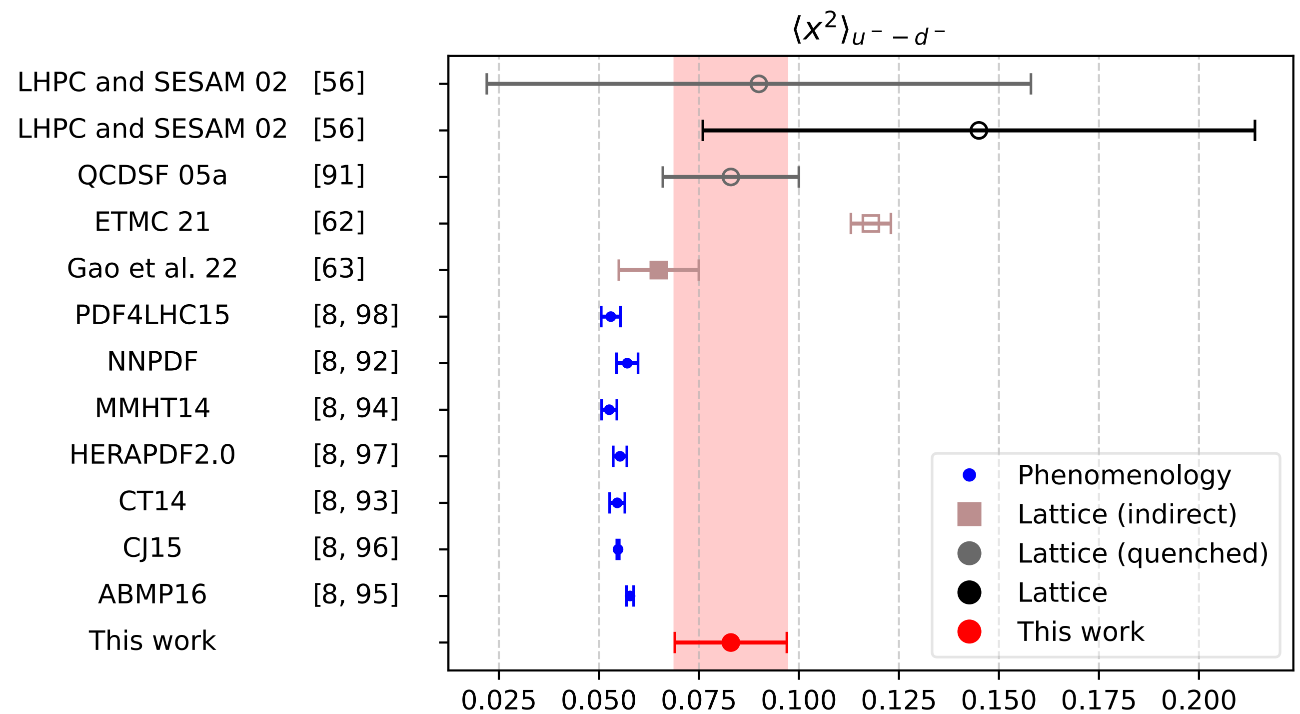
$$\sim \Lambda_{\mathrm{UV}}^{d_1-d_2} = \left(\frac{1}{a}\right)^{d_1-d_2}$$

(UV divergent)

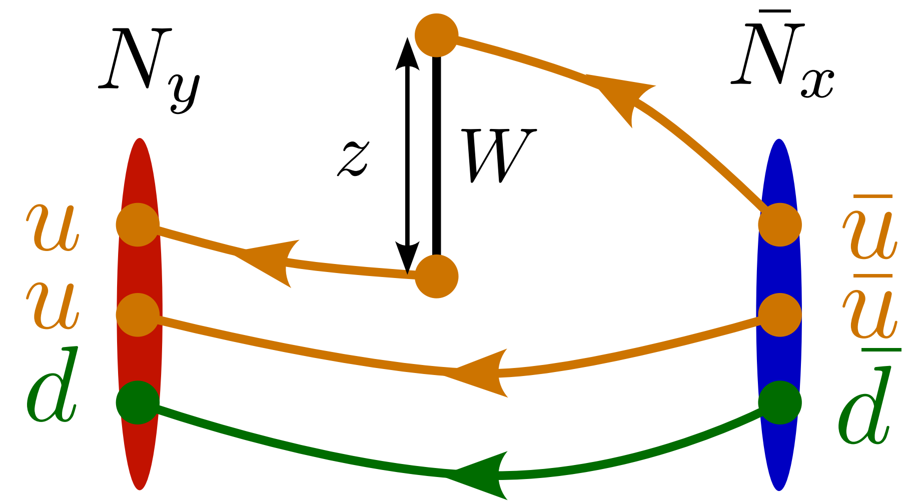
Nucleon 2nd unpolarized Mellin moment [FLAG 2024]



Nucleon 3rd unpolarized Mellin moment [Taggi et al, 2605.02808]



Parton Distributions : x-dependence



[X.Ji, 2013]: PDF can be computed from space-like correlators

- Large-momentum effective theory [X.Ji PRL110: 262002(2013)]: direct matching to PDF in x-space (quasi-PDF)
- "Good lattice cross-sections" [Ma, Qiu (2014)]: PDFs from current-current correlators
- Ioffe-time distribution matching (pseudo-PDF) [A.Radyushkin (2017)]

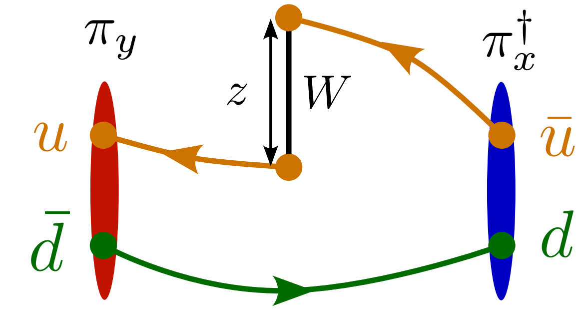
$$\tilde{q}(x, P_z, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu_R}{P_z}, \frac{\mu}{P_z}\right) q(y,\mu) \; + \; O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

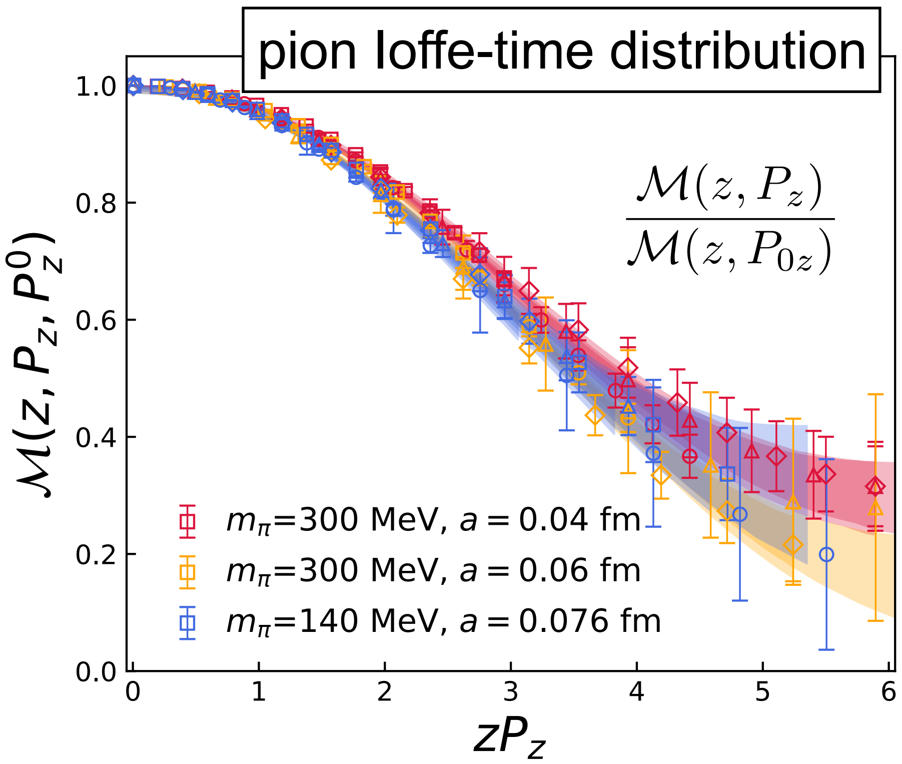
quasi-PDF — matching kernel — light-cone PDF — corrections

- extensive work (un)polarized / transversity PDF, GPD of pion, nucleon – *too much to review*
- exploratory work gluon PDF/GPDs: more complicated matching, uncertainties not clear

Pion PDF & Moments from Ioffe-time Distribution

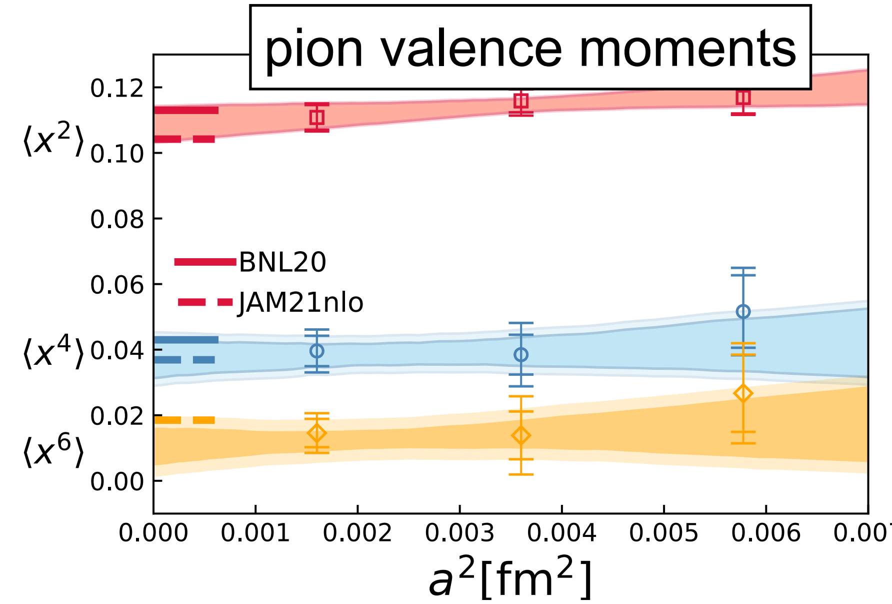
- pion valent-quark moments and PDF $q_v(x)$ [Gao et al, PRD106: 114510)]





- operator product expansion

$$M(z,P_z) \sim \sum_n c_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle + r(aP_z)^2$$



- x-space matching

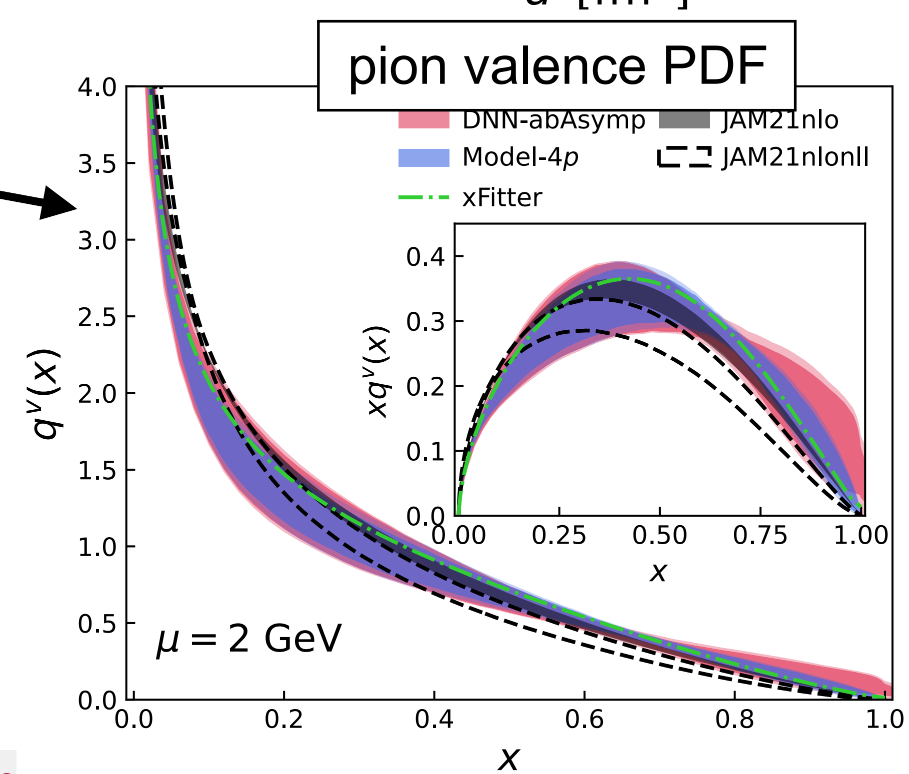
$$\mathcal{M}(z,P_z) = \int_{-1}^{+1} d\alpha\, \bar{\mathcal{C}}(\alpha, \mu^2 z^2)\; Q(\alpha z P_z, \mu) + r(aP_z)^2$$

$$Q(\lambda) = \int_{-1}^{+1} dy\, e^{-iy\lambda} q(y)$$

- 4-parameter model

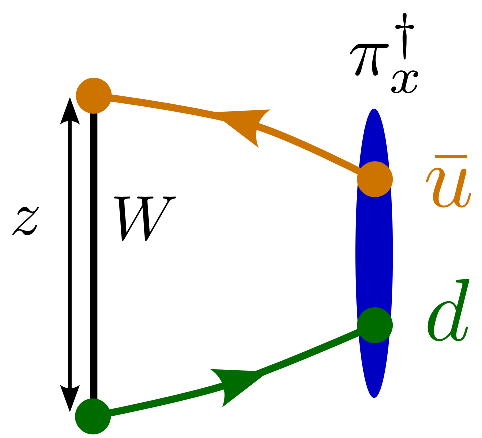
$$q_v(x) \sim \mathcal{N}\, x^\alpha (1 - x^\beta)\, (1 + s\sqrt{x} + tx)$$

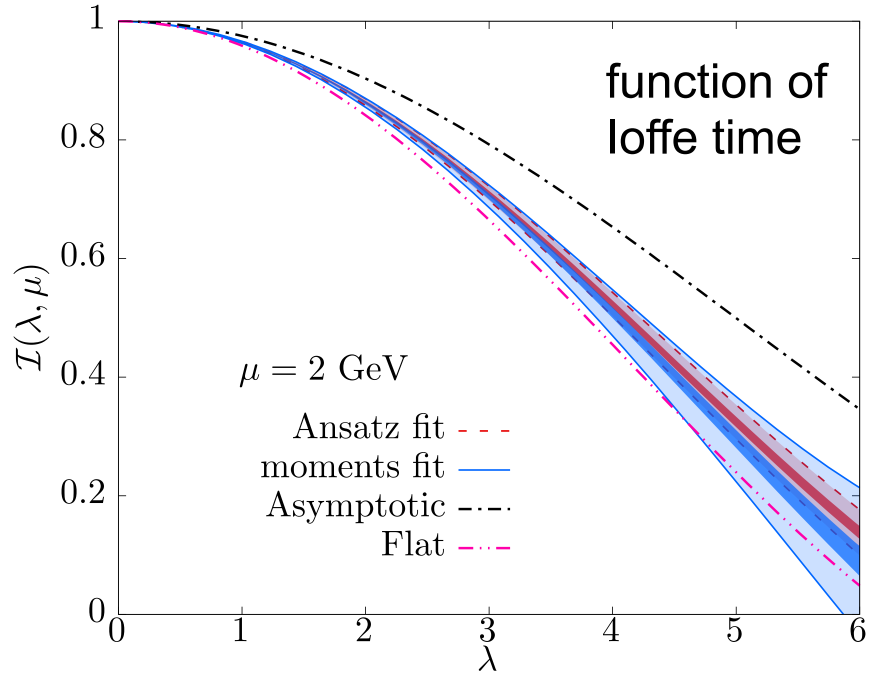
- deep neural network (DNN)



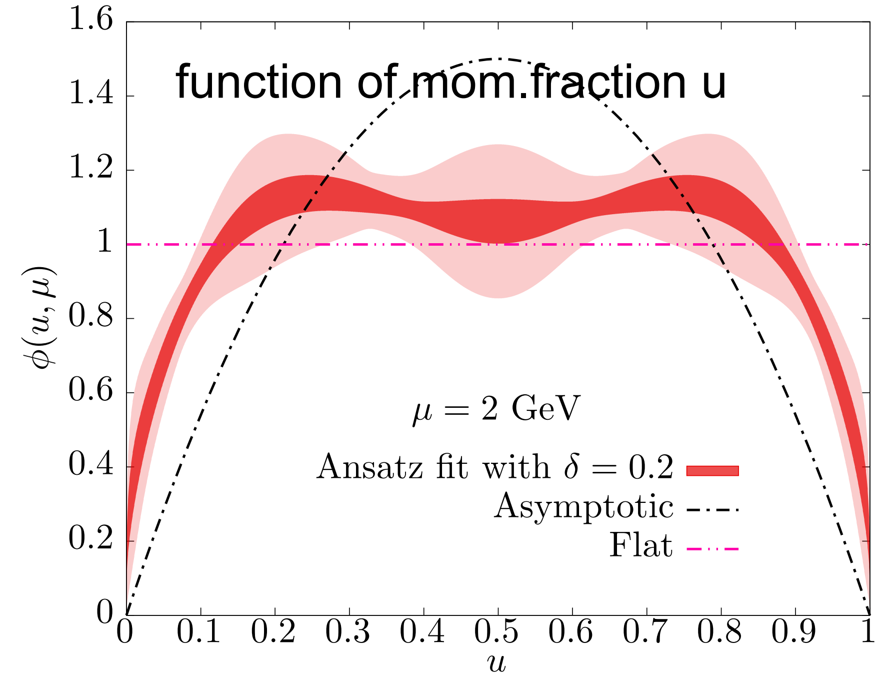
Pion and Kaon Distribution Amplitudes

- Pion distribution amplitude



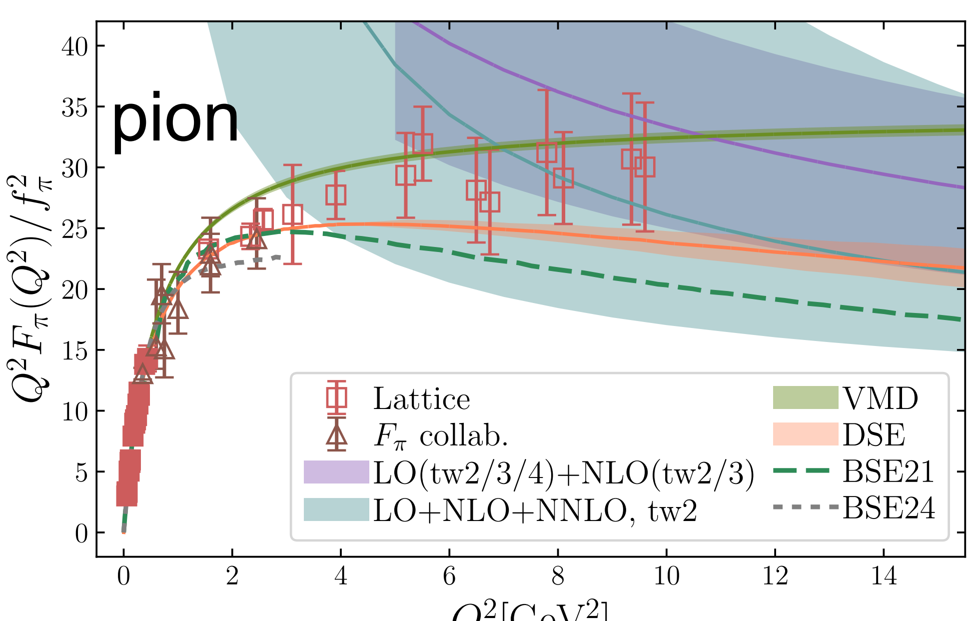


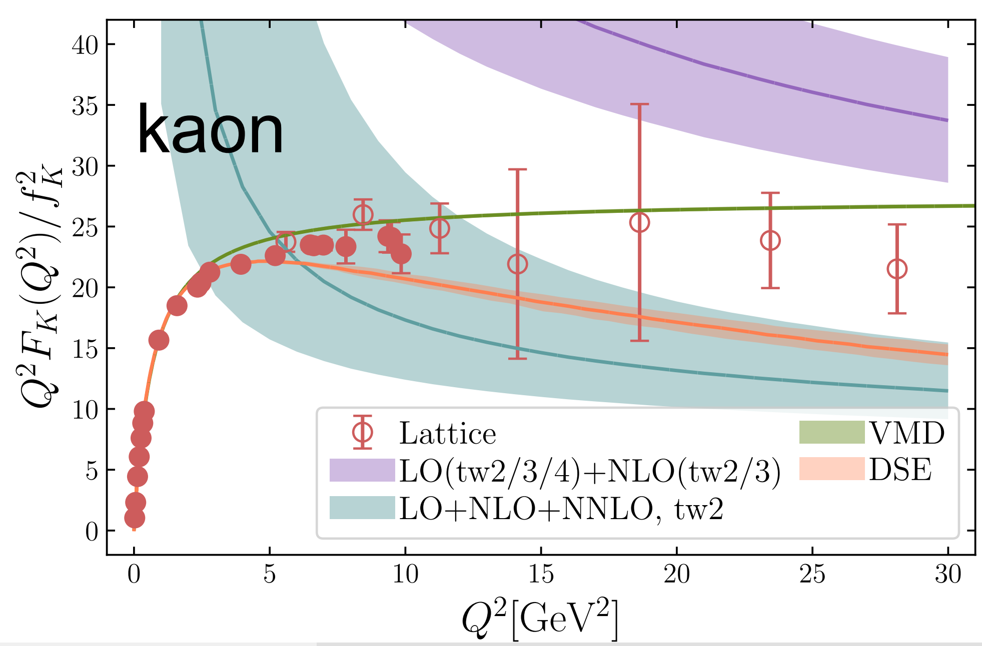
matching



- π,K form factors from Lattice DA + perturbative QCD [Ding et al, PRL133:181902 (2024)]

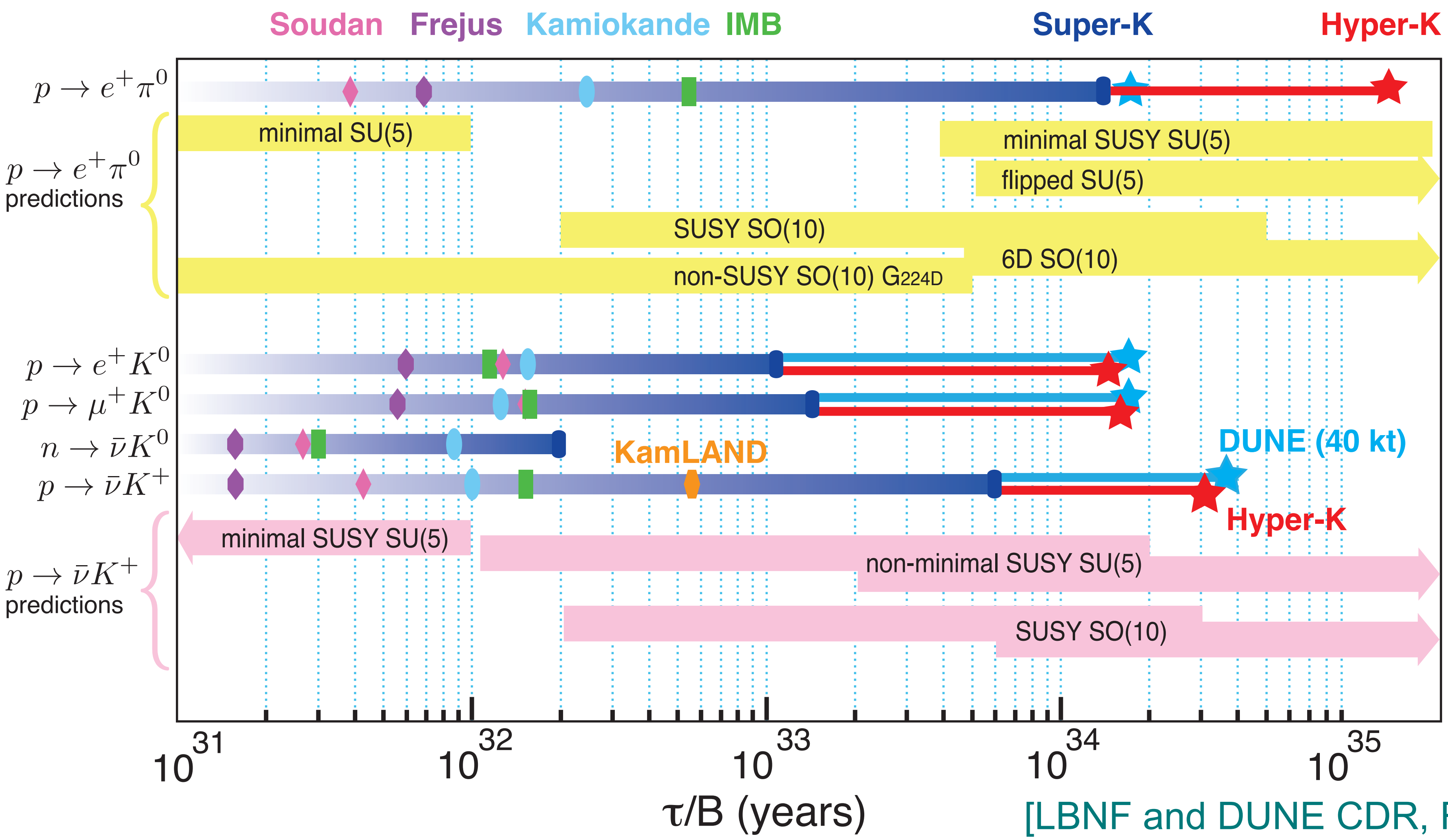
$$F_M(Q^2) = \int_0^1 \int_0^1 dxdy\ \Phi_M^*(y, \mu_F^2) T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \Phi_M(x, \mu_F^2)$$





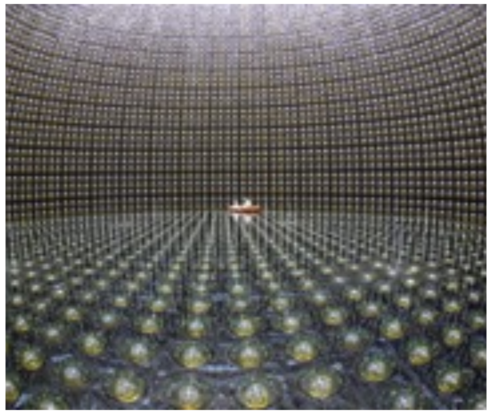
Beyond-SM physics: Does the Proton Decay?

- *Baryon number must be violated by some interaction (e.g. to explain matter-antimatter asymmetry)*
- *Missing piece of Grand-Unified Theories*
- *Limit on nuclear matter stability*



[LBNF and DUNE CDR, R.Acciarri et al (2015)]

Soudan



Super Kamiokande

- Expected x10 improvement on lifetime limit from Hyper-K , DUNE
- Better sensitivity to $p \to \bar{\nu}K^+$ that affects supersymmetric GUT models

Protons Stable due to Topology?

$$\text{Proton lifetime} \qquad \frac{\tau_p}{Br(p\to\pi\bar\ell)} \approx \frac{1.4\cdot 10^{33}\,\text{years}}{|c_I|^2\cdot\big|\langle\pi|\mathcal{O}_{\rm decay}|p\rangle\big|^2}\cdot\left(\frac{\Lambda_{\rm GUT}}{10^{15}\,\text{GeV}}\right)^4$$

$\approx$ GUT

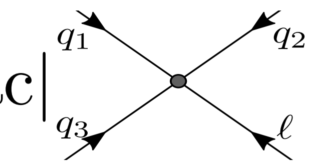
O(1) coupling

decay amplitude at nuclear scale

Why no proton decays seen ?

- ***other*** BNV mechanism ?

  more complicated GUT scenario?
- suppressed ***decay amplitude*** due to QCD dynamics?

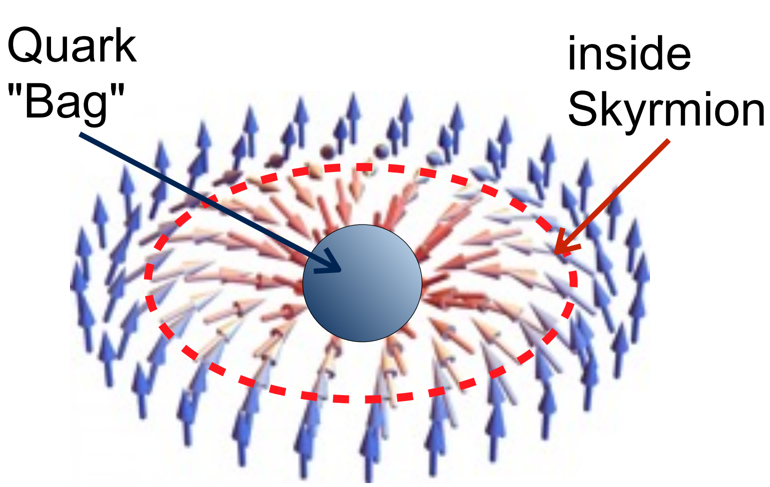
"quark pudding" estimate: $\langle\text{vac}|$  $|N\rangle$

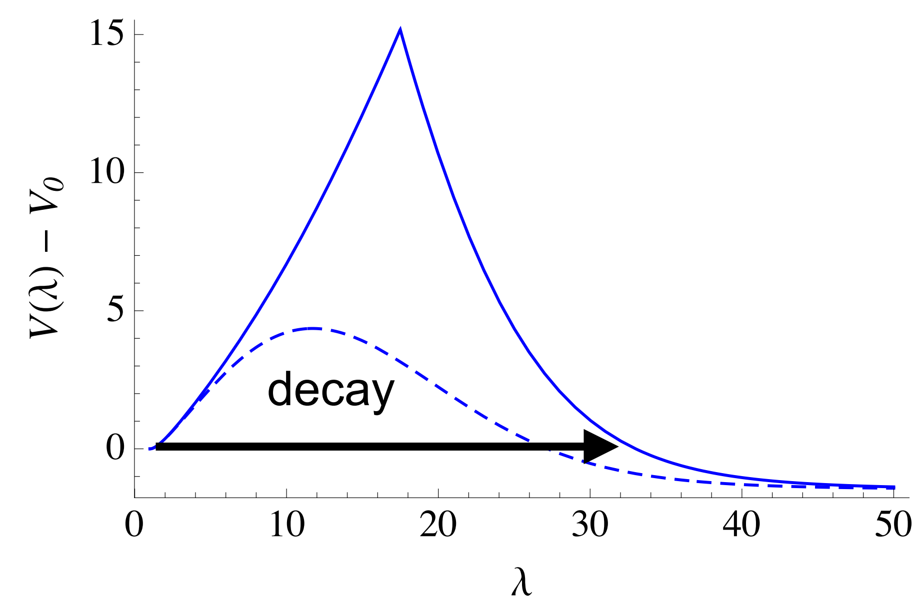
$$\langle\text{vac}|\mathcal{O}^{3q}|N\rangle \sim \rho_q^{3/2}\sqrt{V_N} \sim \frac{1}{V_N} \approx 0.004\,\text{GeV}^3$$

$$\langle\Pi|\mathcal{O}^{3q}|N\rangle \sim \langle\text{vac}|\mathcal{O}^{3q}|N\rangle/f_\pi \approx 0.03\,\text{GeV}^2$$

[A.Martin, G.Stavenga, PRD(2012)]

- proton as a "Chiral Bag" / Skyrmion





- decay = quantum tunneling under topological barrier
- may be suppressed ~ $O(10^{-4}) - O(10^{-12})$

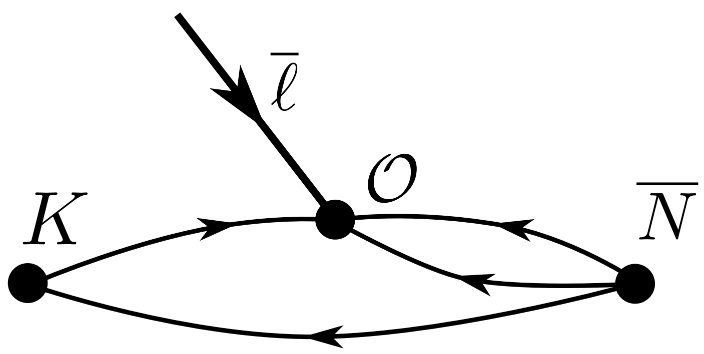
Hadron Correlators in Lattice QCD

Example:

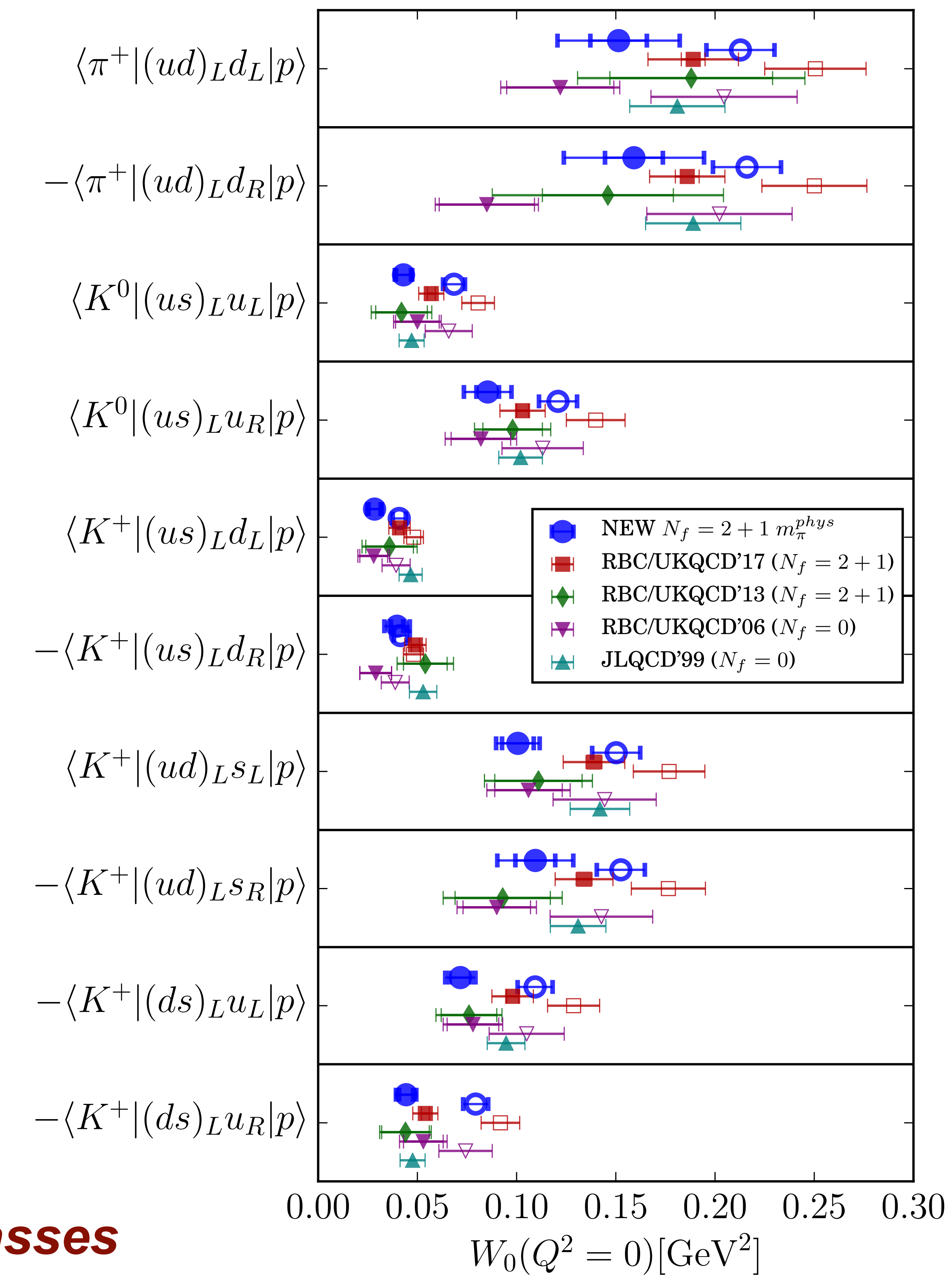
Nucleon-Kaon Matrix Elements:

(need chirally-symmetric fermions)

$$\langle K^+(T)\,[uds]_z\,\overline{N}(0)\rangle$$



Amplitudes $p \to \pi(\bar{\ell}), K(\bar{\ell})$



[Yoo et al, PRD (2022); 2111.01608]

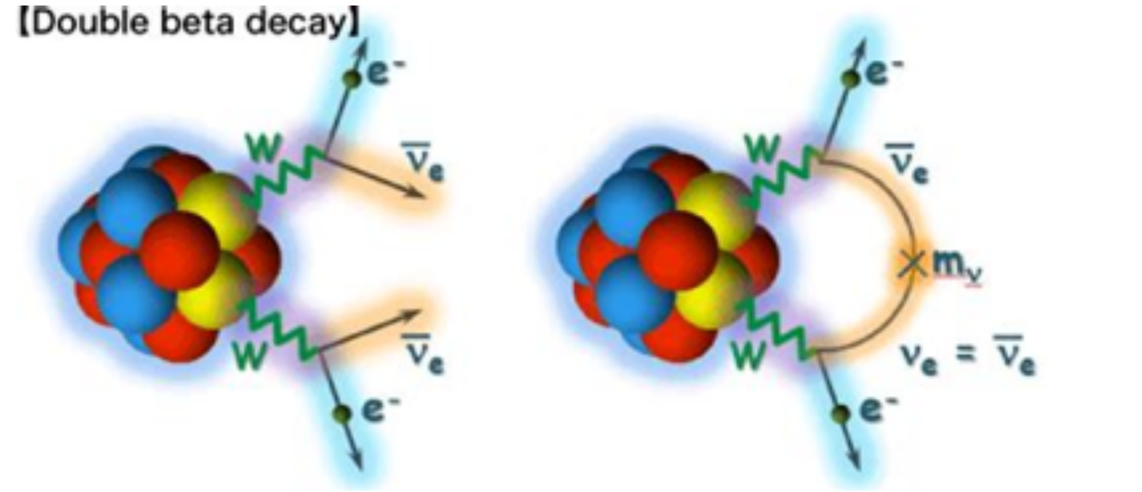
matrix elements ~O(simple estimates)

NO SUPPRESSION at physical quark masses

Neutrinos on Lattice: Double-Beta Decay

Neutrinoless double-beta decay searches

- Neutrino mass mechanism
- mass hierarchy
- leptogenesis (LNV $\Longrightarrow$ BNV thru sphalerons)
- scale of Beyond-SM physics



$$\Gamma^{0\nu\beta\beta} \propto |m_{\beta\beta}|^2 \, |M^{0\nu}|^2$$

Majorana eff.mass

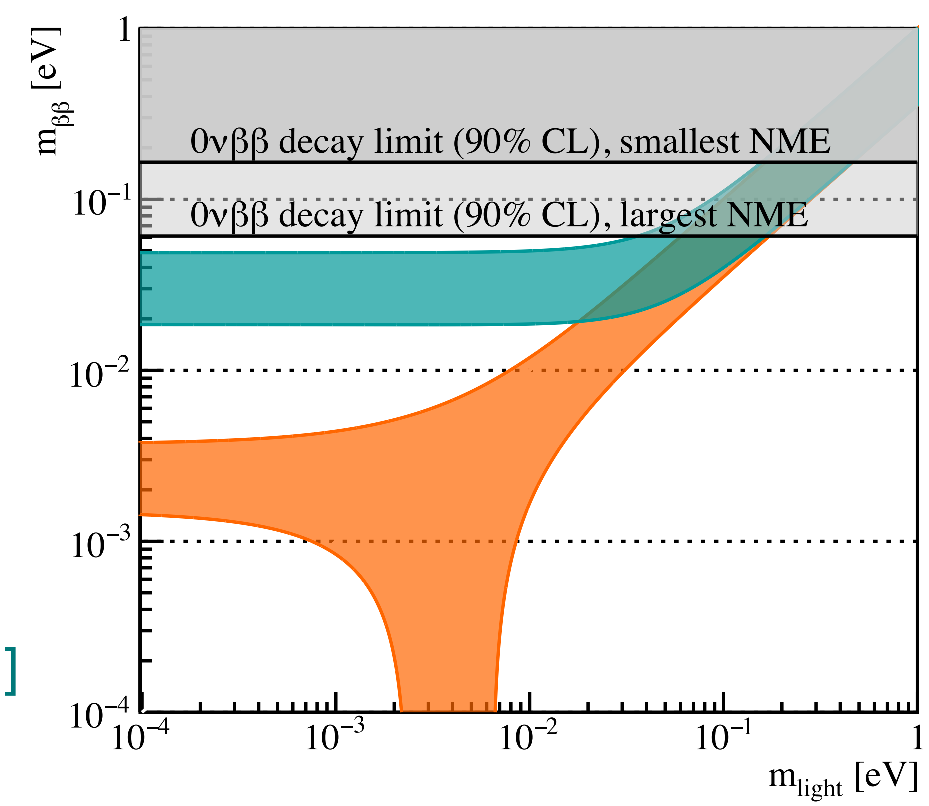
nuclear matrix elements lattice QCD + EFT + Nuclear

Experiments

- current :
  NEMO3, KamLAND-Zen, EXO-200, Majorana, GERDA, CUORE, CUPID
- Next-geneneration nEXO, 1 ton ^{136}Xe (x100 better constraints)

LNV mechanisms:

- light Majorana neutrino? – only in $0\nu2\beta$
- low-scale seesaw?
- BSM at TeV scale?
- neutrino flavor models



[V.Cirigliano, Snowmass 2021 arXiv:2203.12169]

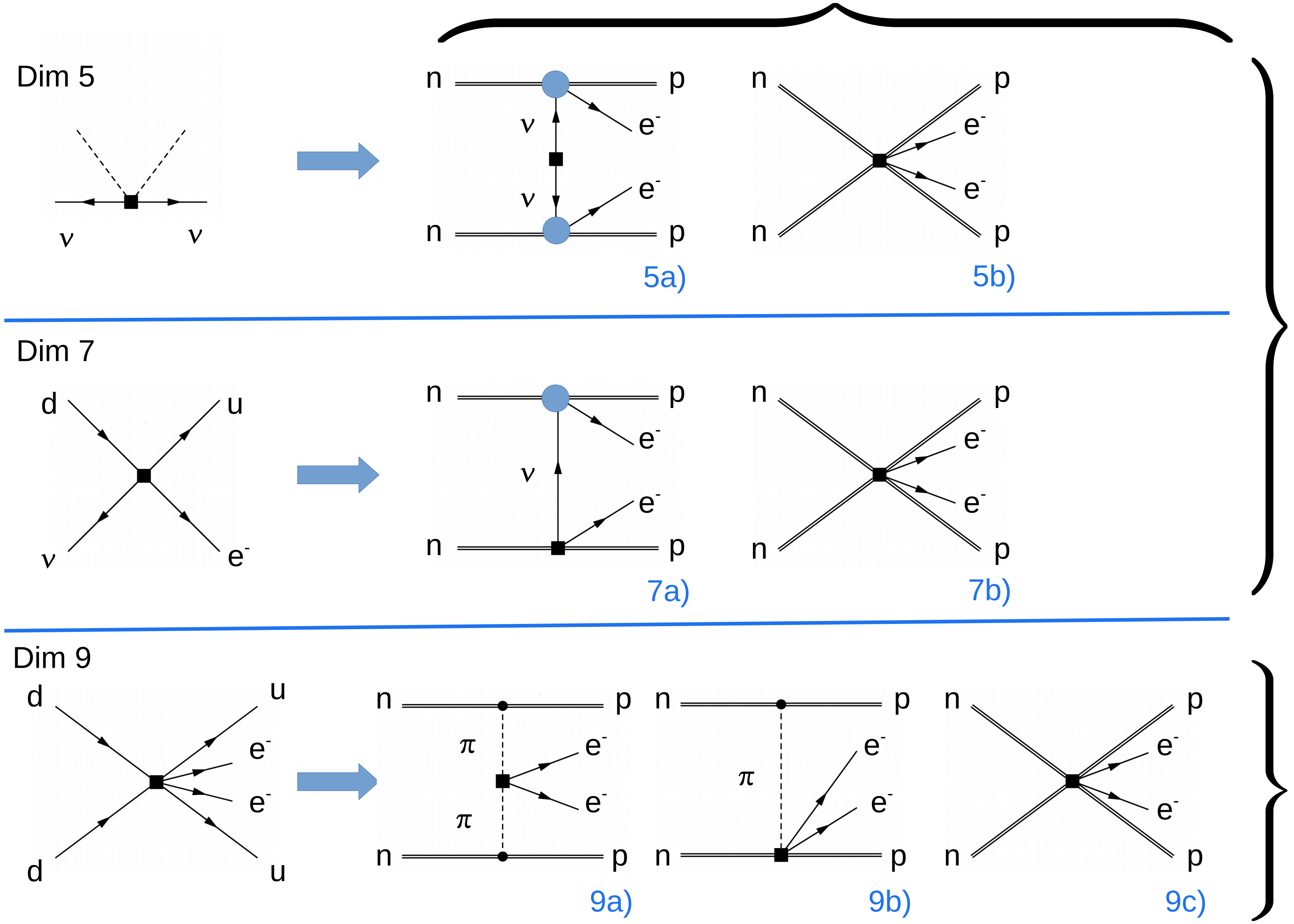
Effective Theories for 0ν2β

Effective Lepton number-violating (LNV) operators in Standard Model fields

$$\mathcal{L}_{\rm LNV} = \sum_i \frac{C_i}{(\Lambda_{BSM})^{{\rm Dim}_i - 4}} \mathcal{O}_i^{\rm LNV}$$

Standard-Model EFT

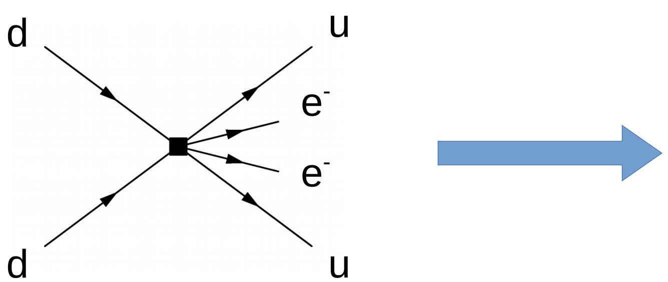
Chiral EFT

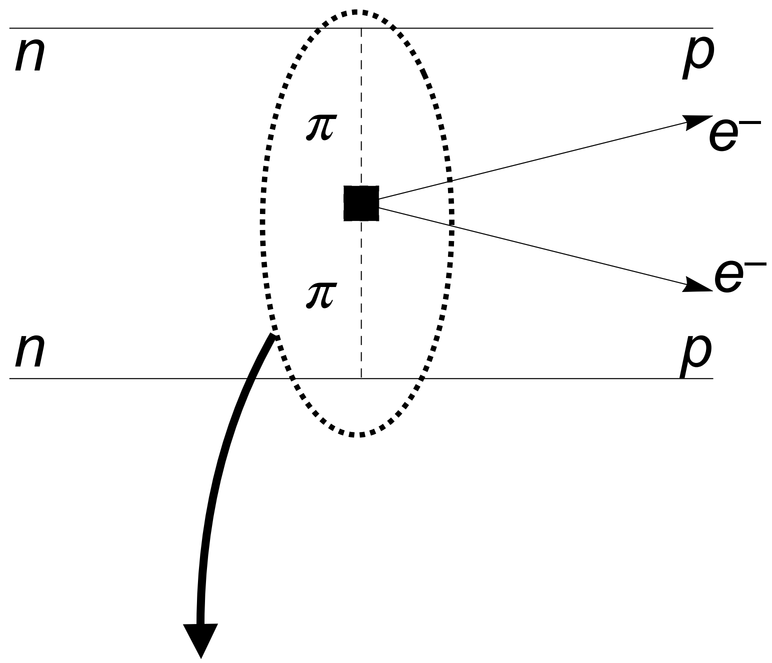


- Long-range neutrino-mediated
- Short-range (pion-mediated)
- + short-range (contact) at each order

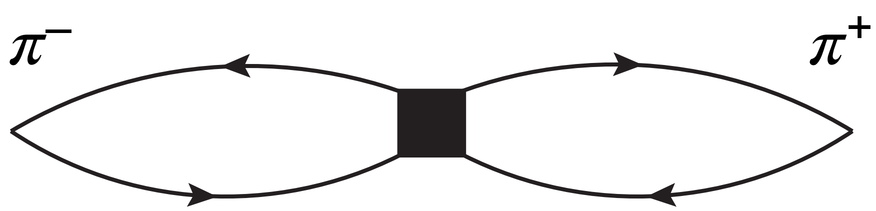
[Cirigliano et al, JHEP12:097(2018) ;1806.02780]

Short-Range (4-quark) LNV in $\pi^- \to \pi^+ e^- e^-$





$\langle \pi^+ | O^{4q} | \pi^- \rangle =$



$$\langle \Omega | \Pi^+(t_f, \mathbf{p}_f) \mathcal{O}(0) \Pi^+(t_i, \mathbf{p}_i) | \Omega \rangle$$

$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^+ \gamma_\mu q_L)(\bar{q}_R \tau^+ \gamma_\mu q_R)$$

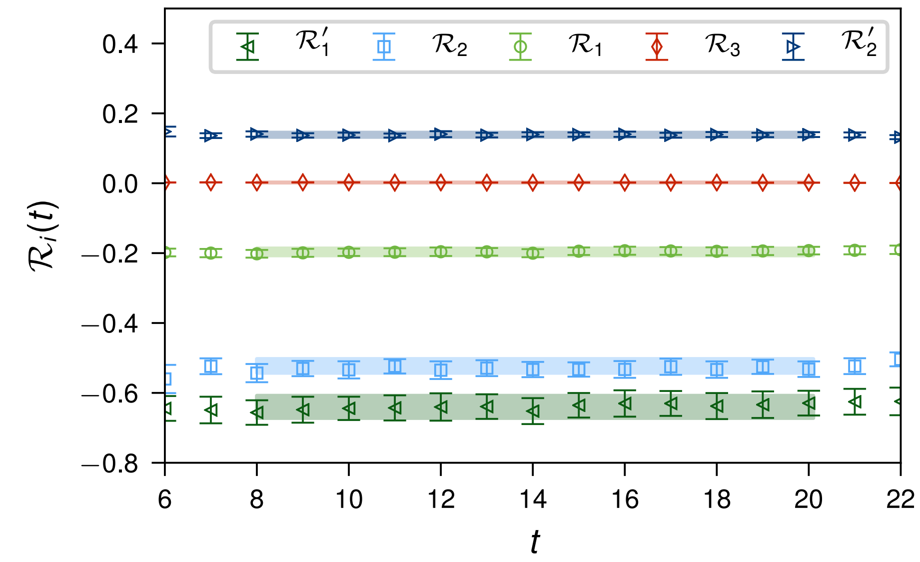
$$\mathcal{O}_{2+}^{++} = (\bar{q}_L \tau^+ q_R)(\bar{q}_R \tau^+ q_L) + \{L \leftrightarrow R\}$$

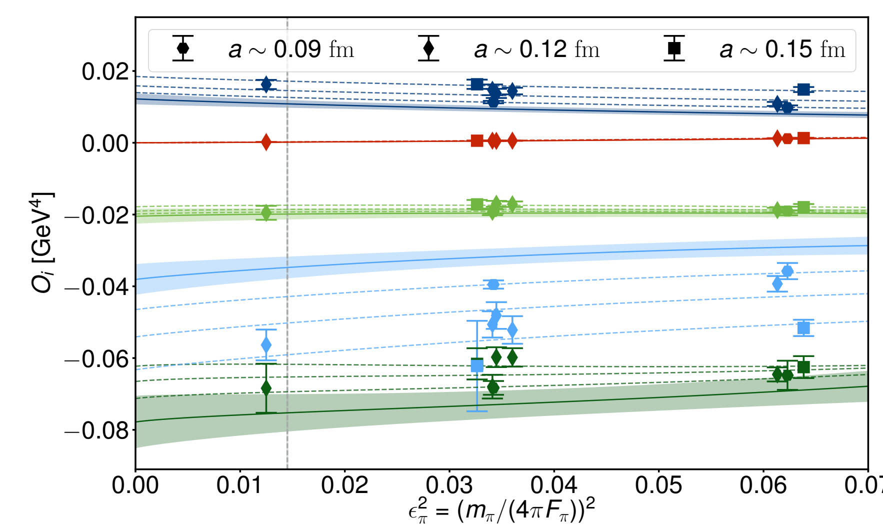
$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^+ \gamma_\mu q_L)(\bar{q}_L \tau^+ \gamma_\mu q_L) + \{L \leftrightarrow R\}$$

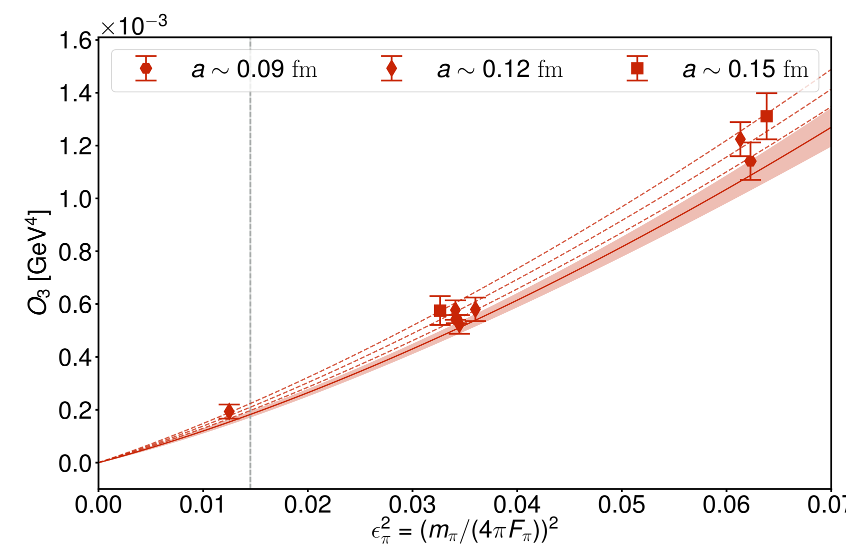
&& color-mixed combinations O_1', O_2'

[Nicholson et al, PRL 2018, 1805.02634]

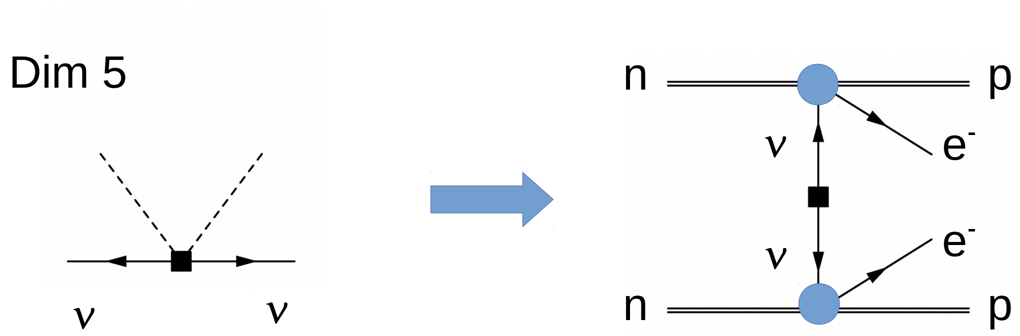
also [Detmold et al, PRD107: 094501 (2023); 2208.05322]







Long-Range LNV in $\pi^- \to \pi^+ e^- e^-$



Direct $nn \to ppe^-e^-$ is challenging

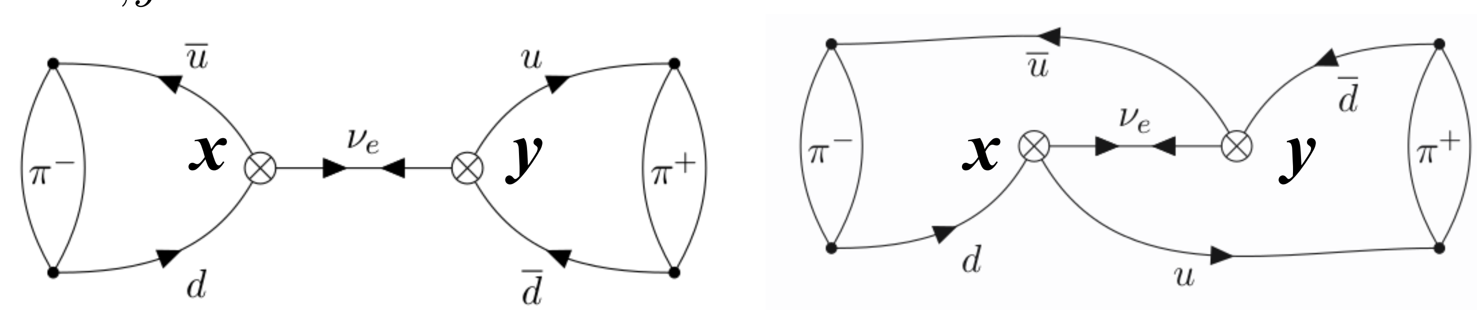
- nn, pp low-lying states
- nonlocal operator in Euclidean time

neutrino propagator

$$\sum_{\vec{x},\vec{y}} S^{\nu}_{\alpha\beta}(x-y)\langle \pi^+(T)\mathcal{T}\{j_{L\alpha}(x)j_{L\beta}(y)\}\,(\pi^-)^\dagger\rangle$$

First step: $\pi^- \to \pi^+ e^- e^-$

- $1 \to 1$ amplitude, large exc. gap
- component of EFT matching



- Low-energy constants for Lepton-violating amplitude $\pi^- \to \pi^+ e^- e^-$ (to be embedded into nuclear models)

[Detmold, Murphy 1811.05554; 2004.07404]

$$g_\nu^{\pi\pi}(770\ \mathrm{MeV}) = -10.78(12)_{\mathrm{stat}}(4)_{\mathrm{fit}}(50)_{\mathrm{FV}}(9)_{\chi\mathrm{PT}},$$
$$S_{\pi\pi} = 1.1054(14)_{\mathrm{stat}}(6)_{\mathrm{fit}}(61)_{\mathrm{FV}}(10)_{\chi\mathrm{PT}},$$
$$M^{0\nu} = 0.01880(6)_{\mathrm{stat}}(2)_{\mathrm{fit}}(10)_{\mathrm{FV}}(2)_{\chi\mathrm{PT}}\ \ \mathrm{GeV}^2$$

[Tuo, Feng, Jin, PRD 2020, 1909.13525]

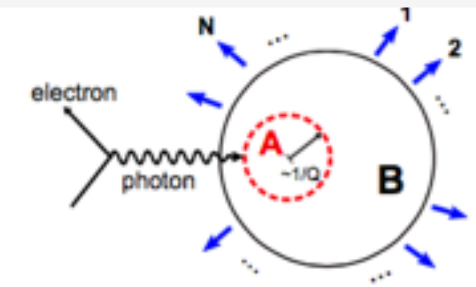
$$g_\nu^{\pi\pi}(\mu)\Big|_{\mu=m_\rho} = -10.89(28)(33)_L(66)_a$$
$$\mathcal{S}_{\pi\pi} = 1.1045(34)_{\mathrm{stat}}(74)_{\mathrm{sys}}.$$

- [Davoudi et al, PRD 2024, 2402.09362] First $nn \to pp\, e^-e^-$ calculation (with heavy unphysical quarks

Entanglement in Hadrons

- EPR paradox at sub-nucleonic scales: [Tu, Kharzeev, Ullrich, arXiv:1904.11974]
  - appear independent in DIS
  - form color-singlets in hadronization

→ **Partons are strongly entangled**
parton density matrix in DIS:



$$\hat{\rho}_A = \mathrm{Tr}_B \, |0\rangle\langle 0|_{hadron}$$

- Entanglement entropy must be gluon-dominated at small-x [Kharzeev, Levin, PRD95:114008(2017)]

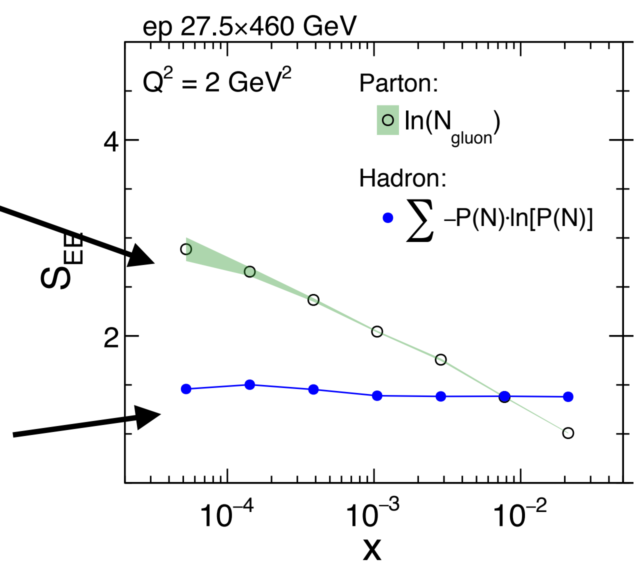
$$S_{\text{hadron}} = -\sum P(N) \log P(N)$$

Test: $\overset{?}{=} \; S_A = -\log[xG(x)]$

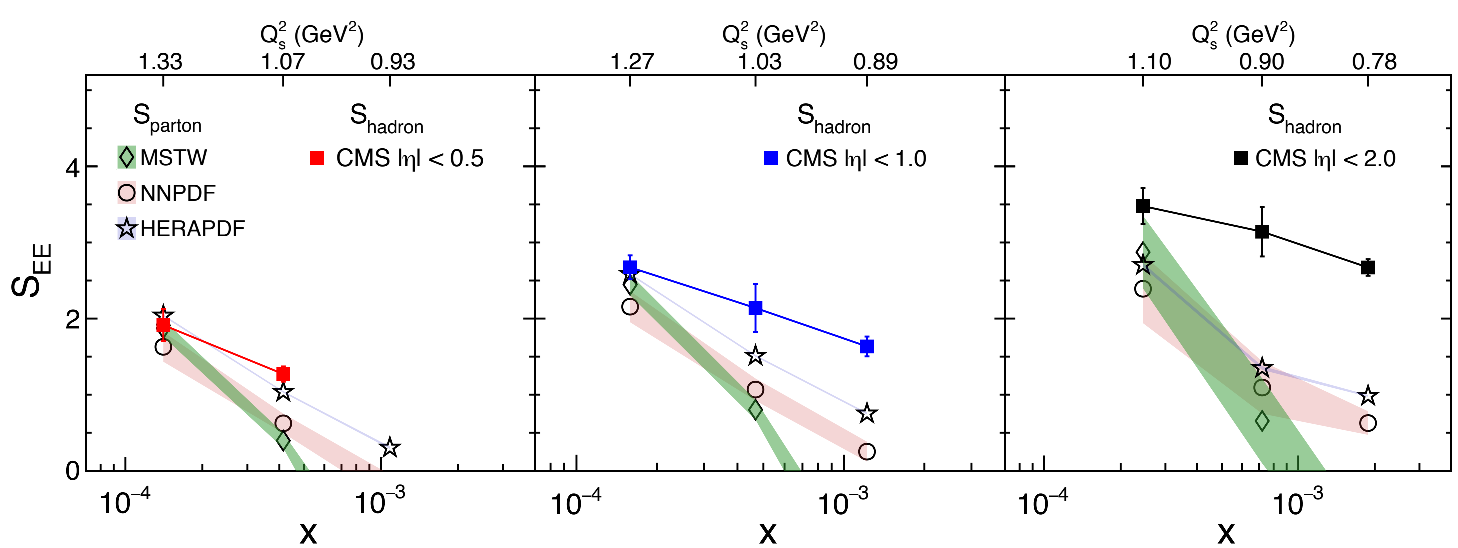
gluon PDF entropy

vs.

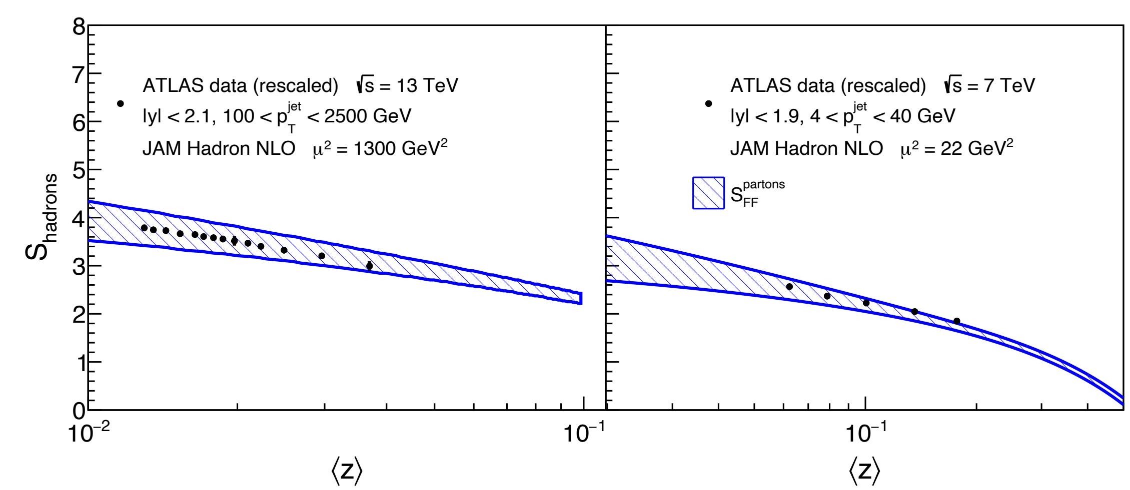
Multiplicity from PYTHIA6(no entanglement)



CMS data [Tu, Kharzeev, Ullrich, PRL124:062001(2020)]



ATLAS data [Datta et al, PRL:134 111902(2025)]

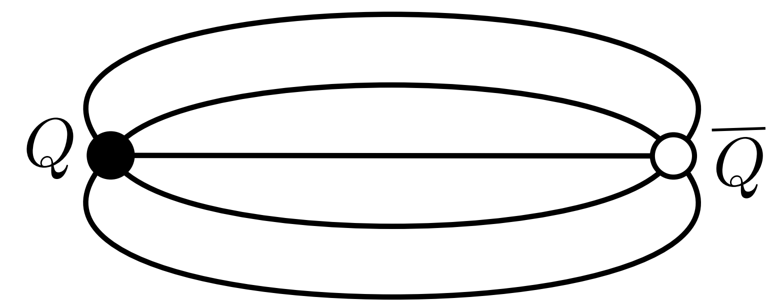


Next at EIC: Compare entropy of multiplicity vs entropy of G(x) at small–x

Color Flux Tube between Quark and AntiQuark

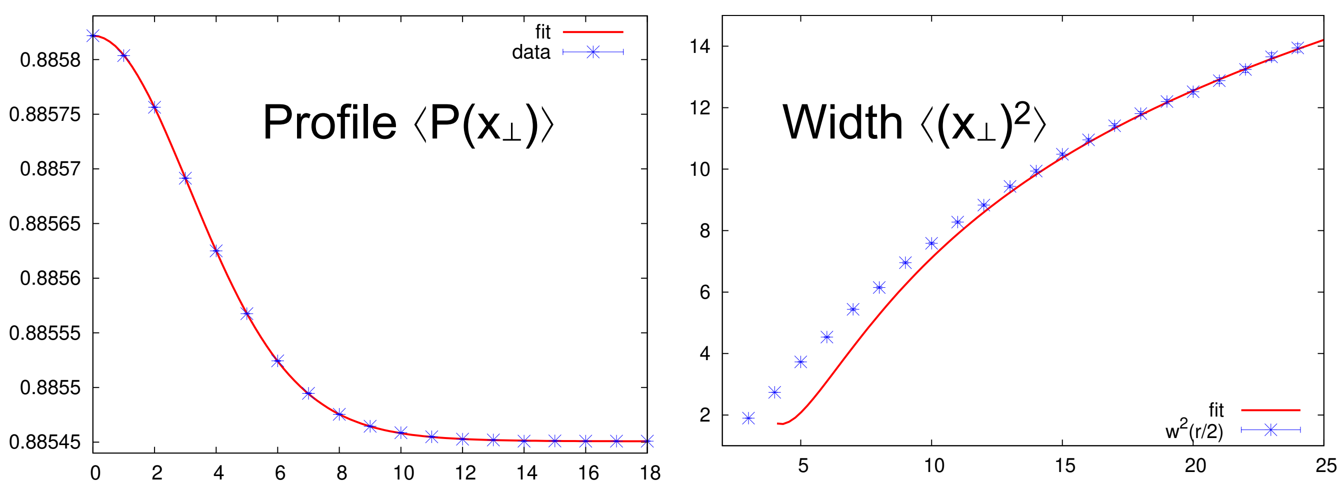
The most prominent IR field configuration in QCD / Yang Mills

- Can be described by a thin vibrating string; e.g. Gaussian profile and width~log(L) due to vibrations [Luscher, Munster, Weisz '81]



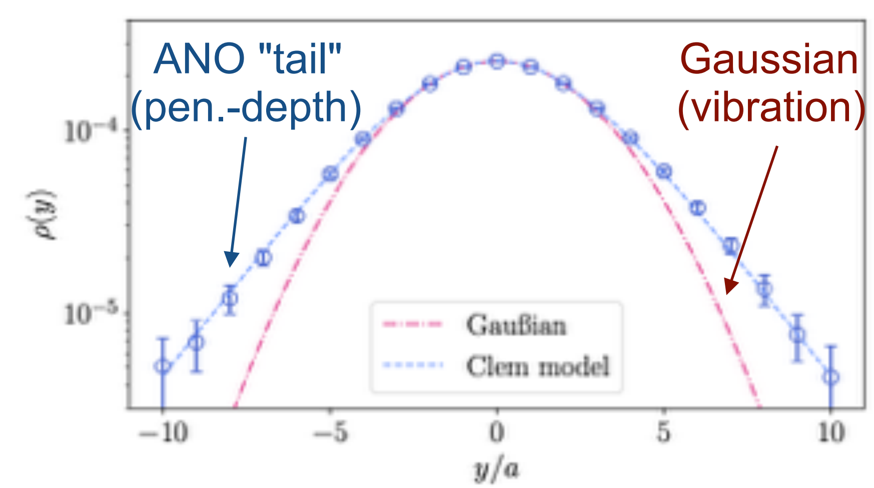
$$P(\vec{x}_\perp) \propto e^{-x_\perp^2/\delta^2}, \quad \delta^2 \propto \frac{1}{\pi\sigma} \log\frac{L}{\lambda}$$

- Action and energy profile: extensively studied on a lattice



(2+1)D SU(2) [Gliozzi, Pepe, Wiese 1002.4888]

Resembles vortex in dual superconductor:



(2+1)D SU(2) [Verzichelli et al, 2501.01740]

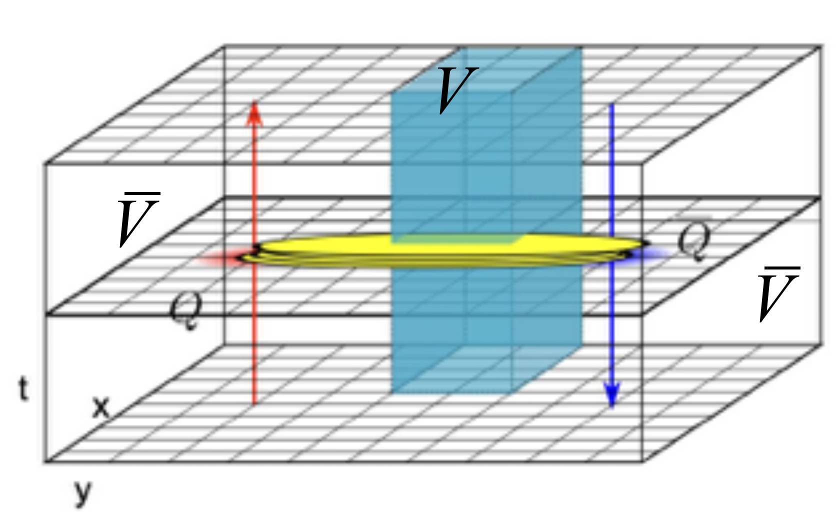
- [Amorosso et al, JHEP(2024), 2411.12818; 2601.17199]
  Flux tube = pure (ground) state in presence of $Q\bar{Q}$;
  Split into $V+\overline{V}$ parts, study quantum entanglement to reveal
  - *String wave function?*
  - *Internal DoFs?*
  - *Intrinsic thickness?*

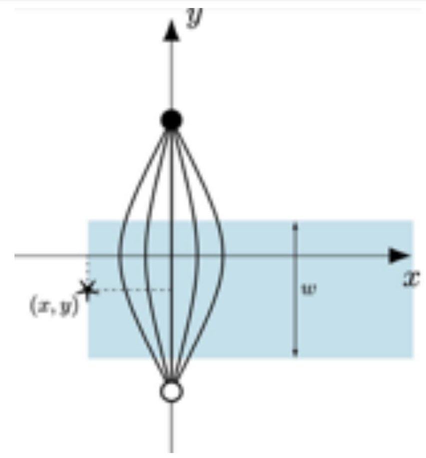
$$S^{vN} = -\mathrm{Tr}\left[\rho_V \log \rho_V\right]$$

where "reduced density matrix"

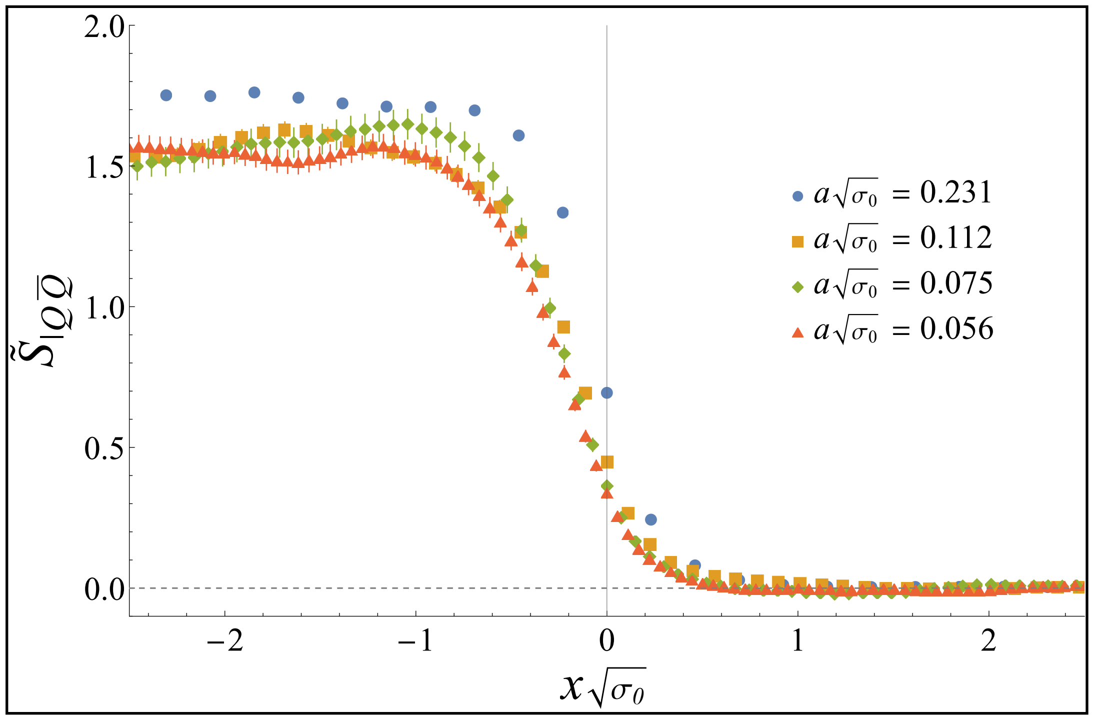
$$\rho_V = \mathrm{Tr}_{\overline{V}}\left[|\Psi_0\rangle\langle\Psi_0|\right]$$

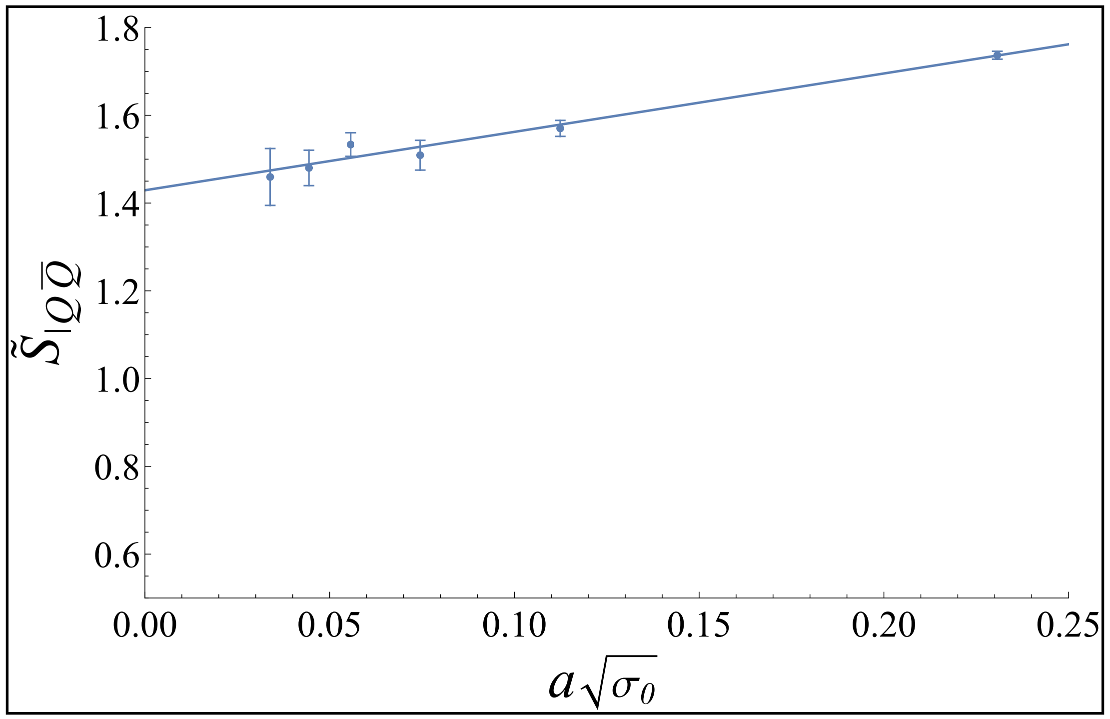
Scaling of FTEE: Finite in Continuum Limit





- x→(+∞) : no cut, all FT in $\bar{V}$
- x=0 : FT half-cut by V
- x→(−∞) : FT cross-cut by V

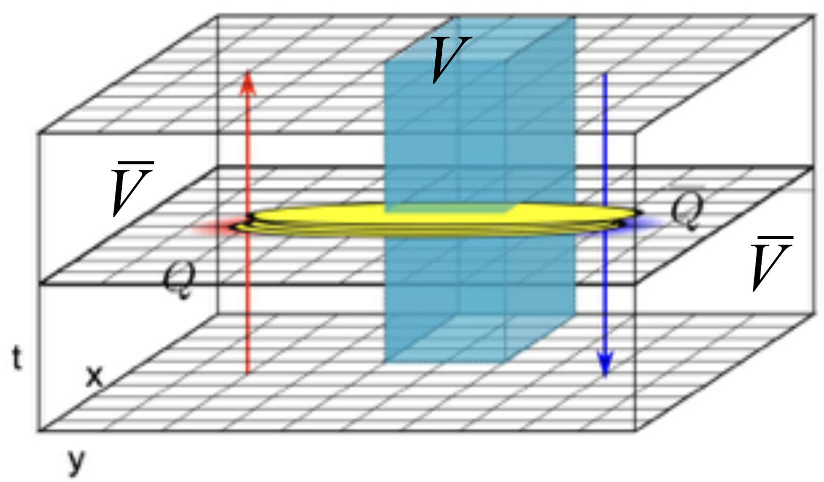


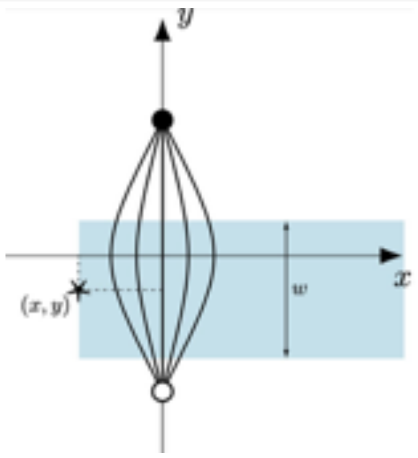


- "Renormalized" $\tilde{S}_{Q\bar{Q}} = (S_{\bar{Q}Q} - S_{vac})_{a\to 0}$: finite in continuum limit
- Flux tube EE = thin <u>vibrating</u> string with <u>color</u>

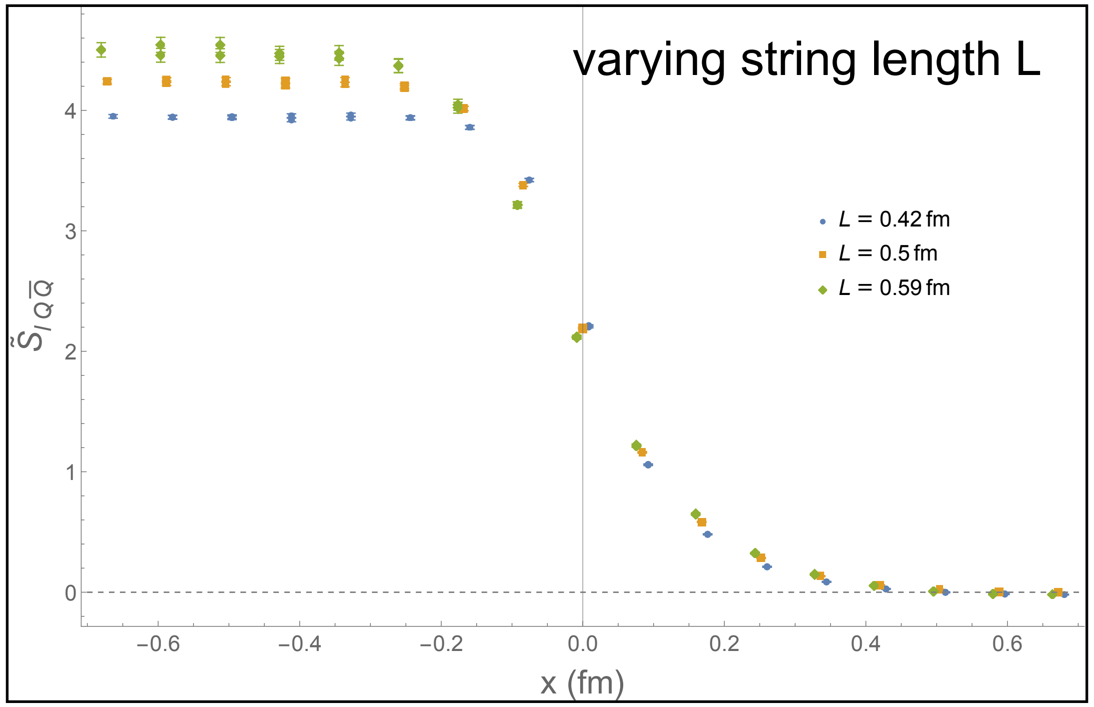
$$\underbrace{S_E^{\max}}_{\text{full cross-cut}} = \underbrace{2\ \log N_c}_{\text{color}} + \underbrace{\frac{1}{4}\ \log\frac{L}{\lambda}}_{\text{vibration}}$$

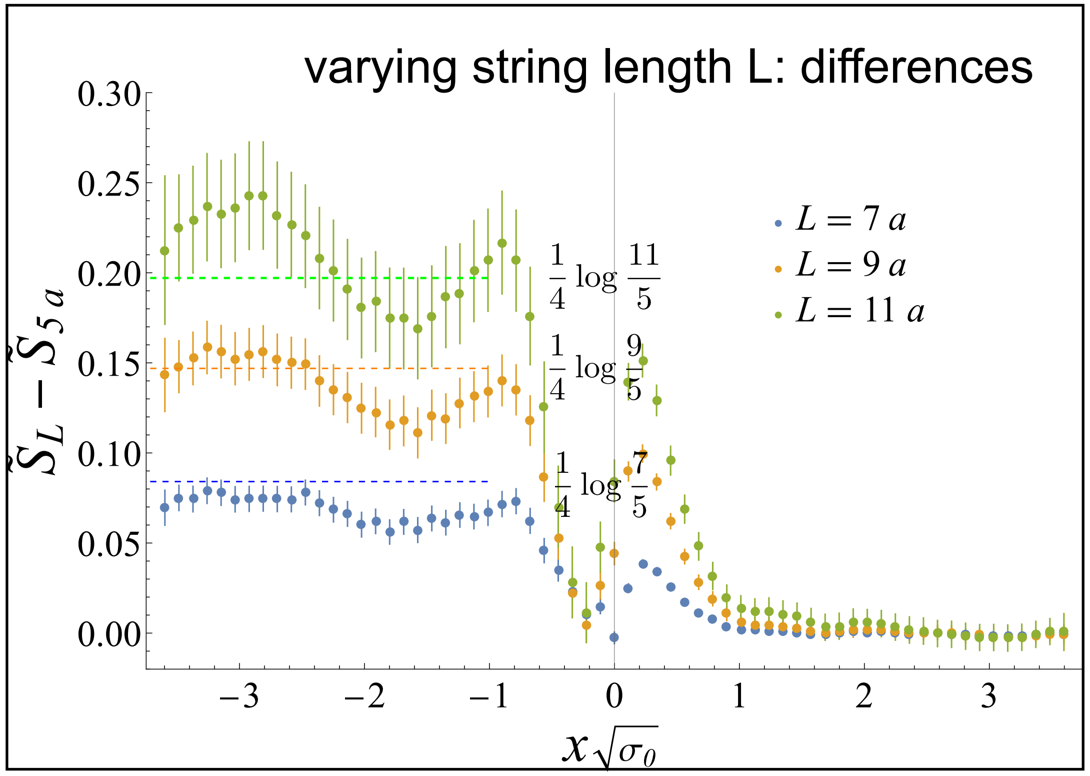
Length dependence





- x→(+∞) : no cut, all FT in V̄
- x=0 : FT half-cut by V
- x→(−∞) : FT cross-cut by V





- "Renormalized" $\tilde{S}_{Q\bar{Q}} = (S_{\bar{Q}Q} - S_{vac})_{a\to 0}$: finite in continuum limit
- Flux tube EE = thin vibrating string with color $\underbrace{S_E^{max}}_{\text{full cross-cut}} = \underbrace{2\log N_c}_{\text{color}} + \underbrace{\frac{1}{4}\log\frac{L}{\lambda}}_{\text{vibration}}$

Summary

- *Lattice QCD methods are mature enough for reliable nucleon structure calculations,* ***and***

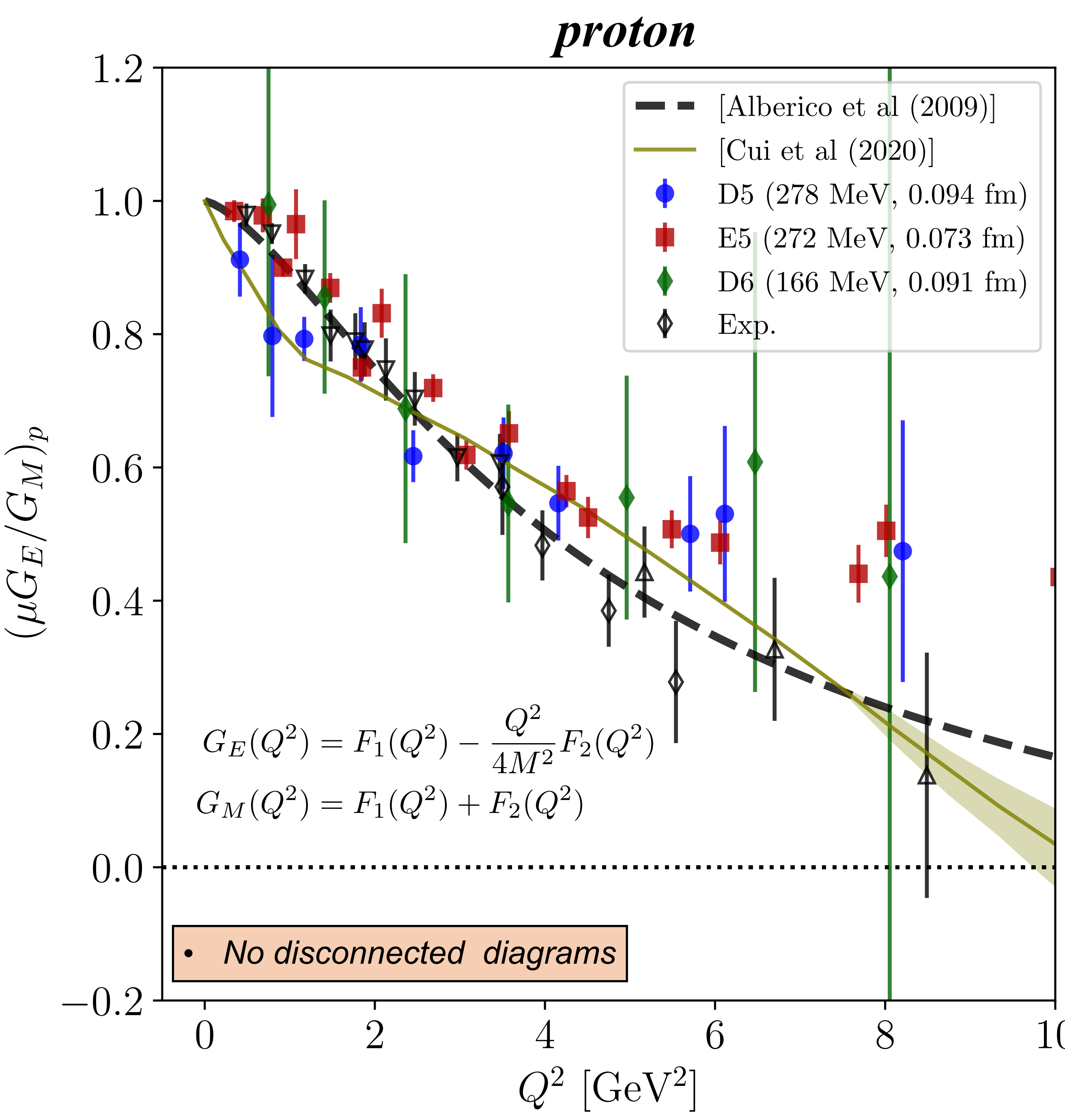
- *... offer multiple avenues to explore nucleon structure to complement experiments, EIC in particular*

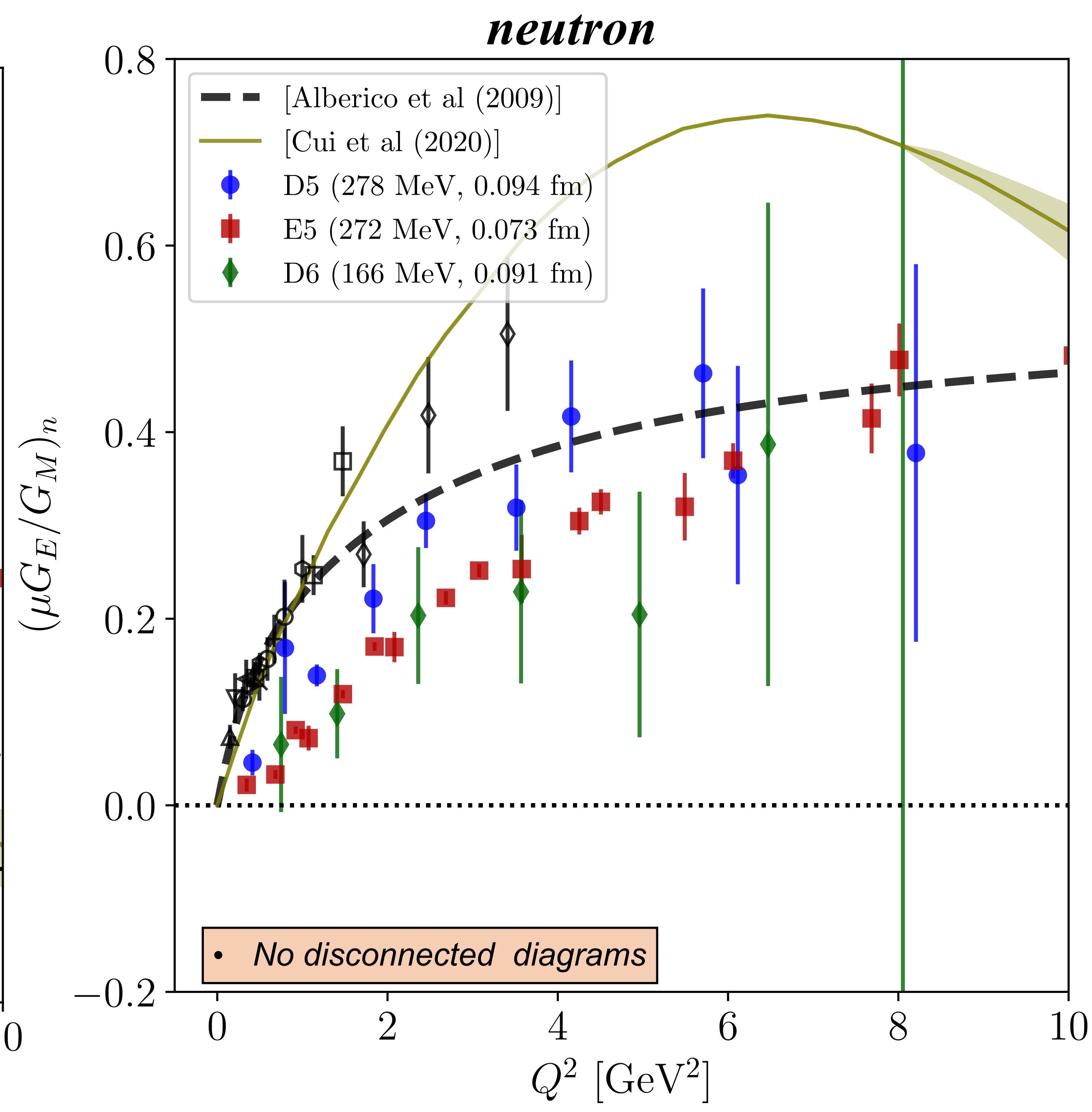
- *... are crucial for experiments that use nucleons as a lab for new physics searches*
  lab for understanding confinement

BACKUP

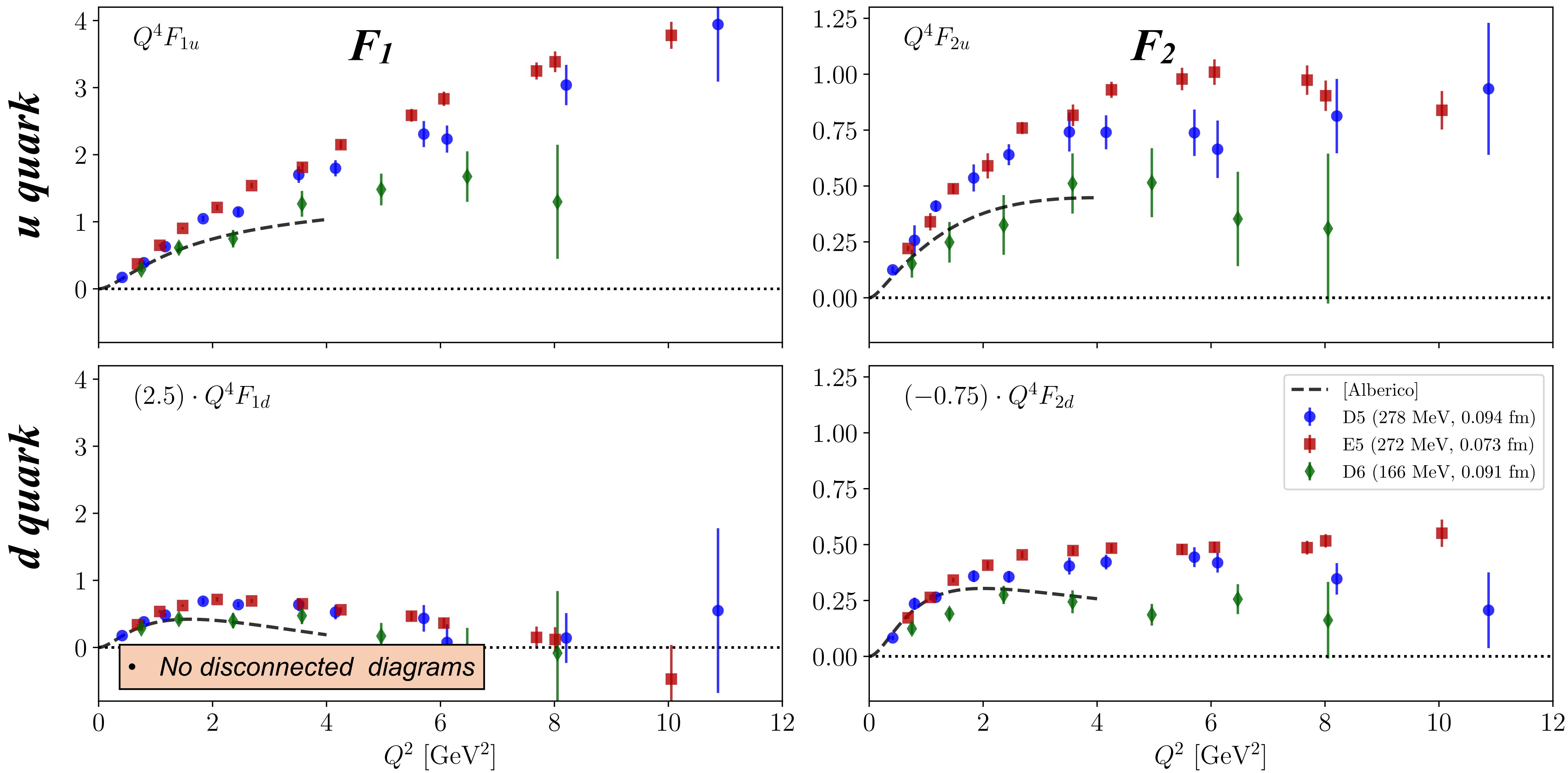
Nucleon Vector Form Factors : High Momentum(2)

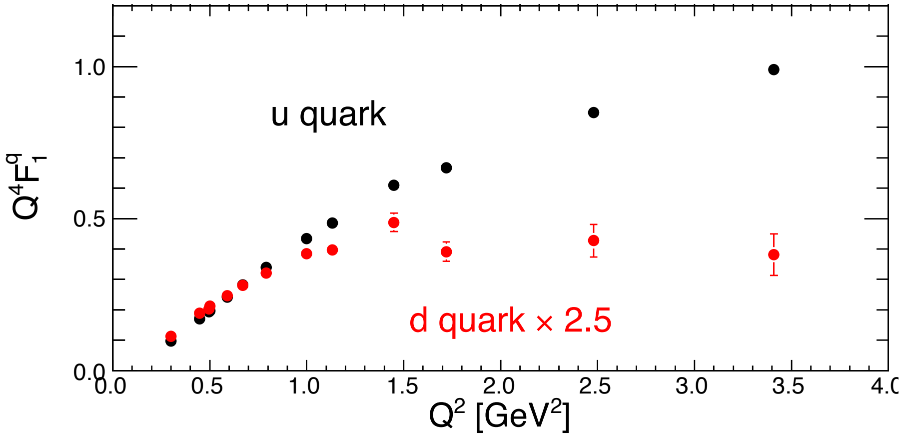
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- *(black points)* Experimental data for proton ($Q^2 \lesssim 8.5$ GeV^2) and neutron ($Q^2 \lesssim 3.4$ GeV^2)

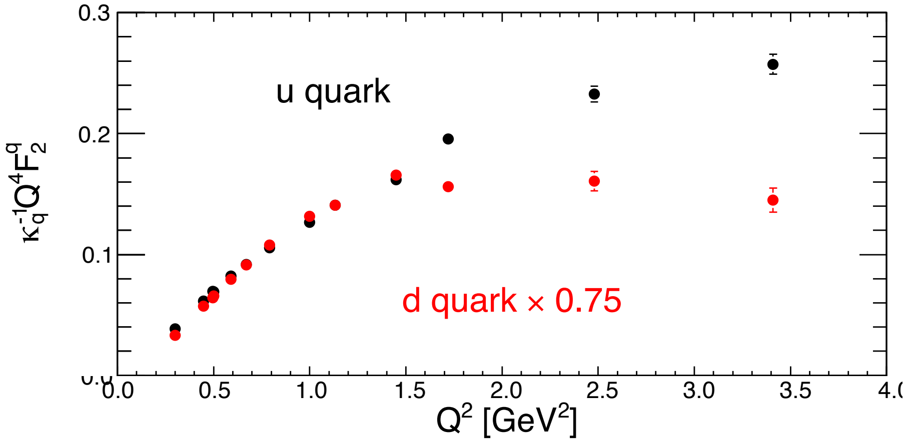




Nucleon Vector Form Factors : High Momentum(3)





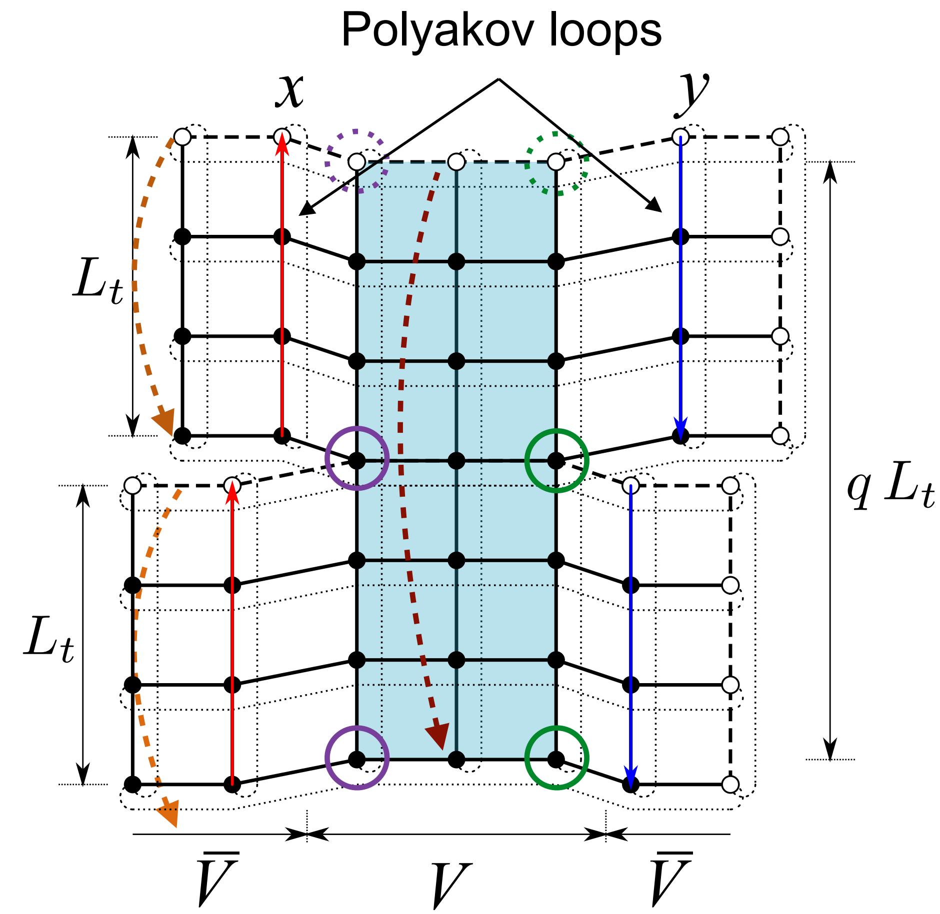


- Reproduce qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

Flux-Tube (Renyi) EE on a Lattice

Approximate entropy by Renyi q-entropy

$$S^{vN} = -\mathrm{Tr}\,[\rho_V\ \log\rho_V] \approx -\frac{1}{q-1}\log\mathrm{Tr}\,[\rho_V^q] = S^{(q)}$$



- vacuum density-matrix $\rho_{\mathrm{vac}}(U_1, U_0)$

$$\left\langle U_1\right|\left(e^{-\hat{H}}\right)^{L_t}\left|U_0\right\rangle \propto \int_{U(0)=U_0}^{U(L_t)=U_1}\mathcal{D}U_{1\ldots(L_t-1)}\,e^{-S[U]}$$

- Static QQ̄ density-matrix $\rho_{Q\overline{Q}}(U_1, U_0)$

$$\left\langle U_1, Q_x^i\overline{Q}_y^j\right|\left(e^{-\hat{H}}\right)^{L_t}\left|U_0, Q_x^k\overline{Q}_y^l\right\rangle \propto \int_{U(0)=U_0}^{U(L_t)=U_1}\mathcal{D}U_{1\ldots(L_t-1)}\,e^{-S[U]}P_x^{ik}\,P_y^{\dagger\,lj}$$

- Partial trace over region Ā:

$$\mathrm{Tr}_{\overline{V}}\,\rho(U_1,U_0) = \frac{1}{Z}\int\mathcal{D}U_{\overline{V}}\,\rho(U_{1,0}|_{\overline{V}} = U_{\overline{V}}))$$

- Final trace: qL_t-periodic BC (vacuum or QQ̄)

$$\mathrm{Tr}_V\left[\left(\rho_V\right)^q\right] = \frac{Z^{(q)}}{(Z)^q}$$

- Renyi-2 entropies of vacuum , QQ̄ pair :

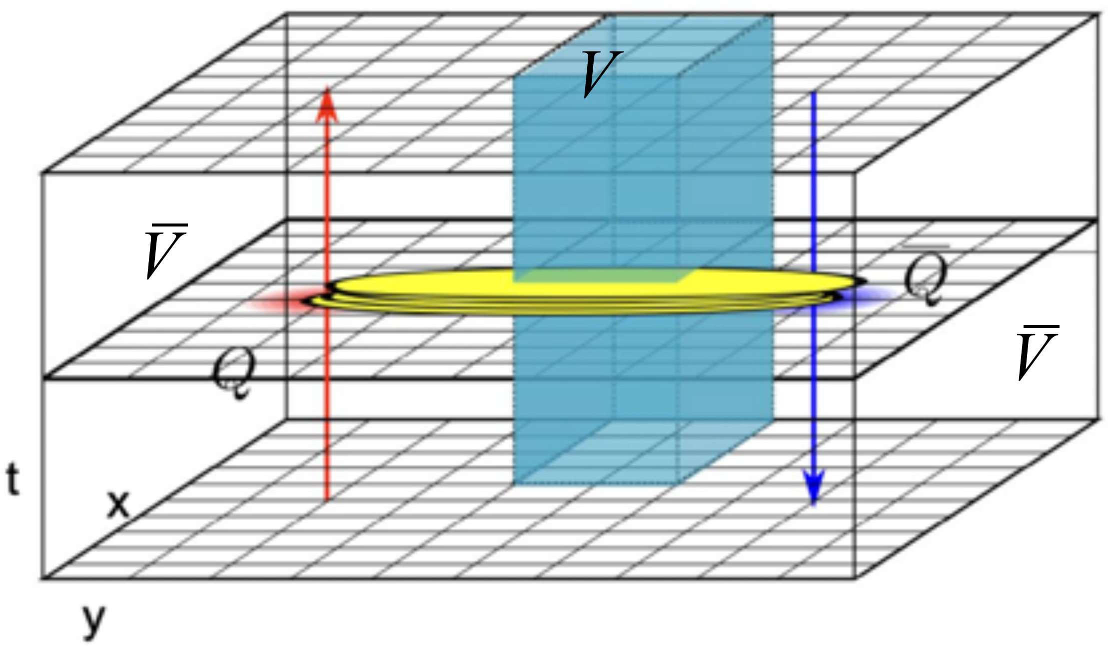
$$S^{(2)}_{\mathrm{vac}} = -\log\frac{Z^{(2)}_{\mathrm{vac}}}{(Z_{\mathrm{vac}})^2}\,,\qquad S^{(2)}_{Q\overline{Q}} = -\log\frac{Z^{(2)}_{Q\overline{Q}}}{(Z_{Q\overline{Q}})^2}$$

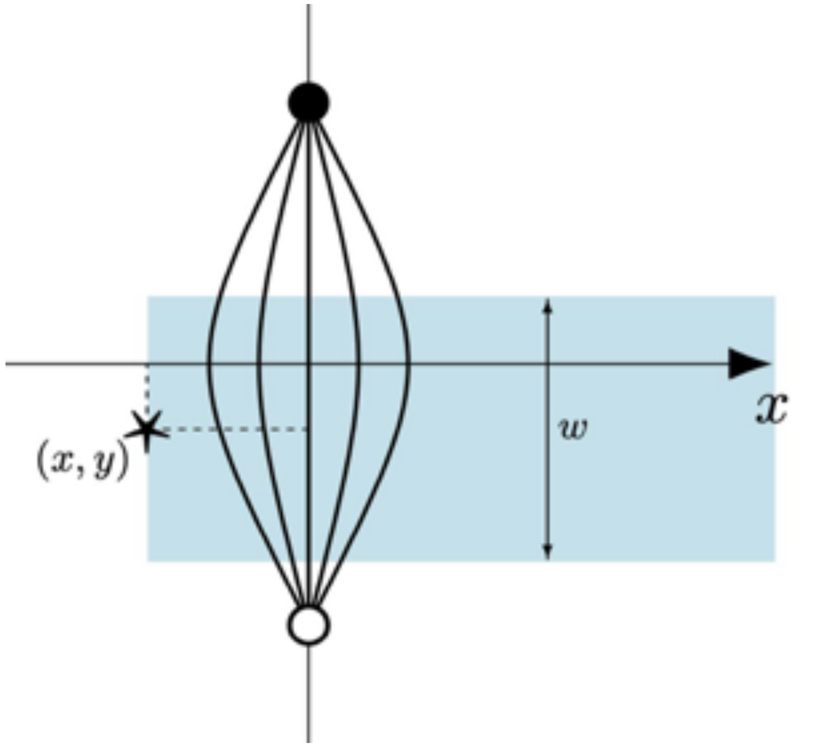
Both UV-divergent!

- Set $L_t \gtrsim 2\,(1\,/\,T_c)$ for "pure ground" state
- Subtlety: gauge transforms must be L_t -periodic at "cusps" [Aoki et al 2016] gauge fixing or discretization issue? [some insight from (1+1)D below]
- Note: 1-replica PL correlator

$$\langle P_x P_y^\dagger\rangle = \frac{Z_{Q\overline{Q}}}{Z_{\mathrm{vac}}} \propto e^{-L_t V_{Q\overline{Q}}(|x-y|)}$$

Partition Space to Cross-cut the Flux Tube





Study entanglement entropy while changing

- L = flux tube length
- w = width of V (blue "slab")
- x = extent of (partial) cross-cut
  - x→(+∞) : no cut, all FT in $\overline{V}$
  - x=0 : FT half-cut by V
  - x→(–∞) : FT cross-cut by V
- y = longitudinal position of the slab
  - y = 0 : symmetric
  - y ≠ 0 : closer to Q (or $\overline{Q}$)