Quasi- and Pseudo-Parton Distribution Functions in the Spectator Diquark Model

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PDFs and Computation on the Light Cone

PDFs are defined by **light-like** separation in Minkowski spacetime: $\xi_M^2 = \xi_0^2 - \xi_1^2 - \xi_2^2 - \xi_3^2 = 0$

$$f_q^{[\Gamma]}(x) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \left\langle P, S \left| \bar{\Psi}_q(\xi^-)\, \Gamma\, W[\xi^-,0]\, \Psi_q(0) \right| P, S \right\rangle_{\xi^+=\xi^\perp=0}$$

(non-forward) **bilocal** matrix elements
require $\xi \neq 0$

PDFs are **light-cone (LC)** distributions

Lattice-QCD works in Euclidean spacetime: $\xi_E^2 = -\xi_0^2 - \xi_1^2 - \xi_2^2 - \xi_3^2 \leq 0$

$\longrightarrow$ we **cannot realize nonzero light-like separation** on the lattice,

LC PDFs are not accessible directly

Proxy lattice distributions: we **can** calculate the same matrix element with **space-like** separation $z^2 \leq 0$

PDFs and Computation on the Light Cone

Proxy lattice distributions: we can calculate the same matrix element with space like separation $z^2 \leq 0$

Quasi-PDF (qPDF): finite momentum [X. Ji, Phys. Rev. Lett. 110, 262002 (2013), doi:10.1103/PhysRevLett.110.262002]

$$f_q^{[\Gamma]}(y, P^z) \sim \int dz\; e^{-iyP^z z} \langle P \,|\, \bar{\Psi}_q(z) \Gamma W[z,0] \Psi_q(0) \,|\, P \rangle$$

$$f^{[\Gamma]}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|}\, C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) f^{[\Gamma]}(x, \mu) + \mathcal{O}\left(\frac{M^2}{y^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-y)^2 P_z^2}\right)$$

departure from LC mixed between kinematic and boost variables: $y \in (-\infty, \infty)$

Pseudo-PDF (pPDF): finite separation (Ioffe time $\nu = P \cdot z$) [A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017), doi:10.1103/PhysRevD.96.034025]

$$f_q^{[\Gamma]}(x, z^2) \sim \int d\nu\; e^{-ix\nu} \langle P \,|\, \bar{\Psi}_q(z) \Gamma W[z,0] \Psi_q(0) \,|\, P \rangle$$

$$f^{[\Gamma]}(x, z^2, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, z^2\mu^2\right) f^{[\Gamma]}(y, \mu) + \mathcal{O}\left(z^2 \Lambda_{QCD}^2\right)$$

departure from LC encoded in boost variable alone: $x \in [-1,1]$

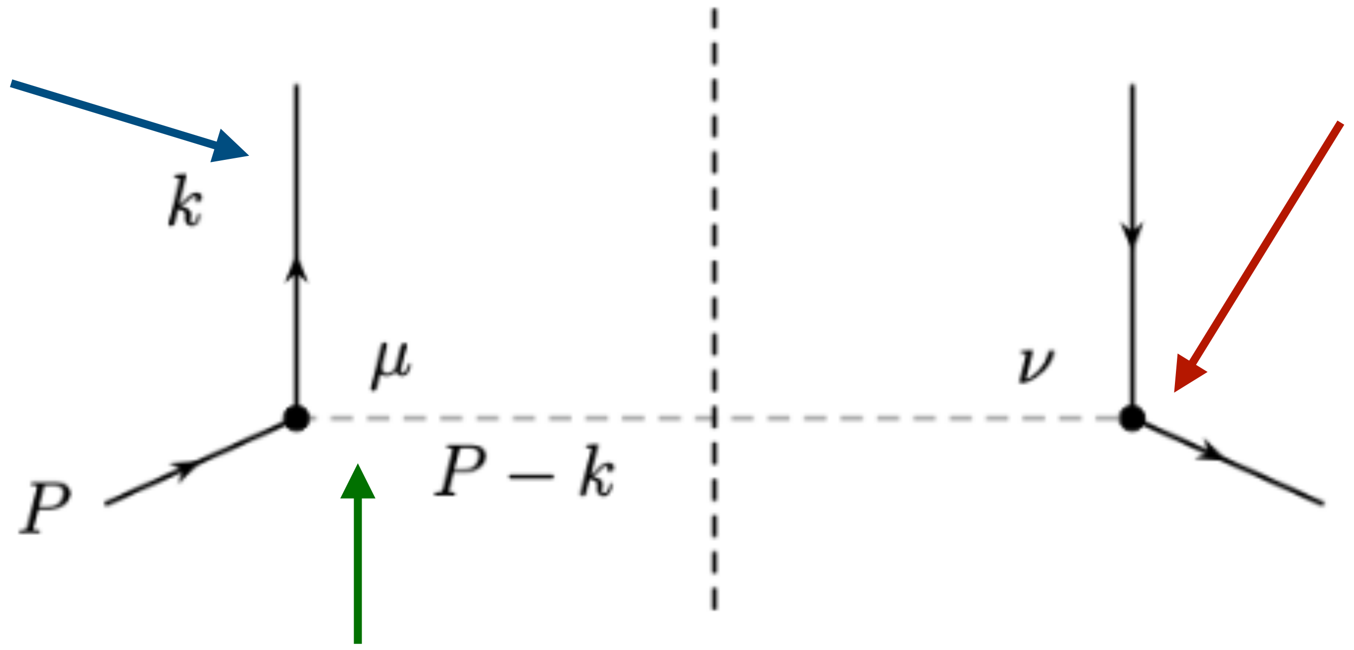
Spectator Model Calculations of PDFs

Unintegrated (projected) quark correlator: $\Phi^{[\Gamma]}(x;P,S) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \left\langle P,S \left| \bar{\Psi}_q(\xi^-)\,\Gamma\, W[\xi^-,0]\Psi_q(0) \right| P,S \right\rangle_{\xi^+=\xi^\perp=0}$

Calculating the PDFs from the quark correlator:

$A(k,P,S) = \frac{iN_A(k)}{k^2 - m^2 + i\epsilon}$

Dirac numerator N_A



$\mathscr{V}(k,P,S) = igI(k^2)V(k,P,S)$

spin structure V

vertex form factor I

$$\Longrightarrow \quad \Phi(k,P,S) \sim \frac{N^{\mu\nu}(k,P,S)\, d_{\mu\nu}(P-k,n)\,|I(k^2)|^2}{(k^2 - m^2 + i\epsilon)^2((P-k)^2 - M_D^2 + i\epsilon)}$$

$D(P-k) = \frac{iN_D(p)}{(P-k)^2 - M_D^2 + i\epsilon}$ polarization structure N_D

Distributions are recovered by performing the respective integrations over k^μ:

PDF $\quad f^{[\Gamma]}(x) = \int d^{d-2}k_\perp \int dk^- \Phi^{[\Gamma]}(k,P,S)\Big|_{k^+=xP^+}$

qPDF $\quad f^{[\Gamma]}(y,P^z) = \int d^{d-2}k_\perp \int dk^0 \Phi^{[\Gamma]}(k,P,S)\Big|_{k^z=yP^z}$

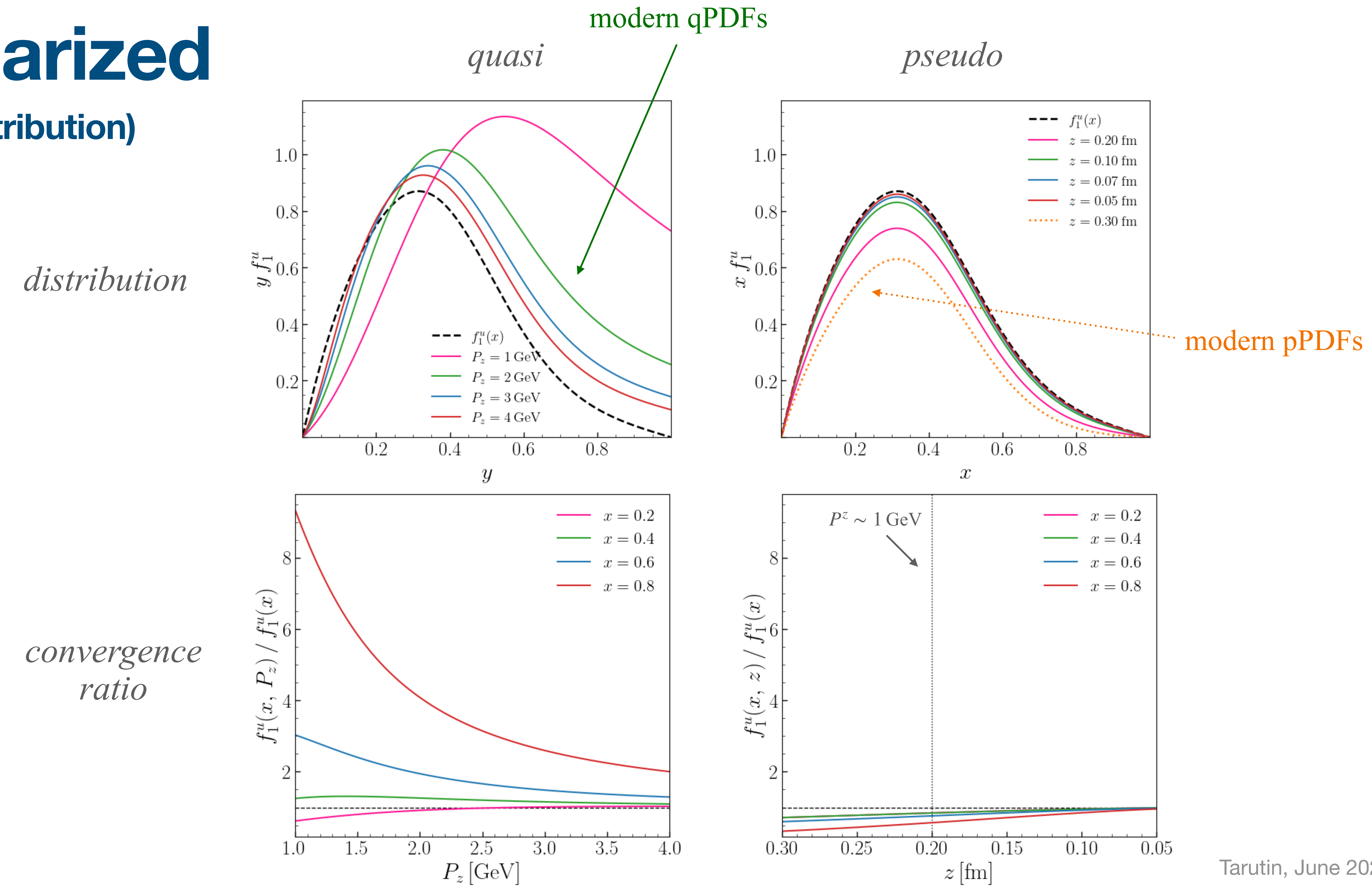
pPDF $\quad f^{[\Gamma]}(x,z^2) = \int d^{d-2}k_\perp\, e^{ik_\perp\cdot z_\perp} \int dk^- \Phi^{[\Gamma]}(k,P,S)\Big|_{k^+=xP^+}$

[A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008), doi:10.1103/PhysRevD.78.074010]

[L. Gamberg, Z.-B. Kang, I. Vitev, and H. Xing, Phys. Lett. B 743, 112 (2015), doi:10.1016/j.physletb.2015.02.021]

Unpolarized

(u-quark distribution)



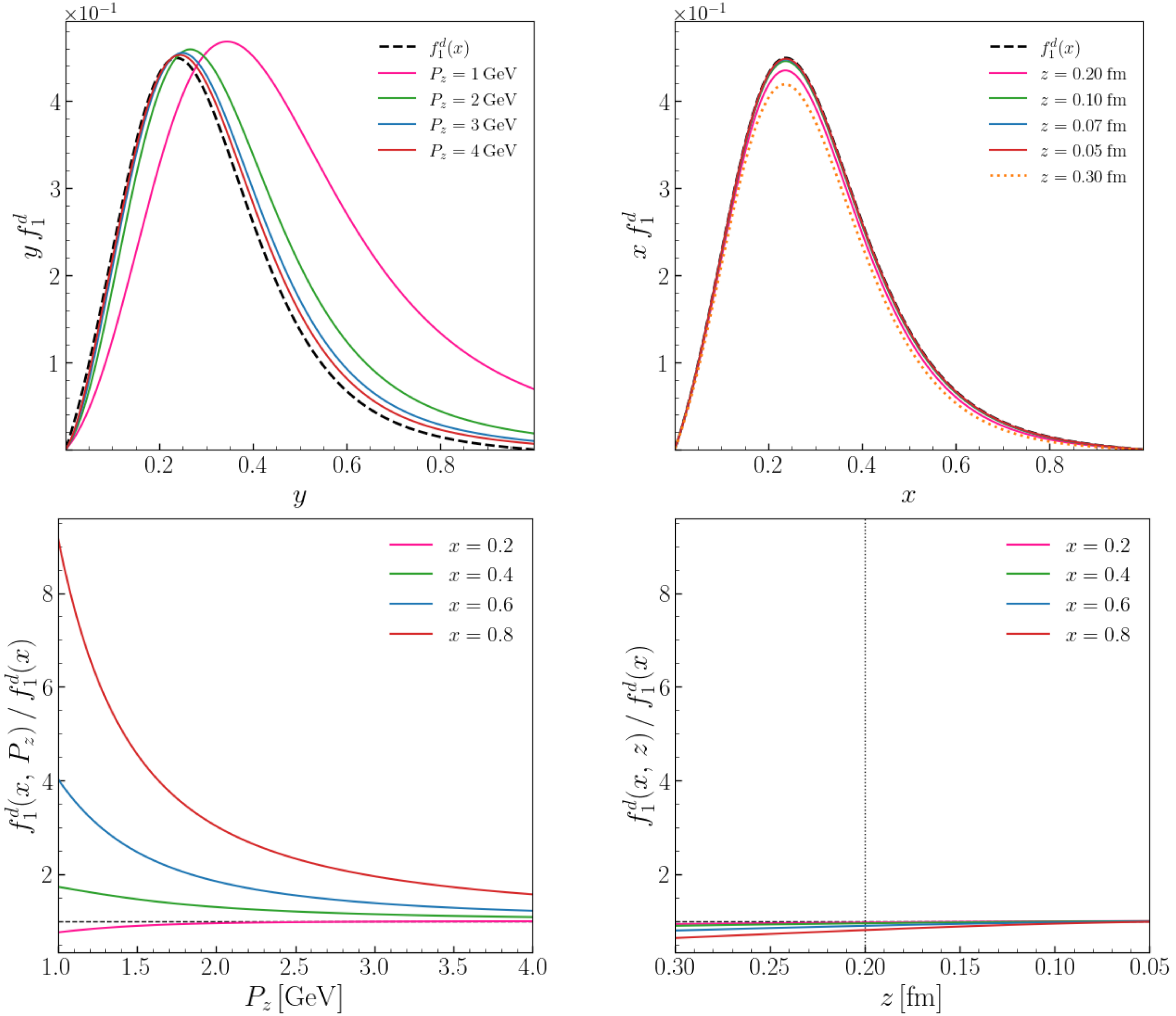
Unpolarized

(d-quark distribution)

quasi *pseudo*

distribution

convergence ratio



Tarutin, June 2026

Helicity

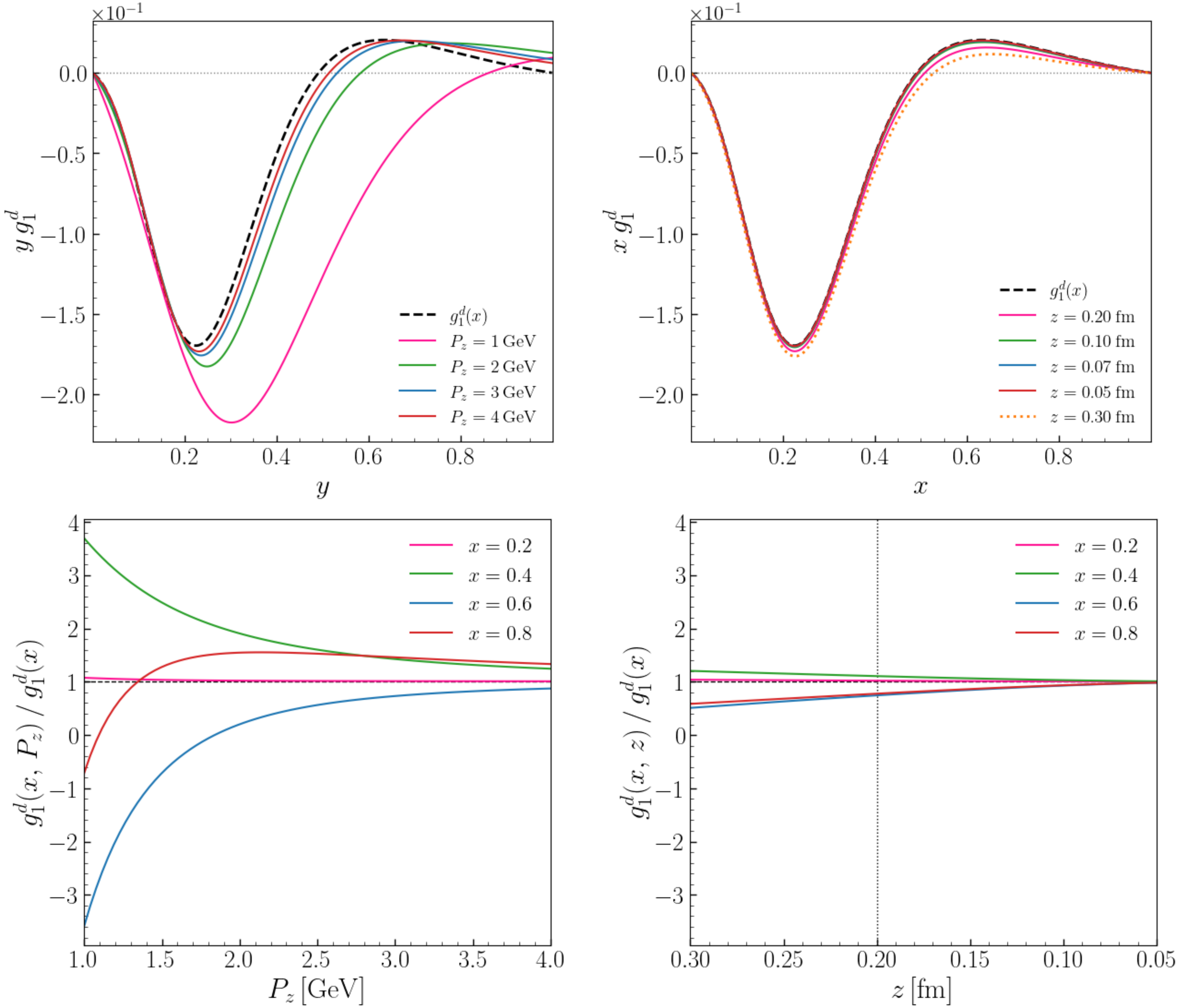
(d-quark distribution)

quasi

pseudo

distribution

convergence ratio



Tarutin, June 2026

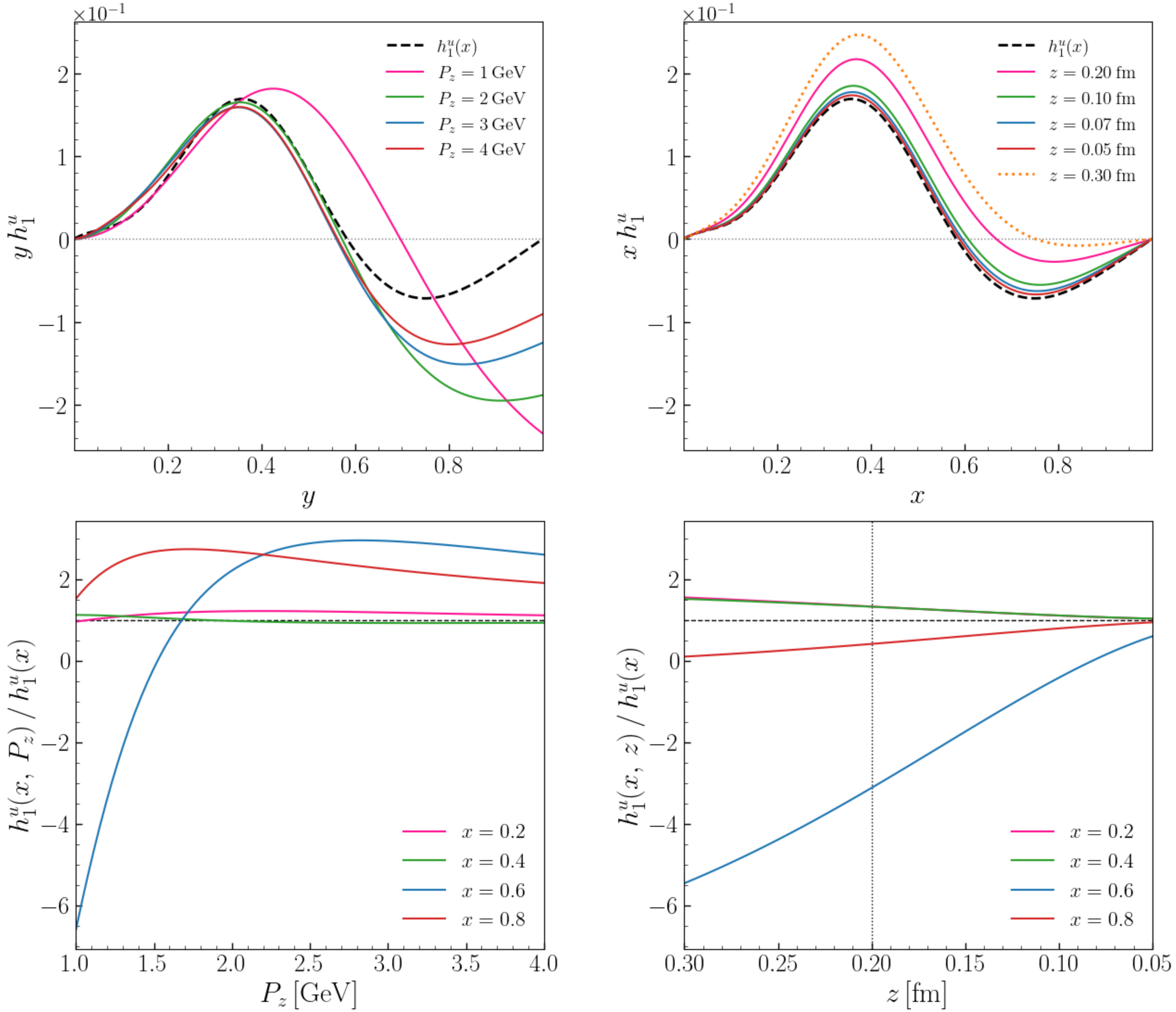
Transversity

(u-quark distribution)

quasi | *pseudo*

distribution

convergence ratio



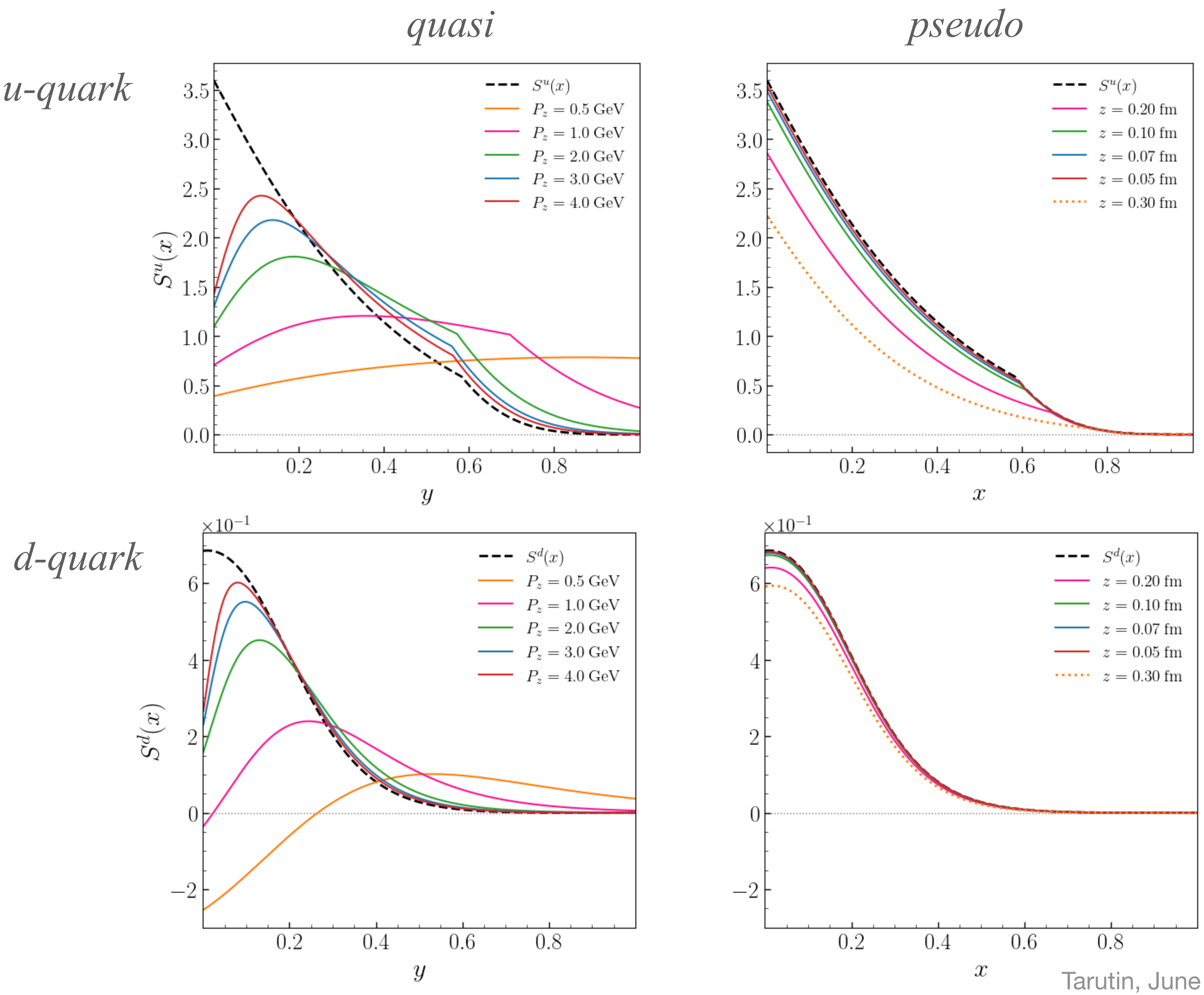
Soffer Bound

The Soffer bound is defined

$$S^q(x) = \frac{1}{2}(f_1^q(x) + g_1^q(x)) - |h_1^q(x)|$$

For LCs PDFs we expect $S^q(x) \geq 0$

[J. Soffer, Phys. Rev. Lett. 74, 1292 (1995), doi:10.1103/PhysRevLett.74.1292]



Conclusion & Outlook

We see two trends:

- The is no universal pointwise advantage between the two constructions; any disparity in the quasi/pseudo-PDF fidelity to the LC PDF further vanishes when comparing realistic P^z and z values ($P^z \sim$ 2 GeV, $z \sim$ 0.3 fm)

- LC convergence is strongly y-dependent for the qPDFs and more uniform for pPDFs; this behavior is attributed to the respective constructions and not the models

Outlook:

- New distributions: higher twist PDFs, GPDs

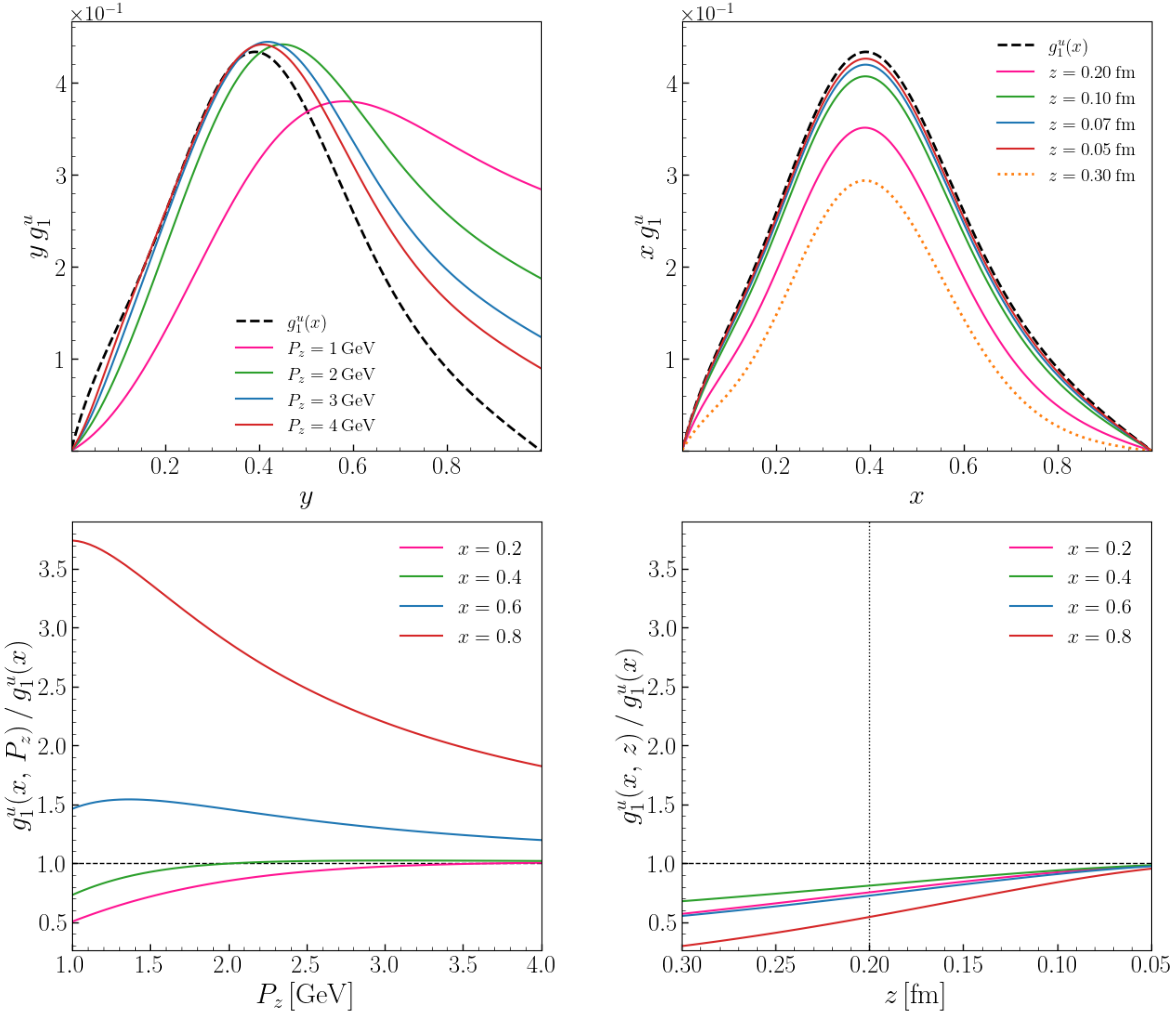
- New Models: different vertex form factors, different models (particularly gauge theories) to investigate other divergence structures

Helicity
(u-quark distribution)

quasi *pseudo*

distribution

convergence ratio

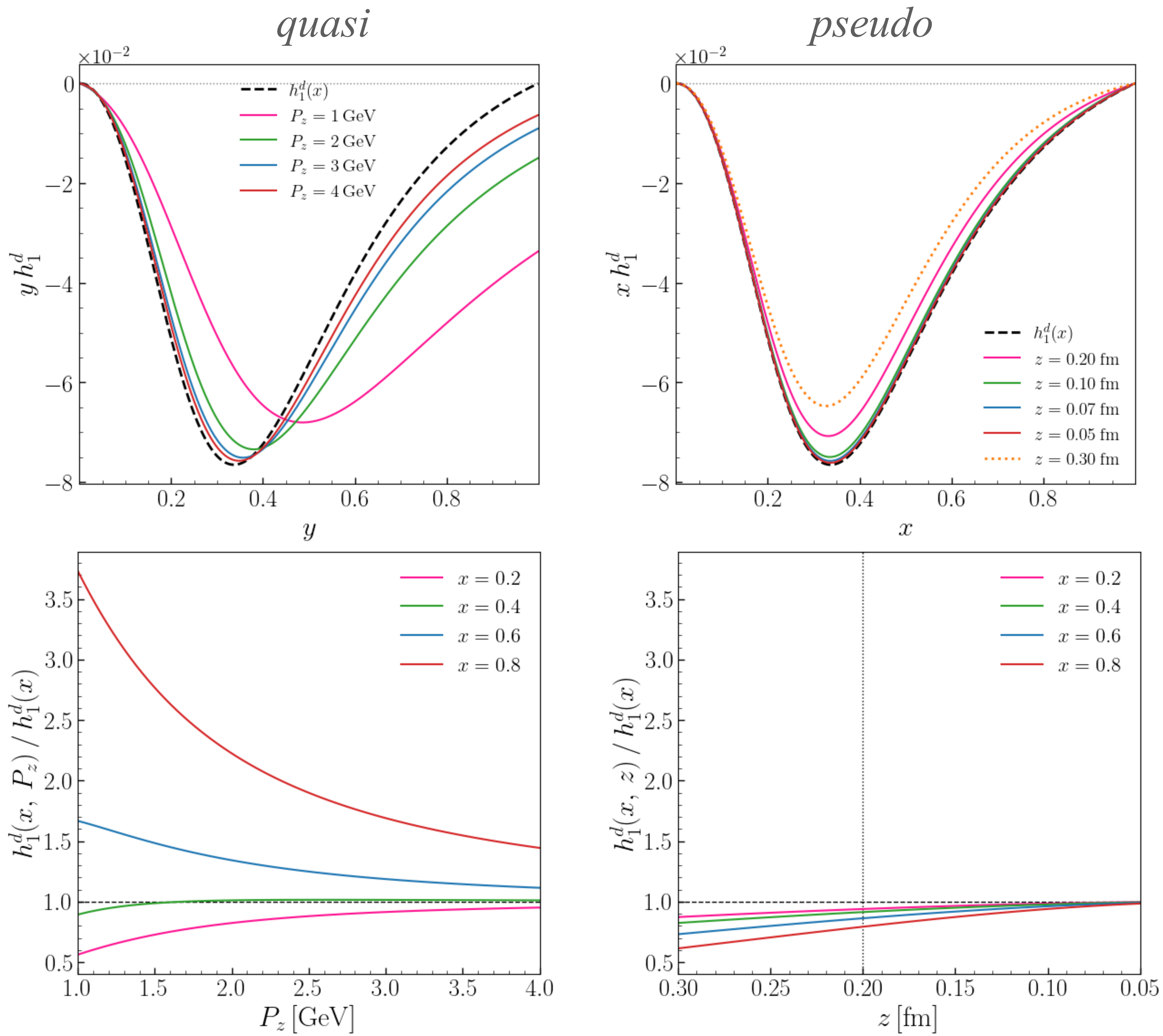


Transversity

(d-quark distribution)

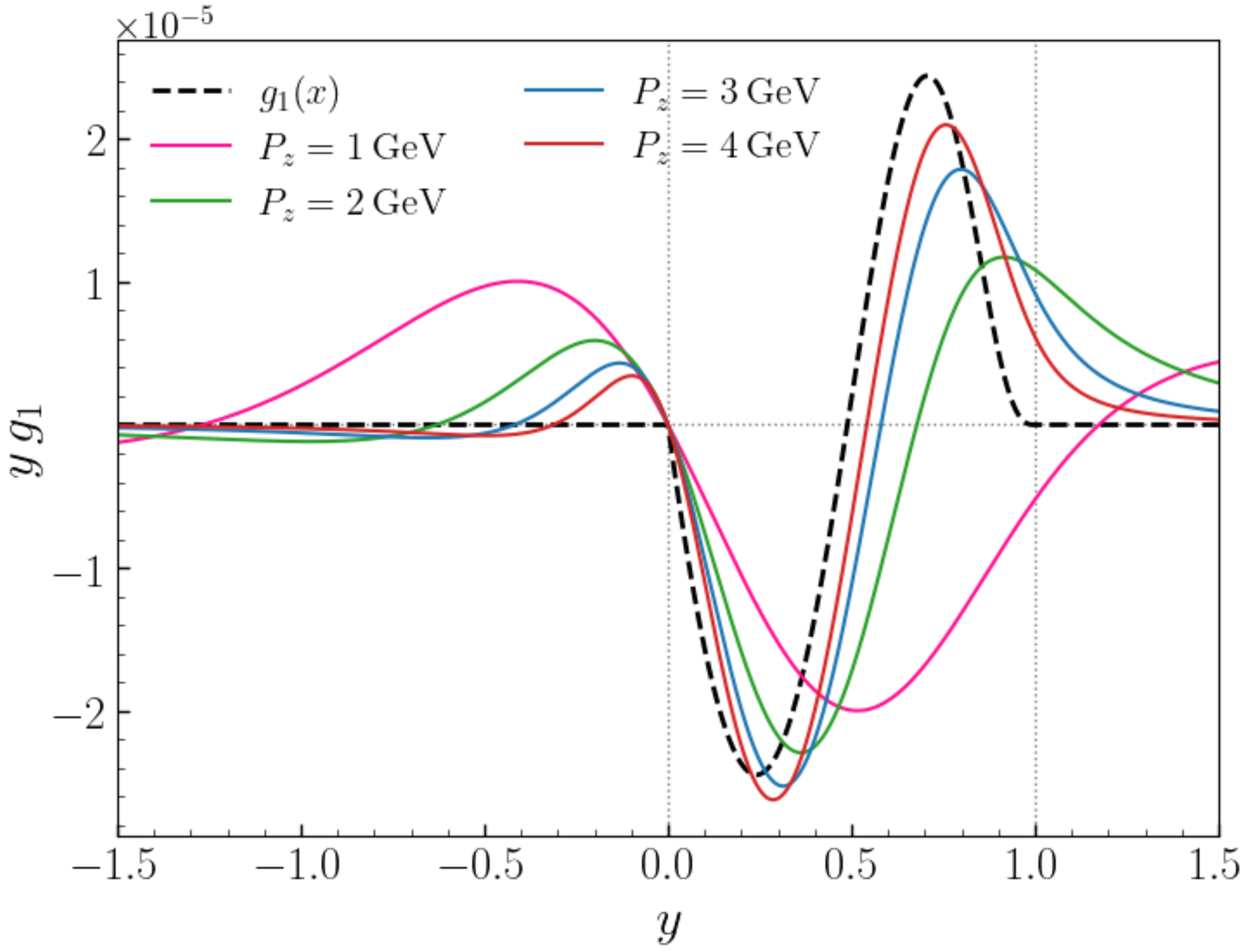
distribution

convergence ratio



Helicity - qPDF Smearing

(scalar-diquark)



Parameter Values

Unless otherwise stated we use the following parameter values:

$$m = 0.33 \text{ GeV}$$

$$M_d = 0.8 \text{ GeV}$$

$$M = 0.938 \text{ GeV}$$

$$\Lambda_X = 1.5 \text{ GeV}$$

$$g = \lambda = \mu = 1$$

Quasi-PDFs

Quasi-PDFs (qPDFs) are finite-momentum analogues of the LC PDFs:

[X. Ji, Phys. Rev. Lett. 110, 262002 (2013), doi:10.1103/PhysRevLett.110.262002]

$$f_q^{[\Gamma]}(y, P^z) \sim \int dz\, e^{-iyP^z z} \langle P | \bar{\Psi}_q(z) \Gamma W[z,0] \Psi_q(0) | P \rangle \quad \text{for finite-momentum fraction } y = \frac{k^z}{P^z}$$

Related to the LC PDFs via a perturbative matching procedure:

finite boost corrections

$$f^{[\Gamma]}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) f^{[\Gamma]}(x, \mu) + \mathcal{O}\left(\frac{M^2}{y^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-y)^2 P_z^2}\right)$$

matching coeffs.
(perturbatively calculable)

The qPDF converges to the PDF in the $P^z \to \infty$ limit (after appropriate matching)

Support properties: $y \in (-\infty, \infty)$ vs LC fraction $x \in [0,1]$ $\longrightarrow$ distribution smearing

Pseudo-PDFs

Pseudo-PDFs (pPDFs) are finite-separation analogues of the LC PDFs:

[A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017), doi:10.1103/PhysRevD.96.034025]

$$f_q^{[\Gamma]}(x, z^2) \sim \int d\nu \; e^{-ix\nu} \langle P \,|\, \bar{\Psi}_q(z) \Gamma W[z,0] \Psi_q(0) \,|\, P \rangle \quad \text{at finite spatial separation } \; z^2$$

Ioffe time: $\nu = P \cdot z$

Related to the LC PDFs via a similar perturbative matching procedure:

$$f^{[\Gamma]}(x, z^2, \mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, z^2\mu^2\right) f^{[\Gamma]}(y, \mu) + \mathcal{O}\left(z^2 \Lambda_{QCD}^2\right)$$

The pPDF converges to the PDF in the $z^2 \to 0$ limit

Support properties: retains the same dependence on LC fraction $x \in [0,1]$

Scalar-Diquark Model

The model:

- Hadron-like model: parent and active states are now spinful, spectator is still scalar
- Spin contributions give nontrivial numerator structure, so all three leading twist PDFs are accessible

Explicit correlator:

$$\Phi_s(k; P, S) = \frac{i g_s^2}{2(2\pi)^4} \frac{(\gamma^\mu k_\mu + m)(\gamma^\nu P_\nu + M)(1 + \gamma_5 \gamma^\rho S_\rho)(\gamma^\delta k_\delta + m)}{(k^2 - m^2 + i\varepsilon)^2 [(P-k)^2 - M_s^2 + i\varepsilon]} |I_s(k^2)|^2$$

[A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008), doi:10.1103/PhysRevD.78.074010]

The UV structure is now regulated by a dipole vertex form factor: $I_s(k^2) = \dfrac{k^2 - m^2}{(k^2 - \Lambda_s^2)^2}$

$\longrightarrow$ Large virtualities are suppressed, so the transverse integrals are finite (trivial matching)

Axial Vector-Diquark Model

The model:

- The spectator now also acquires spin, giving added numerator polarization structure

Explicit correlator: [A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008), doi:10.1103/PhysRevD.78.074010]

$$\Phi^a(k;P,S) = \frac{g_a^2}{2(2\pi)^4}\frac{(\gamma^\rho k_\rho + m)\gamma^\mu\gamma^5(\gamma^\sigma P_\sigma + M)\gamma^\nu\gamma^5(\gamma^\delta k_\delta + m)d_{\mu\nu}(P-k,n)}{(k^2-m^2+i\varepsilon)^2[(P-k)^2 - M_a^2 + i\varepsilon]}\,|I_a(k^2)|^2$$

Polarization tensor: $p = P - k$

light-cone: $$d^{\mu\nu}(p,n) = -g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n} - \frac{p^2 n^\mu n^\nu}{(p\cdot n)^2}, \qquad n^2 = 0$$

spatial: $$d^{\mu\nu} = -g^{\mu\nu} + \frac{n_z\cdot p}{(n_z\cdot p)^2 - n_z^2 p^2}\left(n_z^\mu p^\nu + n_z^\nu p^\mu\right) - \frac{p^2 n_z^\mu n_z^\nu + n_z^2 p^\mu p^\nu}{(n_z\cdot p)^2 - n_z^2 p^2}, \qquad n_z^2 = -1$$

[L. Gamberg, Z.-B. Kang, I. Vitev, and H. Xing, Phys. Lett. B 743, 112 (2015), doi:10.1016/j.physletb.2015.02.021]

Modeling Proton Flavor Content

We can model the proton flavor content (*uud*-composition) by combining the scalar- and vector-diquark channels (spin-isospin decomposition):

$$f^u(x) = c_s^2 f^s + c_a^2 f^a(x), \qquad f^d(x) = c_{a'}^2 f^{a'}(x)$$

The vertex couplings g are fixed by normalizing the distributions and then fitting remaining parameters to phenomenological PDFs:

$$I_X = \pi \int_0^1 dx \int_0^\infty dk_\perp^2 f^{q(X)}(x, k_\perp^2)\Big|_{g_X=1}$$

$$\implies f_{norm}^{q(x)}(x) = \frac{f^{q(x)}(x)}{I_X}$$

Phenomenological inputs for parameters:

[A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008), doi:10.1103/PhysRevD.78.074010]

Channel X	M_X [GeV]	Λ_X [GeV]	c_X
s	0.822	0.609	0.847
a	1.492	0.716	1.061
a'	0.890	0.376	0.880

Pseudo-PDFs and the Ioffe Time Dist.

Pseudo-PDFs are built from the Ioffe time distribution (ITD),

[A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017), doi:10.1103/PhysRevD.96.034025]

$$\mathcal{M}_\alpha(\nu, z^2) = \frac{1}{2P^\alpha}\langle P\,|\,\bar{\Psi}(z)\gamma_\alpha W[0,z]\Psi(z)\,|\,P\rangle, \qquad \nu = P \cdot z$$

where

$$f_1(x, z^2) = \frac{1}{2\pi}\int d\nu e^{-ix\nu}\mathcal{M}(\nu, z^2).$$

The ITD is a function of the invariants ν and z^2 and is therefore frame independent

→ we can built the pPDFs from the correlator in either frame,
the key is to evaluate it at finite separation